Macro C – Reexamination 20 February 2012

Academic aim
At the end of the course, the student should be able to demonstrate:
Understanding of the main model frameworks for long-run macroeconomics. This includes the Diamond model with overlapping generations in discrete time and the Ramsey model in continuous time.

Proficiency in the application of the concepts and methods from these frameworks, including competence in dynamic optimization and dynamic analysis in discrete and continuous time.
Understanding of the role of expectations and basic knowledge of macroeconomic models with forwardlooking expectations under both perfect foresight and uncertainty and rational expectations.
Proficiency in the application of the related concepts and methods.
Competence in analyzing a macroeconomic problem, where the above-mentioned concepts and methods are central, that is competence in solving such models and explaining in economic terms the results and implications and how they derive from the assumptions of the model.

The particularly good performance, corresponding to the top mark, is characterized by a complete fulfilment of these learning objectives.

Problem A: Time inconsistency of monetary policy and delegation

1)

We see from equation (A.1) that if $\pi_t = \pi_t^e = \pi^*$ then $y_t = \bar{y}$. Intuitively, output gap will equal zero if there isn't any surprise inflation (and no supply shocks), which follows from the expectation-augmented Philips curve.

This equilibrium isn't time consistent, which can be showed the following way:

Start by expressing output gap as a function of the inflation rate from equation (A.1):

$$y_t - \bar{y} = \frac{\pi_t - \pi_t^e}{\gamma}$$

Insert this in the social loss function (along with equation (A.3)):

$$SL_t = \left(\frac{\pi_t - \pi_t^e}{\gamma} - \omega\right)^2 + \kappa \cdot (\pi_t - \pi^*)^2$$

The derivative of the social loss function with respect to the inflation rate is given by:

$$\frac{\partial SL_t}{\partial \pi_t} = 2 \cdot \frac{1}{\gamma} \cdot \left(\frac{\pi_t - \pi_t^e}{\gamma} - \omega\right) + 2 \cdot \kappa \cdot (\pi_t - \pi^*)$$

Evaluate this in the initial equilibrium (where $\pi_t = \pi_t^e = \pi^*$ and $y_t = \bar{y}$):

$$\frac{\partial SL_t}{\partial \pi_t} = -2 \cdot \frac{\omega}{\gamma} < 0$$

Since this derivative is negative the government can reduce the social loss by deviating from the announced policy and create surprise inflation. The reason is that initially the inflation rate equals the socially optimal inflation rate (π^*) while output is below the socially optimal level (since by assumption $\bar{y} < y^*$). Thus, starting out from the initial equilibrium an increase in the inflation rate is (to a first order approximation) costless while the increase in output constitutes a first order welfare gain.

2) Setting the derivative found in question 1) equal to zero we can characterize the optimal inflation rate taken the expected inflation as given:

$$\frac{\partial SL_t}{\partial \pi_t} = 2 \cdot \frac{1}{\gamma} \cdot \left(\frac{\pi_t - \pi_t^e}{\gamma} - \omega\right) + 2 \cdot \kappa \cdot (\pi_t - \pi^*) = 0$$

Now, impose the perfect foresight condition ($\pi_t^e = \pi_t$):

$$\frac{1}{\gamma} \cdot (-\omega) + \kappa \cdot (\pi_t - \pi^*) = 0 \Rightarrow$$

$$\pi_t = \pi^* + \frac{\omega}{\kappa \cdot \gamma}$$

which also equals the expected inflation rate (due to perfect foresight). Finally, notice that since $\pi_t^e = \pi_t$ we once again get from (A.1) that $y_t = \bar{y}$.

3)

As showed above the time-consistent equilibrium is characterized by the following:

$$y_t = \bar{y}$$

$$\pi_t = \pi_t^e = \pi^* + \frac{\omega}{\kappa \cdot \gamma}$$

Since $\pi_t > \pi^*$ we see that equilibrium inflation rate exceeds the target inflation rate.

The reason for this inflation bias is that agents in the private sector recognize the basic incentive for the central bank to deviate from the announced policy (of implementing π^*) and create surprise inflation. Thus, it isn't credible that the central bank will implement the target inflation rate. Instead the agents in the private sector raise their inflation expectations to such an extent, that the marginal cost of increasing inflation for the central bank is just equal to the marginal benefit with inflation equal to expected inflation.

