Suggestive solution for
Written Exam for the B.Sc. in Economics 2011-II
Macroeconomics C

Competence description: At the end of the course, the student should be able to demonstrate:

- Understanding of the main model frameworks for long-run macroeconomics. This includes the Diamond model with overlapping generations in discrete time and the Ramsey model in continuous time.
- Proficiency in the application of the concepts and methods from these frameworks, including competence in dynamic optimization and dynamic analysis in discrete and continuous time.
- Understanding of the role of expectations and basic knowledge of macroeconomic models with forwardlooking expectations under both perfect foresight and uncertainty and rational expectations.
- Proficiency in the application of the related concepts and methods.
- Competence in analyzing a macroeconomic problem, where the above-mentioned concepts and methodsare central, that is competence in solving such models and explaining in economic terms the results andimplications and how they derive from the assumptions of the model.

The particularly good performance, corresponding to the top mark, is characterized by a complete fulfilment of these learning objectives.

Problem A

1. **False.** In a PAYG pension system, each young individual in any given period contributes an amount which is immediately used to fund a pension to the old individuals of the same period. Since the young individuals know that they will also receive a pension when they turn old, the system may be seen from the point of view of individuals as a forced saving scheme, and this will unambigously reduce voluntary saving. Since voluntary saving is invested in physical capital, whereas the contributions to the PAYG pension system are immediately paid out as pensions to the currently old, this will reduce investment in physical capital which will thus (in normal circumstances) cause capital and thus real output to decrease over time and to cause total consumption and the real wage (the marginal product of labour) to decrease, while the interest rate (the marginal product of capital) increases.

2. **True.** Assuming that the monopoly does not reset its price, the price chosen will be higher than the one that maximizes profits after the contraction in demand. This price will result in lower demand and thus consumer surplus will have decreased (compared to the situation where the monopoly resets its price). In addition, monopoly profits will have decreased, since the price is not profit maximizing. As a consequence, social surplus will have decreased and it will be socially beneficial to reset the the price if doing so causes an increase in the social surplus which is greater than the cost of changing the cost (the 'menu cost'). However, since it is the decision of the monopoly whether to reset the price, the monopoly will reset the price only if the gain in producer surplus is greater than the menu cost.

Problem B

1. In the long run actual and expected inflation must coincide, $\pi_t = \pi^e_{t,t-1}$. It then follows from the SRAS in eq. (B.4) that (in the absence of shocks, i.e. when $s_t = v_t = u_t = 0$) $y_t = \bar{y}$. From the AD curve in eq. (B.5) it then follows that $\pi_t = \pi^*$.
Figure 2.2 illustrates. In period t_0 the economy is in long run equilibrium at E$_0$. With a lower level of target inflation, the AD curve is given by eq. (B.5) with π^* replaced by π'$< \pi^*$, i.e.
$$y_t = \bar{y} - \alpha_2 h\left(\pi_t - \pi\text{'}\right) \tag{B.5'}$$
Consequently, in period t_0 the AD curve shifts to the left from AD$_0$ to AD$_1$ (which passes through $(\bar{y}, \pi\text{'})$. Since the position of the SRAS curve is determined by $\pi^e_{1,0} = \pi_0 = \pi^*$, the SRAS curve is unaffected and the economy moves to equilibrium at E$_1$ where both inflation and real output have decreased.

