Rettevejledning, Mikro B, august 2012

Problem 1

Jill has a von Neumann Morgenstern utility function U_J on money lotteries constructed by using probabilities and Bernoulli utility function $u_J(x) = x^{½}$, with x being a realized amount.

Jill is offered to enter a lottery which will give her the amount 9 $ with probability 50 % and 25 $ with probability 50 %.

- What is the maximum price Jill is willing to pay to enter this lottery?
- What is the risk premium for Jill?
- Answer the same two questions for Jill's sister, Kate, who has $u_K(x) = x$.
- Please compare and comment the results for the two sisters.

Answer:

Expected utility for Jill is $½·(9)^{½} + ½·(25)^{½} = 4$, hence this lottery is worth $(4)^2 = 16$ $ to her, which is the max price she is willing to pay. The risk premium is the expected value of the lottery (½·(9) + ½·(25) = 17) minus the 16 $, i.e. 1 $. Jill is risk-averse.

Kate is obviously risk-neutral and willing to pay the 17 $, with a risk premium of 0 $.

Problem 2

Al and Bill share a dorm room. Al loves to have music playing when he studies; Bill prefers silence. Let s designate the sound level of music playing, $0 \leq s \leq 1$.
Al's utility function is $u_A(x_A, s) = \ln(s) + ½·x_A$, with x_A being the amount of money available for other consumption. Bill's utility function is $u_B(x_B, s) = x_B – 4·s$. Both of them initially own 5 $. The administrator of the dorm has heard of their conflict, and, having a good economic training, suggests that a perfectly competitive market is opened, such that permits for increasing/decreasing the sound level can be traded.

- Find the Walrasian equilibrium if Al is given the property rights regarding the sound level, i.e. Bill has to buy permits to decrease the sound level from level s = 1.
- Find the Walrasian equilibrium, if, conversely, Bill is given property rights regarding the music level, i.e. Al has to buy permits to increase the sound level from level s = 0.
- Compare the two equilibrium allocations in an Edgeworth box, and comment on the difference in sound level in the two cases.

Answer:

If A has property rights, equilibrium allocation has s = ½, the price for s is 4 and A ends up with $ 7, B with $ 3. The equilibrium price here is determined by Bill's linear preferences (his MRS has a constant absolute value of 4), and quantities are determined by how much Al is willing to trade at the price of 4. With reverse property rights, the equilibrium price and the equilibrium level of s is the same as before, but amounts of money are reversed. That s is the same, independent of property rights, follows Coase's Theorem (2), as both agents have quasi-linear preferences (remember that linear preferences are also quasi-linear!).

Problem 3

Consider the case of a perfectly competitive, and risk-neutral, insurance firm facing two types of customers. There is risky type (type 1) entailing (high) constant marginal costs, MC_1 for each "unit of insurance" the firm provides to such a customer. And there is type 2, who is safer, having lower marginal costs, $0 < MC_2 < MC_1$. For simplicity, assume that all customers have the same, risk-averse preferences.

- Show that in the case of perfect information, such that the company can identify the riskiness of a customer, the equilibrium will have the company offering insurance at prices $p_1 = MC_1$ and $p_2 = MC_2$, respectively, with all type 1 customers becoming fully insured at price p_1, and all type 2 customers becoming fully insured at price $p_{2.}$
- Now assume that the insurance company cannot identify which type a given customer is. Explain why it may, in some cases, happen, that the company sets prices MC_1 and MC_2, respectively, but will offer only partial insurance for customers choosing the low price contract.

Answer:

Perfect competition gives us that prices will be equal to marginal costs, and at actuarially fair prices, with no rationing, both types insure fully. With asymmetric information, risky customers may be tempted to choose the contract aimed at the safer type (Nechyba p. 814). The incentive constraint then forces the company to offer the safer type a contract with price MC_2, but with only partial insurance, sufficiently unattractive that the risky type does not prefer this to full insurance at price MC_1.

Problem 4

A risk-neutral employer is hiring a risk-averse employee for a sales position. The employee's work effort influences the level of sales generated by her. However, chance also plays role, so there is a probability of selling little, even if she works hard, and a probability of selling much, even if she works less hard. A harder work effort costs the employee in terms of disutility.

The employer cannot control the employee's effort. The employer can offer her a contract in which the salary depends on the sales generated. Assume that the employer wants her to choose a high work effort.

- Should the contract offer her the same salary, regardless of sales; yes or no? Please substantiate your answer.
- Should the contract let her salary follow the sales level closely, letting the employer have a constant profit independent of the sales level; yes or no? Please substantiate your answer.

Answer:

The problem here resembles that offered in the note by Birgitte Sloth in which an insurance company has to choose which insurance contract to offer a customer, having to offer a minimum level of expected utility (IR), but at the same time giving incentive to behave cautiously.

