Rettevejledning til

Eksamen på Økonomistudiet, Vinter 2011/2012

Reeksamen

Makro A og Macro A, 2. årsprøve

Efterårssemestret 2011

(Tre-timers prøve uden hjælpemidler)

Målbeskrivelse:

Faget videreudvikler langsigtsdelen af Økonomiske Principper 2, Makro.

I Makro A opstilles og analyseres alternative formelle modeller til forståelse af de langsigtede, trendmæssige tendenser i de vigtigste makroøkonomiske variable, såsom aggregeret indkomst og forbrug (per capita), indkomstfordeling, realløn og realrente, nettofordringsposition overfor udlandet, teknologisk niveau og produktivitet samt ledighed. I sammenhæng hermed præsenteres empirisk materiale under anvendelse af simple statistiske metoder.

Faget bygger op til Makro B ved at beskrive det forankringspunkt, økonomiens fluktuationer foregår omkring. Det bygger også op til Makro C ved at omfatte de mest fundamentale versioner af de langsigtsmodeller, som også indgår i Makro C.

De studerende skal lære de vigtigste såkaldte stiliserede empiriske fakta om økonomisk vækst og strukturel ledighed at kende, og de skal kende til og forstå den række af økonomisk teoretiske modeller, som i kurset inddrages til forklaring af disse fakta og til forståelse af økonomiens trendmæssige udvikling i det hele taget.

En vigtig kundskab, der begyndende skal erhverves i dette kursus, er selvstændig opstilling og analyse af formelle, makroøkonomiske modeller, som af type er som kendt fra faget, men som kan være variationer heraf. Der vil typisk være tale om modeller, som er formulerede som, eller er tæt på at være formulerede som, egentlige generelle ligevægtsmodeller. En del af denne kundskab består i en verbal formidling af en forståelse af modellernes egenskaber.

En anden vigtig kundskab er at kunne koble teori og empiri, så empirisk materiale kan tilvejebringes og analyseres på en måde, der er afklarende i forhold til teorien. Igen er verbal formidling af de konklusioner, der kan drages ud af samspillet mellem teori og empiri, en vigtig del af den beskrevne færdighed.

De typer af modeller, der skal kunne analyseres, omfatter modeller for lukkede såvel som for åbne økonomier, statiske såvel som dynamiske modeller, dynamiske modeller med såvel diskret tid som kontinuert tid. Modellerne skal både kunne analyseres generelt og ved numerisk simulation (sidstnævnte dog kun af ikke-stokastiske dynamiske modeller i diskret tid).

De studerende skal opnå færdigheder i at foretage økonomiske analyser i de typer af modeller, faget beskæftiger sig med, herunder analyser af strukturelle, økonomisk politiske indgreb og formidle analysens indsigter.

Topkarakteren 12 opnås, når de beskrevne færdigheder mestres til en sådan fuldkommenhed, at den studerende er blevet i stand til selvstændigt at analysere nye (fx økonomisk politiske) problemstillinger ved egen opstilling og analyse af varianter af de fra kurset kendte modeller under inddragelse og analyse af relevant empiri og afgive absolut fyldestgørende verbal forklaring af de opnåede analyseresultater.

Problem 1.

Relevant chapters of the text book are Chapter 3 and Chapter 4.

1.1. In usual notation and setting the total factor productivity to one, the long run values of the capital intensity and the real interest rate according to the basic Solow model are:

$$k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$\rho^* = \underbrace{\alpha\left(\frac{s}{n+\delta}\right)^{-1}}_{r^*} - \delta$$

(It is not important to state these equations; the explanations are important). Savings add to capital and thereby to capital per worker, whereas growth in the labour force and depreciation subtracts from capital per worker. Therefore, the higher the savings rate is the more capital per worker will be built up in the long run, and likewise, the higher the labour force growth rate and the rate of depreciation are the less capital per worker will be built up in the long run.

The real interest rate, ρ_t, is the real rental rate of capital, r_t, minus the rate of depreciation, $\rho_t = r_t - \delta$. The real rental rate of capital is given by the marginal productivity of capital which decreases with the capital intensity. Therefore a higher savings capacity as measured by $s/(n+\delta)$, which gives a higher capital intensity in the long run, also gives a lower real rental rate of capital in the long run. This will tend also to give a lower real interest rate ($\partial\rho^*/\partial\delta$ itself is positive if $\alpha/s > 1$, which is plausible). The long run (natural) real rental and interest rates are also increasing in α reflecting that α has a direct positive influence on the marginal productivity of capital.

