Written Exam for the B.Sc. in Economics, Winter 2011/2012
Reexamination
Makro A and Macro A
Second year
February 26, 2012

(3-hours closed book exam)

All questions, 1.1-1.3 and 2.1-2.8, to be answered, and all weighted equally.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e., if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Problem 1: Capital flows, interest rates and national income in opening economies

(In this problem, the focus is on the verbal, intuitive explanations. Formal analysis can, however, be used in the explanations if wanted).

1.1 Explain what determines the capital intensity (the capital-labour ratio) and the real interest rate in a closed economy in the long run according to the basic Solow model.

1.2 Consider two countries which are first both closed economies working in accordance with the basic Solow model (with a zero depreciation rate). Country 1 is more savings strong than Country 2 in the sense that the savings rate relative to the growth rate of the labour force is larger in Country 1. In other respects the two countries are identical. Having adjusted completely to their steady states as closed, the two economies 'open their capital accounts' and come to work as open economies with completely free capital movements between them. What can be expected to happen to capital movements between the two economies, and to capital intensities, real wages and interest rates in the two countries after the opening?

1.3 Is it possible that in the long run both countries obtain higher long run national income per worker than if they had stayed closed? Explain the mechanism behind your answer.

Problem 2: A model of endogenous growth with human capital

(In this problem, formal and computational elements are more important, but verbal, intuitive explanations are still central).

Equations (1) - (8) below make up a model of endogenous growth with human capital.

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\begin{aligned}
Y_t &= \left(K_t^d\right)^{\alpha}\left(H_t^d\right)^{\varphi}\left(A_t L_t^d\right)^{1-\alpha-\varphi}, \quad 0<\alpha<1, \quad 0<\varphi<1, \quad \alpha+\varphi<1 && (1)\\
A_t &= \left(\frac{K_t}{L_t}\right)^{\frac{\alpha}{\alpha+\varphi}}\left(\frac{H_t}{L_t}\right)^{\frac{\varphi}{\alpha+\varphi}} && (2)\\
K_t^d &= K_t && (3)\\
H_t^d &= H_t && (4)\\
L_t^d &= L_t && (5)\\
K_{t+1} &= s_K Y_t + (1-\delta) K_t, \quad 0<s_K<1, \quad 0<\delta<1 && (6)\\
H_{t+1} &= s_H Y_t + (1-\delta) H_t, \quad 0<s_H<1, \quad s_K+s_H<1 && (7)\\
L_{t+1} &= (1+n) L_t, \quad n>-1 && (8)
\end{aligned}
$$

Equation (1) is the representative, individual firm's production function: Aggregate output, Y_t, is produced from the inputs of physical capital, K_t^d, human capital, H_t^d, and labour, L_t^d. Through a productive externality the factor productivity of the individual firm, A_t, depends on the aggregate uses of physical and human capital per worker as described by Equation (2). Equations (3) - (5) are equilibrium conditions stating that the demanded and supplied amounts of physical capital, human capital and labour, respectively, must equate. Equations (6) - (8) describe the accumulation of physical capital, human capital and labour, respectively.

The model's exogenous parameters are α and φ (technical), δ (common depreciation rate for physical and human capital), s_K and s_H (investment rate in physical and human capital, respectively), and n (growth rate of the labour force). The state variables are K_t, H_t and L_t with given, strictly positive initial values K_0, H_0 and L_0, respectively.

Define $k_t \equiv K_t/Y_t$, $h_t \equiv H_t/L_t$ and $y_t \equiv Y_t/L_t$.

2.1 Discuss the productive externality assumed in Equation (1).

2.2 Show that equations (1) - (5) imply the aggregate production function:

$$Y_t = K_t^{\nu} H_t^{1-\nu}, \quad \nu \equiv \frac{\alpha}{\alpha + \varphi}, \tag{9}$$

and the aggregate per worker production function:

$$y_t = k_t^{\nu} h_t^{1-\nu}, \tag{10}$$

and describe and discuss these with respect to the degree of returns to scale and the productivity of labour.

Define $x_t \equiv K_t/H_t = k_t/h_t$, the ratio between physical and human capital (per worker).

2.3 Show that the model implies the following transition equation for x_t:

$$x_{t+1} = \frac{s_K x_t^{\nu} + (1-\delta)\, x_t}{s_H x_t^{\nu} + (1-\delta)} \tag{11}$$

2.4 Show that there is a unique, strictly positive steady state value x^* for x_t:

$$x^* = \frac{s_K}{s_H}, \tag{10}$$

and show that x_t converges to x^* in the long run.

Define the growth rates of physical capital per worker, human capital per worker and output per worker, respectively: $g_t^k \equiv \frac{k_{t+1}-k_t}{k_t}$, $g_t^h \equiv \frac{h_{t+1}-h_t}{h_t}$ and $g_t^y \equiv \frac{y_{t+1}-y_t}{y_t}$.

