Macro III - exam solutions (June 5, 2018)

1 True. Basic real business cycle models assume perfect competitive markets, and no externalities. Thus, the first welfare theorem holds and the decentralized competitive equilibrium is Pareto optimal. Therefore, government intervention cannot increase welfare.

2 False. A reduction in labor taxes increases the young's available income, with no effect on their income when old since they do not work next period and the economy being open implies there is no effect on rate of return of savings. Thus, Ricardian equivalence does not hold in this economy. The initial young want to increase their consumption of goods both when young and when old. Consumption of the initial old is unchanged. Therefore, initial aggregate consumption increases and there is an initial current account deficit.

3 False. In the Meltzer and Richard model only those increases in inequality that make the median voter poorer would lead to an increase in redistribution. In this model the determinant of redistribution is the difference between mean and median income. An increase in inequality (not affecting the mean) can make the median income increase or decrease depending on how incomes become more dispersed.

4 Important: Note that there is a typo in the budget constraint. It should read

$$c_t + k_{t+1} = w_t(1 - x_t) + R_t k_t$$

Students that do not realize there is a mistake here and solve the problem with the given budget constraint should be given full points, unless they reach an unreasonable result and fail to notice this (such as for example finding that capital is negative, or capital is positive, interest rate is positive and capital income is negative).

a) The Lagrangian is given by (note that the problem can be solved using the intertemporal budget constraint)

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \frac{x_t^{1-\epsilon}}{1-\epsilon} + \lambda_t \left(w_t(1 - x_t) + R_t k_t - c_t - k_{t+1} \right) \right]$$

The first order conditions are given by

$$\begin{aligned}
\frac{d\mathcal{L}}{dc_t} &= 0 \longrightarrow \frac{1}{c_t} - \lambda_t = 0, \\
\frac{d\mathcal{L}}{dx_t} &= 0 \longrightarrow x_t^{-\epsilon} - \lambda_t w_t = 0, \\
\frac{d\mathcal{L}}{dk_{t+1}} &= 0 \longrightarrow -\lambda_t + \beta R_{t+1}\lambda_{t+1} = 0
\end{aligned}$$

The Euler equation is given by,

$$\frac{1}{c_t} = \beta R_{t+1}\frac{1}{c_{t+1}}.$$

The interpretation is that the household makes consumption saving choices such that the marginal rate of substutition between current and future consumption equals the marginal rate of transformation, R_{t+1}. An Euler equation can be written for intertemporal leisure choices, but it is not required that the students derive it.

b) In steady state, $c_t = c^*$, $x_t = x^*$ and $k_{t+1} = k^*$. From the Euler equation we find that steady state capital is determined, as usual in this setting, by (here we use that the rental rate for capital, r, is equal to the marginal product of capital, $\alpha\kappa^{\alpha-1}$)

$$R^* = 1 + r^* = 1 + \alpha\left(\frac{k^*}{1-x^*}\right)^{\alpha-1} = 1 + \alpha\kappa^{*\alpha-1} = \frac{1}{\beta}.$$

where $\kappa = \frac{K}{L}$ is the aggregate (or average) capital per unit of labor. Note that $\kappa \equiv \frac{k}{1-x}$, we make this distinction since some results are neater with κ instead of k, but these are equivalent measures in the steady state.

Given κ^* steady state wages are given by $w^* = (1-\alpha)\kappa^{*\alpha}$. Steady state consumption and leisure are characterized by the following system of two equations in two unknowns

$$\begin{aligned}
c^* &= w^*(1-x^*) + \alpha\kappa^{*\alpha-1}k^* = \kappa^{*\alpha}(1-x^*) \qquad (1)\\
c^* &= w^* x^{*\epsilon} = (1-\alpha)\kappa^{*\alpha}x^{*\epsilon}
\end{aligned}$$

Given the initial level of capital, k_0, there is a unique choice of consumption, c_0, and leisure, x_0, such that the dynamics do not violate optimality conditions and converge to the steady state. If consumption initially was higher than c_0 then capital would eventually be depleted such that the Euler equation was violated. Conversely, if consumption initially is lower than c_0 there would be overaccumulation of capital in this economy (the transversality condition would be violated), with the interest rate falling below $\frac{1}{\beta}$ and household could be made better off by eating the excess capital in finite time. Similar reasoning applies to initial choice of leisure as this also satisfies an Euler equation (which would be violated if too much initial leasure is chosen), or would lead to overaccumulation (if too initial leisure is chosen).

