Macro C - exam solutions (Jan 6, 2015)

General remarks

Please grade each item of each question between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

Question 1

Uncertain or false. An announced change in monetary policy does not have real effects in the economy if all prices are flexible (in this case only unannounced policy changes have real effects). But if there are some nominal rigidities, then it is possible to affect output by a pre announced change in monetary policy. In lectures we saw an example of this in the Fischer model of staggered price setting.

Question 2

Uncertain or false. The result (seen in lectures and in Persson and Tabellini) that the outcome under an independent central bank corresponds to a higher ex ante welfare than under a currency peg (or simple rule) for every possible social loss function was derived under the assumption that wage setting was decentralized. We saw, and it is in the textbook, that when there are strong unions that act strategically this may lead to choosing a less conservative central banker. Under certain parameters this could lead to an ex ante loss greater than under a currency peg.

Student also gets points if he/she notices that the standard analysis assumes a currency peg where inflation is always equal to the target level, while it might be the case that inflation is equal to the realized foreign inflation of the euro, the currency Denmark is pegged to (this case was discussed in the lectures). Since euro inflation is stochastic, if supply shocks between Denmark and the eurozone are positively correlated this reduces the ex ante loss of a currency peg and for some parameters it might be a preferable outcome than having an independent central bank.

Problem 1

a) In the budget constraint m appears multiplied by the nominal interest rate, $\pi + r$. Thus, this is the interest forgone by holding money instead of capital. It thus measures the implicit consumption of money services.

The wage and interest rate are determined by imposing equilibrium in factor markets where firms competitively demand labor and capital from households. Maximizing profits

$$\max_{K_t, L_t} \pi^F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t,$$

From FOC of firms' problem of maximizing profits we get

$$\begin{aligned}(1-\alpha)(K_t)^\alpha (L_t)^{-\alpha} &= (1-\alpha)k_t^\alpha = w_t \\ \alpha (K_t)^{\alpha-1}(L_t)^{1-\alpha} &= \alpha k_t^{\alpha-1} = r_t\end{aligned}$$

where we impose that in equilibrium all firms work with the same capital labor ratio, k, which must be equal to the ratio of aggregate capital to labor.

b) Control: c and m, state: a, costate, μ (I use current value Hamiltonian, H^c, obviously student gets full points if using present value Hamiltonian correctly). Hamiltonian is

$$H_t = e^{-(\rho-n)} H_t^c = e^{-(\rho-n)} \left[\ln c + \ln m - \gamma m + \mu_t \left((r_t - n)a_t + w_t + z_t - (c_t + (\pi_t + r_t)m_t)\right)\right]$$

FOC (with no time indexes):

$$\begin{aligned}\frac{1}{c} &= \mu \\ \frac{1}{m} - \gamma &= \mu(\pi + r) \\ \dot{\mu} - (\rho - n)\mu &= -(r-n)\mu \\ \lim_{t\to\infty} a_t \mu_t e^{-(\rho-n)} &= 0\end{aligned}$$

In equilibrium $z = \sigma m$. Steady state characterized by $\dot{a}_t = \dot{m}_t = \dot{\mu}_t = 0$. From FOC this gives $r = \alpha k^{\alpha-1} = \rho$ and thus steady state capital stock, k^*. From $\dot{m}_t = 0$, $\pi = \sigma - n$. From $\dot{a}_t = 0$ this gives $c^* = w^* + rk^* - nk^*$, which does not depend on m, these results are standard. m^* comes from $\frac{1}{m^*} - \gamma = \frac{1}{c^*}(\sigma - n + \rho)$.

Yes, money is superneutral since changes in σ have no effect on k^* or c^*. Optimal quantity of money is obtained when $r + \pi = \sigma - n + \rho = 0$. The Friedman rule applies, we can satiate demand for money in this economy for a finite level of real balances since it is not true that $v'(m)$ is always non-negative. This happens when $m^* = \frac{1}{\gamma}$.

c) Now households pay taxes $\tau_t r_t k_t$ (whose proceeds are rebated lump sum). Thus we need to rewrite the budget constraint as

$$\begin{aligned}
\dot{a}_t &= (r_t - n)a_t + w_t + z_t - (c_t + (\pi_t + r_t)m_t) - \tau_t r_t k_t \\
&= (r_t - n)a_t + w_t + z_t - (c_t + (\pi_t + r_t)m_t) - \tau_t r_t k_t - \tau_t r_t m_t + \tau_t r_t m_t \\
&= (r_t(1-\tau_t) - n)a_t + w_t + z_t - (c_t + (\pi_t + r_t(1-\tau_t))m_t)
\end{aligned}$$

where the lump sum transfer z now will reflect both transfers from printing money and from capital income taxes. Note that the effect of capital income taxes is thus reflected as reducing income from assets and reducing the cost of holding money.

Hamiltonian now is

$$H_t^c = \ln c + \ln m - \gamma m + \mu_t \left((r_t(1-\tau_t) - n)a_t + w_t + z_t - (c_t + (\pi_t + r_t(1-\tau_t))m_t)\right)$$

And FOC:

$$\begin{aligned}
\frac{1}{c} &= \mu \\
\frac{1}{m} - \gamma &= \mu(\pi + r(1-\tau)) \\
\dot{\mu} - (\rho - n)\mu &= -(r(1-\tau) - n)\mu \\
\lim_{t\to\infty} a_t \mu_t e^{-(\rho-n)} &= 0
\end{aligned}$$

d) We now see how capital income taxes affect steady state. Now in equilibrium $z = \sigma m + \tau r k$. Steady state characterized by $r(1-\tau) = \alpha k^{\alpha-1}(1-\tau) = \rho$ and thus steady state capital stock, k^*, depends on the capital income tax, as is to be expected since taxes distort the consumption saving decision. From $\dot{m}_t = 0$, $\pi = \sigma - n$ as before. From $\dot{a}_t = 0$ this gives $c^* = w^* + rk^* - nk^*$, as before, these results are standard. m^* comes from $\frac{1}{m^*} - \gamma = \frac{1}{c^*}(\sigma - n + r(1-\tau)) = \frac{1}{c^*}(\sigma - n + \rho)$.

