Written Exam - Macroeconomics III

(suggested answers)

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Question 1

a The savings problem of a young individual is

$$\max_{c_{1t}, c_{2t+1}} \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1},$$
$$c_{1t} + s_t = w_t(1-\tau),$$
$$c_{2t+1} = s_t\left(1 + r_{t+1}\right) + \tau w_{t+1}.$$

Solving this problem and combining the FOCs yields the Euler equation

$$c_{2t+1} = \frac{1 + r_{t+1}}{1+\rho} c_{1t}.$$

Replace c_{1t} and c_{2t+1} from the budget constraints to obtain the desired equation describing individual savings behavior:

$$s_t = \frac{1}{2+\rho} w_t(1-\tau) - \tau \frac{1+\rho}{2+\rho} \frac{1}{1+r_{t+1}} w_{t+1}.$$

b To derive the capital accumulation equation we use individual savings and replace $k_{t+1} = s_t$ (there is no population growth), and use the equilibrium expressions for wages and rental rates to obtain

$$k_{t+1} = \frac{1}{2+\rho}(1-\alpha)Ak_t^{\alpha}(1-\tau) - \frac{1+\rho}{2+\rho}\frac{(1-\alpha)k_{t+1}}{\alpha}\tau.$$

Thus, we rearrange this expression to obtain

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho}\frac{(1-\alpha)}{\alpha}\tau}\left[\frac{(1-\alpha)(1-\tau)}{2+\rho}Ak_t^{\alpha}\right].$$

Imposing the steady state we get

$$\bar{k} = \left[\frac{1}{1 + \frac{1+\rho}{2+\rho}\frac{(1-\alpha)}{\alpha}\tau}\left(\frac{1}{2+\rho}(1-\alpha)A(1-\tau)\right)\right]^{\frac{1}{1-\alpha}}.$$

c The savings problem of a young individual now reads as

$$\max_{c_{1t},c_{2t+1}} \ln c_{1t} + \frac{1}{1+\rho}\ln c_{2t+1},$$
$$c_{1t} + s_t = w_t(1-\tau),$$
$$c_{2t+1} = (s_t + w_t\tau)(1+r_{t+1}).$$

Solving this problem and combining FOCs yields the Euler equation

$$c_{2t+1} = \frac{1+r_{t+1}}{1+\rho}c_{1t}.$$

Replace c_{1t} and c_{2t+1} from the budget constraints to obtain the desired equation describing individual savings behavior:

$$s_t = k_{t+1} = \left(\frac{1}{2+\rho} - \tau\right) w_t.$$

Thus, as $w_t = (1-\alpha)Ak_t^{\alpha}$:

$$\bar{k}' = \left[\left(\frac{1}{2+\rho} - \tau\right)(1-\alpha)A\right]^{\frac{1}{1-\alpha}}.$$

d As the policy-switch takes place before saving decisions are formulated, the old generation in T finds itself with no pension. Thus, old in T are worse-off.

Question 2

a The representative agent i maximizes the following utility function

$$U_i = C_i - \frac{1}{\beta}L_i^{\beta}, \qquad \beta > 1,$$

subject to the budget constraint

$$PC_i = P_iY_i,$$

where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i. The production function equals

$$Y_i = L_i^{\alpha}, \quad 0 < \alpha < 1.$$

There is monopolistic competition in the goods market. The demand for good i is

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y, \quad \eta > 1,$$

Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\beta} Y_i^{\frac{\beta}{\alpha}}.$$

Maximizing w.r.t. Y_i:

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta}\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}} - \frac{1}{\alpha} Y_i^{\frac{\beta-\alpha}{\alpha}} = 0.$$

After some manipulation we obtain

$$\left(1-\frac{1}{\eta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} = \frac{1}{\alpha} Y_i^{\frac{\beta-\alpha}{\alpha}}.$$

Which, after substituting for $\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}}$ through the demand function, translates into

$$\left(1-\frac{1}{\eta}\right)\frac{P_i}{P} = \frac{1}{\alpha} Y_i^{\frac{\beta-\alpha}{\alpha}}.$$

