Written Exam at the Department of Economics winter 2016-17

Macroeconomics III

Final Exam

February 13, 2017

(3-hour closed book exam)

This exam question consists of 3 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Written Exam - Macroeconomics III
University of Copenhagen
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Question 1

Consider the following version of the Ramsey model. Identical competitive firms maximize the following profit function:

$$\pi^F (K_t, L_t) = K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t$$

where r_t is the interest rate at which firms can borrow capital from households, w_t is the wage rate, L_t and K_t denote the quantities of labor and capital employed by the firm. Assume $\alpha \in (0, 1)$. Capital does not depreciate. Household size is constant. A large number of identical households maximize the following intertemporal utility function, that depends on household consumption C_t:

$$U = \int_0^\infty \frac{C_t^{1-\theta}}{1-\theta} e^{-\rho t} dt,$$

subject to their dynamic budget constraint:

$$C_t + \dot{A}_t = w_t + r_t A_t - T_t.$$

Take $A_0 > 0$ as given. Moreover, $A_t = K_t + B_t^h$ is wealth (upper case variables represent quantities in per capita terms for households), where B_t^h is net lending to other households in the economy, and T_t are lump sum taxes paid (benefits if negative). Although each household size is constant, there is population growth at rate $n > 0$ in this economy since there is an inflow of immigrants, who arrive with no assets. The society has an extreme aversion to inequality and therefore the government provides new immigrants with a transfer, funded by taxes paid by residents (i.e., those not currently migrating), such that wealth is equalized between residents and immigrants. This allows us to assume that at each point in time the economy is populated by a representative household irrespective of when this household entered the economy. Assume $\rho > n$.

a Find the first order conditions for the firms' maximization problem that characterize how much capital and labor a firm demands at given factor prices. As a function of capital per capita, k_t, what is the income that the representative household receives on his/her saving? And for his/her labor services?

b Write and interpret the No-Ponzi game condition for households. What are the control and state variables in households' optimization problem? Write the Hamiltonian, find the first order conditions that characterize the behavior of households, and from these the Euler equation. Give an economic interpretation to this equation.

c Derive the capital accumulation equation in per capita terms. Find the equations that characterize steady state as a function of control and state variables. Draw the phase diagram that describes the dynamics in this economy.

d Assume that the economy is initially in the steady state. Now, unexpectedly, n is permanently increased to $n^{'} > n$ (e.g. think of a civil war in a foreign country). How does this shock affect the $\dot{c} = 0$ and $\dot{k} = 0$ curves? Characterize the new steady state capital per capita, k. What happens with consumption? Explain.

e Now assume there is no change in immigration (i.e., n stays constant), but suddenly the government is forced to use distortionary capital income taxes, i.e. if the household budget constraint is

$$C_t + \dot{A}_t = w_t + (1 - \tau_t) r_t A_t.$$

How does this shock affect the $\dot{c} = 0$ and $\dot{k} = 0$ curves? What happens with consumption? Explain.

Question 2

A representative agent i maximizes utility

$$U_i = C_i - \frac{1}{\lambda} L_i^{\lambda},\ \lambda > 0$$

subject to the budget constraint

$$PC_i = P_i Y_i$$

where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i. The production function equals

$$Y_i = L_i$$

There is monopolistic competition in the goods market. The demand for good i is

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$$

where Y denotes aggregate output and $\eta > 1$. The aggregate demand equation is

$$Y = \frac{M}{P}$$

where M denotes nominal money. Agents have rational expectations. Employ the following notation: $x \equiv \ln X$.

a Derive the equilibrium level of (log) output y and show that the equilibrium (log) aggregate price level equals

$$p = \mu + m$$

where μ is a constant to be derived. [*Hint: assume that each producer charges the same price, and that the price index p equals this common price.*]

b Show that the desired (log) price level for good i equals

$$p_i^* = c + \phi m + (1 - \phi)\, p \tag{1}$$

where ϕ and c are constants to be derived.

c Without loss of generality, set $c = 0$ from now on. Suppose now that the price p_i is fixed for two periods and that price setting is staggered, such that half of prices $p_{i,t}$ are set in period t at level x_t, and the other half were set in period $t - 1$ at level x_{t-1}. Thus, the aggregate price level equals

$$p_t = \frac{1}{2}\left(x_t + x_{t-1}\right)$$

Suppose that the (log) money supply is a white noise. Assuming certainty equivalence (i.e., $x_t = \frac{1}{2}\left(p_{i,t}^* + \mathbf{E}_t\left[p_{i,t+1}^*\right]\right)$), show that the following difference equation can be obtained to define the dynamics of x_t:

$$x_t = \frac{2\phi}{1+\phi} m_t + \frac{1-\phi}{2\left(1+\phi\right)}\left(\mathbf{E}_t\left[x_{t+1}\right] + x_{t-1}\right).$$

d Guess a solution for x_t of the type $x_t = \beta x_{t-1} + \gamma m_t$, and find the equilibrium values of β and γ, conditional on setting $\lambda = 1$.