Guide to answers to Written Exam **Mikro A**

February 2012

(3-hour closed book exam)

Problem 1

A consumer consumes two goods, has the utility function u, has an exogenously determined income I, and faces perfectly competitive markets with price system p.

Consider the statements below. Are they true or false? Please substantiate your answers (e.g. clearly providing a counter-example if you think a statement is false):

- "A good which is inferior, is a Giffen good".
- "If the consumption of a good is increased when income increases, then the good is a luxury good".
- "If both goods yield positive marginal utility, none of them can be a Giffen good".

Answer: All three are false. 1. A Giffen good must be inferior, according to the Slutsky equation, but the contrary is not necessarily true (a good may be inferior, but the substitution effect larger in absolute value than the income effect). 2. Not necessarily true; in order to be a luxury good, consumption must not only increase with higher income, but must do so at a disproportional rate. 3. Also false; "Giffen" has to do with a good being "strongly inferior", not with the good yielding negative marginal utility.

Problem 2

Consider a "Robinson Crusoe" (Koopmans) economy. The consumer has the utility function $u(x_1,x_2) = \text{Min}\{x_1, x_2\}$, with good 1 being time, which can be spent as leisure or used as labor input, and good 2 being food. The producer has the production function $f(\ell) = \ell$, with ℓ being the quantity of labor input, and f designating the maximum level of food output. The consumer's initial endowment is (24,0), i.e. twenty-four hours of time and no food.

- Identify the efficient (Pareto Optimal) state for this economy, i.e. production plan as well as consumption plan.

Assume, instead, that the technology is $g(\ell) = 2\cdot\ell$.

- Now, identify the efficient state with this new production function and compare this state (production function g) with the above (production function f): What has happened to productivity of labor, and what happens to the efficient level of labor input? Comment.

Answer: f-case: Consumption plan: (12,12), production plan (-12,12); g-case: Consumption plan: (16,16) production plan (-8,16). Labor input is reduced, although productivity is doubled; what we observe is a kind of wealth effect: society can afford the consumer working less (the increased "wealth" spills over into higher leisure); with Leontief-preferences there is no substitution effect.

Problem 3:

Consider the following claim; is it true or false, and why?

- "If
  - agent A has quasi-linear preferences represented by the utility function $u_A(x_1,x_2) = x_1^{1/3} + x_2$, and
  - agent B has quasi-linear preferences represented by the utility function $u_B(x_1,x_2) = x_1^{2/3} + x_2$,
  then agents A and B have identical preferences…
  because $x_1^{2/3}$ can be written as a monotonously increasing transformation of $x_1^{1/3}$
  ($x_1^{2/3} = f(x_1^{1/3})$, with $f(t) = t^2$ being a strictly increasing function for positive arguments)".

Answer: The claim is false, as one cannot transform just "part of the utility function". Computing MRS for A and B will show that they have different preferences (B having an MRS-value which is higher in absolute value than A's).

Problem 4:

A consumer lives for two consecutive periods, being young in the first and old in the second. The consumer has an endowment of $e_1 > 0$ as young and $e_2 > 0$ as old, and has monotonously increasing and convex preferences which can be represented by a utility function. The capital market offers the consumer the possibility to save or borrow at an interest rate r.

- Show that the decision to save or borrow corresponds to a life-time/inter-temporal utility maximization problem with $p_1 = (1+r)$ and $p_2 = 1$.
- Explain why savers, in general, may not always want to save more when the interest rate increases
- Show that if…
  - the consumer has Cobb-Douglas preferences of the form $u(x_1,x_2) = x_1 \cdot x_2$
  - the consumer, at the present interest rate, is a saver

  … then his savings will indeed go up, when the interest rate increases.
  Please comment.

Anwer: Wealth effects, making the consumer wealthier when r increases, may increase consumption when young, even though the incentive to save has increased. In the CD case, the saving function is $\frac{1}{2}e_1 - \frac{1}{2}e_2p_2/p_1$*, or* $\frac{1}{2}e_1 - \frac{1}{2}e_2/(1+r)$*. The intuition is that there is a substantial substitution effect, the elasticity of substitution being 1 in the CD-case.*

Problem 5

The firm Polsemaster produces salami using two inputs, meat and fat, and lives in a market environment characterized by perfect competition. The maximum output of salami is given by the production function $g(m,e) = m^{1/3} \cdot f^{1/3}$. The cost of one unit of meat is w_m, the cost of one unit of fat is w_f.

- Solve the cost minimization problem for this producer for a given level of output x.
- Find the conditional demand functions for meat and fat.
- Find the cost functions C(x), AC(x) and MC(x)
- Find an expression for Polsemaster's supply curve

The government introduces, for health reasons, a unit tax on the input fat, hence increasing the cost per unit fat used within food industries.

- Without doing a lot of calculations, but using your economic intuition: How will this affect the cost minimization problem? How will it affect the cost curves? How will it affect Polsemaster's demand for fat? And its supply curve?

Answer: $m(w_m, w_f, x) = (w_f/w_m)^{½}x^{3/2}$, $f(w_m, w_f, x) = (w_m/w_f)^{½}x^{3/2}$, $C(x) = 2(w_m w_f)^{½}x^{3/2}$, $AC(x) = 2(w_m w_f)^{½}x^{½}$, $MC(x) = 3(w_m w_f)^{½}x^{½}$, $S(p) = (p/3)^2/(w_m w_f)$. *The unit cost of using fat has increased, so there will be a substitution from fat to meat, and the MC-curve will move upwards/the supply curve move inwards.*

<u>Problem 6</u>

Paul consumes two goods, beer (good 1) and potato chips (good 2) and has the utility function $u(x_1,x_2) = \ln(x_1) + x_2$.

- Identify the (Hicks-)compensated demand function, having prices and utility level as it arguments
- How does Paul's compensated demand for beer depend on the utility level?
- Identify the expenditure function $E(p_1, p_2, u)$ and show that the derivative of this function with respect to the price of beer is p_2/p_1.

Note: All through this problem, consider only interior solutions, i.e. consumption plans with strictly positive quantities of beer and chips.

Answer: h becomes $(p_2/p_1, u-\ln(p_2/p_1))$; *compensated beer demand does not depend on u; this is because of quasi-linearity of preferences. Expenditure function:* $p_2\cdot(1+u- \ln(p_2/p_1))$. *The good student notes that it is no surprise that the derivative of the expenditure function with respect to the price of beer is exactly the compensated demand for beer; this follows from the envelope theorem.*

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