Reexam summer 2014

Problem 1

Consider Laila that among many other goods and services enjoys leisure when she does not work. She earns a rate of w as an after-tax wage and can buy a composite consumption good at the price of p.

Laila has preferences representable by a quasi-linear utility function of the form $u(f, x) = v(f) + x$ where f is the amount of leisure enjoyed and x the amount of the composite consumption good, while the function $v(\cdot)$ is a differentiable, strictly increasing and concave function.

Laila can work no more than $\bar{L}$ hours and has no endowment of consumption goods initially.

Consider a pair of prices (p, w) such that the choice of Laila, $\big(f(w, p), x(w, p)\big)$, when maximizing utility, is in the interior of the consumption space, which is $\mathbb{R}^2_+$ and $f(w, p) < \bar{L}$.

Can we observe that Laila will work less when the wage rate increases (by a small amount)?

Answer: No, since the preferences are quasilinear and the solution is in the interior, the income effect on the leisure demand is zero and thus only the substitutioneffect works in the demand for leisure; and thus only the substitutioneffect works in the work supply of Laila; but the substitutioneffect is always negative, and as the price of leisure is the wage rate the leisure demand must decrease as the wage rate increases. Aternatively, we have that $\frac{w}{p} = v'(f^*)$, but when the wage rate increases the right hand side, the real wage, also increases, and thus the marginal utility of leisure must increase – which must imply that the consumer should enjoy less leisure, since $v'' < 0$ – and thus work more.

Problem 2

Each month Henning receives dkk 500 from his disability pension, after having been declared unable to work as a consequence of a working accident. Henning consumes, among many other things, Havanna cigars directly imported, which can be acquired at the price of $p_1 = dkk\ 10$ pr cigar.

His preferences are representable by a utility function

$$u(x_1, x_2) = 20 \ln x_1 + 2x_2$$

where x_2 is the consumption of other goods. The price of other goods is normalized to unity, e.g. $p_2 = 1$.

As part of a program to favor former colonies the European Union imposes a tax duty on Havanna Cigars, which means that the price of Havana Cigars rises to dkk 20.

a) Find the minimal amount with which Henning should be compensated to be as well off as before the tax duty.

Assume that the government, in order to compensate Henning, pays him the tax revenue collected from his import of cigars.

b) What should the compensation be? Is Henning as happy as before the tax levied and the compensation? Explain.

Answer: The demand function is $x(p_1, p_2, I) = \left(10\frac{p_2}{p_1}, \frac{I}{p_2} - 10p_2\right)$. Since the Marshall and the Hicks demand is the same off good one the compensated variation and the consumer surplus are identical; we can derive the last by $\Delta CS = \int_{10}^{20} \frac{10}{p_1}\, dp_1 = 10\left(\ln\frac{1}{2} - \ln 1\right) = 10 \ln\frac{1}{2} = -6{,}93$. The tax revenue is now $tx_1 = (20 - 10) * \frac{1}{2} = 5$, the demand with the compensation will be $x' = \left(\frac{1}{2}, 495\right)$ while the original demand is $x = (1{,}490)$, such that the utility is $u' = 20 \ln\frac{1}{2} + 2 * 495 = 976$ compared to the initial level of $u = 20 \ln 1 + 2 * 490 = 980$. We see that the utility decreases, which illustrates the efficiency loss, which is the loss in the tax collection due to the superiority of lump-sum taxes.

Problem 3

Consider the construction firm Digger A/S which sells construction services when building houses. The firm uses a technology given by the production function

$$f(\ell, k) = (\min\{\ell, 2k\})^{\frac{1}{2}}$$

where ℓ is the labour input and k is the number of digging machines. The output is measured as the amount of "cubic-meters hole".

Suppose that the firm can be hired at a price of $p = 10$ per cubic-meter hole, the going wage rate is $w = 2$ while the rental rate of digging machines is $r = 1$. Digger A/S is only a small firm in the industry and thus takes the prices on output and input as given.

a) Determine the cost function of the firm as a function of output
b) How many cubic-meters of holes maximize the profit of the firm? What is the profit of the firm?
c) If the price of holes increases to $p' = 15$, how does this impact the production and profits?
d) If the rental rate increases to $r' = 2$ (with output price $p = 10$), how does this impact the production and profits of the firm.
e) Consider the impact on the labour and capital demand of the firm in c) and d) compared to b). Explain the result.

Answer: We have that $\ell(x) = 2k(x) = x^2$ are the conditional factor demands such that $C(x) = w\ell(x) + rk(x) = 2x^2 + \frac{1}{2}x^2 = \frac{5}{2}x^2$ is the cost function. The profit maximizing production is then $10 = 5x$ or $x^* = 2$ and hence the profit is $\pi^* = 10$. Increasing the price yields an output of $x' = 3$ and profit becomes $\pi' = 15 * 3 - \frac{5}{2} * 9 = \frac{45}{2} = 22{,}5$. An increase in the rental rate alters the cost function to $C(x) = 4x^2$, such that the optimal production becomes $x'' = \frac{5}{4}$ and the profit $\pi'' = \frac{25}{2} - \frac{25}{4} = \frac{25}{4} = 6{,}25$. We see that $\ell^* = 4$,

$\ell' = 9$ and $\ell'' = \frac{25}{8} = 3{,}125$, while $k^* = 2$, $k' = \frac{9}{2}$ and $k'' = \frac{25}{16}$. Thus the increase in output price of one-third more than doubles the labour and capital input, which is due to the decreasing returns to scale such that the increase in production must have increasing proportions of input employment. When the output price increases, the price exceeds the marginal costs, and the firm can increase its profits by hiring more labour and capital to produce more holes and sell it at the higher rate. When the rental rate increases, the marginal costs increases which lowers the optimal production: then, in general, there are two effects on the labour demand: the substitution effect increases the demand, which is zero here, while the lower production decreases the demand for input.

Problem 4

Comment on the following statement:

"In any Pareto efficient allocation the marginal rates of substitution between any two goods must be equal between any two consumers."

Answer: False, since we need to make sure that the allocation is in fact an interior solution of each consumer. Another necessary condition is that the preferences are differentiable, such that the marginal rate of substitution is well-defined for each consumer. A good illustration will suffice to make the argument which should be clear that we consider a boundary solution.

Problem 5

Consider a Koopman economy with a single (representative) consumer with preferences representable by a utility function

$$u(f, x) = f \cdot x$$

and he has at his disposal a production technology given by

$$f(\ell) = \max\{0, \sqrt{\ell - 2}\}$$

The consumer can choose to work or have leisure in maximal $\bar{L} = 5$ hours since the rest must be used for resting.

a) Determine the unique Pareto efficient allocation.
b) Can this Pareto efficient allocation be implemented by a Walrasian equilibrium?

Answer: The Pareto efficient allocation is solved $\max_{\ell \geq 0}(12 - \ell)\sqrt{\ell - 2}$ which has $\ell^* = 3$ and $x^* = 1$, such that the real wage is $\frac{w}{p} = f'(\ell^*) = 1$ which yields a profit of $x^* - \frac{w}{p}\ell^* = -2$ which is negative. Thus, the Pareto efficient allocation cannot be supported by a Walrasian equilibrium. The reason is that the set of production possibilities is non-convex: a convex combination of $(\ell'', x'') = (0,0)$ and $(\ell', x') = (3,1)$, say $\alpha = \frac{1}{2}$ is not feasible production since $f\left(\frac{3}{2}\right) = 0 < \frac{1}{2}$.