Written Exam for M.Sc. in Economics 2010-I

Advanced Microeconomics

23. February 2010

Master course

3 hours written exam. Closed books. All questions should be clearly and briefly answered. Calculations and figures should be clear and understandable. Calculations and figures should be explained.

Exercise 1

Consider an economy with private ownership where there are two goods, two consumers and one firm. The first consumer is described by $X_1 = \mathbb{R}^2_{++}$, $\omega_1 = (10, 0)$, $u_1(x) = \ln x^1 + \ln x^2$ and $\theta_1 = 0.5$. The second consumer is described by $X_2 = \mathbb{R}^2_{++}$, $\omega_1 = (0, 10)$, $u_2(x) = \ln x^1 + \ln x^2$ and $\theta_2 = 0.5$. The firm is described by

$$Y \;=\; \{y \in \mathbb{R}^2 | y^1, y^2 < 1 \text{ and } y^2 \leq \frac{y^1}{y^1 - 1}\}.$$

Let $p \in \mathbb{R}^2_{++}$ denote the price vector.

1.1 State the problems of the consumers and state the problem of the firm.

1.2 Draw Y, solve the problem of the firm and find the profit.

1.3 Find a Walrasian equilibrium.

Exercise 2

Consider an economy with private ownership

$$\mathcal{E}^P = ((X_i, u_i)_{i=1}^I, (Y_j)_{j=1}^J, (\omega_i, \theta_{i1}, \ldots, \theta_{iJ})_{i=1}^I).$$

Suppose that $X_i = \mathbb{R}_+^L$ and $u_i : X_i \to \mathbb{R}$ is continuous representing a monotone and convex preference relation. Suppose that Y_j is closed, $0 \in Y_j$ and Y_j is strictly convex. Suppose that $\omega_i \in \mathbb{R}_{++}^L$ and $\theta_{ij} \in [0,1]$ for all i and $\sum_i \theta_{ij} = 1$ for all j. Let $p \in \mathbb{R}_{++}^L$ denote the price vector.

2.1 State the utility maximization problem (UMP) of consumer i.

2.2 Show that (UMP) has at least one solution.

2.3 Does the utility function $u(x) = b^1 x^1 + \ldots + b^L x^L$, where $b^1, \ldots, b^L \geq 0$ and $\sum_\ell b^\ell > 0$, represents a preference relation that is monotone and convex?

2.4 Define a Walrasian equilibrium for the economy and illustrate it for $L = 2$, $I = 1$ and $J = 1$.

2.5 Define Pareto optimality and show that if $(\bar{p}, (\bar{x}, \bar{y}))$ is a Walrasian equilibrium, then $(\bar{x}, \bar{y})$ is Pareto optimal.

2.6 Suppose that $J = 1$ and

$$Y = \{y \in \mathbb{R}^L | y^1, \ldots, y^{L-1} \leq 0 \text{ and } y^L \leq -\max\{a^1 y^1, \ldots, a^{L-1} y^{L-1}\}\}$$

where $a^1, \ldots, a^{L-1} > 0$. Show that if $(\bar{p}, (\bar{x}, \bar{y}))$ is a Walrasian equilibrium, then

$$\bar{p}_L \leq \frac{\bar{p}_1}{a^1} + \ldots + \frac{\bar{p}_{L-1}}{a^{L-1}}.$$

Exercise 3

Consider an overlapping generation economy. Time extends from $-\infty$ to ∞, there is one good at every date and there is one consumer, who is alive at two dates, in every generation. Consumers are described by their identical consumption sets $X = \mathbb{R}^2_+$, endowment vectors $\omega_t = (\omega_t^y, \omega_t^o) \in X$ and utility functions $u_t : X \to \mathbb{R}$, which are differentiable and represent strongly monotone and convex preference relations.

3.1 Define strong Pareto optimality and discuss other forms of Pareto optimality.

3.2 Define an equilibrium with spot markets and a Walrasian equilibrium.

3.3 Show that if $((p_t)_{t\in\mathbb{Z}}, (x_t)_{t\in\mathbb{Z}})$ is an equilibrium with spot markets, then it is also a Walrasian equilibrium.

Suppose that $\omega_t = (4, 11)$ and $u_t(x) = x^y + 2x^o$, for all t.

3.4 Show that $(x_t)_{t\in\mathbb{Z}}$, where $x_t = (4, 11)$ for all t, is an equilibrium allocation and show that the equilibrium allocation is not strongly Pareto optimal.

3.5 Show that $(x_t)_{t\in\mathbb{Z}}$, where $x_t = (0, 15)$ for all t, is a strongly Pareto optimal allocation.

3.6 Show that $(x_t)_{t\in\mathbb{Z}}$, where $x_t = (0, 15)$ for all t, is an equilibrium allocation.