Exam Macro C - Suggested Solution

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Problem A:

1: Equation (A.1) specifies aggregate demand, which depends positively on stock prices and negatively on goods prices. The reason that higher stock prices stimulate demand is that households respond to the higher market value of their wealth by increasing private consumption and firms respond to the higher stock prices by increasing investment (according to Tobins q-theory). Higher goods prices reduce international competitiveness which reduces net exports and thereby aggregate demand. In addition z is an exogenous shift parameter capturing among others changes in world economic activity and fiscal policy.
Equation (A.2) is the central arbitrage condition, characterizing equilibrium in financial markets. It states that the return to stocks (the left-hand side) must equal the return to bonds (the right-hand side). The return to owning stocks consists of dividends (that part of profit which is paid out to stockholders) and the capital gain on stocks. When dividing by the stock price we get the rate of return to stocks. The return to bonds is simply given by the interest rate, which equals the exogenous foreign interest rate due to perfect capital mobility (and the fact that the exchange rate is fixed).
Equation (A.3) specifies that dividends depend positively on output simply capturing that the source of dividends is profit, and that profit depends positively on output (the state of the economy).
Equation (A.4) is the Philips-curve without supply shocks and with the restriction that expected inflation equals zero (the foreign inflation rate). Thus, inflation depends only on the current output gap, e.g. capturing that when production is high (and unemployment is low) there is a tendency for wages to increase which increases the cost of production and thereby the inflation

rate.
Finally equation (A.5) is the fundamental stock price, which can be derived from equation (A.2) and a transversality condition (stating that stock prices can not approach infinity). The fundamental stock price is simply given by the discounted value of present and all future dividends.

By substituting (A.1) into (A.4) we get:

$$\dot{p}_t = \gamma \cdot (z + \eta \cdot Q_t - \beta \cdot p_t - \bar{y}) \tag{1}$$

Rearranging (A.2) and inserting (A.3) and (A.1) we get:

$$\dot{Q}_t = r^f \cdot Q_t - D_t = r^f \cdot Q_t - \alpha \cdot (z + \eta \cdot Q_t - \beta \cdot p_t) \tag{2}$$

implying:

$$\dot{Q}_t = (r^f - \alpha \cdot \eta) \cdot Q_t + \alpha \cdot \beta \cdot p_t - \alpha \cdot z \tag{3}$$

2: In this model, without any growth in the natural output level, the long run equilibrium is defined by: $\dot{Q}_t = 0$ and $\dot{p}_t = 0$. We can construct the phase diagram by deriving the conditions under which $\dot{p}_t = 0$ and then derive the dynamic evolution of p_t in general, and afterwards do the same for $\dot{Q}_t = 0$.

The condition for $\dot{p}_t = 0$:
Setting $\dot{p}_t = 0$ in equation (1) and solving for Q_t we get:

$$\dot{p}_t = 0 \Rightarrow Q_t = \frac{\bar{y} - z + \beta \cdot p_t}{\eta} \tag{4}$$

which is the upward slooping line in the (p_t, Q_t) space. The reason is that $\dot{p}_t = 0$ requires $y_t = \bar{y}$ and a higher domestic price level tends to reduce y_t (due to the lower international competitiveness which reduces net exports) whereby Q_t must increase in order to stimulate private consumption and investment and thereby output (such that $y_t = \bar{y}$). There is no economic mechanism ensuring that this will actually happen, the line simply states what the relationship between p_t and Q_t must be for the domestic price level to be constant.
If the economy is initially located above (to the left of) this line then stock prices are high (relative to the $p_t = 0$ line), or correspondingly domestic

goods prices are low, implying that aggregate demand is strong such that $y_t > \bar{y}$. According to the Philips curve this situation triggers inflation, such that the domestic price level rises over time. This is illustrated by the horizontal arrows in the figure on page 10. This is the stabilizing mechanism in the Blanchard model.

