

Written Exam for the B.Sc. in Economics, Winter 2010/2011  
Reexamination  
Makro A and Macro A  
Second year  
February 17, 2011  
(3-hours closed book exam)

All questions, 1.1-1.3 and 2.1-2.8, to be answered, and all weighted equally.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e., if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

**Problem 1:** The stylized facts of the long-run growth process in western countries

(In this problem, the focus is on the verbal, intuitive explanations. Formal analysis can, however, be used in the explanations if wanted).

**1.1** State as much as you remember of the three ‘Kaldorian’ stylized facts that characterize economic growth in western countries over the past 100-200 years (i.e., stylized facts 5-7 in Chapter 2 of the text book).

**1.2** Figure 1 on page 5 shows labour’s share of domestic factor incomes for some countries over the period 1970-2005, while Figure 2 on page 6 shows measures of the real interest rate on long-term bonds (as five year moving averages) for some countries over the period 1930-2003. Explain how the regularities evidenced by these figures may be used to motivate that the capital-output ratio is relatively constant in the long run.

**1.3** Explain how the three stylized facts mentioned in Question 1 above have given rise to the concept of ‘balanced growth’ and describe the content of this concept. Discuss how balanced growth is used in growth theory.

## Problem 2: Propagation of productivity shocks

(In this problem, formal and computational elements are more important, but verbal, intuitive explanations are still central).

Equations (1) - (6) below make up a basic Solow model for a closed economy. The model and its notation are as known from Chapter 3 of the textbook, the only difference being that for this problem, we have set the rate of depreciation of capital equal to zero.

$$Y_t = BK_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad B > 0 \quad (1)$$

$$r_t = \alpha B \left( \frac{K_t}{L_t} \right)^{\alpha-1} \quad (2)$$

$$w_t = (1 - \alpha) B \left( \frac{K_t}{L_t} \right)^\alpha \quad (3)$$

$$S_t = sY_t, \quad 0 < s < 1 \quad (4)$$

$$K_{t+1} = S_t + K_t \quad (5)$$

$$L_{t+1} = (1 + n) L_t, \quad n > 0 \quad (6)$$

Equation (1) is a Cobb-Douglas production function: Aggregate output,  $Y_t$ , is produced from the inputs of capital,  $K_t$ , and labour,  $L_t$ , with a constant total factor productivity,  $B$ . Equations (2) and (3) state that the real rental (interest) rate of capital and the real wage rate equal the marginal products of capital and labour, respectively.

Equation (4) assumes that total saving is a given fraction  $s$  of aggregate income, while equations (5) and (6) describe capital accumulation and the evolution of the labour force, respectively.

The exogenous parameters of the model are  $\alpha$ ,  $B$ ,  $s$ , and  $n$ . The state variables are  $K_t$  and  $L_t$  with given, strictly positive initial values  $K_0$  and  $L_0$ .

Define  $k_t \equiv K_t/Y_t$  and  $y_t \equiv Y_t/L_t$ .

**2.1** Show that the per capita (worker) production function is

$$y_t = Bk_t^\alpha \quad (7)$$

Then show that the transition equation and the Solow equation, respectively, for this model are:

$$k_{t+1} = \frac{1}{1+n} [sBk_t^\alpha + k_t] \quad (8)$$

and

$$k_{t+1} - k_t = \frac{1}{1+n} [sBk_t^\alpha - nk_t] \quad (9)$$

**2.2** Illustrate equations (8) and (9) in appropriate diagrams and show that in the long run capital per worker,  $k_t$ , must converge to a specific steady state value,  $k^* > 0$ .

Steady state values of variables are indicated by a \*, e.g., the steady state value of  $r_t$  is  $r^*$ .

**2.3** Show that in steady state

$$y^* = B^{\frac{1}{1-\alpha}} \left( \frac{s}{n} \right)^{\frac{\alpha}{1-\alpha}} \quad (10)$$

$$r^* = \alpha \frac{n}{s} \quad (11)$$

$$w^* = (1-\alpha) B^{\frac{1}{1-\alpha}} \left( \frac{s}{n} \right)^{\frac{\alpha}{1-\alpha}} \quad (12)$$

What is the elasticity of steady state income per worker with respect to the total factor productivity? In view of equation (7), what is the basic mechanism explaining that this elasticity is larger than one?

**2.4** Assume that in period zero the economy is in steady state at given parameter values. Working for the first time in period one the level of technology increases *permanently* from  $B$  to a new and higher level,  $B'$ . Explain and illustrate in diagrams how the economy evolves over time from period one and onwards: First explain what happens

in period one (on impact), e.g., how  $k_t$ ,  $y_t$ ,  $r_t$  and  $w_t$  are affected. Then explain the evolution in the longer run over periods 2, 3, ... for the same variables.

**2.5** Now do what you were asked in Question 2.4 for a *temporary* productivity shock where technology increases from  $B$  to  $B'$  in period one only and then falls back to  $B$  and stays there over all periods 2,3 ... Explain in particular how there can be an effect on  $k_t$ ,  $y_t$ ,  $r_t$  and  $w_t$  in the periods after period one, even though the total factor productivity is back at its old level.

**2.6** Still considering the temporary productivity shock occurring in period one (as in Question 2.5), try somehow, e.g. by a numerical example, to give an impression of how large the effects on capital and income per worker plausibly can be in the periods coming after period one. For instance, relate the relative change in income per worker (from the old steady state level) to the size of the productivity shock.

