Question 1

Two breweries, Tuberg and Carlsborg, are the only competitors in a market for very cheap beer. Both firms produce at a constant marginal cost of 1 and all fixed costs have already been paid and cannot be reversed. The inverse demand curve in the market for very cheap beer is the following:

$$P = 101 - \frac{1}{10}Q$$

where $Q = Q_T + Q_C$ and Q_T is the quantity (in liters) of beer produced by Tuberg and Q_C is the quantity (in liters) of beer produced by Carlsborg. Assume that the two firms are competing in quantities à la Cournot.

a) What is the maximization problem of Carlsborg as a function of Q_C and Q_T?

b) Which quantity of beer will the two firms produce together in equilibrium?

c) Assume that the firms are instead competing in prices à la Bertrand. How many liters will the two firms produce together in equilibrium?

d) Go back to the initial situation and assume that Tuberg and Carlsborg merge. Which quantity of beer will the resulting firm produce?

e) Will the resulting firm in d) have higher, lower or the same profits as the combined profits of Tuberg and Carlsborg in the initial situation? Explain the economic mechanism why profits are different or the same. There is no need to use calculations in your response.

f) Go back to the initial situation and assume that Tuberg now faces constant marginal cost of 3, while Carlsberg continues to produce at marginal cost of 1. Which quantities would the two firms produce in equilibrium?

g) What will happen under Bertrand competition if Tuberg faces constant marginal cost of 3, while Carlsberg continues to face marginal cost of 1? Explain in words whether the overall quantity produced in the market and the price will be different than under c), and – if yes – in which directions they will change relative to c), and why. It is not necessary to calculate the new market price and quantity.

Solution Main exam question 1

a) $\max_{G_C} \quad \pi_C = (101 - \frac{1}{10}(Q_C + Q_T) - 1)Q_C$

b) Derive best response functions ${Q_i}^*(Q_j) = 500 - 0.5 * Q_j$ for $i = C, T$ and $i \neq j$ and either insert into each other or make the argument that the two firms are symmetric and therefore $Q_C = Q_T$. This gives the result that each firm is producing a quantity of 333.33 liters, giving the market quantity 666.66 liters.

c) The two firms will under-cut each other until the price is equal to marginal costs, giving a price of 1. They will not go lower than that because otherwise they would make negative profits; for higher prices a firm would have an incentive to undercut the other firm. Insert into the inverse demand function, giving a total quantity equal to $1,000$ liters.

d) The resulting firm will produce the monopoly quantity of 500 liters.

e) The resulting firm will have higher profits than the combined profits of Tuberg and Carlsborg in the initial situation. The quantities that would maximize the joint profit of the two firms in the initial situation would be half of the monopoly quantity for each firm. However, this cannot be an equilibrium under Cournot competition, as each firm would have an incentive to make a one-sided deviation and increase profits by producing more. The merger of the two firms makes them a monopolist that makes decisions as one entity, and therefore can maximize the joint profits of the two firms. Alternatively, one could argue that under Cournot competition, firms do not fully incur (internalize) the cost of an increase in their produced quantity in the form of having a lower price (the "cost" of lowering the price occurs to both firms). One could also argue that, since the joint quantity is different when the two firms make decisions together than when they make decisions separately, the combined profit has to be higher, as they might as well have continued producing the old quantity. One could also argue that the Cournot case entails some competition, which brings the equilibrium outcome closer to perfect competition, which features zero profits.

f) Tuberg's new best response function becomes $Q_T(Q_C) = 490 - 0.5 * Q_C$, while Carlsborg's best response function remains $Q_C(Q_T) = 500 - 0.5 * Q_T$ as before. Inserting into each other gives the new quantities $Q_T = 320$ and $Q_C = 340$.

g) Under Bertrand competition, Tuberg will no longer want to offer beer at a price of 1, as it will make negative profits in this case, so it has to raise its price to 3. Carlsberg will play a best response to that, which means it will undercut Tuberg by a tiny amount ε and capture the entire market. The overall market price will increase, and the overall market quantity will decrease relative to the situation under c).

Question 2

Suppose you are a member of the successful newcomer band "The Public Good Providers". "The Public Good Providers" have three fans, Claus, Søren and Niels, who have already purchased tickets for a concert of your band. Assume that there are no other guests at the concert. If your band practices, this increases the quality Q of the concert. Claus, Søren and Niels consider paying your band for practicing, and your band has offered them to practice as many hours as they want in return for receiving one kroner per hour. Music quality will be 0 without practice and increase by one unit with each hour of practice.

