Macro III - exam solutions (June 7, 2017)

1 False. Monopolistic power in price setting leads to inefficiencies in the use of resources (lower output and higher prices). But, with no barriers to change prices (i.e. with no nominal rigidities), money is still neutral.

2 False or uncertain. The real exchange rate is the price of a basket of goods in one country relative to the price of the *same* basket in another country. Thus, assuming that across countries the productivities of tradables and non-tradables respectively grow at the same rate, real exchange rates are constant, and thus the statement is false. An uncertain answer is also correct if it relates the uncertainty to the potentially different productivity growth rates in the tradable sector across countries (countries with higher productivity growth in tradables tend to experience real exchange appreciation, i.e. the price of the basket of goods increases faster than on other countries).

3 False. A distortionary capital income tax discourages saving by reducing its net rate of return. Thus, after the replacement of the distortionary tax by a lump sum alternative saving will increase. The desire to smooth consumption will make consumers to increase their saving, and thus reduce consumption, at the announcement and not wait for the tax to be replaced.

4 a) Characterizing individual saving behavior requires setting up the problem of workers.

$$\begin{aligned} \max_{s_t, c_{1t}, c_{2t+1}} \quad & \ln(c_{1t}) + \beta \ln(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t} = w_t(1-\tau) - s_t \\ & c_{2t+1} = s_t r_{t+1} + \tau w_{t+1} \end{aligned}$$

Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \beta r_{t+1} c_{1t} \tag{1}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{\beta}{1+\beta} w_t(1-\tau) - \frac{1}{(1+\beta)} \frac{1}{r_{t+1}} \tau w_{t+1} \tag{2}$$

To get capital accumulation, since there is no population growth, we replace individual savings with next period capital per worker $k_{t+1} = s_t$, and we use equilibrium expressions for wage and interest rates, $w = (1-\alpha)Ak^{\alpha}$, $r = \alpha A k^{\alpha-1}$)

$$k_{t+1} = \left[\frac{\beta}{1+\beta}(1-\alpha)Ak_t^{\alpha}(1-\tau) - \frac{1}{(1+\beta)}\frac{(1-\alpha)k_{t+1}}{\alpha}\tau\right]$$

Combining terms with k_{t+1}

$$k_{t+1} = \frac{1}{\left[1+\frac{1}{(1+\beta)}\frac{1-\alpha}{\alpha}\tau\right]}\frac{\beta}{1+\beta}(1-\alpha)Ak_t^{\alpha}(1-\tau)$$

From here imposing steady state we get the following

$$k^* = \left[\frac{1}{\left[1+\frac{1}{(1+\beta)}\frac{1-\alpha}{\alpha}\tau\right]}\frac{\beta}{1+\beta}(1-\alpha)A(1-\tau)\right]^{\frac{1}{1-\alpha}}.$$

b) The shock is such that in the first period the ratio of workers to retirees is $1+n$, and in all subsequent periods is 1. What is different in the setup is that migrants do not contribute, nor benefit, from social security. Thus, to keep track of capital accumulation we need to aggregate the potentially different saving behavior of natives and immigrants.

The government chooses contribution rate τ' such that the initial old receive the same benefits. Since the presence of immigrants increases the workforce for a given level of capital (initial steady state level k^*) this reduces the wage. Since the contribution base for social security is not affected (immigrants do not contribute), the new contribution rate is determined by

$$\tau' w_t = \tau'(1-\alpha)A\left(\frac{k^*}{1+n}\right)^{\alpha} = \tau'\frac{w^*}{(1+n)^{\alpha}} = \tau w^*$$

Thus,

$$\tau' = \tau(1+n)^{\alpha}.$$

To see the effect on capital accumulation we need to characterize the saving behavior of immigrants, s_t^m, as that of residents, s_t^r, will still be determined by equation (2). But for immigrants, the Euler equation, (1), holds as well, such that the only change in (2) is that they pay no taxes (and receive no benefit). Thus, it is straightforward to derive

$$s_t^m = \frac{\beta}{1+\beta}w_t$$

Combining both saving functions and remembering that there is no population growth (the measure of savers in period t, $1+n$, is the same as the measure of workers in period $t+1$), we find the following expression for capital accumulation

$$k_{t+1} = \frac{\beta}{1+\beta}w_t - \frac{1}{1+n}\frac{\beta}{1+\beta}w_t\tau' - \frac{1}{1+n}\frac{1}{(1+\beta)}\frac{1}{r_{t+1}}\tau' w_{t+1}$$

Where the term $\frac{1}{1+n}$ reflects the fraction of workers that contribute to, and benefit from, social security.

