Written Exam for the B.Sc. in Economics summer 2011

Macro A

Final Exam

24 June 2011

(3-hour closed book exam)

All questions, 1.1-1.3 and 2.1-2.8, to be answered.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem 1: Short questions

(In these problems the focus is on the verbal, intuitive explanations. Formal analysis can, however, be used in the explanations if wanted).

1.1) Explain why there will necessarily exist unemployment in the general equilibrium model of union wage setting.

1.2) Explain why positive growth in output per worker can not be sustained in the basic Solow model without technological growth.

1.3) Explain the concept conditional convergence. Does the empirical evidence tend to support conditional convergence?

Problem 2: **Social infrastructure and endogenous growth**

(In this problem formal and computational elements are more important, but verbal, intuitive explanations are still important)

In the following, the idea that social infrastructure (to be interpreted as the quality of institutions and government policies) affect society's ability to convert inputs into output, is pursued. Consider the following model for a closed economy, where Λ_t represents social infrastructure at time t.

1) $Y_t = K_t^{\alpha} \cdot L_t^{1-\alpha} \cdot \Lambda_t^{\phi}$

2) $K_{t+1} = K_t \cdot (1-\delta) + I_t$

3) $I_t = S_t$

4) $S_t = s \cdot Y_t$

5) $L_{t+1} = L_t \cdot (1+n)$

6) $C_t = Y_t - S_t$

Notation is the same as in the book. The parameters of the model are.

$0 < \alpha < 1$, $\phi \geq 0$, $\delta \geq 0$, $n > -1$ and $0 < s < 1$.

Finally social infrastructure is considered as being endogenous, such that Λ_t depends positively on prosperity, capturing the idea that richer countries can afford to invest more in better institutions:

7) $\Lambda_t = Y_t^{\lambda}$

where $\lambda \geq 0$.

Now we consider the case where $0 \leq \lambda \cdot \phi < 1 - \alpha$

2.1)

Interpret briefly equation 1) - 6). Show that:

8) $Y_t = K_t^{\alpha/(1-\chi)} \cdot L_t^{(1-\alpha)/(1-\chi)}$

where we have defined $\chi = \lambda \cdot \phi$. What is the interpretation of χ? Show that the production function exhibits increasing returns to scale with respect to capital and labour together when $\chi > 0$ and explain why this is the case.

In order to ease the analysis of the model we now define a new variable:

9) $A_t \equiv Y_t^{\chi/(1-\alpha)}$

which can be interpreted as measuring labour augmenting technology. With this definition the production function can be written as:

10) $Y_t = K_t^{\alpha} \cdot (A_t \cdot L_t)^{1-\alpha}$

Notice that 9) and 10) implies that the change in A_t over time is given by:

$$11)\ \frac{A_{t+1}}{A_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\frac{\alpha\cdot\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{1-\chi}}$$

Further we now define:

$\tilde{k}_t = \dfrac{K_t}{L_t \cdot A_t}$ and $\tilde{y}_t = \dfrac{Y_t}{L_t \cdot A_t}$

Notice that $\tilde{y}_t = \tilde{k}_t^{\alpha}$ and as an implication that the inverse capital output ratio is given by:

$$12)\ \frac{Y_t}{K_t} = \frac{\tilde{y}_t}{\tilde{k}_t} = \tilde{k}_t^{\alpha-1}$$

2.2)

Show that $\dfrac{\tilde{k}_{t+1}}{\tilde{k}_t} = \dfrac{L_t}{L_{t+1}} \cdot \dfrac{K_{t+1}}{K_t} \cdot \dfrac{A_t}{A_{t+1}}$ and as an implication that the transition curve for $\tilde{k}_t$ is given by:

$$13)\ \tilde{k}_{t+1} = \tilde{k}_t \cdot \left(\frac{1}{1+n}\right)^{\frac{1}{1-\chi}} \cdot \left(1-\delta+s\cdot\tilde{k}_t^{\alpha-1}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}}$$

(hint: use equation 2), 3), 4) 5), 11) and 12)). Illustrate the transition curve (you may take as given that the economy will actually converge towards steady state). What does this transition curve reduce to when $\chi = 0$?

