Suggested solutions to the IO (BSc) resit exam on August 14, 2009
VERSION: September 3, 2009

Solution Q1

a) (i) Standard calculations yield

$$q^* = \frac{a}{b(n+1)}. \tag{1}$$

Market price equals

$$p^* = a - nb\frac{a}{b(n+1)} = \frac{a}{n+1}, \tag{2}$$

which is decreasing in n and approaches 0 (=MC) as $n \to \infty$. Consumer surplus is

$$(a - p^*)\, nq^*/2 = b\,(nq^*)^2/2 = \frac{(na)^2}{2b(n+1)^2}$$

and aggregate profits are

$$p^* nq^* = \frac{na^2}{b(n+1)^2}.$$

Therefore, total surplus (the sum of the two above) is

$$\begin{aligned} TS &= \frac{(na)^2}{2b(n+1)^2} + \frac{na^2}{b(n+1)^2} = \frac{(na)^2 + 2na^2}{2b(n+1)^2} \\ &= \frac{a^2 n(n+2)}{2b(n+1)^2}, \end{aligned}$$

which is increasing in n and approaches $a^2/2b$ as $n \to \infty$ (the largest possible TS, where output is so large that all gains from trade are exploited).

a) (ii) There are two **Cournot-Nash equilibria:** $(q_1^*, q_2^*) = (4, 0)$ and $(q_1^*, q_2^*) = \left(\frac{8}{3}, \frac{8}{3}\right) \approx (2.67, 2.67)$. This should be proven.

b) Standard reason: allocative inefficiency (failure to exploit gains from trade).

- The *standard argument* for why a monopoly (or lack of competition more generally) is undesirable is that it gives rise to an *allocative inefficiency* — the monopoly sets a price that implies that some gains from trade are left unexploited. Some consumers in the economy have a valuation for the good that exceeds the firm's cost of producing it. Hence, if the firm and the consumer agreed to trade at some price between the valuation and the cost, both parties would gain. However, they do not trade.

- *Rent seeking:* In an article published in 1967, Tullock argued that the welfare losses due to a monopoly are not properly measured by the black triangle — the losses are actually bigger. Tullock argued that, to the black triangle, we should add (at least parts of) the rectangle representing the monopoly profits. Why? In Tullock's own words: "Surely we should expect that with a prize of this size dangling before our eyes, potential monopolists would be willing to invest large resources in the activity of monopolizing." That is, Tullock argued that: (i) Firms will lobby or pressure a government in an attempt to win the monopoly. (ii) The resources that the firms are willing to spend in an attempt to win the monopoly may possibly add up to the whole monopoly profit. (iii) The costs of these activities are wasted, so we should indeed add them to the social cost of having a monopoly.

c) The Herfindahl index is defined as the sum of the squared market shares, that is, as

$$H = \sum_{i=1}^{n} s_i^2,$$

where s_i is firm i's market share and there are n firms in the market. Applying this definition yields

$$\begin{aligned} H &= \left(\frac{1}{3}\right)^3 + 4 \times \left(\frac{1}{6}\right)^2 \\ &= \left(\frac{1}{3}\right)^3 + 4 \times \left(\frac{1}{2 \times 3}\right)^2 \\ &= \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 \\ &= \frac{2}{9} \approx 0.22, \end{aligned}$$

That is, the Herfindahl index for this market equals $\frac{2}{9} \approx 0.22$.

d) Green-Porter:

- The key model ingredient is that a firm cannot observe its rival's price. It is also assumed that demand fluctuates stochastically (which makes inference about the rival's chosen price non-trivial). Otherwise it is a standard duopoly model with price-setting firms, interacting over an infinite horizon. One can, as in a standard repeated game, sustain a collusive equilibrium if the firms care sufficiently much about future profits. However, in this model, collusion cannot be sustained perfectly (i.e., all the time). For whenever demand is low, the trigger strategy must specify a punishment phase to start, because the firms cannot distinguish between demand being low and the rival having cheated (a firms only sees that it didn't sell anything, without knowing the reason). Because of this logic, another condition needed for some collusion to be possible is that the probability of a low state is not too large.

