Written Exam for the M.Sc. in Economics winter 2013-2014

Advanced Microeconomics

Master's Course

06NOV2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

This exam question consists of 6 pages in total including this page.

Advanced Microeconomics, Fall 2013

3 hours closed book exam

Assumptions are enclosed and there should be **6** pages in your problem set (including the title page).

There are 3 problems. Problem A has a weight of approximately 50%, B a weight of approximately 20% and C has a weight of approximately 30% in the marking process. All subproblems have the same weight except those in Problem B which have twice the weight of the others.

Below

$$\begin{aligned} \mathbb{R}^k_+ &= \left\{ x \in \mathbb{R}^l \mid x_h \geq 0 \text{ for } h = 1, 2, \ldots, k \right\} \text{ and} \\ \mathbb{R}^k_{++} &= \left\{ x \in \mathbb{R}^l \mid x_h > 0 \text{ for } h = 1, 2, \ldots, k \right\} \end{aligned}$$

for $k = 1, 2, \ldots$.

Problem A

(a) Give a graphic example of a production possibility set Y in $\mathbb{R}^2$ which satisfies Assumption P1 but where there is a production on the boundary of Y which is not an efficient production

(b) The production possibility set $Y \subset \mathbb{R}^L$ exhibits non-increasing returns to scale. What does this mean?

(c) Assume that the binary relation R on $\mathbb{R}^L$ is represented by the function $S : \mathbb{R} \longrightarrow \mathbb{R}$ in the sense that: $x\mathrm{R}y$ if and only if $S(x) \geq S(y)$. Show that R is a transitive and total relation.

(d) Assume that Arrow's assumptions for a SWF are satisfied and that Schedule 1 below is mapped to a ranking for society with a above b. What can be concluded about society's ranking of a and b for Schedule 2?

Schedule 1			Schedule 2		
b	c	a	c	b	c
c	b	c	b	c	a
a	a	b	a	a	b

(e) Let $\mathcal{E} = (\mathbb{R}^2_+, u^i, \omega^i)_{i\in\{a,b\}}$ be a pure exchange economy. Define what is meant by a fair allocation in $\mathcal{E}$.

(f) Let $\xi(p_1, p_2, \mathrm{w}) = \left(\frac{3}{4}\frac{\mathrm{w}}{p_1}, \frac{1}{4}\frac{\mathrm{w}}{p_2}\right)$ be the demand function of a consumer in a private ownership (pure exchange) economy with commodity space $\mathbb{R}^2$ and $\mathrm{w} = p_1\omega_1 + p_2\omega_2, \omega_1, \omega_2 > 0$. Will the consumer's excess demand function satisfy the Gross Substitutes assumption? Does you answer depend on the initial endowment ω of the consumer?

(g) Show by a graphic example, in $\mathbb{R}^2$, that when a consumer has a "thick" indifference curve (that is, an indifference "curve" with non-empty interior) then, for suitable prices and wealth, there might be a vector x which is a solution to the UMP (Utility Maximization Problem) which is not a solution to the EMP (Expenditure Minimization Problem).

(h) Define a market equilibrium (or, if you so prefer, an equilibrium with transfers).

(i) Let $(\bar{x}, \bar{y}, p)$ be a Walras equilibrium for an economy $\mathcal{E} = \{(u, \omega), Y, \alpha\}$ (one consumer, one producer) where $\alpha = 1$ (the consumer owns the producer). State the conditions that $(\bar{x}, \bar{y}, p)$ must satisfy. Be careful when defining the consumer´s wealth.

(j) Define what is meant by quasi-linear preferences.

Problem B

(a) Let the consumption possibility set be $X = \mathbb{R}^L_+$ and let $p \in \mathbb{R}^L_{++}, \mathrm{w} > 0$. Show that the budget set

$$B(p, \mathrm{w}) = \left\{x \in \mathbb{R}^L_+ \mid px \leq \mathrm{w}\right\}$$

is a non-empty, compact and convex set. **Hint:** You may use that the intersection of closed sets is a closed set and that the intersection of convex sets is a convex set. It is immediate that $B(p, \mathrm{w})$ is lower bounded (in the vector ordering). Is it also upper bounded (in the vector ordering)? Is it bounded in norm (that is, there is $k \in \mathbb{R}$ such that: $x \in B(p, \mathrm{w})$ implies $\|x\| \leq k$)?

(b) Let u represent the rational preference relation $\succsim$ on $\mathbb{R}_+^L$. Let $\varphi : \mathbb{R} \longrightarrow \mathbb{R}$ be a strictly increasing function. Show that the composition of u and φ; the function $\varphi \circ u$ with values $\varphi(u(x))$ also represents $\succsim$.