1.2 Before the opening, the savings strong Country 1 will have a higher capital intensity and therefore a lower real interest rate than Country 2. Upon opening capital will flow from Country 1 to Country 2 taking advantage of the higher interest rate in the latter. Thereby the capital intensities, the marginal productivities of capital and therefore the interest rates will be equalized between the two countries. The new and lower capital intensity in Country 1 will imply lower real wages there, while the new and higher capital intensity in Country 2 will imply higher real wages there. In a new steady state, Country 1 will be a net creditor while Country 2 will be a net debtor.

1.3 It is indeed possible for both countries to obtain higher national income per worker. Before the opening, the total, international amount of capital is not allocated efficiently between the countries because the marginal productivities of capital are different between the countries. More production can be obtained by the same amounts of inputs by moving capital from where it has a low marginal productivity (Country 1) to where it has a high marginal productivity (Country 2). This is exactly what free capital movements can accomplish: capital seeks to the high interest rates which are just reflections of high marginal productivities of capital.

Problem 2.

Model repeated for convenience:

$$\begin{aligned}
Y_t &= \left(K_t^d\right)^{\alpha}\left(H_t^d\right)^{\varphi}\left(A_t L_t^d\right)^{1-\alpha-\varphi}, \quad 0<\alpha<1, \quad 0<\varphi<1, \quad \alpha+\varphi<1 && (1)\\
A_t &= \left(\frac{K_t}{L_t}\right)^{\frac{\alpha}{\alpha+\varphi}}\left(\frac{H_t}{L_t}\right)^{\frac{\varphi}{\alpha+\varphi}} && (2)\\
K_t^d &= K_t && (3)\\
H_t^d &= H_t && (4)\\
L_t^d &= L_t && (5)\\
K_{t+1} &= s_K Y_t + (1-\delta) K_t, \quad 0<s_K<1, \quad 0<\delta<1 && (6)\\
H_{t+1} &= s_H Y_t + (1-\delta) H_t, \quad 0<s_H<1, \quad s_K+s_H<1 && (7)\\
L_{t+1} &= (1+n) L_t, \quad n>-1 && (8)
\end{aligned}$$

2.1 It seems natural to compare to the model of endogenous growth based on productive externalities known from the text book. In this, there is no human capital in the production function, Eq. (1) here, and the productive externality goes from the use of physical capital, K_t, to factor productivity, A_t, in the form: $A_t = K_t^{\phi}$, where $0 < \phi \leq 1$, and $\phi = 1$ implies (truly) endogenous growth, whereas $\phi < 1$ implies semi-endogenous growth. The motivation for the externality is "learning by doing": Workers become more productive from working with capital and the increased skills are kept if the worker in question is separated from the capital. Since in the long run workers are not tied to a specific firm, the use of aggregate capital gives rise to a productive effect that is external to the individual firm.

There are three distinguishing features here: 1) The externality goes from both physical and human capital. The learning by doing argument can be extended to human capital assuming that workers become more productive from working with more educated colleagues. The motivation thus seems as good or bad as in the case of only physical capital. 2) The externality arises from capital per worker, not from capital in total. One can argue in favour of either. It is well known that the per-worker formulation is a way of getting rid of very problematic scale effects, but at a cost, see below. 3) The two exponents in Eq. (2) add up to one. A more general formulation would be:

$$A_t = \left[\left(\frac{K_t}{L_t}\right)^{\frac{\alpha}{\alpha+\varphi}}\left(\frac{H_t}{L_t}\right)^{\frac{\varphi}{\alpha+\varphi}}\right]^{\phi}, \quad \phi \leq 1,$$

so the formulation in (2) is the particular case, $\phi = 1$. This means that the assumed productive externality is very strong and one would expect that the model can exhibit (truly) endogenous growth.

