Macro C suggestive solution – May 2012

Academic aim

- Understanding of the main model frameworks for long run macroeconomics. This includes the Diamond model with overlapping generations in discrete time and the Ramsey model in continuous time.
- Proficiency in the application of the concepts and methods from these frameworks, including competence in dynamic optimization and dynamic analysis in discrete and continuous time.
- Understanding of the role of expectations and basic knowledge of macroeconomic models with forwardlooking expectations under both perfect foresight and uncertainty and rational expectations.
- Proficiency in the application of the related concepts and methods.
- Competence in analyzing a macroeconomic problem, where the above mentioned concepts and methods are central, that is competence in solving such models and explaining in economic terms the results and implications and how they derive from the assumptions of the model.

The particularly good performance, corresponding to the top mark, is characterized by a complete fulfilment of these learning objectives

Problem A

1)

The social loss function (which is also the loss function of the government) is given by:

$$SL_t = \kappa \cdot (\pi_t - \pi^*)^2 + (y_t - y^*)^2$$

The loss function of the central bank is given by:

$$SL_t^{cb} = (\kappa + \varepsilon) \cdot (\pi_t - \pi^*)^2 + (y_t - y^*)^2$$

where $\varepsilon > 0$ reflects that the central bank is more adverse towards deviations of the inflation rate form its target (π^*) than society.

Since a fraction β of the responsibility for the conduct of monetary policy is delegated to the central bank, the loss function of the combined policy maker is given by:

$$SL_t^* = (1-\beta)\cdot SL_t + \beta \cdot SL_t^{cb} =$$

$$(1-\beta)\cdot(\kappa\cdot(\pi_t-\pi^*)^2+(y_t-y^*)^2)+\beta\cdot\left((\kappa+\varepsilon)\cdot(\pi_t-\pi^*)^2+(y_t-y^*)^2\right) =$$

$$(\kappa+\beta\cdot\varepsilon)\cdot(\pi_t-\pi^*)^2+(y_t-y^*)^2$$

Substituting equation (A.4) and then (A.1) in we get:

$$SL_t^* = (\kappa+\beta\cdot\varepsilon)\cdot(\pi_t-\pi^*)^2+(y_t-\bar{y}-\omega)^2 =$$

$$(\kappa+\beta\cdot\varepsilon)\cdot(\pi_t-\pi^*)^2+\left(\pi_t-\pi_{t,t-1}^e-s_t-\omega\right)^2$$

The combined policy maker chooses the rate of inflation in order to minimize SL_t^*. The first order condition for minimizing SL_t^* is given by:

$$\frac{\partial SL_t^*}{\partial \pi_t} = 2\cdot(\kappa+\beta\cdot\varepsilon)\cdot(\pi_t-\pi^*)+2\cdot\left(\pi_t-\pi_{t,t-1}^e-s_t-\omega\right)=0 \Rightarrow$$

$$(\kappa+\beta\cdot\varepsilon)\cdot(\pi_t-\pi^*)+\left(\pi_t-\pi_{t,t-1}^e-s_t-\omega\right)=0$$

Which basically states the marginal cost of increasing inflation must equal the marginal benefit, which consists of higher output (due to higher surprise inflation since $\pi_{t,t-1}^e$ is taking as given).

2)

Imposing rational expectations in the first order condition (i.e. taking expectations conditional on the information available at time $t-1$, when inflation expectations are formed, and imposing: $E[\pi_t|I_{t-1}] = \pi_{t,t-1}^e$) we get:

$$(\kappa+\beta\cdot\varepsilon)\cdot(E[\pi_t|I_{t-1}]-\pi^*)+\left(E[\pi_t|I_{t-1}]-\pi_{t,t-1}^e-E[s_t|I_{t-1}]-\omega\right)=0 \Rightarrow$$

$$(\kappa+\beta\cdot\varepsilon)\cdot\left(\pi_{t,t-1}^e-\pi^*\right)+\left(\pi_{t,t-1}^e-\pi_{t,t-1}^e-\omega\right)=0 \Rightarrow$$

$$\pi_{t,t-1}^e = \pi^* + \frac{\omega}{\kappa+\beta\cdot\varepsilon}$$

where we have also used the absence of auto-correlation in supply shocks (i.e. $E[s_t|I_{t-1}] = 0$) and the fact that agents know at time $t-1$ what they will expect at time $t-1$ (i.e. $E\left[\pi^e_{t,t-1}|I_{t-1}\right] = \pi^e_{t,t-1}$).

