LM August 2018

(1) 1) $L:\mathbb{R}^2 \to \mathbb{R}^4$ så $\underline{\underline{n=2}}$, $\underline{\underline{m=4}}$

2) $Lx = \underline{0}$

$$L \longleftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$N(L) = \{\underline{0}\}$ så L <u>er</u> injektiv.

3)

$$R(L) = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\} \left(= \text{span}\{v_1, v_2\}\right)$$

$\dim R(L) = 2 < 4 = \dim \mathbb{R}^4$, L ej surjektiv

Dim.satn: $2 - 0 = 2$

4) $(3,2,a,b) \in R(L)$, dvs

$$\left[\begin{array}{cc|c} 1 & 4 & 3 \\ 1 & 3 & 2 \\ 0 & 2 & a \\ 0 & 1 & b \end{array}\right]$$ er konsistent

$$\left[\begin{array}{cc|c} 1 & 0 & 2-a-b \\ 0 & 1 & b \\ 0 & 0 & 1-b \\ 0 & 0 & a-2b \end{array}\right],$$

Så er $1-b=0$ og
$a-2b=0$

dvs $\underline{\underline{a=2}}$, $\underline{\underline{b=1}}$

5) $LX = Y$

$$\left[\begin{array}{cc|c} 1 & 4 & Y_1 \\ 1 & 3 & Y_2 \\ 0 & 2 & Y_3 \\ 0 & 1 & Y_4 \end{array}\right] \leftrightarrow \left[\begin{array}{cc|c} 1 & 0 & Y_2-3Y_4 \\ 0 & 1 & Y_4 \\ 0 & 0 & Y_1-Y_2-Y_4 \\ 0 & 0 & Y_3-2Y_4 \end{array}\right]$$

Heraf fås

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} Y_2-3Y_4 \\ Y_4 \end{bmatrix} \quad \text{og} \quad \left.\begin{matrix} Y_1-Y_2-Y_4 = 0 \\ Y_3-2Y_4 = 0 \end{matrix}\right\} \text{forat } Y \in R(L)$$

6) Vi løser

$$\alpha_1 V_1 + \alpha_2 V_2 = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \text{ dvs } \left[\begin{array}{cc|c} 1 & 4 & 3 \\ 1 & 3 & 2 \\ 0 & 2 & a \\ 0 & 1 & b \end{array}\right] \text{ hvor } a=2, b=1$$

fra 4) fås $(\alpha_1, \alpha_2) = (-1, 1)$ som er koordinater m.h.t. V_1, V_2.

7)

$$t\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} t+4s \\ t+3s \\ 2s \\ s \end{bmatrix}$$

er koordinaten m.h.t. standard basen.

2

1) $v_1 \cdot v_3 = 0$, $v_2 \cdot v_3 = 0$, så mulig v_3 er f.eks $(1,1,0)$.

2) $$P_A(\lambda) = \begin{vmatrix} 1-\lambda & & \\ & -1-\lambda & \\ & & 2-\lambda \end{vmatrix}$$
$$= (1-\lambda)(-1-\lambda)(2-\lambda)$$
$$= -(1-\lambda)(1+\lambda)(2-\lambda)$$
$$= -(1-\lambda^2)(2-\lambda) = \underline{-\lambda^3+2\lambda^2+\lambda-2}$$

3) $\det(A) = 1\cdot(-1)\cdot 2 = -2 \neq 0$, så A er inv.

4) $$A^{-1} v_3 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$

5) $$e^A(v_1+v_2+v_3) = e^A v_1 + e^A v_2 + e^A v_3$$
$$= e^1 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + e^{-1} \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + e^2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e+e^{-1}+e^2 \\ -e-e^{-1}+e^2 \\ e-2e^{-1} \end{bmatrix}$$

3

$$\int (\cos(x)+\sin(2x))\sin(3x)\,dx =$$

$$\int \cos(x)\cdot\sin(3x) + \sin(2x)\sin(3x)\,dx =$$

$$\int \left(\frac{e^{ix}+e^{-ix}}{2}\right)\left(\frac{e^{i3x}-e^{-i3x}}{2i}\right)dx$$

$$+\int \left(\frac{e^{i2x}-e^{-i2x}}{2i}\right)\left(\frac{e^{i3x}-e^{-i3x}}{2i}\right)dx$$

$$= \frac{1}{4i}\int e^{i4x}-e^{-i2x}+e^{i2x}-e^{-i4x}\,dx$$

$$-\frac{1}{4}\int e^{i5x}-e^{-ix}-e^{ix}+e^{-i5x}\,dx$$

$$= \frac{1}{2}\int \frac{e^{i4x}-e^{-i4x}}{2i}+\frac{e^{i2x}-e^{-i2x}}{2i}\,dx$$

$$-\frac{1}{2}\int \frac{e^{i5x}+e^{-i5x}}{2}-\frac{e^{ix}+e^{-ix}}{2}\,dx$$

$$= \frac{1}{2}\int \sin(4x)+\sin(2x)\,dx - \frac{1}{2}\int \cos(5x)-\cos(x)\,dx$$

$$= \frac{1}{2}\left(-\frac{1}{4}\cos(4x)-\frac{1}{2}\cos(2x)\right)-\frac{1}{2}\left(\frac{1}{5}\sin(5x)-\sin(x)\right)+k$$

$$= -\frac{1}{8}\cos(4x)-\frac{1}{4}\cos(2x)-\frac{1}{10}\sin(5x)+\frac{1}{2}\sin(x)+d.$$