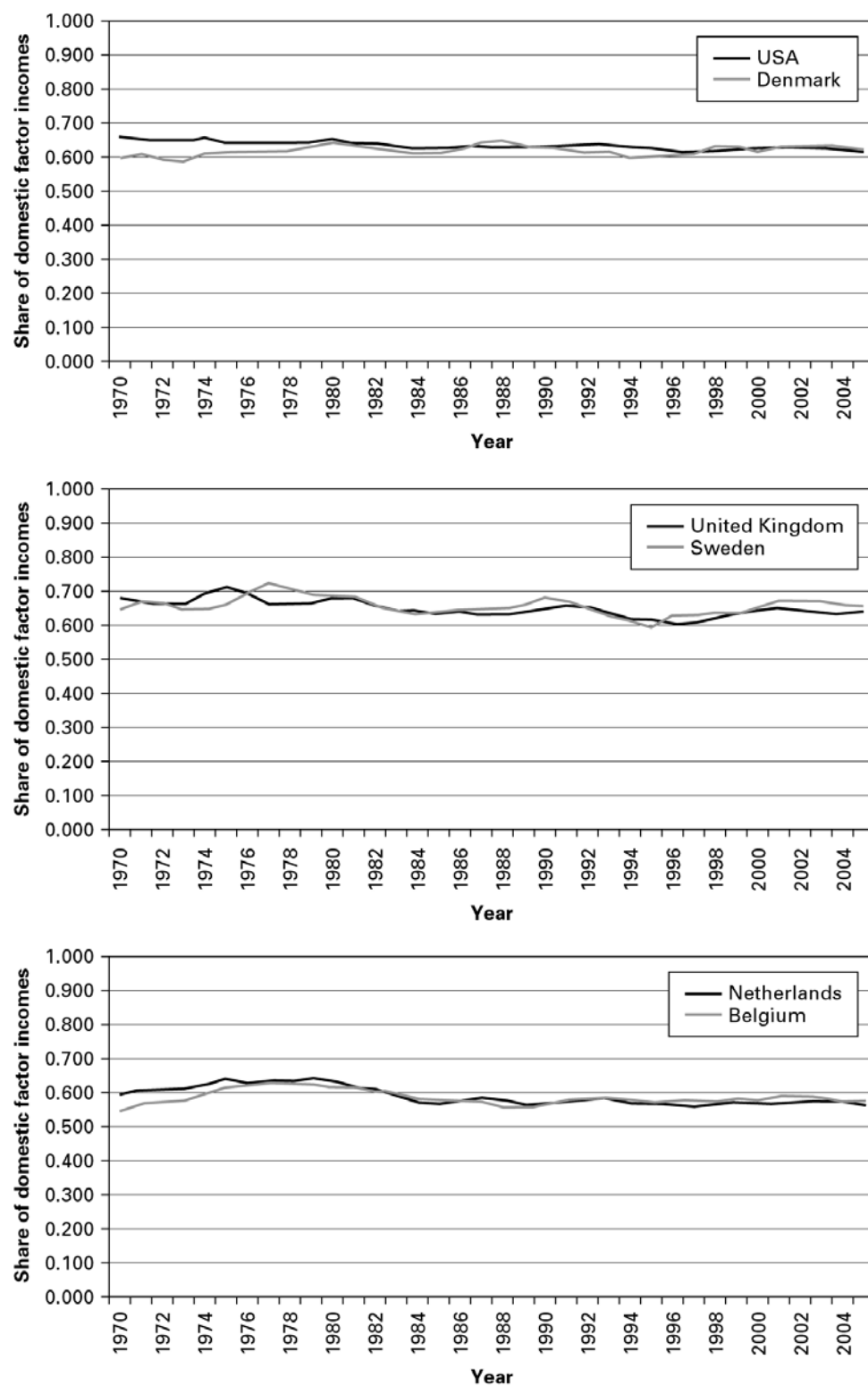
Assume now that the economy considered were instead functioning as a small open economy with free capital mobility (as in Chapter 4 of the text book). The model's parameters are the same as above, and in addition there is a given and constant international real interest rate  $\bar{r}$ . To ensure comparability we assume that the international real interest rate is the same as the economy would itself create in the long run if it were a closed economy, that is,  $\bar{r} = \alpha n/s$ .

**2.7** Explain how the small open economy would react to a temporary productivity shock, again focusing first on the period the shock occurs and then on the succeeding periods. The difference to the reactions of the closed economy should be emphasized. (You are not supposed to set up a formal model for the open economy etc. It is OK just to give verbal explanations).

It is often considered a merit of models with capital or wealth accumulation that temporary shocks can have longer lasting effects through the mechanisms of “propagation” (dansk: “udbredelse” eller “forplantning”) analyzed above. This may help explaining how random, temporary and uncorrelated shocks can cause more persistent behaviour of macroeconomic variables such as income and consumption over the business cycle.

**2.8** In view of your findings above discuss how convincing you find capital and/or wealth accumulation as a business cycle mechanism for propagation of productivity shocks.

**Figure 1.** Labour's share of domestic factor incomes.



**Figure 2.** Real interest rates on long-term bonds, five years moving averages

