Written Exam for the B.Sc. or M.Sc. in Economics, Winter 2011/2012

Operations Research

Elective Course

January 24th 2012

3-hour open book exam

CORRECTION GUIDE
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RETTEVEJLEDNING

Part 1

Question 1.1:

We have to find the Economic Order Quantity (EOQ) in the case with stochastic demand, lead time and backorders.

The following parameters are knows:
K = \$75 (order cost)
h = \$4 * 20% = \$0.80 (yearly stocking cost per item)
L = 3/ 52 (Lead time)
D – yearly demand – is N(12*52; 16*52) = N(624; 832)
X – demand during leadtime – is N(12*3; 16*3)=N(36; 48)

Now we can calculate the Wilson q – the economic order quantity:
q* = sqrt(2*K*E(D)/h)=sqrt(2*75*624/0.80=342

and r* is found where P{X>=r*}=h*q/(cB*E(D)).

But now we notice that no backorder cost, cB, has been given. We can therefore not say <u>when</u> the order should be places, only that every time an order is placed, the quantity should be 342 bottles.

Question 1.2:
Here we choose an arbitrary ordering policy, namely with q*=342 and r*=50

Since SLM2 is the expected number of cycles where we have to take backorders, we have that
SLM2 = P{X>r}*E(D)/q = (1-Φ((36-50)/sqrt(48))*624/342 =
Φ(-2.02)*624/342 = 0.0216*624/342 = 4%.

Part 2

Question 2.1:

The operational researcher is presented with an LP model which seems to have at least 5 decision variables (for the 5 products which are produced) and only 3 constraints. An LP model will always have as many variables (regular variables or slack variables) in basis as there are constraints. Since 5 products are produced (are above their lower bound) this indicates that the current production plan probably isn't optimal.
In the unlikely (but possible) case where the LP model has multiple optimal solutions, the solution found could indeed be optimal if the solution is one of the multitude of optimal solutions.

Question 2.2:

Now the operational researcher receives the additional information that there in fact exists an upper bound on each decision variable, and that this upper bound is reached. In the typical case he will therefore be able to conclude that the upper bounds are binding and that the 3 original constraints are not binding.

However, special cases can be set up where one or more of the other restrictions are binding as well. In this case the LP model will be degenerated.

Part 3

Question 3.1:

First we note that we have a maximization problem instead of the typical minization problem in the transportation model. We then note that the transportation problem is balanced.

One method of dealing with this is to transform the matrix to use for minimazation by replacing each profit $c_{i,j}$ with K-$c_{i,j}$.

Since we produce 120 in total in all feasible solutions, the real profit is then 120*K minus the new objective function value.

Here we choose K=42 (the largest profit element) and get the following "cost" matrix:

1	15	14	18
2	13	0	19
5	12	15	21

A first basic feasible solutions is found using the VAM:

20	0	0	30
0	10	30	10
0	20	0	0

Using the transportation simplex, we calculate the dual variables for the rows and columns and then the reduced costs for all non-basic cells. And we find that this initial solution is in fact optimal. However, the reduced cost for $x_{2,1}$ is zero and it can therefore alternatively be entered into basis. In that case $x_{2,4}$ must exit. Then we have the following solution:

10	0	0	40
10	10	30	0
0	20	0	0

The two alternative optimal solutions have the total cost of 120*42-1120=3920.

Question 3.2:

When it is no longer possible for any factory to produce more than one product, the problem can be formulated as an maximum assignment problem. Here we need to invent a dummy factory.

The first task is to determine how many units that can be produced in each assignment of a product to a factory. That is the minimum of the factory capacity and the sales of products:

20	30	30	40
20	30	30	40
20	20	20	20
20	30	30	40

The 4^{th} row is the dummy row.