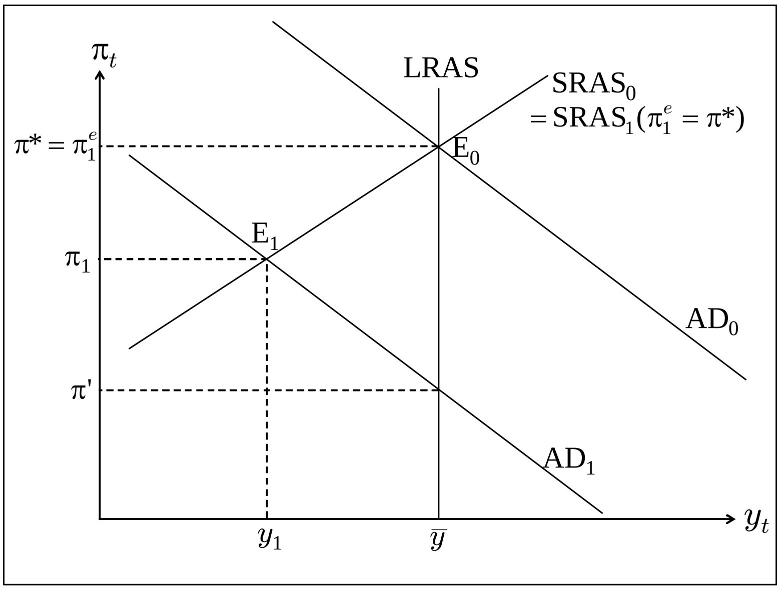

Figure B.1

The economic reason is that the lower target rate of inflation has caused the fiscal policy authorities to decrease public spending. This will decrease goods market demand and lead to lower real output in equilibrium. The decrease in real output decreases firms' use of labour which causes the marginal product of labour to increase and this leads firms' marginal costs to increase at a lower pace. This induces firms to increase prices at a lower pace, i.e. it leads to lower inflation.

As shown in figure B.2, in period t_2 the AD curve will remain at AD_1 (it could be added that the lower target rate of inflation may be seen as a permanent negative shock to demand), while the SRAS curve will now shift downwards to SRAS_2 due to expected inflation being given by $\pi^e_{2,1} = \pi_1$ and thus being lower than expected inflation in period t_1. This brings the economy to the equilibrium at E_2, where real output has increases, while inflation decreases further.

The economic reason is that the decrease in expected inflation (from $\pi^e_{1,0} = \pi^*$ to $\pi^e_{2,1} = \pi_1$) will cause the labour union to reduce the growth rate of nominal wages which will in turn make the marginal costs of the firm increase at a slower pace. Consequently firms will increase their prices at a slower pace, i.e. inflation will be reduced. The reduction in inflation will then fiscal authorities to increase public spending and this increases goods market demand through increased investment and possibly increased private consumption which finally leads to increased real output.

In period t_3 also, we have $\pi^e_{3,2} = \pi_2 < \pi^e_{2,1}$ and consequently this process of downward-shifting SRAS curves will continue until the economy has reached the new long run equilibrium at E_{LR} where real output is once again equal to its natural level, while inflation has fallen permanently to its new target level.

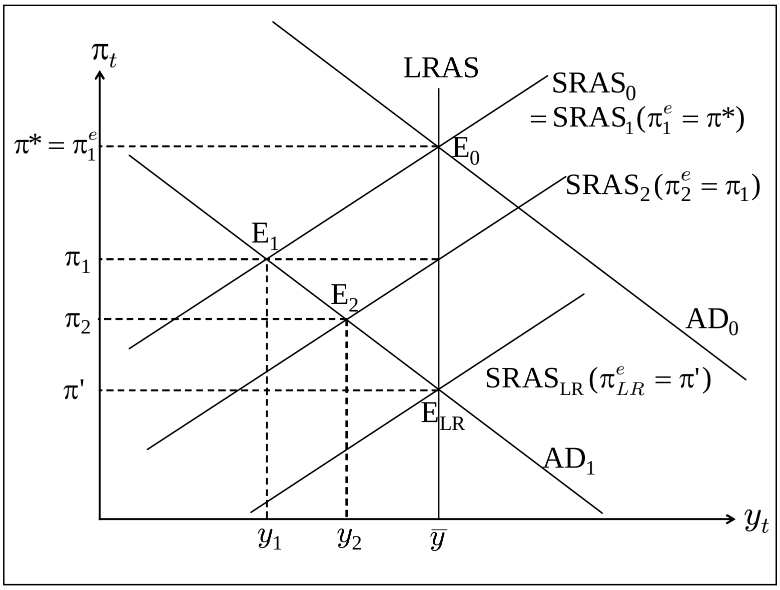

Figure B.2

2. Solving models with rational expectations can be done using the following three-step procedure:

Step 1: Here we solve the model for the endogenous variables, the expectations of which enter the model. Specifically, this means solving for π_t.