c) An increase in ϵ corresponds to an increase in the concavity of preferences for leisure. Capital per unit of labor is unaffected since the condition that determines κ^* only depends on β. Combining the two equations that characterize c^* and x^* we get that

$$\frac{x^{*\epsilon}}{1-x^*} = \frac{1}{1-\alpha}.$$

Since both x^* and ϵ are between zero and one, an increase in ϵ reduces the left hand side of this equation for the initial value of x^*. Thus, x^* has to increase. This implies that c^* is lower (from (1)). Finally, note that because κ^* is capital per unit of labor, and labor supply is reduced, capital per worker, k^*, is lower.

5 a) We start with the following equations that characterize this economy

$$\begin{aligned} y_t &= m_t - p_t && (2)\\ p_t &= \frac{1}{2}(p_t^1 + p_t^2) && (3)\\ p_t^i &= \phi E[m_t|I_{t-i}] + (1-\phi)E[p_t|I_{t-i}], && (4) \end{aligned}$$

Substitute (3) into (4) when $i = 1$ and take expectations given I_{t-2}

$$\begin{aligned} E[p_t^1|I_{t-2}] &= E\left[\phi E[m_t|I_{t-1}] + (1-\phi)E[\frac{1}{2}(p_t^1+p_t^2)|I_{t-1}]|I_{t-2}\right] \\ &= \phi E[m_t|I_{t-2}] + (1-\phi)E[\frac{1}{2}(p_t^1+p_t^2)|I_{t-2}] \iff \\ E\left[p_t^1\left(\frac{1}{2}+\frac{1}{2}\phi\right)|I_{t-2}\right] &= E\left[p_t^1\left(\frac{1+\phi}{2}\right)|I_{t-2}\right] = \phi E[m_t|I_{t-2}] + \frac{1-\phi}{2}E[p_t^2|I_{t-2}] \iff \\ E\left[p_t^1|I_{t-2}\right] &= \frac{2\phi}{1+\phi}E[m_t|I_{t-2}] + \frac{1-\phi}{1+\phi}p_t^2 && (5) \end{aligned}$$

Note that p_t^2 is known at time $t-2$ and is given by

$$p_t^2 = \phi E[m_t|I_{t-2}] + (1-\phi)E[\frac{1}{2}(p_t^1+p_t^2)|I_{t-2}]$$

This can be rewritten as

$$p_t^2 = \frac{2\phi}{1+\phi} E[m_t|I_{t-2}] + \frac{1-\phi}{1+\phi} E[p_t^1|I_{t-2}]$$

Substituting in (5) gives

$$p_t^2 = E[m_t|I_{t-2}] \tag{6}$$

Finally, to find p_t^1 we replace the above solution for p_t^2 into

$$p_t^1 = \frac{2\phi}{1+\phi} E[m_t|I_{t-1}] + \frac{1-\phi}{1+\phi} p_t^2$$

and get

$$p_t^1 = E[m_t|I_{t-2}] + \frac{2\phi}{1+\phi} \left(E[m_t|I_{t-1}] - E[m_t|I_{t-2}]\right) \tag{7}$$

Substituting (6) and (7) in (3)

$$\begin{aligned} p_t &= \frac{1}{2} \left[E[m_t|I_{t-2}] + E[m_t|I_{t-2}] + \frac{2\phi}{1+\phi} \left(E[m_t|I_{t-1}] - E[m_t|I_{t-2}]\right) \right] \\ &= E[m_t|I_{t-2}] + \frac{\phi}{1+\phi} \left(E[m_t|I_{t-1}] - E[m_t|I_{t-2}]\right) \qquad (8) \end{aligned}$$

Substituting (8) into (2)

$$y_t = m_t - E[m_t|I_{t-2}] - \frac{\phi}{1+\phi} \left(E[m_t|I_{t-1}] - E[m_t|I_{t-2}]\right) \tag{9}$$

c) Monetary shocks persist for two period, i.e. they have real effects when they occur and one period after that. The reason for this is that a shock that takes place in period t will affect prices set for two periods. Since these prices cannot be adjusted later to new developments in aggregate demand, this price stickiness has real effects. In equation (9) the persistence is given by the fact that $E[m_t|I_{t-1}] - E[m_t|I_{t-2}]$ has an effect on output in period t even though this is information that is known before the realization of aggregate demand in the period.