2.2 Inserting (2)-(5) into (1) gives:

$$Y_t = K_t^{\alpha} H_t^{\varphi}\left(\overbrace{\left(\frac{K_t}{L_t}\right)^{\frac{\alpha}{\alpha+\varphi}}\left(\frac{H_t}{L_t}\right)^{\frac{\varphi}{\alpha+\varphi}}}^{=A_t} \cdot L_t\right)^{1-\alpha-\varphi}$$

Rewriting:

$$\begin{aligned}
Y_t &= K_t^{\alpha} H_t^{\varphi} \left(K_t^{\frac{\alpha}{\alpha+\varphi}} H_t^{\frac{\varphi}{\alpha+\varphi}} L_t^{1-\frac{\alpha}{\alpha+\varphi}-\frac{\varphi}{\alpha+\varphi}} \right)^{1-\alpha-\varphi} \\
&= K_t^{\alpha} H_t^{\varphi} \left(K_t^{\frac{\alpha}{\alpha+\varphi}} H_t^{\frac{\varphi}{\alpha+\varphi}} L_t^{1-1} \right)^{1-\alpha-\varphi} \\
&= K_t^{\alpha+\frac{\alpha}{\alpha+\varphi}(1-\alpha-\varphi)} H_t^{\varphi+\frac{\varphi}{\alpha+\varphi}(1-\alpha-\varphi)} \\
&= K_t^{\frac{\alpha(\alpha+\varphi)+\alpha(1-\alpha-\varphi)}{\alpha+\varphi}} H_t^{\frac{\varphi(\alpha+\varphi)+\varphi(1-\alpha-\varphi)}{\alpha+\varphi}} \\
&= K_t^{\frac{\alpha}{\alpha+\varphi}} H_t^{\frac{\varphi}{\alpha+\varphi}}
\end{aligned}$$

This shows:

$$Y_t = K_t^{\nu} H_t^{1-\nu}, \quad \nu \equiv \frac{\alpha}{\alpha+\varphi} \tag{9}$$

Dividing on both sides by L_t gives:

$$\frac{Y_t}{L_t} = \frac{K_t^{\nu} H_t^{1-\nu}}{L_t^{\nu} L_t^{1-\nu}} = \left(\frac{K_t}{L_t}\right)^{\nu} \left(\frac{H_t}{L_t}\right)^{1-\nu} \iff$$

$$y_t = k_t^{\nu} h_t^{1-\nu} \tag{10}$$

There are constant returns to scale to the (produced) inputs of physical and human capital alone. From the derivation it is clear that this is due to the hidden parameter ϕ being one. It thus follows from the assumption of a very strong productive externality. In Eqs. (9) and (10) there is not a coefficient, B say, on $K_t^{\nu} H_t^{1-\nu}$ or $k_t^{\nu} h_t^{1-\nu}$, where B depends positively on labour input, that is, there is no problematic scale effect. From the derivation it is clear that this follows from the externality arising from capital per worker (and not just from capital). On the other hand, the same feature combined with the degree of the externality being as strong as it is implies that at the aggregate level labour input is not productive at all. The positive productivity in the individual firm present in Eq. (1) cancels out in the aggregate production function (9) by the L_t in the denominators of the externality (2).

2.3 By definition:

$$x_{t+1} = \frac{K_{t+1}}{H_{t+1}}$$

Inserting (6) and (8) and then (9) gives:

$$x_{t+1} = \frac{s_K Y_t + (1-\delta) K_t}{s_H Y_t + (1-\delta) H_t} = \frac{s_K K_t^{\nu} H_t^{1-\nu} + (1-\delta) K_t}{s_H K_t^{\nu} H_t^{1-\nu} + (1-\delta) H_t}$$

Dividing by H_t in both numerator and denominator gives:

$$x_{t+1} = \frac{s_K K_t^{\nu} H_t^{-\nu} + (1-\delta) \frac{K_t}{H_t}}{s_H K_t^{\nu} H_t^{-\nu} + (1-\delta)} = \frac{s_K \left(\frac{K_t}{H_t}\right)^{\nu} + (1-\delta) \frac{K_t}{H_t}}{s_H \left(\frac{K_t}{H_t}\right)^{\nu} + (1-\delta)}$$

Using here $x_t = K_t/H_t$ gives:

$$x_{t+1} = \frac{s_K x_t^{\nu} + (1-\delta) x_t}{s_H x_t^{\nu} + (1-\delta)} \tag{11}$$

