Guide[1] to answers, Written Exam for the B.Sc. or M.Sc. in Economics

Microeconomics A, 2nd Year

February 2015

Problem 1

Consider a consumer who can consume non-negative, continuous quantities of two goods. The consumer has an exogenous income I, and faces the price system p.

- Prove that it is not possible to have two different consumption plans both maximizing preferences, when the consumer's preferences are strictly convex.

Answer: Suppose that both x and x' both maximize preferences, i.e. $px^* \leq I$, $px' \leq I$ and x* and x' are both weakly preferred to all consumption plans x that satisfy $px \leq I$. Any convex combination of the two, $x(\lambda)=\lambda x^*+(1-\lambda)x'$,with $0 \leq \lambda \leq 1$, will also satisfy the budget constraint, as $p[\lambda x^*+(1-\lambda)x'] = \lambda px^*+(1-\lambda)px' \leq I$. By strict convexity, however, $x(\lambda)$ will, , for $0 < \lambda < 1$, be strictly preferred to both x* and x", contradicting that these two both maximize preferences. Hence, we cannot find two different consumption plans maximizing preferences when preferences are strictly convex.*

Problem 2

Please comment on the following statement:
"When assessing the welfare impact on a consumer from a price increase for a certain good, measuring the change in Consumer's Surplus will tend to exaggerate the welfare impact when the good is normal, and, conversely, under-estimate it when the good is inferior".

Answer: When the good is normal, we know from the Slutsky equation that the derivative of the Marshall demand wrt price will be greater, in absolute value, than the derivative of the Hicksian demand (the Hicksian (compensated) demand curve is steeper (seen from the quantity axis) than the (uncompensated) Marshall demand curve). The most correct measure for the welfare impact is the Equivalating Variations measured by the change in area (integral) behind the Hicksian demand curve, and, using the above facts, this will be smaller than the change in the area behind the Marshall demand curve, identical to the change in CS. See Nechyba's graph 10.9.

Problem 3

Consider the following statements for a consumer and for a certain good. If you think the statement is true, please argue why. If you think the statement is false, please provide a clear counter-example

- a) If the consumer has an exogenous income, and the good is normal, we can be certain that a price increase will lower his or her consumption of the good
- b) If the consumer's income stems from private ownership of an endowment, and the good is normal, we can be certain that a price increase will lower his or her consumption of the good

[1] What is presented here is not a full, satisfactory answer to the problems, but indicates the correct results and important points to be made.

Answer: a) is true, as a price increase will erode the real value of the exogenous income, making the consumer poorer. Hence, the income effect will, like the substitution effect, lower the consumption of the good. However, b) is false, as the consumer may be a seller of the good the price of which has risen. Here, a price increase will increase the purchasing power, and with the good being normal, the over-all income effect will run counter to, and may dominate, the substitution effect. A predominant case would be the good being labor/leisure.

Problem 4

Consider the hairdressing salon ToughCut offering stylish haircuts, selling to a market characterized by perfect competition. It does so using two inputs, both of which we, for simplicity, assume may be used in continuous, non-negative quantities:

- Labor, the quantity of which can be changed in the short run
- Capital, the quantity of which can only be changed in the long run.

The production function is given by y = Min {L, K}. At the moment, the firm has a fixed quantity of capital which is $\underline{K}$ = 64, and it faces a wage rate of 4, whereas the rental price for capital is 4. The price for a haircut is p.

- a) Please identify the firm's short run supply curve, expressing this mathematically as well as in a clear diagram
- b) Please identify the firm's long run supply curve, expressing this mathematically as well as in a clear diagram
- c) Please compare and comment on the two supply curves

Answer: For output less than 64, output is constrained by the quantity of labor, so a haircut can be produced at constant MC and AVC of 4. Output cannot exceed 64, due to the capital constraint. Hence, the short-term supply curve is horizontal, at price 4, up to output 64, and vertical at output 64 (for prices larger than 4). In the long run, each haircut requires one unit of each input, the average and marginal cost in the long run hence being 8, so the long run supply curve is horizontal at price 8. The hairdresser is able to supply cheaper haircuts (at low output levels) in the short than in the long run, because only (variable) labor costs need to be covered.

Problem 5

Consider, for a consumer, the concept of "expenditure function".

- a) Define the expenditure function
- b) Describe how it is used in measuring changes in welfare for consumers
- c) Describe its mathematical properties

Answer: a) For a consumer with utility function u, the expenditure function is defined as the minimum income required, at price system p, to ensure the consumer the utility level u, i.e. e(p,u) has a price system (;-vector) and a utility level as arguments and results in a real number (income). The expenditure function is related to the Hicksian compensated demand, h(p,u), as e(p,u) = p·h(p,u), with h(p,u) solving the problem min p·x s.t. u(x)≥u.
b) The expenditure function can be used to determine how to compensate the consumer for price changes, for instance caused by new taxes. It is also used to measure compensating and equivalating variations, and to measure dead-weight loss.

c) The expenditure function is increasing (or at least non-decreasing) in u and in the coordinates of p. It is homogenous of degree 1 with respect to prices. The derivative of e wrt. p_m *is* $h_m(p, u)$ *(Shephard's Lemma).*

<u>Problem 6</u>

Consider a pure exchange economy consisting of two consumers who both can consume continuous non-negative quantities of two goods: Food (good 1) and drinks (good 2). There is a total initial endowment in the economy of 10 units of food and 20 units of drinks. Consumer A has the utility function $u_A(x_{1A}, x_{2A}) = 8 \cdot x_{1A} + 16 \cdot x_{2A}$, while B has $u_B(x_{1B}, x_{2B}) = 2 \cdot x_{1B} + x_{2B}$.

- Identify the efficient allocations for this economy
- Describe, for each allocation where this is possible, how an efficient allocation can be implemented as a market equilibrium (given by a set of market prices, using drinks as a numeraire, and two individual incomes)

Answer: The efficient allocations consist of the vertical left-hand side of the Edgeworth box and the upper horizontal side, i.e. A consumes all of good 2 and/or B consumes all of good 1. Formally, the allocations are $x_A = (0, x_{2A})$, $x_B = (10, 20 - x_{2A})$, $0 \leq x_{2A} \leq 20$, *and* $x_A = (10 - x_{1B}, 20)$, $x_B = (x_{1B}, 0)$, $0 \leq x_{1B} \leq 10$. *For the first group of allocations, the price system is determined by B's preferences, the relative price of good 1 being 2, so allocation* $x_A = (0, x_{2A})$, $x_B = (10, 20 - x_{2A})$ *and price system (2,1) implicate incomes* $I_A = x_{2A}$, *and* $I_B = 40 - 2x_{2A}$. *For the second group, the price system is determined by A's preferences, the relative price of good 1 being ½, so allocation* $x_A = (10 - x_{1B}, 20)$, $x_B = (x_{1B}, 0)$ *and price system (½,1) implicate* $I_A = 25 - ½x_{1B}$, *and* $I_B = ½x_{1B}$.

Mtn, 19 February 2015