LM August 2019



① 1) $V \subseteq U$ og $\dim(V) =$ antal basis-vektorer $= 2$.

2) $u_1 + u_2 = (1,1,0,0)$ mht $u_1 u_2 u_3 u_4$

$$L(u_1+u_2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ mht } v_1, v_2$$

3)

$$(2,1) = 2v_1 + 1v_2 = 2(u_1 - u_2) + 1(u_1 - u_3 + u_4)$$
$$= 3u_1 - 2u_2 - u_3 + u_4$$
$$= (3,-2,-1,1) \text{ mht } u_1, u_2, u_3, u_4.$$

4) $LX = \underline{0}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \longleftrightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} x_3 = s \\ x_4 = t \end{array}$$

$$x_2 = -x_4 = -t$$
$$x_1 = -x_3 = -s$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{w_1} + t \underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}}_{w_2}, \; s,t \in \mathbb{R} \quad \left(\text{mht } u_1 u_2 u_3 u_4\right)$$

Ej inj.

$N(L) = \operatorname{span}\{w_1, w_2\}$. $4 - 2 = 2$

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5) $\underline{x} =$
$-2u_1 + u_2 + 2u_3 - u_4 = (-2, 1, 2, -1)$ mht $u_1, u_2, u_3, u_4$

$$L\underline{x} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \underline{0}$$

dvs $\underline{x} \in N(L)$.

$$\alpha_1 w_1 + \alpha_2 w_2 = \underline{x}$$

$$\left[\begin{array}{cc|c} -1 & 0 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right] \Leftrightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Dvs $\underline{x} = (2, -1)$ mht. $w_1, w_2$.

6) $v_1 + v_2 = (1, 1)$ mht $v_1, v_2$

$L\underline{x} = v_1 + v_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{array}\right] \Leftrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

Dvs $x_3 = s$, $x_4 = t$

$x_2 = 1 - t$, $x_1 = -s$ så

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad s, t \in \mathbb{R}.$$

mht $u_1, u_2, u_3, u_4$

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2)

i) Da A sym er $v_1 \perp v_3$ og $v_2 \perp v_3$ så

$V_1 \cdot (x_1, x_2, x_3) = x_1 = 0$ og

$v_2 \cdot (x_1, x_2, x_3) = \cancel{x_1} \cdot x_2 + 2x_3 = 0$

Dvs $v_3 = (0, -2, 1)$ er en mulighed.

2) Da $Av_1 = \begin{bmatrix} a & 0 & 0 \\ 0 & a & a \\ 0 & a & b \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

er $v_1$ egenvektor hørende til $\lambda = a$.

$Av_2 = \begin{bmatrix} a & 0 & 0 \\ 0 & a & a \\ 0 & a & b \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3a \\ a+2b \end{bmatrix} = \lambda \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

Heraf ses, at $v_2$ er egenvektor hørende til $\lambda = 3a$. Så må $a + 2b = 3a \cdot 2$ dvs $2b = 5a$.

3) $\text{spor} = 2a + b = a + 3a + \lambda_3$, dvs

$-2a + b = \lambda_3 \Leftrightarrow -4a + 2b = 2\lambda_3$

$-4a + 5a = 2\lambda_3 \Leftrightarrow \lambda_3 = \frac{a}{2}$.

dvs $a, 3a$ og $\frac{a}{2}$.

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4) $\det(A) = a \cdot 3a \cdot \frac{a}{2} \neq 0$, dvs $A$ inv.

5) $e^{A}(v_1+v_2) = e^{A}v_1 + e^{A}v_2 = e^{a}v_1 + e^{3a}v_2$

$(e^{a},0,0) + (0,e^{3a},2e^{3a}) = (e^{a},e^{3a},2e^{3a})$

(3) $\int \sin^2(2x)\sin(3x)dx$

1) $= \frac{-1}{8i}\int (e^{i4x}+e^{-i4x}-2)(e^{i3x}-e^{-i3x})dx$

$= -\frac{1}{4}\int \sin(7x) - 2\sin(3x) - \sin(x)\, dx$

$= -\frac{1}{4}\left(-\frac{1}{7}\cos(7x) + \frac{2}{3}\cos(3x) + \cos(x)\right) + K$

2) $z^2 = x^2 - y^2 + i2xy = \sqrt{3} - i$ $\qquad x^2 - y^2 = \sqrt{3}$, $\quad 2xy = -1$

$y = -\frac{1}{2x}$ så $x^2 - \left(-\frac{1}{2x}\right)^2 = \sqrt{3}$

$x^2 - \frac{1}{4x^2} = \sqrt{3} \iff 4x^4 - 4\sqrt{3}x^2 - 1 = 0$ så

$\underline{u = x^2 > 0}$ $\quad u = \frac{4\sqrt{3} \pm \sqrt{16\cdot 3 + 16}}{8} = \frac{4\sqrt{3}+8}{8}$

$x^2 = \frac{\sqrt{3}}{2} + 1$

$x = \pm\sqrt{\frac{\sqrt{3}}{2}+1}$, $\quad z = \pm\left(\sqrt{\frac{\sqrt{3}}{2}+1} + i\frac{-1}{2\sqrt{\frac{\sqrt{3}}{2}+1}}\right)$

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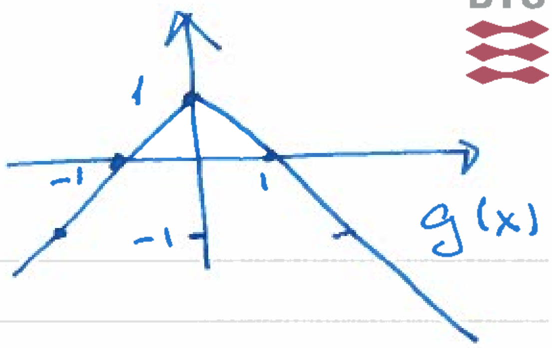
4) $g(x) = -|x|+1$

$|g(x)| < 1$




$-1 < -|x|+1 < 1$

$-1 < -|x|+1$ og $-|x|+1 < 1$

$|x| < 2$ $\quad$ $0 < |x|$

dvs $\underline{-2<x<2}$ $\quad$ $\underline{x \neq 0}$

1) dvs $Dm(f) = ]-2,0[ \cup ]0,2[$ .

2) $f(x) = \dfrac{1}{1-(-|x|+1)} = \dfrac{1}{|x|}$

3) Da $g$ er voksende i $]-2,0[$ og aftagende i $]0,2[$ gælder det samme for $f$.

4) For $x \to \pm 2$ vil $f(x) \to \frac{1}{2}$ $\quad$ $Vm = \underline{\underline{]\frac{1}{2},\infty[}}$

For $x \to 0^{\pm}$ vil $f(x) \to \infty$

åbenlyst ikke injektiv (da $f(x) = f(-x)$.)

5) $f(x) = y$ $\quad$ $\dfrac{1}{|x|} = y \Leftrightarrow |x| = \dfrac{1}{y}$

$\Leftrightarrow \underline{\underline{x = \pm \dfrac{1}{y}}}$