ANSWERS

A1. Consider the following statement: "*In pre-industrial societies population density is a good proxy for the level of technological development*." Do you agree or disagree? Explain why.

Readings: In particular Ashraf and Galor (2008) and Weisdorf (2008).

Both agree and disagree can be defended. The nature of the argument is what matters for full credit.

In pre-industrial societies fertility may well have been determined along "Malthusian lines".

The traditional narrative is that increasing income, at the level of the household, instigates higher fertility. In a one good economy (Ashraf and Galor), a higher level of technological sophistication may have such an effect. As fertility rises population size increases, and thereby works to lower average productivity in the next generation due to diminishing returns to labor input. In this manner income is kept at a low level, in spite of step-wise increases in technology. Hence, in this setting long-run steady state income will be independent of the level of technology. But, population density in steady state will be higher in more technologically sophisticated societies. Against this background the statement can be viewed as sensible.

Things are a bit more complicated, however, if we allow for a *two-sector economy* (Weisdorf). In general fertility depends on the level of income as well as the relative price of children; typically associated with the price of nutrition in pre-industrial societies. In Ashraf and Galor the price of children is parametrically fixed. More generally, however, the relative price of children (relative price of food) is endogenous to the relative level of productivity across sectors.

Accordingly, innovations in manufacturing will (given competitive markets) work so as to increase the price of nutrition which may *lower* fertility. Hence, whereas a higher level of technology in agriculture solely will work so as to increase population density, better technologies in the non-agricultural sector may well do the opposite thus *increasing* living standards. The latter is true even with step-wise (once over) technological change.

With this in mind a high population density may either be taken to mean a high level of technological sophistication in agriculture, a low level of productivity in manufacturing, or a combination of the two. Either way, it is difficult to make the assertion that "population density is higher in more technologically advanced societies".

A2. Discuss the problems associated with estimating the *impact* of longevity on growth, and how these problems may potentially be over come.

Readings: The main case is Acemoglu and Johnson's article.

The methodological discussion required for this question is similar to the one laid out in *2009-I, Question A2*. Only "fertility" is to be replaced by longevity. Moreover, the explanation for the slopes needs to be adjusted. *A priori* one would expect that income raises longevity (better nutrition, for instance), and that longevity increases prosperity (by increasing planning horizons, for instance). Hence, both curves would be upward sloping on *a priori* grounds. But aside from that the identification problem, and potential remedy in the form of 2SLS estimation can be laid out in similar manner to what is described in the answers to 2009-I, Q A2.

In addition to these considerations the student should also talk about the empirical study by Acemoglu and Johnson, which develops an interesting identification strategy by invoking the international epidemiological transition (IET). IET was unleashed by the discovery of penicillin, in particular.

The basic ideas is that countries where mortality *ex ante* was higher, within diseases that *ex post* penicillin was curable, would experience the largest increases in life expectancy in the decades to follow. They show that the initial mortality rate within a nation, within diseases that are curable with penicillin, indeed is strongly correlated with subsequent changes in life expectancy. They also show, as a falsification test of their instrument, that the initial mortality rate (within …) do not correlate with changes in longevity prior to the discovery of penicillin. It also strengthens the case in favor of the exclusion restriction.

Using this identification strategy AJ estimate a large positive impact of increases in adult mortality rates on population growth and growth in GDP. But a negative impact on GDP per capita. Hence, in contrast to prevailing wisdom, their estimates suggest that rising longevity does not increase growth, at least over the time horizon examined (60 years).

THE ANALYTICAL QUESTIONS RELATE TO TORVIK (2001)

B1. Profits in the N-sector are given by $\Pi_N = pA_N f(\eta) - w\eta$. Note that Differentiating wrt η gives $pA_N f'(\eta) = w$. The FOC from the T-sector is analogous.

Since we assume free mobility across the two sectors. $pA_N f'(\eta) = w = A_T g'(1-\eta)$. Rearrangement immediately gives

$$p = \frac{A_T}{A_N}\frac{g'(1-\eta)}{f'(\eta)}$$

In a (p,η) diagram the above expression has a positive slope, and provides combinations of prices and employment shares that are consistent with equilibrium in the labor market.

The intuition for the slope is that as p goes up, the value of the marginal product of labor in the N sector rises, which, due to free mobility, works to pull workers into the N sector. The process continues until wages once again are equalized.

An increase in the relative level of productivity shifts the line upward. The intuition is simple: if the relative level of productivity

B2. The problem is to

$$\max U = \frac{\sigma}{\sigma-1}\left[c_T^{\frac{\sigma-1}{\sigma}} + c_N^{\frac{\sigma-1}{\sigma}}\right], s.t. \mathrm{Y} = \mathrm{pc_N} + \mathrm{c_T}$$

Straight forward computations leads to $c_N = \frac{Y}{\left(p^{\sigma-1}+1\right)p}$.

