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Number of questions: This exam consists of 2 questions.

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1. **The UIP and the Risk Premium**

   This question relates to the learning objectives: Describe and explain Covered Interest Rate Parity (CIP), Uncovered Interest Rate Parity (UIP), and Purchasing Power Parity (PPP) and be able to summarize the empirical evidence on these parity conditions; and describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, and Lucas asset pricing model) and use these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models. The question first considers the UIP and CIP relations and the empirical evidence on these relations. Then we apply the Lucas asset pricing model to derive an expression for the risk premium that empirical evidence suggest should be added to UIP. The last two questions relates to the explicit expression of the risk premium.

   Consider the UIP relation

   $$E_t s_{t+1} - s_t = i_t - i_t^*$$

   where notation is standard. Combining UIP with CIP $f_t^{(1)} - s_t = i_t - i_t^*$ we find that

   $$E_t s_{t+1} = f_t^{(1)}$$

   (a) Explain the rationale behind UIP and CIP and what the combined relation implies.

   Answer:

   CIP is an arbitrage relationship between interest rate differentials and forward and spot exchange rates. If domestic and foreign assets are identical and there are no transaction costs or barriers to arbitrage across countries, then arbitrage ensures that any interest rate differential is reflected in movements of exchange rates at the same maturity. In the CIP relation stated above, s_t is the spot exchange rate, f_t^1 is the forward exchange rate at time t for delivery at time $t+1$, i_t is the domestic interest rate and i_t^* is the foreign interest rate.

UIP states that if agents are risk–neutral and have rational expectations, then the return from holding one currency instead of another should be offset by the opportunity cost of holding bonds in a certain currency, i.e., the expected change in spot exchange rates must be equal to the interest differential.

Combination of CIP and UIP implies that under the assumption of risk–neutral agents and rational expectations the forward exchange rate agreed upon at time t for delivery of currency at time $t+1$ must be equal to the spot rate at time $t+1$. Any deviation from this reflects market inefficiencies.

(b) Summarize the empirical evidence on UIP and CIP.

Answer: CIP: The main result from the literature is that there are very few profitable trading opportunities, in pther words, CIP tends to hold on average. At the same time we often observe a maturity effect such that the existence of profitable opportunities is an increasing function of the length of the period to maturity of the underlying financial instruments. An alternative way of testing CIP is to examine whether news announcements affect exchange rates or the CIP relation. Using this approach the standard result is that new information immediately is reflected in the prices to eliminate any arbitrage opportunity. The overall conclusion from the empirical literature is that we cannot reject CIP.

UIP: In the empirical literature we often reject this hypothesis implying that the foreign exchange market is not efficient. One so called *stylized fact* is that the exchange rate moves in the opposite direction, instead of a depreciated currency if home interest rate is increased (see the UIP relation above), the exchange rate tends to appreciate. This empirical regularity is called forward premium puzzle. One explanation for the failure of UIP is the existence of a risk premium (or the assumption that home and foreign bonds are not identical), agents are not risk neutral and they require a risk premium, a higher rate of return to compensate for the risk of holding foreign currency. The empirical result is that there exists a risk premium and that it is time-varying. Tests of UIP also consider the assumption of rational expectations. Empirical tests suggest that this assumption also is rejected. Overall result is that we reject UIP, there exists a time-varying risk premium and we reject rational expectations.

(c) Consider the following Euler equations derived from a 2 country, 2 goods, 2 assets infinite horizon endowment model with cash-in-advance constraints:

$$U_{Y_t} = \gamma \mathrm{E}_t \left[(1 + r_t) \, U_{Y_{t+1}} \right]$$

and

$$U_{Y_t} = \gamma \mathrm{E}_t \left[(1 + r_t^*) \, U_{Y_{t+1}} \right].$$

where notation is standard. Show that

$$\mathrm{E}_t \left[(r_t - r_t^*) \frac{U_{Y_{t+1}}}{U_{Y_t}} \right] = 0 \tag{1}$$

Answer: To derive the result we first note that the preferences are identical across the two countries, i.e., γ is equal across the two countries. We can use this assumption to find that the Euler equations can be combined in the following way

$$\mathrm{E}_t\left[(1+r_t)\,\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]=\mathrm{E}_t\left[(1+r_t^*)\,\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]$$

or

$$\mathrm{E}_t\left[(1+r_t)\,\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]-\mathrm{E}_t\left[(1+r_t^*)\,\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]=0$$

which will give us the result

$$\mathrm{E}_t\left[(r_t-r_t^*)\,\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]=0$$

