Written Exam for the B.Sc. in Economics Summer 2010, re-exam

Macro A

Final Exam

11 August, 2010

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Exercise 1

Consider the following economy:

$$Y_t = K_t^{\alpha}\left(A_t L_t\right)^{\beta} X_t^{\kappa}, \quad \alpha+\beta+\kappa=1$$

$$K_{t+1} = sY_t + (1-\delta)K_t, \qquad K_0 \text{ given}$$

$$L_{t+1} = \left(1+n\right)L_t, \quad L_0 \text{ given}$$

$$A_{t+1} = \left(1+g\right)A_t, \quad A_0 \text{ given}$$

Y_t is national output, K_t is the capital stock, A_t is a productivity parameter which grows at rate *g*, L_t is the population size which grows at rate *n*. *s* is the savings rate and δ is the depreciation rate.

a. Explain why a steady state in which K_t / L_t stays constant cannot exist if *g=0*.

b. Assume a balanced growth path exists in which the ratio $z_t = K_t / Y_t$ is constant. Use the production function to show that the growth rate of per-capita income $y_t = Y_t / L_t$ is

$$g_t^y \cong \frac{\beta g - \kappa n}{1-\alpha} = \frac{\beta g - \kappa n}{\beta + \kappa}.$$

Explain why the growth may eventually become negative.

The question now is whether the economy converges to a balanced growth path in which $z_t = K_t / Y_t$ stays constant. Consider that the law of motion of the ratio $z_t = K_t / Y_t$ is given by

$$z_{t+1} = \left(\frac{1}{\left(1+n\right)\left(1+g\right)}\right)^{\beta}\left[s+\left(1-\delta\right)z_t\right]^{1-\alpha} z_t^{\alpha}.$$

c. Show graphically whether the economy converges to a balanced growth path.

The following regression equation is frequently used to estimate the impact of the economies' structural characteristics (such as the savings rate and the population growth rate) and the availability of land per worker on GDP per worker. The GDP data is taken from the year 2000 and the land per worker data from the year 1994. The savings rate and the population growth rate are average annual values over the period 1960-2000.

$$\ln y_{00}^i \cong \gamma_0 + \gamma_1\left[\ln s^i - \ln\left(n^i + 0.075\right)\right] + \gamma_2 \ln\left(\frac{X^i}{L_{94}^i}\right).$$

The regression results are $\gamma_0 = 2.71, \gamma_1 = 1.35$, and $\gamma_2 = 0.38$.

d. Show analytically whether the sign of the estimated coefficients is compatible with a balanced growth path of the economy.

Exercise 2

Consider the following model of a two-sector economy:

$$Y_t = K_t^{\alpha}\left(A_t L_{Yt}\right)^{1-\alpha}$$
$$A_{t+1} - A_t = \rho A_t^{\phi} L_{At}^{\lambda}$$
$$K_{t+1} = sY_t + \left(1-\delta\right)K_t$$
$$L_{Yt} + L_{At} = L_t$$
$$L_{At} = s_R L_t.$$
$$L_t = L$$

The notation is as in exercise 1. s_R is the fraction of population which works in the research and development (R&D) sector of the economy. L_{Yt} is the number of workers in the final goods sector and the residual $L_{At} = L_t - L_{Yt}$ works in the R&D sector.

Assume that $\phi = \lambda = 1$.

a. Determine the growth rate of the productivity term A_t as a function of total population. Which feature does the growth rate exhibit? Explain whether this model can be classified as an endogenous growth model.

b. Derive the steady state of the economy. Write the law of motion for the capital intensity $\tilde{k}_t = K_t /(A_t L)$ and show graphically whether the economy converges to this steady state.

c. Use the capital accumulation equation and denote the growth rate of the productivity term A_t as g. Show that the steady state value of $\tilde{k}_t = K_t /(A_t L)$ is

$$\tilde{k}^* = \left(\frac{s}{g-\delta}\right)^{\frac{1}{1-\alpha}}(1-s_R).$$

d. How does the steady state level of $\tilde{k}_t$ relate to the fraction of the population which works in the R&D sector? Provide an economic explanation.

e. Now, assume the government wants to increase s_R. Explain verbally with the help of the steady state level of per-capita income $y_t = Y_t / L_t$ which effects the government has to balance when setting s_R so as to maximize y_t. A formal derivation of the optimal s_R is not necessary.

Now assume that $0 < \phi < 1$ and $\lambda = 1$.

f. Compute first the ratio of the growth rate of the productivity term A_t g_{t+1} / g_t and from there the law of motion for the growth rate. Does there exist a steady state growth rate, i.e. a situation in which $g_{t+1} / g_t = 1$? Illustrate your finding with the help of a figure.