2.4 Setting $x_{t+1} = x_t = x$ in (11) and solving for x restricting to $x > 0$ gives:

$$
\begin{aligned}
x = \frac{s_K x^{\nu} + (1-\delta)\, x}{s_H x^{\nu} + (1-\delta)} &\Longleftrightarrow \\
x\left[s_H x^{\nu} + (1-\delta)\right] = s_K x^{\nu} + (1-\delta)\, x &\Longleftrightarrow \\
s_H x^{1+\nu} + (1-\delta)\, x = s_K x^{\nu} + (1-\delta)\, x &\Longleftrightarrow \\
s_H x^{1+\nu} = s_K x^{\nu} &\Longleftrightarrow \\
x = \frac{s_K}{s_H} \equiv x^* &
\end{aligned} \tag{12}
$$

To show convergence to x^* from any initial $x_0 > 0$ it suffices to show that:

1) The transition curve, i.e., x_{t+1} as a function of x_t, passes through (0,0): It follows directly from (11) that for $x_t = 0$, also $x_{t+1} = 0$.

2) The transition curve is increasing. This follows directly from a rewriting of (11):

$$
x_{t+1} = \frac{s_K + (1-\delta)\, x_t^{1-\nu}}{s_H + (1-\delta)\, x_t^{-\nu}}
$$

3) The transition curve has a unique, strictly positive intersection with the 45°−line: Shown above.

4) This intersection is from above which can be shown as follows. By differentiation:

$$
\begin{aligned}
\frac{dx_{t+1}}{dx_t} &= \frac{\left[s_H x_t^{\nu} + (1-\delta)\right]\left[s_K \nu x_t^{\nu-1} + (1-\delta)\right] - \left[s_K x_t^{\nu} + (1-\delta)\, x_t\right] s_H \nu x_t^{\nu-1}}{\left[s_H x_t^{\nu} + (1-\delta)\right]^2} \\
&= \frac{(1-\delta)\, s_K \nu x_t^{\nu-1} + \left[s_H x_t^{\nu} + (1-\delta)\right](1-\delta) - (1-\delta)\, x_t s_H \nu x_t^{\nu-1}}{\left[s_H x_t^{\nu} + (1-\delta)\right]^2} \\
&= (1-\delta)\, \frac{s_K \nu x_t^{\nu-1} + s_H x_t^{\nu} + (1-\delta) - s_H \nu x_t^{\nu}}{\left[s_H x_t^{\nu} + (1-\delta)\right]^2} \\
&= (1-\delta)\, \frac{s_K \nu x_t^{\nu-1} + (1-\delta) + (1-\nu)\, s_H x_t^{\nu}}{\left[s_H x_t^{\nu} + (1-\delta)\right]^2}
\end{aligned}
$$

For $x_t \to 0$, this derivative goes to infinity because of the presence of the term $s_K \nu x_t^{\nu-1}$ in the numerator and $0 < \nu < 1$.

2.5 We start from the definition of g_t^k and proceed by using definitions and model relations:

$$
\begin{aligned}
g_t^k &= \frac{k_{t+1} - k_t}{k_t} = \frac{k_{t+1}}{k_t} - 1 \\
&= \frac{\frac{K_{t+1}}{L_{t+1}}}{\frac{K_t}{L_t}} - 1 = \frac{\frac{s_K Y_t + (1-\delta) K_t}{(1+n) L_t}}{k_t} - 1 \\
&= \frac{1}{1+n} \frac{s_K y_t + (1-\delta)\, k_t}{k_t} - 1 \\
&= \frac{1}{1+n}\left[s_K \frac{y_t}{k_t} + (1-\delta)\right] - 1
\end{aligned}
$$

Finally we insert that from (10), $y_t/k_t = k_t^{\nu} h_t^{1-\nu}/k_t = k_t^{\nu-1} h_t^{1-\nu} = (k_t/h_t)^{\nu-1}$:

$$
g_t^k = \frac{1}{1+n}\left[s_K \left(\frac{k_t}{h_t}\right)^{\nu-1} + 1 - \delta\right] - 1 \tag{13}
$$

By similar operations:

$$g_t^h = \frac{1}{1+n}\left[s_H\left(\frac{k_t}{h_t}\right)^{\nu} + 1 - \delta\right] - 1 \tag{14}$$