(d) Assume that PPP holds ($S_t=\frac{P_t}{P_t^*}$) and write the Fisher equation as $1+r_t=(1+i_t)\,\frac{P_t}{P_{t+1}}$. Show that the equation (1) can be written as

$$\mathrm{E}_t\left[\frac{F_t^{(1)}-S_{t+1}}{P_{t+1}}\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]=0 \tag{2}$$

Answer: To derive the result we first take the difference between the real interest rates in the two economies such that

$$r_t-r_t^*=(1+i_t)\,\frac{P_t}{P_{t+1}}-(1+i_t^*)\,\frac{P_t^*}{P_{t+1}^*}.$$

Then we use the CIP condition

$$\frac{F_t^{(k)}}{S_t}=\frac{1+i_t}{1+i_t^*}$$

and the PPP condition to rewrite the real interest rate difference as

$$r_t-r_t^*=\frac{(1+i_t)\,P_t}{F_t^{(1)}}\left[\frac{F_t^{(1)}-S_{t+1}}{P_{t+1}}\right]$$

and then we insert this into the condition in equation (1) such that

$$\mathrm{E}_t\left[\frac{(1+i_t)\,P_t}{F_t^{(1)}}\left[\frac{F_t^{(1)}-S_{t+1}}{P_{t+1}}\right]\frac{U_{Y_{t+1}}}{U_{Y_t}}\right]=0$$

where we note that $\frac{(1+i_t)P_t}{F_t^{(1)}}\neq 0$ such that we obtain equation (2).

(e) Assume the following CRRA utility function

$$U(C) = \frac{1}{1-\phi} C_t^{1-\phi} \tag{3}$$

where ϕ is the coefficient of relative risk aversion. Show that equation (2) can be written as

$$\mathrm{E}_t\left[\frac{F_t^{(1)} - S_{t+1}}{P_{t+1}}\left(\frac{1}{C_{t+1}}\right)^{\phi}\right] = 0. \tag{4}$$

Answer: Differentiate the utility function with respect to C_t and C_{t+1} and insert into equation (2)

$$\mathrm{E}_t\left[\frac{F_t^{(1)} - S_{t+1}}{P_{t+1}}\left(\frac{C_t}{C_{t+1}}\right)^{\phi}\right] = 0$$

and since $C_t^{\phi} \neq 0$ and known at time t we obtain equation (4), i.e.,

$$\mathrm{E}_t\left[\frac{F_t^{(1)} - S_{t+1}}{P_{t+1}}\left(\frac{1}{C_{t+1}}\right)^{\phi}\right] = 0.$$

(f) In order to derive a risk premium we assume that all variables in equation (4) are joint log–normally distributed. Take logs of equation (4) using that if $z = \ln Z \sim N(\mu_z, \sigma_z^2)$ then $\mathrm{E}[Z] = \mathrm{E}[\exp(z)] = \exp\left(\mu_z + \frac{1}{2}\sigma_z^2\right)$ and show that the equation can be written in the following logarithmic form

$$\mathrm{E}_t[s_{t+1}] - f_t^{(1)} = \phi \mathrm{cov}(s_{t+1}, c_{t+1}) - \frac{1}{2}\mathrm{var}(s_{t+1}) + \mathrm{cov}(s_{t+1}, p_{t+1}). \tag{5}$$

Answer: We are given the hints that $E_t[Z] = E_t[\exp(z)]$ where $z = \ln Z$. Using the hints we can rewrite (4) as

$$E_t\left[exp(\overbrace{f_t - p_{t+1} - \phi c_{t+1}}^{z_1}) - exp(\underbrace{s_{t+1} - p_{t+1} - \phi c_{t+1}}_{z_2})\right] = 0$$

We also know that $\mathrm{E}[Z] = \mathrm{E}[\exp(z)] = \exp\left(\mu_z + \frac{1}{2}\sigma_z^2\right)$ and we have the definitions of the mean $E_t[z] = \mu_z$ and the variance $\sigma_z^2 = E_t[z - E_t[z]]^2$.
We start by computing the means. They are given by

$$E[z_1] = E[f_t - p_{t+1} - \phi c_{t+1}]$$

and

$$E[z_2] = E[s_{t+1} - p_{t+1} - \phi c_{t+1}]$$

Using the definition of the variance we find that

$$\sigma_{z_1}^2 = E\left[f_t - p_{t+1} - \phi c_{t+1}\right]^2 = \sigma_{f_t}^2 + \sigma_{p_{t+1}}^2 + \phi 2\sigma_{c_{t+1}}^2 -$$
$$2cov(f_t, p_{t+1}) - 2\phi cov(c_{t+1}, f_t) + 2\phi cov(c_{t+1}, p_{t+1})$$

and a similar expression holds for z_2

$$\sigma_{z_2}^2 = E\left[s_{t+1} - p_{t+1} - \phi c_{t+1}\right]^2 = \sigma_{s_{t+1}}^2 + \sigma_{p_{t+1}}^2 + \phi 2\sigma_{c_{t+1}}^2 -$$
$$2cov(s_{t+1}, p_{t+1}) - 2\phi cov(c_{t+1}, s_{t+1}) + 2\phi cov(c_{t+1}, p_{t+1})$$