The condition for $\dot{Q}_t = 0$:
Setting $\dot{Q}_t = 0$ in equation (3) and solving for Q_t we get:

$$\dot{Q}_t = 0 \Rightarrow Q_t = \frac{\alpha \cdot z - \alpha \cdot \beta \cdot p_t}{r^f - \alpha \cdot \eta} \tag{5}$$

which is a downward slopping line (since by assumption $r^f > \alpha \cdot \eta$) The reason is the following:
Since the line requires $\dot{Q}_t = 0$ then equation (A.2) implies that:

$$\frac{D_t}{Q_t} = r^f \Rightarrow D_t = r^f \cdot Q_t \tag{6}$$

Now, an increase in Q_t by one unit increases the right hand side (the required total return on stocks) by r^f units. The left hand side also increases (since higher stock prices increase output which increases dividends) but only by the amount $\alpha \cdot \eta$. Since by assumption $r^f > \alpha \cdot \eta$ dividends doesn't increase by enough, and we need a further increase in dividends. This can only happen through lower domestic goods prices which stimulates net exports and thereby output and dividends. This explains the negative relationship between Q_t and p_t whenever $r^f > \alpha \cdot \eta$.
If the economy is initially located above the line then p_t is high, implying that net exports are low. Thus, output and dividends are low. In order to compensate for the low dividends there must be a capital gain on owning stocks in order for the financial arbitrage equation to be satisfied. This is the destabilizing mechanism in the Blanchard model.

The dynamics of the economy implies that the model is saddle path stable, implying that for each p_t there is one and only value of Q_t which ensures convergence towards the long run equilibirum. At each point in time p_t is predetermined, while Q_t is free to jump. We will assume that Q_t adjusts to the point on the saddle path (shown as SS in the figure). Paths starting out above the saddle path implies that stock prices will tend towards infinity, which violates the transversality condition and is a fragile path (if investors begin to believe that the stock market will crash at some point in time, say time T, they will act in way which places the economy on the saddle path immediately). Paths starting out below the saddle path can be ruled out whenever there is free disposal, since stocks prices will eventually become negative.

3: Now, let's consider an unanticipated increase in $\bar{y}$. The effects are illustrated in the figure on page 11. As the (unancitipated) increase in $\bar{y}$ is implemented, the economy jumps from E to A, which is located on the new saddle path. Afterwards the economy converges towards the new long run equilibrium (E'). The economic intuition is as follows:
At the time of implementation stock prices increase since financial investors realize, that the higher output level in the future will increase dividends. The prospect of future increases in dividends immidiately induces investors to buy stocks which drives up the stock price according to equation (A.5). Stock prices undershoot in the short run due to the assumption: $r^f > \alpha \cdot \eta$. At A output has increased (as a result of the increase in stock prices) but output gap is negative ($y_t < \bar{y}$) since output has increased by less than $\bar{y}$. As a result there is now a excess supply and prices begin to fall, according to the Philips curve. This explains why p_t falls from A to E'. The falling goods prices improves international competitiveness and increases net exports, which gradually increases output. The increase in output increases dividends (according to (A.3)), and this gradual and fully anticipated increase in dividends increases stock prices (according to (A.5)).
4: Now let's consider an anticipated increase in $\bar{y}$. We can determine the movements of the economy by noticing four things:
- At time t_0 p_t is predetermined and will only adjust gradually over time
- Q_t can jump but if so it must be at time t_0 where financial investors receive new information
- At time t_1 the economy must be located somewhere on the new saddle path
- Between time t_0 and t_1 the dynamics of the economy is guided by the old dynamic system

These properties imply that the economy must jump from E to A (see the figure on page 12) at time t_0. This is the announcement effect. Stock prices increase since financial investors realize that the higher output in the future will increase future dividends which increases stock prices already at time t_0 (according to equation (A.5)). The increase in stock prices increases output (due to higher private consumption and investment), such that $y_t > \bar{y}$ at point A (remember that $\bar{y}$ hasn't increased yet). The positive output gap triggers inflation such that p_t increases from point A to point B. Stock prices increase since the economy moves closer in time to when dividends increase. The economy reaches the point B exactly at time t_1. Afterwards the economy converges towards E' as described in 3).

