Written Exam for the B.Sc. or M.Sc. in Economics winter 2014-15

Operations Research

CORRECTIONAL GUIDE

Elective Course

January 19th, 2015

(3-hour open book exam)

You may write your exam paper in Danish or in English.

Part 1

Question 1.1: State the optimal solution and solution value in the Optimal Simplex tableau.

We read from the Optimal Tableau that the optimal solution is $x^* = (10, 10, 0, 20)$ with $z^* = 115$. We also see that all three constraints are binding.

Question 1.2: Suggest a LP problem which could have been the one solved by Simplex above.

We presume that this is a maximization problem since all reduced costs are non-negative in the tableau which claims to be optimal. Reverse engineering the Initial tableau we get:

Maximize $z = 0.5x_1 + 3x_2 + x_3 + 4x_4$
subject to

$$x_1 + x_2 + x_3 + x_4 \leq 40$$
$$2x_1 + x_2 - x_3 - x_4 \geq 10$$
$$-x_2 + x_4 \geq 10$$
$$\text{All } x_i \geq 0.$$

Other initial LP's may be solved using these tableaus. For instance, variable substitution etc. may have been applied beforehand.

Question 1.3: How much can the objective function coefficient to x_1 deviate without changing the optimal solution?

We use the direct approach:

z	x_1	x_2	x_3	x_4	s_1	e_2	e_3	RHS
0	1	0	-1/2	0	0	-1/2	-1/2	10
1	0	0	4	0	7/2	3/2	1	115
Θ	0	0	-1/2	0	0	-1/2	-1/2	-10

For the reduced costs to be non-negative we have that:
$4 - 1/2\ \Theta \geq 0$ or $\Theta \leq 8$
$3/2 - 1/2\ \Theta \geq 0$ or $\Theta \leq 3$
$1 - 1/2\ \Theta \geq 0$ or $\Theta \leq 2$
So of all reduced costs should be non-negative, we have that $\Theta \leq 2$, that is $c_1 \leq 5/2$

Question 1.4: Formulate the dual LP to the LP found in question 1.2 and give the optimal solution.

Following the guidelines of Winston in how to set up the dual LP model we first change the primal model to a max normal problem by multiplying constraint 2 and 3 by -1:

Maximize $z = 0.5x_1 + 3x_2 + x_3 + 4x_4$
subject to

$$x_1 + x_2 + x_3 + x_4 \leq 40$$

$-2x_1 - x_2 + x_3 + x_4 \leq -10$
$x_2 - x_4 \leq -10$
All $x_i \geq 0$.

We then apply the regular transformation rules and get:

Minimize $w = 40y_1 - 10y_2 - 10y_3$
subject to

$y_1 - 2y_2 \geq 0.5$
$y_1 - 1y_2 + y_3 \geq 3$
$y_1 + y_2 \geq 1$
$y_1 + y_2 - 1y_3 \geq 4$
$y_1\ y_2, y_3 \geq 0$

This Dual LP will have the optimal solution y = (3.5, 1.5, 1) with w* = 115, which is read as the dual variables in the Optimal (Primal) Simplex Tableau.
Further comments may include verification of the optimal dual value of 115 (40*3.5-10*1.5-10*1) and noticing that constrains 1, 2 and 4 are binding which corresponds to primal variables x_1, x_2 and x_4 being in basis.

Part 2

Question 2.1: Formulate a cutting plane which can be added to the Simplex tableau

$5.1\ x_1 – 3.3\ x_2 + 1\ x_3 – 3.9\ s_1 + 0.5\ s_2 = 42.2$

We split the constraint into integer and fractional parts:

$5\ x_1 + 0.1\ x_1 – 4\ x_2 + 0.7x_2 + 1\ x_3 – 4\ s_1 + 0.1\ s_1 + 0.5\ s_2 = 42 + 0.2$

We then gather the integer parts on the left and the fractional parts on the right:

$5\ x_1 – 4\ x_2 + 1\ x_3 – 4\ s_1 – 42 = 0.2 – 0.1\ x_1 – 0.7x_2 – 0.1\ s_1 – 0.5\ s_2$

The rights hand side must be ≤ 0 which gives us the cut:

$– 0.1\ x_1 – 0.7x_2 – 0.1\ s_1 – 0.5\ s_2 \leq –0.2$

Question 2.2: If instead a Branch & Bound algorithm was underway, which constraints would now be added?

We are not told which variable is in basis in the constraint, but since x_3 has the coefficient of 1, it can only be that one. We notice that x_3=42.2. We would therefore formulate two sub-problems with each of the following constraints added: $x_3 \leq 42$ and $x_3 \geq 43$

Part 3

Question 3.1: Formulate a balanced Transportation model to maximize the profit.

We notice that the problem is not balanced. The shopping centers can at most sell 900 items and the warehouses can at most supply 1100 items. Since both of these are max limits, we may use the lower number (900) for actual numbers of batches, are use a dummy shopping center for the remaining 200 batches.
This, however, will only make sense if full delivery is profitable when subtracting the costs from the income. Therefore we'll need to look at the profit matrix, ($p_{i,j}$):

10-M	10-M	9-M	9-M	14-M	0
12-M	12-M	11-M	11-M	16-M	0
11-M	11-M	10-M	10-M	15-M	0

Except for the dummy warehouse, the profit elements all depend on M and we notice that as long as M is between 5 and 8, all profit elements are non-negative. It therefore makes sense to formulate the problem as a transportation model where all 1100 (including 200 dummy) batches are transported. Since the objective is maximization and not minimization we may change the sign and add a sufficiently high value to all profits so that they represent a positive "cost". Or we may simple reverse the optimality criterion in the transportation Simplex. Either way, we are not asked to solve the problem.

The column demands are 100, 200, 300, 200, 100 and 200 (for the dummy) and the row supply is 300, 400 and 400.

Question 3.2: Find an initial basic feasible solution using Vogel's method

Vogel's method may easily be used when the objective is maximization since the rule "take most where the difference between the best and the second best is largest" equally works for minimization and maximization.

10-M	10-M	9-M	9-M	14-M (1)	0
12-M	12-M	11-M	11-M	16-M	0
11-M	11-M	10-M	10-M	15-M	0

Below we show the outcome of Vogel's. After the number of batches, we show in parenthesis the iteration number of the decision:

				100 (1)	200 (7)
100 (2)	200 (3)	100 (4)			
		200 (5)	200 (6)		0 (8)

Question 3.3: What would happen if M increases to 18?

Inserting M=18 in the profit table above we notice, that all profits are non-positive. Therefore, we can only lose money by actually transporting batches. Luckily, given the problem formulation, we may opt not to transport at all since the limits given were max limits and we can assume 0 for lower limits.