Written Exam Economics Summer 2019

Econometrics II

Solution Guide

Part 1: Hand-In Assignment #2
Relationship Between Stock Prices and Economic Growth

The Case: The purpose of Part 1 is to analyze the relationship between the development in the real economy, measured by the growth rate of the gross domestic product (GDP), and the stock market.

The Data: Graphs of the data and relevant transformations must be shown in the exam. Some comments on the stationarity of the time series must be given. It could be noted that the time series for nominal and real stock market returns are almost identical.

Econometric Theory: The econometric theory must include the following:

(1) A precise definition and interpretation of the vector autoregressive model.
(2) A description of the estimator used, e.g. a discussion of the maximum likelihood (ML) principle.
(3) A description of the sufficient conditions for consistent estimation and valid inference.
(4) The theory must be presented precisely and in a logical order with a consistent and correct notation.

Empirical Results: The empirical results must include the following:

(1) A presentation and discussion of the relevant empirical results, so that the reader is able to understand the steps carried out in the process as well as the conclusions made.
(2) A description of the model selection process based on a general-to-specific approach, information criteria, or both.
(3) A discussion of whether the assumptions for consistent estimation and valid inference are satisfied for the estimated models. Specifically, this includes misspecification testing.
(4) A clear conclusion to the main question and a discussion of the limitations of the approach used to reach the conclusion. Specifically, the conclusion regarding the relationship between GDP growth and stock market returns.

Part 2: Hand-In Assignment #4
Macroeconomic News Announcements and International Exchange Rates

The Case: The goal of Part 2 is to analyze the impact of news announcements on the conditional mean and variance of the bilateral exchange rate between US dollar and the euro.

The Data: Graphs of the data and relevant transformations must be shown. The graphs should emphasize the presence of volatility clustering to motivate the ARCH modelling framework.

Econometric Theory: The econometric theory must include the following:

(1) A precise definition and interpretation of the models considered and their properties. Specifically, some variants of the ARCH or GARCH model must be presented and interpreted. The interpretation of included explanatory variables should be discussed. Some remarks on asymmetry could be given.
(2) A description of the estimator used, e.g. the maximum likelihood (ML) estimation principle. This part may reference to the ML theory in Part 1.
(3) A description of the sufficient conditions for consistent estimation and valid inference.
(4) The theory must be presented precisely and in a logical order with a consistent and correct notation.

Empirical Results: The empirical results must include the following:

(1) A presentation and discussion of the relevant empirical results, so that the reader is able to understand the steps carried out in the process as well as the conclusions made.
(2) A description of the model selection process based on a general-to-specific approach, information criteria, or both.
(3) A discussion of whether the assumptions for consistent estimation and valid inference are satisfied for the estimated models. Specifically, this includes misspecification testing.
(4) A clear conclusion to the main question and a discussion of the limitations of the approach used to reach the conclusion. Specifically, the conclusion regarding the impact of news announcements on the expected change and the volatility of the exchange rate.

Part 3
The Phillips Curve

#5.1 Theoretical Assignment

This part considers the expectation augmented Phillips curve,

$$\pi_t = \alpha_1 + \alpha_2 u_t + \alpha_3 \pi^e_{t+1|t} + \eta_t, \quad t = 1, 2, ..., T, \tag{5.1}$$

where π_t is the inflation rate measured as the change in consumer prices from the year before, u_t is the unemployment rate as a percentage of the labour force measuring the business cycle, and $\pi^e_{t+1|t}$ is the expected inflation in the next period, $t+1$, given the information available at the beginning of period t. Finally, η_t measures other things affecting inflation, and it is assumed to be i.i.d. with $E(\eta_t) = 0$, $E(\eta_t^2) = \sigma^2$, and uncorrelated with u_t and $\pi^e_{t+1|t}$.

(1) First, assume that we have a survey measure of expected inflation,

$$\pi^{\text{survey}}_{t+1|t} = \pi^e_{t+1|t} + v_t,$$

where v_t is a measurement error, uncorrelated with u_t and η_t.

