Written Exam for the B.Sc. or M.Sc. in Economics winter 2011-2012

Game Theory

Final Exam/ Elective Course/ Master's Course

19. January 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Problem 1

In a TU game *(N,v)*, one can define the (Harsanyi) dividends $\Delta_v(S)$, for *S* any coalition, inductively (on the size of coalitions) by $\Delta_v(\{i\}) = v(\{i\})$ for each player *i* and $\Delta_v(S) = v(S) - \sum_{T \subset S, T \neq S} \Delta_v(T)$ for coalitions with more than one player.

A *selector* is a function α, which to each coalition *S* gives a player $\alpha(S)$ from the coalition *S*. The *selector value* belonging to α is the vector $m^\alpha(v)$ with *i*th coordinate

$$m_i^\alpha(v) = \sum_{S: i = \alpha(S)} \Delta_v(S).$$

Show that $m^\alpha(v)$ is an pre-imputation in the game *(N,v)*, i.e. a Pareto optimal payoff vector.

Give an example of a game (with at least 4 essential players) where $m^\alpha(v)$ does not belong to the core for some selector α.

Problem 2

A Nash equilibrium (possibly in mixed strategies) is *essential* if for all games with the same number of pure strategies, if all payoffs are close enough, then there is a Nash equilibrium with optimal strategies close to the original ones.

Give an example of a game where each player has at least three pure strategies, and which has no essential Nash equilibria.

Problem 3

A group of individuals have access to a technology with constant returns to scale, producing a single commodity using several input commodities, which have to be inserted in fixed proportions. Each individual has a certain amout of each of the input commodities.

The individuals contemplate going together and then dividing the income from the sale of the output. Explain that this gives rise to a cooperative TU game (a *production game),* which is superadditive.

Show that the game is balanced.

A game is *totally balanced* if each subgame *(S,v)* (with player set *S* and the same value *v(T)* of the characteristic function for subsets *T* of *S*) is balanced. Explain that a production game is totally balanced.