Inserting (B.5) into (B.4) produces

$$\begin{aligned}
\pi_t &= \pi^e_{t,t-1} + \gamma\left(z_t - \alpha\left(\pi_t - \pi^*\right)\right) + s_t \\
&= \pi^e_{t,t-1} + \gamma z_t - \gamma\alpha\pi_t + \gamma\alpha\pi^* + s_t \Leftrightarrow \\
\pi_t &= \frac{1}{1+\gamma\alpha}\pi^e_{t,t-1} + \frac{\gamma\alpha}{1+\gamma\alpha}\pi^* + \frac{\gamma}{1+\gamma\alpha}z_t + \frac{1}{1+\gamma\alpha}s_t \qquad (1)
\end{aligned}$$

Step 2: Using the result from step 1, we now form expectations, using the assumption of rational expectations

Based on (1) we thus get, using the fact that s_t and v_t, and conseqently z_t, are white noise and therefore have zero means,

$$\begin{aligned}
\pi^e_{t,t-1} &= E\left(\pi_t \middle| I_{t-1}\right) \\
&= E\left(\frac{1}{1+\gamma\alpha}\pi^e_{t,t-1} + \frac{\gamma\alpha}{1+\gamma\alpha}\pi^* + \frac{\gamma}{1+\gamma\alpha}z_t + \frac{1}{1+\gamma\alpha}s_t \middle| I_{t-1}\right) \\
&= \frac{1}{1+\gamma\alpha}\pi^e_{t,t-1} + \frac{\gamma\alpha}{1+\gamma\alpha}\pi^* \Rightarrow \\
(1+\gamma\alpha)\,\pi^e_{t,t-1} &= \pi^e_{t,t-1} + \gamma\alpha\pi^* \Rightarrow \\
\pi^e_{t,t-1} &= \pi^* \qquad (2)
\end{aligned}$$

Step 3: Inserting the result from step 2 into the result from step 1 we get

$$\begin{aligned}
\pi_t &= \frac{1}{1+\gamma\alpha}\pi^* + \frac{\gamma\alpha}{1+\gamma\alpha}\pi^* + \frac{\gamma}{1+\gamma\alpha}z_t + \frac{1}{1+\gamma\alpha}s_t \Rightarrow \\
&= \pi^* + \frac{\gamma}{1+\gamma\alpha}z_t + \frac{1}{1+\gamma\alpha}s_t \\
&= \pi^* + \frac{\gamma}{1+\gamma\frac{\alpha_1 b}{1+\alpha_1 k}}\frac{v_t}{1+\alpha_1 k} + \frac{1}{1+\gamma\frac{\alpha_1 b}{1+\alpha_1 k}}s_t \\
&= \pi^* + \frac{\gamma}{1+\alpha_1 k+\gamma\alpha_1 b}v_t + \frac{1+\alpha_1 k}{1+\alpha_1 k+\gamma\alpha_1 b}s_t \\
&= \pi^* + \frac{1}{1+\alpha_1 k+\gamma\alpha_1 b}\left(\gamma v_t + (1+\alpha_1 k)\,s_t\right) \qquad (B.9)
\end{aligned}$$

And now to the analysis of the shock. Based on the expression in eq. (B.9) and the fact that v_t and s_t are white noise it follows that $\pi_{1,0} = E\left[\pi_1 \left| I_0\right.\right] = E\left[\pi^* + \frac{1}{1+\alpha_1 k+\gamma\alpha_1 b}\left(\gamma v_t + (1+\alpha_1 k)\,s_t\right.\right.$ π^*. It then follows from eq. (B.4) that the SRAS will in period t_1 remain at its position from period t_0 just as was the case in question 1. The AD curve also shifts left-ward to AD_1 just as in question 1 and thus the situation is as depicted in Figure B.1 with real output and inflation decreasing for the same economic reasons as in question 1.

In period t_2 the situation is different, however: based on eq. (B.9) we find that $\pi_{2,1} = E\left[\pi_2 \left| I_1\right.\right] = E\left[\pi\text{'} + \frac{1}{1+\alpha_1 k+\gamma\alpha_1 b}\left(\gamma v_t + (1+\alpha_1 k)\,s_t\right)\left| I_1\right.\right] = \pi\text{'}$ and thus the SRAS curve from period t_2 and onwards has shifted to $\mathrm{SRAS_{LR}}$ as shown in Figure B.2 and with the AD curve being unaffected at AD_1, the economy will from period t_2 onwards be in the new long-run equilibirum $\mathrm{E_{LR}}$.