Taking logs and rearranging to obtain y_i^*:

$$y_i^* = \frac{\alpha}{\beta-\alpha}(p_i - p) + \frac{\alpha}{\beta-\alpha}\left[\ln\left(1-\frac{1}{\eta}\right) - \ln\left(\frac{1}{\alpha}\right)\right].$$

We aggregate to find y:

$$y = \frac{\alpha}{\beta-\alpha}\ln\left(\alpha\frac{\eta-1}{\eta}\right).$$

b We then compute the following derivative

$$\frac{\partial y}{\partial \eta} = \frac{\alpha}{\beta-\alpha}\frac{1}{\eta(\eta-1)} > 0, \text{ as } \beta > \alpha \text{ and } \eta > 1.$$

Interpretation: as the degree of substitutability among the goods traded in the monopolistically competitive market increases, the deadweight loss due to imperfect competition drops, reflecting into higher equilibrium output.

c Assuming certainty equivalence:

$$x_t = \frac{1}{2}\left(p_{i,t}^* + \mathbf{E}_t\left[p_{i,t+1}^*\right]\right).$$

Thus

$$\begin{aligned} x_t &= \frac{1}{2}\left(m_t + y + \mathbf{E}_t\left[m_{t+1} + y\right]\right) \\ &= y + \frac{1}{2}\left(m_t + \mathbf{E}_t\left[m_{t+1}\right]\right), \end{aligned}$$

Clearly, higher (contemporaneous and expected) money supply (m) increases the desired price, thereby x_t.

d Derive an expression for aggregate price inflation:

$$\begin{aligned} \pi_t &= p_t - p_{t-1} \\ &= \frac{1}{2}\left(x_t + x_{t-1}\right) - \frac{1}{2}\left(x_{t-1} + x_{t-2}\right) \\ &= \frac{1}{2}x_t - \frac{1}{2}x_{t-2} \\ &= \frac{1}{2}\left[y + \frac{1}{2}\left(m_t + \mathbf{E}_t\left[m_{t+1}\right]\right)\right] - \frac{1}{2}\left[y + \frac{1}{2}\left(m_{t-2} + \mathbf{E}_{t-2}\left[m_{t-1}\right]\right)\right] \\ &= \frac{1}{4}\left(m_t + \mathbf{E}_t\left[m_{t+1}\right]\right) - \frac{1}{4}\left(m_{t-2} + \mathbf{E}_{t-2}\left[m_{t-1}\right]\right). \end{aligned}$$

Now, use the fact that $m_t = \rho m_{t-1} + \varepsilon_t$, obtaining:

$$\begin{aligned} \pi_t &= \frac{1}{4}\left(m_t + \mathbf{E}_t\left[m_{t+1}\right]\right) - \frac{1}{4}\left(m_{t-2} + \mathbf{E}_{t-2}\left[m_{t-1}\right]\right) \\ &= \frac{1}{4}\left(m_t + \mathbf{E}_t\left[\rho m_t + \varepsilon_{t+1}\right]\right) - \frac{1}{4}\left(m_{t-2} + \mathbf{E}_{t-2}\left[\rho m_{t-2} + \varepsilon_{t-1}\right]\right) \\ &= \frac{1+\rho}{4}\left(m_t - m_{t-2}\right) \\ &= \frac{1+\rho}{4}\left(\underbrace{\rho m_{t-1} + \varepsilon_t}_{=m_t} - m_{t-2}\right) \\ &= \frac{1+\rho}{4}\left(\underbrace{\rho^2 m_{t-2} + \rho\varepsilon_{t-1}}_{=\rho m_t - 1} + \varepsilon_t - m_{t-2}\right) \\ &= \frac{1+\rho}{4}\left[\left(1-\rho^2\right) m_{t-2} + \varepsilon_t + \rho\varepsilon_{t-1}\right]. \end{aligned}$$