Written Exam for the B.Sc. in Economics Summer 2010-RE

Microeconomics A

Final Exam

18 August 2010

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Question 1

a) Consider the statement: *if a quantity tax is put on a commodity produced by a profit maximizing firm then the firm's profit per unit is reduced by the level of the tax* Comment this statement.

b) Assume that a firm has a supply curve that can be described by $S(p) = \gamma + \delta p$ and the market demand this firm is facing is $D(p) = \alpha - \beta p$. Assume that the firm is a *price taker* (i.e. its decisions do not influence the price). Find the market equilibrium and discuss how the equilibrium is influenced by the parameters β and δ in the model.

c) Assume that a quantity tax *t* is put on the commodity. Analyse how this influence the equilibrium and relate the answer to the discussion of the parameters in question b) and your answer in question a).

d) If the tax revenue is paid back (lump sum subsidies) to the producers and consumers relative to their tax burden, are the consumers and producers then affected by the tax – discuss and illustrate.

Answers

a) This is not always the case. It depends on the elasticities of supply and demand curves. If perfect elastic supply then the burden is put on consumers. If inelastic supply then all is put on the producer.

b) $p^* = (\alpha - \gamma)/(\beta + \delta)$, $q^* = (\alpha \cdot \delta + \beta \cdot \gamma)/(\beta + \delta)$. *The higher β the higher the sensitivity of consumers; this reduces equilibrium price and quantity. Higher δ increases supply elasticity and reduces price and increases quantity in equilibrium*

c) The simple linear analysis illustrating the answer in a). $p_S^* = (\alpha - \gamma - \beta t)/(\beta + \delta)$, $p_D^* = (\alpha - \gamma + \delta t)/(\beta + \delta)$, $q^* = (\alpha \cdot \delta + \beta \cdot \gamma - \beta\delta t)/(\beta + \delta)$. *The tax reduce price for producers and when more sensitive demand this increase the reduction. Price for consumers increases and more so when supply is more elastic. The quantity is reduced and by more so when either supply or demand is elastic.*

d) Yes, the tax revenue is on the reduced quantity. There is a dead weight loss to consider. This should be shown in a diagram showing the impact on the producers' surplus and the consumers' surplus.

Question 2

A consumer has monotone increasing, continuous and strictly convex preferences. Consider the following statement:

If the consumer is a net supplier in the market of a good and the price increases then this consumer is better off if he remains a seller.

If you think this statement is true then prove it. If you think it is false then argue why or give a counterexample.

Answer:

This can be shown using revealed preferences. It is easily seen in a diagram, which is also enough if the diagram is accompanied by an explanation.

Question 3

Consider an economy of two commodities with private property rights. Show that if a consumer with monotone increasing, continuous and strictly convex preferences is a net buyer in one market then he must be a net supplier in the other market.

Answer

Use Walras' law: pz(p)=0. If $z_1(p)>0$ then $p_1 z_1(p)>0$, but then $p_2 z_2(p)$ must be negative for Walras'law to be satisfied. Hence $z_2<0$.

Question 4

Describe using the Slutsky equation how we can conclude whether demand increases or decreases following a price increase in an economy with exogenous income. You should especially account for the different types of goods that we can come across. As part of your answer you should also explain/prove why the substitution effect is always negative.

Answer

The proof of the negative substitution effect can be proved using revealed preferences. The bundle chosen before a price change x_0 is revealed preferred to any bundle in the budget set. After the price change we know the consumer will choose either x_0 or a bundle that is not in the original budget set – remember that the consumer is compensated in income such that the "new" budget set is intersecting the original budget set in x_0 (Slutsky compensation). Hence, demand must be smaller. A diagram should assist this proof.
When we know that the substitution effect is negative, then normal goods will always be reduced since the income effect is also negative. Inferior goods have positive income effect and if the effect is strong enough then demand may increase (Giffen good).

Question 5

James can consume two goods: bread (commodity 1) and butter (commodity 2). James' preferences can be represented by the utility function $u(x_1, x_2) = x_1 x_2 + x_1$.

a) Solve James problem assuming that he has an exogenous income *m* and when the prices are exogenous.
b) If the price on bread changes, what is then the Hicks compensated demand function?
c) Consider a specific case, where *m=100* and prices $(p_1, p_2)=(20,20)$. Since there is a growing nutrition problem in the country, the government has decided to tax butter such that the price increases to 25. How does that influence James' demand? Split the change into income and substitution effects (use the Hicks compensation).
d) How is James welfare measured as the *Compensating variation* influenced?

