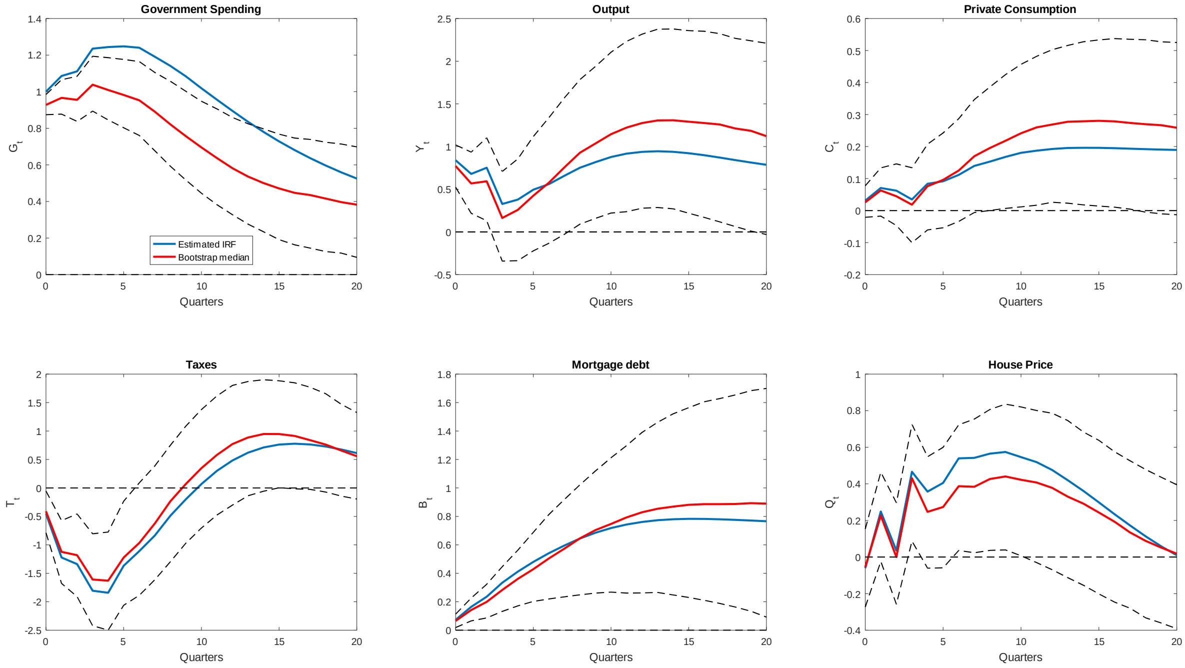
Part 1

Question 1 Codes for the estimation of VAR models have been made available (during the course). Standard transformations implies taking the natural logarithm of all variables. For taxes, a constant needs to be added to avoid taking logs of a negative number (results may differ slightly depending on the choice of this constant). For the results that follow, all log variables have been multiplied by 100. A quadratic trend can be applied by setting the option 'tt = 3' in the codes distributed. The estimation itself (coefficient matrices etc.) need not be reported.

Question 2 In order to apply a Choleski decomposition to identify a government spending shock, it is crucial that government spending is ordered first in the recursive ordering. This amounts to an implicit assumption that government spending cannot respond to changes in any other variable within-quarter. This is the identifying assumption proposed by Blanchard and Perotti (2002), who defend this assumption on two grounds: First, the *inside lag* associated with changes to government spending (i.e., the time it takes to decide on, pass, and implement new legislation about government expenditure) is rather long; in particular, longer than a quarter (the frequency of this dataset). Second, unlike government revenues, there is no automatic and within-quarter response of government spending to GDP (or any other variable in the system). These are the crucial assumptions students should mention. In addition, it is implicitly assumed that agents learn about government spending shocks only when they show up in the data. Students are not expected to discuss this assumption here, as it is the topic of Question 7. Concerning the ordering of the remaining variables, some different choices can be defended. For example, it is not obvious whether output or consumption should rank first, or how mortgage debt should place. It seems natural, however, to rank the house price last, as it is (potentially) a fast-moving variable. Students are expected to see this. The results below are based on the following ordering: $[G_t, Y_t, C_t, T_t, B_t, Q_t]$.

