Written Exam for M.Sc.

Investment Theory

Master Course

6th January 2014

3 hours closed books exam

Question 1

Consider an investment project with investment cost $I > 0$ and exit cost $E > 0$. The project is to enter a market with perfect competition. The cost of producing $Y \geq 0$ units of output is

$$C(Y) \;=\; \frac{1}{4}Y^2 - H$$

where $H > 0$ is a fixed cost independent of output. The price of output is $P > 0$ where

$$dP \;=\; \alpha P dt + \sigma P dz.$$

The interest rate is $r > 0$. There is an asset with price $Q > 0$, where

$$dQ \;=\; (\alpha + \delta) Q dt + \sigma Q dz$$

and $\delta > 0$, and no dividend.

The project can be repeated in perpetuity. Let $F(P)$ be the value of the option to invest and $J(P)$ the value of the active firm.

(a) Give an example of an investment project that fits the above project.

(b) State possible strategies for the project. Use the strategies to relate $F(P)$ and $J(P)$.

(c) What is the profit of an active firm?

(d) Find the dividend rate of a portfolio consisting of one unit of the option to invest and minus n units of the asset. Find a differential equation in $F(P)$. Find $F(P)$ up to undetermined constants.

(e) Find the dividend rate of a portfolio consisting of one unit of the active project and minus n units of the asset. Find a differential equation in $J(P)$. Find $J(P)$ up to undetermined constants.

(f) Find an assumption on the parameters of the project that ensures that the $J(P)$ is economically meaningful.

(g) Interpret your expressions for $F(P)$ and $J(P)$.

(h) Show how the optimal strategies and the undetermined constants for $F(P)$ and $J(P)$ can be found and discuss economic consequences of the strategies.