Written Exam for the B.Sc. in Economics Summer 2010

Macro A- Solution

Final Exam

Date: 2 June 2010

(3-hour closed book exam)

Exercise 1

a.

Answer: Firm profits are

$$\pi_t = BK_t^{\alpha} L_t^{1-\alpha} - w_t L_t - rK_t \,.$$

Differentiating w.r.t. L_t and K_t yields

$$(1-\alpha)BK_t^{\alpha} L_t^{-\alpha} = w_t \text{ and } \alpha BK_t^{\alpha-1} L_t^{1-\alpha} = r \,.$$

Capital demand is defined by the first-order conditions. Households supply their savings in the international capital market. Since the country is small (relative to the rest of the world) domestic capital supply has no impact on the interest rate in the international capital market, *r*. Capital demand (see the second first-order condition) is thus not affected by the domestic savings rate.

b.

Answer: Insert domestic savings into $V_{t+1} = S_t + V_t$, dividing by L_{t+1} and rearranging yields

$$v_{t+1} = \frac{1}{1+n}\left(v_t + \frac{sY_t^n}{L_t}\right) = \frac{1}{1+n}\left(v_t + sy_t^n\right)$$

Now, divide disposable income Y_t^n by L_t. Rearranging gives:

$$y_t^n = y_t + \bar{r}\, f_t$$

$$= y_t + \bar{r}(v_t - k_t)$$

$$= w^* + \bar{r}\, v_t$$

The second equation follows from inserting $V_t = K_t + F_t$, expressed in per-capita levels. Now, insert the expression for *w** into the expression for v_{t+1} which yields

$$v_{t+1} = \frac{1+s\bar{r}}{1+n} v_t + \frac{sw^*}{1+n}. \qquad (*)$$

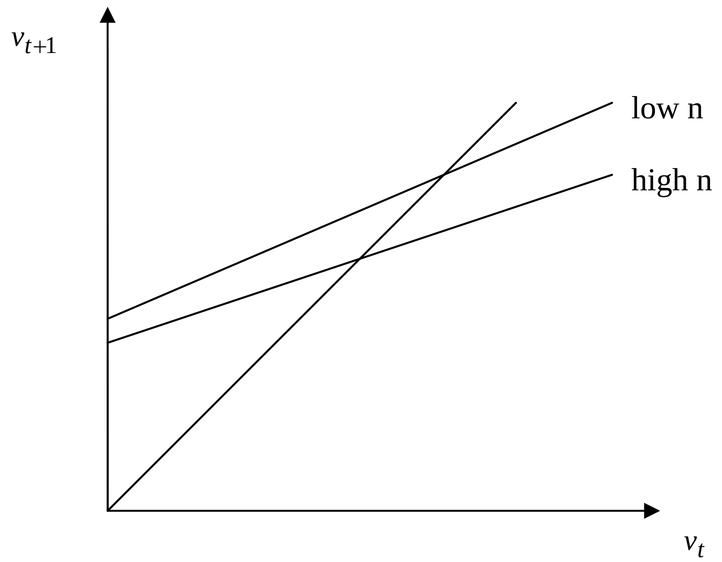
c.

Answer: Setting $v^* = v_t = v_{t+1}$, equation (*) yields a level of v^* equal to

$$v^* = \frac{s}{n - s\bar{r}} w^* = \frac{s/n}{1 - s\bar{r}/n} w^*, \text{ where } \frac{s\bar{r}}{n} < 1$$

d.

Answer: The graphical illustration is as follows:



The intercept and the slope of the wealth accumulation equation (*) drop. The steady state level of per-capita wealth thereby drops. The intuition is that wealth has to be shared among more households.

e.

Answer: The per-capita production function reads $y_t = Bk_t^{\alpha}$. The first-order condition for physical capital is

$$\alpha BK_t^{\alpha-1} L_t^{1-\alpha} = r,$$

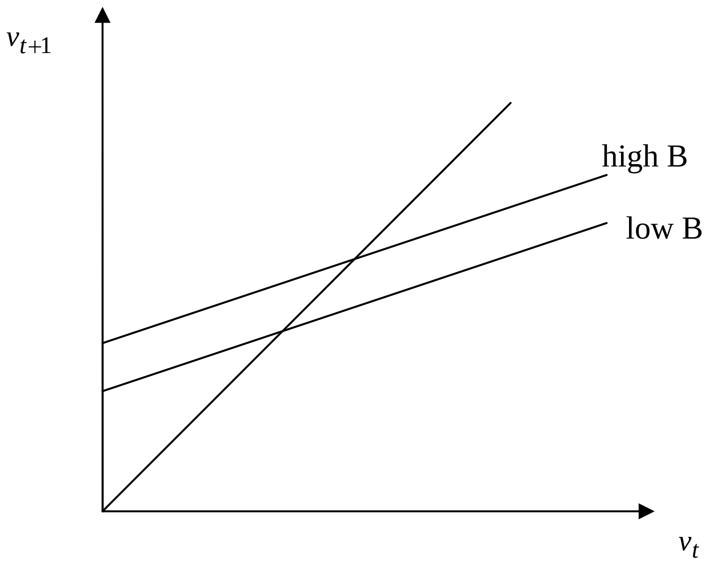
which can be rewritten to

$$\alpha B k_t^{\alpha-1} = r \, . \tag{**}$$

Equation (**) which pins down k_t. Thus, the capital intensity is not influenced by the population growth rate *n* and so is per-capita GDP. The intuition is that a higher *n* increases the amount of labor which is used in production. This raises the marginal productivity of capital above the interest rate *r* and, thus, implies an inflow of capital until the marginal productivity of capital is aligned to the interest rate. The capital intensity reaches its initial level. It is the mobility of capital which neutralizes the effect of *n* on per-capita GDP.

f.

