Written Exam - Macroeconomics III
(suggested answers)

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Question 1

Note that in model per capita and per household quantities are the same ($C = c$, $A = a$)

a The wage and interest rate are determined by imposing equilibrium in factor markets where firms competitively demand labor and capital from households. Thus, the student needs to maximize profit function for firms

$$\max_{L_t, K_t} K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t$$

From the FOCs of firms' problem of maximizing profits we get

$$\begin{aligned} (1-\alpha) K_t^{\alpha} L_t^{-\alpha} &= (1-\alpha) k_t^{\alpha} = w_t \\ \alpha K_t^{\alpha-1} L_t^{1-\alpha} &= \alpha k_t^{\alpha-1} = r_t \end{aligned}$$

where we impose that in equilibrium all firms work with the same capital labor ratio, k_t, which must be equal to the ratio of aggregate capital to labor. Market wage, $(1-\alpha)k_t^{\alpha}$, and interest rate r_t *times* saving per capita as payments to households. Thus household income on saving is $k_t r_t = \alpha k_t^{\alpha}$.

b The no-Ponzi game condition is

$$\lim_{t\to\infty} e^{-\int_0^t r_s ds} A_t \geq 0$$

The intuition is that this rules out schemes in which one household issues debt and rolls it over forever. The condition allows for debt, but total debt cannot increase at a rate faster than the interest rate (thus it cannot be rolled over entirely).

Control (C) and state variables (A). Hamiltonian:

$$H_t^c = \frac{C_t^{1-\theta}}{1-\theta} + \mu_t \left(w_t + r_t A_t - C_t \right)$$

with $\mu_t \equiv \lambda_t e^{-\rho t}$. FOCs:

$$\begin{aligned} \frac{dH_t^c}{dC_t} &= C_t^{-\theta} - \mu_t = 0 \\ \dot{\mu}_t &= -\frac{dH_t^c}{dA_t} + \rho \mu_t = -\mu_t (r_t - \rho) \\ \lim_{t\to\infty} e^{-\rho t} \mu_t A_t &= 0 \end{aligned}$$

Note that the law of motion of the state variable is also a FOC. The Euler equation, or Keynes-Ramsey condition:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r_t - \rho)$$

Correct interpretation is that consumption (in per capita terms) is increasing/falling over time as long as interest rate is above/below rate at which future consumption is discounted, and that for CRRA preferences, the instantaneous elasticity of substitution (inverse of coefficient of relative risk aversion), measuring the response of consumption growth rate to a given difference between r_t and ρ, is $\frac{1}{\theta}$.

c Steady state is characterized by $\dot{k}_t = \dot{c}_t = 0$. IMPORTANT: since per capita variables are the same as household variables $\dot{A}_t = \dot{a}_t$, there is no need to do $\frac{\dot{A}_t}{L_t}$ as in the basic Ramsey model. Thus $\dot{k}_t = 0$ implies that $c_t = w_t + r_t a_t - n a_t = (k_t)^\alpha - n k_t$. $\dot{c}_t = 0$ implies that $r_t = \alpha k_t^{\alpha-1} = \rho$. This pins down the steady state saving per capita ($k^* = \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}}$).

The phase diagram should have: $\dot{k}_t = 0$ and $\dot{c}_t = 0$ curves and the local dynamics of the variables in the four quadrants they define. It should also have the saddle path of convergent dynamics to the steady state (and this correctly identified as the intersection of the $\dot{k}_t = 0$ and $\dot{c}_t = 0$ curves). It is the same as in the basic Ramsey model.

d This shock affects the $\dot{k}_t = 0$ but not the $\dot{c}_t = 0$ curve. The former shifts downwards in a non-parallel way. The new steady state features lower c and same amount of k. The correct phase diagram analysis of the effect of shock has consumption jumping down and capital per capita unaffected, up to the new steady state. Since k does not change in the short or the long run, there is no crowding out of investment. Intuition is that this is due to households adapting to being permanently poorer since they have to pay higher taxes for the larger inflow in immigrants.

