Suggested answers for written Exam for the B.Sc. or the M.Sc. in Economics, Summer 2011

Makro B / Macro 3

Final Exam

August 23, 2011

(3-hour closed-book exam)

Academic Aim: The aim of the course is to describe and explain the macroeconomic fluctuations in the short run, i.e. the business cycles around the long run growth trend, as well as various issues related to this, and to teach the methodology used in formulating and solving formal models explaining these phenomena. Students are to learn the most important stylized facts about business cycles and to acquire knowledge about theoretical dynamic models aimed at explaining these facts. In connection with this, the aim is to make students familiar with the distinction between deterministic and stochastic models. Furthermore, students are to gain an understanding of the distinction between the impulses initiating a business cycle and the propagation mechanisms that give business cycles a systematic character. Finally students are to learn how to use the models for analyzing the effects of macroeconomic stabilization policy under various assumptions regarding the exchange rate regime.To obtain a top mark in the course students should at the end of the course be able to demonstrate full capability of using the techniques of analysis taught in the course as well as a thorough understanding of the mechanisms in the business cycle models for open and closed economies, including the ability to use relevant variants and extensions of the models in order to explain the effects of various shocks and the effects of macroeconomic stabilization policies under alternative monetary and exchange rate regimes.

Problem A

$$(r+\varepsilon)V_t = D_t^e + V_{t+1}^e - V_t, \tag{A.1}$$

1. Equation (A.1) is an arbitrage condition stating that the required return from investing in shares must equal those of investing in bonds plus a risk premium because investing in a bond is considered less risky than investing in shares. If r is the market rate of interest on bonds (which we here for simplicity assume to be constant), the opportunity cost of shareholding is rV since this is the interest income that the shareholder could have earned during period t if she had sold her shares at the initial market value V_t and invested the the amount in bonds. The risk premium reflects that stock prices are in general more volatile than bond prices and interest payments, so in the absence of a risk premium, the risk averse investor would prefer to hold "low-risk" bonds rather than "high-risk" shares if expected returns from these investments were identical.

   Re-writing equation (A.1) involves

$$\begin{aligned}(r+\varepsilon)V_t &= D_t^e + V_{t+1}^e - V_t \Longleftrightarrow \\ V_t(1+r+\varepsilon) &= D_t^e + V_{t+1}^e \Longleftrightarrow \\ V_t &= \frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon} \end{aligned} \tag{A.2}$$

2. Equation (A.4) simply states that the market value of the firm equals the present discounted value of the expected future dividends paid out by the firm. It is called "structural" because the valuation of the firm reflects the firm's fundamental ability to generate future cash flows to its owners. To derive (A.4), start with (A.2), lead it one period to obtain a measure of V_{t+1}^e and plug it into (A.2)

$$V_t = \frac{D_t^e + \left(\frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon}\right)}{1+r+\varepsilon}$$

$$
\begin{aligned}
V_t &= \frac{D_t^e + \left(\frac{D_{t+1}^e + V_{t+2}^e}{1+r+\varepsilon}\right)}{1+r+\varepsilon} \\
&= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{V_{t+2}^e}{(1+r+\varepsilon)^2} \\
&= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + ... + \frac{V_{t+n}^e}{(1+r+\varepsilon)^n}
\end{aligned}
$$

and when applying $\lim_{n\to\infty} V_{t+n}^e / (1+r+\varepsilon)^n = 0$ we get

$$
= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + ... \tag{A.4}
$$

Equation (A.4) proposes three sources for stock prices volatility:

1. Fluctuations in the (growth rate) of expected future real dividends D_t^e.
2. Fluctuations in the (expected) real interest rate r.
3. Fluctuations in the required risk premium on shares, ε.

The theory is compatible with rational behavior although it says nothing about how expectations are formed. All that is being said is how stock prices relate to future dividends. How expectations on r, D and ε are formed are not modelled.

3. Equation (A.5) is an auxillary equation (definition) that is introduced to make it possible to link the objective of the owners maximization problem, V_t, with the decision variable, I_t. It simply defines the ratio between the market value of the firm and the replacement value of the firm's capital stock.

   Equation (A.6) postulates that investors believe the current share price per unit of capitel one period from now is equal to the value of today, e.g. investors form static expectations on q_{t+1}^e.

   Equation (A.7) is a bookkeeping identity stating that next periods capital stock is equal to what is left of the previous period's capital stock after depreciation plus what was being invested in the previous period.