2.3)

Define the steady state in this model and show that the steady state values of $\tilde{k}_t$ and $\tilde{y}_t$ are given by:

$$14)\ \tilde{k}^* = \left(\frac{s}{(1+n)^{(1-\alpha)/(1-\alpha-\chi)} - (1-\delta)} \right)^{\frac{1}{1-\alpha}}$$

$$15)\ \tilde{y}^* = \left(\frac{s}{(1+n)^{(1-\alpha)/(1-\alpha-\chi)} - (1-\delta)} \right)^{\frac{\alpha}{1-\alpha}}$$

In order to secure meaningful expressions it is assumed that: $(1+n)^{(1-\alpha)/(1-\alpha-\chi)} > 1-\delta$.

Explain how $\tilde{k}_t$ and $\tilde{y}_t$ depends on s, n and δ.

2.4)

Explain why the growth rates of A_t, $k_t = \frac{K_t}{L_t}$ and $y_t = \frac{Y_t}{L_t}$ are all equal in steady state, implying

that: $g_{se} = \frac{A_{t+1}}{A_t} - 1 = \frac{k_{t+1}}{k_t} - 1 = \frac{y_{t+1}}{y_t} - 1$

Show that this common growth rate is given by (*hint:* use equation 11), 5) the definition of k_t and k_{t+1} and the fact that the growth rates of k_t and A_t are equal in steady state):

$g_{se} = (1+n)^{\chi/(1-\alpha-\chi)} - 1$

Explain why positive steady state growth can not be sustained when $n = 0$ or $\chi = 0$. Explain further why g_{se} depends positively on n. Is this a weak or a strong scale effect?

2.5)

Show that the steady state level of y_t and c_t are given by (*hint:* for the derivation of y_t^* use equation 9) and the definitions of $\tilde{y}_t$ and y_t):

$$y_t^* = \left(\tilde{y}^*\right)^{(1-\alpha)/(1-\alpha-\chi)} \cdot L_t^{\chi/(1-\alpha-\chi)} = \left(\frac{s}{(1+n)^{(1-\alpha)/(1-\alpha-\chi)} - (1-\delta)} \right)^{\frac{\alpha}{1-\alpha-\chi}} \cdot \left(L_0 \cdot (1+n)^t\right)^{\chi/(1-\alpha-\chi)}$$

(the last derivation just uses equation 15) and the fact that: $L_t = L_0 \cdot (1+n)^t$).

$$c_t^* = (1-s) \cdot \left(\frac{s}{(1+n)^{(1-\alpha)/(1-\alpha-\chi)} - (1-\delta)} \right)^{\frac{\alpha}{1-\alpha-\chi}} \cdot \left(L_0 \cdot (1+n)^t\right)^{\chi/(1-\alpha-\chi)}$$

Explain why there are counteracting effects from an increase in n on y_t^* and c_t^*.

2.6)

Illustrate the relationship between s and c_t^* in a diagram and explain why this relationship is non-monotonic. Define the golden rule value of s and show that this is given by:

$$s^{GR} = \frac{\alpha}{1-\chi}$$

Explain why s^{GR} is larger than in the case where $\chi = 0$.

For the rest of the exam the case where $\chi = 1-\alpha$ and $n = 0$ is considered.

2.7)

Show that in this case output is given by:

$$Y_t = K_t \cdot \theta$$

where $\theta = L^{(1-\alpha)/\alpha}$. Further show that the growth rate of y_t, k_t and c_t is given by:

$$g_e = s \cdot \theta - \delta$$

Illustrate in appropriate diagrams. Explain why an increase in s now results in a higher growth rate.

2.8)

Discuss whether the properties of this model (with $\chi = 1-\alpha$) are plausible. Does this model predict convergence of any kind?