e The analysis here is basic textbook analysis on distortionary capital income taxation. Since resources from taxes are returned to the economy (not to the paying households but the new immigrants) the $\dot{k}_t = 0$ is not affected by whether taxation was lump sum or distortionary. But taxes discourage savings since the $\dot{c}_t = 0$ is now given by $r_t(1-\tau) = \alpha k_t^{\alpha-1}(1-\tau) = \rho$. So steady state saving per capita is lower ($k^{*\prime} = \left(\frac{\alpha(1-\tau)}{\rho}\right)^{\frac{1}{1-\alpha}}$). Consumption jumps up on impact as households respond to the lower after tax return by reducing savings (and thus have to increase consumption). From the new level consumption will embark on a downward trajectory consistent with the Euler equation, up to reaching the new steady state with lower capital and lower consumption.

Question 2

a Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\lambda} Y_i^{\lambda}$$

Maximizing w.r.t. Y_i:

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta}\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}} - Y_i^{\lambda-1} = 0$$

After some manipulation we obtain

$$\left(1 - \frac{1}{\eta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} = Y_i^{\lambda-1}$$

Which translates into

$$\left(1 - \frac{1}{\eta}\right)\frac{P_i}{P} = Y_i^{\lambda-1}$$

Taking logs and rearranging to bring y_i on the LHS:

$$y_i = \frac{1}{\lambda - 1}(p_i - p) + \frac{1}{\lambda - 1}\ln\left(1 - \frac{1}{\eta}\right) \tag{1}$$

We know that each producer charges the same price, and that the general price index equals this common price. Thus:

$$y = \frac{1}{\lambda - 1}\ln\left(\frac{\eta - 1}{\eta}\right)$$

Since $y = m - p$:

$$p = m - y = m - \frac{1}{\lambda - 1}\ln\left(\frac{\eta - 1}{\eta}\right)$$

Therefore

$$\mu = -\frac{1}{\lambda - 1}\ln\left(\frac{\eta - 1}{\eta}\right)$$

b As to the desired price at the individual level, we resort to (1). Imposing the notation $\phi \equiv \lambda - 1$, we obtain:

$$p_i^* = \phi m + (1 - \phi) p + c$$

where $c = \phi\mu$.

c Set $\phi\mu = 0$. Assuming certainty equivalence:

$$x_t = \frac{1}{2}\left(p^*_{i,t} + \mathbf{E}_t\left[p^*_{i,t+1}\right]\right)$$

Thus

$$x_t = \frac{1}{2}\left(\phi m_t + (1-\phi)\,p_t + \phi \mathbf{E}_t\left[m_{t+1}\right] + (1-\phi)\,\mathbf{E}_t\left[p_{t+1}\right]\right)$$

Recall now that $p_t = \frac{1}{2}\left(x_t + x_{t-1}\right)$ and that m_t is a white noise shock. The equation above becomes

$$x_t = \frac{\phi}{2}m_t + \frac{1}{2}\left((1-\phi)\,\frac{1}{2}\left(x_t + x_{t-1}\right) + \frac{1}{2}\left(1-\phi\right)\mathbf{E}_t\left[\left(x_{t+1} + x_t\right)\right]\right)$$

which translates into

$$x_t = \frac{\phi}{1+\phi}m_t + \frac{1-\phi}{2\left(1+\phi\right)}\left(\mathbf{E}_t\left[x_{t+1}\right] + x_{t-1}\right) \tag{2}$$

d Guess a solution for x_t of the type:

$$x_t = \beta x_{t-1} + \gamma m_t$$

Plug this into (2) to eliminate $\mathbf{E}_t\left[x_{t+1}\right]$:

$$x_t = \frac{2\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta}m_t + \frac{1-\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta}x_{t-1}$$

from which we can infer

$$\begin{aligned}\gamma &= \frac{2\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta}\\ \beta &= \frac{1-\phi}{2\left(1+\phi\right) - \left(1-\phi\right)\beta}\end{aligned}$$

Thus, we need to solve the system above, conditional on $\lambda = 1$, which implies $\phi = 0$:

$$\begin{aligned}\gamma &= 0\\ \beta &= \frac{1}{2-\beta}\end{aligned}$$

so that β solves the quadratic equation

$$\beta^2 - 2\beta + 1 = 0$$

whose solution is $\beta = 1$.