## Written Exam for the B.Sc. or M.Sc. in Economics summer 2014

# **Monetary Economics: Macro Aspects**

Master's Course

16 June

(3-hour closed-book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total

Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

## **QUESTION 1:**

Evaluate whether the following statements are true or false. Explain your answers.

- (i) There are never limits on how much seigniorage a public sector can extract in a model with a conventional money demand function.
- (ii) In the basic New-Keynesian model, shocks to productivity  $A_t$ , in the aggregate production function  $Y_t = A_t N_t$  (where  $Y_t$  is output and  $N_t$  is labor), pose an inflation—output trade off for the monetary policymaker.
- (iii) In the Poole (1970) model, the case for a interest-rate operating procedure gets weaker, all things equal, when a relationship between the broad money supply and base money is introduced.

### **QUESTION 2:**

Assume a flex-price, closed economy in discrete time, where households maximize

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t, n_t\right)$$

with

$$u(c_t, m_t, n_t) \equiv \frac{(c_t m_t)^{1-\Phi}}{1-\Phi} + \frac{(1-n_t)^{1-\eta}}{1-\eta}, \qquad \Phi > 0, \quad \eta > 0,$$

subject to the budget constraints

$$f(k_{t-1}, n_t) + \tau_t + (1 - \delta) k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} = c_t + k_t + m_t,$$
(1)

where  $c_t$  is consumption,  $m_t$  is real money balances at the end of period t,  $n_t$  is fraction of time spent working,  $k_{t-1}$  is physical capital at the end of period t-1,  $\tau_t$  are monetary transfers from the government,  $0 < \delta < 1$  is the depreciation rate of capital, and  $\pi_t$  is the inflation rate. The function f is defined as

$$f(k_{t-1}, n_t) = k_{t-1}^{\alpha} n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

- (i) Discuss why money may enter the utility function, and describe (1) in detail.
- (ii) Derive the relevant first-order conditions for optimal choices of c, m, k, and n subject to (1) and the definition

$$a_t \equiv \tau_t + \frac{1}{1 + \pi_t} m_{t-1}$$

For this purpose, use the value function  $V(a_t, k_{t-1}) = \max \{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\}$ . Interpret the first-order conditions intuitively, and show that they can be combined (along with the expressions for the partial derivatives of the value function) into

$$u_m(c_t, m_t, n_t) + \frac{\beta}{1 + \pi_{t+1}} u_c(c_{t+1}, m_{t+1}, n_{t+1}) = u_c(c_t, m_t, n_t)$$
 (2)

$$u_c(c_t, m_t, n_t) = \beta R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}),$$
(3)

$$-u_n(c_t, m_t, n_t) = u_c(c_t, m_t, n_t) f_n(k_{t-1}, n_t),$$
(4)

where  $R_t \equiv f_k(k_t, n_{t+1}) + 1 - \delta$  is the gross real interest rate, which equals  $(1+i_t)/(1+\pi_{t+1})$ , with  $i_t$  being the nominal interest rate.

(iii) With the specific functional forms for u and f, examine the properties of the steady state using (2), (3), and (4) together with the national account identity  $c^{ss} = k^{ss^{\alpha}} n^{ss^{1-\alpha}} - \delta k^{ss}$ . Assess under which circumstances the model exhibits superneutrality or not. For that purpose focus on whether changes in  $m_t$  (induced by changes in inflation and nominal interest rates) have real effects. Explain intuitively the transmission mechanism that leads to potential non-superneutrality.

#### **QUESTION 3:**

Consider the following log-linear model of a closed economy:

$$x_t = \mathbf{E}_t x_{t+1} - \sigma^{-1} \left( \hat{i}_t - \mathbf{E}_t \pi_{t+1} \right), \qquad \sigma > 0, \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

$$\widehat{i}_t = \phi \pi_t, \qquad \phi > 1, \tag{3}$$

where  $x_t$  is the output gap,  $\hat{i}_t$  is the nominal interest rate's deviation from steady state, and  $\pi_t$  is goods-price inflation,  $e_t$  is a mean-zero "cost-push" shock without serial correlation.  $E_t$  is the rational-expectations operator conditional upon all information up to and including period t.

- (i) Explain the economics of (1) and (2) with focus on the underlying micro-economic foundations. What does (3) represent? Explain.
- (ii) Derive the solutions for  $x_t$  and  $\pi_t$ . [Hint: Conjecture that the solutions are linear functions of  $e_t$ , and use the method of undetermined coefficients.] Comment on the role of the policy parameter  $\phi$  in terms of the output gap's and inflation's dependence on  $e_t$ , and discuss whether the parameter can be chosen such that the output gap and inflation are stabilized completely.

Assume that a welfare-relevant loss function can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left( \pi_t^2 + \lambda x_t^2 \right), \qquad \lambda > 0.$$
 (4)

(iii) Derive the welfare-optimal values of  $x_t$  and  $\pi_t$  under discretionary policymaking [hence, equation (3) no longer applies]. For this purpose, treat  $x_t$  as the policy instrument, and show that the relevant first-order condition for optimal policy together with (2) yield the difference equation

$$\pi_t = \frac{\lambda \beta}{\kappa^2 + \lambda} E_t \pi_{t+1} + \frac{1}{1 + \kappa^2 / \lambda} e_t.$$
 (5)

Find the unique solutions for  $\pi_t$  and  $x_t$ , and discuss whether commitment of the central bank can improve on policymaking.