We then multiply these quantities with the profits of each combination of factory and product to get the profit matrix for the maximum assignment:

820	810	840	960
800	870	1260	920
740	600	540	420
0	0	0	0

The maximum in this matrix is 1260 and we transform the problem to a minimum weight assignment problem by using the costs 1260 minus the profit:

440	450	420	300
460	390	0	340
520	660	720	840
1260	1260	1260	1260

Here we first subtract the minimum of each row for the row and then the minimum of each column from the column and we get the follow matrix:

140	150	120	0
460	390	0	340
0	140	200	320
0	0	0	0

A 0-assignment is easily located and we see that Factory 1 must produce Product 4, Factory 2 must produce Product 3, Factory 3 must produce Product 1 and the dummy produces Product 2. So Product 2 is the product not produced.
The total profit is 960+1260+740=2960.

We notice, that this quite a bit less than the profit of 3920 in question 1 and that it therefore must likely is desireable for the company to continue with the more complicated production plan.

Part 4

Question 4.1:

We define $g_t(d)$ as the grade the student gets from allocating *d* days for exam t. $g_t(d)$ is given in the table.

We can then write the recursion formula expressing the grade sum from exam *t* up to exam *T*(=4) when there are *i* days left for preparation, as follows:

$f_t(i) = \max\{g_t(j)+f_{t+1}(i-j)\}$, s.t. $1 <= j <= i + t - 4$

$f_{t=T=4}(i) = g_{t=4}(i)$ indicating no benefit of unused days. $f_1(7)$ is to optimal grade sum.

In this formular *j* is the number of days allocated to exam number *t*. This is upper bound because we cannot use so many days that there are not at least one day left for each of the rest of the exams.

The optimality principle says in this case that the number of days allocated to one of earlier exams is irrelevant in determining how to allocate the remaining days for the remaining exams.

Question 4.2:

t=4:
$f_4(1)=6$
$f_4(2)=7$
$f_4(3)=9$
$f_4(4)=9$

t=3:

$$f_3(2)=\max \{ g_3(1) + f_4(1) = 2 + 6 = 8 \} = 8$$

$$f_3(3)=\max \begin{cases} g_3(1) + f_4(2) = 2 + 7 = 9 \\ g_3(2) + f_4(1) = 4 + 6 = 10 \end{cases} = 10$$

$$f_3(4)=\max \begin{cases} g_3(1) + f_4(3) = 2 + 9 = 11 \\ g_3(2) + f_4(2) = 4 + 7 = 11 \\ g_3(3) + f_4(1) = 7 + 6 = 13 \end{cases} = 13$$

$$f_3(5)=\max \begin{cases} g_3(1) + f_4(4) = 2 + 9 = 11 \\ g_3(2) + f_4(3) = 4 + 9 = 13 \\ g_3(3) + f_4(2) = 7 + 7 = 14 \\ g_3(4) + f_4(1) = 8 + 6 = 14 \end{cases} = 14$$

t=2:

$$f_2(3)=\max \{ g_2(1) + f_3(2) = 5 + 8 = 13 \} = 13$$

$$f_2(4)=\max \begin{cases} g_2(1) + f_3(3) = 5 + 10 = 15 \\ g_2(2) + f_3(2) = 5 + 8 = 13 \end{cases} = 15$$

$$f_2(5)=\max \begin{cases} g_2(1) + f_3(4) = 5 + 13 = 18 \\ g_2(2) + f_3(3) = 5 + 10 = 15 \\ g_2(3) + f_3(2) = 6 + 8 = 14 \end{cases} = 18$$

$$f_2(6)=\max \begin{cases} g_2(1) + f_3(5) = 5 + 14 = 19 \\ g_2(2) + f_3(4) = 5 + 13 = 18 \\ g_2(3) + f_3(3) = 6 + 10 = 16 \\ g_2(4) + f_3(2) = 9 + 8 = 17 \end{cases} = 19$$

t=1:

$$f_1(7)=\max \begin{cases} g_1(1) + f_2(6) = 3 + 19 = 22 \\ g_1(2) + f_2(5) = 5 + 18 = 23 \\ g_1(3) + f_2(4) = 6 + 15 = 21 \\ g_1(4) + f_2(3) = 7 + 13 = 20 \end{cases} = 23$$

The optimal grade sum of 23 is attainable by using 2 days for the first exam, 1 day for the second exam, 3 days for the third exam and 1 day for the fourth exam.