However, while Søren places great importance on the music being of high quality, Niels places not so much importance on it, and Claus does not care at all about the quality of the music and only goes to the concert to hang out with the others. They have the following utility functions, where x_C, x_S and x_N are the amounts of money Claus, Søren and Niels "consume" in the end, respectively:

$$u_C(x_C) = x_C$$
$$u_S(x_S, Q) = x_S + 4\sqrt{Q}$$
$$u_N(x_N, Q) = x_N + 2\sqrt{Q}$$

where u_C denotes Claus' utility function, u_S denotes Søren's utility function, and u_N denotes Niels' utility function. Niels, Claus and Søren each have an endowment of 100 kroner.

a) What levels of the public good of music quality Q^C, Q^S, Q^N will each of the three contribute if they make decisions individually and take the other players' decisions as given? What is the total amount provided?

b) What is the socially optimal level of music quality?

c) Your band understands that Claus, Niels and Søren will buy less music quality than the socially optimal level. Your band therefore decides to try to overcome this problem by letting them pay Lindahl prices. What are the Lindahl prices t^C, t^S, t^N that each of the three fans would have to pay in the Lindahl equilibrium?

d) Why may Claus, Niels and Søren be unwilling to truthfully provide the information you need to calculate the socially optimal level of the public good and the Lindahl prices? Explain in words.

Your band decides to set up the Vickrey-Clarke-Groves mechanism to elicit the willingness to pay for music quality from Claus, Niels and Søren. For simplicity, assume that music quality can be either 0 or 16. Your band decides to offer to provide music quality of 16 at a total price of 15, where each of the three would have to pay 5 kroner for the provision of the public good.

e) What are Claus', Niels' and Søren's net utilities, n_C, n_N and n_S, for going from music quality of 0 to music quality of 16 if each of them has to pay 5 kroner for the provision of the public good?

f) Assume that the Vickrey-Clarke-Groves mechanism is successful in the sense that all three report their true net utilities. Will the public good be purchased? Which agent is pivotal? What is the Clarke tax that this agent will have to pay?

Solution Main exam question 2

a) Solve the budget constraint for money and replace in the utility functions. Take derivatives of the resulting function w.r.t. Q^i, an individual's contribution to the public good (e.g. for Søren differentiate

$100 - Q^S + 4\sqrt{Q^C + Q^S + Q^N}$ w.r.t. Q^S). Alternatively, directly use $|MRS_i| = c$ for each of them. All approaches yield the following *overall* quantities of the public good that would be optimal for each of them: Claus $Q^C + Q^S + Q^N = 0$; Niels $Q^C + Q^S + Q^N = 1$; Søren $Q^C + Q^S + Q^N = 4$. In a Nash equilibrium, Claus will not contribute anything. (For Claus one could also directly argue that he would contribute zero because the public good does not appear in his utility function.) If Niels contributes 1, then Søren would contribute 3 to obtain an overall level of 4. However, then Niels would have an incentive to go to zero, as he only aims for an overall amount of 1. When he provides 0, Søren will increase his contribution to 4, which is the Nash equilibrium where both are playing best responses to each other. The total amount provided is $Q = 4$.

b) Set up the Langrangian and maximize one fan's utility under the constraints that the utility of the others' will be at least fixed levels $\bar{u_1}$ and $\bar{u_2}$. Alternatively, directly use the social optimality condition $|MRS_C| + |MRS_S| + |MRS_N| = c$. Each approach gives an optimal level of $Q^* = 9$.

c) The Lindahl prices are equal to the absolute of the marginal rates of substitution at the socially optimal quantity, so $t^C = 0, t^S = \frac{2}{3}, t^N = \frac{1}{3}$.

d) Claus, Niels and Søren would have to provide you with information on their MRS at the socially optimal quantity $Q^* = 9$. They understand that this will influence the individual Lindahl prices they have to pay, so they would have an incentive to lie and report a lower MRS, and free-ride on the contributions of the others.

e) This is basically just plugging in into the different utility functions. $n_C = 0 - 5 = -5$, $n_S = 16 - 5 = 11$ and $n_N = 8 - 5 = 3$.

f) The total net utility is positive at $-5 + 11 + 3 = 9 > 0$, so the public good will be purchased. To find out which agent is pivotal, we remove one agent at a time and examine whether the decision to buy the good would change. If we remove Claus, it would not change as the sum of the others' net utility is still positive. If we remove Niels, the decision would not change as the sum of the others' net utility is still positive. If we remove Søren, the decision would change, as the sum of the net utilities of the other two is -2, so we would not end up buying the public good. This means that Søren is pivotal. Søren therefore has to pay a Clarke tax of 2, equal to the absolute of the sum of the net utility of the other two.

Question 3

Mette, Asger and Jeanet want to go to the movies together. The movie preferences of the three are described in the following table.

Person/Movie	Star Wars Episode 23	Fast and Furious 17	Transformers 9
Mette	1	3	2
Asger	2	1	3
Jeanet	3	2	1

The numbers represent the ranking of Mette, Asger and Jeanet, respectively, i.e. in the first row, 1 stands for Mette's favorite movie and 3 for her least preferred movie and so on.

a) Mette suggests to apply the Democracy Social Choice Function (SCF) and find the optimal decision through pairwise voting. That is, the friends will vote between two movies, and the winning movie goes on to the next round, where there is a vote between that movie and another option which it has not yet won over. The process is repeated until there is an option that has won over all the other options. Explain what problem occurs in this process and how it arises.

b) Jeanet suggests to over-come this problem by forbidding voting on options that have already lost in a vote, and that she decides the order of the voting. The other two agree. Which pair of movies is Jeanet going to suggest for the first round of voting? Is she going to succeed in getting to watch her favorite movie in case all three vote according to their preferences? Explain what new issue occurs due to the change in the voting rules.

c) Asger thinks a step ahead and notices that, despite the fact that the voting is done in the order suggested by Jeanet, he can improve the outcome for himself through smart voting behavior. Which movie does he have to vote for (against his preferences) and in which voting round does he have to do so in order to improve his situation? Which movie would the three friends watch in that case? Assume that the other two will vote in accordance with their true preferences and that Asger knows this.

d) Can we apply the median voter theorem in the situation described under a)? Why or why not?

