Written Exam for the B.Sc. in Economics winter 2015-16

Microeconomics B (II)

Final Exam

19. January 2016

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total

Problem 1

In order to generate a revenue for the government you are asked to submit a proposal for imposing either a tax as a registration fee on vehicles or as taxation of land.

Consider a per unit tax and assume that the current annual sales of cars is equal to the number of acres of land taxable by a land tax. Assume also that the markets for cars and land are both characterized by competitive behavior both on the seller and buyer side.

Explain what determines the size of the revenue from taxing either car sales or land for a given per unit tax with respect to the price sensitivity of demand and supply. Which kind of tax is likely to yield the highest revenue? Why?

Solution:

Since the sales of the two alternatives are of equal size the relative revenue is determined by the demand and supply elasticity of each market; the lower the elasticity, ceritus paribus, the lower the change in the equilibrium quantity and thus the revenue is greater. From the equilibrium condition $P_D(q) - t = P_S(q)$ we can obtain the change in equilibrium quantity as $\frac{dq}{dt} = -\frac{1}{\varepsilon_S - \varepsilon_D}$ where $\varepsilon_i = \frac{dP_i}{dq}\frac{dq}{P_i}$ is the elasticity. In general one would expect the supply of land to have a lower price elasticity than the supply of cars. Thus, ceritus paribus the land tax is expected to yield the highest revenue. Good illustrations of the conclusion suffices, if the revenue is emphasized og shows the conclusion. It is quit misleading to discuss both the tax incidence and/or the deadweight loss since this is part of the question posed in the problem.

Problem 2

Consider a situation with two roommates; A is a smoker and B is a non-smoker. The two roommates have incomes as follows: A has \$100 and B has \$200. They have utility functions:

$$U_A(M_A, S_A) = M_A + \ln S_A$$

for roommate A and

$$U_B(M_B, S_B) = M_B + 3 \ln S_B$$

where M is money and S is related to the smoke intensity in the room. S_A is the quantity of smoke and in the case for B we interpret S_B as the non-existence of smoke. Smoke, S, is measured on a scale from 0 to 1, such that possible allocations must satisfy $S_A + S_B = 1$.

According to the dormitory rules no smoking is allowed. However, roommates can bribe each other and trade the right to air (smoke filled or clean).

a) Determine the Walrasian equilibrium if trade in smoke permits takes place.

b) What would happen if students were allowed to smoke as much as possible?

Solution:

a) An Walrasian equilibrium in this economy is a price vector and an allocation such that each roommate maximize utility given the budget constraint and there is market clearing. The income/wealth of roommate A is his money income $\bar{M}_A = 100$, while the wealth of roommate B is $\bar{M}_B = 200 + p$. The budget constraints are then $M_A + pS_A = \bar{M}_A$ and $M_B + pS_B = \bar{M}_B$. Market demands are $S_A(p) = \frac{1}{p}$ and $S_B(p) = \frac{3}{p}$ for the smoking permits, such that the equilibrium price $p^* = 4$ and thus the allocation is $(S_A, M_A) = \left(\frac{1}{4}, 99\right)$ and $(S_B, M_B) = \left(\frac{3}{4}, 201\right)$.

b) Now the wealth of each roommate are $\bar{\bar{M}}_A = 100 + p$ and $\bar{\bar{M}}_B = 200$. The equilibrium quantity of smoking is unchanged due to quasi-linear preferences of both, and a reference to Coases' theorem, and also the equilibrium price of smoking permits. While the spending on other goods changes to $M_A' = 103$ and $M_B' = 197$.

Problem 3

Consider a company, Medicals Ltd, that has been granted a patent for a pharmaceutical to alleviate pains following a surgical operation. There are two types of consumers, 1 and 2, and: $D_1(p) = \max\{100 - p, 0\}$ and $D_2(p) = \max\{50 - p, 0\}$ are the aggregate demands of the two types, measured as number of prescriptions demanded by customers.

The Medicals Ltd can produce each prescription at a constant cost of $\$10$.

As a marketing expert, having your own marketing consultancy, you can carry out a survey to determine some objective characteristics that will allow Medical Ltd to discriminate perfectly between the two types of consumers, hence being able to act as a 3rd degree price discriminator.

Without your survey Medical Ltd must charge a uniform price to both types of consumers.

a) What will be the maximal amount that Medical Ltd would be willing to pay for your service?
b) What happens to the amount in a) if the demand of type 1 is $D_1(p) = \max\{200 - p, 0\}$?