The time-consistent equilibrium is showed in the figure below. The iso-welfare curves represent combinations of y_t and π_t which ensure a constant social loss. Mathematically, they can be expressed the following way:

$$\pi_t = \pi^* \pm \sqrt{\frac{\overline{SL_t} - (y_t - y^*)^2}{\kappa}}$$

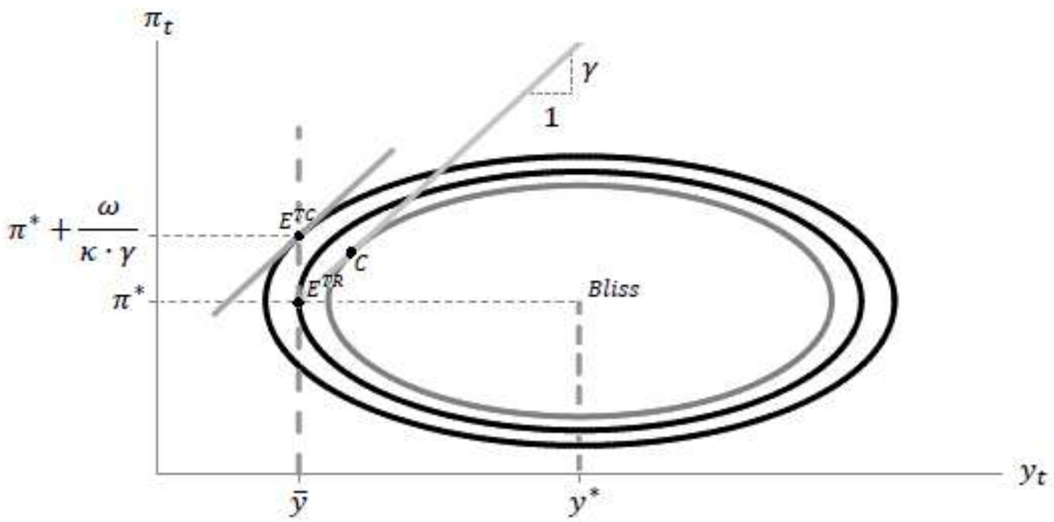
Iso-welfare curves closer to the bliss point (the first best outcome where $y_t = y^*$ and $\pi_t = \pi^*$) are consistent with a lower social loss. Starting out from the Taylor rule equilibrium, E^{TR}, (the second best outcome where $y_t = \bar{y}$ and $\pi_t^e = \pi_t = \pi^*$) the policymaker has an incentive to create surprise inflation and move the economy from E^{TR} to C, where the SRAS curve with $\pi_t^e = \pi^*$ is just tangent to an iso-welfare curve (the grey). However, agents in the private sector recognize the incentive to create surprise inflation and thus raise their inflation expectations, such that:

$$\pi_t^e = \pi^* + \frac{\omega}{\kappa \cdot \gamma}$$

The perfect foresight equilibrium (the time consistent equilibrium) can be found where the new SRAS curve, i.e. the line given by:

$$\pi_t = \pi^* + \frac{\omega}{\kappa \cdot \gamma} + \gamma \cdot (y_t - \bar{y})$$

is just tangent to an iso-welfare curve and where $y_t = \bar{y}$ (since this is a requirement for a perfect foresight equilibrium according to equation A.1). This time consistent equilibrium is the third best outcome and is denoted by E^{TC}. Social loss in the time-consistent equilibrium is higher than in the Taylor rule equilibrium, due to the higher inflation rate. Thus, the policymaker would prefer to end up in the Taylor rule equilibrium but as showed above that isn't a time consistent equilibrium.



Also we see from the expression for the equilibrium inflation rate that the inflation bias vanishes in two limiting cases:

- When $\omega \to 0$, i.e. when the socially optimal output level equals the natural output level (no supply side distortions or political pressure for higher employment). In this case there isn't any gain from trying to create surprise inflation, since the economy is initially in the first best outcome.
- When $\kappa \to \infty$, i.e. when the policy maker only cares about stabilizing inflation around the target inflation rate.