Inserting the expression for $\pi^e_{t,t-1}$ back in the first order condition we find the implied value of π_t :

$$(\kappa + \beta \cdot \varepsilon) \cdot (\pi_t - \pi^*) + \left(\pi_t - \pi^* - \frac{\omega}{\kappa + \beta \cdot \varepsilon} - s_t - \omega\right) = 0 \Rightarrow$$

$$\pi_t = \pi^* + \omega \cdot \frac{1}{\kappa + \beta \cdot \varepsilon} + s_t \cdot \frac{1}{1 + \kappa + \beta \cdot \varepsilon}$$

which also implies that:

$$E[\pi_t] = \pi^* + \omega \cdot \frac{1}{\kappa + \beta \cdot \varepsilon} = \pi^e_{t,t-1}$$

and:

$$\pi_t - \pi^e_{t,t-1} = s_t \cdot \frac{1}{1 + \kappa + \beta \cdot \varepsilon}$$

which can be inserted in equation (A.1) to yield:

$$y_t - \bar{y} = \pi_t - \pi^e_{t,t-1} - s_t = s_t \cdot \frac{1}{1 + \kappa + \beta \cdot \varepsilon} - s_t = -s_t \cdot \frac{\kappa + \beta \cdot \varepsilon}{1 + \kappa + \beta \cdot \varepsilon} \Rightarrow$$

$$y_t = \bar{y} - s_t \cdot \frac{\kappa + \beta \cdot \varepsilon}{1 + \kappa + \beta \cdot \varepsilon}$$

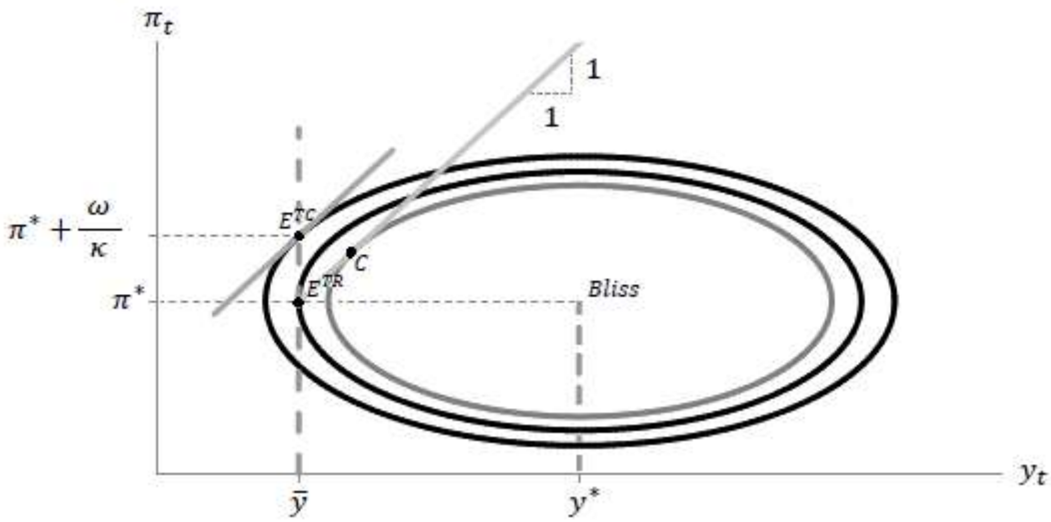
3)

The inflation bias is generated by the basic incentive for the policy makers to increase output beyond the natural level (since the socially optimal output level exceeds the natural output level by assumption) by creating surprise inflation. Since the agents in the private sector recognize this incentive, they will expect a high inflation rate, which translates into a demand (e.g. by unions) for high wage growth and thus high growth in the costs of production and high inflation.

Let's illustrate the inflation bias in the case without delegation and supply shocks, i.e. when: $s_t = \varepsilon \cdot \beta = 0$.