B3.

Market clearing in the N-market requires that demand equals supply

$$\frac{Y}{\left(p^{\sigma-1}+1\right)p} = A_N f(\eta)$$

The resource constraint of the economy is

$$Y = A_T R + X_T + pX_N$$

Inserting the production functions and inserting Y into the equilibrium condition yields the result

$$p = \left\{\frac{A_T}{A_N}\left[\frac{g(1-\eta)+R}{f(\eta)}\right]\right\}^{\frac{1}{\sigma}}$$

This equations provides combinations of (p,η) that are consistent with equilibrium in the N-market. The slope is negative. The intuition is that as η goes up excess supply arises in the N-market. For market clearing, prices will have to decline.

An increase in relative productivity shifts this curve upwards. The basic intuition is that higher productivity is consistent with higher demand.

B4.

(i) In the intersection between the two curves we find the general equilibrium. More specifically, we find the equilibrium in the labor market and the market for N goods. By Walsras' law, equilibrium prevail in the T-market as well.

(ii) The "Natural Resource Curse" referrers to the empirical regularity that resource rich economies tend to grow slowly. Counter examples abound, suggesting a potentially complex association. However, a number of theoretical explanations have been forwarded in the literature.

The present model captures one example which also is referred to as "Dutch disease" or "deindustrialization". As is clear, if R increases the N-demand curve shifts up, implying a higher relative price on N-goods, and fewer people in the T-sector. If the T sector is more productive, output per worker in the aggregate could fall. In this way the model may capture a resource curse scenario.

B5. Observe that the effect is directly comparable The impact of changes in relative A on the labor market curve is

$$\frac{\partial p}{\partial \lambda} = \frac{p}{\lambda}, \lambda \equiv A_T / A_N$$

Whereas the impact on the N-market clearing curve is

$$\frac{\partial p}{\partial \lambda} = \frac{1}{\sigma} \frac{p}{\lambda}, \lambda \equiv A_T / A_N$$

Hence, if $\sigma = 1$ the two curves shift upwards to exactly the same extent, nullifying a net impact on η. If $\sigma < 1$ the net impact of the employment share in the N sector is *positive*, and vice-versa if $\sigma > 1$.

When the relative level of productivity in the T sector expands the consumption possibility set increases in the economy. However, at a more subtle level, N-goods become relatively more costly. Hence if the two goods are "easily substitutable" ($\sigma > 1$) the outward shift is not quite as large as when the two goods are hard to substitute ($\sigma < 1$). In the latter case the outward shift of the N-good demand curve is "large" leading to, on net., an increase in η.

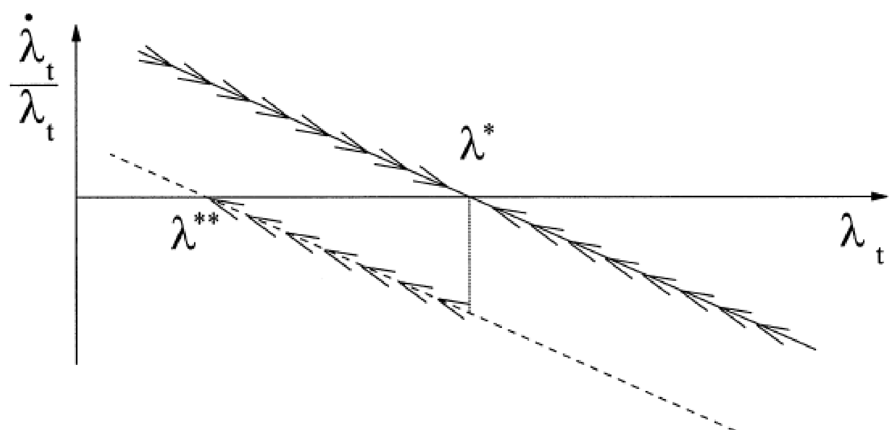
B6. The two differential equations provide the law of motion for sector specific knowledge. The idea is that, due to Learning-by-doing, more employed leads to more ideas. Observe also, that the model admits spillovers across sectors. The strength of the spillover effects are captured by δ_j, j=T,N.

B7. Use the definition $\lambda \equiv A_T / A_N$, and differentiate wrt time. This gives $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{A}_T}{A_T} - \frac{\dot{A}_N}{A_N}$.