2.6 We have shown that $x_t = k_t/h_t$ converges to $x^* = s_K/s_H$ in the long run. Therefore from (13):

$$\begin{aligned} g_t^k &\rightarrow \frac{1}{1+n}\left[s_K\left(x^*\right)^{\nu-1} + (1-\delta)\right] - 1 = \frac{1}{1+n}\left[s_K\left(\frac{s_K}{s_H}\right)^{\nu-1} + (1-\delta)\right] - 1 \\ &= \frac{1}{1+n}\left[s_K^{\nu} s_H^{1-\nu} + (1-\delta)\right] - 1 \equiv g_e \end{aligned} \tag{15}$$

and (14):

$$\begin{aligned} g_t^h &\rightarrow \frac{1}{1+n}\left[s_H\left(x^*\right)^{\nu} + (1-\delta)\right] - 1 = \frac{1}{1+n}\left[s_H\left(\frac{s_K}{s_H}\right)^{\nu} + (1-\delta)\right] - 1 \\ &= \frac{1}{1+n}\left[s_K^{\nu} s_H^{1-\nu} + (1-\delta)\right] - 1 \equiv g_e \end{aligned} \tag{15}$$

From (10) follows:

$$\frac{y_{t+1}}{y_t} = \frac{k_{t+1}^{\nu} h_{t+1}^{1-\nu}}{k_t^{\nu} h_t^{1-\nu}} = \frac{k_{t+1}^{\nu}}{k_t^{\nu}} \cdot \frac{h_{t+1}^{1-\nu}}{h_t^{1-\nu}} = \left(\frac{k_{t+1}}{k_t}\right)^{\nu} \cdot \left(\frac{h_{t+1}}{h_t}\right)^{1-\nu} \Leftrightarrow$$

$$1 + g_t^y = \left(1 + g_t^k\right)^{\nu} \cdot \left(1 + g_t^h\right)^{1-\nu}$$

Since both of g_t^k and g_t^h go to g_e in the long run, the right hand side goes to $(1+g_e)^{\nu} \cdot (1+g_e)^{1-\nu} = 1 + g_e$. Therefore the left hand side also goes to $1 + g_e$ and hence $g_t^y \rightarrow g_e$.

2.7 The model can exhibit (truly) endogenous growth, not only semi-endogenous growth: It is seen directly from (15) that the common growth rate g_e of k_t, h_t and y_t can be positive also if $n = 0$. This is due to the character of the productive externality where the hidden parameter ϕ has been set equal to one.

For a plausible numerical evaluation on annual basis one could let $n = 0$, $s_K = s_H = 0.2$ (investment rates in physical capital around 20 per cent are reasonable and it is also reasonable to assume somewhat similar investment rates in physical and human capital) and $\delta = 0.1$ (admittedly a bit high). In that case $g_e = 0.1$ corresponding to an annual growth rate of income per worker of 10 per cent. This is extremely high compared to empirical growth rates and not realistic over very long periods. Clearly, for plausible parameter values $g_e > 0$ is reasonable. The too high (endogenous) growth rate we end up with is a reflection of the productive externality assumed being very (and too) strong.

The model does not have a problematic scale effect where positive growth of the labour force ($n > 0$) implies exploding growth in income per worker. This is a consequence of the productive externality arising from K_t/L_t and H_t/L_t rather than from just K_t and H_t. On the other hand, the same feature combined with the strong degree of the externality is responsible for labour input not being productive at all in aggregate production as explained in Question 2.2.

2.8 Figure 1 builds on an assumption of $\nu = \frac{1}{2}$. This is plausible since it follows from $\alpha = \varphi$, normally considered a reasonable assumption. Clearly, the figure is in nice accordance with the results above: the main result is exactly that $s_K^{\nu} s_H^{1-\nu}$ has a positive long run influence on the growth rate of income per worker. Hence, the empirical material presented does not contradict the theory of endogenous growth considered. It does not prove it either. First, in the exogenous growth model with human capital an increase in the investment rates would give a period of higher *transitory* growth in income per worker. It could be that the countries with high average investment rates systematically have experienced increases in these. The picture could therefore simply reflect transitory growth effects from a model of exogenous growth. Second, the correlation shown in Figure 1 could be an expression of reversed causality or spurious correlation. These are important reservations. Nevertheless, the correlation of Figure 1 *does* in itself give some reason to be interested in the model of (truly) endogenous growth considered.