4)

The social loss of the combined policy maker can be found by calculating the weighted loss function given by:

$$SL_t^* = (1-\beta) \cdot SL_t + \beta \cdot SL_t^{cb}$$

Inserting the expression for the social loss (equation (A.2)) and the loss function of the central bank we get:

$$SL_t^* = (1-\beta) \cdot \left(\overbrace{(y_t - \bar{y} - \omega)^2 + \kappa \cdot (\pi_t - \pi^*)^2}^{SL_t} \right) + \beta \cdot \left(\overbrace{(y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2}^{SL_t^{cb}} \right) =$$

$$(1-\beta) \cdot ((y_t - \bar{y})^2 + \omega^2 - 2 \cdot \omega \cdot (y_t - \bar{y}) + \kappa \cdot (\pi_t - \pi^*)^2) +$$

$$\beta \cdot ((y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2) =$$

$$(y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2 + (1-\beta) \cdot \omega \cdot \left(\omega - 2 \cdot (y_t - \bar{y})\right)$$

When $\beta = 0$ we get:

$$SL_t^* = (y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2 + \omega^2 - 2 \cdot \omega \cdot (y_t - \bar{y}) =$$

$$(y_t - \bar{y} - \omega)^2 + \kappa \cdot (\pi_t - \pi^*)^2 = SL_t$$

which corresponds to the case analyzed above, where no authority is delegated to the independent central bank.

Conversely, when $\beta = 1$ we get:

$$SL_t^* = (y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2 = SL_t^{cb}$$

which corresponds to the case where all authority is delegated to the central bank. In this case the 'combined policy maker' will want to stabilize output around the natural output level, and the inflation bias will vanish.

5)

Inserting the expression for the output gap found above in the expression for the social loss of the combined policy maker we get:

$$SL_t^* = \left(\overbrace{\frac{\pi_t - \pi_t^e}{\gamma}}^{y_t - \bar{y}}\right)^2 + \kappa \cdot (\pi_t - \pi^*)^2 + (1-\beta) \cdot \omega \cdot \left(\omega - 2 \cdot \left(\overbrace{\frac{\pi_t - \pi_t^e}{\gamma}}^{y_t - \bar{y}}\right)\right)$$

The first order condition for the optimal choice of π_t is given by:

$$\frac{1}{\gamma} \cdot \frac{\pi_t - \pi_t^e}{\gamma} + \kappa \cdot (\pi_t - \pi^*) - (1-\beta) \cdot \omega \cdot \frac{1}{\gamma} = 0$$

Imposing perfect foresight ($\pi_t^e = \pi_t$) we get:

$$\pi_t = \pi^* + (1-\beta) \cdot \frac{\omega}{\kappa \cdot \gamma}$$

Once again $y_t = \bar{y}$, which follows from the fact that $\pi_t^e = \pi_t$ and equation (A.1).

Notice that the inflation bias is now given by:

$$\pi_t - \pi^* = (1-\beta) \cdot \frac{\omega}{\kappa \cdot \gamma}$$

We see that a higher value of β reduces the inflation bias. The reason is that more delegation implies that the incentive (for the combined policy maker) to create surprise inflation is weakened. Also, we should notice that the equilibrium value of y_t (corresponding to a zero output gap) is independent of the degree of commitment. Thus, the value of β which minimizes the social loss function is clearly given by $\beta = 1$, since this corresponds to a zero inflation bias. This is the case of *complete delegation*.

As discussed in chapter 22 in *Introducing Advanced Macroeconomics: Growth and Business Cycles* (Sørensen and Whitta Jacobsen 2010) this result isn't robust. In particular, when supply shocks are introduced into the model an increase in the amount of delegation will also have a cost since stabilization policy is distorted. With a positive amount delegation output will respond too much to supply shocks compared to what is socially optimal. Thus, there is a trade-off between credibility and flexibility and delegating responsibility for the conduct of monetary policy to an independent central bank with preferences with doesn't match the preferences of the public isn't a 'free lunch'.