Solution Main exam question 3

a) The suggested procedure will lead to a Condorcet cycle, meaning that the proposed decision rule may lead to intransitive social preferences even if all individual preferences are transitive (and total). To see this, consider the comparison Star Wars against Fast and Furious. Fast and Furious will win, as Asger and Jeanet prefer it to Star Wars. Now Fast and Furious is compared to Transformers. Transformers will win as it is preferred by Mette and Jeanet. Now Transformers is compared to Star Wars. Star Wars will win, as Mette and Asger prefer it, and we are back at a point where we would have to compare Star Wars to Fast and Furious. We do not get a Condorcet winner. We could also arrive at this result by doing the votes in different order.

b) Jeanet will suggest that they first vote on Star Wars vs Fast and Furious. Fast and Furious will win and they will vote on Fast and Furious against Transformers. Transformers will win and the voting

will be over, as Star Wars has already lost a vote. In this way Jeanet will succeed in watching her first choice. This illustrates that the Democracy SCF with pairwise voting and forbidding votes on options that have already lost overcomes the problem of Condorcet cycles. However, this comes at the cost of creating a new problem, namely giving "Agenda Setting Power" to the agent deciding about the order in which votes over different options occur.

c) Asger prefers both Star Wars and Fast and Furious to Transformers. If Asger votes for Star Wars in the first round even though he prefers Fast and Furious, the second round will be a vote of Star Wars against Transformers, which is preferred by Mette and Asger, so Star Wars wins, and Asger ends up with his second choice instead of his third choice.

d) The median voter theorem tells us that if preferences are rational (total and transitive) and single-peaked, then the ideal point of the median voter is socially optimal and will be a Condorcet winner. In the situation under a) we do not have a Condorcet winner – we get a Condorcet cycle. The problem is that the individual preferences are not single-peaked (even though they are rational). This is because there is no objective numeric ordering of the different options, such that we can say "the ideal point of Mette is below the ideal point of Asger" or similarly. This implies that we cannot identify a median voter.

Question 4

Do you agree or disagree with the following statements? Explain your answers.

a) "Franchising contracts will typically involve no information rents to the agent, i.e. the agent will typically earn her reservation utility. This tends to be the case regardless of whether the agent is risk-neutral or risk-averse, and of whether the revenue is risky or not."

b) "In the grand scheme of things, allowing firms to take out patents for new products they have invented through research and innovation is a bad thing since it leads to market power." (Note: A patent guarantees a firm to be the only seller of a new product, typically for a duration of a few years.)

Solution Main exam question 4

a) This statement is true. In principal agent models where the effort of the agent cannot be observed after the contract has been signed, one way to induce positive effort of the agent is franchising. Franchising means that the principal allows the agent to produce and keep the profits to herself, but the agent in turn has to pay a fixed amount to the principal. The worker gets the full marginal return of her work and is therefore "residual claimant". This will ensure that the agent chooses the level of effort optimally. The principal will then charge the agent the highest amount such that the agent still accepts the offer, which means that the agent will earn her reservation utility in expectation. Otherwise the principal

could increase her profits by charging a higher price and the agent would still accept. This will be true no matter whether the agent is risk-averse or not, and no matter whether there is a random component driving the revenue or not. If the agent is risk-averse and there is a random component, however, the agent will have a lower willingness to pay for the franchising contract. This will lower the principal's expected profits, so there will be information cost to the principal. However, the agent will still earn her reservation utility in expectation, and therefore not earn any information rents here.

b) On the one hand, allowing one firm to be the sole seller of a particular product indeed leads to welfare losses compared to a case where this product is offered by many firms, since this firm will likely behave as monopolist and charge a price that is too high and produce a suboptimally low quantity. However, there will most likely still be a welfare improvement from having one monopolist selling the new product compared to a situation where the product is not produced at all. Patents will provide an incentive to firms to invest in research and innovation and develop new products. To see this, note that investing in research and innovation creates positive externalities for other firms if those other firms are allowed to also produce and sell the newly invented products. This means that every individual firm will have an incentive to underinvest in research and innovation. Allowing firms to take out patents removes those positive externalities to other firms, increasing each firm's incentive to invest in research and innovation. One could invoke the Coase theorem and argue that patents assign property rights to a good whose property rights were previously undefined, and could therefore improve efficiency. One could also argue that without earning positive profits as in the monopoly case, firms would not be able to pay for the research and innovation, so they may not do it. In practice, one could come up with policies that limit patents for a certain duration, or one could regulate the new monopolies in various ways to make sure that welfare losses are minimized.