For the purpose of illustration let's start out at the Taylor rule equilibrium (the second-best outcome marked E^{TR} in the diagram below) where output equals the natural level and actual and expected inflation equals the inflation target. Given that $\pi^e_{t,t-1} = \pi^*$ the policymaker has an incentive to create higher inflation (by setting lower interest rates) and thereby increase output beyond $\bar{y}$. The gain from higher output will exceed the cost of higher inflation, since the inflation rate is initially optimal while the output level is suboptimal (since $\omega > 0$). The policymaker will thus increase inflation such that the economy moves to point C in the diagram below, where the SRAS-curve with $\pi^e_{t,t-1} = \pi^*$ is just tangent to an iso-welfare curve. This is the cheating outcome (the outcome which minimizes social loss given that $\pi^e_{t,t-1} = \pi^*$), and consistent with a iso-welfare curve witch is closer to bliss (the ideal outcome where $\pi_t = \pi^*$ and $y_t = y^*$) than the Taylor rule equilibrium.

However, the cheating outcome is *not* consistent with rational expectations, since the agents in the private sector recognize the incentive to create higher inflation and thus raise their inflation expectations. The economy ends up in the point E^{TC}, which is the time-consistent (rational expectations) equilibrium. In this outcome the policy maker doesn't have any incentive to increase inflation further, and inflation is fully foreseen, which further implies that $y_t = \bar{y}$ (according to equation A.1 with $s_t = 0$). As a consequence of the higher inflation the economy ends up at an iso-welfare curve which is further way from bliss than the initial Taylor rule equilibrium. The time-consistent equilibrium is thus third-best.



In general (i.e. with supply shocks and delegation) the inflation bias is given by:

$$E[\pi_t] - \pi^* = \omega \cdot \frac{1}{\kappa + \beta \cdot \varepsilon}$$

We see that an increase in the amount of effective delegation ($\beta \cdot \varepsilon$) decreases the inflation bias. An increase in β implies that a larger part of the responsibility for the conduct of monetary policy is delegated to the inflation bank which is more adverse towards a high inflation rate than the government (given that $\varepsilon > 0$), which implies that the incentive for the combined policymaker to create surprise inflation is weakened. An increase in ε implies that the central bank is more adverse towards inflation which also weakens the incentive for the combined policymaker to generate surprise inflation (given that $\beta > 0$).

In the special case where $\varepsilon \to \infty$ the inflation bias will entirely vanish (given that $\beta > 0$).

4)

The cost of delegation consists of a distortion of stabilization policy, since a higher amount of effective delegation increases the variance of output according to the expression derived in question 2). When the responsibility for monetary policy is delegated to the central bank output will fluctuate too much (compared to what is socially optimal), since the central bank is more concerned about a stable inflation rate relative to a stable output level than society. This is the trade-off between flexibility and credibility. If there isn't any supply shocks, i.e. $\sigma_s^2 = 0$ this trade-off will disappear.

5)

In this case we know from the policy ineffectiveness proposition that the policy maker will not be able to affect the time path of output (since the combined policy maker has no informational advantage compared to the agents in the private sector). Thus, there is no cost of delegating authority to the central bank while there is still a benefit from a lower inflation bias. Thus, there is no trade-off between flexibility and credibility and an increase in the amount of delegation is thus always beneficial in this case.

Problem B

The problem of the representative household is to choose a consumption path $(c_t)_{t=0}^{\infty}$ in order to maximize the intertemporal utility function:

$$U_0 = \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} \cdot e^{-(\rho - n)\cdot t} dt$$

subject to the intertemporal budget constraint, which can be written as a combination of the equation governing the evolution of household wealth per worker:

$$\dot{a}_t = (r_t - n) \cdot a_t + w_t - T_t - c_t$$

and a No Ponzi game condition (restricting the asymptotic evolution of a_t), taking initial wealth (a_0) as given.

The No Ponzi game condition can be ignored when deriving the optimal *growth rate* of consumption over time. We solve the problem using *optimal control theory*.