Problem B: A public sector in the Diamond model

1)

The problem of each young household at time t is to choose c_{1t} (consumption as young) and c_{2t+1} (consumption as old) so as to maximize the intertemporal utility function:

$$U_t = lnc_{1t} + \frac{1}{1+\rho} \cdot lnc_{2t+1}$$

subject to the intertemporal budget constraint:

$$c_{1t} + \frac{1}{1 + r_{t+1} \cdot (1-\tau)} \cdot c_{2t+1} = w_t - T$$

The Langrarian is given by:

$$\mathcal{L} = lnc_{1t} + \frac{1}{1+\rho} \cdot lnc_{2t+1} - \lambda \cdot \left[c_{1t} + \frac{1}{1 + r_{t+1} \cdot (1-\tau)} \cdot c_{2t+1} - w_t + T\right]$$

The first order conditions with respect to c_{1t} and c_{2t+1} are given by:

$$\frac{\partial \mathcal{L}}{\partial c_{1t}} = \frac{1}{c_{1t}} - \lambda = 0 \Rightarrow \frac{1}{c_{1t}} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial c_{2t+1}} = \frac{1}{1+\rho} \cdot \frac{1}{c_{2t+1}} - \lambda \cdot \frac{1}{1 + r_{t+1} \cdot (1-\tau)} = 0 \Rightarrow \frac{1}{1+\rho} \cdot \frac{1}{c_{2t+1}} = \lambda \cdot \frac{1}{1 + r_{t+1} \cdot (1-\tau)}$$

Combining these two first order conditions we get that:

$$\frac{c_{2t+1}}{c_{1t}} = \frac{1 + r_{t+1} \cdot (1-\tau)}{1+\rho}$$

which is the discrete time version of the Keynes Ramsey rule. Combining the Keynes Ramsey rule with the intertemporal budget constraint we can derive the optimal consumption level as young.

$$c_{1t} + \frac{1}{1 + r_{t+1} \cdot (1-\tau)} \cdot \frac{1 + r_{t+1} \cdot (1-\tau)}{1+\rho} \cdot c_{1t} = w_t - T \Rightarrow$$

$$c_{1t} + c_{1t} \cdot \frac{1}{1+\rho} = w_t - T_t \Rightarrow c_{1t} = \frac{1+\rho}{2+\rho} \cdot (w_t - T)$$

Finally, we can derive saving of each young household from equation (B.2):

$$s_t = w_t - T - c_{1t} = w_t - T - \frac{1+\rho}{2+\rho} \cdot (w_t - T) = \frac{1}{2+\rho} \cdot (w_t - T)$$

The Keynes Ramsey rule states that consumption will grow over time whenever the after-tax interest rate (which can be interpreted as the return to postponing consumption) exceeds the rate of impatience (which can be interpreted as the cost of postponing consumption).

Equation (B.5) states that the optimal consumption pattern over time implies that consumption in the young age is a constant fraction of after-tax lifetime earnings. The (average and marginal) propensity to consume out of lifetime earnings is independent of the after-tax interest rate since utility is assumed to be logarithmic.

2)

Using the definition of k_t and inserting the expression for K_{t+1} along with expression for the growth of the labour force we get:

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{L_t \cdot s_t}{L_t \cdot (1+n)} = \frac{s_t}{1+n}$$

Now insert the expression for the saving of each young household derived above:

$$k_{t+1} = \frac{s_t}{1+n} = \frac{\frac{1}{2+\rho} \cdot (w_t - T)}{1+n} = \frac{w_t - T}{(2+\rho) \cdot (1+n)}$$