Question 3 The VAR model is only partially identified: While a government spending shock has been identified, we have not made assumptions about how to identify shocks to the other variables in the system. It is of course possible to give shocks to other variables using the recursive ranking, but we have not discussed whether the recursiveness assumption is reasonable to identify these shocks. A tax shock is a good example of this: As mentioned above, tax revenues can be expected to respond instantly and automatically to changes in GDP (and consumption, for that matter). The recursiveness assumption is therefore not applicable or appropriate for tax shocks, the effects of which would be heavily biased if we were to impose the unrealistic assumption that all within-period effects are zero.

Question 4 The impulse-responses of all six variables, including the bootstrapped 68% confidence bands and the median of the bootstrapped impulse-responses, are plotted in Figure 1. In Figure 1, the responses of all variables have been normalized in order to display a unit shock to G_t, and the response of output has further been normalized by the ratio of government spending to output, so as to directly observe the fiscal



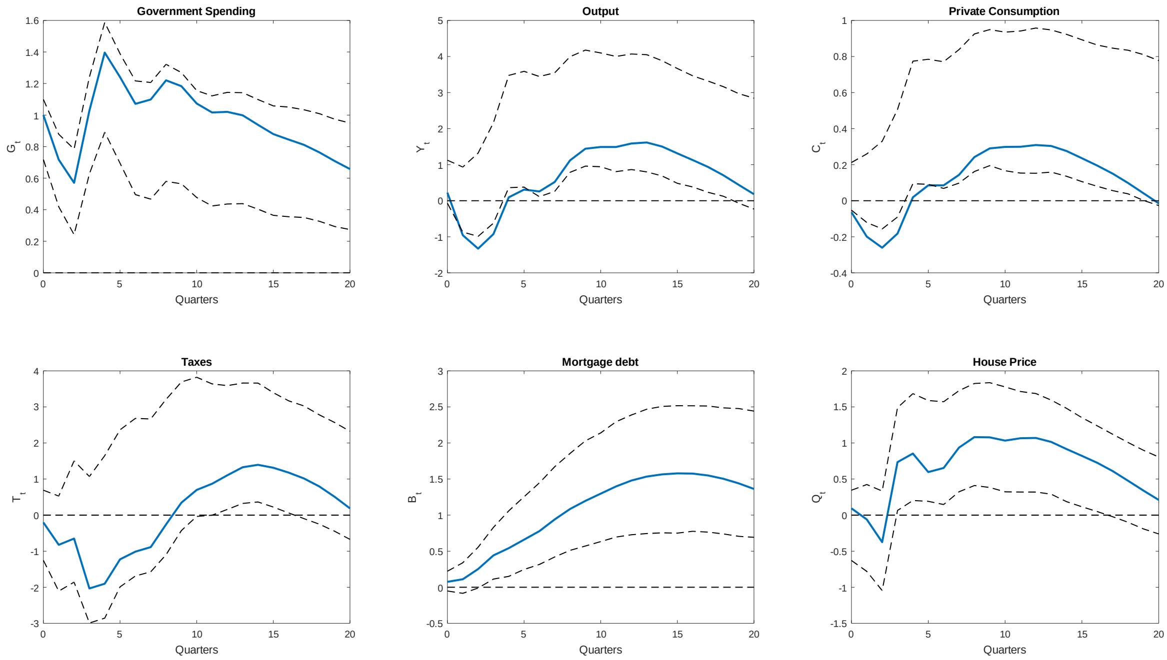
multiplier. The output multiplier is slightly below 1 according to the estimated IRF, both on impact and at the peak, which occurs after around 3 years. According to the bootstrapped median, the multiplier peaks slightly above 1. The responses of output and consumption are in line with those in the empirical literature studied in class, in particular Blanchard and Perotti (2002): Output increases on impact and peaks even later, while private consumption increases, and significantly so for some quarters 2-4 years after the shock. The net tax revenue declines, indicating that the increase in G_t must be debt-financed. Concerning the last two variables, a significant increase in both can be appreciated: real mortgage debt increases significantly and very persistently, while the house price increases as well, though in a less persistent manner. These responses will be discussed below. Finally, the median bootstrapped responses are rather close to the estimated IRFs in all cases, with the exception of government spending itself. For all other variables, the estimated IRF stays well within the bootstrapped confidence bands in all periods. This is a sign of a well-specified model.