The economic reason for the immediate jump from E_1 (see Figure B.2) to the new long-run equilibrium in period t_2 is the fact that with expectations being rational and the fact that the new target rate of inflation is now known, the expected inflation will decrease in period t_2 to π'. By the economic mechanisms following from lower expected inflation described above, this immediately moves the economy to the new long-run equilirium.
Comparing with the evolution in question 1 this is one instance where rational expectations are probably more realistic than static expectations, since in the latter case, expected inflation systematically deviates from (is higher than) actual inflation during the entire adjustment proces from t_1 onwards, despite the fact that is has been publicly announced that the target rate of inflation has been decreased. With rational expectations, there are only expectation errors in period t_1 where the target rate is unexpectedly changed.

3. According to eq. (B.2) a higher value of k corresponds to a higher degree of responsiveness in fiscal policy to deviations in real output from its natural level. It is readily seen (B.8) that a higher value of k will decrease the effect on y_t from both demand and supply shocks, i.e. $v_t \neq 0$ and $s_t \neq 0$.
Following a demand shock (a positive one, to be specific), the economic effects are:

$$v \uparrow \rightarrow y \uparrow \underset{k}{\rightarrow} g \downarrow \rightarrow y \downarrow \tag{3}$$

The second part of the process whichs counteracts the initial effect on real output, is stronger the stronger the value of k and thus a higher value of k stabilises real output.
Following a supply shock (negative to be specific), the economic effects are:

$$s \uparrow \rightarrow \pi \uparrow \rightarrow g \downarrow \rightarrow y \downarrow \underset{k}{\rightarrow} g \uparrow \rightarrow y \uparrow \tag{4}$$

and once again it follows directly that a stronger response through fiscal policy to changes in real output will tend to stabilise real output.
With respect to the effect on inflation, eq. (B.9) shows a high value of k to also stabilise inflation following a demand shock. The reason is that (compare with (3) above, and in the following L denotes employment, MP_L the marginal product of labour, and MC firms' marginal costs)

$$v \uparrow \rightarrow y \uparrow \begin{array}{l} \nearrow \;\; L \uparrow \rightarrow MP_L \downarrow \rightarrow MC \uparrow \rightarrow \pi \uparrow \\ \\ \underset{k}{\searrow} \;\; g \downarrow \rightarrow y \downarrow \rightarrow L \downarrow \rightarrow MP_L \uparrow \rightarrow MC \downarrow \rightarrow \pi \downarrow \end{array} \tag{5}$$

Since inflation is thus basically affected by a demand shock *through* the effect from the demand shock on real output, stabilising real output will also stabilise inflation.
However, following a supply shock there is a trade-off between stabilising real output and inflation. The reason may be seen from eq. (B.5) according to which a supply shcok will have to affect either π_t or y_t or a combination, and the smaller the effect on one, the

larger will be the efefct on the other. This may also be seen by analysing the economic mechanisms following the supply shock which are (compare with (4))

$$s\uparrow\rightarrow\pi\uparrow\rightarrow g\downarrow\rightarrow y\downarrow \quad \begin{array}{l} \nearrow \quad L\downarrow\rightarrow MP_L\uparrow\rightarrow MC\downarrow\rightarrow\pi\downarrow \\[1em] \underset{k}{\searrow} \quad g\uparrow\rightarrow y\uparrow\rightarrow L\uparrow\rightarrow MP_L\downarrow\rightarrow MC\uparrow\rightarrow\pi\uparrow \end{array}$$

This shows that the part of the proces that tends to stabilise real output will amplify the increase in inflation.

4. The point to note is that k does not enter the solution for neither y_t nor π_t and thus the part of fiscal policy depending on the expected output gap has absolutely no effect. This is an instance of the so-called Policy Ineffectiveness Proposition (PIP) according to which active demand mangement policies cannot affect real output (and employment) when expectations are rational. The economic reason is that, as seen from (B.5), if y_t is to be affected systematically (for reasons other than stochastic shocks), actual and expected inflation will have to deviate, $\pi_t \neq \pi^e_{t,t-1}$, i.e. it should be possible for policy makers to create surprise inflation. However, when private agents have rational expectations and specifically, when they know the policy rule in (B.10), they are able to fully foresee the part of fiscal policy that depends on the expected real output gap, and conseqently this part of policy will not be able to create the necessary surprise inflation.
However, as seen from question 3, this is not the case when fiscal policy depende on the actual output gap (or more generally, if policy is conducted using information on the output gap that private agents, even having rational expectations, cannot fully foresee), and in that case the solution for real oputput depended on the value of the parameter k, i.e. the PIP did not hold.