At first, write up the present value Hamiltonian:

$$\mathcal{H} = \frac{c_t^{1-\theta} - 1}{1-\theta} \cdot e^{-(\rho-n)\cdot t} + \lambda_t \cdot \left((r_t - n) \cdot a_t + w_t - T_t - c_t\right)$$

The intratemporal first order condition with respect to the control variable (c_t) is given by:

$$\frac{\partial \mathcal{H}}{\partial c_t} = c_t^{-\theta} \cdot e^{-(\rho-n)\cdot t} - \lambda_t = 0$$

From this first order condition we can derive the growth rate of λ_t:

$$\lambda_t = c_t^{-\theta} \cdot e^{-(\rho-n)\cdot t} \Rightarrow ln\lambda_t = -\theta \cdot lnc_t - (\rho - n) \cdot t \Rightarrow$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \frac{dln\lambda_t}{dt} = -\theta \cdot \frac{\dot{c}_t}{c_t} - (\rho - n)$$

The intertemporal first order condition with respect to the state variable (a_t) is given by:

$$\frac{\partial \mathcal{H}}{\partial a_t} = -\dot{\lambda}_t \Rightarrow \lambda_t \cdot (r_t - n) = -\dot{\lambda}_t$$

from which it follows:

$$\frac{\dot{\lambda}_t}{\lambda_t} = -(r_t - n)$$

Equating the two expressions for $\dot{\lambda}_t/\lambda_t$ above we get:

$$-\theta \cdot \frac{\dot{c}_t}{c_t} - (\rho - n) = -(r_t - n) \Rightarrow$$

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\theta}$$

which is the Keynes Ramsey rule (the Euler equation) describing the optimal evolution of consumption over time (in addition to this Euler condition the optimal solution also contains a transversality condition, which ensures that the level of consumption is consistent with the No Ponzi game condition).

The Keynes Ramsey rule describes the tradeoff of the representative household when it has to choose the optimal consumption pattern. On the one hand, the household prefers to smooth out consumption (since $\theta > 0$) which tends to make consumption constant over time. This motive is captured by the denominator in the expression. On the other hand, the numerator describes the net return to postponing consumption, which consists of the interest rate (the return to saving and thereby postponing consumption) minus the rate of time preference (the cost of postponing consumption).

If $r_t > \rho$ the representative household will take advantage of the high return to postponing consumption and accept that consumption is growing over time.

2)

Equation (B.2) states that in equilibrium the real interest rate equals the marginal product of capital net of depreciation. The equation reflects an arbitrage equation (of households), stating that the return to placing saving in physical capital (given by the rental rate net of depreciation, i.e. $R_t - \delta$) must equal the return to placing saving in financial capital (given by the interest rate: r_t), and an optimality condition stating that firms demand physical until the real rental rate equals the marginal product of capital, i.e. $R_t = f'(k_t)$

Equation (B.3) is the capital accumulation equation stating that the source of investment is saving (given by the part of income/output which is not consumed) since we consider a closed economy. Depreciation and labour force growth tends to decrease the amount of capital per effective worker, such that we need to subtract replacement investment.

In order to construct the phase diagram we can start by deriving the condition for $\dot{c}_t = 0$. From the Keynes Ramsey rule we see that:

$$\dot{c}_t = 0 \Rightarrow r_t = \rho$$

(since we ignore the trivial case where $c_t = 0$).

From equation (A.2) we see that $r_t = \rho$ implies that:

$$f'(k_t) - \delta = \rho$$

Thus we can define the steady state capital stock as:

$$f'(k^*) - \delta = \rho$$

which is illustrated in figure 2 as the vertical line.

Now let's consider the case where k_t isn't equal to k^*:

If $k_t < k^*$ then we know that $r_t = f'(k_t) - \delta$ is above ρ due to diminishing returns to capital ($f''(k_t) < 0$). From the Keynes Ramsey rule this implies that: $\dot{c}_t > 0$, since households will take advantage of the relative high return to saving and postpone consumption. If conversely $k_t > k^*$ then $r_t = f'(k_t) - \delta$ is below ρ and in that case $\dot{c}_t < 0$ since households bring forward consumption and plan with a consumption level which is falling over time.

The second steady state requirement is that $\dot{k}_t = 0$ which from (B.3) implies:

