University of Copenhagen
Department of Economics
Emiliano Santoro, Laura Sunder-Plassmann

Written exam
Macroeconomics C

August 4, 2015

Number of questions: This exam consists of 2 questions.

---

1. Consider an economy where individuals live for two periods, and population is constant. Identical competitive firms maximize profit

$$\max_{K_t, L_t} AK_t^{\alpha} L_t^{1-\alpha} - R(t)K(t) - w(t)L(t)$$

where R_t is the rental rate on capital, w_t is the wage rate, L_t and K_t denote the quantities of labor and capital employed by the firm, and $A > 0$ is total factor productivity. Assume $\alpha \in (0, 1)$. Capital depreciates fully, that is $\delta = 1$. Utility for young individuals born in period t is

$$U_t = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1}$$

with $\rho > -1$. c_{1t} denotes consumption when young, c_{2t+1} consumption when old. Young agents work one unit of time (ie their labor income is equal to the wage they receive). Old agents do not work, receive income from their savings and social security benefits. The return on savings is r_{t+1}.

Suppose the government runs an unfunded (pay-as-you-go) social security system in which the young contribute a fraction $\tau \in (0, 1)$ of their wages to the system, and these contributions are paid out in the same period to the current old.

(a) Find the first order conditions for the firm's maximization problem that characterize how much capital and labor a firm demands at given factor prices.

(b) Set up and solve the individual's problem of optimal intertemporal allocation of resources. Derive the Euler equation. Show that individual savings behavior is characterized by

$$s_t = \frac{1}{2+\rho} w_t(1-\tau) - \frac{1+\rho}{2+\rho} \frac{1}{1+r_{t+1}} \tau w_{t+1}$$

(c) Show that the capital accumulation equation that gives k_{t+1} as a function of k_t is given by

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho} \frac{(1-\alpha)}{\alpha} \tau} \left(\frac{1}{2+\rho} (1-\alpha) A k_t^{\alpha} (1-\tau) \right)$$

Find the level of capital in steady state. Can the economy be dynamically inefficient in this steady state? Explain.

Assume that the economy is initially in the steady state. Now unexpectedly at time $t = T$ the social security system is dismantled: No contributions are raised an no benefits are paid, neither in the present nor at any point in the future.

(d) What is the expression for the new steady state capital level? What are the effects of the shock on capital accumulation k_{T+1} and consumption when young c_{1T} (compared to consumption and capital in the original steady state)? Explain.

(e) Do the young in the period T benefit from this policy change? The old? Explain.

2. Consider an economy with aggregate demand $y = m - p$, where y denotes real income, m is the amount of nominal money balances and p is the aggregate price. As to the supply side, a fraction $(1-q)$ of the population of firms sets prices in a flexible manner, while the remaining fraction q has rigid prices. Let p^f denote the price set by a representative flexible-price firm and p^r the price set by a representative rigid-price firm. Flexible-price firms set their prices after m is known, while rigid-price firms set their prices before m is known (and thus must form expectations on m and p). All variables are in logarithmic terms.

Suppose flexible-price firms set

$$p^f = (1-\phi)p + \phi m$$

while rigid price-firms set

$$p^r = (1-\phi)E[p] + \phi E[m]$$

where $0 \leq \phi \leq 1$ measures the degree of real rigidity in the economy (how responsive prices are to aggregate demand), expectations are subject to the information known when fixed-price firms set prices and $p = qp^r + (1-q)\,p^f$, with $0 \leq q \leq 1$.

(a) Find p^f in terms of p^r, m and the parameters of the model (ϕ and q).

(b) Find p^r in terms of $E[m]$ and the parameters of the model.

(c) Show that the equilibrium y and p are, respectively:

$$\begin{aligned} y &= (m - E[m])\,\frac{q}{\phi + (1-\phi)\,q} \\ p &= E[m] + (m - E[m])\,\frac{\phi\,(1-q)}{\phi + (1-\phi)\,q} \end{aligned}$$

(d) What are the equilibirum values of y and p as $q \to 0$? Explain.

Solution:

1. (a) FOCs of the profit function in per capita terms are

$$\begin{aligned} R_t &= \alpha A k_t^{\alpha-1} \\ w_t &= (1-\alpha) A k_t^{\alpha} \end{aligned}$$

Given full depreciation, we have $R_t - \delta = R_t - 1 = r_t$.

(b) The savings problem of a young individual is

$$\max_{c_{1t},c_{2t+1}} \ln c_{1t} + \frac{1}{1+\rho}\ln c_{2t+1}$$
$$c_{1t} + s_t = w_t(1-\tau)$$
$$c_{2t+1} = s_t r_{t+1} + \tau w_{t+1}$$

Solving this problem and combining FOCs yields the Euler equation

$$c_{2t+1} = \frac{r_{t+1}}{1+\rho} c_{1t}$$

Replace c_{1t} and c_{2t+1} from the budget constraints to obtain the desired equation describing individual savings behavior.