(a) Assume that v_t is i.i.d.
The assumption implies that

$$\pi^e_{t+1|t} = \pi^{\text{survey}}_{t+1|t} - v_t,$$

such that model is given by

$$\begin{aligned} \pi_t &= \alpha_1 + \alpha_2 u_t + \alpha_3 (\pi^{\text{survey}}_{t+1|t} - v_t) + \eta_t \\ \pi_t &= \alpha_1 + \alpha_2 u_t + \alpha_3 \pi^{\text{survey}}_{t+1|t} + w_t, \end{aligned} \tag{5.2}$$

or

$$\pi_t = x_t'\beta + w_t$$

with a combined error term given by

$$w_t = \eta_t - \alpha_3 v_t,$$

and vectors

$$\beta = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \quad \text{and} \quad x_t = \begin{pmatrix} 1 \\ u_t \\ \pi^{\text{survey}}_{t+1|t} \end{pmatrix}.$$

The equation in (5.2) is a linear equation in observed variables.

(i) To use OLS to consistently estimate the parameters β, we need the moment condition

$$E\left(x_t w_t\right) = 0.$$

Under the given assumptions, this is violated, however, because w_t includes v_t, which is correlated with $\pi_{t+1|t}^{\text{survey}}$. In particular

$$\begin{aligned}
\operatorname{cov}(\pi_{t+1|t}^{\text{survey}}, w_t) &= \operatorname{cov}(\pi_{t+1|t}^{\text{survey}}, \eta_t - \alpha_3 v_t) \\
&= \operatorname{cov}(\pi_{t+1|t}^{e} + v_t, \eta_t - \alpha_3 v_t) \\
&= \operatorname{cov}(\pi_{t+1|t}^{e} \eta_t) - \alpha_3 \operatorname{cov}(\pi_{t+1|t}^{e} v_t) + \operatorname{cov}(v_t, \eta_t) - \alpha_3 V(v_t) \\
&= -\alpha_3 V(v_t),
\end{aligned}$$

because remaining covariances are zero.

We conclude that the OLS estimator is inconsistent, whenever $V(v_t) \neq 0$.

(ii) An valid instrument for $\pi_{t+1|t}^{\text{survey}}$ could be the lagged value, $\pi_{t|t-1}^{\text{survey}}$. When the measurement error is i.i.d. (such that v_t and v_{t-1} are uncorrelated), the new instrument, $\pi_{t|t-1}^{\text{survey}}$, is uncorrelated with w_t. In particular

$$\begin{aligned}
\operatorname{cov}(\pi_{t|t-1}^{\text{survey}}, w_t) &= \operatorname{cov}(\pi_{t|t-1}^{\text{survey}}, \eta_t - \alpha_3 v_t) \\
&= \operatorname{cov}(\pi_{t|t-1}^{e} + v_{t-1}, \eta_t - \alpha_3 v_t) \\
&= \operatorname{cov}(\pi_{t|t-1}^{e} \eta_t) - \alpha_3 \operatorname{cov}(\pi_{t|t-1}^{e} v_t) + \operatorname{cov}(v_{t-1}, \eta_t) - \alpha_3 \operatorname{cov}(v_{t-1}, v_t) \\
&= 0.
\end{aligned}$$

To derive an estimator, define the list of instruments

$$z_t = \begin{pmatrix} 1 \\ u_t \\ \pi_{t|t-1}^{\text{survey}} \end{pmatrix},$$

and the population moment conditions

$$E\left(z_t w_t\right) = E\left(z_t(\pi_t - x_t'\beta)\right) = 0.$$

The corresponding sample moments conditions are given by

$$\frac{1}{T}\sum_{t=1}^{T} z_t(\pi_t - x_t'\hat{\beta}) = 0,$$

such that the method of moments estimator (or instrumental variables estimator) is given by

$$\hat{\beta} = \left(\frac{1}{T}\sum_{t=1}^{T} z_t x_t'\right)^{-1} \frac{1}{T}\sum_{t=1}^{T} z_t \pi_t.$$

The good solution notes that this requires that the first term is invertible, i.e. that $\pi_{t|t-1}^{\text{survey}}$ is a valid instrument for $\pi_{t+1|t}^{\text{survey}}$. This requires that

$$\operatorname{cov}(\pi_{t+1|t}^{\text{survey}}, \pi_{t|t-1}^{\text{survey}}) \neq 0,$$

which would be violated if the survey measure is *not* serially correlated.

