Macro C - exam solutions (May 28, 2014)

General remarks

Please grade each question/item between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

In Problem 2, items a) and b), an answer that assumes $\gamma = 0$ should get half the points.

1 Question 1

False. When individuals live for two periods and the rate of population growth is higher than the rate of return of a storage technology there can exist a monetary equilibrium, an equilibrium where money has value since it facilitates transactions. Depending on how new money that is printed is distributed in the economy money can be superneutral (this happens when new money is introduced through interest payments to money holders). Thus it is not necessary for consumers to have an infinite planning horizon.

2 Question 2

Uncertain or false. The result (seen in lectures and in Persson and Tabellini) that the outcome under an independent central bank corresponds to a higher ex ante welfare than under a currency peg (or simple rule) for every possible social loss function was derived under the assumption that wage setting was decentralized. We saw, and it is in the textbook, that when there are strong unions that act strategically this may lead to choosing a less conservative central banker. Under certain parameters this could lead to an ex ante loss greater than under a currency peg.

Student also gets points if he/she notices that the standard analysis assumes a currency peg where inflation is always equal to the target level, while it might be the case that inflation is equal to the realized foreign inflation of the euro, the currency Denmark is pegged to (this case was discussed in the lectures). Since euro inflation is stochastic, if supply shocks between Denmark and the eurozone are positively correlated this reduces the ex ante loss of a currency peg and for some parameters it might be a preferable outcome than having an independent central bank.

3 Problem 1

This problem is exactly as seen in problem set 13, except that now the fraction of prices that is fixed, q, is stochastic.

The prices are set according to the following equations (1) and (2)

$$p^f = E[p_i^*|m] = (1-\phi)E[p|m] + \phi m \tag{1}$$

$$p^r = E[p_i^*] = (1-\phi)E[p] + \phi E[m] \tag{2}$$

where $\phi \in [0,1]$ (this was not specified in the question but was explained as such in lecture and in problem set, there is no penalty if not assumed by student). The average price is given, after the realization of q, by

$$p = qp^r + (1-q)p^f \tag{3}$$

where $q \in (0,1)$.

a) Substituting (3) into (1)

$$\begin{aligned} p^f = (1-\phi)E[p|m] + \phi m = (1-\phi)\big(E[q]p^r + (1-E[q])p^f\big) + \phi m \quad &\Leftrightarrow \\ p^f\big(1-(1-\phi)(1-E[q])\big) = (1-\phi)E[q]p^r + \phi m \quad &\Leftrightarrow \\ p^f = \frac{(1-\phi)E[q]p^r + \phi m}{1-(1-\phi)(1-E[q])} = \frac{(1-\phi)E[q]p^r + \phi m}{\phi + (1-\phi)E[q]} = \frac{\big[(1-\phi)E[q] + \phi\big]p^r - \phi p^r + \phi m}{\phi + (1-\phi)E[q]} & \\ p^f = p^r + (m - p^r)\frac{\phi}{\phi + (1-\phi)E[q]} & \end{aligned} \tag{4}$$

Substituting (3) into (2)

$$p^r = (1-\phi)E[p] + \phi E[m] = (1-\phi)E[qp^r + (1-q)p^f] + \phi E[m]$$

Substituting in (1)

$$p^r = (1-\phi)E[q]p^r + (1-\phi)(1-E[q])p^r + E[m]\frac{\phi(1-\phi)(1-E[q])}{\phi+(1+\phi)E[q]} - p^r\frac{\phi(1-\phi)(1-E[q])}{\phi+(1-\phi)E[q]} + \phi E[m]$$
$$= (1-\phi)p^r - p^r\frac{\phi(1-\phi)(1-E[q])}{\phi+\phi(1-\phi)E[q]} + \phi E[m]\left[1+\frac{(1-\phi)(1-E[q])}{\phi+(1-\phi)E[q]}\right] \quad \Leftrightarrow$$
$$p^r\left[1-(1-\phi)+\frac{\phi(1-\phi)(1-E[q])}{\phi+(1-\phi)E[q]}\right] = p^r\left[\phi+\frac{\phi(1-\phi)(1-E[q])}{\phi+(1-\phi)E[q]}\right]$$
$$= p^r\phi\left[1+\frac{(1-\phi)(1-E[q])}{\phi+(1-\phi)E[q]}\right] = \phi E[m]\left[1+\frac{(1-\phi)(1-E[q])}{\phi+(1-\phi)E[q]}\right] \quad \Leftrightarrow$$
$$p^r = E[m] \tag{5}$$

