Written Exam - Macroeconomics III

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Question 1

Consider an economy where individuals live for two periods, and the population is constant. Identical competitive firms maximize their profits employing a Cobb-Douglas technology that combines labor, L_t, and capital, K_t, so that $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, with $\alpha \in (0,1)$. Assume full capital depreciation (i.e., $\delta = 1$). Under these assumptions, profit maximization leads to:

$$\begin{aligned} 1 + r_t &= \alpha A k_t^{\alpha - 1}, \\ w_t &= (1-\alpha) A k_t^{\alpha}, \end{aligned}$$

where r_t is the (net) rental rate of capital, w_t is the wage rate, and k_t denotes capital in per-worker units.

Utility for young individuals born in period t is

$$U_t = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1},$$

with $\rho > -1$. c_{1t} denotes consumption when young, c_{2t+1} consumption when old. Young agents spend their entire time endowment, which is normalized to one, working. Suppose the government runs an unfunded (pay-as-you-go) social security system, according to which the young pay a contribution d_t that amounts to a fraction $\tau \in (0,1)$ of their wages. Thus, the contributions are paid out in the same period to the current old. The latter do not work, and sustain their consumption though their savings and the social security benefits. Thus, the budget constraints in each period of life read as:

$$\begin{aligned} c_{1t} + s_t &= w_t (1 - \tau), \\ c_{2t+1} &= s_t (1 + r_{t+1}) + d_{t+1}. \end{aligned}$$

a Set up and solve the individual's problem of optimal intertemporal allocation of resources. Derive the Euler equation. Show that individual saving behavior is characterized by

$$s_t = \frac{1}{2+\rho} w_t (1-\tau) - \tau \frac{1+\rho}{2+\rho} \frac{1}{1+r_{t+1}} w_{t+1}.$$

b Show that the capital accumulation equation that gives k_{t+1}, as a function of k_t, is given by

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho} \frac{(1-\alpha)}{\alpha} \tau} \left[\frac{(1-\alpha)(1-\tau)}{2+\rho} A k_t^{\alpha} \right].$$

Show also that, in the steady state, the amount of capital-per-worker is

$$\bar{k} = \left[\frac{1}{1 + \frac{1+\rho}{2+\rho}\frac{(1-\alpha)}{\alpha}\tau} \frac{(1-\alpha)(1-\tau)A}{2+\rho} \right]^{\frac{1}{1-\alpha}}.$$

c Suppose that, at time T, before saving decisions are made, the government decides to switch to a fully funded social security system according to which the young pay a contribution d_T that amounts to a fraction $\tau \in (0,1)$ of their wages. These contributions are then paid out in the next period, together with the accrued interest rate. The budget constraints in each period of life now read as:

$$\begin{aligned} c_{1t} + s_t &= w_t (1-\tau), \\ c_{2t+1} &= (s_t + w_t \tau)(1 + r_{t+1}), \quad \text{for } t \geq T. \end{aligned}$$

Show that the new steady-state capital-per-worker, which is denoted by $\bar{k}'$, is such that

$$\bar{k}' = \left[\left(\frac{1}{2+\rho} - \tau \right) (1-\alpha) A \right]^{\frac{1}{1-\alpha}}, \quad \text{where } \tau < \frac{1}{2+\rho} \text{ is implicitly imposed.}$$

d Is the older generation at time T better off or worse off, after the social security system has been changed?

Question 2

The representative agent i maximizes her utility function

$$U_i = C_i - \frac{1}{\beta} L_i^{\beta}, \qquad \beta > 1,$$

subject to the budget constraint

$$PC_i = P_i Y_i,$$

where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i. The production function equals

$$Y_i = L_i^{\alpha}, \quad 0 < \alpha < 1.$$

There is monopolistic competition in the goods market. The demand for good i is

$$Y_i = \left(\frac{P_i}{P} \right)^{-\eta} Y,$$

where Y denotes aggregate output and $\eta > 1$ is the elasticity of substitution in the demand for differentiated goods. The aggregate demand equation is

$$Y = \frac{M}{P},$$

where M denotes money supply. Agents have rational expectations. Employ the following notation: $x \equiv \ln X$.

a After taking logs of the first order condition from the utility maximization problem, derive the optimal production y_i^* as a function of the relative price $p_i - p$.

b Impose homogeneity, and show that $y = \frac{\alpha}{\beta - \alpha} \ln \left(\alpha \frac{\eta - 1}{\eta} \right)$, which increases in η. Provide an economic interpretation to this result.

c Suppose now that individual prices are fixed for 2 periods, and that price-setting is staggered, such that 1/2 of the prices are set in period t at the level x_t, and 1/2 were set in period $t-1$ at the level x_{t-1}. Thus, the aggregate price level equals

$$p_t = \frac{1}{2} \left(x_t + x_{t-1} \right).$$

Assuming certainty equivalence (i.e., $x_t = \frac{1}{2} \left(p_{i,t}^* + \mathbf{E}_t \left[p_{i,t+1}^* \right] \right)$, where $p_{i,t}^* = m_t + y$ denotes the optimal reset price), show that the equilibrium reset price, x_t, depends on m_t and $\mathbf{E}_t \left[m_{t+1} \right]$.

d Suppose that the (log) money supply follows an AR(1) process: $m_t = \rho m_{t-1} + \varepsilon_t$, where ε_t is a white noise. Show that aggregate price inflation, $\pi_t = p_t - p_{t-1}$, is a MA(1) process taking the following form:

$$\pi_t = \frac{1 + \rho}{4} \left(\varepsilon_t + \varepsilon_{t-1} \right).$$