Here, offering the same salary in all states will give her no incentive to work hard, hence breaking the Incentive Constraint (IC). Letting her bear all risk will, due to her risk aversion, cost her in terms of expected utility, which will have to be compensated by a higher average salary (to honor the Individual Rationality Constraint (IR), harming the expected profits of the employer; the fact that the employer is risk neutral implies that he should bear at least some risk. (Note that in special cases (for example, if working hard is very costly in terms of disutility, and/or if it only maginally increases the probability of high sales, it may be more profitable for the employer to settle for a contract allowing the employee to work less hard, discarding the IC).

Problem 5

W.Mart, the local grocer, faces two types of customers, both of whom like to eat chocolate. Mr. A, who has a job, has the demand function $D_A(p) = \text{Max}\ \{10 – p, 0\}$ where p is the unit price (in $) of chocolate, whereas the unemployed Mr. B has the demand function $D_B(p) = \text{Max}\ \{5 – p, 0\}$. W.Mart has constant marginal costs of 1 $ when selling one unit of chocolate. Assume for simplicity there are no fixed costs. There are no other shopping options around, so W.Mart has a monopoly position.

- If the shop has to set one price (per unit of chocolate), what should this price be, how much chocolate will be sold to each of the customers, and how much profit will be made?
- The daughter of the owner of W.Mart has studied microeconomics and suggests to her father that he should instead offer a discount to customers who are unemployed (Mr. B can do this by presenting a statement from his unemployment insurance company). Which prices should be set for the two customers, and how much profit will be made?

Answer:

Aggregate demand is $10 – p$ for $5 \leq p \leq 10$ and $15 – 2 \cdot p$ for $0 \leq p \leq 5$. This gives us the inverse aggregate demand function $10 – x$ for $0 \leq x \leq 5$, and $7½ – ½ \cdot x$ for $5 \leq x \leq 15$. So the MR function or inverse aggregate demand is $10 – 2 \cdot x$ for $0 \leq x \leq 5$, and $7½ – x$ for $5 \leq x \leq 15$.

For the case of one common price, we have MR = MC at x = 4½ (with only A buying) and at x = 6½ (A and B are both buying). The former yields price 5.5, revenue 24.75, and profits 20.25, the latter has price 4.25, revenue 27.625, and profits 21.125, hence being the profit maximizing solution.

Looking at the customers separately, we easily get $MR_A = 10 – 2 \cdot x_A$ and $MR_B = 5 – 2 \cdot x_B$. The third-degree price discrimination solution is to charge A the price 5½ and charge B the price 3. A will buy 4½, B will buy 2, and profits will be (24.75+6-6.5) = 24.25. It is unsurprising that profits are higher than before, as price discrimination allows the seller to exploit that A has a lower absolute elasticity in his demand.

Problem 6

Alice and Bridget share an office, and both appreciate having fresh flowers in the vase standing in the window. These flowers constitute a public good. One unit of flowers can be financed by spending 1 $. Let G be the quantity of flowers (for simplicity, treat this as a continuous variable, and let x be money available for other consumption, after having contributed to the flowers. Ann's preferences are represented by the utility function $u_A(x_A,G) = \ln(x_A) + \ln(G)$, while Bridget has utility function $u_B(x_B,G) = \ln(x_B) + 3 \cdot \ln(G)$. Initially, Alice and Briget have amounts e_A and e_B, respectively.

Now, assume that the quantity of flowers will be determined by voluntary donations.

- Show that Ann's best-response donation, taking Bridget's donation g_B as given, can be expressed by $R_A(g_B) = \text{Max}\ \{½(e_A-g_B), 0\}$, and comment on this expression.
- Identify Bridget's best-response donation function and predict the outcome (Nash equilibrium) for the special case when $e_A = e_B$.
- Will the resulting quantity of flowers be efficient?
- Is it possible, in the general case, that one of the agents free-rides completely?

Answer:

Alice needs to maximize $\ln(e_A-g_A) + \ln(g_A+g_B)$ wrt. g_A, taking g_B as given. FOC is $1/(e_A-g_A) = 1/(g_A+g_B)$. Solving this for g_A, and taking into account the impossibility of negative donations, we get the desired expression. We see that Alice wants to donate more, the wealthier she is, and to donate less the more Bridget has already donated. Also note that in some cases, Alice chooses the corner solution; to contribute nothing. Similaerly, we obtain for Bridget: $R_B(g_A) = \text{Max}\ \{(3e_B-g_A)/4, 0\}$. In Nash equilibrium, with $e_A = e_B$, we obtain $g_B^* = 5e/7$, $g_A^* = e/7$, $G^* = 6e/7$. This level of the public good will be inefficiently low; the sum of MRS's being larger than 1 (being 2, in fact). As preferences are not quasi-linear, initial wealth distribution matters. A free-rides completely, donating nothing, if B is relatively wealthy ($e_B > (4/3) \cdot e_A$), with B having no incentive to reduce donations.

Ref.: mtn, 9. juli 2012