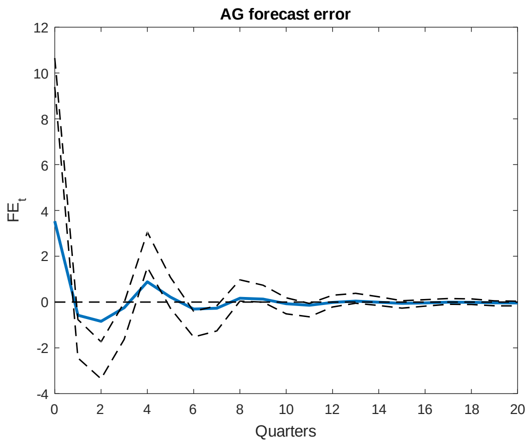
Question 5 If there is evidence of a positive within-quarter effect on government spending of an increase in output, this would in principle preclude the use of a Choleski decomposition to identify government spending shocks. This case would be similar to a tax shock in Question 3 and the arguments presented there. Instead, we could resort to the identification scheme of Blanchard and Perotti (2002), which allows for such contemporaneous effects (and only collapses to a Choleski decomposition if all such effects are zero). In their setup, this would amount to the case of $b_1 \neq 0$. To proceed, we would then require outside information about the exact size of this elasticity. If indeed $b_1 > 0$ as assumed in this question, our Choleski-based estimate of the fiscal multiplier from the previous question would be seriously upward biased. This point is made, e.g., by Caldara and Kamps (2017). Intuitively, this can be explained as follows: If $b_1 > 0$, the increase in output resulting from a positive government spending shock will in itself lead to an increase in government spending. As a result, the total increase in government spending will be larger than in the absence of this automatic effect, while the increase in output is the same, ignoring a small second-round effect. In other words, the denominator in $\frac{\partial Y_t}{\partial G_t}$ is raised, which reduces the fraction itself, and thus the fiscal multiplier on output.

Question 6 The increases in the real house price and real mortgage debt was noted above. The latter suggests that the increase in private consumption is to some extent debt-financed. More generally, the responses of house prices and mortgage debt are well in line with Keynesian views about the effects of fiscal policy: An increase in government spending leads to an increase in economic activity and output by stimulating aggregate demand. Apparently, this includes the demand for housing, the supply of which is fixed in the short run, leading to an increase in housing prices. Mortgage debt also increases, either because housing purchases are typically debt-financed, and/or because the increase in housing prices allows some households to extract housing equity to be used for consumption (smoothing).

Question 7 We now add FE_t to the set of variables in the VAR model. Importantly, this variable should be ordered first in the Choleski ordering, so as to be consistent with the interpretation of it as a forecast error, cf. Auerbach and Gorodnichenko (2012). The ordering of the remaining variables is as above. To generate an unanticipated shock to government spending, students then need to give a shock to FE_t itself. The responses of the remaining variables are illustrated in Figure 2 (note that students are not asked to plot the median of the bootstrapped IRFs in this case). As the figure shows, the previous results are generally confirmed using this approach, at least from a qualitative viewpoint. First, observe that government spending increases also in this case, confirming that we have in fact identified a shock to *unforecasted* government spending followed by an *actual* increase in this variable. The persistence of the shock is comparable to that in Figure 1. Output again displays a significant increase, though this time only after an initial (but insignificant) drop. A similar pattern can be observed for private consumption. The output multiplier again peaks around 1, so the previous results are confirmed also quantitatively. Furthermore, we observe also in this case increases in mortgage debt and the house price, with the former again more persistent than the latter. Finally, while the pattern of tax revenues is rather similar to above, the initial drop is now no longer statistically significant. Students can note that unlike the previous SVAR model, we now see some variables going outside their confidence bands in a few periods (this happens for output and consumption). For completeness, the response of FE_t itself is shown in Figure 3. Students are not expected to comment on this.

Question 8 The correlation coefficient between the two residuals is 0.46 (this can be obtained from the estimated variance-covariance matrix or by simply computing the correla-


Government Spending
Output
Private Consumption
Taxes
Mortgage debt
House Price
Quarters


AG forecast error
FE
Quarters

tion coefficient between the two residual vectors). This is notable - for comparison, the correlations between the FE_t-residuals and the residuals from the remaining equations are all below 0.05 in absolute terms, i.e., practically zero. It is also worth recalling that since FE_t is ranked first, the residuals from this equation by definition equal the structural shocks, and thus the shocks used to identify government spending shocks. This seems to imply that these shocks are not really orthogonal, and thus questions the success of the identification strategy. If the correlation between the two residual series had been 1, we would be giving the exact same shocks as in the original VAR, and the results would have been identical. Had the correlation been 0, the shocks would be truly orthogonal (as they are for the other variables). A correlation of 0.46 indicates that we are dealing with a middle ground between these two extremes.

