Macro III - exam solutions (August 13, 2018)

1 False. The real exchange rate is the price of a basket of goods in one country relative to the price of the *same* basket in another country. Thus, since the productivities of tradables and non-tradables respectively grow at the same rate across countries, real exchange rates are constant. Thus, the statement is false.

2 False. In the Calvo model, only a fraction of firms are assumed to be able to change their prices in each period (or instant, in the original continuous-time version). This is the origin of price stickiness in this model.

3 False or uncertain. An aging population would require either that benefits be cut while contributions remain constant, or benefits remain constant and contributions increase, or a mix of benefit cuts and contribution increases. It is through the political process that society chooses the way in which a social security system adapts to aging. And if benefits are not to be reduced, it is not necessary to give workers incentives to increase private savings.

4 a) The Lagrangian is given by (note that the problem can be solved using the intertemporal budget constraint)

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \frac{x_t^{1-\epsilon}}{1-\epsilon} + \lambda_t \left(w_t(1-x_t) + R_t k_t - c_t - k_{t+1} \right) \right]$$

The first order conditions are given by

$$\begin{aligned}
\frac{d\mathcal{L}}{dc_t} &= 0 \longrightarrow \frac{1}{c_t} - \lambda_t = 0, \\
\frac{d\mathcal{L}}{dx_t} &= 0 \longrightarrow x_t^{-\epsilon} - \lambda_t w_t = 0, \\
\frac{d\mathcal{L}}{dk_{t+1}} &= 0 \longrightarrow -\lambda_t + \beta R_{t+1} \lambda_{t+1} = 0
\end{aligned}$$

The Euler equation is given by,

$$\frac{1}{c_t} = \beta R_{t+1} \frac{1}{c_{t+1}}.$$

The interpretation is that the household makes consumption saving choices such that the marginal rate of substitution between current and future consumption equals the marginal rate of transformation, R_{t+1}. An Euler equation can be written for intertemporal leisure choices, but it is not required that the students derive it.

In steady state, $c_t = c^*$, $x_t = x^*$ and $k_{t+1} = k^*$. From the Euler equation we find that steady state capital is determined, as usual in this setting, by (here we use that the rental rate for capital, r, is equal to the marginal product of capital, $\alpha\kappa^{\alpha-1}$)

$$R^* = 1 + r^* = 1 + \alpha \left(\frac{k^*}{1-x^*}\right)^{\alpha-1} = 1 + \alpha{\kappa^*}^{\alpha-1} = \frac{1}{\beta}. \tag{1}$$

where $\kappa = \frac{K}{L}$ is the aggregate (or average) capital per unit of labor. Note that $\kappa \equiv \frac{k}{1-x}$, we make this distinction since some results are neater with κ instead of k, but these are equivalent measures in the steady state.

Given κ^* steady state wages are given by $w^* = (1-\alpha){\kappa^*}^{\alpha}$. Steady state consumption and leisure are characterized by the following system of two equations in two unknowns

$$\begin{aligned} c^* &= w^*(1-x^*) + \alpha{\kappa^*}^{\alpha-1}k^* = {\kappa^*}^{\alpha}(1-x^*) \qquad (2) \\ c^* &= w^*{x^*}^{\epsilon} = (1-\alpha){\kappa^*}^{\alpha}{x^*}^{\epsilon} \qquad (3) \end{aligned}$$

b) An increase in ϵ corresponds to an increase in the concavity of preferences for leisure. Capital per unit of labor is unaffected since the condition that determines κ^* only depends on β. Combining the two equations that characterize c^* and x^* we get that

$$\frac{{x^*}^{\epsilon}}{1-x^*} = \frac{1}{1-\alpha}. \tag{4}$$

Since both x^* and ϵ are between zero and one, an increase in ϵ reduces the left hand side of this equation for the initial value of x^*. Thus, x^* has to increase. This implies that c^* is lower (from (2)). Finally, note that because κ^* is capital per unit of labor, and labor supply is reduced, capital per worker, k^*, is lower.

c) The presence of capital income taxes reduces the return perceived by households from saving. Denoting τ the (presumed constant) capital income tax, equation (1) is now given by

$$R^* = 1 + r^*(1-\tau) = 1 + \alpha \left(\frac{k^*}{1-x^*}\right)^{\alpha-1}(1-\tau) = 1 + \alpha{\kappa^*}^{\alpha-1}(1-\tau) = \frac{1}{\beta}.$$

Thus, unambiguously κ^* is depressed.