Problem B:

1: Equation (B.1) describes aggregate demand. It states that a higher real exchange rate (corresponding to a weaker domestic currency) and a lower domestic interest rate tends to make output gap positive. β once again measures the price elasticity of net exports, while ϵ measures the sensitivity of aggregate demand to the interest rate (working through the effects on investment and probably also consumption e.g. due to asset price effects). The reason that r^f is included on the right-hand side is that the equation is linearized around the long run equilibrium where the domestic interest rate must equal r^f. Once again z is an exogenous shift parameter which captures changes in (among others) fiscal policy, consumer confidence and world economic activity.
Equation (B.2) is the equation describing equilibrium in the money market (also linearized around the long run equilibrium). The left hand side is the (log of) real money supply, where m is an exogenous policy instrument while the domestic price level is endogenous. The right hand side is the demand for (real) money, depending positively on the level of output (the transaction motive) and negatively on the interest rate (which is the oppurtunity cost of holding money). By assumption the expected inflation rate is equal to zero, which implies that the real interest rate equals the nominal interest rate. The parameter ε measures the sensitivity (semi-elasticity) of real money demand with respect to the interest rate.
Equation (B.3) is an arbitrage equation stating that domestic and foreign bonds must pay the same rate of return. The equation is also known as the uncovered interest rate parity. Whenever there is a (expected) depreciation of the domestic currency this increases the return to holding bonds denominated in terms of the foreign currency. Thus, the expected (and actual) change in the value of the domestic currency is included on the right hand side.
Equation (B.4) is equal to equation (A.4) and the interpretation has already been discussed.

2: Taking e_t^n and p_t as given equation (B.1) and (B.2) represents two equations in two unknowns , y_t and r_t. From (B.2) we get:

$$r_t - r^f = \frac{y_t - \bar{y} - (m - p_t)}{\varepsilon} \tag{7}$$

and inserting this in (B.1):

$$y_t - \bar{y} = z + \beta \cdot (e_t^n - p_t) - \epsilon \cdot \frac{y_t - \bar{y} - (m - p_t)}{\varepsilon} \tag{8}$$

implying:

$$(y_t - \bar{y}) \cdot \frac{\varepsilon + \epsilon}{\varepsilon} = z + \beta \cdot e_t^n - p_t \cdot \frac{\beta \cdot \varepsilon + \epsilon}{\varepsilon} + \frac{\epsilon}{\varepsilon} \cdot m \Rightarrow \tag{9}$$

$$y_t - \bar{y} = \frac{z \cdot \varepsilon + \varepsilon \cdot \beta \cdot e_t^n - p_t \cdot (\beta \cdot \varepsilon + \epsilon) + \epsilon \cdot m}{\varepsilon + \epsilon} \tag{10}$$

Inserting this in equation (7):

$$r_t - r^f = \frac{1}{\varepsilon} \cdot \frac{z \cdot \varepsilon + \varepsilon \cdot \beta \cdot e_t^n - p_t \cdot (\beta \cdot \varepsilon + \epsilon) + \epsilon \cdot m}{\varepsilon + \epsilon} - \frac{m - p_t}{\varepsilon} \tag{11}$$

implying:

$$r_t - r^f = \frac{z + \beta \cdot e_t^n + (1 - \beta) \cdot p_t - m}{\varepsilon + \epsilon} \tag{12}$$

which proves (B.5) and (B.6). This (pseudo-)equilibrium is illustrated in the figure on page 13.

The IS-curve describes the equilibrium in the goods market, and defines a downward slooping relationship since a higher interest rate reduces demand and thereby output. The LM-curve describes the equilibrium in the money market and is upward slooping since a higher output level stimulates money demand requiring a higher interest rate (since the money supply is constant). An increase in z (a positive demand shock) moves the IS-curve to the right and increases output which incrases money demand and thereby the interest rate. An increase in m (a higher money supply) moves the LM-curve downward and reduces the interest rate which increases output.