2.5 Show that in any period:

$$g_t^k = \frac{1}{1+n}\left[s_K\left(\frac{k_t}{h_t}\right)^{\nu-1} + 1 - \delta\right] - 1, \tag{11}$$

and

$$g_t^h = \frac{1}{1+n}\left[s_H\left(\frac{k_t}{h_t}\right)^{\nu} + 1 - \delta\right] - 1. \tag{12}$$

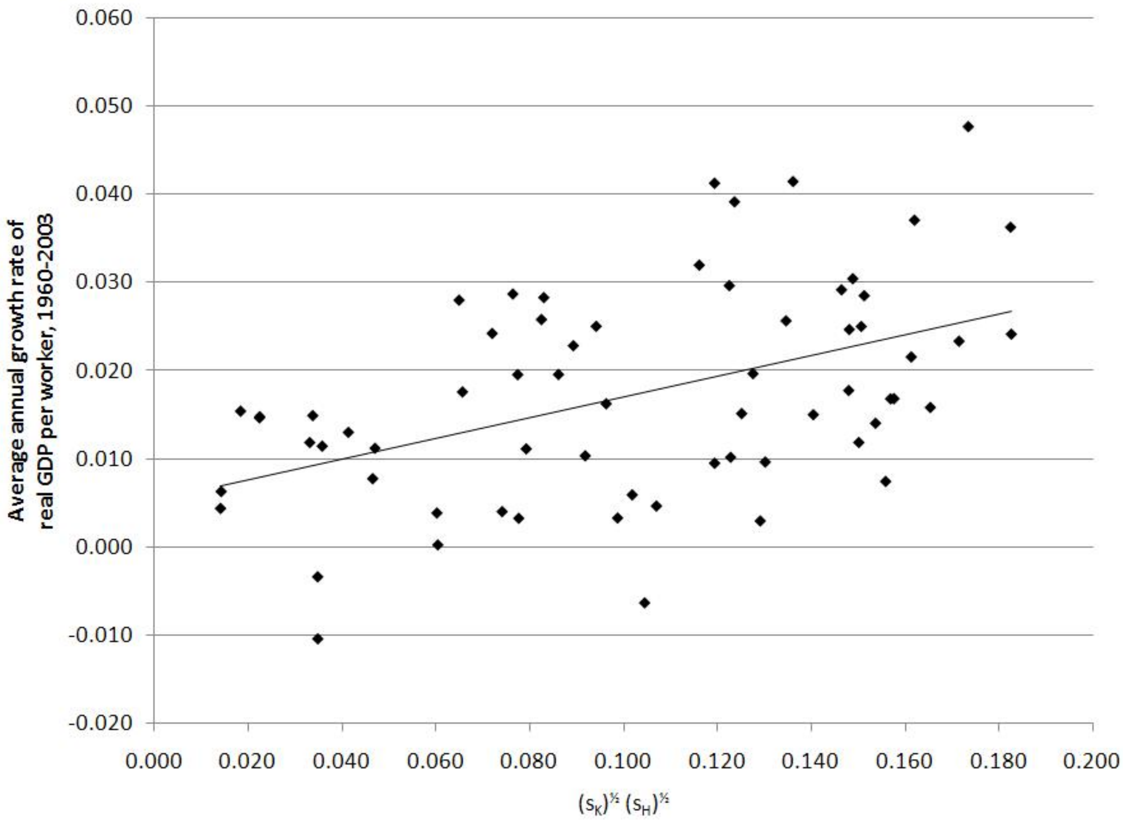
2.6 Show that in the long run all of g_t^k, g_t^h and g_t^y converge to:

$$g_e \equiv \frac{1}{1+n}\left[s_K^{\nu} s_H^{1-\nu} + 1 - \delta\right] - 1 \tag{13}$$

2.7 Comment on your results with respect to the possibility of endogenous growth in this model. What would be a reasonable numerical evaluation of g_e, and, in particular, is $g_e > 0$ reasonable? Also comment on scale effects.

2.8 Comment on Figure 1 below in connection with the results obtained from the present model.

Figure 1. Scatter plot: Average annual growth rates of real GDP per worker against geometric average of investment rates in physical and human capital, 1960-2003, across 65 representative countries.



Note: Along the horizontal axis is $(s^i_K)^{\frac{1}{2}}\,(s^i_H)^{\frac{1}{2}}$, where s^i_K and s^i_H are the average investment rate in physical capital and in human capital of country i over the period considered, respectively. The indicated line is a line of best fit estimated by OLS.

Source: Table A in Sørensen, P.B and H.J. Whitta-Jacobsen, Introducing Advanced Macroeconomics: growth and Business Cycles, second edition, McGrawHill, 2010.