$$c_t = f(k_t) - g - k_t \cdot (n + \delta)$$

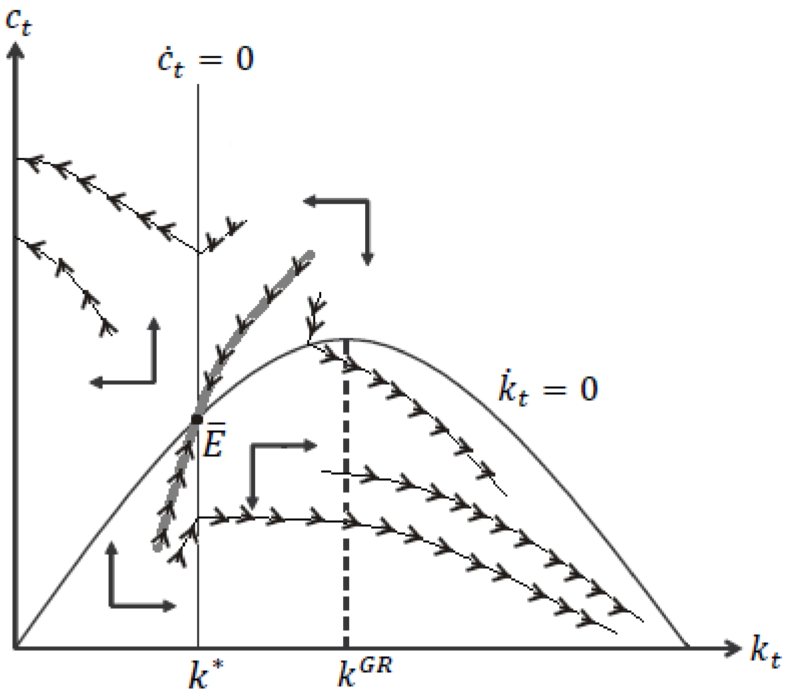
which is a concave relationship, as illustrated in the diagram below. Given that k_t is constant the level of consumption is maximized when k_t attains the golden rule level characterized by:

$$f'(k^{GR}) = n + \delta$$

The golden rule capital stock is higher than the steady state level since we have assumed from the beginning that: $\rho > n$ (which is necessary in order to secure that utility is bounded).

If c_t is initially above the $\dot{k}_t = 0$-curve then saving isn't sufficient to cover replacement investment and k_t falls over time. If c_t is below the $\dot{k}_t = 0$-curve then the amount of saving is more than sufficient to cover replacement investment and k_t increases over time.

The movements of the economy are illustrated in the diagram below.



The steady state is saddle path stable, i.e. the economy only converges to steady state if the economy initially starts out on the saddle path. Paths starting out above the saddle path can be ruled out since k_t will fall and eventually reach zero implying that output reach zero and that consumption jumps to zero. This can never be consistent with utility maximization. Paths starting out below the saddle path violate the transversality condition since in this case consumption will over time converge towards zero while the capital stock converges towards a positive value. Households realize that this inefficient overaccumulation can never be optimal, and thus these paths can be ruled out.

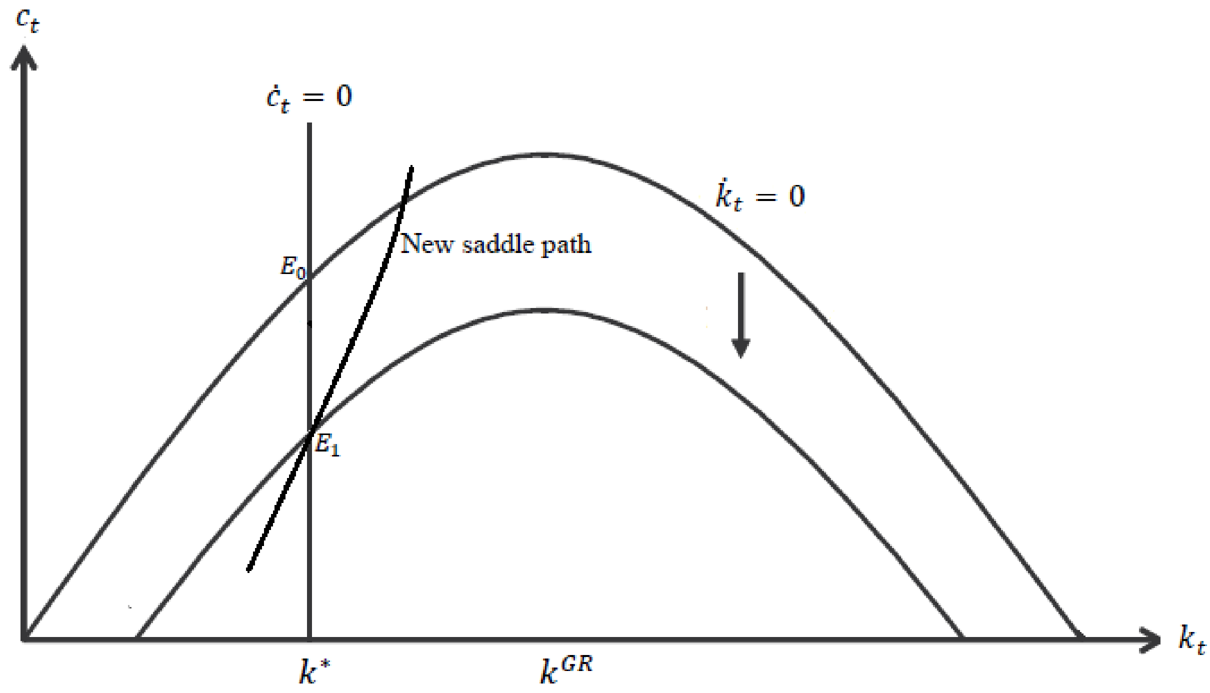
3)