b) Substituting (4) and (5) into (3)

$$p = qp^r + (1-q)p^f = qp^r + (1-q)p^r + (m-p^r)\frac{\phi(1-q)}{\phi+(1-\phi)E[q]} = p^r + (m-p^r)\frac{\phi(1-q)}{\phi+(1-\phi)E[q]}$$
$$= E[m] + (m-E[m])\frac{\phi(1-q)}{\phi+(1-\phi)E[q]} \tag{7}$$

Since $y = m - p$, substituting (7) into this relation

$$y = m - E[m] - (m-E[m])\frac{\phi(1-q)}{\phi+(1-\phi)E[q]} = (m-E[m])\left(1-\frac{\phi(1-q)}{\phi+(1-\phi)E[q]}\right)$$
$$= (m-E[m])\left(\frac{\phi+(1-\phi)E[q]-\phi(1-q)}{\phi+(1-\phi)E[q]}\right) = (m-E[m])\left(\frac{E[q]+\phi(q-E[q])}{\phi+(1-\phi)E[q]}\right) \tag{8}$$

From this equation we see that only unanticipated changes in m have an effect on output. The reason for this is that output is given by the difference between m and the price level, and the latter fully reflects anticipated changes in m (changes in $E[m]$). This is expected in a static model where money is neutral.

c) This requires looking at (8), in particular at

$$\frac{\phi q + (1-\phi)E[q]}{\phi+(1-\phi)E[q]}$$

This it is straightforward that an increase in q increases the response of output to an unanticipated change in m (keeping $E[q]$ constant). The reason for this is that an increase in q means there are less flexible prices in the price level. Thus there is a smaller response of the price level to the change in m (only flexible prices respond to shocks), and thus a larger response of output.

From the same equation we have that, given a value of q and since $q < 1$, then an increase in $E[q]$ increases the response of output to an unanticipated change in m (this requires $1 - \phi > 0$, thus no penalty if student arrives at an ambiguous result from not assuming this). The reason for this is that an increase in the expected importance of fixed prices makes flexible prices less responsive to shocks (see equation (4), and its derivation where $E[q]$ shows as weight on fixed prices on the expectation of the price level).

4 Problem 2

a) The profit function for firms is given

$$K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t^L K_t$$

The FOC of firms' problem of maximizing profits (taking factor prices as given) are

$$\begin{aligned}
(1-\alpha)(K_t)^{\alpha}(L_t)^{-\alpha} - w_t &= 0 \\
\alpha(K_t^j)^{\alpha-1}(L_t^j)^{1-\alpha} - r_t^L &= 0
\end{aligned}$$

In what follows, since population is constant we can assume that $L_t \equiv 1$, such that the capital labor ratio, $k \equiv \frac{K}{L}$ satisfies $k_t = K_t$ (not necessary for any result).

The wage and interest rates are determined by imposing equilibrium in factor markets where firms competitively demand labor from households and capital from intermediaries. Thus

$$\begin{aligned}
w_t &= (1-\alpha)k_t^{\alpha} \\
r_t^L &= \alpha k_t^{\alpha-1}
\end{aligned}$$

The income of the old from saving is the product of saving times the rate of interest on savings, which is $r_t^D = r_t^L(1-\gamma)$, and since capital is only a fraction of household saving, $k_t = s_{t-1}(1-\gamma)$ (where s_{t-1} is saving done in $t-1$ when the old were workers), thus the income from saving is $(1-\gamma)\alpha k_t^{\alpha-1}\frac{k_t}{1-\gamma} = \alpha k_t^{\alpha}$. The income that workers receive from their labor services, as a function of k, is just $(1-\alpha)k_t^{\alpha}$. Note that workers pay taxes out of labor income, so an answer that labor income is net of taxes (and thus $(1-\alpha)k_t^{\alpha}(1-d)$) is also correct.

b) This requires setting up the problem of workers.