Problem C

1. From eq. (C.6) it follows that

$$\dot{p} = 0 \Rightarrow \gamma\eta Q - \gamma\beta p + \gamma(d - \bar{y}) = 0 \Leftrightarrow$$

$$\dot{p} = 0 : Q = \frac{\beta}{\eta}p - \frac{1}{\eta}(d - \bar{y}) \tag{1}$$

where thus eq. (1) is the equation for the $\dot{p} = 0$ locus (Goods Market, GM curve).
From eq. (C.7) we get

$$\dot{Q} = 0 \Rightarrow \left(r^f - \alpha\eta\right)Q + \alpha\beta p - \alpha d = 0 \Leftrightarrow$$

$$\dot{Q} = 0 : Q = -\frac{\alpha\beta}{r^f - \alpha\eta}p + \frac{\alpha}{r^f - \alpha\eta}d \tag{2}$$

(Provided that $r^f \neq \alpha\eta$, which holds since $r^f > \alpha\eta$.) Eq. (2) is the equation for the $\dot{Q} = 0$ locus (the Asset Market, AM curve).
The phase diagram is constructed by first drawing the loci where $\dot{p} = 0$ and $\dot{Q} = 0$, i.e. the loci given by eqs. (1) and (2).
Eq. (1) shows the $\dot{p} = 0$ locus to be an upward-sloping line with a slope of $\frac{\beta}{\eta} > 0$ (in a (p, Q)-diagram) and an intercept with the vertical axis of $-\frac{1}{\eta}(d - \bar{y})$, while eq. (2) shows the $\dot{Q} = 0$ to be a downward-sloping line with a slope of $-\frac{\alpha\beta}{r^f - \alpha\eta} < 0$ (when $r^f > \alpha\eta$), and an intercept with the vertical axis equal to $\frac{\alpha}{r^f - \alpha\eta}d$. Given that the two loci are stated to intersect in the positive orthant, they must look as in figure C.1.
The directions of motions indicated by the arrows in figure 1 are found from eqs. (C.6) and (C.7) in the following way: Beginning at any point on the $\dot{p} = 0$ locus moving either vertically up (increasing Q with p unchanged) or horizontally to the left (decreasing p with Q unchanged), it may be concluded from eq. (C.6) that at the new point it is the case that $\dot{p} > 0$, i.e. that p will be increasing over time. This is indicated in figure 1 by the horizontal and rightward-pointing arrows to the left of (above) the $\dot{p} = 0$ locus. In a similar manner it may be concluded that $\dot{p} < 0$ to the right of (below) the $\dot{p} = 0$ locus explaining the horizontal leftward-pointing arrows there.
Beginning at any point on the $\dot{Q} = 0$ locus moving either vertically up or horizontally to the right, it follows from eq. (C.7) that at these points it is the case that $\dot{Q} > 0$ (moving vertically up, i.e. increasing Q with p unchanged, one should use the assumption that $r^f > \alpha\eta$). Consequently Q is increasing over time above (to the right of) the $\dot{Q} = 0$ locus which explains the upward-pointing vertical arrows there. In a similar manner one can explain the downward-pointing vertical arrows to the left of (below) the $\dot{Q} = 0$ locus indicating that at these points Q will be decreasing over time.

