Exam Microeconomics A – June 2015

Problem 1

True or false? In each case explain your answer.

1) The marginal rate of substitution of a consumer measures the change in utility from consuming proportionally more of every good.
2) A consumer with rational and monotone preferences will always spend his or her entire budget.
3) A firm will choose to hire labour such that the market value of the marginal product of labour equals the wage rate.
4) A consumer with rational, monotone and convex preferences will always buy more of a good if his/her income increases.

Answer:

1) False; the marginal rate of substitution measures the slope of the indifference curve, and thus the subjective exchange rate, i.e. the willingness to exchange one good for another.
2) True; if not then the consumer can buy more of any good and obtain a more preferable bundle: if $p \cdot x^* < I$, then let $z_i = \frac{1}{N}\frac{I - p \cdot x^*}{p_i}$ and thus $x^* + z \succ x^*$ by monotonicity.
3) True; in the short run $MP_l = f'(l)$ and in the long run $MP_l = f_l'(l,k)$; if $pMP_l > w$ the firm can increase profits by hiring more labour, while $pMP_l < w$ implies that profits can be increased by lowering the labour input, thus $MP_l = \frac{w}{p}$
4) False; there can exist inferior goods.

Problem 2

Consider a consumer, Gerda, with rational, strictly monotone and convex preferences, representable by a utility function $u(x_1, x_2)$. She can buy goods at the prices $p = (p_1, p_2) \gg 0$ and she has a fixed income of $I > 0$. At the going prices she will consume a strictly positive amount of both goods.

The government has proposed to levy a per-unit tax $t$ on good 1.

1) Can we be sure that Gerda will decrease her consumption of good 1?
2) If Gerda has the utility function $u_G(x_1, x_2) = \ln x_1 + x_2$, what is the effect of the tax the her demand for good 1 (find $\frac{\partial x_1(p_1, p_2, I)}{\partial t}$)? (Assume, for simplicity, that her income is sufficient to ensure she is consuming strictly positive quantities of each of the two goods).

An economic consultant hired by the government, however, suggests that the tax instead should be in the form of a lump-sum tax.

3) Explain and/or illustrate why you as an economist would prefer the lump sum tax. Explain the concept of a dead-weight loss.
4) Derive the dead-weight loss in the case the Gerda has the utility function in question 2) and if prices are $p = (1,1)$, tax rate $t = 1$ and Gerda's income is $I = 20$.

Answer:

1) The taxation will increase the price of good 1; but if good 1 is a Giffen good for Gerda, then the tax levy will increase her consumption of good 2. A nice graphical illustration is sufficient.
2) We have $x_1(p_1, p_2, I) = \frac{p_2}{p_1}$ where $p_1$ is the after-tax price, thus $x_1(p_1 + t, p_2, I) = \frac{p_2}{p_1+t}$, and thus $\frac{\partial x_1}{\partial t} = -\frac{p_2}{(p_1+t)^2} < 0$. Thus if $p = (1,1)$ and $t = 1$, then $x_1^* = 1$ and $x_1' = \frac{1}{2}$, while $\frac{dx}{dt} = -\frac{1}{4}$.
3) A lump-sum tax will imply that the slope of the budget line is unchanged, but it will cross the budget line of 1) exactly at the optimal choice in 1) when the two revenues equals. Using a revealed preference argument the consumer is better off using the lump-sum tax. We have obtained the same revenue, but the consumer is better off; thus the lump-sum tax is more efficient than the per unit tax. The EV measures how much Gerda would pay to avoid the tax, and thus we could collect the revenue $EV > T$ and Gerda would be as well off.
4) The death weight loss is equal to the EV less the revenue: since Gerda has quasi-linear preferences we find $EV = \Delta CS = \int_{p_1}^{p_1+t} x_1(p_1, p_2)\, dp_1 = \int_{p_1}^{p_1+t} \frac{p_2}{p_1}\, dp_1 = [p_2 \ln q]_{p_1}^{p_1+t} = p_2(\ln(p_1 + t) - \ln p_1) = p_2 \ln \frac{p_1+t}{p_1} = p_2 \ln\left(1 + \frac{t}{p_1}\right)$ while the revenue is $t x_1(p_1 + t, p_2) = \frac{tp_2}{p_1+t} = \frac{p_2}{\left(1+\frac{p_1}{t}\right)}$, and hence the death weight loss is $EV - T = p_2\left(\ln\left(1 + \frac{t}{p_1}\right) - \frac{1}{\left(1+\frac{p_1}{t}\right)}\right)$ so if $p = (1,1)$ and $t = 1$, then $DWL = \ln 2 - \frac{1}{2} = 0.193$.