Insert the two laws of motion, and rearrange so as to obtain

$$\dot{\lambda} / \lambda = v\left(1-\delta_T\right) - \eta\left(\lambda, R\right)\left[u\left(1-\delta_N\right) + v\left(1-\delta_T\right)\right] \equiv \Pi\left(\lambda\right)$$

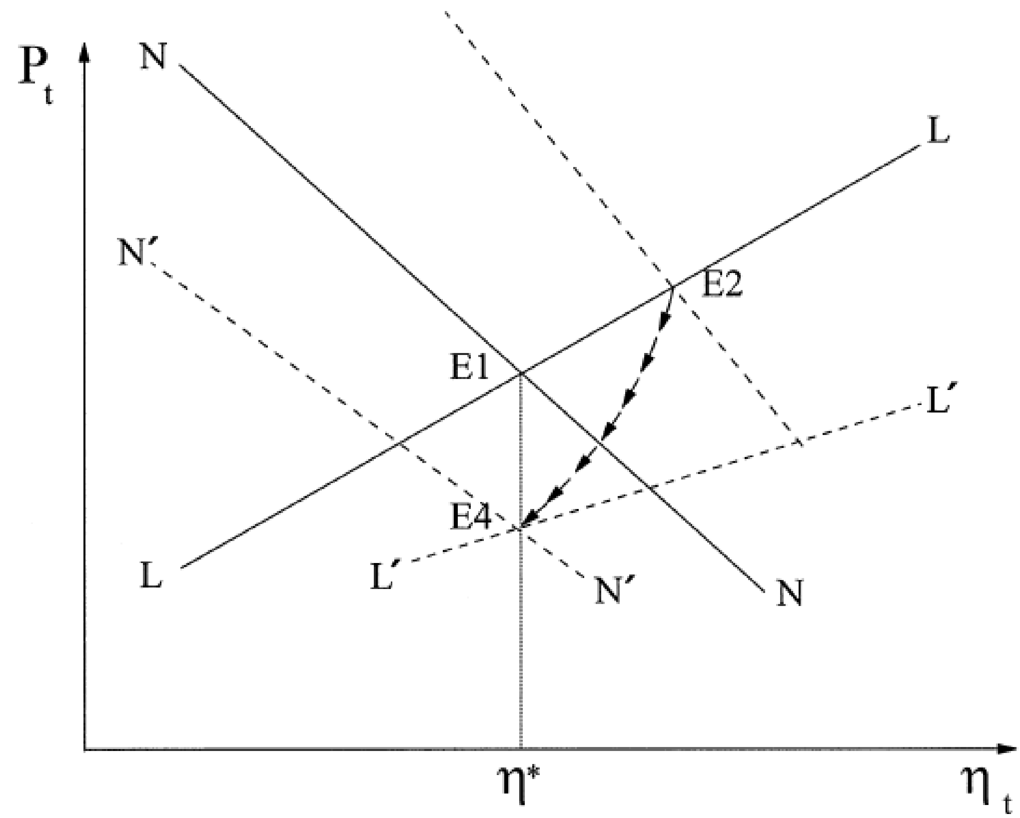
B8. If s<1 we have that $\Pi'\left(\lambda\right) < 0$, as $\eta'_{\lambda}\left(\lambda, R\right) > 0$ (cf discussion in B5.) The resulting Phase diagram is illustrated below; the model is stable and the steady state λ is unique.



Source: Torvik, p. 295.

The steady state employment share is obtained by examining the differential equation for $\dot{\lambda} / \lambda = 0 \Rightarrow \eta^* = \frac{v\left(1-\delta_T\right)}{\left[u\left(1-\delta_N\right) + v\left(1-\delta_T\right)\right]}$. The key novel finding is that changes in R does not affect the allocation of labor in the long-run; only productivity parameters influences η.

B9.



The initial move, from a change in R is to E2 in the figure. With a η above steady state, however, relative productivity in the T sector declines; more people in the N-sector raises productivity growth. The process continues until η* is once again attained. Interestingly, the windfall gain leads to lower relative prices in the long-run; the short run price increase is only temporary.

B10.

From the resource constraint we have

$\frac{Y}{A_T} = R + X_T + \frac{pX_N}{A_T}$. Since, in the long-run, η and λ (and R) are constants, we can conclude that output (per capita) grows at the same rate as A_j , j=T,N, in the steady state.

$$\left(\dot{A}_T / A_T\right)^* = \frac{v\left(1-\delta_T\right)\left(\delta_N u - v\right)u}{\left[u\left(1-\delta_N\right)+v\left(1-\delta_T\right)\right]} + v.$$

In the long-run the growth rate is clearly unaffected by R. Hence the model does not imply a Natural resource curse in this case. But there could be a level effect. In general, however, this depends on whether the spillover effect is sufficiently large. If so, a resource inflow will always imply a higher level of income. Contingent on small spillovers the economy may suffer a lower level of income, if it is hit by a windfall gain.