We see from these equations that, when the nominal interest rate is positive (i.e. $\sigma - n + \rho > 0$) the demand for money in steady state is decreasing in τ, since taxes affect negatively c^*. As noted above, since taxation is distortionary, it affects k^* and c^*. From the equations that determine steady state we see that k^* and c^* are still not affected by σ. Thus, money still is superneutral. Capital income taxes affect money demand, and they affect steady state capital stock and consumption. But they do not affect the fact that the latter are independent of the rate of money growth.

Problem 2

a) Characterizing individual saving behavior requires setting up the problem of workers.

$$\begin{aligned}\max_{c_{1t},c_{2t+1}} \quad & \ln(c_{1t}) + \frac{1}{1+\rho}\ln(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t} = w_t(1-\tau^s) - s_t \\ & c_{2t+1} = s_t r_{t+1} + \tau w_{t+1}\end{aligned}$$

Note that in second period consumption we have sr and not $s(1+r)$ because there is 100% depreciation of capital, so only the return to savings enters consumption. Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \frac{r_{t+1}}{1+\rho}c_{1t}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{1}{2+\rho}w_t(1-\tau^s) - \left(\frac{1+\rho}{2+\rho}\right)\frac{1}{(r_{t+1})}\tau^s w_{t+1}$$

b) To get capital accumulation we replace individual savings with next period capital per worker $k_{t+1} = s_t$ (note there is no $\frac{1}{1+n}$ term since there is no population growth), and we use equilibrium expressions for wage and interest rates, $w = (1-\alpha)Ak^\alpha$, $r = \alpha Ak^{\alpha-1}$)

$$k_{t+1} = \left[\frac{1}{2+\rho}(1-\alpha)Ak_t^\alpha(1-\tau^s) - \left(\frac{1+\rho}{2+\rho}\right)\frac{(1-\alpha)k_{t+1}}{\alpha}\tau^s\right]$$

Combining terms with k_{t+1}

$$k_{t+1} = \frac{1}{\left[1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau^s\right]}\frac{1}{2+\rho}(1-\alpha)Ak_t^\alpha(1-\tau^s)$$

From here imposing steady state we get the following

$$k^* = \left[\frac{1}{\left[1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau^s\right]}\frac{1}{2+\rho}(1-\alpha)A(1-\tau^s)\right]^{\frac{1}{1-\alpha}}.$$

The economy can be dynamically inefficient in this steady state if $r^* < 1$, since the rate of return of social security is $1+n = 1$. It is up to parameters whether the economy will be dynamically efficient or not.

c) This shock increases the disposable income of the young because they do not pay contributions, but reduces their future income coming from social security when they retire. The effects on saving, capital accumulation and consumption will depend on the impact of these direct effects, plus the general equilibrium effects working through prices (only interest rate for first generation).

Let's see how the shock affects budget constraints of the young born at the time of dismantling the social security, $t = 0$:

$$\begin{aligned} \max_{c_{10},c_{21}} \quad & \ln(c_{10}) + \frac{1}{1+\rho}\ln(c_{21}) \\ \text{s.t.} \quad & c_{10} = w^* - s_0 \\ & c_{21} = s_0 r_1 \end{aligned}$$

Note that we do not need to calculate the gross return on debt, which has to be the same as on capital (but debt does not depreciate as capital so the net return will be lower than r_1). From Euler equation we get individual savings

$$s_0 = \frac{1}{2+\rho}w^*$$

To find the impact on capital accumulation we need to take into account that the young must buy the debt, and since the initial old receive same level of benefits as if the social security system was not dismantled, the level of debt is $b_1 = w^*\tau^s$

$$\begin{aligned} k_1 + b_1 &= s_0 = \frac{1}{2+\rho}w^* \\ k_1 &= w^*\left(\frac{1}{2+\rho} - \tau^s\right) = w^*\frac{1}{2+\rho}\left[1 - \tau^s(2+\rho)\right] \end{aligned}$$

Before the shock, capital accumulation in the steady state was

$$k^* = w^*\frac{1}{2+\rho}\left[1 - \tau^s - \tau^s\frac{1+\rho}{r^*}\right] = w^*\frac{1}{2+\rho}\left[1 - \tau^s\left(1 + \frac{1+\rho}{r^*}\right)\right]$$

Thus, if the economy was dynamically efficient in the initial steady state (i.e. $r^* > 1$) then $k_1 < k^*$ (conversely for dynamically inefficient economies).

For consumption in the first period now we have $c_{10} = \frac{1+\rho}{2+\rho}w^*$ while in the previous steady state

$$c_1^* = w^*\frac{1+\rho}{2+\rho}(1-\tau^s) + w^*\frac{1+\rho}{2+\rho}\frac{\tau^s}{r^*} = w^*\frac{1+\rho}{2+\rho}\left[1 - \tau^s + \frac{\tau^s}{r^*}\right]$$

Thus, if the economy was dynamically efficient in the initial steady state, then $c_{10} > c_1^*$. From this result we can conclude that when the economy was dynamically efficient, the young benefit from this policy ($k_1 < k^*$ implies $r_1 > r^*$ so from Euler equation $c_{21} > c_2^*$). Since the old have the same level of consumption they are indifferent about this policy.