Written Exam for the M.Sc. in Economics summer 2013

Auctions

Final Exam

June 6, 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total including this page.

This exam contains 4 exercises. For each of the exercises, the indicative weighting of the exercise in the grading of the exam is listed in parenthesis.

Exercise 1: True or false statements (20%)
State whether each of the following 3 statements are true or false and give a short explanation.

Question 1a: An owner of a painting sells it in a second-price sealed-bid auction. Upon learning the bids she regrets not using a first price auction.

Statement: As the highest bid was larger than the second highest bid, she would have made a higher revenue if she had chosen a first-price sealed-bid auction instead.

Question 1b: At the venue, Jyske Bank BOXEN, a security guard finds a one-of-a-kind unisex jacket worn by Justin Bieber thrown into a corner during concert. Realizing the fans might value the jacket, the guard decides to sell the jacket through a sealed-bid auction at the local fair the following weekend.

Around 100 girls and a few boys are interested in buying the jacket. Their values can be described as private values. However they all work part time jobs at the minimum wage. Thus, all bidders are budget constrained compared to the value they place on the jacket.

Statement: The symmetric equilibrium in a first-price sealed-bid auction has a higher expected revenue than the symmetric equilibrium in a second-price sealed-bid auction.

Question 1c: 4 bidders are bidding for 2 goods in a Vickrey auction. Their bids are as shown in the table below.

	Bid 1	Bid 2
Bidder 1	20	15
Bidder 2	10	10
Bidder 3	14	12
Bidder 4	6	5

Statement: Bidder 1 wins both goods at a total price of 24.

Exercise 2: The Revenue Equivalence Principle (25%)

The Revenue Equivalence Principle states that if values are independently and identically distributed and all bidders are risk neutral, then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue to the seller. It follows from the Revenue Equivalence Principle that all auctions that live up to it have an expected payment per bidder of:

$$m^A(x) = \int_0^x yg(y)dy$$

A seller has considered what auction format to use for selling an object. The seller has decided to use what he calls a *contribution auction*. In a contribution auction, the bidder with the highest bid wins the object and all bidders that participate in the auction pay 50% of their bid (irrespectively of whether they win the auction).

The seller has carried out a market survey showing that bidders are risk neutral and that bidders' values are independently and identically distributed according to the uniform distribution on the interval [0,1000].

Question 2a: Does the contribution auction satisfy the Revenue Equivalence Theorem?

Question 2b: Find a symmetric and increasing equilibrium strategy.

The seller now carries out some further market research and finds out that there are three bidders in the auction.

Question 2c: Calculate the expected revenue for the seller.

Exercise 3: An auction for oil and gas rights (25%)

The Ministry of Environment wants to set up an auction for oil and gas extraction rights to a sea zone in the North Sea. The exact amount of oil and gas in the sea zone is unknown. All major oil companies have been allowed to conduct some geological oil and gas explorations (searches of the sea zone). The explorations have given each company an estimate of the expected amount of oil and gas in the sea zone.

Question 3a: Describe the setup in terms of auction theory

Initially a second-price sealed-bid auction is chosen.

Question 3b: Describe a symmetric and increasing equilibrium strategy for a bidding oil company.

The Ministry of Finance is concerned about the choice of auction format. Some economists at the Ministry of Finance have written a technical note recommending an open English auction.

Question 3c: Comment on this recommendation.

Exercise 4: A Combinatorial Clock Auction (30%)

A government is about to hold a spectrum auction. In the auction 14 lots of spectrum will be sold. The government has decided to use a Combinatorial Clock Auction format for the spectrum auction. The auction will have three stages: the Primary/Clock Stage, the Supplementary Stage and the Assignment Stage.

Question 4a: Explain briefly how each of the three stages work and what the purpose of each stage is.

The auction has five bidders that have submitted the bids below.

	4 lots	3 lots	2 lots	1 lot
Bidder 1	100	0	0	0
Bidder 2	120	0	0	0
Bidder 3	0	90	0	0
Bidder 4	0	70	0	0
Bidder 5	80	0	0	0

Question 4b: How many lots do each of the bidders win?

Question 4c: How much does each of the bidders pay?