Macro III - exam solutions (August 9, 2019)

General remarks

Please grade each item of each question between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

1 False or uncertain. An aging population would require either that benefits be cut while contributions remain constant, or benefits remain constant and contributions increase, or a mix of benefit cuts and contribution increases. It is through the political process that society chooses the way in which a social security system adapts to aging. And if benefits are not to be reduced, it is not necessary to give workers incentives to increase private savings.

2 False. Ricardian equivalence relates to whether a change in the finance policy of the government has real effects or not. So Ricardian equivalence might fail to hold even if there is no change in the current account. Furthermore, if the economy is adequately described by an OLG population structure with no bequest motive, then a reduction in current taxes will affect consumption behavior and cause a deficit in the current account.

3 False. A distortionary capital income tax discourages saving by reducing its net rate of return. Thus, after the replacement of the distortionary tax by a lump sum alternative saving will increase. The desire to smooth consumption will make consumers to increase their saving, and thus reduce consumption, at the announcement and not wait for the tax to be replaced.

4 a) The Lagrangian is given by (note that the problem can be solved using the intertemporal budget constraint)

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \frac{x_t^{1-\epsilon}}{1-\epsilon} + \lambda_t \left(w_t(1-x_t) + R_t k_t - c_t - k_{t+1} \right) \right]$$

The first order conditions are given by

$$\begin{aligned}
\frac{d\mathcal{L}}{dc_t} &= 0 \longrightarrow \frac{1}{c_t} - \lambda_t = 0, \\
\frac{d\mathcal{L}}{dx_t} &= 0 \longrightarrow x_t^{-\epsilon} - \lambda_t w_t = 0, \\
\frac{d\mathcal{L}}{dk_{t+1}} &= 0 \longrightarrow -\lambda_t + \beta R_{t+1}\lambda_{t+1} = 0
\end{aligned}$$

The Euler equation is given by,

$$\frac{1}{c_t} = \beta R_{t+1}\frac{1}{c_{t+1}}.$$

The interpretation is that the household makes consumption saving choices such that the marginal rate of substutition between current and future consumption equals the marginal rate of transformation, R_{t+1}. An Euler equation can be written for intertemporal leisure choices, but it is not required that the students derive it.

In steady state, $c_t = c^*$, $x_t = x^*$ and $k_{t+1} = k^*$. From the Euler equation we find that steady state capital is determined, as usual in this setting, by (here we use that the rental rate for capital, r, is equal to the marginal product of capital, $\alpha\kappa^{\alpha-1}$)

$$R^* = 1 + r^* = 1 + \alpha\left(\frac{k^*}{1-x^*}\right)^{\alpha-1} = 1 + \alpha\kappa^{*\alpha-1} = \frac{1}{\beta}. \tag{1}$$

where $\kappa = \frac{K}{L}$ is the aggregate (or average) capital per unit of labor. Note that $\kappa \equiv \frac{k}{1-x}$, we make this distinction since some results are neater with κ instead of k, but these are equivalent measures in the steady state.

Given κ^* steady state wages are given by $w^* = (1-\alpha)\kappa^{*\alpha}$. Steady state consumption and leisure are characterized by the following system of two equations in two unknowns

$$\begin{aligned}
c^* &= w^*(1-x^*) + \alpha\kappa^{*\alpha-1}k^* = \kappa^{*\alpha}(1-x^*) && (2) \\
c^* &= w^* x^{*\epsilon} = (1-\alpha)\kappa^{*\alpha}x^{*\epsilon} && (3)
\end{aligned}$$

b) An increase in ϵ corresponds to an increase in the concavity of preferences for leisure. Capital per unit of labor is unaffected since the condition that determines κ^* only depends on β. Combining the two equations that characterize c^* and x^* we get that

$$\frac{x^{*\epsilon}}{1-x^*} = \frac{1}{1-\alpha}. \tag{4}$$

Since both x^* and ϵ are between zero and one, an increase in ϵ reduces the left hand side of this equation for the initial value of x^*. Thus, x^* has to increase. This implies that c^* is lower (from (2)). Finally, note that because κ^* is capital per unit of labor, and labor supply is reduced, capital per worker, k^*, is lower.

c) The presence of capital income taxes reduces the return perceived by households from saving. Denoting τ the (presumed constant) capital income tax, equation (1) is now given by

$$R^* = 1 + r^*(1-\tau) = 1 + \alpha\left(\frac{k^*}{1-x^*}\right)^{\alpha-1}(1-\tau) = 1 + \alpha {\kappa^*}^{\alpha-1}(1-\tau) = \frac{1}{\beta}.$$

Thus, unambiguously κ^* is depressed.

There is no change in household income, since they receive the receipts of taxation as a lump sum transfer. Thus, equation (2) derived from the resource constraint does not change. And the choice between consumption and leisure is also unaffected as capital taxation only affects intertemporal choices. Thus, equation (3) does not change. From the analysis done in b), (see (4)), we know that x^* is unaffected by capital income taxation. Thus, k^* is lower (as $\kappa = \frac{k}{1-x}$), and c^* is also lower as can be seen from (2).