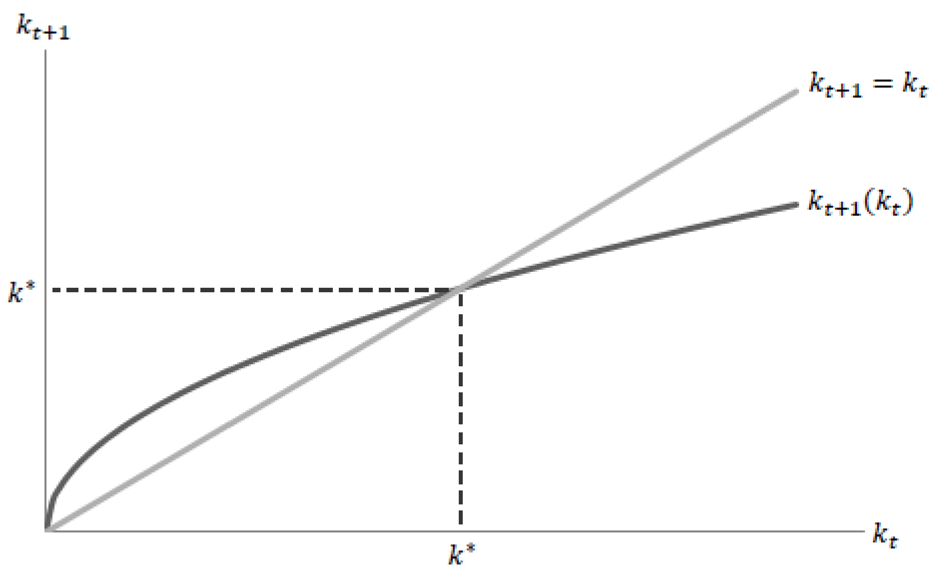
Finally insert the expression for the wage rate:

$$k_{t+1} = \frac{\overbrace{(1-\alpha) \cdot k_t^{\alpha}}^{w_t} - T}{(2+\rho) \cdot (1+n)}$$

which is the transition curve determining k_{t+1} uniquely as function of k_t (for a given value of T).

The positive relationship between k_t and k_{t+1} can be explained the following way: An increase in k_t will increase w_t (since the marginal product of labor, and thereby the wage rate, depends positively on the amount of capital in the economy) and thereby lifetime earnings of the young households. Each young household responds by increasing consumption as young, but by less than the increase in the wage rate (due to basic desire to smooth out consumption). Thus, saving of each young household increase, which increases capital accumulation. The relationship is concave due to the assumption of diminishing returns to capital.

The transition curve is illustrated below (in the case where $T = 0$):



3)

When $T = 0$ the transition curve is given by:

$$k_{t+1} = \frac{(1-\alpha)\cdot k_t^\alpha}{(2+\rho)\cdot(1+n)}$$

The steady state is by definition characterized by k_t being constant (as illustrated above), i.e.:

$$k_{t+1} = k_t = k^* \Rightarrow$$

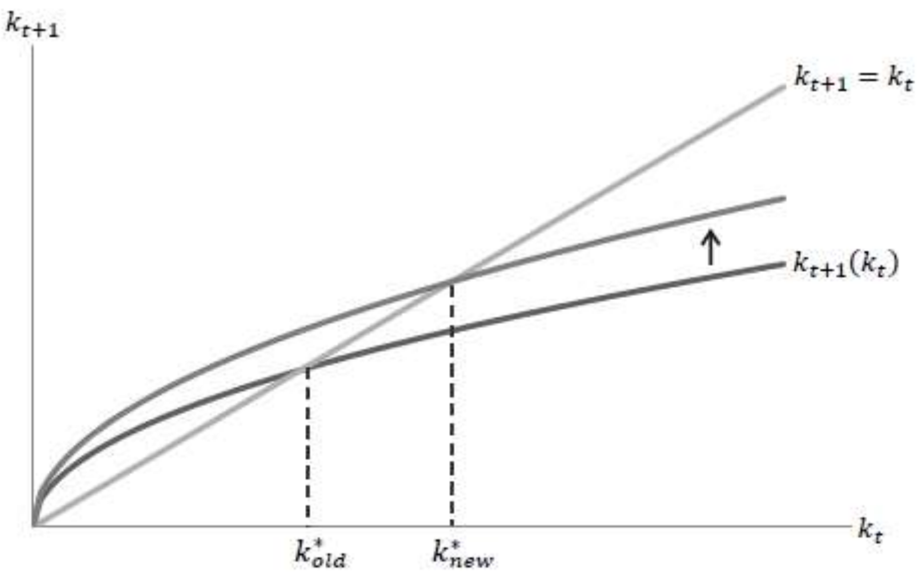
$$k^* = \frac{(1-\alpha)\cdot(k^*)^\alpha}{(2+\rho)\cdot(1+n)} \Rightarrow (k^*)^{1-\alpha} = \frac{1-\alpha}{(2+\rho)\cdot(1+n)} \Rightarrow$$

