Guideline to Answers in
Written Exam for the B.Sc. or M.Sc. in Economics autumn 2011-2012

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Final Exam

January 2012

Problem 1

Consider Tom who consumes bread (commodity 1) and beer (commodity 2) in non-negative quantities. Tom's preferences can be represented by the utility function $u(x_1,x_2) = v(x_1) + x_2$, with v being a twice differentiable function, $v'(x_1) > 0$, $v''(x_1) < 0$, and $v'(x_1) \to \infty$ for $x_1 \to 0$. Consider a certain consumption plan $(\underline{x}_1,\underline{x}_2)$, with both quantities being strictly positive.

- What happens to the marginal rate of substitution (MRS) if the quantity of bread is increased?
- What happens to MRS if the quantity of beer is increased?
- What happens to MRS if both quantities are doubled?
- Show that the interior solution to Tom's utility maximization problem, when the price system is p, and his income is I, is $x_1(p_1, p_2, I) = (v')^{-1}(p_1/p_2)$ and $x_2(p_1, p_2, I) = [I - p_1 \cdot x_1(p_1, p_2, I]/p_2$
- Is bread a normal good? Inferior? Giffen good?
- Is beer a normal good? Inferior?

Note: When answering the questions, please disregard corner solutions and focus on consumption plans with strictly positive quantities of bread and beer, i.e. interior solutions.

Answer: MRS falls in absolute value when bread quantity increases, but is constant as beer quantity increases (because of quasi-linearity); it decreases when both quantities are doubled. The expression for the interior solution to the consumer's utility maximization problem follows easily from the FOC $v'(x_1) = p_1/p_2$. Hence, bread is neither normal nor inferior, hence non-luxury, non-Giffen (no income effect). Beer is a normal good, not inferior.

Problem 2

Consider an Edgeworth exchange economy with private ownership. In the economy, there are two goods, good 1 being food, good 2 being housing. Consumer A has the initial endowment (2,8), i.e. two units of food and eight units of housing. Consumer B has the initial endowment (8,2). The preferences of A can be represented by the utility function $u_A(x_1,x_2) = x_1^3 \cdot x_2$, and similarly we have $u_B(x_1,x_2) = x_1 \cdot x_2^3$.

- Using housing as numeraire, i.e. letting p_2 be 1, find the Walrasian (competitive) equilibrium allocation for this economy and the corresponding equilibrium price for food.

Answer: The equilibrium price is 1, agent A consuming (7½,2½), B consuming (2½,7½), in equilibrium.

Problem 3

Consider a firm which has the production function $f(\ell) = \ln(1+\ell)$, where ℓ is the quantity of labor input. The price received for each unit of output is p, and w is the price of labor.

- Solve the firm's profit maximization problem
- Express the firm's supply of output, and its demand for labor, as functions of price and wage
- Express the maximum profit level as a function of price and wage

Answer: $\ell(p,w) = p/w - 1$, $y(p,w) = \ln(p/w)$, $\pi(p,w) = p\cdot\ln(p/w) - p + w$

Problem 4

Anthony and Beatrice both consume M different goods, and both have convex and differentiable preferences. At the price system p*, both Anthony and Beatrice consume a positive quantity of each of these M goods, although their consumption patterns naturally differ.

- Verify that it is not possible to re-allocate consumption goods between them in a way that makes at least one of them better off while not harming the other.

Answer: For every pair of goods, each of the consumers will have an MRS which is equal to the relative price, hence they will have identical MRS'es.

Problem 5

Discuss, as a micro-economist, the following statement which might be found in a journalistic article printed in the business pages of a newspaper:

- "It is absolutely obvious, and a universal truth, that in order to induce people to work more, taxes on income must be reduced, such that the after-tax wage is increased, because this will strengthen the incentive to work more".

Answer: The journalist has a point, alluding to what micro-economists would call the substitution effect, but completely overlooks the wealth effect, ref. the Slutsky equation in case of private ownership/endogenous income. The latter may mean that higher wages imply a greater demand for leisure, i.e. lower labor supply.

Problem 6

A firm, existing in a world of perfect competition, uses labor and capital to produce food, having the production function f.

- Define and comment on the concept of TRS (technical rate of substitution)
- If the producer chooses a production plan, with positive quantities of both production factors, for which TRS equals the relative factor prices, is this a necessary condition for profit maximization?
- Is it a sufficient condition?

Answer: TRS is the ratio between the marginal products, designating the slope of the isoquant curve. It is a necessary condition, as it is necessary to ensure cost minimization, which again is necessary to obtain maximum profits. It is, however, not sufficient, as the quantity of output in that production plan may not be at a level where MC = p.

Mtn, 13. jan. 2012