We also note that the forward exchange rate is known at time t so therefore is a constant implying that $\sigma_{f_t}^2 = 0$ and that $cov(f_t, p_{t+1}) = cov(c_{t+1}, f_t) = 0$. Putting it all together we then find that

$$f_t = E\left[s_{t+1}\right] + \frac{1}{2}\sigma_{s_{t+1}}^2 - cov(s_{t+1}, p_{t+1}) - \phi cov(s_{t+1}, c_{t+1})$$

and we have thus derived the result.

(g) What can we learn about the risk premium and deviations from UIP from the expression above? Can the risk premium explain large portions of the excess returns? Explain!

Answer: The equation above implies that there is a wedge between future spot exchange rates and the forward rate even in case agents are risk neutral. This result follows, however, from the log–linearization of a non–linear first order condition. The risk premium is given by ϕcov (s_{t+1}, c_{t+1}) and as can be ssen in the equation, the risk premium is increasing in the coefficient of relative risk aversion and in the covariance between spot rates and consumption.

The empirical literature typically find that a very large (unrealistically large) coefficient of relative risk aversion or an extremely high covariance between spot rates and consumption is needed to explain the deviation between future spot rates and forward rates.

2. **Second generation currency crisis model**

This question relates to the learning objective: describe, explain and compare first-, second- and third-generation models of currency crises and apply these models to analyze actual currency crises.

Consider the Obstfeld second generation currency crisis model comprised of the following equations:

$$\mathcal{L} = \theta \dot{p}^2 + (y - \tilde{y})^2 + C(\dot{s}) \tag{6}$$
$$y = \bar{y} + \dot{p} - \dot{p}^e - v \tag{7}$$
$$\tilde{y} - \bar{y} = k > 0 \tag{8}$$
$$s = p - p^* \tag{9}$$

where notation is standard.

(a) Explain the underlying rationale behind these equations.

Answer: Equation (6) is the policy rule where θ is the weight on inflation, $\dot{p}$ is inflation, y is actual output whereas $\tilde{y}$ is the output target chosen by the policymaker. Equation (7) is a standard aggregate supply function where $\bar{y}$ is the natural level of output, $\dot{p}^e$ is expected inflation and υ is the output shock which is assumed to be a white noise sequence. Equation (8) defines whether there is a difference between target and natural output, this difference is k. It is assumed that k is positive such that there is an output target bias. There is a temptation to cheat in the model. The policymaker finds that by devaluating the currency such that inflation increases will raise output and the loss function will be minimized. Since the model is known, the market knows that speculative attacks may lead to devaluations. Finally, equation (9) is the PPP relation which is assumed to hold.

(b) Explain the assumed sequencing of events in the model and how currency crises are generated.

Answer: Sequencing: Private agents choose $\dot{s}^e$ before the shock υ hits the economy implying that the expectation is formed prior to observing $\dot{s}$. The monetary authority can choose $\dot{s}$ after observing both $\dot{s}^e$ and υ. This sequencing opens up for cheating.

Fundamentals, monetary authorities preferences and the shocks determine the equilibrium and whether there are multiple equilibria. A sudden shock to expectations trigger a speculative attack and we may end up in a currency crisis. There must be a temptation to abandon the fixed exchange rate or devalue the currency. To generate crises, shocks must lead to sudden jumps in expectations and even small shocks can lead to jumps from an initial equilibrium compatible with a fixed exchange rate to an equilibrium where the fixed exchange rate regime must fall.