(c) To derive the capital accumulation equation we use individual savings and replace $k_{t+1} = s_t$ (there is no population growth term here since by assumption $n = 0$), and use the equilibrium expressions for wages and rental rates to obtain

$$k_{t+1} = \frac{1}{2+\rho}(1-\alpha)Ak_t^\alpha(1-\tau) - \frac{1+\rho}{2+\rho}\frac{(1-\alpha)k_{t+1}}{\alpha}\tau$$

Combine terms with k_{t+1} we get the desired expressions. Imposing steady state we get

$$\bar{k} = \left[\frac{1}{1+\frac{1+\rho}{2+\rho}\frac{(1-\alpha)}{\alpha}\tau}\left(\frac{1}{2+\rho}(1-\alpha)A(1-\tau)\right)\right]^{\frac{1}{1-\alpha}}$$

The economy can be dynamically inefficient in this steady state if $r^* < 1$ since the rate of return on social security is $1 + n = 1$.

(d) From the previous part we know that the new steady state satisfies

$$\bar{k}_{NEW} = \left[\frac{1}{2+\rho}(1-\alpha)A\right]^{\frac{1}{1-\alpha}} > \bar{k}$$

In period T we have that the wage is given by $\bar{w} = (1-\alpha)A\bar{k}^\alpha$ since capital is predetermined, and capital accumulation satisfies

$$k_1 = \frac{1}{2+\rho}\bar{w}$$

Thus $k_1 > \bar{k}$. The effect on consumption when young in T is ambiguous. Note that

$$\begin{aligned}
\bar{c} &= \bar{w}(1-\tau) - \bar{s} \\
&= \bar{w}(1-\tau) - \left(\frac{1}{2+\rho}\bar{w}(1-\tau) - \frac{1+\rho}{2+\rho}\frac{1}{\bar{r}}\tau\bar{w}\right) \\
&= \bar{w}\frac{1+\rho}{2+\rho} - \tau\bar{w}\frac{1+\rho}{1+\rho}\left(1-\frac{1}{\bar{r}}\right)
\end{aligned}$$

and

$$\begin{aligned} c_{1T} &= \bar{w} - k_{T+1} \\ &= \bar{w} - \frac{1}{2+\rho}\bar{w} \\ &= \bar{w}\frac{1+\rho}{2+\rho} \end{aligned}$$

so

$$c_{1T} - \bar{c} = \tau\bar{w}\frac{1+\rho}{2+\rho}\left(1 - \frac{1}{\bar{r}}\right)$$

If the economy is dynamically inefficient such that $\bar{r} < 1$, then the shock reduces first period consumption relative to the original steady state. In this case, the shock makes the young worse off by reducing the size of the social security which had a higher rate of return. The income effect leads them to allocate fewer resources to first and second period consumption. This explains the reduction in first period consumption.

(e) From the above reasoning, the young are better off only when the economy initially was dynamically efficient, ie when $\bar{r} > 1$. The old are clearly always worse off since they do not receive contributions.

2. The model economy can be summarized by the following equations:

$$p^f = (1-\phi)p + \phi m \tag{1}$$

$$p^r = (1-\phi)E[p] + \phi E[m] \tag{2}$$

$$p = qp^r + (1-q)\,p^f \tag{3}$$

$$y = m - p \tag{4}$$

where $0 \leq \phi \leq 1$ and $0 \leq q \leq 1$.

(a) Substituting (3) into (1):

$$p^f = (1-\phi)\left(qp^r + (1-q)\,p^f\right) + \phi m$$

and rearranging so as to bring p^f on the left-hand side of the equality:

$$p^f = p^r + \frac{\phi}{\phi + (1-\phi)\,q}\,(m - p^r) \tag{5}$$

(b) Substituting (3) into (2):

$$p^r = (1-\phi)E\left[qp^r + (1-q)\,p^f\right] + \phi E[m]$$

and rearranging so as to bring p^r on the left-hand side of the equality:

$$p^r = E[m] \tag{6}$$

(c) Substituting (5) and (6) into (3):

$$p = E[m] + (m - E[m])\,\frac{\phi\,(1-q)}{\phi + (1-\phi)\,q}$$

Substituting the latter into (4):

$$y = (m - E[m])\,\frac{q}{\phi + (1-\phi)\,q}$$

(d) As q tends to zero, rigid-price firms vanish, and prices are only set in a flexible manner, so that $p = m$ and $y = 0$. As a result, the unanticipated component of aggregate demand $(m - E[m])$ is no longer relevant, as all firms are able to set prices after observing the actual m.