$$\begin{aligned}
\max_{c_{1t}, c_{2t+1}} \quad & \ln(c_{1t}) + \frac{1}{1+\rho} \ln(c_{2t+1}) \\
\text{s.t.} \quad & c_{1t} = w_t(1-d) - s_t \\
& c_{2t+1} = s_t r_{t+1}^D + d w_{t+1}
\end{aligned}$$

Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \frac{r_{t+1}^D}{1+\rho} c_{1t}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{1}{2+\rho} w_t(1-d) - \left(\frac{1+\rho}{2+\rho}\right) \frac{1}{(r_{t+1}^D)} d w_{t+1}$$

To get capital accumulation we replace individual savings with next period capital per worker $k_{t+1} = (1-\gamma)s_t$ (note there is no $\frac{1}{1+n}$ term since there is no population growth), and we use equilibrium expressions for wage and interest rates from a)

$$k_{t+1} = (1-\gamma)\left[\frac{1}{2+\rho}(1-\alpha)k_t^\alpha(1-d) - \left(\frac{1+\rho}{2+\rho}\right)\frac{(1-\alpha)k_{t+1}}{(1-\gamma)\alpha} d\right]$$

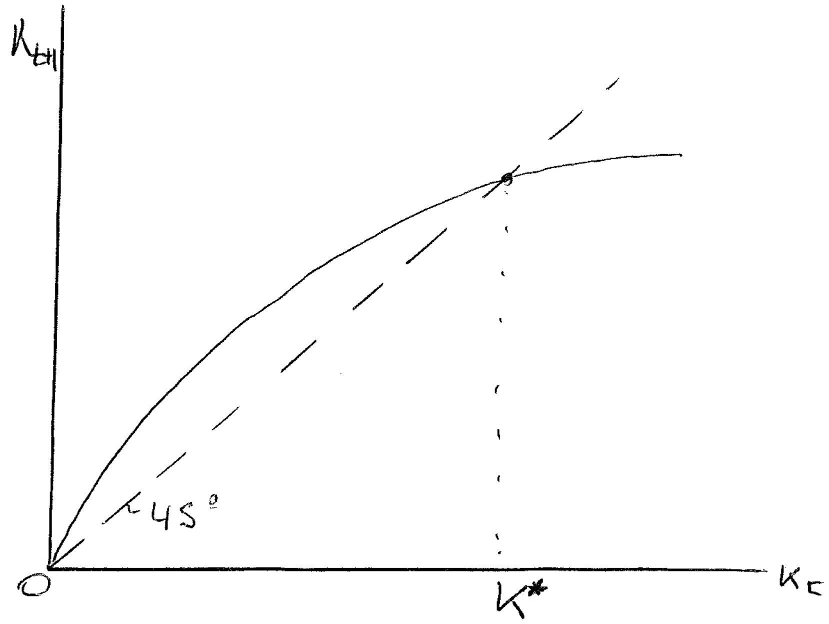
Combining terms with k_{t+1}

$$k_{t+1} = \frac{1}{\left[1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha} d\right]} \frac{1}{2+\rho}(1-\gamma)(1-\alpha)k_t^\alpha(1-d)$$

From here imposing steady state we get the following

$$k^* = \left[\frac{1}{\left[1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha} d\right]} \frac{1}{2+\rho}(1-\gamma)(1-\alpha)(1-d)\right]^{\frac{1}{1-\alpha}}.$$