(c) Under the assumption that the shocks are uniformly distributed we can derive the following expression for the expected exchange rate

$$\mathrm{E}(\dot{s}) = \frac{1}{1+\theta}\left\{\left[1 - \frac{\bar{\upsilon} - \underline{\upsilon}}{2V}\right](\dot{s}^e + k) - \frac{\bar{\upsilon}^2 - \underline{\upsilon}^2}{4V}\right\} \tag{10}$$

where $\bar{\upsilon} = \sqrt{\bar{C}(1+\theta)} - k - \dot{s}^e$ is the devaluation trigger, $\underline{\upsilon} = -\sqrt{\underline{C}(1+\theta)} - k - \dot{s}^e$ is the revaluation trigger and V $(-V)$ is the largest (smallest) possible value of υ, and where the cost functions of devaluations and revaluations are defined as

$$\begin{aligned} C(\dot{s}) &= 0 \quad \text{if} \quad \dot{s} = 0 \\ C(\dot{s}) &= \bar{C} > 0 \quad \text{if} \quad \dot{s} > 0 \\ C(\dot{s}) &= \underline{C} > 0 \quad \text{if} \quad \dot{s} < 0. \end{aligned}$$

such that a devaluation or a revaluation will occur if

$$\frac{(\dot{s}^e + \upsilon + k)^2}{1+\theta} > C(\dot{s}).$$

Illustrate the model in the E($\dot{s}$)–$\dot{s}^e$–plane. Provide a detailed discussion about how this graph is constructed.

Answer: We have to consider three cases, small, intermediate and large values of $\dot{s}^e$! The reason is that the slope of the expression above depends on the relative size of $\dot{s}^e$.

Consider first small values of $\dot{s}^e$. According to UIP: $\dot{s}^e = i - i^*$ so that the minimum value must be $\dot{s}^e = -i^*$ (expected revaluation). In this case $\underline{\upsilon} > -V$ and $\bar{\upsilon} < V$ implying that $-V < \underline{\upsilon} \leq \bar{\upsilon} < V$. From above we have that

$$\bar{\upsilon} = \sqrt{\bar{C}(1+\theta)} - k - \dot{s}^e$$

and

$$\underline{\upsilon} = -\sqrt{\underline{C}(1+\theta)} - k - \dot{s}^e.$$

The slope of the E($\dot{s}$)–curve is therefore

$$\frac{\partial \mathrm{E}(\dot{s})}{\partial \dot{s}^e} = \frac{1}{1+\theta}$$

since the partial derivatives of $\bar{\upsilon}$ and $\underline{\upsilon}$ with respect to $\dot{s}^e$ are both equal to -1.

Intermediate values of $\dot{s}^e$: Increase the value of $\dot{s}^e$. This implies that $\underline{\upsilon}$ falls until it is equal to $-V$. An increase in $\dot{s}^e$ will reduce $\bar{\upsilon}$ so that it will depart from V. We assume that $\dot{s}^e$ increases but not such that $\bar{\upsilon} \to -V$. This implies that $-V < \bar{\upsilon} < V$. In this case $\frac{\partial \underline{\upsilon}}{\partial \dot{s}^e} = 0$ since $\underline{\upsilon}$ cannot fall any further (the limit is $-V$). But $\frac{\partial \bar{\upsilon}}{\partial \dot{s}^e} = -1$. We then find that the slope for this range of $\dot{s}^e$ is given by

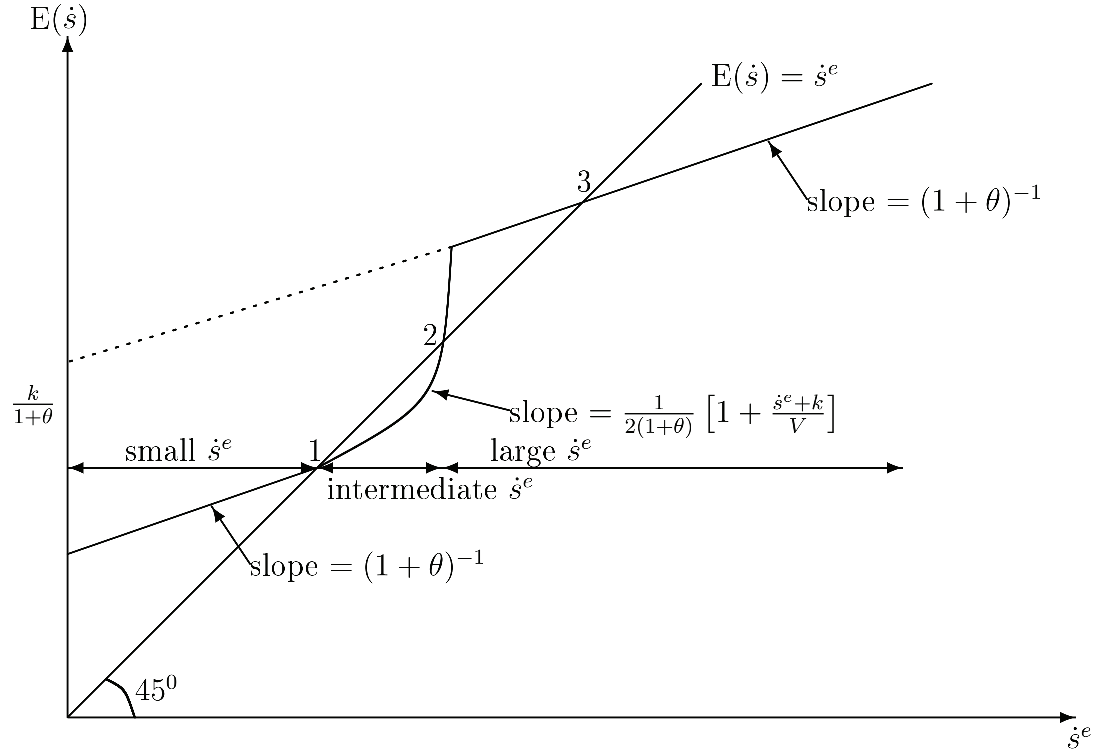
$$\frac{\partial \mathrm{E}(\dot{s})}{\partial \dot{s}^e} = \frac{1}{2(1+\theta)}\left(1 + \frac{\dot{s}^e + k}{V}\right)$$

