Mikro B, Juni 2014
Guide to answers[1]

Problem 1

Consider a von Neumann-Morgenstern agent with the Bernoulli utility function u(x), where x is realized income, and consider income lotteries with two possible outcomes, x_L and x_H, $0 < x_L < x_H$, each of them having probability 50 %. We assume u is continuous and strictly increasing.

- a) Give a formal expression for the concept "certainty equivalent" for such a lottery, and illustrate in a diagram for a risk-averse agent
- b) Calculate the risk premium for an agent with $u(x) = x^2$, and with $x_L = 0$ and $x_H = 10$

Answer: a) $CE = u^{-1}(½u(x_L)+ ½u(x_H))$, i.e. the certain income with same expected utility as the income lottery.
b) In this concrete lottery, with expected utility of 50 (=½·0+½·100), we have $CE = (50)^{½} = 7.07$, so with the expected value of the lottery being 5, the risk premium is –2.07; negative because the agent is a risk-lover, as u" >0.

Problem 2

In central Copenhagen, the market for lunch sandwiches has a supply side with the supply function S(p) = 10·p, p being the price of one sandwich, measured in DKK.
People working have the demand function $D_w(p) = \text{Max}\{600–5·p, 0\}$, and students have the demand function $D_s(p) = \text{Max}\{400–5·p, 0\}$. The market is characterized by perfect competition.

- a) Find the equilibrium price in this market, and the number of sandwiches consumed in equilibrium by workers and students, respectively.
- b) Calculate Producers' Surplus, as well as Consumers' Surplus for workers and students, respectively.
- c) The Minister of Education succeeds in introducing a subsidy of 20 DKK for each sandwich purchased by a student. Answer questions from a) and b) after the introduction of this subsidy.
- d) Comment on this statement from the minister: "This is obviously a good initiative, as students will receive more nutrition, and sandwich sellers will have better business conditions"

Answer: a) Inverse demand functions are $P_w(q) =120–q/5$, $P_s(q) =80–q/5$. Aggregate demand is 1000–10·p (for prices below 80). Equilibrium becomes p = 50, $q_w = 350$, $q_s = 150$.
b) PS = 12500, CS_w =12250, CS_s =2250, aggregate surpluses (gains from trade) being 27000.
c) For students, the inverse demand shifts, with the subsidy, to 100–q/5, and the new aggregate demand becomes 1100–10·p (for prices below 100). New equilibrium price is 55, and new quantities are 325 and 225, respectively. PS increases to 15125, CS increases from 2250 to 5063 for students, but falls from 12250 to 10563 for workers. The sum of surpluses increases to 30750, 3750 higher than before, whereas the subsidy costs the tax-payers 4500.
d) Students will eat more (and gain a larger CS), yes, and sellers will prosper, but workers suffer from a higher sandwich price, and there is a deadweight loss of 750 because the subsidy distorts the price signal.

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[1] Note that this guide is only indicative and does not provide full answers to the problems; this document merely outlines the correct mathematical results and the most important points to be made.

Problem 3

Consider the market for car insurance in a country with 1 million drivers, half of them good drivers, the other half bad. A good driver will incur a marginal cost of 1,000 $ for the insurance company with whom the driver becomes a customer; a bad driver will incur a marginal cost of 2,000 $. An insurance company is assumed to have no fixed costs.

The number of good drivers, x_g, wanting to buy the insurance depends on p, the price of insurance, with $x_g = D_g(p) = \text{Max}\{500{,}000 - 200 \cdot p, 0\}$, and similarly $x_b = D_b(p) = \text{Max}\{500{,}000 - 200 \cdot p, 0\}$ for the bad drivers.

The supply side is characterized by fierce competition, allowing only zero profits.

- a) What will the market outcome be if it is obvious to insurance companies which drivers are good and which are bad?
- b) What will the market outcome be if it is impossible for insurance companies to see who is a good driver and who is a bad driver? Compare to a) from a welfare economic point of view
- c) What will happen if insurance companies can, at a certain cost per driver, perform a test and conclude with certainty whether the driver is good or bad?