The dynamic evolution of the economy is illustrated in the phase diagram below (for illustrational-purposes it is assumed that $g = 0$ initially - but that isn't important).

When g increases the economy immediately jumps from the old steady state to the new steady state, i.e. from point E_0 to E_1 in the diagram, since this is the only way that the economy can be located at the new saddle path immediately after the increase in g. There is no change in the capital stock, since it is determined by the condition:

$$f'(k^*) - \delta = \rho$$

which is unaffected by the increase in g. The economic effects are explained below:



The increase in g implies an equivalent increase in lump-sum taxes (T_t). Households respond to the prospect of an increase in taxes (and thus a corresponding fall in after-tax income) in all future periods by reducing private consumption by an equal amount.

The increase in public consumption will everything else equal reduce total saving per worker (given by $s_t = y_t - c_t - g = f(k_t) - c_t - g$) and thereby investment and capital accumulation. However, the fact that private consumption falls by an amount equal to the increase in g fully offsets the reduction in saving, implying that saving is unaffected by the increase in g.

4)

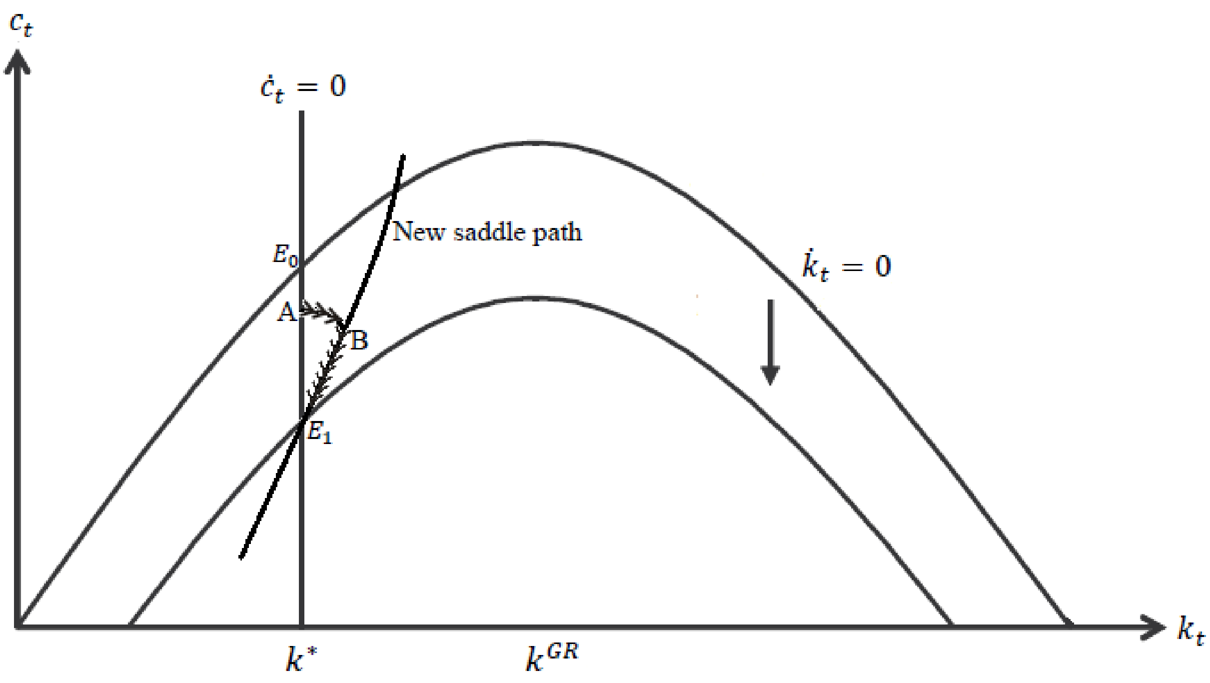
We now consider an increase in g which is announced in advance. The increase in g is announced at time t_0 and implemented at time $t_1 > t_0$.