   Equation (A.8) states that expected dividends are equal to profits minus what is spend on investing in the firm's capital stock. Investment costs are the sum of the acquisition cost (I_t) and the costs associated with installing new capital $c(I_t)$.

Equation (A.9) are obtained in the following way. First lead (A.5) one period

$$q_{t+1}^e = \frac{V_{t+1}^e}{K_{t+1}},$$

we note that K_{t+1} is predetermind from period t which is why we drop on expectations on this variable. Now substitute (A.6) into this expression and re-arrange

$$\begin{aligned} q_t &= \frac{V_{t+1}^e}{K_{t+1}} \Longleftrightarrow \\ V_{t+1}^e &= q_t K_{t+1} \end{aligned}$$

Finally substitute (A.7) into this expression

$$V_{t+1}^e = q_t \left[K_t (1-\delta) + I_t\right] \tag{S.1}$$

Substituting (S.1) and (A.8) into (A.2) gives us (A.9)

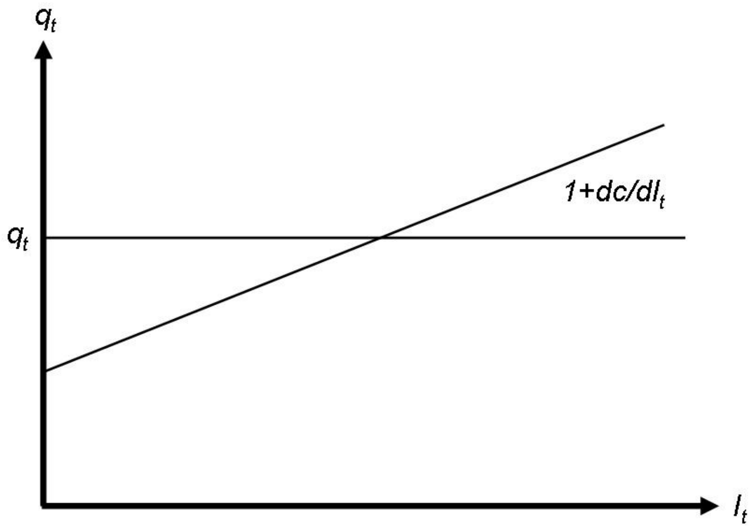
$$\begin{aligned} V_t &= \frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon} \\ &\quad \frac{\overbrace{\Pi_t^e - I_t - c(I_t)}^{D_t^e,\ \text{cf.(A.8)}} + \overbrace{q_t \left[K_t (1-\delta) + I_t\right]}^{V_{t+1}^e,\ \text{cf. (S.1)}}}{1+r+\varepsilon} \end{aligned} \tag{A.9}$$

4. The first-order condition is obtained when taking the derivative of V_t as given by (A.9) with respect to I_t

$$\begin{aligned} \frac{\partial V_t}{\partial I_t} &= \frac{-1 - c'(I_t) + q_t}{1+r+\varepsilon} = 0 \Longleftrightarrow \\ \overbrace{q_t}^{\substack{\text{expected} \\ \text{capital gain}}} &= \overbrace{1 + c'(I_t)}^{\substack{\text{forgone} \\ \text{dividend}}} \end{aligned} \tag{A.10}$$

This expression states that the firm will invest up to the point where the rise in its market value induced by an extra unit of investment is exactly equal to the sum of the acquisition and instllation cost of buying and installing an additional unit of capital. The higher the firm is valued, the higher is q_t and the more the firm will invest. The slower installation costs rise the more will be invested. Graphically, the first-order condition can be illustrated in the following way:

Figure S.1



This illustration assumes that marginal costs are linear. This is the case for the particular functional form of $c(\cdot)$ presented in the textbook, but other forms are possible as well. The optimal level of investments are found where investment costs intersects with q_t.

It is easily seen that without installation costs it would be optimal to invest without any upper limit (assumed that $q_t > 1$).

5. From (A.4) one can show that the introduction of a proportional tax on dividends will reduce the stock price of the firm. Let V_t^0 be the market value of a firm in the beginning of period t in the situation where no taxes are introduced. Likewise, let V_t^1 be the market value when a proportional tax

on dividiends have been introduced. Using (A.4) we then have

$$\begin{aligned} V_t^0 &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + ... > \\ & \frac{(1-\tau)\,D_t^e}{1+r+\varepsilon} + \frac{(1-\tau)\,D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{(1-\tau)\,D_{t+2}^e}{(1+r+\varepsilon)^3} + ... = V_t^1, \end{aligned}$$

where τ is the tax rate.