Answers

a) *The demand is split into two parts – an inner solution where $p_2 \leq m$ and a boundary solution where $p_2>w$. The inner solution is* $(x_1, x_2) = \left(\frac{m+p_2}{2p_1}, \frac{m-p_2}{2p_2}\right)$ *and the boundary solution is* $(x_1, x_2) = \left(\frac{m}{p_1}, 0\right)$
b) *The Hicks compensation ensures that the same utility level can be maintained after the price change. Hence, with the new prices what is the demand that maintains the utility level at u^0?*

This can be found by solving the cost minimization problem min $p_1x_1+p_2x_2$ s.t. $u(x_1,x_2)=u^0$. Where p_1 is equal with the new price. The solution to this problem is

$h_1=\left(\frac{u^0p_2}{p_1}\right)^{½}$, $h_2=\left(\frac{u^0p_1}{p_2}\right)^{½}-1$ *when $p_2\leq u^0p_1$ and $h_2=u^0$, $h_1=0$ otherwise*

c) $(x_1^0,x_2^0)=(3,2)$ *and* $(x_1^1,x_2^1)=\left(\frac{25}{8},\frac{3}{2}\right)$, *$u^0=9$, so $h_1=(9*25/20)^{½}=(45/4)^{½}$ and $h_2=(9*20/25)^{½}-1=(36/5)^{½}-1$. Hence, the substitution effect is $(3-(45/4)^{½}$; $2-(36/5)^{½}+1)\approx$ (-0.35; 0,32) and the income effect is* $\left(\frac{25}{8}+0.35,\frac{3}{2}-0.32\right)$ *and the total effect is (1/8 ;-½)*

d) *We have that $e(p^1,u^0) = 2(u^0\,p_1^0\,p_2^1)^{½} - p_2^1$* and thus that *$CV=m-(2(u^0\,p_1^0\,p_2^1)^{½} - p_2^1)$*

Question 6

Consider the Edgeworth economy with two goods and two consumers. The consumption possibilities are in R^2_{++}. The two consumers (Allen and Betty) have preferences that can be described by

$u_A(x_{1A},x_{2A})=\alpha_A\ln(x_{1A})+(1-\alpha_A)\ln(x_{2A})$ and

$u_B(x_{1B},x_{2B})=\alpha_B\ln(x_{1B})+(1-\alpha_B)\ln(x_{2B})$

where $\alpha_A=\alpha_B$. The total endowment in the economy is $\omega=(\omega_1, \omega_2)$, where both $\omega_2>0$ and $\omega_1>0$.

a) Find the Walras equilibrium in this economy.

There is also a firm in the economy that can transform good 1 to good 2 using the production function $f(q)=q$, where q is the total quantity of input of good 1. Assume that $\omega_A=(15,6)$ and that $\omega_B=(3,6)$. Also normalise the price of good 1 to 1. Allen's share of the firm is γ and Betty's share is $(1-\gamma)$.

b) Find the Walras equilibrium in the case where $\gamma=0.5$

c) Is it possible that Pareto optimal allocation, where Betty gets everything can be implemented as a market equilibrium – how?

Answers

a) *This is straight forward to find the equilibrium in the Edgeworth economy that is outlined. Since both have CD utility functions with the same elasticities we can state the demand functions as $x_{1A}=\alpha\, m/p$ etc. The equilibrium is found by equalizing the excess demand (net demand) of the two in one market. Then Walras law ensures that the other market is also cleared*

b) *Insert the specific values specified in the question. The solution price is determined from the supply side to be (1,1), this gives a zero profit and income levels of 21 and respectively. In equilibrium the demand are (10½,10½) and (4½,4½) respectively. The firm uses 4 units of input to give 3 units of output,*
c) *Yes, this is a typical setting for the second welfare theorem. We need to specify the ownership of endowments and the firm. In the case here there is no profit to consider.*

There i sprivate property in the economy such that Der er privat ejendomsret i økonomien, og ejendomsforholdene er givet ved, at Arno initialt ejer $k \cdot \omega$, mens Birger ejer $(1 - k) \cdot \omega$, hvor $0 < k < 1$.

5a) Find Walras-ligevægts-priserne for denne økonomi, idet du normaliserer priserne således, at prisen på vare 2 sættes til 1, så prissystemet har formen (p,1). Kommentér den betydning, som parameteren k har for ligevægtspriserne, især for $k \to 0$ og for $k \to 1$, og giv en intuitiv forklaring på dette.

5b) Antag at der i økonomien nu introduceres en virksomhed, der bruger frugt som input og har juice som output. Virksomhedens produktionsfunktion har formen $y = q$, hvor $q \geq 0$ er mængden af frugt, der bruges som input, og y er mængden af juice-output. Antag, at $k = ½$ samt at hver af de to forbrugere ejer halvdelen af virksomheden. Find Walras-ligevægtspriserne for økonomien med de to forbrugere samt virksomheden – og angiv produktionsomfanget i ligevægt.

Something with an Edgeworth/Koopmans economy
a) Find the equilibrium
b) Introduce production – find equilibrium
c) Point out a specific PO allocation and ask whether this is possible to implement as Walras eq. – they must find the prices and ownership share of the firm