Combining terms with k_{t+1}, and replacing $\tau' = \tau(1+n)^{\alpha}$, we get

$$k_{t+1} = \frac{1}{\left[1+\frac{1}{1+n}\frac{1}{(1+\beta)}\frac{1-\alpha}{\alpha}\tau(1+n)^{\alpha}\right]}\frac{\beta}{1+\beta}w_t\left[1-\frac{1}{1+n}\tau(1+n)^{\alpha}\right]$$

Thus, it is clear that the long run effect of the shock is to increase capital accumulation (the dynamic equation corresponds to a system in which everybody contributed, and benefited, the equivalent tax rate $\tau^{eq} = \tau(1+n)^{\alpha-1} < \tau$).

The effect in the first period is ambiguous since the initial wage is reduced to $\frac{w^*}{(1+n)^{\alpha}}$. For small initial social security systems (τ close to zero), the wage effect dominates and capital accumulation initially decreases. But for large initial social security systems, (τ close to one), the reduction in the equivalent contribution rate dominates and capital accumulation initially increases.

c) Since the presence of immigrants increases the workforce for a given level of capital in the first period, this increases the interest rate. This makes the old to be strictly better off since they have the same level of benefits but a higher capital income. The disposable income of the young residents in the first period is given by $w_t(1-\tau') = (1-\alpha)A\left(\frac{k^*}{1+n}\right)^{\alpha}(1-\tau(1+n)^{\alpha}) < w^*(1-\tau)$. Thus, the disposable income of the initial young generation of residents is lower than in the initial steady state.

5 a) Since the objective function is quadratic, the optimal policy rule is linear. In principle it is given by

$$\pi = \psi + \psi_{\theta}\theta + \psi_{\epsilon}\epsilon + \psi_{\epsilon^{eu}}\epsilon^{eu}$$

and expected inflation given information available to the private sector and the policy rule is

$$\pi^e = \psi + \psi_{\theta}\theta$$

The objective function to minimize is then given by

$$\frac{1}{2}\left\{E\left[\psi+\psi_\theta\theta+\psi_\epsilon\epsilon+\psi_{\epsilon^{eu}}\epsilon^{eu}\right]^2+\lambda E\left[\theta-(1-\psi_\epsilon)\epsilon+\psi_{\epsilon^{eu}}\epsilon^{eu}-\bar{x}\right]^2\right\}$$

Thus, it is immediate that $\psi = 0$ and $\psi_{\epsilon^{eu}} = 0$. The latter reflects that supply shocks in Europe have no direct effect on Denmark's output and thus optimal policy should not respond to them, while the former is due to anchoring expected inflation around the target of zero inflation.

We complete the optimal policy rule by noting that $\psi_\theta = 0$ since this shock is already incorporated into private sector inflation expectations, and the only non-trivial coefficient is ψ_ϵ, whose optimal value trades off the effect of supply shocks on output and inflation. The first order condition gives

$$\psi_\epsilon = \frac{\lambda}{1+\lambda}.$$

Thus, equilibrium outcomes are given by

$$\pi = \frac{\lambda}{1+\lambda}\epsilon$$
$$x = \theta - \frac{1}{1+\lambda}\epsilon$$

b) Under a credible peg of the krone to the euro, $\pi^e = 0$ and $\pi = \pi^{eu} = \frac{1}{1+\lambda^{eu}}\epsilon^{eu}$. Thus, equilibrium output is given by

$$x = \theta + \frac{1}{1+\lambda^{eu}}\epsilon^{eu} - \epsilon$$

Since the equilibrium outcome characterized in a) is the optimal one, the equilibrium under a credible peg is never better than it. In facto, it will be worse since it leaves the economy exposed to Europe's supply shock.

c) Now, $\rho = \frac{E[\epsilon\epsilon^{eu}]}{\sigma_\epsilon\sigma_{eu}}$. The only term of the social loss function that would be affected is the one related to output deviations from target. This is now given by

$$E\left[\theta+\frac{1}{1+\lambda^{eu}}\epsilon^{eu}-\epsilon-\bar{x}\right]^2 = \sigma_\theta^2 + \left(\frac{1}{1+\lambda^{eu}}\right)^2\sigma_{eu}^2+\sigma_\epsilon^2+\bar{x}^2-2\left(\frac{1}{1+\lambda^{eu}}\right)\sigma^e\sigma^{eu}\rho$$

Thus, the social loss will be lowest the highest the positive correlation between supply shocks, i.e. when $\rho = 1$. The reason for this is that in this case the peg reduces the most the exposure of output to Denmark's supply shock.