- That is, in this model (in contrast to the one by Rotemberger and Saloner), collusion breaks down when demand is low (hence price war during recessions). The intuition for this is the reasoning above, namely, that a firm cannot observe its rival's price and therefore it cannot distinguish between demand being low and the rival having deviated. So the trigger strategy must specify that the collusion breaks down whenever there is a low-demand state — if it did not, the players would have an incentive to deviate.

e) Downward- and upward-sloping best reply functions, respectively. (Answers in terms of the sign of the cross-derivative of the payoff function would also be fine) Examples: The results of comparative statics exercises; whether there is a first- or a second-mover advantage in a particular game.

f) *Explain briefly the conjectural-variations approach to modelling an oligopoly.*

- The idea is to assume that (in, say, a duopoly) the firms believe that a change in one firm's output leads to a change in the rival's output, even though the firms' choices are otherwise modelled as being simultaneous. The degree to which the rival's output changes is captured by a parameter, the conjectural variations parameter. This parameter is typically assumed to be constant (and often also identical across firms). As this parameter takes various values, the outcome of the model (the equilibrium quantities) can be made identical to, for example, the outcome under Cournot or Bertrand competition or the collusive outcome. The approach is therefore used as a reduced-form way of capturing a family of different models with different degrees of competition.

Solution Q2

a)

- First note that the costumers will always be willing to travel (also the longest possible distance A-D) in order to buy the good. The only equilibrium is where both firms locate at B. One way of showing this is to note that an equilibrium configuration must belong to one of the following three categories:

    1. The firms choose different locations and these are *not* next to each other.
    2. The firms choose different locations and these *are* next to each other.
    3. The firms choose the same locations.

- One can work through the categories and investigate whether any firm would have an incentive to deviate unilaterally from the configuration in question.

- The other way of finding the Nash equilibria is to calculate the number of customers the firms would get for all possible configurations and then construct a game matrix (given that the market price is exogenously given, this will give the same result as if constructing a matrix with profit levels in the entries). By inspecting this matrix, one can easily verify that (B,B) is the only equilibrium. The matrix will look like this:

$$\begin{array}{ccccc} & A & B & C & D \\ A & 9000, 9000 & 7000, 11000 & 9500, 8500 & 12000, 6000 \\ B & 11000, 7000 & \boxed{9000, 9000} & 12000, 6000 & 14000, 4000 \\ C & 8500, 9500 & 6000, 12000 & 9000, 9000 & 16000, 2000 \\ D & 6000, 12000 & 4000, 14000 & 2000, 16000 & 9000, 9000. \end{array}$$

- In the equilibrium (B,B), each firm's profit equals $9000 \times (1 - 0.5)$ =DKK 4,500. The consumers' surplus equal 5-1=DKK 4 for those located at B; 5-1-1=DKK 3 for those located at A and C; and 5-1-2=DKK 2 for those located at D. Therefore, total consumer surplus for each group is equal to:

$$7,000 \times \text{DKK } 3 = \text{DKK } 21,000 \qquad \text{for the A customers;}$$

$$5,000 \times \text{DKK } 4 = \text{DKK } 20,000 \qquad \text{for the B customers;}$$

$$4,000 \times \text{DKK } 3 = \text{DKK } 12,000 \qquad \text{for the C customers;}$$

$$2,000 \times \text{DKK } 2 = \text{DKK } 4,000 \qquad \text{for the D customers.}$$

  And total consumer surplus for all customers is equal to:

$$5,000 \times \text{DKK } 4 + (7,000 + 4,000) \times \text{DKK } 3 + 2,000 \times \text{DKK } 2 = \text{DKK } 57,000.$$

b)

- One way of identifying the Nash equilibria under this new assumption is to modify the game matrix above, by multiplying by 0.5 to get profits instead of sales and then subtracting 2000 from the payoff of a firm that chooses B. This yields

$$\begin{array}{ccccc} & A & B & C & D \\ A & \boxed{4500, 4500} & 3500, 3500 & 4750, 4250 & 6000, 3000 \\ B & 3500, 3500 & 2500, 2500 & 4000, 3000 & 5000, 2000 \\ C & 4250, 4750 & 3000, 4000 & 4500, 4500 & 8000, 1000 \\ D & 3000, 6000 & 2000, 5000 & 1000, 8000 & 4500, 4500. \end{array}$$