To make the argument complete one should actually start with recasting the arbitrage condition in the following way

$$(r+\varepsilon)\,V_t = (1-\tau)\,D_t^e + V_{t+1}^e - V_t,$$

but the students are not required to do this.

If the tax is introduced in two years' time (and assuming that each period constitutes one year) we see that this also leads to a decrease in the market value of the firm, although this decrease is smaller than in the case where the tax is immidiately introduced

$$\begin{aligned} V_t^0 &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + ... > \\ & V_t^2 \equiv \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{(1-\tau)\,D_{t+2}^e}{(1+r+\varepsilon)^3} + ... > \\ & \frac{(1-\tau)\,D_t^e}{1+r+\varepsilon} + \frac{(1-\tau)\,D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{(1-\tau)\,D_{t+2}^e}{(1+r+\varepsilon)^3} + ... = V_t^1, \end{aligned}$$

where V_t^2 is the market value in the case where the tax on dividends are introduced two years from now.

1. This is an open question, not directly discussed at the lectures, designed to indicate how the students are able to use the model in a practical matter. As shown above, taxation of dividends leads to lower market values of firms. Using the insight obtained in this problem, we know that this will decrease the market value of the firm relative to the replacement value of the firm. In other words: q will fall, leading to a lower level of investments and ultimately to a lower level of capital in the economy. With lower levels of capital, the marginal product of labour falls and with this, the general wage level decreases as well.

Problem B

1. Equation (B.1) is called the nominal interest rate parity and comes from the arbitrage condition

$$(1+i) = \left(1+i^f\right)\left(\frac{E^e_{+1}}{E}\right). \tag{S.2}$$

The left-hand side of (S.2) measures the amount of wealth accruing to an investor at the end of the current period if she invests one unit of the domestic currency in the domestic capital market at the beginning of the period. As an alternative to such a domestic investment, the investor could have bought $1/E$ units of the foregin currency at the start of the period for the purpose of investment in the foreign capital market. At the end of the period she would then have ended up with an amount of wealth $(1/E)\left(1+i^f\right)$. At the time of the investment she believes this to be worth $\left(E^e_{+1}/E\right)\left(1+i^f\right)$ in domestic values. Therefore equation (S.2) says that a domestic and a foreign investment must generate the same expected end-of-period wealth and hence must yield the same expected rate of return. If not, investors would immediately sell bonds with the lowest expected rate of return (which drives down (up) the prices (interests) on theses bonds) and buy bonds with the highest expected rate of return (which drives up (down) the prices (interests) on these bonds). This arguments requires perfect capital mobility.

The students can give this explanation from (B.1) and do not need to state (S.2) above. However, (B.1) is approximated from (S.2) by taking logs;

$$\begin{aligned} \ln(1+i) &= \ln\left(1+i^f\right) + \ln E^e_{+1} - \ln E \Rightarrow \\ i &= i^f + e^e_{+1} - e, \end{aligned} \tag{B.1}$$

where we used that $\ln(1+i) \approx i$. When the nominal exchange is credibly fixed, there is no expected change in the nominal exchange rate so $\Delta e^e_{+1} \equiv e^e_{+1} - e = 0$ and (B.1) reduces to

$$i = i^f.$$

Hence, the domestic central bank cannot set the nominal interest rate independently from the decisions made by the foreign central bank. This leaves monetary policy impotent – the central bank has no possibilities to stabilize business cycles through monetary policy.

2. Equation (B.2) is the AD curve of the economy. According to this – since $\beta_1 > 0$ – higher domestic inflation will be associated with lower aggregate demand for domestic output. This is because higher dometic inflation erodes the international competitiveness of domestic producers by reducing the real exchange rate. Hence, a rise in π raises the relative price of domestic goods, thereby reducing net exports.

   Equation (B.3) the SRAS of the economy. This states a positive association between domestic output and inflation. Firms set prices as a mark-up over marginal costs. As labour input increases, the marginal product of labour decreases and marginal costs increase which leads to increased levels of inflation.