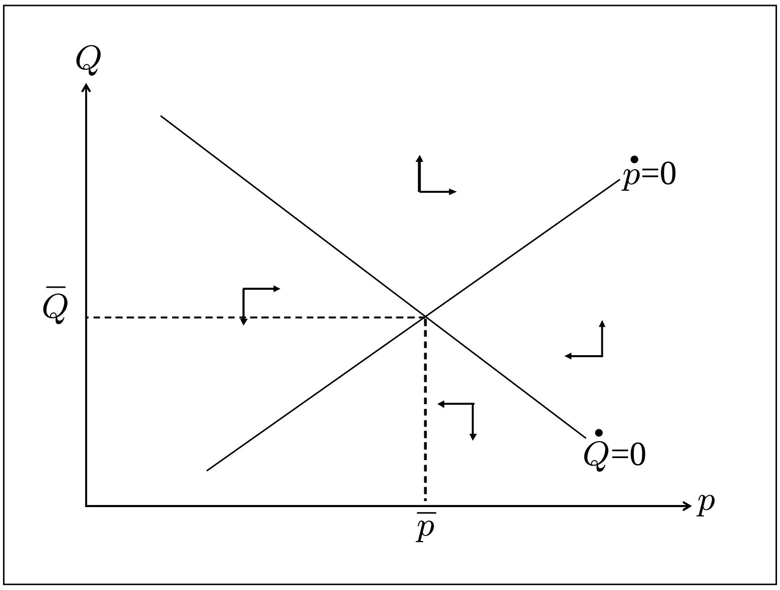

Figure C.1

The directions of motion shown in figure C.1 reveal that the economy is saddle point stable meaning that there is a unique path, the negatively sloped saddle path SS in figure C.2, approaching (from either side) the long run equlibrium at E, where p and Q are constant, while all other paths are diverging from the long run equilibrium at E. The diverging paths may be thought of as bubbles in the stock price where the stock price is eventually ever increasing or decresing due to self-fullfilling expectations. However, since there is rational expectations and no uncertainty which together imply perfect foresight, such evolutions may be ruled out on the argument that rational agents will not believe the stock price to be forever increasing or decreasing.[1] Since at any point in time p is predetermined, while Q is free to jump/adjust because, as argued in question 2, it is based on the future, we can conclude that for any initial value of p, Q will be chosen such that the economy is on the saddle path and evolves along this to the long run equilibrium at point E.

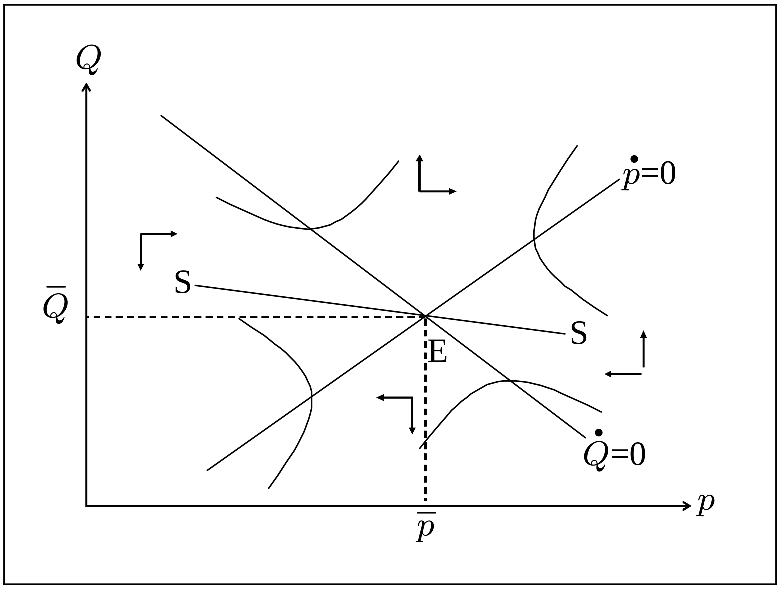

Figure C.2

1. $e^{-r(s-t)}$ is the continous time discount factor used when discounting from time s back to time t. Thus equation (C.8) states that the equlibrium stock price is the sum (integral) of all present and future discounted dividends between time t and the future time T plus the discounted value of the stock price at the future time T (which is known since the model assumes perfect foresight). This is easily interpretable: When buying shares one basically buys the right to a future stream of dividend income and the payment that one is willing to make (the stock price) is the total discounted value of this stream of

[1] It could be added that an evolution where the economy follows a diverging path for a while and then jumps to the saddle path is also not consistent with rational behaviour, since with perfect foresight, the time of the jump would be known to all agents. If therefore, e.g., the stock price were to increase discretely (jump) all agents would want to purchase shares the instant before the jump. This, however, would drive up the price the instant before, making agents want to purchase shares even earlier making the price increase even earlier and so forth. By continuing to drive this argument backwards in time, it may be concluded that such an evolution could never get started.

dividends plus whatever the share is worth in the future and can thus be sold for.