An increase in p_t shifts both the IS-curve and the LM-curve. The LM-curve moves upwards since the lower real money supply (M/P_t) tends to increase the interest rate. The IS-curve moves to the left since the higher domestic price level reduces international competitiveness and thereby net exports and output. This tends to reduce the interest rate due to the lower money demand. The total effect is that the interest rate increases whenever $\beta < 1$, since in this case the shift in the IS-curve is relatively small.
3: First, let's derive the condition under which $\dot{p}_t = 0$. From equation (B.4) we see that this requires $y_t = \bar{y}$. Setting the left hand side of equation (9) equal to zero, we get:

$$z \cdot \varepsilon + \varepsilon \cdot \beta \cdot e_t^n - p_t \cdot (\beta \cdot \varepsilon + \epsilon) + \epsilon \cdot m = 0 \tag{13}$$

implying:

$$e_t^n = \frac{p_t \cdot (\beta \cdot \varepsilon + \epsilon) - \epsilon \cdot m - z \cdot \varepsilon}{\varepsilon \cdot \beta} \tag{14}$$

which is an upwards slooping line. The reason being that an increase in the price level reduces output (through lower net exports and a lower real money supply which increases the interest rate and thereby reduces aggregate demand), requiring the domestic currency to devaluate (i.e. an increase in e_t^n) in order to boost net exports and thereby output. Once again, if the economy is initially to the left of the $\dot{p}_t = 0$-line the price level will increase over time since in this case $y_t > \bar{y}$.

The condition for $\dot{e}_t^n = 0$:
From (B.3) we see that $\dot{e}_t^n = 0$ requires $r_t = r^f$. Setting the left hand side of equation (12) equal to zero:

$$z + \beta \cdot e_t^n + (1 - \beta) \cdot p_t - m = 0 \Rightarrow e_t^n = \frac{m - z - (1 - \beta) \cdot p_t}{\beta} \tag{15}$$

which is a downward slooping line (since $\beta < 1$). The reason is that an increase in p_t will tend to increase the interest rate (since $\beta < 1$ by assumption), but since $\dot{e}_t^n = 0$ requires $r_t = r^f$ we need a fall in the interest rate. This can only be achieved by a appreciation of the domestic currency which reduces output and thereby the demand for money and the interest rate.
If the economy is initially located above the $\dot{e}_t^n = 0$-line the domestic interest rate is above r^f (since at this point e_t^n is high implying that international

competitiveness is strong and that output and money demand is high or correspondingly that p_t is high implying a high domestic interest rate since we assume $\beta < 1$). With $r_t > r^f$ holders of domestic currency must experience a capital loss, i.e. $\dot{e}_t^n > 0$ in order for domestic and foreign bonds to be equally attractive.
The movements of the economy are illustrated in the diagram on page 14. The model is once again saddle path stable and the economy will only converge towards the steady state if it starts out somewhere on the saddle path. Paths starting out either above or below the saddle path can be ruied out for similar reasons as in question (A.2).

4: The long run equilibrium is defined by: $\dot{p}_t = 0$ and $\dot{e}_t^n = 0$. Setting the expressions for e_t^n found in question 3) equal to each other we get:

$$\frac{p_t \cdot (\beta \cdot \varepsilon + \epsilon) - \epsilon \cdot m - z \cdot \varepsilon}{\varepsilon \cdot \beta} = \frac{m - z - (1-\beta) \cdot p_t}{\beta} \Rightarrow \tag{16}$$

$$p_t \cdot \epsilon - \epsilon \cdot m = \varepsilon \cdot m - \varepsilon \cdot p_t \Rightarrow p_t = m \tag{17}$$

Inserting this in equation (15) (or (14)) we get:

$$e_t^n = \frac{m - z - (1-\beta) \cdot m}{\beta} = m - \frac{z}{\beta} \tag{18}$$