In order to determine the dynamic evolution of the economy we notice four properties:

1) k_t is predetermined (determined by past accumulation of capital) and will only adjust gradualle over time

2) c_t is a jump variable. However, c_t can only jump at time t_0 (when the new information arrives) since households prefer a smooth consumption path and thus dislike discrete changes in consumption
3) At time t_1, when the increase in g is actually implemented, the economy must be located somewhere on the new saddle path (in order for the economy to converge towards the new steady state over time)
4) Between time t_0 and t_1 the economy is guided by the old dynamics, i.e. the dynamic system corresponding to the initial value of g

The resulting movements of the economy are illustrated in the phase diagram below. At time t_0 the economy jumps from point E_0 to A. Between announcement and implementation the economy moves from A to B, which is reached exactly at time t_1. After time t_1 the economy converges towards the new steady state along the new saddle path (the movement from B to E_1). The economic effects are explained below.



At time t_0 the economy jumps from E_0 to A. This is an announcement effect and steems from the fact that households realize that taxes will increase in the future (after t_1). Since the level of consumption depends on the present value of all future after-tax wage income, given by:

$$\int_{t=0}^{\infty} (w_t - T_t) \cdot e^{-\int_0^t (r_s - n) ds} \, dt$$

households reduce consumption, but by less than the increase in g (since T_t only increases for $t \geq t_1$). A longer time span between announcement and implementation (i.e. $t_1 - t_0$) makes the initial jump smaller.

The fall in private consumption implies that total saving (given by $s_t = f(k_t) - c_t - g$ as earlier mentioned) increases. The increase in saving increases investment and thereby capital accumulation. Thus, the capital stock increases from point A to B. In spite of the gradual increase in the capital stock (and thereby output and real wages) consumption falls from A to B. This fall can be explained in two ways:

- The fact that the economy moves closer in time to when taxes are actually increased implies that the present value of after-tax income tends to fall over time (the higher taxes are gradually discounted with lower discount rates)
- The increase in the capital stock (after point A) implies that the marginal product of capital falls, implying that the real interest rate is below the rate of time preference, which from the Keynes Ramsey rule implies that consumption must be falling over time.

At time t_1, when the increase in g is implemented, the economy is located at point B. The increase in public consumption reduces saving, which implies that saving isn't sufficient to cover replacement investment whereby the capital stock falls over time (from B to E_1). Consumption doesn't jump downwards at time t_1 since the increase in taxes was fully foreseen. However, consumption keeps falling gradually over time as the marginal product net of depreciation is still below the rate of time impatience. This can also be explained by the fact that the present value of future labour income falls over time since the gradual fall in the capital stock implies that the real wage is falling over time.

In the long run the economy converges towards E_1, where the capital stock is back at its old level. At E_1 the marginal product of capital net of depreciation is once again equal to rate of time preference and consumption is once again constant over time. Consumption has fallen (compared to the old steady state level) by an amount equal to the increase in taxes (public consumption).

5)

In this case the increase in public consumption is not financed by lump-sum taxes but instead by a distorting tax (on the return to capital) which reduces the return to saving and thereby the return to accumulating capital.

In addition to the movement of the $\dot{k}_t = 0$-curve (illustrated above) the vertical $\dot{c}_t = 0$-line moves to the left. Initially (right at time of implementation) the level of consumption may either decrease or increase (there are offsetting substitution and income effects due to the lower after-tax interest rate - but if consumption falls it will fall by less than the increase in public consumption due to the substitution effect), but in the long run consumption and the capital stock unambiguously falls. The fall in consumption is greater than in question 3) since the steady state capital stock also falls (which reduces the steady state wage level and household wealth and thus consumption).