In question 1 it was argued that the economy will always be moving along a path that eventually reaches the saddle path and thus ends up in long run equilibrium where $Q = \bar{Q}$, i.e. is constant. We thus have

$$\lim_{T\to\infty} e^{r(T-t)} Q(T) = \lim_{T\to\infty} e^{r(T-t)} \bar{Q} = \bar{Q} \lim_{T\to\infty} e^{r(T-t)} = 0$$

and using this when letting $T = \infty$ in eq. (C.8) we find

$$Q(t) = \int_t^\infty D(s)\, e^{-r(s-t)} ds \tag{3}$$

just saying that the stock price is the sum of all present and future discounted dividends. This is known as the *fundamental value* of the shares.

It could be added that in the long run equilibrium the price level is constant, it follows from eq. (C.1) that $y = \bar{y}$ and it then follows from eq. (B.5) that $D = \alpha\bar{y}$, i.e. dividends will be constant. Using this together with the fact that $r = r^f$, we get from eq. (3)

$$Q(t) = \int_t^\infty D(s)\, e^{-r(s-t)} ds = \int_t^\infty \alpha\bar{y} e^{-r^f(s-t)} ds$$

with the solution to the last integral being $\frac{\alpha\bar{y}}{r^f}$ which is exactly the long run value, $\bar{Q}$, and which can alternatively be obtained by calculating the intercept between the $\dot{p} = 0$ and $\dot{Q} = 0$ loci.

2. The phase diagram is shown in figure C.3. As is seen from eqs. (1) and (2) the $\dot{Q} = 0$ locus is unaffected, while the $\dot{p} = 0$ locus decreases its intercept with the vertical axis. Thus, the long run equlibrium changes from E_1 to E_2 as shown in figure C.3. (In figure C.3 the original $\dot{p} = 0$ locus is dotted while the new is denoted $\dot{p} = 0_{new}$. Figure C.3 only shows the directions of motion associated with the *new* loci.)

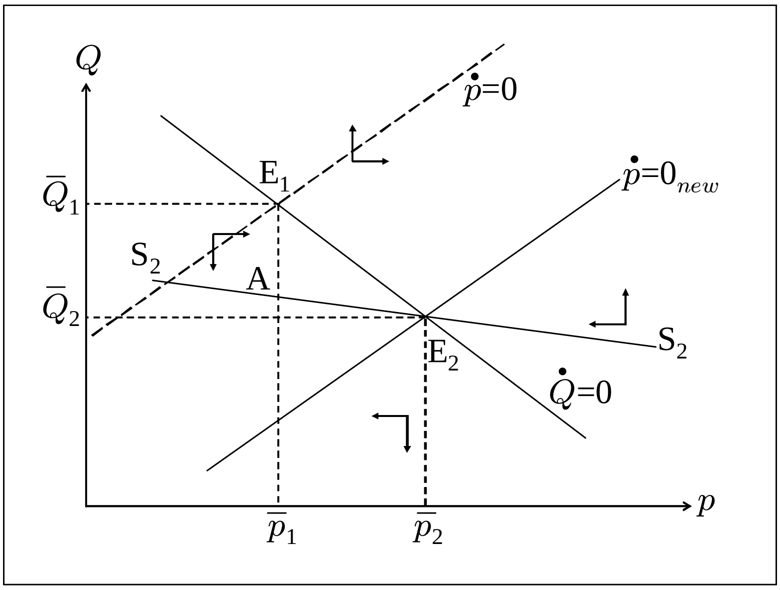

Figure C.3

We now make the following considerations:

a) Before time t_0 the economy is at point E_1 and thus at time t_0 p is predetermined at $\bar{p}_1$, while Q is free to jump, since, as argued abopve, it depends on the future.

b) Sooner or later the economy must reach the new saddle path, S_2S_2, since otherwise it will forever follow one of the diverging paths which we have ruled out.

c) If Q is to jump at any point in time it must be at time t_0, since otherwise the jump would be expected/known meaning that agents would sit around waiting for capital gains or losses and this is not compatible with rational behaviour.