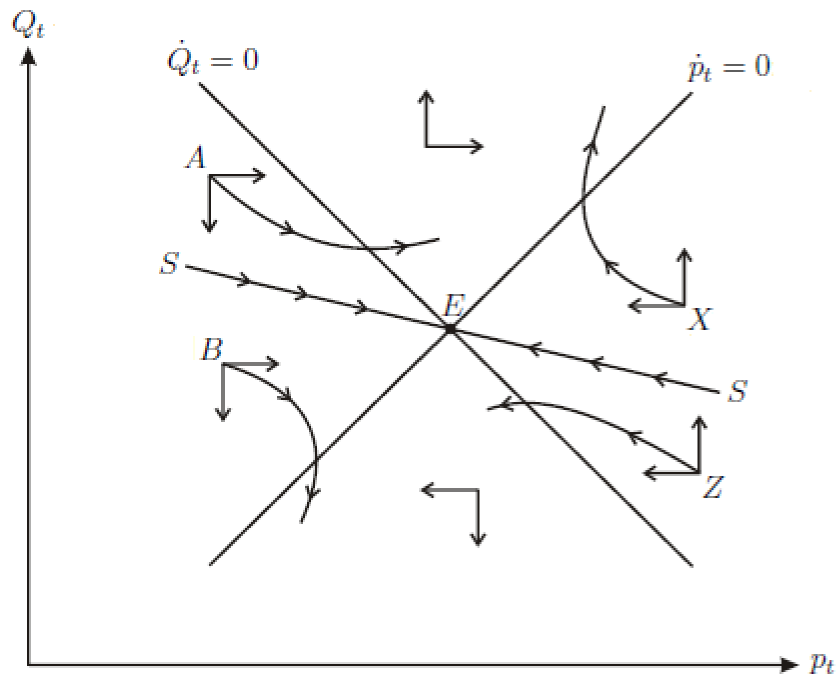
The fact that the long run (steady state) value of p_t is given by m also follows directly from the equilibrium condition for the money market (equation (B.2)) with output gap equal to zero and the interest rate equal to the foreign interest rate. Also, the long run value of the exchange rate follows from the goods market equilibrium (B.1) with $y_t = \bar{y}$, $r_t = r^f$ and $p_t = m$.
A fiscal expansion will in the long run crowd out net exports by appreciating the domestic currency. A fiscal expansion will not affect the long run domestic price level, since the price level equilibrates money supply and demand. A fiscal expansion affects neither in the long run.
5: Now, let's consider the effects of a fiscal expansion. According to equation (14) and (15) an increase in z will shift both the $\dot{p}_t = 0$-line and the $\dot{e}_t^n = 0$-line downwards. Also, we know from above that the domestic price level is unaffected in the long run. Thus, the economy can only be located on the new saddle path right after the increase in z by jumping directly from the old long run equilibrium (E) to the new long run equilibrium (E') (see the diagram on page 15). Thus, there is no short run expansion following the fiscal

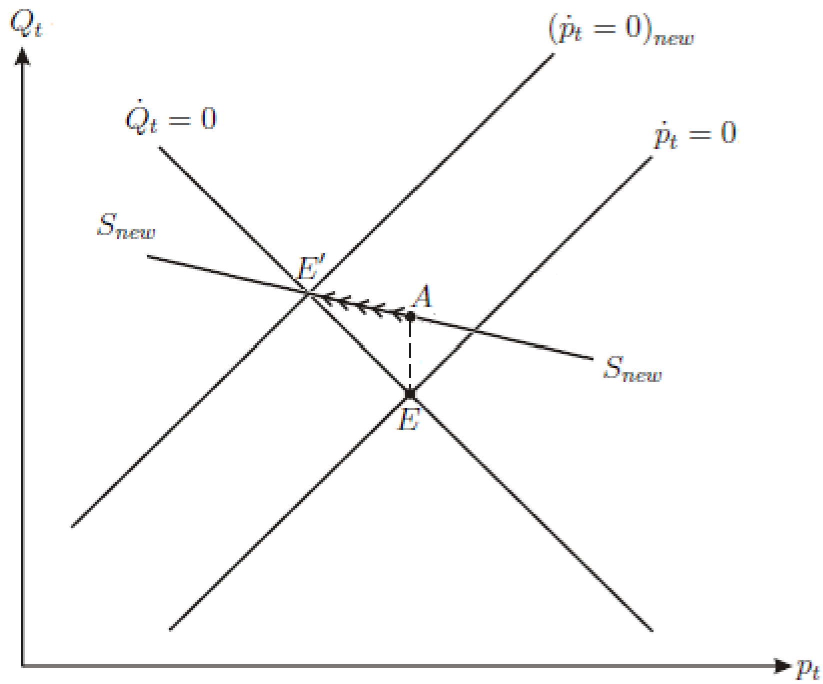
expansion. The fiscal expansion tends to increase the domestic interest rate, but financial investors respond to this by switching from foreign to domestic bonds, which increases the demand for domestic currency and result in a appreciation of the currency. As a result of the appreciation net exports and output falls which reduces the demand for money and thereby the interest rate. The net result is that the interest rate and output is unaffected by the increase in z and that the economy immidiately jumps to the new long run equilibrium.