Problem 3

Heinrich is a consumer that has a utility function $u(f, c) = f \cdot \sqrt{c}$ in a Koopmans economy, in which the production technology is given by $y = 2\sqrt{l}$. Heinrich owns no initial amount of the consumption good, but has 12 hours to use either for labour or leisure time.

1) Find the Pareto efficient allocation of this economy.
2) What is the Walrasian equilibrium in, when Heinrich fully owns the firm?

Answer:

1) The condition for efficiency: The problem $\max_l u(L - l, f(l))$ gives $-u_l' + u_c' f' = 0$ such that $MRS = \frac{u_l'}{u_c'} = f'$, thus we have here $MRS(L - l, 2\sqrt{l}) = MP(l)$ or $MRS = -\frac{MU_f}{MU_c} = -2\frac{c}{f}$ and $MP_l = \frac{1}{\sqrt{l}}$ becomes $2\frac{2\sqrt{l}}{L-l} = \frac{1}{\sqrt{l}}$ or $L - l = 4l$ such that $l = \frac{12}{5}$.
2) We have that $w = MP_l$ when we normalize $w = \sqrt{\frac{5}{12}}$.

Problem 4

A farmer produces pigs for export by using labour, $l$, and capital, $k$, using the production function $y = f(l, k) = \min\{2l, k\}$. The farmer can hire labour at the wage rate $w > 0$ and capital at a rental rate of $r > 0$. He exports the pigs on the world market where he can sell each pig at the price $p > 0$.

1) Derive the cost function in the long run.

2) Derive the long-run supply function of the farmer.

Answers:

1) Cost minimization yields $y = 2l = k$ $c(w, r; y) = \left(\frac{w}{2} + r\right) y$ is the cost function
2) If $p > \frac{w}{2} + r$ then the farmer supply an infinite amount, $p < \frac{w}{2} + r$ supplies zero, and if $p = \frac{w}{2} + r$ then supply any arbitrary amount.

Problem 5

Viggo lives in two periods and consumes a consumption good in both periods, we denote by $c_1$ his consumption today and $c_2$ his consumption tomorrow. He earns income today of $e_1 > 0$ but nothing tomorrow. He can borrow or lend at an interest rate of $r$. Viggo obtains utility in the form of $u(c_1, c_2) = \ln c_1 + 2 \ln c_2$.

1) Show that Viggo can choose any consumption plan $(c_1, c_2)$ that satisfies $c_1 + \frac{c_2}{1+r} = e_1$
2) Derive the saving function $s(r, e_1)$
3) How will Viggo's utility be affected by an increase in the interest rate? Explain.

Answers

1) The savings is $s = e_1 - c_1$ which can be spend tomorrow $c_2 = (1 + r)s$, since there is no income tomorrow, and thus $c_2 = (1 + r)(e_1 - c_1)$ or the stated
2) The optimal consumption choice is $c_1 = \frac{1}{3} e_1$ thus he saves $s = \frac{2}{3} e_1$
3) Since $c_2 = \frac{2}{3}(1 + r)e_1$ an increase in the interest rate increases Viggo's utility; as a lender the value of this savings in the future increases and thus he obtains more consumption possibilities.