Applying these considerations to figure C.3 it is seen that is must be the case that exactly at time t_0 when $\bar{y}$ is decreased, the economy jumps from E_1 to A and then moves continously along the new saddle path to the new long run equlibrium at E_2 where Q is lower, while p is higher.
The economic intuition is the following: When it becomes known that natural output has decreased, it follows that in the future long-run equilibrium where $\dot{p} = 0$ requires $y = \bar{y}$, dividends, $D = \alpha y$, will have decreased. This immediately decreases the stock price since, according to question 2, it equals the present discounted value of future dividends, and this explains (part of) the initial jump from E_1 to A. There is a second effect, however, since according to the goods market eqilibrium in eq. (C.2) a decrease in Q will decrease demand and thus real output. This in turn will decrease dividends according to eq. (C.5) and will cause a further decrease in Q. In principle it could be that this latter effect would immediately decrease y al the way to the new value of $\bar{y}$ but one can show that when $r^f > \alpha\eta$ this will not be the case. (If instead $r^f < \alpha\eta$ the effect would one the other hand be so strong that Q would initially undershoot its long-run level).
At point A y has thus decreased but not al the way to its new natural level. Consequently we have that $y > \bar{y}$. According to the SRAS curve in eq. (C.1) this will cause the price level to be increasing over time. This increase will then worsen the competitiveness of domestic goods and over time real output and thus dividends and consequently the stock price will be decreasing. This explains the economic mechanisms along the new saddle path from A to E_2.

4. The phase diagram is shown in figure C.4 where now only the directions of motion (dotted) associated with the *original* $\dot{p} = 0$ locus are shown.
In addition to considerations a)-c) i question 3 above we now also have:

d) Between time t_0 and time t_1 the economy is governed by the original directions of motion and from time t_1 by the new directions of motion.

Applying these considerations to figure C.4 we conclude that the economy must at time t_0 jump from E_1 to A which is above the new saddle path. Between time t_0 and time t_1 the economy then moves according to the original directions of motion from A to B which is reached exactly at time t_1 after which point in time the economy moves along the new saddle path to the new long run equlibrium at E_2.
The economic intuition is the following: At time t_0 it is learned that at the future time t_1 the natural output will decrease and it is thus known that in the new long-run equilibrium y and thus dividends will be lower and this immediately decreases Q (causes the jump from E_1 to A). However, the decrease will be smaller than in question 3 becuase it it not until later that natural output will decrease, i.e. the lower dividends will be in effect in the more distant future, making the present discounted value higher.
The decrease (jump) in Q immediately decreases real oputput, y, due to decreased goods market demand. However, since $\bar{y}$ has not yet decreased (and since the economy was in equilibrium E_1 where $\dot{p} = 0$ and thus $y = \bar{y}$) it must be the case that $y < \bar{y}$ and according to the SRAS the economy will now experience a decreasing price level over time. This is what happens when the economy moves from A to B. At the same time Q is decreasing due to the fact that time evolves and the lower future dividends are moving closer in time thereby decreasing the present discounted value.
At time t_1 the economy reaches point B and $\bar{y}$ is decreased. This now puts the economy in a situation where $y > \bar{y}$ and causes the price level to start increasing, while Q is still decreasing for the reason mentioned before. This explains the movement along the saddle path from B to E_2 just as in question 3.

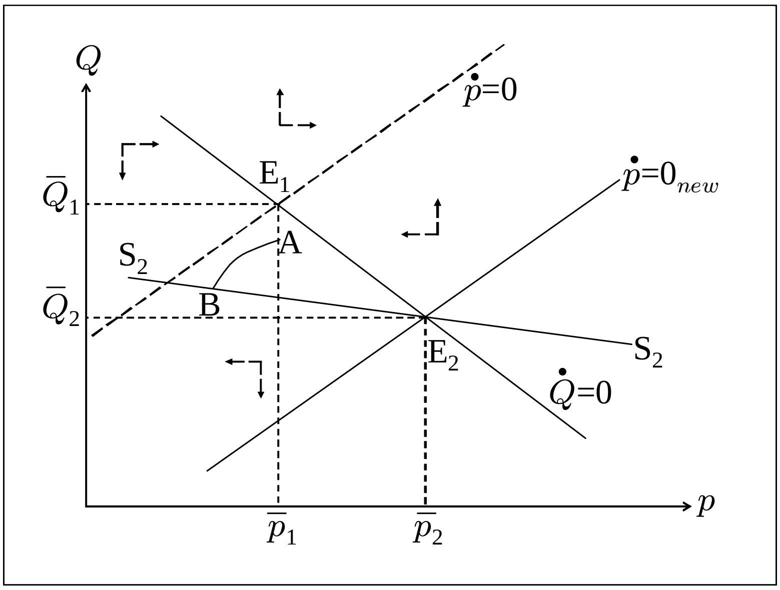

Figure C.4