6: Now let's consider a fiscal expansion which is announced at time t_0 and implemented at time $t_1 > t_0$. Once again, we can notice four properties that the dynamic evolution of the economy must obey:

- At time t_0 p_t is predetermined and will only adjust gradually over time
- e_t^n can jump but if so it must be at time t_0 where financial investors receive new information
- At time t_1 the economy must be located somewhere on the new saddle path
- Between time t_0 and t_1 the dynamics of the economy are guided by the old dynamic system

These properties imply that the economy must jump from E to A at time t_0 (see the diagram on page 16). This is the announcement effect, which steems from financial investors knowledge that the domestic currency eventually will appreciate. Thus, they respond by selling foreign currency and buying domestic currency, in order to benefit from the appreciation. This behaviour increases the demand for domestic currency and makes the domestic currency appreciate already at time t_0. As a result net exports and output falls, and $y_t < \bar{y}$ at point A. Thus, domestic goods prices fall over time (as a consequence of the excess suply). The currency appreciates since the domestic interest rate has fallen below r^f (due to lower money demand), which can only constitute an equilibrium if domestic bondholders receive a capital gain on domestic currency. Intuitively, as the economy moves closer in time to when the fiscal expansion is actually implemented the consequenzes become more important (as they are discounted by a lower discount rate). The economy reaches point B exactly at time t_1. At this point in time z increases implying that y_t increases above $\bar{y}$. Thus p_t increases (as a result of the excess demand). The gradual increase in the price level increases the domestic interest which makes investors gradually switching from forreign to domestic bonds, making the currency appreciate even further (untill the economy reaches E'). The gradual increase in p_t and fall in e_t^n implies that output gradually falls over time untill it reaches $\bar{y}$ at point E'.




Q_t
$(\dot{p}_t = 0)_{new}$
$\dot{Q}_t = 0$
$\dot{p}_t = 0$
S_{new}
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A
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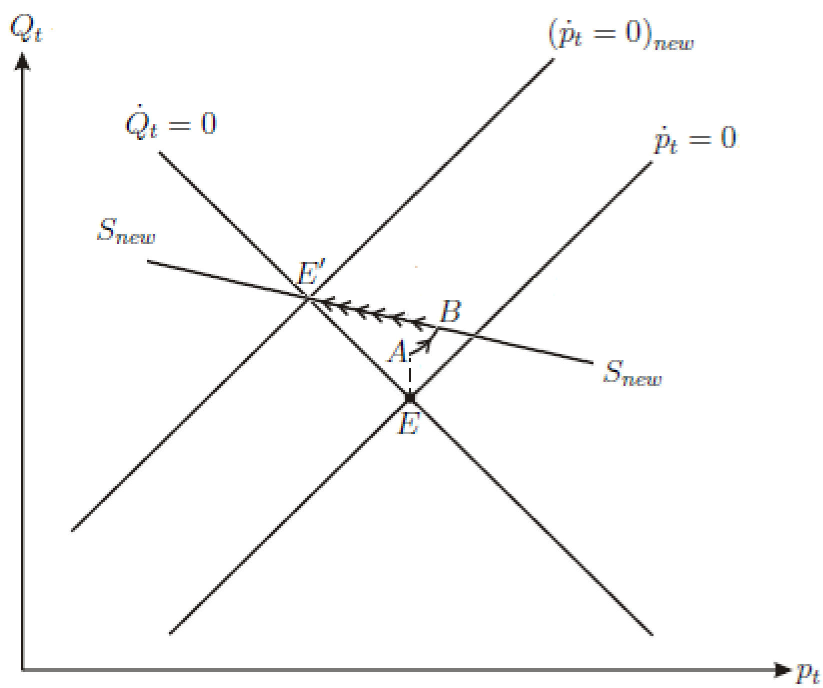
p_t



