Eksamensopgave januar 2013, rettevejledning

Problem 1

A consumer, consuming two goods, both in strictly positive, and continuous quantities, has preferences which can be represented by the utility function $u(x_1,x_2) = x_1^{a} \cdot x_2^{(1-a)}$, where $0 < a < 1$.

- Show that the consumer's elasticity of substitution is 1

Answer: The absolute value of MRS is $[a \cdot x_2]/[(1-a) \cdot x_1]$, so when x_2/x_1 increases by 1 per cent, MRS will change by 1 per cent; hence the elasticity of substitution is 1.

Problem 2

- 2a) Define and describe the Hicksian compensated demand function for a consumer who has the strictly quasi-concave and monotonically increasing utility function u.
- 2b) For which purposes can the Hicksian demand function be used by economists?

Answer: With u being strictly quasi-concave, there exists a unique solution h(p,u) to the problem "Minimize $p \cdot x$ s.t. $u(x) \geq \underline{u}$". The Hicksian demand function h(p,u) identifies the consumption plan the consumer would choose at price system, if he or she were compensated in income to be able to achieve utility level $\underline{u}$. It is used to measure the substitution effect of price changes. It has the quality that "quantities move in the opposite direction of price changes". It is used to measure the substitution effect of price changes, and to measure welfare changes implied by price changes (integral; change in area behind the H-demand curve), which it does in a more appropriate way than consumer's surplus (when preferences are not quasi-linear).

Problem 3

Consider a Koopmans economy with one consumer whose 24 hours can be used as labor in the manufacturing unit producing a consumption good (good 2) or enjoyed as leisure (good 1). The manufacturing unit has the production function $x = l$, with l being the number of labor hours (input), and x being the output quantity of the consumption good. The consumer's consumption plan consists of leisure, f, and the consumption good.

- 3a) Find the efficient (Pareto Optimal) allocation(s) if the consumer has the utility function $u(f,x) = f \cdot x$, where f and x indicate the quantities of leisure and consumption good
- 3b) Find the efficient (Pareto Optimal) allocation(s) if the consumer has the utility function $u(f,x) = f$
- 3c) Find the efficient (Pareto Optimal) allocation(s) if the consumer has the utility function $u(f,x) = x$
- 3d) Find the efficient (Pareto Optimal) allocation(s) if the consumer has the utility function $u(f,x) = f + x$

Comment on the results found.

Answer: The allocations are (mentioning consumption plan first, then production plan):

- *(12,12), (12,12)*
- *(24,0), (0,0)*
- *(0,24), (24,24)*

- *All convex combinations of the latter two extreme cases*

In the first case, there is a trade-off between the two goods, and with the nice Cobb-Douglas preferences, the consumer ends up with a nicely balanced consumption plan. In the second case, the consumer appreciates only leisure, so no production takes place. In the third case, leisure is not appreciated at all, so all hours are spent working. In the fourth case, all possible allocations are efficient, as the highest possible indifference curve coincides with the graph of the production function, in the Koopmans diagram.

Problem 4:

Consider a consumer who has the utility function $u(x_1,x_2) = x_1^{½} + x_2$, has the exogenously given money income I and meets the market prices (p_1,p_2).

- 4a) Present the Lagrange problem corresponding to utility maximization, and solve the problem, hence finding the Marshall demand function (barring corner solutions and focusing solely on interior solutions).
- 4b) Present the Lagrange problem corresponding to expenditure minimization, and solve the problem, hence finding the Hicksian compensated demand function (barring corner solutions and focusing solely on interior solutions).

Answer: The two Lagrangian expressions are, respectively, $L_{umax}(x,\lambda) = x_1^{½} + x_2 - \lambda\cdot(p_1\cdot x_1 + p_2\cdot x_2 - I)$ *and* $L_{emin}(x,\mu) = p_1\cdot x_1 + p_2\cdot x_2 - \mu\cdot(x_1^{½} + x_2 - u)$. *FOC are* $[\partial u(x_1,x_2)/\partial x_1]/[\partial u(x_1,x_2)/\partial x_2] = p_1/p_2$ *and* $p_1\cdot x_1 + p_2\cdot x_2 - I = 0$, *and* $[\partial u(x_1,x_2)/\partial x_1]/[\partial u(x_1,x_2)/\partial x_2] = p_1/p_2$ *and* $x_1^{½} + x_2 - u = 0$. *The solutions are:*

$x_1(p,I) = [p_2^2/(4\cdot p_1^2)]$

$x_2(p,I) = [(I/p_2) - p_2/(4\cdot p_1)]$

$h_1(p,u) = [p_2^2/(4\cdot p_1^2)]$

$h_2(p,u) = [u - p_2/(2\cdot p_1)]$

Problem 5:

Explain and comment on the Second Welfare Theorem.

Answer: A good response reveals a clear understanding of the two concepts (efficiency vs. market equilibrium with transfers) and the difference between them. It explains how the theorem tells us that an efficient allocation can be implemented in a way conforming with perfect competition markets by finding the right price system and individual income levels. Furthermore, it illustrates how lack of convexity (for one or more consumers and/or producers) may make such an implementation impossible.

Problem 6:

Consider an Edgeworth economy with two consumers, Arnie and Bernie, having the utility functions, $u_A(x_{1A},x_{2A}) = x_{1A}^{a}\cdot x_{2A}^{(1-a)}$ and $u_B(x_{1B},x_{2B}) = x_{1B}^{b}\cdot x_{2B}^{(1-b)}$, with $0 < a,b < 1$.

The economy is characterized by private ownership, Arnie owning the initial endowment (e_{1A},e_{2A}) and Bernie owning (e_{1B},e_{2B}).

- 6a) Identify the Walrasian equilibrium, using good 2 as numeraire, find the equilibrium value for the price of good 1.
- 6b) Will the Walrasian equilibrium allocation be efficient (Pareto Optimal)?
- 6c) What happens with the equilibrium price, if e_{1A} increases? Is this intuitive?

Answer: Solving $a\cdot[p_1\cdot e_{1A} + e_{2A}]/p_1 + b\cdot[p_1\cdot e_{1B} + e_{2B}]/p_1 = e_{1A} + e_{1B}$ we obtain $p_1{}^* = [a\cdot e_{2A} + b\cdot e_{2B}]/[(1-a)\cdot e_{1A} + (1-b)\cdot e_{1B}]$. Yes, the equilibrium allocation will be efficient, ref. First Welfare Theorem. An increase of Arnie's initial endowment of good 1 will decrease the equilibrium price, as good 1 becomes less scarce.

Ref.: mtn 8. december 2012