  In the new equilibrium (A,A), each firm's profit again equals $9000 \times (1 - 0.5)$ =DKK 4,500. The consumers' surplus equal 5-1=DKK 4 for those located at A; 5-1-1=DKK 3 for those located at B; 5-1-2=DKK 2 for those located at C; and. 5-1-3=DKK 1 for those located at D. Therefore, total consumer surplus for each group is equal to:

$$7,000 \times \text{DKK } 4 = \text{DKK } 28,000 \qquad \text{for the A customers;}$$

$$5,000 \times \text{DKK } 3 = \text{DKK } 15,000 \qquad \text{for the B customers;}$$

$$4,000 \times \text{DKK } 2 = \text{DKK } 8,000 \qquad \text{for the C customers;}$$

$$2,000 \times \text{DKK } 1 = \text{DKK } 2,000 \qquad \text{for the D customers.}$$

  And total consumer surplus for all customers is equal to:

$$\text{DKK } 28,000 + \text{DKK } 15,000 + \text{DKK } 8,000 + \text{DKK } 2,000 = \text{DKK } 53,000.$$

c)

- One example is one firm at A and the other at C. This will make the total amount of travel costs for the consumers smaller than the equilibrium configuration in a). In particular, the consumers' surplus equals 5-1=DKK 4 for those located at A and C; and 5-1-1=DKK 3 for those located at B and D. Therefore, total consumer surplus is equal to

$$(7,000 + 4,000) \times \text{DKK } 4 + (5,000 + 2,000) \times \text{DKK } 3 = \text{DKK } 65,000.$$

  Another possible answer would be the configuration (A,B), which would yield the total consumer surplus DKK $64,000$, and therefore also beat (B,B) and (A,A)

Solution Q3
a) and b)

- A monopoly firm produces a single good.
- Constant average (and marginal) cost of production: c.
- There is a continuum of consumers, each with the following utility:

$$\begin{cases} \theta V(q) - T(q) & \text{if buying a quantity } q > 0 \\ 0 & \text{if not buying.} \end{cases}$$

  - θ is a taste parameter.
  - $V(\cdot)$ is an increasing function.
  - $T(q)$ is the amount the consumer must pay the firm if buying a quantity q.

- A given consumer's taste parameter θ is either high or low:

$$\theta \in \{\theta_1, \theta_2\}, \qquad \text{with } 0 < \theta_1 < \theta_2.$$

  - *Asymmetric information*: The consumer knows his own θ, but the firm does not know it.
  - The firm only knows that a fraction λ of all consumers are of type θ_1, and the others (a fraction $1-\lambda$) are of type θ_2.

- The firm offers two price-quantity bundles to the consumers:
  - (q_1, T_1) is directed to the type-θ_1 consumers.
  - (q_2, T_2) is directed to the type-θ_2 consumers.

How to Solve the Problem: A Five-Step Recipe

1. Show that PC-L and IC-H imply PC-H, so we can ignore PC-H.
2. *Guess* that IC-L doesn't bind.
3. Inspect the problem and note that the two remaining constraints must bind. Therefore we can plug them into the objective function.
4. Solve the resulting unconstrained problem.
5. Verify that the solution satisfies IC-L (i.e., that the guess at (2) was correct). [Doing this step is not required as the question stated that we may assume that IC-L doesn't bind.]

Analysis

- The firm chooses q_1, q_2, T_1, and T_2 so as to maximize its expected profit,

$$\Pi^m = \lambda (T_1 - cq_1) + (1 - \lambda)(T_2 - cq_2),$$

subject to four constraints:

 - Type-θ_1 consumers must prefer their bundle to no bundle at all:

$$\theta_1 V(q_1) - T_1 \geq 0. \tag{IR-1}$$

 - Type-θ_2 consumers must prefer their bundle to no bundle at all:

$$\theta_2 V(q_2) - T_2 \geq 0. \tag{IR-2}$$

 - Type-θ_1 consumers must prefer their bundle to the bundle directed to the type-θ_2 consumers:

$$\theta_1 V(q_1) - T_1 \geq \theta_1 V(q_2) - T_2. \tag{IC-1}$$

 - Type-θ_2 consumers must prefer their bundle to the bundle directed to the type-θ_1 consumers:

$$\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1. \tag{IC-2}$$

- We can simplify the problem:

 - If IR-1 and IC-2 are satisfied, so is IR-2. We can therefore ignore IR-2.

   * *Proof:*

$$\begin{aligned} \theta_2 V(q_2) - T_2 &\overset{\text{By IC-2}}{\geq} \theta_2 V(q_1) - T_1 \\ &\underset{\text{By } \theta_2 > \theta_1}{>} \theta_1 V(q_1) - T_1 \underset{\text{By IR-1}}{\geq} 0 \end{aligned}$$

   which means that IR-2 is satisfied. □

 - At the optimum, IC-1 is not binding. According to the question, we do not need to prove this.