Part 2

Question 1 To find the budget constraint of the government, consolidate the budget constraints of the household and that of the entrepreneur. Thus, impose the aggregate resource constraints:

$$\begin{aligned} c_t^{'} + q_t\left(h_t^{'} - h_{t-1}^{'}\right) + \frac{R_{t-1}b_{t-1}^{'}}{\pi_t} + T_t^{'} &= b_t^{'} + w_t^{'}L_t^{'} + F_t \\ c_t + q_t\left(h_t - h_{t-1}\right) + \frac{R_t b_{t-1}}{\pi_t} + w_t^{'}L_t &= \frac{Y_t}{X_t} + b_t \end{aligned}$$

Sum up the two constraints on each side:

$$\begin{aligned} & c_t^{'} + \underbrace{q_t\left[\underbrace{h_t^{'} + h_t}_{=H} - \underbrace{\left(h_{t-1}^{'} + h_{t-1}\right)}_{=H}\right]}_{=0} + \frac{R_{t-1}}{\pi_t}\underbrace{\left(b_{t-1}^{'} + b_{t-1}\right)}_{=0} + c_t + w_t^{'}L_t + T_t^{'} \\ = & \underbrace{b_t^{'} + b_t}_{=0} + w_t^{'}\underbrace{L_t^{'}}_{=L_t} + \underbrace{F_t}_{=\left(1-\frac{1}{X_t}\right)Y_t} + \frac{Y_t}{X_t} \end{aligned}$$

So that

$$T_t^{'} = \underbrace{Y_t - c_t^{'} - c_t}_{=g_t}.$$

Question 2 Assuming zero inflation (so that $R = 1/\beta$), the steady state will be described by

$$h = \frac{H}{1 + \frac{j}{1-\beta}\frac{\left(X - v + \frac{\gamma v(1-\beta)m}{1-\gamma_e}\right) - \zeta}{\frac{\gamma v}{1-\gamma_e}}}$$

$$\frac{qh}{Y} = \frac{\gamma v}{1-\gamma_e}\frac{1}{X}$$
$$\frac{b}{Y} = \frac{\beta m \gamma v}{1-\gamma_e}\frac{1}{X}$$
$$\frac{c}{Y} = \frac{v}{X}\frac{(1-\gamma)(1-\beta m)}{(1-\gamma_e)}$$
$$\frac{c^{'}}{Y} = \frac{1}{X}\left(X - v + \frac{\gamma v (1-\beta) m}{1-\gamma_e}\right) - \zeta$$

where $\gamma_e \equiv m\beta + (1-m)\gamma$. Otherwise, the log-linear system reads as:

$$\begin{aligned}
\hat{Y}_t &= (c/Y)\,\hat{c}_t + \left(c^{'}/Y\right)\hat{c}^{'}_t + \zeta \hat{g}_t \\
\hat{c}^{'}_t &= E_t \hat{c}^{'}_{t+1} - \widehat{rr}_t \\
c\hat{c}_t &= b\hat{b}_t + Rb\left(\hat{\pi}_t - \hat{R}_{t-1} - \hat{b}_{t-1}\right) + (vY/X)\left(\hat{Y}_t - \hat{X}_t\right) - qh\Delta\hat{h}_t \\
\hat{q}_t &= \gamma_e E_t \hat{q}_{t+1} + (1-\gamma_e)\left(E_t \hat{Y}_{t+1} - \hat{h}_t - E_t \hat{X}_{t+1}\right) - m\beta\widehat{rr}_t - (1-m\beta)\,E_t \Delta \hat{c}_{t+1} \\
\hat{q}_t &= \beta E_t \hat{q}_{t+1} + \iota \hat{h}_t + \hat{c}^{'}_t - \beta E_t \hat{c}^{'}_{t+1} \\
\hat{b}_t &= E_t \hat{q}_{t+1} + \hat{h}_t - \widehat{rr}_t \\
\hat{Y}_t &= \frac{\eta v}{\eta - (1-v)}\hat{h}_{t-1} - \frac{1-v}{\eta-(1-v)}\left(\hat{X}_t + \hat{c}^{'}_t\right) \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t \\
\hat{R}_t &= r_R \hat{R}_{t-1} + (1-r_R)\left((1+r_\pi)\,\hat{\pi}_{t-1} + r_Y \hat{Y}_{t-1}\right) + \hat{e}_{R,t} \\
\hat{g}_t &= \rho^g \hat{g}_{t-1} + u^g_t \\
\hat{g}_t &= \hat{T}^{'}_t
\end{aligned}$$

