Written Exam for the M.Sc. in Economics summer 2014

Auctions

Final Exam

June 10, 2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total

This exam contains 3 exercises. For each of the exercises, the indicative weighting of the exercise in the grading of the exam is listed in parenthesis.

Exercise 1: True or false statements (25%)

State whether each of the following 3 statements are true or false and give a short explanation.

Question 1a:

A fan of the pop singer Justin Bieber has gotten hold of a signed poster from the competing pop group One Direction. The fan has absolutely no interest in the signed poster. He therefore decides to sell it in a first-price sealed bid auction and wonders if he should set a minimum starting price.

Statement: Since the sellers does not value the signed poster, $x_0 = 0$, the optimal reserve price is 0.

Question 1b:

Three bidders with uniformly distributed values on [0,10] are participating in a standard auction. The expected payment of a bidder with value zero is zero, such that the expected payment is

$$m^A(x) = \int_0^x yg(y)dy$$

Statement: The expected revenue to the seller in this standard auction is

$$E[R^A] = N * E[m^A(X)] = 10/3$$

Question 1c:

Consider the Combinatorial Clock Auction format that is typically used for spectrum auctions. In one such auction a bidder is informed that her individual opportunity cost is 100 while the price she has to pay is 120.

Statement: Such a case can occur because bidders can be required to pay more than their individual opportunity costs when the joint opportunity costs of a group of bidders exceeds the sum of the individual opportunity costs of the bidders.

Exercise 2: Copenhagen Real Estate Market (35%)

A young couple is looking to buy an apartment in Copenhagen to be able to move in together. They have found the ideal compromise between size, location, price – and it has a balcony facing south-west. The bylaws governing the Danish real estate market stipulates usage of a first-price sealed bid auction. The couple's preferences are similar to other buyers' preferences and can be described as risk neutral private values.

Question 2a: Explain a symmetric and increasing bidding strategy of the couple and in particular why it is optimal to bid below their value.

The couple has recently lost out on three separate apartments sold to other buyers. The couple really would like to buy this ideal apartment and having been burned, they have shifted their preferences to a utility function of constant relative risk aversion, $u(x) = x^{\alpha}$. In fact practically all buyers on the market have shifted to similar preferences.

Question 2b: Explain what impact the shift in risk appetite has for the revenue to the seller of the ideal apartment

The main concern for the Ministry of Housing is efficiency in the market, but the Ministry is getting increasingly worried about a new house price bubble. Therefore the Ministry of Housing has commissioned an auction expert to write an expert report looking into other selling options for the market. The report recommends adopting the second-price auction for the real estate market. This adoption can take the pressure out of a potential price bubble, and once preferences are back to the 'normal' long-term risk neutrality then real estate prices will not 'on average' be affected. All the while the report claims that efficiency is unaffected.

Question 2c: Explain the reasoning behind the report

The influential lobby group, NCCS, National Coalition of Concerned Sellers consisting of risk-averse sellers are countering the idea adopting a second-price auction due to the effects caused by the risk preferences of buyers (short-term risk aversion and long-term risk neutrality).

Question 2d: Explain why the NCCS are opposed to the idea of adopting a second-price auction

Recently, magazines and newspapers have run articles about an increasing trend of middle-aged couples moving to back Copenhagen. After their grown-up children have left home, couples in their 50's sell their suburban houses freeing up substantial equity to put as down-payment for an apartment in Copenhagen. This shift has caused an asymmetry among the previously homogenous bidders. Specifically, middle-aged couples have values that are stochastically higher than the young couples.

Question 2e: Explain how this affects the young couple and what the impact is on efficiency

Exercise 3: Multiple object auctions (40%)

Nynne is employed at the student café Cafelitten at the Department for Economics. She has learned about auctions and is eager to apply her knowledge when selling coffee to students in Cafelitten. One late afternoon, two students enter Cafelitten and each request two cups of coffee. However, Nynne only has two cups of coffee left to sell. She decides to sell the two cups of coffee in an auction.

Nynne considers the two students to be symmetric bidders in her coffee auction. She estimates that for both bidders the value of the first cup of coffee is uniformly distributed between 10 and 20, $X_1^i \sim u[10,20]$, and the value of the second cup of coffee is uniformly distributed between 0 and 10, $X_2^i \sim u[0,10]$.

Nynne considers using either a Vickrey auction, a Uniform-price auction or a Discriminatory auction to sell the two cups of coffee.

Question 3a: Argue why it is an equilibrium strategy for a student is to bid its actual values, i.e. $\beta(\boldsymbol{x}) = \boldsymbol{x}$, if Nynne chooses a Vickrey auction.

Question 3b: Find the expected revenue in the Vickrey auction, $E[R^V]$.

Question 3c: Can one use the multi-unit Revenue Equivalence Principle to argue that the expected revenue in the Vickrey auction, $E[R^V]$, must be the same as the expected revenue in the Uniform-price auction, $E[R^U]$, and the expected revenue in the Discriminatory auction, $E[R^D]$? Why/why not?

After looking more closely in the coffee pot, Nynne realises that she has three cups of coffee to sell instead of two. Each student is still only interested in at most two cups of coffee. She decides to sell the three cups of coffee using a Discriminatory auction.

Question 3d: Find an equilibrium bidding strategy in the Discriminatory auction [Hint: You may use this expression for the expected payment of a bidder with value vector (x_1, x_2) in the Vickrey auction $m^V(x_1, x_2) = \int_0^{x_2} y f_2(y) dy$.].