- The simplified problem: choose q_1, q_2, T_1, and T_2 so as to maximize

$$\Pi^m = \lambda (T_1 - cq_1) + (1 - \lambda)(T_2 - cq_2), \tag{3}$$

subject to

$$\theta_1 V(q_1) - T_1 \geq 0 \tag{IR-1}$$

and

$$\theta_2 V(q_2) - T_2 \geq \theta_2 V(q_1) - T_1. \tag{IC-2}$$

- Note that since Π^m is increasing in T_1 and T_2, both constraints must bind at the optimum.
  - We thus have
  $$T_1 = \theta_1 V(q_1) \tag{4}$$
  and
  $$\begin{aligned} T_2 &= \theta_2 V(q_2) - \theta_2 V(q_1) + T_1 \\ &= \theta_2 V(q_2) - \theta_2 V(q_1) + \theta_1 V(q_1) \\ &= \theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1). \end{aligned} \tag{5}$$

- We can now simplify the problem further.
  - By plugging (4) and (5) into (3), we have
  $$\begin{aligned} \Pi^m &= \lambda [\theta_1 V(q_1) - cq_1] + \\ & (1-\lambda)[\theta_2 V(q_2) - (\theta_2 - \theta_1) V(q_1) - cq_2], \end{aligned}$$
  - The problem now amounts to maximizing this expression for Π^m w.r.t. q_1 and q_2 (without having to take any constraints into account).

- The FOC w.r.t. q_1:
$$\frac{\partial \Pi^m}{\partial q_1} = \lambda [\theta_1 V'(q_1) - c] - (1-\lambda)(\theta_2 - \theta_1) V'(q_1) = 0 \Leftrightarrow$$
$$\theta_1 V'(q_1^*) = c \underbrace{\left[1 - \frac{(1-\lambda)}{\lambda} \frac{(\theta_2 - \theta_1)}{\theta_1}\right]^{-1}}_{>1} \tag{6}$$

- The FOC w.r.t. q_2:
$$\frac{\partial \Pi^m}{\partial q_2} = (1-\lambda)[\theta_2 V'(q_2) - c] = 0$$
or
$$\theta_2 V'(q_2^*) = c \tag{7}$$

Conclusions

1. The high-demand consumers buy the socially optimal quantity (marginal utility equals marginal cost). [By (7).]
2. The low-demand consumers buy less than the socially optimal quantity (marginal utility > marginal cost). [By (6).]
   - Intuition:

    - The monopolist wants to extract the high-demand consumer's large surplus.
    - An obstacle to this: If the high type gets too little, he can choose the low type bundle instead.
    - To prevent this, the monopolist makes the low-type's bundle less attractive by offering those consumers less.
    - This works because high-demand consumers suffer more from a reduction in consumption than low-demand consumers.

3. Low-demand consumers derive no net surplus, while high-demand consumers derive a positive net surplus.

4. The relevant personal arbitrage constraint is to prevent high-demand consumers from choosing the L-bundle.

- (7) and (6) imply a *larger spectrum of consumption patterns*, compared to first best:

$$\boxed{\underbrace{q_1^* < \overbrace{D_1(c) < q_2^* = D_2(c)}^{\text{q-distance under FB}}}_{\text{q-distance under FB}}}$$

- Other applications include a **quality choice** of the monopolist.
  - In that application, the implication is a broader *quality* spectrum offered to the consumers.

- In both cases, the intuitive reason is that the monopolist wants to ensure that the high types don't want to choose the low types' bundle.
  - The intended first-class passengers mustn't want to buy second-class tickets instead, so let's make second class sufficiently uncomfortable!

c)
Different kinds of price discrimination

- Pigou's (1920) taxonomy of price discrimination:
  - First-degree (or perfect) price discrimination:
    * The producer discriminates across sold units and consumers — captures the whole consumer surplus.
  - Second-degree price discrimination:
    * Per-unit prices of the good differ, but these prices are the same for all consumers (e.g., quantity discounts).
  - Third-degree price discrimination:
    * Per-unit prices of the good are the same for a given consumer, but different consumers pay different prices (e.g., student discounts).

	No discr. across units	Discr. across units
No discr. across consumers	No P.D.	2nd degree P.D.
Discr. across consumers	3rd degree P.D.	1st degree P.D.