Exam summer 2014

Problem 1

Consider a consumer, Anna, who is fond of pizza (good 1) and diet soda (good 2). Each month she spends an amount of dkk 300 on pizza and soda altogether. Her preferences on pizza and diet soda are representable by a utility function $u(x_1, x_2) = x_1^2 x_2$. Initially, one pizza costs dkk 40 while one diet soda costs dkk 20. To improve public health the government imposes a tax of dkk 10 per pizza to reduce the fat intake.

a) How much would Anna be willing to pay in order to prevent a tax on pizzas before the price change?
b) What is the efficiency loss of this tax? Explain why the efficiency loss occurs.

Answer: Since $EV = I - E(p, u')$, at the new prices the consumer buys $x' = (4,5)$, the utility level after the tax levied on pizza is $u' = (4)^2 5 = 80$, and $\min p_1 x_1 + p_2 x_2$ such that $x_1^2 x_2 = 80$ gives $E(p, 80) = 80^{\frac{1}{3}}\left(2^{\frac{1}{3}} + 2^{-\frac{2}{3}}\right) p_1^{\frac{2}{3}} p_2^{\frac{1}{3}}$ such that $EV = 300 - 80^{\frac{1}{3}}\left(2^{\frac{1}{3}} + 2^{-\frac{2}{3}}\right)(40)^{\frac{2}{3}}(20)^{\frac{1}{3}} \cong 41{,}46$. The tax revenue is $10 * 4 = 40$, but then there is an efficiency loss of $T - EV = 1{,}46$. This is the loss due to the fact that there is an economic loss due to the substitution effect is non-zero, so there is an alternative way of collecting the revenue with a lump-sum tax and making the consumer better off – this is a loss in efficiency.

Problem 2

Consider a market where there are two types of consumers, *students* who consume beers and other goods, with a utility function $u_S(x_1, x_2) = 4\sqrt{x_1} + x_2$ and *other people* who have a utility function $u_O(x_1, x_2) = 2\sqrt{x_1} + x_2$, where x_1 is the amount of beers enjoyed by a consumer and x_2 all other goods.

A student has income only from SU which amounts to $I_S = 20$, while the *other people* has an income of $I_O = 100$. Normalize the price of other goods to unity, $p_2 = 1$.

The breweries can produce beer at a fixed marginal costs, $MC = 2$, and endures no fixed nor quasi-fixed costs.

a) Derive the individual demand functions for beer and the aggregate demand function.
b) Find the price and quantity of beers in equilibrium.
c) What happens to the equilibrium in the beer market if the government increases the SU to $I_S = 30$? Explain your result.

Answer: We have that $x_{1S}(p_1, I_S) = \frac{4}{p_1^2}$ for students and $x_{1O}(p_1, I_O) = \frac{1}{p_1^2}$ for other people such that the aggregate demand is $X(p_1) = 5\frac{1}{p_1^2}$. The equilibrium price must equal the marginal costs that are constant, thus the price must be $p_1^* = MC = 2$ and the equilibrium quantity $X^*(2) = \frac{5}{4}$ with demand of each consumer type $x_{1S}^* = 1$ and $x_{1O}^* = \frac{1}{4}$. Since the preferences are quasi-linear, and the solution is in the interior, the income effect is zero and hence the equilibrium is independent of the income increase of the students.

Problem 3

Consider a company, *PølseKompagniet Hansen & Søn A/S*, which produces sausages to be consumed for lunch and breakfast. The company uses a technology with the production function $f(\ell, k) = \ell^{\frac{1}{2}} k^{\frac{1}{4}}$. Each sausage can be sold at a price of $p = 2$, the wage rate is $w = 1$ and the rental rate is $r = 1$.

a) Determine the profit maximizing production plan of *Pølsekompagniet A/S*, when the firm is tied up on a contract of the capital level at $\bar{k} = 1$.

Assume that Niels Henning, the chief engineer of *PølseKompagniet A/S*, invents some ingenious new technology for sausage production. This enables them to produce twice as many sausages with the same amount of input as before, which means that the new production function becomes $g(\ell, k) = 2\ell^{\frac{1}{2}} k^{\frac{1}{4}}$.

b) What is the effect on the firm's profit after this invention? Comment on the effect.