2) $(3+i2)z + 7 - i10 = i8(1-i)$

$$(3+i2)z + 7 - i10 = 8 + i8$$

$$(3+i2)z = 1 + i18$$

$$z = \frac{(1+i18)(3-i2)}{(3+i2)(3-i2)}$$

$$z = \frac{3 - i2 + i54 + 36}{9+4}$$

$$z = \frac{39 + i52}{13} = \underline{\underline{3+i4}}$$

4 $\sum_{n=0}^{\infty} \left(\frac{1}{x^2-4x+5}\right)^n$

1) $g(x) = \frac{1}{x^2-4x+5} = \frac{1}{(x-2)^2+1}$

$|g(x)| < 1$, dvs $\frac{1}{(x-2)^2+1} < 1$ $\Updownarrow$

($-1 < g(x)$ er automatisk opfyldt da $g(x) > 0$)

$1 < (x-2)^2 + 1$ $\Updownarrow$

$0 < (x-2)^2$ $\Updownarrow$

$\underline{\underline{x \neq 2}}$

Veldef for

$\underline{x \in]-\infty; 2[\cup]2, \infty[} = M$

2) $f(x) = \dfrac{1}{1 - \frac{1}{x^2-4x+5}}$, $\underline{x \in M}$.

3) f og g samme monotoniforhold.

$$g(x) = (x^2-4x+5)^{-1}$$

$$g'(x) = -1\,(x^2-4x+5)^{-2} \cdot (2x-4) = 0$$

dvs $x = 2$, som $\underline{\text{ikke ligger i } M}$.

Dvs $\underline{\text{ingen ekstrema}}$.

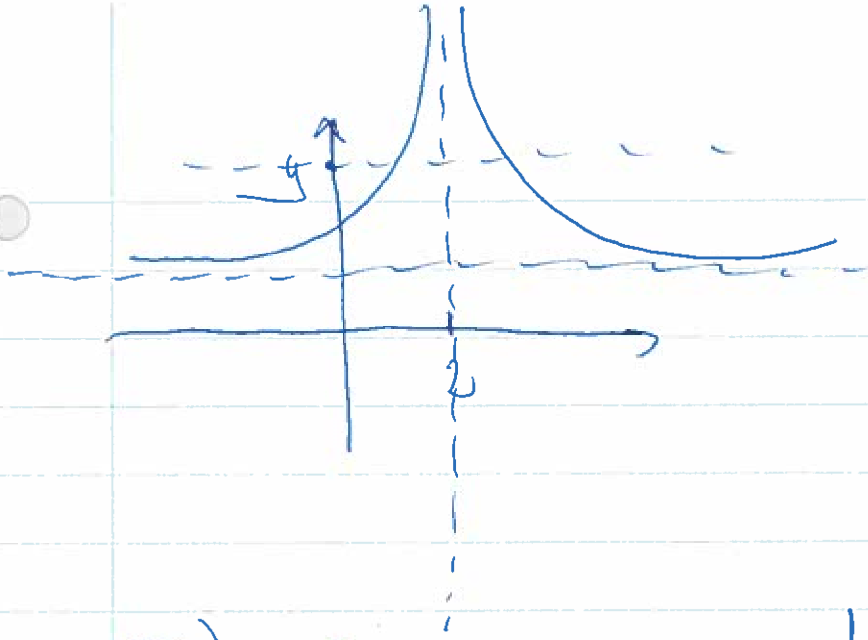
for $x < 2$ er $g'(x) > 0$ og
$x > 2$ er $g'(x) < 0$

dvs

		2	
f'	$+$		$-$
f	$\nearrow$		$\searrow$

4) For $x \to \pm\infty$ vil $f(x) \to 1$
For $x \to 2^{\pm}$ vil $f(x) \to \infty$

$$\underline{Vm(f) =]1, \infty[}$$



f er oplagt ej injektiv da $f(x)=y$ har 2 løsninger for $y>1$.

5) $f(x) = y \iff \frac{1}{x^2-4x+5} = \frac{y-1}{y}$.

dvs $$x^2-4x+5 = \frac{y}{y-1}$$

$$x^2-4x+\left(5-\frac{y}{y-1}\right) = 0$$

$$x = \frac{4 \pm \sqrt{16-4\left(5-\frac{y}{y-1}\right)}}{2}$$

$$x = 2 \pm \sqrt{4-\left(5-\frac{y}{y-1}\right)}$$

som kan reduceres til

$$x = 2 \pm \sqrt{\frac{1}{y-1}}$$