Q_t
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$\dot{Q}_t = 0$
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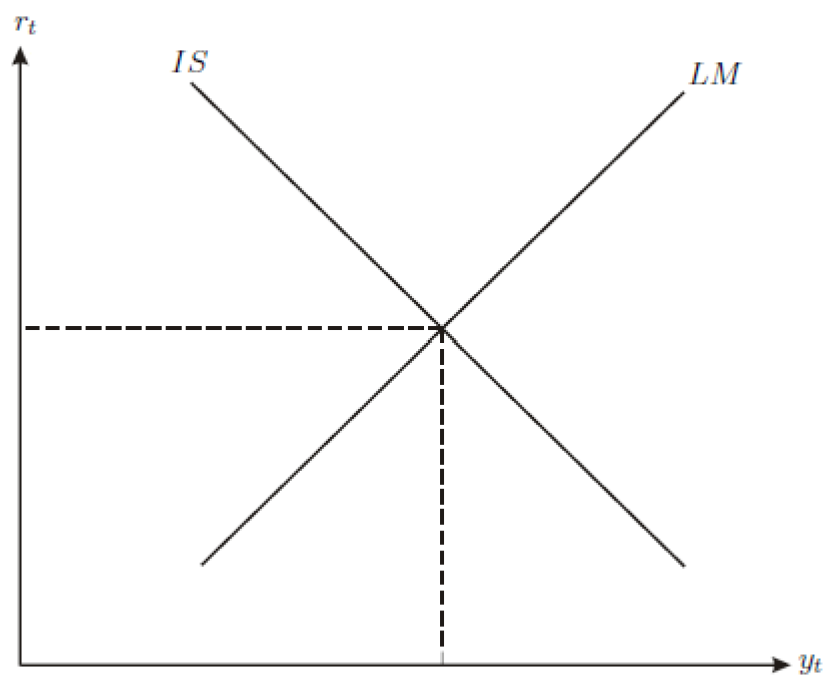
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IS
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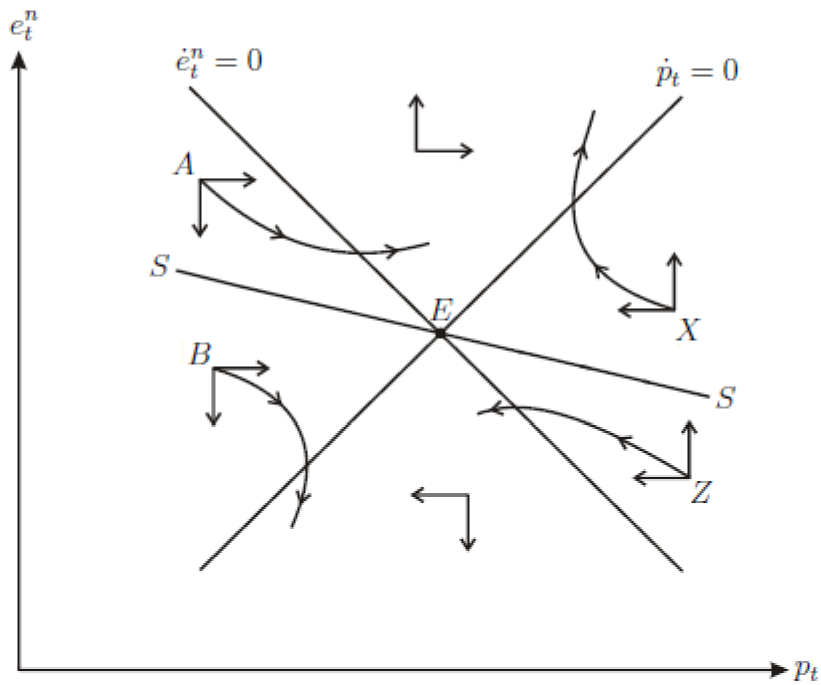
y_t