(b) Next, assume that v_t follows an MA(1) process,

$$v_t = \xi_t + \rho\xi_{t-1}, \quad \text{with} \quad \xi_t \text{ i.i.d.}$$

Now the model is

$$\pi_t = x_t'\beta + w_t$$

with

$$w_t = \eta_t - \alpha_3\xi_t - \alpha_3\rho\xi_{t-1}.$$

(i) Using the same line of arguments as above, the OLS estimator is not consistent.

(ii) In this case, the instrument $\pi^{\text{survey}}_{t|t-1}$ is not valid either, because

$$\begin{aligned}
\text{cov}(\pi^{\text{survey}}_{t|t-1}, w_t) &= \text{cov}(\pi^{\text{survey}}_{t|t-1}, \eta_t - \alpha_3\xi_t - \alpha_3\rho\xi_{t-1}) \\
&= \text{cov}(\pi^e_{t|t-1} + \eta_{t-1} - \alpha_3\xi_{t-1} - \alpha_3\rho\xi_{t-2}, \eta_t - \alpha_3\xi_t - \alpha_3\rho\xi_{t-1}) \\
&= \text{cov}(\alpha_3\xi_{t-1}, \alpha_3\rho\xi_{t-1}) \\
&= \alpha_3^2\rho V(\xi_{t-1}) \\
&\neq 0.
\end{aligned}$$

Instead, we may use $\pi^{\text{survey}}_{t-1|t-2}$ as an instrument. In particular

$$\begin{aligned}
\text{cov}(\pi^{\text{survey}}_{t-1|t-2}, w_t) &= \text{cov}(\pi^{\text{survey}}_{t|t-2}, \eta_t - \alpha_3\xi_t - \alpha_3\rho\xi_{t-1}) \\
&= \text{cov}(\pi^e_{t|t-2} + \eta_{t-2} - \alpha_3\xi_{t-2} - \alpha_3\rho\xi_{t-3}, \eta_t - \alpha_3\xi_t - \alpha_3\rho\xi_{t-1}) \\
&= 0,
\end{aligned}$$

because ξ_t is i.i.d. We may therefore use

$$z_t = \begin{pmatrix} 1 \\ u_t \\ \pi^{\text{survey}}_{t-1|t-2} \end{pmatrix}.$$

The good solution may note that for a weakly dependent time series, this instrument is typically weaker. The sample moment conditions is given by

$$\frac{1}{T}\sum_{t=1}^{T} z_t(\pi_t - x_t'\hat{\beta}) = 0.$$

(2) Next, consider the case of forward-looking and rational expectations,

$$\pi^e_{t+1|t} = E(\pi_{t+1} \mid \mathcal{I}_t), \tag{5.3}$$

where

$$\mathcal{I}_t = \{\pi_{t-1}, \pi_{t-2}, ...\pi_1, u_t, u_{t-1}, u_{t-2}, ..., .u_1\}, \tag{5.4}$$

denotes the information set available before the price-setting decision at time Given the information set $\mathcal{I}_t$, we may use the decomposition

$$\pi_{t+1} = E(\pi_{t+1} \mid \mathcal{I}_t) + v_t = \pi^e_{t+1|t} + v_t,$$

with $E(v_t \mid \mathcal{I}_t) = 0$. Inserting the observed value produces

$$\begin{aligned}\pi_t &= \alpha_1 + \alpha_2 u_t + \alpha_3 \pi^e_{t+1|t} + \eta_t \\ &= \alpha_1 + \alpha_2 u_t + \alpha_3(\pi_{t+1} - v_t) + \eta_t \\ &= \alpha_1 + \alpha_2 u_t + \alpha_3 \pi_{t+1} + w_t\end{aligned}$$