There is no change in household income, since they receive the receipts of taxation as a lump sum transfer. Thus, equation (2) derived from the resource constraint does not change. And the choice between consumption and leisure is also unaffected as capital taxation only affects intertemporal choices. Thus, equation (3) does not change. From the analysis done in b), (see (4)), we know that x^* is unaffected by capital income taxation. Thus, k^* is lower (as $\kappa = \frac{k}{1-x}$), and c^* is also lower as can be seen from (2),

5 a) Treating the economy as closed the problem is identical to the basic Diamond economy where the young are endowed with $1 + t_1$ units of time instead of 1. Individual optimal lifetime choices are characterized by (this is straightforward from the maximization problem either by setting a Lagrangian, or replacing in utility U_t the expression for consumptions from budget constraints: $c_{1t} = w_t(1 + t_1) - s_t$, and $c_{2t+1} = s_t r_{t+1}$)

$$\begin{aligned} c_{1t} &= \frac{1}{1+\beta} w_t(1+t_1), \\ c_{2t+1} &= \frac{\beta}{1+\beta} w_t(1+t_1) r_{t+1}. \end{aligned}$$

Capital accumulation comes from young's saving. Given that population grows at rate $1 + n$, $k_{t+1} = \frac{s_t}{1+n}$, where k is capital per worker (note that students might work with capital per units of labor, $\frac{k}{1+t_1}$, this is equally valid, but most results are easier with k), and s is saving per worker (equal to labor income minus first period consumption, i.e. $\frac{\beta}{1+\beta} w_t(1 + t_1)$). Thus,

$$k_{t+1} = \frac{\beta}{1+\beta} w_t(1+t_1) = \frac{\beta}{1+\beta}(1-\alpha)A\left(\frac{k_t}{1+t_1}\right)^{\alpha}(1+t_1),$$

where wages are given by the marginal product of labor, $(1-\alpha)A\left(\frac{k_t}{1+t_1}\right)^{\alpha}$, and the relevant capital ratio is capital per units of labor. In steady state $k_t = k_{t+1} = k^*$. Thus,

$$k^* = \left[\frac{\beta}{1+\beta}(1-\alpha)A(1+t_1)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}.$$

b) Now the budget constraints are given by $c_{1t} = w_t - s_t$, and $c_{2t+1} = s_t r_{t+1}(1 + t_2)$. Given logarithmic preferences, workers will save same fraction $\frac{\beta}{1+\beta}$ of labor income.

Optimal consumptions are given by

$$
\begin{aligned}
c_{1t} &= \frac{1}{1+\beta}w_t, \\
c_{2t+1} &= \frac{\beta}{1+\beta}w_t(1+t_2)r_{t+1}.
\end{aligned}
$$

Note that while in a) the consumption of retirees (given wages) was higher by $1+t_1$, and now it is higher by $1+t_2$, these terms represent different mechanisms: The first was working through an increase in saving due to higher income, while the second is given by the higher effective return on saving.

Capital accumulation and steady state capital are given by,

$$
\begin{aligned}
k_{t+1} &= \frac{\beta}{1+\beta}w_t = \frac{\beta}{1+\beta}(1-\alpha)Ak_t^{\alpha} \\
k^* &= \left[\frac{\beta}{1+\beta}(1-\alpha)A\right]^{\frac{1}{1-\alpha}}.
\end{aligned}
$$

Thus, clearly steady state capital is lower in this case. The reason is that, as mentioned above, when benefits accrue to retirees, this has no effect on saving behavior. When workers benefit, they have a higher labor income, and thus higher saving which on the aggregate will result in higher capital (note that although in this case the interest rate will be lower, saving is unaffected by this under logarithmic preferences).

c) From the previous analysis, when trade benefitted retirees there is no effect on capital accumulation (either in the short or long run). If trade benefited workers, then a trade war will result in lower capital accumulation, with a lower steady state capital stock.

If trade benefited workers, these are clearly worse off by a trade war as they will have lower labor income and lower lifetime utility. Note that wages go up a bit since capital per units of labor goes up. But this cannot compensate the decrease in time endowment. Retirees are also worse off since interest rates are lower given that capital per units of labor is higher.

If trade benefited retirees, these are clearly worse off by a trade war as they a lower return on their capital income. Workers are also worse off since they will have the same labor income, and a lower return from their saving. Thus, in this model a trade war is unambiguously a bad idea (keep in mind the model is too simple to understand current affairs).