$$k^* = \left(\frac{1-\alpha}{(2+\rho)\cdot(1+n)}\right)^{1/(1-\alpha)}$$

Using that $y_t = k_t^\alpha$ we further get:

$$y^* = \left(\frac{1-\alpha}{(2+\rho)\cdot(1+n)}\right)^{\alpha/(1-\alpha)}$$

A fall in ρ

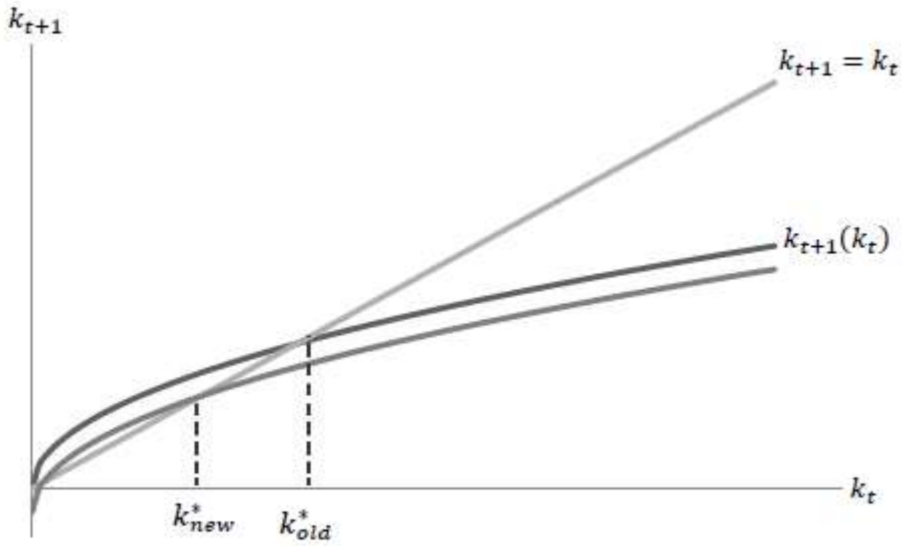


A fall in ρ implies that each young household becomes more patient, and thus want to consume more in their old age. Thus, saving of each young household increase, which increases the capital stock in the next period. The higher capital stock will increase the wage rate and as explained above this will induce the young generation to increase first period consumption by less than the increase in wage rate whereby saving of each young household increases, which further increases capital accumulation and so on. This mechanism dies out over time due to diminishing returns to capital, and the economy ends up in the new steady state (k^*_{new}) with a higher capital intensity.

4)

The consequences of an increase in T (with the revenue being used for financing public consumption) are showed in the figure below. The intuition is as follows:

An increase in T decreases the (after-tax) lifetime earnings of each young household. Due to the desire for consumption smoothing each young household respond by decreasing consumption in both periods. Thus, consumption in the young period decreases by *less* than the increase in T and consequently saving of each young household (given by $s_t = w_t - T - c_{1t}$) decreases. The lower amount of saving implies that the capital stock (per worker) decreases, which further decreases the wage rate and thereby depresses capital accumulation further. Once again these effects die out over time (due to diminishing returns to capital) and the economy ends up in the new steady state (k^*_{new}) with a lower capital intensity.



5)

From equation (B.6) and the transition curve it follows that an increase in τ doesn't affect saving behavior or capital accumulation. The reason for this (somewhat paradoxical) result is due to the assumption of log-utility, which implies that the substitution and the income effects exactly cancel each other out. These two effects are explained below:

Substitution effect: An increase in τ reduces the after-tax interest rate, i.e. the after-tax return to saving. Thus, saving becomes less attractive and consumption tends to increase.

The income effect: The lower after-tax interest rate also implies that the young household must save more in order to obtain a given income level in the old period. This tends to increase saving by the young generation and thereby decrease consumption.

In the log-utility case these two effects will cancel out. This will however not be the case with a more general utility function, e.g. the CIES-utility function:

$$U(c) = \frac{c^{1-\theta} - 1}{1 - \theta}$$

where log-utility represents the special case with $\theta = 1$.