Vejl. løsn. Lineære Modeller
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Opg 1

1+2+3: Vi løser $Lx = y$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & y_1 \\ 1 & 1 & 2 & y_2 \\ 1 & 1 & 2 & y_3 \\ 1 & 1 & 0 & y_4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & y_1 \\ 0 & 0 & 2 & y_2 - y_4 \\ 0 & 0 & 0 & y_3 - y_2 \\ 1 & 1 & 0 & y_4 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & y_1 \\ 0 & 1 & 0 & y_4 - y_1 \\ 0 & 0 & 1 & \frac{1}{2}y_2 - \frac{1}{2}y_4 \\ 0 & 0 & 0 & y_3 - y_2 \end{array}\right]$$

Heraf ses at 1) L er injektiv, $N(L) = \{0\}$.

2) L er ikke surjektiv, en basis for $R(L)$ er søjlerne

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \quad \dim R(L) = 3.$$

3)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_4 - y_1 \\ \frac{1}{2}y_2 - \frac{1}{2}y_4 \end{bmatrix}$$

og $y_3 - y_2 = 0$ er åbenbart kravet for at $y \in R(L)$.

Opg 2

1) $A = QDQ^T$ med $D = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ og

$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$.

Heraf fås $A = \begin{bmatrix} \frac{1}{2}(a+b) & \frac{1}{2}(a-b) \\ \frac{1}{2}(a-b) & \frac{1}{2}(a+b) \end{bmatrix}$

2) $f(A) = Q f(D) Q^T$ giver så

$$f(A) = \begin{bmatrix} \frac{1}{2}(f(a)+f(b)) & \frac{1}{2}(f(a)-f(b)) \\ \frac{1}{2}(f(a)-f(b)) & \frac{1}{2}(f(a)+f(b)) \end{bmatrix}$$

3) $\det A^7 = \det D^7 = a^7 b^7 = \underline{\underline{(ab)^7}}$

4) $\det e^{f(A)} = \underline{\underline{e^{f(a)} e^{f(b)} = e^{f(a)+f(b)}}}$

$e^{f(A)} v_1 = e^{f(a)} v_1 = (e^{f(a)}, e^{f(a)})$

Opg 3

1) $$\int \sin(ax)\cos(bx)\,dx = \int \left(\frac{e^{iax}-e^{-iax}}{2i}\right)\left(\frac{e^{ibx}+e^{-ibx}}{2}\right)dx$$

$$= \frac{1}{4i}\int e^{i(a+b)x} + e^{i(a-b)x} - e^{-i(a-b)x} - e^{-i(a+b)x}\,dx$$

$$= \frac{1}{2}\int \sin(a+b)x + \sin(a-b)x\,dx$$

$$= -\frac{1}{2}\left(\frac{\cos(a+b)x}{a+b} + \frac{\cos(a-b)x}{a-b}\right) + k.$$

2) Da $(2+i)^2 = 4-1+4i = 3+4i$

$(1-2i)(1+2i) = 1+4 = 5$

er ligningen

$$(3+4i)z - (3+4i) = 5z$$

$$\Updownarrow$$

$$(-2+4i)z = 3+4i$$

$$\Updownarrow$$

$$z = \frac{3+4i}{-2+4i} = \frac{(3+4i)(-2-4i)}{(-2+4i)(-2-4i)}$$

$$z = \frac{10-20i}{20} = \underline{\underline{\frac{1}{2} - i}}$$

Opg 4

1) $\left|\frac{1-x}{2-x}\right| < 1$ for $(1-x)^2 < (2-x)^2$,

dvs $x^2-2x+1 < x^2-4x+4$

$$2x < 3$$

$$\underline{\underline{x < \frac{3}{2}}}$$

2) $f(x) = \frac{1}{1-\left(\frac{1-x}{2-x}\right)} = \underline{\underline{2-x, \text{ for } x<\frac{3}{2}}}$

3) $f'(x) = -1$ så f er mon. aftagende og dermed injektiv.

4) $Vm(f) = \left]\frac{1}{2}, \infty\right[$ idet

$f(x) \to \frac{1}{2}$ for $x \to \frac{3}{2}^-$

5) $f(x) = y \iff \cancel{}\, 2-x = y$

$\iff \underline{\underline{2-y = x}}$

for $y > \frac{1}{2}$.