Individual Take-Home Exam for the M.Sc. in Economics 2009-II

ADVANCED MACROECONOMETRICS

Final Project Examination, 2009
June 24, 10:00 – June 26, 10:00

PLEASE NOTE that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

FOCUS ON EXAM CHEATING: In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Practical Information

Note the following formal requirements:

- This is an *individual* examination. You are not allowed to cooperate with other students or other people, see the *focus on exam cheating* above.
- The assignment consists of Sections 2-7 with 20 questions to be answered. *Please answer all questions.*
- In the assessment of your exam paper the sections are (as a guiding principle) given the following weights: Section 2 (20%), Section 3 (15%), Section 4 (20%), Section 5 (15%), Section 6 (10%), and Section 7 (20%).
- The exam paper should not exceed 20 pages. A maximum of 20 pages of supporting material (graphs, estimation output, etc.) can accompany the paper as appendices. You may refer to the computer output in the appendices when answering the questions. Also you may add clarifying comments in the output as part of your answer.
- All pages must be numbered consecutively and marked with your *exam number*. You should *not* write your name on the exam paper.
- *Your paper must be uploaded on the course page in Absalon before June 26 at 10:00.* The exam paper (including supporting material) must be in *PDF-format* and collected in *one file only*; the uploaded file must be named `1234.pdf`, where `1234` is your exam number.

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

1 Background

This project examination deals with econometric models for the monetary sector of the economy and the economic setup is close the discussion in the textbook by Katarina Juselius (*The Cointegrated VAR Model: Methodology and Applications*, Oxford University Press, 2006). The purpose is to assess your ability to use statistical procedures to make inference on the equilibrium structures and the dynamic adjustment properties, as well as your ability to interpret the results.

The data set you are given consists of the five variables

Money	:	The nominal broad money stock (M3).
Prices	:	The deflator to real income ($2000 = 1$).
RIncome	:	Real income (GDP).
Rm	:	Short interest rate (deposit rate in percent *p.a.*).
Rb	:	Long interest rate (10 year bond rate in percent *p.a.*).

All variables are observed quarterly from $1977:1$ to $2006:1$. For the empirical analysis, define the following transformed variables

$$\begin{aligned} m_t^n &= \log(\mathsf{Money}_t) \\ p_t &= \log(\mathsf{Prices}_t) \\ m_t &= \log(\mathsf{Money}_t/\mathsf{Prices}_t) = m_t^n - p_t \\ y_t &= \log(\mathsf{RIncome}_t) \\ dp_t &= 4 \cdot \Delta p_t \\ R_{mt} &= \mathsf{Rm}_t/100 \\ R_{bt} &= \mathsf{Rb}_t/100 \end{aligned}$$

where m_t is the natural log of real money stock and dp_t is the quarterly log inflation rate multiplied with 4 to be comparable with the annual interest rates (i.e. the yield from year to year).

Most of the empirical analysis considers the $p = 5$ dimensional data vector

$$x_t = (m : y : dp : R_m : R_b)_t' .$$

Anticipating unit roots in the variables, economic theory may suggest different candidates for equilibrium relationships between the variables.

First, the expectations hypothesis of interests rates implies that interest rates with different maturities should share the same stochastic trend and that the interest rate spread should hence be stationary, i.e.

$$R_{mt} - R_{bt} = u_{1t}, \tag{1.1}$$

where u_{1t} is a stationary process, $u_{1t} \sim I(0)$. Furthermore, the non-stationarity of the nominal interest rates could be driven by inflation alone suggesting the stationarity of real

interest rates, i.e.

$$R_{mt} - dp_t = u_{2t} \quad \text{and} \quad R_{bt} - dp_t = u_{3t}, \tag{1.2}$$

with $u_{2t}, u_{3t} \sim I(0)$.

Secondly, simple theories for the monetary sector of the economy suggest that velocity is approximately constant,

$$y_t - m_t = u_{4t}, \quad u_{4t} \sim I(0). \tag{1.3}$$

Alternatively, money and income may cointegrate with the opportunity cost of holding money in an LM-curve relationship

$$m_t = \phi_1 y_t - \phi_2(R_{bt} - R_{mt}) + u_{5t}, \quad u_{5t} \sim I(0). \tag{1.4}$$

Specific theories furthermore suggest that income enters homogeneously, i.e. $\phi_1 = 1$, so that inverse velocity cointegrates with the opportunity cost.

Thirdly, the demand side may be characterized by a negative IS-curve relationship between output (possibly de-trended to account for the autonomous growth in productivity and wages) and the real interest rate

$$y_t = -\phi_3(R_{bt} - dp_t) + u_{6t}, \quad u_{6t} \sim I(0). \tag{1.5}$$

Below you are guided through a full cointegration analysis of the variables in x_t. In the analysis of the long-run structure we consider (1.1), (1.2), (1.3), (1.4), and (1.5) as *theoretical* candidates for cointegrating relationships and look for an identified cointegration space that can be interpreted in terms of the outlined theories. The candidates are shown without deterministic terms, but it could be the case that additional deterministic components are needed to balance the autonomous growth in the processes.

Regarding the institutional setup of your specific country, you are informed that the restrictions on the international flows of capital were changes from January 1992.

2 The Statistical Model

Consider the $p-$dimensional vector autoregressive model, VAR(k), given by

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + ... + \Pi_k x_{t-k} + \mu_0 + \mu_1 t + \phi D_t + \epsilon_t, \tag{2.1}$$

for $t = 1, 2, ..., T$. The initial values, $x_{-k+1}, ..., x_{-1}, x_0$, are considered fixed for the statistical analysis and the error term is assumed to be independently Gaussian distributed, $\epsilon_t \sim N(0, \Omega)$, where Ω is the covariance matrix. The vector D_t contains potential dummy variables to be included in the empirical analysis.