Answer: a) In the separating equilibrium, 300,000 good drivers each pay 1,000 $ for insurance, 100,000 bad drivers each pay 2,000 $
b) In the pooling equilibrium, 200,000 good and 200,000 bad drivers each pay 1,500 $, and there is a DWL as good drivers under-insure and bad drivers over-insure; the DWL is two times ½·100.000·500, i.e. 50 million $.
c) When the screening cost per driver is not too large, it will be profitable for insurance companies to screen costumers, hence obtaining a separating equilibrium. The cost of screening will be added to the price good drivers pay. When screening becomes sufficiently costly, the separating equilibrium may be a worse solution for all parties: Insurance companies still receive zero profits, bad drivers obviously pay more than in the pooling equilibrium, but, with sufficiently high screening costs, good drivers are also paying more than in the pooling equilibrium.

Problem 4

- a) Please present the economic problem "Tragedy of the Commons"; please do so formally, with diagrams, and provide economic intuition as well as some perspective as to the relevance of this case to our present economic society.

Answer: The central quality of the model is that with common ownership, the marginal peasant who considers letting his animal enter the commons, looks at the average benefit and average cost. The social optimizer, however, looks at the marginal ditto. With decreasing social marginal net benefit (net of cost), the consequence is that the level of activity will become inefficiently high. Contemporary economic examples: Over-fishing; traffic congestion.

Problem 5

Consider the market for ice-cream. On the production side, this product can be produced at constant marginal and average costs of 10 per ice-cream, with no fixed costs. On the customer side, the demand is given by the function $D(p) = \text{Max}\{110 - p, 0\}$.

Suppose that the supply side has a monopoly producer.

- a) Find, in equilibrium, the quantity sold, the market price, customers' surplus (CS), producer's surplus (PS), and dead-weight loss (DWL).
- b) Let the government introduce a tax of 10 per unit of ice-cream. Find, in equilibrium, the quantity sold, the price paid by customers and the price received by the monopoly, respectively. Also identify customers' surplus (CS), producer's surplus (PS), tax-revenue (R), and dead-weight loss (DWL)

Answer:
a) Quantity is 50, price is 60, CS = 1250, PS = 2500, DWL = 1250.
b) Quantity is 45, the price paid by customers is 65, and the price received by the monopolist is 55. CS = 1012½, PS = 2025, R = 450, DWL = 1512½.

<u>Problem 6</u>

Arthur and Bill each sell drinks on campus on Friday nights. The market demand for drinks is given by the function D(p) = Max {60 – p, 0}. When each of the producers has chosen a quantity of drinks to produce and supply to the market, the price of drinks will be determined as the market-clearing level, given the total quantity supplied by Arthur and Bill. Assume, for simplicity, that both of them have constant marginal costs which are zero, MC = 0.

Assume that Arthur can act as a Stackelberg leader, having the advantage of being able to enter the market and determine his quantity prior to Bill.

Please determine the outcome, in terms of the following seven variables:
- The quantity produced and sold by each producer
- The market price
- Producer's surplus for each producer
- Consumers' surplus
- Deadweight loss.

Do so in in each of the following three cases.

- a) Both producers have fixed costs being zero.
- b) Now, suppose that both producers have fixed costs of 25 (in the sense that entering the market and starting production and sales takes an investment of 25).
- c) Finally, suppose that both producers have fixed costs of 250.

Answer: a) A produces 30, B produces 15, price is 15, Π_A = PS_A =450, Π_B =PS_B =225.
b) With FC of 25, A should deter B from entering by producing 50; then, B stays out, price becomes10, and PS_A =500, Π_A =475, Π_B =PS_B =0.
c) With FC of 250, B will never enter, and A can safely act as a monopolist, quantity 30, price 30, PS_A = 900, Π_A = 650.

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