with the combined error term

$$w_t = \eta_t - \alpha_3 v_t.$$

(a) Using the arguments above, we cannot use OLS.
Instead, we introduce a set of $R \geq 3$ instruments, $z_t \in \mathcal{I}_t$. Under the model assumptions and rational expectations, it holds that

$$E(w_t \mid z_t) = E(\eta_t - \alpha_3 v_t \mid z_t) = 0.$$

This implies an unconditional population moment conditions

$$g(\theta) = E(z_t w_t) = 0,$$

with corresponding sample moments

$$g_T(\beta) = \frac{1}{T}\sum_{t=1}^{T} z_t w_t = \frac{1}{T}\sum_{t=1}^{T} z_t(\pi_t - x_t'\beta).$$

This is a system of R equations with $K = 3$ unknowns.

(i) If $R = K = 3$, we may use the MM estimator as above.
If $R > 3$, we define the criteria function

$$Q_T(\beta) = g_T(\beta)' W_T g_T(\beta),$$

where W_T is an $R \times R$ symmetric positive definite weight matrix. The GMM estimator is defined as the minimizer,

$$\hat{\beta}(W_T) = \arg\min g_T(\beta)' W_T g_T(\beta),$$

which depends on the weight matrix.
The optimal estimator is obtained for W_T^{opt}, with the property that

$$W_T^{\text{opt}} \xrightarrow{p} W^{\text{opt}} = S^{-1} \quad \text{with} \quad S = V(\sqrt{T} g_T(\beta)).$$

The good solution gives some intuition for this result.

(ii) To find the GMM estimator, we may rewrite using matrices

$$g_T(\beta) = \frac{1}{T}\sum_{t=1}^{T} z_t(\pi_t - x_t'\beta) = \frac{1}{T} Z' \left(\Pi - X\beta\right),$$

with stacked matrices

$$\underset{(T\times 1)}{\Pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_T \end{pmatrix}, \quad \underset{(T\times K)}{X} = \begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_T' \end{pmatrix}, \quad \text{and} \quad \underset{(T\times K)}{Z} = \begin{pmatrix} z_1' \\ z_2' \\ \vdots \\ z_T' \end{pmatrix}.$$

Then the GMM criteria function is

$$\begin{aligned} Q_T(\beta) &= g_T(\beta)' W_T g_T(\beta) \\ &= \left(T^{-1} Z' (\Pi - X\beta)\right)' W_T \left(T^{-1} Z' (\Pi - X\beta)\right) \\ &= T^{-2} \left(\Pi' Z W_T Z' \Pi - 2\beta' X' Z W_T Z' \Pi + \beta' X' Z W_T Z' X \beta\right). \end{aligned}$$

The first order condition for the GMM estimator is

$$\frac{\partial Q_T(\beta)}{\partial \beta} = -2T^{-2} X' Z W_T Z' \Pi + 2T^{-2} X' Z W_T Z' X \beta = 0$$

such that

$$\hat{\beta}(W_T) = \left(X' Z W_T Z' X\right)^{-1} X' Z W_T Z' \Pi.$$

The good solution may note that if we choose the i.i.d. estimator for S, the GMM estimator coincides with two-stage least squares.

#5.2 Empirical Assignment

(1) This part should estimate the forward-looking model using GMM based on monthly data for the US.
The solution should explain the overall approach and the choice of instruments.
There should be a presentation of the results in terms of statistical significance and also economic significance (e.g. expected signs).

(2) The solution should explain the test for the over-identifying restrictions. It should be noted, in particular, that it does *not* test the identifying restrictions but tests over-identification given that identification works. The null hypothesis is that all moment conditions contain information that point in the same direction.

(3) Finally, a hybrid-model of the form

$$\pi_t = \alpha_1 + \alpha_2 u_t + \alpha_3 E(\pi_{t+1} \mid \mathcal{I}_t) + \alpha_4 \pi_{t-1} + \eta_t,$$

should be estimated.
Again, the statistical significance and also economic significance should be discussed and it should be concluded whether the Phillips curve seems to be primarily forward looking or backward looking.