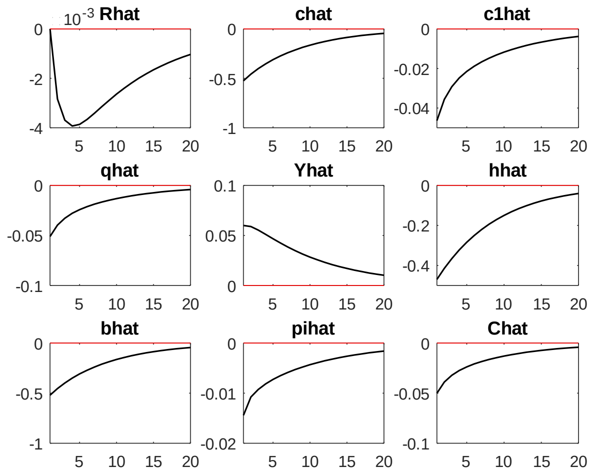
where $\iota \equiv (1-\beta)\,h/h'$, $\kappa = (1-\theta)(1-\beta\theta)/\theta$, $\widehat{rr}_t = \hat{R}_t - E_t \hat{\pi}_{t+1}$.

$$\frac{(1-\beta)}{(1-\frac{h}{H})}\frac{h}{H}$$

Question 3 Figure 4 shows the impulse-responses of the requested variables. Here the student should recall the crowding out effect on private consumption: Higher government spending means higher taxes, reducing the lifetime income of households. The interpretation of other variables is rather standard.

Question 4 As for the perfect comovement between $\hat{q}_t$ and $\hat{c}^{'}_t$, it is useful to consider households' Euler equation for land (eq. (4) in Iacoviello, 2005), which is solved forward to yield:

$$\lambda_t q_t = \mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i j_{t+i} \left(h^{'}_{t+i}\right)^{-1} \right\} \equiv \Upsilon_t. \tag{1}$$



where λ_t denotes the marginal utility of nondurable consumption. Now consider the case where there are no land demand shocks; e.g., $j_t = j$, $\forall\ t$. Since land does not depreciate, $h_t^{'}$ is effectively an "idealized durable" in the sense of Barsky *et al.* (2007). This means that the intertemporal elasticity of substitution in land demand is close to infinity. Any short-term movements in $h_t^{'}$ affect the right-hand side of (1) relatively little, as β is close to one. Hence, we can make the approximation:

$$\lambda_t q_t = \Upsilon_t \approx \Upsilon. \tag{2}$$

This condition shows that movements in housing prices (in absence of land-demand shocks) mirror movements in λ_t, i.e., households' marginal utility of consumption. Hence:

$$q_t \approx c_t^{'}\Upsilon. \tag{3}$$

Question 5 Here students should note that the DSGE model is unable to reproduce some key empirical findings from the SVAR model. While both models predict a positive comovement between aggregate consumption, mortgage debt and housing prices conditional on a fiscal shock, the direction of this movement in the DSGE model is at odds with the empirical evidence from the SVAR. While the SVAR-results are generally in line with a Keynesian interpretation of the effects of fiscal policy, as discussed in Question 6 in part 1, the DSGE model gives quite different predictions as far as concerns these variables. Students should point out that this mismatch is widely recognized as far as concerns consumption, as also studied during the course. The responses of house prices and mortgage debt to fiscal policy have received much less attention, and while there are recent papers looking at this, students are not expected to point to these.

Question 6 Figure 5 plots the response of housing prices to a government spending shock for the four different values of ρ^g requested. As can be seen, the drop in housing prices is larger when the fiscal shock is more persistent. The explanation is straightforward in light of the responses to the previous questions (in particular, Question 3). A

more persistent increase in government spending means that also taxes increase more persistently, thus exerting a larger negative effect on the permanent income of households. In other words, the negative wealth effect is larger. This leads to a larger drop in households' consumption, and thus also in housing prices, cf. the explanation provided in Question 4.

Question 7 In light of the answer to Question 4, while potentially being able to produce an increase in aggregate consumption, adding rule-of-thumb consumers to the present framework is not going to affect the tight connection between Ricardian households' consumption and the housing price (as expressed by (3)), so that the latter would decrease in any case.

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