Solution:

a) The profit with 3rd degree price discrimination: $q_1 = 45$ and $q_2 = 20$, and prices $p_1 = 55$ and $p_2 = 30$. Thus, the profit is $\pi^3 = 45^2 + 20^2 = 2425$; without price discrimination he must choose how many segments to service: if only the high demanders $\pi_1 = 2025$ and if both segments are serviced the quantity is $Q = 65$ and price $p = 42{,}5$, thus the profit is $\pi_2 = 2112{,}5$. Thus the company's maximum willingness to pay for the ability to exhibit perfect discrimination is $2425 - 2112{,}5 = 312{,}5$.
b) The price discrimination profit $\pi^3 = 9425$ while non-discrimination profit becomes $\pi_1 = 9025$ such that $9425 - 9025 = 400$, It is important to note that it is no longer profitable to include type 2 in the non-discriminatory case. Hence the income increase increases the willingness to pay for the marketing. The intuition is that with rich customers becoming richer, the costs of not being able to discriminate increases since it is no longer possible to service the low-demand costumers.

Problem 4

Two producers of dairy products, Arly AS and Thoese ApS, sell milk facing the following demand functions, as these two companies are the only two operating in the milk market and hence constitute a duopoloy with milk products that are not completely homogenous

$$D_A(p_A, p_T) = 90 - 2p_A + p_T$$

and

$$D_T(p_A, p_T) = 90 + p_A - 2p_T$$

where p_A is the price that a consumer pays for an Arly milk and p_T the price of a Thoese milk. Both producers set a price per liter and will accordingly satisfy the resulting demand for their milk. Both producers can produce a liter of milk at a constant extra cost of $9.

a) Find the equilibrium price and demand for milk for both producers.
b) Compare with the competitive equilibrium.

The government now imposes a fat content tax of $6 on each liter of milk and which is paid by the producers.

c) Find and explain what happens to the equilibrium price, quantity and profits.
d) Compare with the change in a competitive equilibrium. In particular, in which case is the change in quantities strongest?

Solution:

a) Since the milk from the two producers is not perfect substitutes, the two producers can charge different prices. Since both producers will set prices the situation is described by a Bertrand competition: each will choose his price that maximize his profit given the best choice of price by his/her competitor. The reaction function is (by symmetry) $R_i(p_j) = \frac{2c+90}{4} + \frac{1}{4}p_j$ since $(p_1 - c)(-2) + 90 - 2p_1 + p_2 = 0$ and thus a Nash equilibrium $p_A^* = p_T^* = \frac{2c+90}{3} = 36$, and quantities $q_A^* = q_T^* = 90 - \frac{2c+90}{3} = 54$. Some uses the Bertrand solution that prices are pushed down to marginal costs, as is the result of the Bertrand-paradox. However, the result does not apply here, since we in this case consider inhomogeneous products. Referring to the result however does give some credit.
b) The competitive equilibrium is $p_A = p_T = c = 9$ and $q_A = q_T = 90 - c = 81$
c) The tax changes the reaction function: the profit of say Arly is $(p_A - t - c)D_A(p_A, p_T)$ and thus $R_A(p_T) = \frac{2(t+c)+90}{4} + \frac{1}{4}p_T$. Here $p_A^{**} = p_T^{**} = 40$ and $q_A^{**} = q_T^{**} = 50$; while the profits become $\pi_A^{**} = \pi_T^{**} = (p^{**} - c - t)q^{**} = 25 * 50 = 1250$, while the profit before the tax is $\pi_A^* = \pi_T^* = 27 * 54 = 1458$, such that there is a drop in profits by 208. The tax increases the base price the firms will charge; but not the price reaction of the other firm's price changes. Note, however, that the reaction function does not change in one-to-one since $dR = \frac{1}{2}dt$.
d) The competitive case $p_A = p_T = c + t = 15$ and $q_A^c = q_T^c = 90 - 15 = 75$. Thus the change is strongest in the case of competitive markets: the price increases by 6 in the competitive case compared to 4 in the monopolistic case; on the other hand, the quantities change by -6 in the competitive case and -4 in the monopolistic case. Note that the producers carries a part of the tax incidence in the case of oligopoly, while in the case of competitive markets only the consumers carry the tax incidence. The weaker competition thus implies that prices and quantities changes by less. Note, however, that when we consider percentage changes the two cases are equal.

