Written Exam for the B.Sc. or M.Sc. in Economics, Winter 2010/2011

Operations Research

Elective Course

January 21st 2011

3-hour open book exam

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Part 1

Consider the following LP model:

$$
\begin{array}{ll}
\text{Max} & z = 6\,x_1 + 1\,x_2 \\
\text{S.t.} & 1\,x_1 + 1\,x_2 \leq 3 \\
 & 3\,x_1 + 1\,x_2 \leq 6 \\
 & x_1, x_2 \geq 0
\end{array}
$$

<u>Question 1.1:</u>
Set up the model in a simplex tableau and perform one simplex-iteration.

<u>Question 1.2:</u>
Is the solution optimal? If it is optimal, perform a sensitivity analysis on both right-hand-side coefficients. If it is not optimal, perform one additional simplex-iteration. Comment on the results.

<u>Question 1.3:</u>
Set up the dual model to the LP model above.

<u>Question 1.4:</u>
Set up the dual model in a simplex tableau so that it can be solved directly with the dual simplex algorithm. Perform one dual simplex-iteration and comment on the results.

Part 2

Consider the following cost matrix for a minimum cost assignment problem:

4	3	1	2	2
5	3	2	2	2
2	2	5	3	3
4	3	1	2	2
4	3	1	2	2

<u>Question 2.1:</u>
Solve the assignment problem above using the Hungarian method.

<u>Question 2.2a:</u>
Remodel the assignment problem above as a transportation problem using less that 5 suppliers and less than 5 demanders.

<u>Question 2.2b:</u>
It is often described (for instance by Winston) how Vogels Method for finding a basic feasible solution to the transportation problem typically finds a better solution than the Northwest Corner Method. But can it also be worse? Explain.

Part 3

Mr. Frenzy arrives in Las Vegas with the intention to see as many shows as possible within the next 48 hours. From the general program he creates a list of shows which start and end within the next 48 hours, and he notes the start and end times of each show. In total, there are n shows and we assume that all shows start and end at the hour so show j starts at the beginning of hour $s(j)$ and ends at the end of hour $e(j)$, with all $s(j)$ and $e(j)$ being integers.

We require that a selected show must be seen in its entirety and that two timely overlapping shows cannot both be selected.

<u>Question 3.1</u>
Formulate a dynamic programming model which will determine the maximum number of shows that Mr. Frenzy can see.
(*Hint: This is most easily done without a state-variable*)

<u>Question 3.2</u>
Mr. Frenzy realises that he is in fact on a budget. Show number j costs $w(j)$ and he can spend no more than a total of W on shows. Reformulate the model of Question 3.1 to encompass this budget constraint, but still maximize the number of shows Mr. Frenzy can see.

Part 4

An energy company wishes to sell the power its offshore wind farm produces at the market price before the power is actually produced. Of course, the exact production for a given hour in the future is not known exactly but it can be described by the stochastic variable $\boldsymbol{D}$.

The production, $\boldsymbol{D}$, in a given hour ahead is estimated to be uniform distributed with support in the interval $[a, b]$ given the current weather forecast. The cumulative distribution function of the production is therefore:

$$P\{\boldsymbol{D} \leq d\} = (d-a)/(b-a) \text{ for } a \leq d \leq b$$

(outside this interval we obviously have that $P\{\boldsymbol{D} \leq d\} = 0$ for $d < a$ and $P\{\boldsymbol{D} \leq d\} = 1$ for $d > b$)

If the company sells too much power in advance they will need to buy back the difference. This will cost them the market price plus a penalty of 5 Euro per MWh.

If, on the other hand, they sell too little power they can sell the rest at a discount (loss) of 2 Euro per MWh compared to the market price.

<u>Question 4.1:</u>

Given a and b, how much power should the company sell at market price?

The energy company also needs to repair the wind mills when they break down. They have one repair ship for this, and the ship can work on only one wind mill at the time. It can be assumed that the repair time is exponentially distributed with an average repair time of 3 days. The transportation time between wind mills is negligible. The waiting-time until the next breakdown of a mill in the wind farm is likewise exponentially distributed with an average of 20 breakdown per year.

<u>Question 4.2:</u>

How many wind mills can be expected to be non-producing after break-down at any given moment in time due to repair or waiting for the repair ship to arrive?