e_t^n
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$\dot{p}_t = 0$
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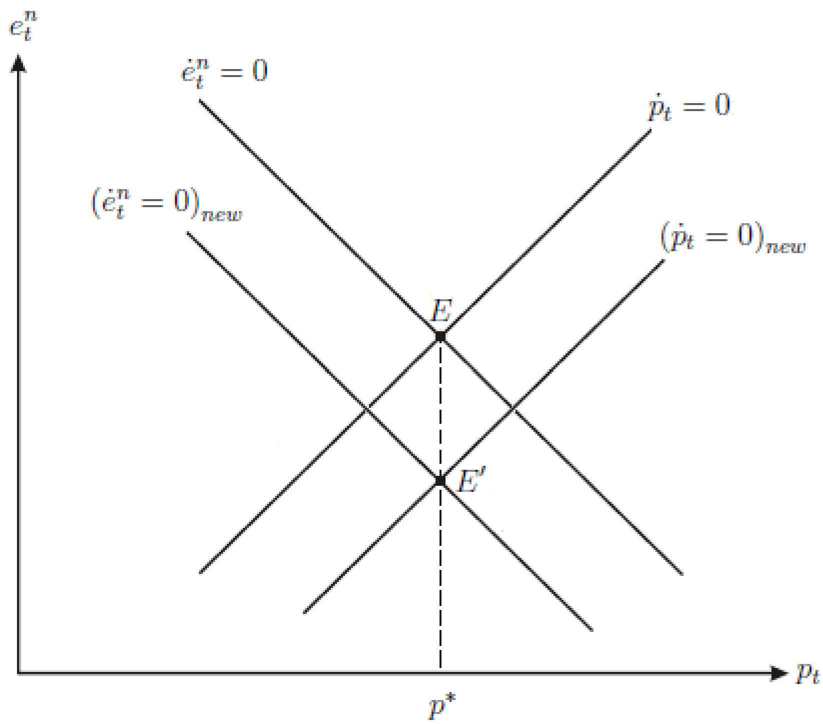
X
B
S
Z
p_t



e_t^n
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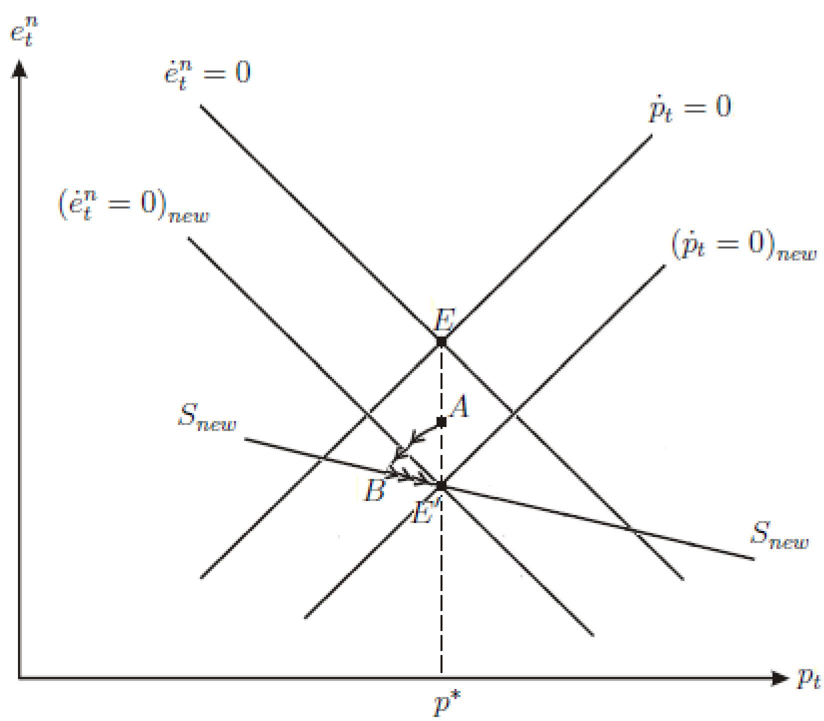
$\dot{p}_t = 0$
$(\dot{e}_t^n = 0)_{new}$
$(\dot{p}_t = 0)_{new}$
E
E'
p_t
p^*


e_t^n
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A
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S_{new}
p_t
p^*