[1] Construct the transformed variables defined above and perform a graphical analysis of the time series.

Comment on the time series behavior of the variables in x_t and look for indications of the structures indicated by the relations in (1.1), (1.2), (1.3), (1.4), and (1.5).

[2] Specify and estimate a suitable statistical model for x_t similar to (2.1).
You need to consider issues like: (*i*) the relevant deterministic specification, (*ii*) the appropriate order, k, of the model, (*iii*) potential misspecification of the model relative to the underlying assumptions, (*iv*) the potential need for dummy variables to account for outliers and level shifts, and (*v*) the constancy of the parameters of the empirical model.
In practice it may be necessary to iterate between the steps above before you reach your preferred model, but in the exam paper you only need to present the preferred model and outline the procedure you apply. Note that it may be impossible to find a model that is acceptable in all directions, just do as well as you can.

[3] State the stability condition for the VAR model, i.e. when time series generated from the equations in (2.1) will be stationary. Analyze whether the stability condition of the unrestricted VAR model seems to be accepted for the empirical model.

3 The Cointegration Rank

Now consider the VAR model in error correction form (VECM):

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + \mu_0 + \mu_1 t + \phi D_t + \epsilon_t. \qquad (3.1)$$

for $t = 1, 2, ..., T$.

[4] State the characteristic polynomial to (2.1) or (3.1) for your preferred model and explain how the presence of a unit root in the characteristic polynomial is related to the reduced rank of Π.

[5] Discuss how the likelihood ratio test for the cointegration rank is calculated. Explain your preferred treatment of the trend and other deterministic terms in the testing procedure.

[6] Determine the cointegration rank, $r = Rank(\Pi)$, by using *all* the available sources of information.

4 Testing Hypothesis

Independent of your preferred cointegration rank in Section 3, continue with a cointegration rank of $r = 2$ in the following.

[7] Impose the reduced rank, $\Pi = \alpha\beta'$, in (3.1) and estimate the cointegrated VAR (CVAR) model. Comment on the results.

[8] Perform tests for *long-run exclusion* for all variables in the model, including deterministic terms, i.e. the hypothesis that the variables does not enter any of the long-run relationships. Explain how the hypotheses are tested and how to calculate

the degrees of freedom. Comment on the implications in relation to the theoretical candidates in Section 1.

[9] Perform tests for *stationarity* of the variables in x_t, possibly allowing for deterministic terms. Explain how the hypotheses are tested and how to calculate the degrees of freedom. Comment on the results in relation to the graphical analysis.

[10] Test whether each of the theoretical candidates for cointegrating relationships, (1.1), (1.2), (1.3), (1.4), and (1.5) are stationary for your data, possibly allowing for deterministic terms. Discuss whether the estimated relationships can be interpreted from economic theory (i.e. whether the signs and magnitudes of the estimated coefficients and the error correction make sense), and compare with the tentative conclusions from the graphical analysis.

5 Identification

[11] Consider a restricted cointegration space defined by $\beta = (\beta_1 : \beta_2) = (H_1\varphi_1 : H_2\varphi_2)$. Give the formal conditions under which the structure is *identifying* the two cointegrating relationships.

[12] Based on the findings in Section 4, try to identify the cointegrating relationships in the empirical model. Give an economic interpretation of the long-run structure and the equilibrium adjustment.

[13] Perform a recursive estimation and discuss whether the identified cointegrating parameters are stable over time.

6 The Moving Average Representation

[14] Estimate and interpret the Granger representation for your preferred identified structure.

[15] Perform tests for *weak exogeneity* of the variables in x_t. Explain how the Granger representation changes in the presence of weakly exogenous variables. Is it possible in the model above that all variables except m_t are simultaneously weakly exogenous?

7 I(2) Analysis

The I(1) data vector considered so far,

$$x_t = (m_t : y_t : dp_t : R_{mt} : R_{bt})',$$

is a transformation of a data set in levels

$$y_t = (m_t^n : p_t : y_t : R_{mt} : R_{bt})'.$$

In this section we consider the original *nominal data* in y_t.

[16] Estimate a VAR model for y_t using the same specification of dummy variables as in preferred model for x_t in (2.1).
Scrutinize the empirical model for signs of double unit roots and report any evidence suggesting that the nominal variables are integrated of order two, $y_t \sim I(2)$.

[17] Formally define the cointegrated I(2) VAR model as a sub-model of the general VAR. Explain how the different I(2) models are nested and how the cointegration ranks (r, s) can be determined.

[18] Try to calculate the likelihood ratio tests for the cointegration ranks (r, s_1). Note that if you have included level shifts in the model, the asymptotic tables in CATS are not valid, and the results are only indicative.

[19] Impose the reduced ranks $(r, s_1) = (2, 2)$, allowing for $s_2 = p - r - s_1 = 1$ I(2) trend in the model, and estimate the model. Look at the estimate of $\beta_{\perp 2}$ and explain how the single I(2) trend seems to affect the variables.

[20] Carefully explain how the real data we have previously analyzed, x_t, can be derived from a *nominal-to-real transformation* applied to the nominal data, y_t. Explain the restrictions that should be imposed on the cointegrated I(2) VAR model to make the transformation valid and test the appropriate restrictions in CATS. Comment on the results.