   Equation (B.4) comes from the definition of the real exchange rate

$$E^r \equiv \frac{EP^f}{P},$$

   where P^f and P denote the foreign and domestic price level respectively. Taking logs and subtracting the laggede values and restricting E to be constant ($\bar{E}$) leads to (B.4):

$$\begin{aligned} \ln E^r - \ln E^r_{-1} &= \ln E + \ln P^f - \ln P - \ln \bar{E} - \ln P^f_{-1} - \ln P_{-1} \\ e^r &= e^r_{-1} + \pi^f - \pi \end{aligned} \tag{B.4}$$

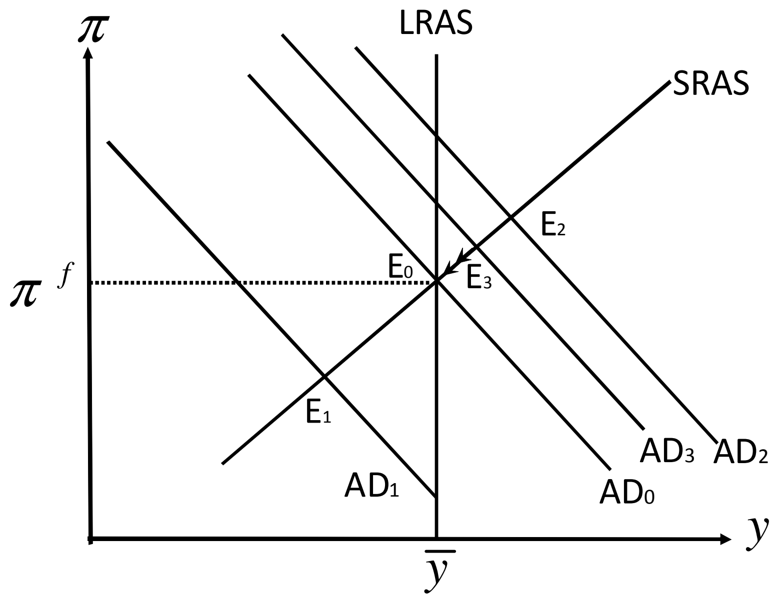
   The students do not need to show this, but should be able to tell that the real exchange rate depreciates when foreign foreign inflation exceeds domestic inflation *et vice versa*. A depreciation of the real exchange rate (a higher value of e^r) leads to an improvement of competitiveness *et vice versa*.

   The role of s is to capture aggregate supply shocks. The role of z is to capture aggregate demand shocks. In the current model, theses shocks include changes in foreign real output, y^f, and in the foreign real interest rate, r^f, changes in domestic public spending, g, and in the private sector's confidence. All these shocks are real shocks and one might note that one further demand-shock channel is present in the model through the level of foreign inflation, π^f, in the AD curve.

3. Figure S.2 shows the convergence towards the long-run equilibrium $\left(\bar{y}, \pi^f\right)$ following one-period demand shock. In period 1, when the negative demand

shock has hit the economy, the economy finds itself in a recession in the short-run equilibrium E_1 with a negative output gap $(y_1 - \bar{y} < 0)$. Also, inflation is below its long-run level $\left(\pi_1 - \pi^f < 0\right)$ which is because the low level of employment associated with the negative output gap leads to a high level of marginal product of labour and therefore a low level of marginal costs. In the next period, the demand shock is gone which tends to shift the AD curve back to its original position. However, the real exchange rate has depreciated because of the low inflation level in period 1 and because of this improvement in competitiveness, period 2's demand curve shifts to AD_2 and the short-run equilibrium shifts to E_2. Now we have the opposite situation, because now domestic inflation is higher than abroad. Therefore, competitiveness detoriates over time (high levels of employment $\Rightarrow$ low increases in MPL $\Rightarrow$large increases in MC $\Rightarrow$high levels of inflation)and gradually the demand curve falls back towards AD_0. This proces continues as long as domestic inflation is above that of the international economy.

Figure S.2



This adjustment mechanism differs substantially from that of the closed economy. In the closed economy, it is a requirement that the Taylor principle is met to estabilish the negative slope of the AD curve which is nescessary for the economy to converge towards long-run equlibrium. The Taylor principle states that when inflation rises, nominal interest rates must me increased more than one-for-one with inflation to ensure that the real interest rate increases. By rasing the real interest rate the central bank is able to reduce private demand (the sum of private consumption and private investments). Likewise, when inflation falls, the nominal interest rate should be lowered more than one-for-one, so that the real interest rate is reduced which will stimulate private demand. The Taylor principle cannot be met under a fixed-exchange rate regime since the central bank is not allowed to change the interest rate when inflation changes.

