Written Exam for M.Sc. in Economics

Investment Theory

15. August 2011

Master Course

3 hours written exam. Closed Books. All questions should be clearly and briefly answered. Calculations and figures should be clear and understandable. Calculations and figures should be explained.

Exercise 1.

Consider a firm in a market. The profit of the firm is $P - C$ where

$$dP = \alpha P dt + \sigma P dz$$

and $C > 0$. The firm has the option to expand its capacity. The cost of the expansion is $J > 0$ and the profit of the firm is $(1 + K)(P - C)$, where $K > 0$, after the expansion.

The interest rate is $r > 0$ and there is a portfolio with no dividend and price Q where

$$dQ = (\alpha + \delta)Q dt + \sigma Q dz$$

with $\delta > 0$.

Let $V(P)$ be the value of the firm before the expansion and let $W(p)$ be the value of the firm after the expansion.

(a) Interpret the expressions for the profit of the firm before and after the expansion. Give an example of an investment project that fits the above project.

(b) State a possible cutoff strategy for option to expand. Use the strategy to relate $V(P)$ and $W(P)$ and discuss their properties.

(c) Find differential equations in $V(P)$ and $W(P)$ that can be used to determine $V(P)$ and $W(P)$.

(d) Find $W(P)$.

(e) Find $V(P)$ up to undetermined constants.

(f) Interpret your expressions for $V(P)$ and $W(P)$.

(g) Find the optimal strategy for the option to expand.

(h) Find the effect of a small change in the interest rate on the optimal strategy.