Written Exam at the Department of Economics winter 2017-18

Macroeconomics III

Final Exam

January 9, 2018

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 4 pages in total

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Written Exam - Macroeconomics III

University of Copenhagen

January 9, 2018

Question 1

Consider an economy where individuals live for two periods and the population is constant. The utility for young individuals born in period t is

$$\frac{\left(c_{1t}C_{1t}^{-\alpha}\right)^{1-\sigma}}{1-\sigma}+\frac{1}{1+\rho}\frac{\left(c_{2t+1}C_{2t+1}^{-\alpha}\right)^{1-\sigma}}{1-\sigma}, \quad \rho>-1$$

where c_{1t} is consumption when young, c_{2t+1} is consumption when old, and C_{1t} and C_{2t+1} are average consumptions, of young and old respectively, which are taken as given by individuals. Young agents work a unit of time (i.e., their total labor income is equal to the wage rate). Old agents do not work and must provide consumption through saving. Production for firm i that hires labor and capital is given by

$$Y_t^i = AN_t^i + BK_t^i, \quad A, B > 0$$

where K^i and N^i are the amounts of capital and labor hired by the firm (since there is no population growth, take the aggregate amount of labor, N, to be normalized to one). Markets for factors are competitive, resulting in factors being rewarded their marginal products, r_t and w_t. There is no depreciation of capital in production.

a Find w_t and r_t. Is the economy dynamically efficient?

b Show that when $\sigma > 1$ and $0 < \alpha < 1$ preferences have the property that an increase in aggregate consumption (for either young or old) raises the marginal utility of individual consumption. This is known as "keeping up with the Joneses". Interpret.

c Find savings and capital accumulation in the steady state [*hint: impose that individuals in each generation consume at the generation's average level, i.e.* $c_1 = C_1$ and $c_2 = C_2$].

d Is consumption higher or lower than the optimal level [hint: take the perspective of a planner that considers $c_{1t} = C_{1t}$ and $c_{2t+1} = C_{2t+1}$]? Interpret.

Suppose now that, at t_0, the government starts a pay-as-you-go social security system in which the young contribute an amount τ that is received by the old (you might think of τ as a subsidy).

e Is the social security reform supported by both the young and the old? Explain.

Question 2

Assume a continuum of identical households, whose total number is normalized to one. A representative agent i maximizes utility

$$U_i = C_i - \frac{1}{\lambda} L_i^{\lambda},\ \lambda > 0$$

subject to the budget constraint

$$PC_i = P_i Y_i$$

where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i. The production technology is:

$$Y_i = L_i^{\alpha}, \quad 0 < \alpha < 1$$

There is monopolistic competition in the goods market. The demand for good i is

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$$

where Y denotes aggregate output and $\eta > 1$. The aggregate demand equation is

$$Y = \frac{M}{P}$$

where M denotes nominal money. Agents have rational expectations. The following notation applies, for a generic non-negative variable X: $x \equiv \ln X$.

a Set up the utility maximization problem and provide the relevant first order condition for the representative household.

b Show that the desired (log) price level equals

$$p^* = c + \phi m + (1 - \phi) p \tag{1}$$

where ϕ and c are constants to be derived. [*hint: assume that each producer charges the same price, so that $p_i^* = p^*$. Moreover, since households are all the same and their total number is normalized to one, $y_i = y$.*]

c How is real rigidity, as measured by ϕ, affected by α? Provide some intuition.

From now on set $c = 0$, without loss of generality. Assume that a fraction $(1 - q)$ of the population of firms sets prices in a flexible manner, while the remaining fraction q has rigid prices. Let p^f denote the price set by a representative flexible-price firm and p^r the price set by a representative rigid-price firm. Flexible-price firms set their prices after m is known, while rigid-price firms set their prices before m is known (and thus must form expectations on m and p). All variables are in logarithmic terms.

Suppose flexible-price firms set

$$p^f = \phi m + (1-\phi)p$$

while rigid-price firms set

$$p^r = \phi E[m] + (1-\phi)E[p]$$

Expectations are subject to the information known when fixed-price firms set prices (thus, $p^r = E[p^r]$). Finally, $p = qp^r + (1-q)\,p^f$, with $0 \le q \le 1$.

d Find p^f in terms of p^r, m and the parameter of the model. Then find p^r in terms of $E[m]$ and the parameters of the model.

e Show that equilibrium y and p are, respectively:

$$\begin{aligned} y &= (m - E[m])\,\frac{q}{\phi + (1-\phi)\,q} \\ p &= E[m] + (m - E[m])\,\frac{\phi\,(1-q)}{\phi + (1-\phi)\,q} \end{aligned}$$

f What are the equilibrium values of y and p as $q \to 0$? Explain.