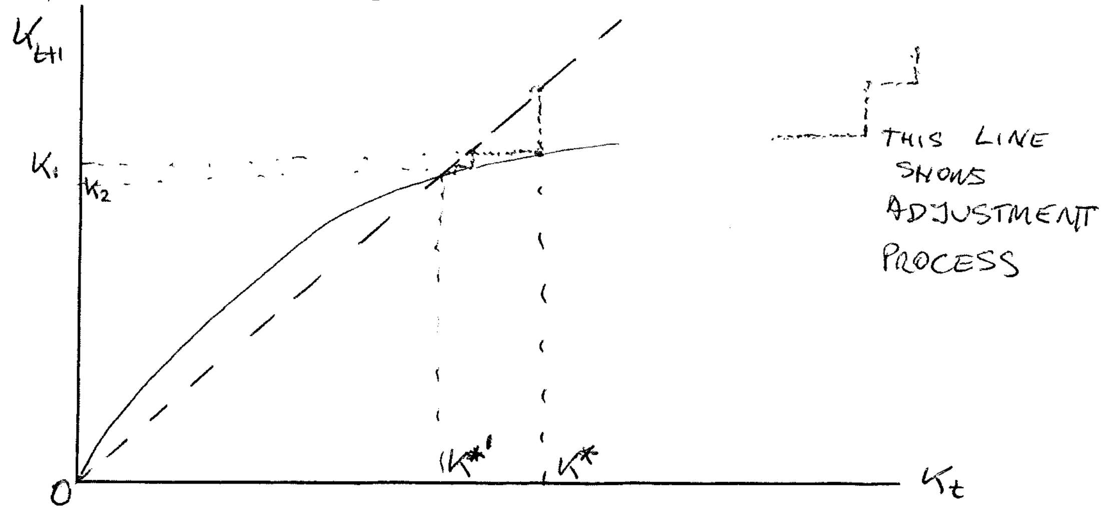
Graphically



c) From b) above we know that the new steady state satisfies

$$k^{*\prime} = \left[\frac{1}{\left[1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}d\right]} \frac{1}{2+\rho}(1-\gamma')(1-\alpha)(1-d) \right]^{\frac{1}{1-\alpha}} < k^*,$$

where $\gamma' > \gamma$ is the new fraction of deposits that intermediaries must store. Graphically



In the first period we have that wage is equal to $w^* = (1-\alpha)(k^*)^\alpha$ and capital accumulation satisfies

$$k_1 = \frac{1}{\left[1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}d\right]} \frac{1}{2+\rho}(1-\gamma')w^*(1-d)$$

because $\gamma' > \gamma$, $k_1 < k^*$. Since $k_1 = (1-\gamma')s_0$ this means that on impact (in the first period) saving is unaffected ($s_0 = s^*$). Thus on impact there is no effect on first period consumption of workers (and obviously no change in consumption of the old). The reason for this is that when the shock takes place it affects capital next period, not output today, and the effects on individual savings of lower capital tomorrow (through decreased future benefits and decreased interest rate on deposits) exactly cancel out in this case with logarithmic utility and Cobb-Douglas production function. Thus saving is unaffected and from Euler equation we see that all the effect of lower interest rate on deposits translates in reduce consumption next period when old (something that students are not required to acknowledge since the question only refers to first period consumption) As more resources are stored there is a decrease in capital accumulation that will reduce future wages and consumption. Given the assumptions of logarithmic utility and Cobb-Douglas production function this adjustment process is gradual and monotonous.

d) This requires to consider a new, reduced, contribution to social security, $d' < d$ that offsets the effects of the increase in γ on capital accumulation. Thus we want

$$k_1 = k^{*'} = k^* = \frac{1}{\left[1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}d'\right]}\frac{1}{2+\rho}(1-\gamma')w^*(1-d').$$

Since $k^{*'} = k^*$ implies $(k^{*'})^{1-\alpha} = (k^*)^{1-\alpha}$, from the expression for steady state we get that

$$\frac{1-d'}{1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}d'} = \frac{1-d}{1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}d}\left(\frac{1-\gamma}{1-\gamma'}\right)$$

Since $\frac{1-\gamma}{1-\gamma'} > 1$ this indeed shows that $d' < d$. (there is no need to explicitly solve for d' from above expression, the solution is trivial since the equation is linear, but messy).

It is straightforward to show that steady state welfare must be lower since output is unchanged ($k^{*'} = k^*$), and this must cover consumption of young and old and saving. But saving is higher to sustain capital at same level, therefore consumption must be lower, and thus welfare must be lower than before the shock. Also since the policy implies a reduction of contributions on impact, the initial old are worse off.

I have noticed that the wording of the question is a bit ambiguous regarding the welfare comparison. In the previous paragraph I did the comparison against old steady state. But if a student compares welfare under this policy with welfare given the shock but no policy change that is valid as well. In that case in steady state welfare with lower contributions is higher, since output is higher, thus providing more consumption for both young and old. But the initial old are still worse off since with the reduction in contributions they consume less.