Answer: We can solve for the cost function given capital level $C(x; \bar{k}) = w(\bar{k})^{\frac{1}{2}} x^2 = (\bar{k})^{\frac{1}{2}} x^2 = x^2$ and thus the optimal production solves $2 = 2x^*$ and hence $x^* = 1$, and thus the costs $C(x^*) = 1$ while profits $\pi^* = 2 - 1 = 1$. The invention has the impact that the total and the marginal costs in the short run are reduced by a fourth, $C_g(x) = C_f\left(\frac{1}{2}x\right) = \frac{1}{4}x^2$ and thus the output is $\frac{1}{2}x^2 = 2 \Leftrightarrow x' = \sqrt{4} = 2$, and the costs are then $C_g(x') = 1$ such that new profit is $\pi' = 4 - 1 = 3$. The innovation increases the output and the profits of the firm, while the employment at the firm increases. The innovation reduces the marginal costs and thus the supply curve of the firm; since the price is unchanged and costs of the optimal supply, the profit must increase.

Problem 4

Comment on the following statement:

"A competitive firm will always produce such that the marginal costs exactly equal the market price, whenever the price exceeds the total average costs."

Answer: False, since in the short run the price must only exceed the average variable costs, thus neglecting the fixed cost: the profit $\pi_{sr} = px - VC(x) - F \geq -F = \pi_{sr}(0)$ if and only if $p \geq AVC(x) = \frac{VC}{x}$.

Problem 5

Consider two consumers, Karen and Jørgen, who each live in the same two periods: "today" and "tomorrow". In each period they can consume the same aggregate consumption good, and denote by c_1 the consumption today and c_2 the consumption tomorrow, and a pair $c = (c_1, c_2)$ is called a consumption plan. Karen has preferences on consumption plans given by $u_K(c_{1K}, c_{2K}) = c_{1K} c_{2K}$ while Jørgen has a utility function given by $u_J(c_{1J}, c_{2J}) = \min\{c_{1J}, c_{2J}\}$.

Karen has more income today than tomorrow, while Jørgen has less income today than tomorrow. Thus, Karen has an initial endowment of $e_K = (4{,}0)$, while Jørgen has an initial endowment of $e_J = (1{,}5)$.

a) Determine the Walras equilibrium of this economy with the price of consumption tomorrow normalized $p_2 = 1$.
b) Determine the equilibrium interest rate, and the savings of both Karen and Jørgen.

Assume alternatively that Jørgen instead of the above utility function had a utility function given by $\tilde{u}_J(c_{1J}, c_{2J}) = \min\{c_{1J}, 2c_{2J}\}$.

c) Determine the Walras equilibrium of this economy with the price of consumption tomorrow normalized $p_2 = 1$. Compare with your result in a) and explain.

Answer: In equilibrium the Walrasian prices becomes the solution to the equation $\frac{1}{2}\frac{I_K}{p_1} + \frac{I_J}{p_1+1} = 5$ where $I_K = 4p_1$ and $I_J = p_1 + 5$, which is satisfied whenever $p_1 = 1$: $I_K = 4$ and $I_J = 6$ then $\frac{1}{2}\frac{4}{p_1} + \frac{6}{p_1+1} = \frac{1}{2}\frac{4}{1} + \frac{6}{1+1} = 2 + 3 = 5$. Thus, the equilibrium interest rate, with $p_1 = 1 + r$, is then $r = 0$, and the saving of Karen is $s_K^* = e_{1J} - c_{1J}^* = 5 - 2 = 3 > 0$ and of Jørgen $s_J^* = e_{1J} - c_{1J}^* = 0 - 3 = -3$. Thus Jørgen borrows from Karen and the equilibrium interest rate is zero. With the new utility function we obtain an equilibrium such that $\frac{1}{2}\frac{I_K}{p_1} + \frac{2I_J}{2p_1+1} = 5$ then the equilibrium price is $p_1 = \frac{7}{4}$ since $I_K = 7$ and $I_J = \frac{27}{4}$, such that the aggregate demand becomes $\frac{1}{2}\frac{7}{\frac{7}{4}} + 2\frac{\frac{27}{4}}{\frac{7}{2}+1} = 2 + 3 = 5$. The new interest rate is then $r = p_1 - 1 = \frac{3}{2} - 1 = \frac{1}{2}$, and the savings becomes $s_K = 5 - 2 = 3$ and $s_J = 0 - 3 = -3$. We see that new preferences represents an impatience of Jørgen, since consumption tomorrow only counts as half consumption today. This represents an increase in the demand of Jørgen for consumption today, which then increases the price of consumption today e.g. the interest rate increases; the savings of Karen increases due to the substitution and ordinary income effect, but the increased value of savings increases the wealth, which is spend on consumption today cancelling the other effects, while the savings of Jørgen is also unchanged since the two income effects cancel each other (the substitutioneffect is zero).