Written Exam - Macroeconomics III

University of Copenhagen
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Question 1

Identical competitive firms maximize the following profit function:

$$\pi(k_t) = k_t^{\alpha} - \omega k_t, \ \omega > 0$$

where k_t denotes the total amount of capital per-capita employed by the firm. Assume $0 < \alpha < 1$.

A large number of identical households/investors maximize the following intertemporal utility function, that depends on per-capita consumption, c_t:

$$\max_{c_t, s_{t+1}} U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta},$$

subject to their dynamic budget constraint:

$$c_t + s_{t+1} q_t = s_t (q_t + d_t),$$

where s_t denotes the amount of shares of a portfolio comprising the stocks of the productive firms. The price of each stock at time t is denoted by q_t. In every period, this portfolio also pays a per-share dividend that is denoted by $d_t = \pi(k_t)$. We impose $0 < \beta < 1$ throughout the exercise.

a Find the first-order condition for firms' profit-maximization problem.

b Write the Lagrangian and find the first-order conditions that characterize the behavior of the households/investors, and from these the Euler equation. Give an economic interpretation to this equation.

c Determine the steady-state share price, q. How is this affected by the marginal cost of production, ω? Interpret.

d Go back to the original model. Assume the government imposes a dividend tax (denote the tax rate with $\tau \in (0,1)$), so that households' budget constraint becomes

$$c_t + s_{t+1} q_t = s_t [q_t + (1-\tau) d_t].$$

How does this affect the steady-state share price, q?

Question 2

Consider the following model of monetary policy: the government controls inflation directly (i.e. $\pi_t = m_t$, where π_t is the rate of inflation and m_t is the rate of growth of money supply) and its instantaneous loss function is

$$L(\pi_t, x_t) = \frac{1}{2}\left[\pi_t^2 - \lambda\left(x_t - \bar{x}\right)\right]$$

where $x_t = \theta_t + \pi_t - \pi_t^e$. The following notation applies:

π_t^e:	expected rate of inflation
x_t:	output level
θ_t:	potential output (white noise with variance σ_θ^2)
$\bar{x}$:	policy output target

We assume that potential output is stochastic and that its realizations are observed by both the public and the policy maker before expectations are formed by the private sector. Parameter $\lambda > 0$ measures the relative importance of output fluctuations around the target, $\bar{x}$, relative to inflation fluctuations.

a Show that the optimal policy under commitment implies $\pi_t^C = 0$ and $x_t^C = \theta_t$ [hint: i) recall that the loss function is quadratic, thus the optimal policy rule is linear and can be guessed to be of the form $\pi_t = \psi + \psi_\theta \theta_t$; ii) recall that the loss to be minimized is the unconditional one].

b Show that the optimal policy under discretion implies $\pi_t^D = \lambda$ and $x_t^D = \theta_t$. The *inflation bias* increases in λ: explain why.

c Now set $\bar{x} = 0$ and assume there are two periods, i.e. $t = 1, 2$. Compute the optimal strategy at time $t = 1$ for a government that is expected to ensure π_1^C, but decides to deviate from the announced strategy (hint: the policy maker takes $\pi_1^e = 0$ as given, when minimizing the loss function).

d Keep assuming $\bar{x} = 0$. What are the benefits and costs from deviating from commitment at time $t = 1$ and playing discretion at time $t = 2$ (hint: the benefit at $t = 1$ is the difference between the loss under commitment and the loss under the deviation strategy, while the cost at $t = 2$ is computed as the difference between the loss under discretion and the loss under commitment)? Is it convenient to deviate at $t = 1$?