

# **Written Exam for M.Sc. in Economics 2009-II**

## **Investment Theory**

**22.-23. June 2009**

### **Master course**

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

### **Focus on Exam Cheating**

In case of presumed exam cheating, which is observed by either the examination administration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview

the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In cases of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for a limited period of time or permanent expulsion.

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The Study and Examinations Office  
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24 hours written exam. All questions should be clearly and briefly answered. Calculations and figures should be clear and understandable. Calculations and figures should be explained.

### Exercise 1

Consider an investment project with a possibility to expand. Let  $I_P > 0$  be the investment cost for the project. The dividend of an active project is  $P$  coming from selling one unit of an output where

$$dP = \alpha_P P dt + \sigma_P P dz_P.$$

Moreover if the project is active, then it is possible to expand. Let  $I_Q > 0$  be the cost of expansion. The dividend of the expansion is  $Q$  (in addition to  $P$ ) coming from selling one unit of another output where

$$dQ = \alpha_Q Q dt + \sigma_Q Q dz_Q.$$

Let  $\rho \in [-1, 1]$  be the correlation of  $P$  and  $Q$ . Suppose that the interest rate is  $r > 0$  and the convenience yield of holding one unit of the  $P$ -output is  $\delta_P > 0$  and of the  $Q$ -output is  $\delta_Q > 0$ .

- 1.1 Discuss the form of the optimal strategy.
- 1.2 Find the value of getting  $P$  forever and find the value of getting  $Q$  forever.
- 1.3 Let  $F_Q(Q)$  be the value of the option to expand. Find the dividend rate of a portfolio consisting of one unit of the option to expand minus  $x$  units of the  $Q$ -output.
- 1.4 Find  $F_Q(Q)$  and the optimal strategy for the option to expand. Discuss the form of  $F_Q(Q)$ .

- 1.5 Let  $V_P(P, Q)$  be the value of an active project including the option to expand. Find  $V_P(P, Q)$  and discuss the form of  $V_P(P, Q)$ .
- 1.6 Let  $F_P(P, Q)$  be the value of the option to invest. Find the dividend rate of a portfolio consisting of one unit of the option to invest minus  $y$  units of the  $P$ -output and  $z$  units of the  $Q$ -output.
- 1.7 Find  $F_P(P, Q)$  (hint: guess that  $F_P(P, Q) = A_P P^{\varepsilon_P} + A_Q Q^{\varepsilon_Q}$ ) and discuss the form of  $F_P(P, Q)$ .
- 1.8 Find the optimal strategy for the option to invest.
- 1.9 Find a case of an investment project, that has the option to invest and the option to expand, and explain your case.
- 1.10 Discuss the results in 1.1-1.8 in relation to your case.

## Exercise 2

Consider an investment project with waiting time. Let  $I > 0$  be the investment cost for the project with waiting time and let  $V$  be the value of the project where

$$dV = \mu(\bar{V} - V)Vdt + \sigma Vdz$$

where  $\mu, \bar{V} > 0$ . If the investor pays  $I$  at date  $t$ , then she gets  $V$  at date  $t + T$  where  $T > 0$ , so she gets  $V(t + T)$ . Let  $J > I$  be the investment cost for the project without waiting time. If the investor pays  $J$  at date  $t$ , then she gets  $V$  at date  $t$ , so she gets  $V(t)$ . Let  $\rho > 0$  be the discount rate of the investor.

- 2.1 Give an example that fits with the project and discuss the form of the optimal strategy.

- 2.2 Let  $F_I(V)$  be the value of the option to invest with waiting time. Find a differential equation to determine  $F_I(P)$
- 2.3 Find  $F_I(V)$  and discuss the form of  $F_I(V)$ .
- 2.4 Find the optimal strategy for the project with waiting time.
- 2.5 Let  $F_J(V)$  be the value of the option to invest without waiting time. Find a differential equation to determine  $F_J(V)$
- 2.6 Find  $F_J(V)$  and discuss the form of  $F_J(V)$ .
- 2.7 Find the optimal strategy for the project without waiting time.
- 2.8 Compare the optimal strategy for the project with and without waiting time.
- 2.9 Find the optimal strategy for the project.