5 a) Characterizing individual saving behavior requires setting up the problem of workers.

$$\begin{aligned} \max_{s_t, c_{1t}, c_{2t+1}} \quad & \ln(c_{1t}) + \beta\ln(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t} = w_t(1-\tau) - s_t \\ & c_{2t+1} = s_t r_{t+1} + \tau w_{t+1} \end{aligned}$$

Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \beta r_{t+1} c_{1t} \tag{5}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{\beta}{1+\beta} w_t(1-\tau) - \frac{1}{(1+\beta)}\frac{1}{r_{t+1}}\tau w_{t+1} \tag{6}$$

To get capital accumulation, since there is no population growth, we replace individual savings with next period capital per worker $k_{t+1} = s_t$, and we use equilibrium expressions for wage and interest rates, $w = (1-\alpha)Ak^\alpha$, $r = \alpha A k^{\alpha-1}$)

$$k_{t+1} = \left[\frac{\beta}{1+\beta}(1-\alpha)Ak_t^\alpha(1-\tau) - \frac{1}{(1+\beta)}\frac{(1-\alpha)k_{t+1}}{\alpha}\tau\right]$$

Combining terms with k_{t+1}

$$k_{t+1} = \frac{1}{\left[1 + \frac{1}{(1+\beta)}\frac{1-\alpha}{\alpha}\tau\right]}\frac{\beta}{1+\beta}(1-\alpha)Ak_t^\alpha(1-\tau)$$

From here imposing steady state we get the following

$$k^* = \left[\frac{1}{\left[1 + \frac{1}{(1+\beta)}\frac{1-\alpha}{\alpha}\tau\right]}\frac{\beta}{1+\beta}(1-\alpha)A(1-\tau)\right]^{\frac{1}{1-\alpha}}.$$

b) The shock is such that in the first period the ratio of workers to retirees is $1+n$, and in all subsequent periods is 1. What is different in the setup is that migrants do not contribute, nor benefit, from social security. Thus, to keep track of capital accumulation we need to aggregate the potentially different saving behavior of natives and immigrants.

The government chooses contribution rate τ' such that the initial old receive the same benefits. Since the presence of immigrants increases the workforce for a given level of capital (initial steady state level k^*) this reduces the wage. Since the contribution base for social security is not affected (immigrants do not contribute), the new contribution rate is determined by

$$\tau' w_t = \tau'(1-\alpha)A\left(\frac{k^*}{1+n}\right)^\alpha = \tau'\frac{w^*}{(1+n)^\alpha} = \tau w^*$$

Thus,

$$\tau' = \tau(1+n)^\alpha.$$

To see the effect on capital accumulation we need to characterize the saving behavior of immigrants, s_t^m, as that of residents, s_t^r, will still be determined by equation (6). But for immigrants, the Euler equation, (5), holds as well, such that the only change in (6) is that they pay no taxes (and receive no benefit). Thus, it is straightforward to derive

$$s_t^m = \frac{\beta}{1+\beta}w_t$$

Combining both saving functions and remembering that there is no population growth (the measure of savers in period t, $1+n$, is the same as the measure of workers in period

$t+1$), we find the following expression for capital accumulation

$$k_{t+1} = \frac{\beta}{1+\beta} w_t - \frac{1}{1+n} \frac{\beta}{1+\beta} w_t \tau' - \frac{1}{1+n} \frac{1}{(1+\beta)} \frac{1}{r_{t+1}} \tau' w_{t+1}$$

Where the term $\frac{1}{1+n}$ reflects the fraction of workers that contribute to, and benefit from, social security.

Combining terms with k_{t+1}, and replacing $\tau' = \tau(1+n)^\alpha$, we get

$$k_{t+1} = \frac{1}{\left[1 + \frac{1}{1+n} \frac{1}{(1+\beta)} \frac{1-\alpha}{\alpha} \tau (1+n)^\alpha \right]} \frac{\beta}{1+\beta} w_t \left[1 - \frac{1}{1+n} \tau (1+n)^\alpha \right]$$

Thus, it is clear that the long run effect of the shock is to increase capital accumulation (the dynamic equation corresponds to a system in which everybody contributed, and benefited, the equivalent tax rate $\tau^{eq} = \tau(1+n)^{\alpha-1} < \tau$).

The effect in the first period is ambiguous since the initial wage is reduced to $\frac{w^*}{(1+n)^\alpha}$. For small initial social security systems (τ close to zero), the wage effect dominates and capital accumulation initially decreases. But for large initial social security systems, (τ close to one), the reduction in the equivalent contribution rate dominates and capital accumulation initially increases.

c) Since the presence of immigrants increases the workforce for a given level of capital in the first period, this increases the interest rate. This makes the old to be strictly better off since they have the same level of benefits but a higher capital income. The disposable income of the young residents in the first period is given by $w_t(1-\tau') = (1-\alpha)A\left(\frac{k^*}{1+n}\right)^\alpha (1 - \tau(1+n)^\alpha) < w^*(1-\tau)$. Thus, the disposable income of the initial young generation of residents is lower than in the initial steady state.