Problem C

Let $\mathcal{E} = \{(X, u), Y, \omega, \alpha\}$ be a private ownership economy with a single consumer who owns the single producer so that $\alpha = 1$ and

$$\begin{aligned} X &\in [0, \infty[\times[-1, \infty[\\ Y &= \left\{y \in \mathbb{R}^2 \mid y_2 \leq \log(1 - y_1), y_1 \leq 0\right\} \\ u(x_1, x_2) &= x_1(x_2 + 1) \\ \omega &= (\omega_1, 0) \end{aligned}$$

(a) The set Y satisfies Assumption P1. Exhibit the set $Y + \{\omega\}$ in a diagram. State and solve the Producer Problem (PMP) for prices $p = (p_1, p_2) \in \mathbb{R}_{++}^2$. **Hint**: Distinguish the cases $p_2 \geq p_1$ and $p_2 < p_1$.

(b) Find the maximal profit given $p = (p_1, p_2) \in \mathbb{R}_{++}^2$.

(c) Plot the indiffence curve corresponding to utility level 1. Solve the Consumer Problem as $p = (p_1, p_2) \in \mathbb{R}_{++}^2$ and wealth is given by w > 0. **Hint:** You may consider rewriting the budget restriction as $p_1x_1 + p_2(x_2 + 1) \leq \mathrm{w} + p_2$ if you recall the solution for a Cobb-Douglas utility function.

(d) Find the consumer´s wealth given $p = (p_1, p_2) \in \mathbb{R}_{++}^2$.

(e) Derive the market balance condition for good 2. If $p = (p_1, p_2)$ satisfies this condition is then (p_1, p_2) an equilibrium price system?

(f) Put $p_2 = 1$, $\omega_1 = 1$ and find an equation for p_1 from the market balance condition in (c). Is $p_1 = 1$ a solution to this equation?

Assumptions on Producers

Assumption P1: *The production set $Y \subset \mathbb{R}^L$ satisfies*

(a) $0 \in Y$ (Possibility of inaction)

(b) *Y is a closed subset of $\mathbb{R}^L$* (Closedness)

(c) *Y is a convex set* (Convexity)

(d) $Y \cap (-Y) = \{0\}$ (Irreversibility)

(e) *If $\bar{y} \in Y$, $y \in \mathbb{R}^L$ and $y \leq \bar{y}$ then $y \in Y$* (Free disposal, downward comprehensive)

Assumption P2:*(constant returns to scale) If the vector $y \in Y$ and $\lambda \in [0, +\infty[$ then $\lambda y \in Y$.*

Assumptions on Consumers

Assumption F1.
The consumption set $X \subset \mathbb{R}^L$ satisfies:

(a) X is a non-empty set.

(b) X is a closed set

(c) X is a convex set

(d) X is a lower bounded set (in the vector ordering)

(e) X is upward comprehensive ($x \in X$ and $\nabla \in \mathbb{R}^L_+$ implies $x + \nabla \in X$)

Monotonicity assumptions

Assumption of **weak monotonicity**
$\mathbf{F2}^0 : x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow x^1 \succsim x^2$

$\mathbf{F2}^0 : x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow u(x^1) \geq u(x^2)$

In the interpretation: "at least as much of each commodity is at least as good".

Assumption of **monotonicity** (MWG Def. 3.B.2)

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow x^1 \succ x^2$

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow u(x^1) > u(x^2)$

In the interpretation: "more of each commodity is better".

Assumption of **strict (or strong) monotonicity** (MWG Def. 3.B.2)

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow x^1 \succ x^2$

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow u(x^1) > u(x^2)$

The preference relation $\succsim$ is **locally non-satiated** if: Given $x \in X$ and $\varepsilon > 0$ there is $x' \in X$ such that $x' \succ x$ and $\| x' - x \| < \varepsilon$. (Definition 3.B.3, MWG)

A preference relation, $\succsim$, is a **convex preference relation** if and only if, for $x \in X$, the set $\{x' \in X \mid x' \succsim x\}$ is a convex set. (MWG Def. 3.B.2).

If u represents $\succsim$ then $\succsim$ is a convex preference relation if and only if u is a quasi-concave function.

We want to consider also a stronger convexity assumptions

F3: A preference relation, $\succsim$, is a **strictly convex** preference relation if: $x^1, x^2, x^3 \in X$, $x^1 \succsim x^2$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)x^2$ for some $t \in]0,1[$ implies $x^3 \succ x^2$.

F3: The utility function is **strictly quasi-concave** if: $x^1, x^2, x^3 \in X$, $u(x^1) \geq u(x^2)$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)x^2$ for some $t \in]0,1[$ implies $u(x^3) > u(x^2)$.