Written Exam for the B.Sc. or M.Sc. in Economics autumn 2012-2013

Advanced Microeconomics

Master's Course

02NOV2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Advanced Microeconomics, Fall 2012

3 hours closed book exam

Anders Borglin, who is responsible for the exam problems, can be reached during the exam on +46 735 754176. Assumptions are enclosed and there should be **6 pages** in your problem set

There are 3 problems. The problems B and C have the same weight in the marking process and Problem A has half the weight of Problem B.

Below

$$
\begin{aligned}
\mathbb{R}^k_+ &= \left\{x \in \mathbb{R}^l \mid x_h \geq 0 \text{ for } h = 1, 2, \ldots, k\right\} \text{ and} \\
\mathbb{R}^k_{++} &= \left\{x \in \mathbb{R}^l \mid x_h > 0 \text{ for } h = 1, 2, \ldots, k\right\}
\end{aligned}
$$

for $k = 1, 2, \ldots$.

Problem A

(a) What is meant by a rational preference relation?

(b) The production possibility set Y exhibits non-decreasing returns to scale. What does this mean?

(c) Give a graphic example of a consumption possibility set in $\mathbb{R}^2$ where commodity 1 is indivisible. Is your consumption possibility set a convex set..

(d) Assume that Arrow's assumptions for a SWF are satisfied and that Schedule 1 below is mapped to a ranking for society with a above b. What can be concluded about society's ranking of a and b for Schedule 2?

Schedule 1			Schedule 2		
b	c	a	c	b	c
a	b	c	b	a	a
c	a	b	a	c	b

(e) Let $\mathcal{E} = (\mathbb{R}^2_+, u^i, \omega^i)_{i \in \{a,b\}}$ be a pure exchange economy where consumers satisfy assumptions F1,F2 and F3. How is the total (aggregate) excess demand defined for this economy?

(f) Let $\xi(p_1, p_2, \mathrm{w}) = \left(\frac{1}{4}\frac{\mathrm{w}}{p_1}, \frac{3}{4}\frac{\mathrm{w}}{p_2}\right)$ be the demand function of a consumer in a private ownership (pure exchange) economy with $\mathrm{w} = p_1\omega_1 + p_2\omega_2, \omega_1, \omega_2 > 0$. Will the consumer's excess demand function satisfy the Gross Substitutes assumption? Does you answer depend on the initial endowment ω of the consumer?

Problem B

(a) Consider an economy $\mathcal{E} = \left((\mathbb{R}^L_+, u^i)_{i\in\{a,b\}}, Y, \omega\right)$ where the (only) producer satisfies P1 and the consumers satisfy F1 and F2. Let $\left((\bar{x}^i)_{i\in\{a,b\}}, \bar{y}\right)$ be a Pareto optimal allocation. Show that $\bar{y}$ is an efficient production in Y. (**Hint**: Argue by contradiction.)

(b) State and prove The First Theorem of Welfare Economics for a pure exchange economy (without private ownership). (**Hint**: Argue by contradiction.) Where do you need Assumption F2?

Problem C

Let $\mathcal{E} = \{(X, u), Y, \omega\}$ be a private ownership economy with a single consumer (who owns the single producer) and

$$\begin{aligned} X &= \left\{x \in \mathbb{R}^2 \mid x_1 \geq 2, x_2 \geq 0\right\} \\ Y &= \left\{y \in \mathbb{R}^2 \mid y_2 \leq 2(-y_1)^{1/2}, y_1 \leq 0\right\} \\ u(x_1, x_2) &= (x_1 - 2)\, x_2 \\ \omega &= (4, 0) \end{aligned}$$

(a) Does Y satisfy Assumption P1? State and solve the Producer Problem for prices $p = (p_1, p_2) \in \mathbb{R}^2_{++}$ and find the maximal profit.

(b) Solve the Consumer Problem as $p = (p_1, p_2) \in \mathbb{R}^2_{++}$ and wealth is given by $\mathrm{w} \geq 2p_1$. **Hint:** You may consider rewriting the budget restriction as $p_1(x_1 - 2) + p_2 x_2 \leq \mathrm{w} - 2p_1$ if you recall the solution for a Cobb-Douglas utility function.

(c) Assume that wealth is now given by the value of initial endowment and profits. Derive the market balance condition for good 1. If $p = (p_1, p_2)$ satisfies this condition is then (p_1, p_2) an equilibrium price system?

(d) Put $p_1 = 1$ and find p_2 from the market balance condition in (c).

(e) Find the Walras equilibrium for $\mathcal{E}$. Check that both markets balance.

Assumptions on Producers

Assumption P1: *The production set $Y \subset \mathbb{R}^L$ satisfies*

(a) $0 \in Y$ (Possibility of inaction)

(b) *Y is a closed subset of $\mathbb{R}^L$* (Closedness)

(c) *Y is a convex set* (Convexity)

(d) $Y \cap (-Y) = \{0\}$ (Irreversibility)

(e) *If $\bar{y} \in Y$, $y \in \mathbb{R}^L$ and $y \leq \bar{y}$ then $y \in Y$* (Free disposal, downward comprehensive)

Assumption P2:*(constant returns to scale) If the vector $y \in Y$ and $\lambda \in [0, +\infty[$ then $\lambda y \in Y$.*

Assumptions on Consumers

Assumption F1.
The consumption set $X \subset \mathbb{R}^L$ satisfies:

(a) X is a non-empty set.

(b) X is a closed set

(c) X is a convex set

(d) X is a lower bounded set (in the vector ordering)

(e) X is upward comprehensive ($x \in X$ and $\nabla \in \mathbb{R}^L_+$ implies $x + \nabla \in X$)

Monotonicity assumptions

Assumption of **weak monotonicity**
$\mathbf{F2}^0$: $x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow x^1 \succsim x^2$

$\mathbf{F2}^0$: $x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow u(x^1) \geq u(x^2)$

In the interpretation: "at least as much of each commodity is at least as good".

Assumption of **monotonicity** (MWG Def. 3.B.2)

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow x^1 \succ x^2$

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow u(x^1) > u(x^2)$

In the interpretation: "more of each commodity is better".

Assumption of **strict (or strong) monotonicity** (MWG Def. 3.B.2)

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow x^1 \succ x^2$

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow u(x^1) > u(x^2)$

The preference relation $\succsim$ is **locally non-satiated** if: Given $x \in X$ and $\varepsilon > 0$ there is $x' \in X$ such that $x' \succ x$ and $\| x' - x \| < \varepsilon$. (Definition 3.B.3, MWG)

A preference relation, $\succsim$, is a **convex preference relation** if and only if, for $x \in X$, the set $\{x' \in X \mid x' \succsim x\}$ is a convex set. (MWG Def. 3.B.2).

If u represents $\succsim$ then $\succsim$ is a convex preference relation if and only if u is a quasi-concave function.

We want to consider also a stronger convexity assumptions

F3: A preference relation, $\succsim$, is a **strictly convex** preference relation if: $x^1, x^2, x^3 \in X$, $x^1 \succsim x^2$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)\,x^2$ for some $t \in \,]0,1[$ implies $x^3 \succ x^2$.

F3: The utility function is **strictly quasi-concave** if: $x^1, x^2, x^3 \in X$, $u(x^1) \geq u(x^2)$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)\,x^2$ for some $t \in \,]0,1[$ implies $u(x^3) > u(x^2)$.