

# Written Exam for the B.Sc. in Economics 2011-I-R

## Macro C

### Final Exam

17 February 2011

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

**All questions of all three problems should be answered**

**Problem A**

For each of the following statements, provide an *economic explanation* of whether the statement is true or false.

1. In the Ramsey Model without growth in labour efficiency, the long-run growth rate of aggregate real output will equal the growth rate of population,  $n$ , but the real interest rate is not necessarily equal to  $n$  in the long run.
2. In models with rational expectations, changes in the money supply cannot have lasting real effects.
3. If in the government social loss function,  $SL = (y_t - y^*)^2 + \kappa (\pi_t - \pi^*)^2$ , the target rate for real output is equal to the natural level,  $y^* = \bar{y}$ , the Policy Ineffectiveness Proposition will not hold.

**Problem B**

Consider the OLG Model where the problem of an individual born in period  $t$  is

$$\max_{c_{1t}, c_{2t+1}, s_t} U_t = \ln c_{1t} + (1 + \rho)^{-1} \ln c_{2t+1} \quad (\text{B.1})$$

$$\text{s.t.} \quad c_{1t} + s_t = w_t - T_t \quad (\text{B.2})$$

$$c_{2t+1} = (1 + r_{t+1}) s_t \quad (\text{B.3})$$

where  $w_t$  is the real wage per unit of labour and  $T_t$  is a lump sum tax levied on each young individual. Firms produce according to the Cobb-Douglas technology

$$F(K_t, A_t L_t) = (K_t)^\alpha (A_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1 \quad (\text{B.4})$$

where the size of generation  $t$ ,  $L_t$ , is assumed to grow at rate  $n > 0$ , while labour efficiency/technology,  $A_t$ , grows at rate  $g$ . Firms' first order condition for optimal use of labour lead to

$$A_t (1 - \alpha) k_t^\alpha = w_t \quad (\text{B.5})$$

where  $k_t \equiv \frac{K_t}{A_t L_t}$ . Finally, it is assumed that the lump sum tax is indexed to the labour efficiency index,  $T_t = \tau A_t$ .

1. Show that the first order conditions for solving the individual utility maximisation problem lead to

$$\frac{c_{2t+1}}{c_{1t}} = \frac{1 + r_{t+1}}{1 + \rho} \quad (\text{B.6})$$

Comment and relate it to the Keynes Ramsey rule in the Ramsey Model.

2. Show that individual saving is given by

$$s_t = \frac{1}{2 + \rho} (w_t - T_t) \quad (\text{B.7})$$

and comment on the fact that saving is independent of the interest rate. Also show that  $k_t$  evolves according to

$$k_{t+1} = \frac{1}{1+g} \frac{1}{1+n} \frac{1}{2+\rho} ((1-\alpha) k_t^\alpha - \tau) \quad (\text{B.8})$$

and show that when  $\tau = 0$ , the economy will converge to a unique steady state.

3. Assume that in period  $t_0$  the economy is in steady state with  $\tau = 0$ , when government decides to introduce public spending and to finance it by lump sum taxes in the form of a positive and constant value of  $\tau$ .

Analyse how this will affect the economy over time from period  $t_0$  and onwards with respect to capital, production, the real wage, the real interest rate and consumption.

4. What would be the effects if *instead* government decides from period  $t_0$  and onwards to finance public spending by introducing a value added tax that would change the budget constraints of the individual to

$$(1 + \tau^c) c_{1t} + s_t = w_t \quad (\text{B.9})$$

$$(1 + \tau^c) c_{2t+1} = (1 + r_{t+1}) s_t \quad (\text{B.10})$$

where  $\tau^c$  is the constant value added tax rate? Explain.

## Problem C

Consider the following version of the Dornbusch Model in usual notation. All variables are functions of time but for ease of exposition, reference to time is not made explicit. Expectations are assumed to be rational and there is no uncertainty.

$$\dot{p} = \gamma (y - \bar{y}), \quad \gamma > 0 \quad (\text{C.1})$$

$$r = r^f + \dot{e} \quad (\text{C.2})$$

$$m - p = y - \delta r, \quad \delta > 0 \quad (\text{C.3})$$

$$y = \beta (e - p) - \eta r + z, \quad \beta, \eta > 0 \quad (\text{C.4})$$

The economy may be described by the following two differential equations

$$\dot{p} = \frac{\gamma\beta\delta}{\eta + \delta} e - \gamma \frac{\beta\delta + \eta}{\eta + \delta} p + \frac{\gamma\eta}{\eta + \delta} m + \frac{\gamma\delta}{\eta + \delta} z - \gamma \bar{y} \quad (\text{C.5})$$

$$\dot{e} = \frac{\beta}{\eta + \delta} e + \frac{1 - \beta}{\eta + \delta} p - \frac{1}{\eta + \delta} m + \frac{1}{\eta + \delta} z - r^f \quad (\text{C.6})$$

and it is assumed that values of parameters and exogenous variables are such that the  $\dot{p} = 0$  and  $\dot{e} = 0$  loci intersect in the positive orthant. You can also use the fact that solving equations (3) and (4) for  $y$  and  $r$  produces:

$$y = \frac{\beta\delta}{\eta + \delta}e - \frac{\beta\delta + \eta}{\eta + \delta}p + \frac{\eta}{\eta + \delta}m + \frac{\delta}{\eta + \delta}z \quad (\text{C.7})$$

$$r = \frac{\beta}{\eta + \delta}e + \frac{1 - \beta}{\eta + \delta}p - \frac{1}{\eta + \delta}m + \frac{1}{\eta + \delta}z \quad (\text{C.8})$$

The following analysis should be carried out under the assumption that  $\beta > 1$ .

1. Construct the phase diagram of the economy. Comment.

*Hint:* You will need to show that the  $\dot{e} = 0$  locus is positively sloped but flatter than the  $\dot{p} = 0$  locus.

Assume now that the economy is in a long-run equilibrium when, at time  $t_0$ , there is an *unexpected and permanent increase* in the foreign interest rate  $r^f$ .

2. Use the phase diagram to analyse the effects on the economy from time  $t_0$  and onwards. Provide explanations of the relevant economic effects.

Assume instead that the economy is in long-run equilibrium, when, at time  $t_0$ , it is *announced* that at time  $t_1 > t_0$  there will be a *permanent increase* in  $r^f$ .

3. Use the phase diagram to analyse the effects on the economy from time  $t_0$  and onwards. Provide explanations of the relevant economic effects.