4. (B.6) simply states that when $a > 0$ government consumption, g, is allowed to fluctuate systematically around its long-run value $\bar{g}$. When the output gap is negative, government consumption is raised above its long-run value, while government consumption is tightened when the output gap is positive. From (B.2*) we see that a positive value of a affects the slope of the AD curve. More specifically we have that since

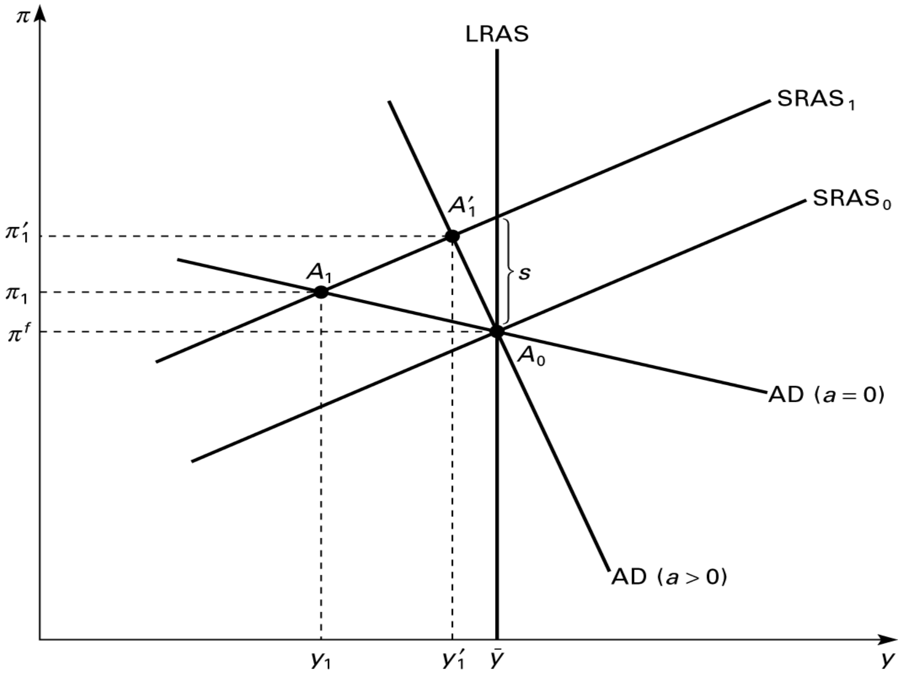
$$\left(\frac{1+\beta_3 a}{\beta_1}\right) > \left(\frac{1}{\beta_1}\right),$$

the AD curve is steeper when fiscal policy reacts systematically to the output gap. The economic intution is that when inflation rises, the competitiveness of the economy is impared. However, when parts of the fall in private demand is counteracted by an increased level of public consumption, the overall drop in total output will be lower.

Figure S.3 illustrates the situation where the economy is in long-run equilibrium $\left(\bar{y}, \pi^f\right)$ in period 0 and hit by a temporary negative supply shock in period 1. This negative supply shock causes the SRAS curve to shift from

$SRAS_0$ to $SRAS_1$.

Figure S.3



When fiscal policy is passive $(a = 0)$, the short-run equilibrium shifts to (y_1, π_1) and when fiscal policy is countercyclical $(a > 0)$ the short-run equilibrium shifts to $\left(y_1', \pi_1'\right)$. In other words: the steeper the AD curve the smaller will be the effect on real output and the larger will be the effect on inflation.According to the SRAS curve in eq. (B.2) a negative supply shock will have to *either* affect inflation positively, affect real output negatively or a combination of both. The less inflation is affected, the larger will be the effect on real output *et vice versa*. The higher the value of a, i.e. the more focused fiscal policy is on stabilizing real output around its long run level, the higher must thus be the effect on inflation. Hence the conclusion is that in case of a supply shock policy makers have to choose between stabilizing real output *or* inflation.

This is not the case when shocks are due to demand. An illustration is given in figure S.4. A demand shock shifts down the AD curve by the vertical distance $\hat{z}/\beta_1$ The economic intuition is the following: A demand shock will directly affect goods market demand and thereby equilibrium real output.

Real output in turn affects inflation through the supply side of the economy. Consequently, by stabilizing real output around its long run level, inflation will also be stabilized (and by stabilizing inflation, real output will be stabilized through monetary policy).

Figure S.4