Written Resit Exam for M.Sc. in Economics

Winter 2010/2011

Advanced Microeconomics

16. February 2011

Master course

3 hours written exam with closed books

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Exercise 1:

Consider an economy with L goods and I consumers. Consumers have identical consumption sets $X = \mathbb{R}^L$. Consumers are described by their endowment vectors $\omega_i \in \mathbb{R}^L$ and utility functions $u_i(x_i) = -a_i^1 e^{-x_i^1} - \ldots - a_i^L e^{-x_i^L}$ where $a_i^\ell > 0$ for all ℓ.

1.1 State the utility maximization problems of the consumers (UMP). Find the demand functions of the consumers.

1.2 Define Pareto optimality the economy. Define Walrasian equilibria for the economy.

1.3 For $I = 1$ find a Walrasian equilibrium.

1.4 For $L = 2$ and $I = 2$ find the set of Pareto optimal allocatons.

Exercise 2:

Consider a pure-exchange economy with $L \geq 1$ goods, one consumer and one firm. The consumer is described by her consumption set $X = \mathbb{R}^L_+ = \{v \in \mathbb{R}^L | v^\ell \geq 0 \text{ for all } \ell\}$, initial endowment vector $\omega \in \mathbb{R}^L_{++} = \{v \in \mathbb{R}^L | v^\ell > 0 \text{ for all } \ell\}$, and preference relation $\succeq$, where $\succeq$ is rational, strongly monotone, strictly convex and continuous. The firm is described by its production set $Y \in \mathbb{R}^L$, where Y is convex and compact.

2.1 State the utility maximization problem of a consumer (UMP). Show that for every $p \in \mathbb{R}^L_{++}$ there exists a unique solution to (UMP).

2.2 Define feasible allocations for the economy. Define Pareto optimality for the economy.

2.3 Show that there exists a unique Pareto optimal allocation for the economy.

2.4 Suppose that the preference relation can be represented by a differentiable utility function $u : X \to \mathbb{R}$. Let $(\bar{x}, \bar{y})$ be the Pareto optimal allocation for the economy. Suppose that $\bar{x} \in \mathbb{R}^L_{++}$. Find a Walrasian equilibrium for the economy.

Exercise 3:

Consider a stationary overlapping generations economy with time going from $-\infty$ to ∞, one good per date and one consumer, who lives for two dates, per generation. Consumers are described by their identical consumption sets $X = \mathbb{R} \times \mathbb{R}_+$, endowment vectors $\omega \in X$ and utility functions $u(x^y, x^o) = x^y + 2\sqrt{x^o}$. The market structure is spot markets and money where $p_t > 0$ is the price of the good at date t.

3.1 State the utility maximization problem of consumer t. Find the solution.

3.2 Define equilibria for the economy. Show that there exists an equilibrium $((\bar{p}_t)_{t\in\mathbb{Z}}, (\bar{x}_t)_{t\in\mathbb{Z}})$ where $\bar{x}_t = \omega$ for all t.

3.3 Define strong Pareto optimality for the economy. Show that no allocation is strongly Pareto optimal. Explain you answer.

3.4 Find a difference equation such that if $(p_t)_{t\in\mathbb{Z}}$ is a solution to the equation, then there exists an equilibrium $((\bar{p}_t)_{t\in\mathbb{Z}}, (\bar{x}_t)_{t\in\mathbb{Z}})$ such that $\bar{p}_t = p_t$ for all t. Explain your answer.