implying that the slope is increasing in $\dot{s}^e$.

Large values of $\dot{s}^e$: If $\dot{s}^e$ is very large, then $\underline{\upsilon} = -V$ but $\bar{\upsilon} = -V$. This implies that revaluations are precluded with certainty $\Pr\{\upsilon \leq \underline{\upsilon}\} = 0$ and $\upsilon \geq -V$ or in other words $\Pr\{\upsilon \geq \bar{\upsilon}\} = 1$. Insert $\bar{\upsilon} = \underline{\upsilon}$ into the equation above and compute the slope

$$\frac{\partial \mathrm{E}(\dot{s})}{\partial \dot{s}^e} = \frac{1}{1+\theta}.$$

We can now put all this together and illustrate the model in the following graph.



In the graph, there are three equilibria, points 1, 2 and 3! Equilibrium 3 is the zero commitment solution $\mathrm{E}(\dot{s}) = \frac{k}{\theta}$ which must be located above $\frac{k}{1+\theta}$. Note also that equilibrium 1 implies a certain devaluation. Expected devaluation is equal to zero only if there is an equilibrium at origo. If agents devaluation expectations suddenly increase (given that we are in equilibrium 1), we will move along the curve towards equilibrium 2, which is unstable. Depending on the shock v we may end up at point 3, the insight is that speculation can move the economy to equilibrium 3. Note that it is not necessarily the case that we have multiple equilibria. If $k\downarrow$ or $\theta\uparrow$, then $\frac{k}{1+\theta}\downarrow$. The obvious solution is to set $\tilde{y} = \bar{y} \Rightarrow k = 0$ or to increase the weight on inflation in the objective function (appoint a more conservative central bank).

(d) Can this model be used to explain the ERM crisis (Britain's exit from the ERM in 1992)? If so why? If not why not?

Answer: The model is often used to illustrate what happened during the ERM crisis. The reason is that most fundamentals in ERM countries were consistent with the exchange rate but sudden shocks or unexpected events made market participants revise their expectations. Even small or minor events had large effects on expectations and therefore triggered speculation. Among explanations to why the crisis broke out are: The German unification in 1990, large and persistent inflation differences, and the result of the Danish referendum on the Maastricht Treaty. These events, and in particular the result of the Danish referendum, were considered as negative news. There was an immediate reaction on the foreign exchange market. Speculative attacks initially were targeted against the lira and the British Pound and without strong support from the Bundesbank, Italy and the UK left ERM I. Speculative attacks then targeted the Irish punt, the Portuguese escudo and the Spanish peseta and spread to the Belgian franc, the Danish krone and the French franc. Finally, ERM I broke down in August 2, 1993. These event illustrate that sudden events can have a tremendous effect of expectations and trigger speculation even in cases when fundamentals are in line with the exchange rate.

Moreover, the ERM crisis provides an example of the impossible trinity, fixed exchange rate, monetary policy independence, and full capital mobility cannot be had at the same time. The liberalization of financial markets in EU countries together with fix exchange rates was incompatible with divergent monetary policies. At the same time, Germany was going through its unification. This may also have contributed to the breakdown of the ERM system.

(e) Can this model be used to explain the Asian crisis? If so why? If so why not?

Answer: No! One could argue that there was a large negative real shock that shifted the economy from an equilibrium with low expected depreciation to an equilibrium with high expected depreciation. But the underlying mechanism is quite different. In the 2nd generation model the reason for the shift is that the government is tempted to abandon the peg in order to boost output. It is not clear that that was what was going on in Asia.