Written Exam at the Department of Economics winter 2016-17

Macroeconomics III

Final Exam

January 9, 2017

(3-hour closed book exam)

This exam question consists of 3 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Written Exam - Macroeconomics III
University of Copenhagen
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Question 1

Consider an economy where individuals live for two periods, and population is constant. Identical competitive firms maximize profit

$$\max_{K_t, L_t} AK_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t,$$

where R_t is the rental rate on capital, w_t is the wage rate, L_t and K_t denote the quantities of labor and capital employed by the firm, and $A > 0$ is total factor productivity. Assume $\alpha \in (0, 1)$. Capital depreciates fully, that is $\delta = 1$. Utility for young individuals born in period t is

$$U_t = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1},$$

with $\rho > -1$. c_{1t} denotes consumption when young, c_{2t+1} consumption when old. Young agents work one unit of time (i.e., their labor income is equal to the wage they receive). Old agents do not work, receive income from their savings and social security benefits. The gross return on savings is $1 + r_{t+1} (= R_t)$.
Suppose the government runs an unfunded (pay-as-you-go) social security system in which the young contribute a fraction $\tau \in (0, 1)$ of their wages to the system, and these contributions are paid out in the same period to the current old.

a Find the first order conditions for firms' maximization problem that characterize how much capital and labor a firm demands at given factor prices.

b Set up and solve the individual's problem of optimal intertemporal allocation of resources. Derive the Euler equation. Show that individual savings behavior is characterized by

$$s_t = \frac{1}{2+\rho} w_t (1-\tau) - \tau \frac{1+\rho}{2+\rho} \frac{1}{1+r_{t+1}} w_{t+1}.$$

c Show that the capital accumulation equation that expresses k_{t+1}, as a function of k_t, is given by

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho} \frac{(1-\alpha)}{\alpha} \tau} \left[\frac{(1-\alpha)(1-\tau)}{2+\rho} A k_t^{\alpha} \right].$$

d Find the level of capital in the steady state.

e Assume that the economy is initially in the steady state. Unexpectedly, at time $t = T$, the government decides to dismantle the social security system from period $T+1$ onwards: No contributions will be raised an no benefits will be paid, at any point in the future. Importantly, the policy shift is communicated to the public after savings decisions have been formulated. What is the expression for the new steady state capital level? Will the stock of capital be higher at $T+2$, as compared with $T+1$?

f Is the old generation at time $T+1$ better-off or worse-off, after the social security system has been dismantled?

g Has the economy higher chances of being dynamically inefficient with or without a pay-as-you-go social security system? Explain.

Question 2

Consider the following model of monetary policy: the government controls inflation directly (i.e. $\pi_t = m_t$, where π_t is the rate of inflation and m_t is the rate of growth of money supply) and its instantaneous loss function is

$$L(\pi_t, x_t) = \frac{1}{2}\left[\pi_t^2 + \lambda\left(x_t - \bar{x}\right)^2\right]$$

where $x_t = \theta_t + \pi_t - \pi_t^e$. The following notation applies

π_t^e:	expected rate of inflation
x_t:	output level
θ_t:	potential output
$\bar{x}$:	policy output target

We assume that potential output is stochastic and that its realizations are observed by both the public and the policy maker before expectations are formed by the private sector. Parameter $\lambda > 0$ measures the relative importance of output fluctuations around the target, $\bar{x}$, relative to inflation fluctuations.

a Show that the optimal policy under commitment implies $\pi_t^C = 0$ and $x_t^C = \theta_t$ [hint: i) recall that the loss function is quadratic, thus the optimal policy rule is linear and can be guessed to be of the form $\pi_t = \psi + \psi_\theta \theta_t$; ii) recall that the loss to be minimized is the unconditional one].

b Show that the optimal policy under discretion implies $\pi_t^D = -\lambda\left(\theta_t - \bar{x}\right)$ and $x_t^D = \theta_t$. The *inflation bias* increases in the target $\bar{x}$: explain why.

c Assume that potential output cannot be observed before expectations are formed. The goal of the central bank is still to minimize the loss function. However, the monetary policy stance should now result as ex-post optimal given both π_t^e and θ_t (as the latter is not observed until after expectations are formed). Assume that $E[\theta_t] = 0$. Show that the optimal policy under discretion now implies $\pi_t^{D*} = \frac{\lambda}{1+\lambda}\left[\lambda(1+\bar{x}) - \theta_t\right]$ and $x_t^{D*} = \frac{\lambda^2}{1+\lambda} + \frac{1}{1+\lambda}\theta_t - \frac{\lambda}{1+\lambda}\bar{x}$. Under $\lambda = 0$ it is possible ensure that $\pi_t^{D*} = \pi_t^D$ and $x_t^{D*} = x_t^D$. Explain why this is the case.