Problem 5

Svend is a hard working custodian at the national gallery, and he has an annual income of 300 thousand dkk after-tax. Due to expected government spending cuts on the culture budgets he expects to be unemployed in the next year with a probability of 10 pct. In the event of a year of unemployment he can only receive an income in the form of cash benefit from the government in total of 100 thousand dkk.

Svend has preferences on lotteries that satisfies the expected utility hypothesis and he has a bernoulli function $u(x) = \sqrt{x}$.

Coincidently, Svend receives a telephone call from an insurance company, that offers him an unemployment insurance. He is offered to pay an insurance premium of 30 thousand dkk in annual premium and in return the company pays him 250 thousand dkk in the case of unemployment. If he accepts the insurance, in the event of unemployment and receiving the insurance amount, the amount will be deducted from his cash benefits and he will receive no income from the government.

a) Does Svend accept the offer by the insurance company?
b) How much would Svend be willing to pay in annual premium for the insurance amount?

Solution:

a) Since $16.59 \approx 0.1 * \sqrt{100} + 0.9 * \sqrt{300} > 0.1 * \sqrt{220} + 0.9 * \sqrt{270} \approx 16.27$ Svend will not accept the insurance.
b) We must find the amount: ρ that satisfies $16.59 = 0.1 * \sqrt{250 - \rho} + 0.9 * \sqrt{300 - \rho}$ and by inspecting we obtain $\rho \approx 19.5$ thousand (an approximate amount suffices). Thus Svend would be willingly to pay 19.500 dkk for the insurance. Note that this is not the risk premium since the insurance does not cover the full loss of income. The risk premium is the amount that Svend would pay to obtain the certain amount equal to the expected value of the lottery. However, an answer that mentions the risk premium/certainty-equivalence must give some point solution – however, not a perfect solution.

Problem 6

A bank is considering lending money to an entrepreneur who faces two possible projects, both of which demand an investment of 5 million $. The entrepreneur has nowhere else to go, so the bank is in a strong bargaining position, basically being able to extract all gains from trade.

In project A, there is an 80% chance of earning profits of 10 million, and a 20% chance of earning a zero profit. In project B, there is a 40% chance of earning 22 million, and a 70% chance of earning a zero profit.

The entrepreneur has no initial capital and needs to borrow the full amount of five million from the bank. The bank and the entrepreneur are both risk-neutral. There is limited liability, such that in the event of a failure of a project controlled by the entrepreneur, the bank must write off its claims.

a) If the bank can control the entrepreneur's choice of project, on which terms should the bank offer the entrepreneur to borrow the million?

b) If the bank cannot observe the choice of project, what is the maximal amount the bank can charge while making sure that the entrepreneur chooses the same project as the bank? How does the lack of control affect the bank's expected profits?

Solution: ERROR in exam: the sum of the probabilities in project B does not sum to one. The students were given the new distribution: 40 pct. probability of success and 60 pct. probability of failure. If the student answered the question using this distribution the correct answer would be

a) The bank would want the entrepreneur to take on project B since $E[x_A] = 8 < 8.8 = E[x_B]$
b) There is no moral hazard since the inequality $\pi_A(G_A - R) < \pi_B(G_B - R)$ always hold, and the bank's and entrepreneur's interests are always aligned.

Some students, in realizing this non-problem, used instead the distribution: 30 pct. of success and 70 pct. of failure. Using this distribution the correct answer would be:

a) Since $E[x_A] = 8$ million and $E[x_B] = 6.6$ million, the bank should choose A-project and require a repayment of $R = 10$ mio. And the expected profit is $E[x_A] - 5 = 3$ mio.
b) To take into account the incentive constraint, the payment R must satisfy that the expected profit of the entrepreneur when choosing the A project is larger than when choosing the B project: $\pi_A(G_A - R) \geq \pi_B(G_B - R)$, such that $R \leq \bar{R} = \frac{\pi_A G_A - \pi_B G_B}{\pi_A - \pi_B} = \frac{8-6.6}{0.8-0.3} = 2.8$ is the maximal repayment consistent with the incentive compatibility constraint. The profit drops from 3 mio. to -2.2 mio. which is thus an information cost of the bank. Now the project is no longer profitable when taking into account the information costs. Taking the probabilities of project B to be 40% of success (and 70% of failure) there is no moral hazard issue: both the bank and the entrepreneur's interests are aligned.

Both answers are regarded as equally correct in assessing the grade.