Answer: A higher B increases w^*. To see this, solve for the wage rate from the first-order conditions in exercise 1a. This gives $w^* = (1-\alpha)B^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\bar{r}}\right)^{\frac{\alpha}{1-\alpha}}$.



A higher productivity parameter raises the intercept of the wealth accumulation equation (*). The steady state level thereby increases. Intuitively, a higher wage leads to higher disposable income. More income is saved and thereby goes into the accumulation of wealth.

Exercise 2

a.

Answer: Dividing the accumulation equation

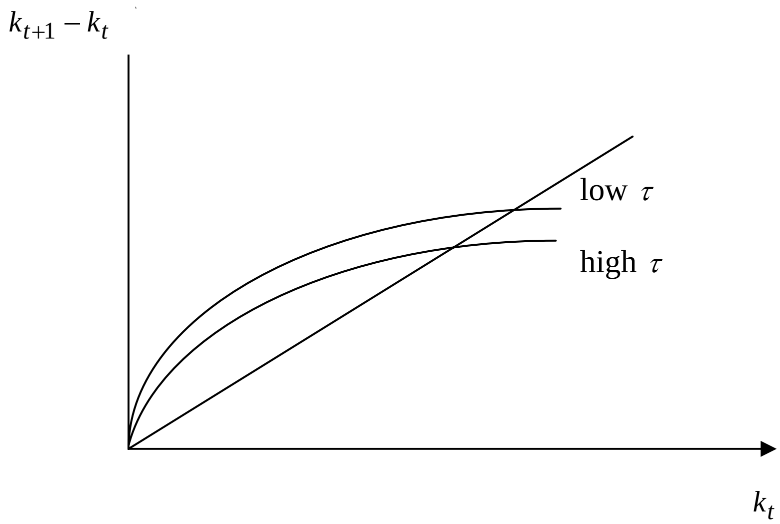
$$K_{t+1} = S_t + (1-\delta)K_t$$

by $L_t = L$ and inserting the production function gives

$$k_{t+1} = s(1-\tau)k_t^{\alpha} + (1-\delta)k_t \, .$$

Subtracting k_t from both sides gives

$$k_{t+1} - k_t = s(1-\tau)k_t^{\alpha} - \delta k_t$$



The figure displays two savings function where the upper one entails a lower tax rate. After the rise in the tax rate the savings function shifts downward which implies a lower steady state capital intensity. The intuition is that a higher tax rate lowers savings and thereby investment. The capital intensity in steady state must drop.

b.
Answer: Inserting $A_t = G_t$ and $G_t = \tau Y_t$ into the production function yields

$$Y_t = K_t^{\alpha} (\tau Y_t)^{1-\alpha} L^{1-\alpha} \, .$$

Solving for Y_t yields

$$Y_t = K_t (\tau L)^{\frac{1-\alpha}{\alpha}} \, .$$

Subsequently, we define $A = (\tau L)^{\frac{1-\alpha}{\alpha}}$. The production function has constant returns to scale w.r.t. capital.

c.

Answer: Divide

$$K_{t+1} = S_t + (1-\delta)K_t$$

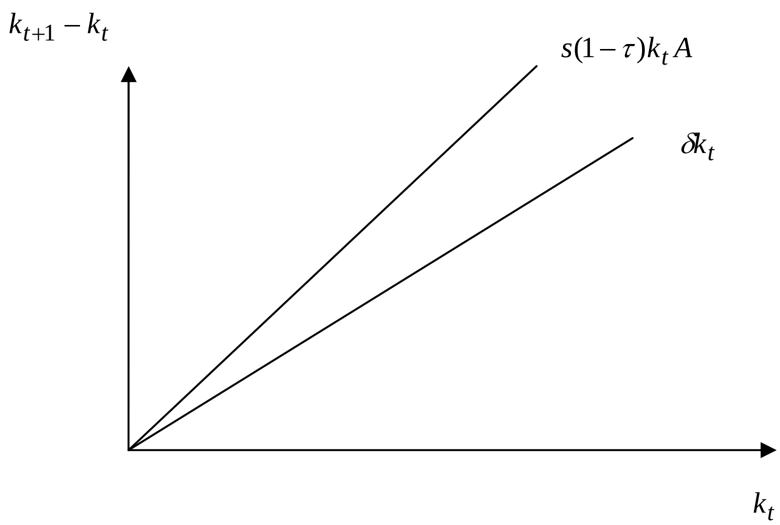
by $L_t = L$ and insert the production function. This gives

$$k_{t+1} = s(1-\tau)k_t A + (1-\delta)k_t .$$

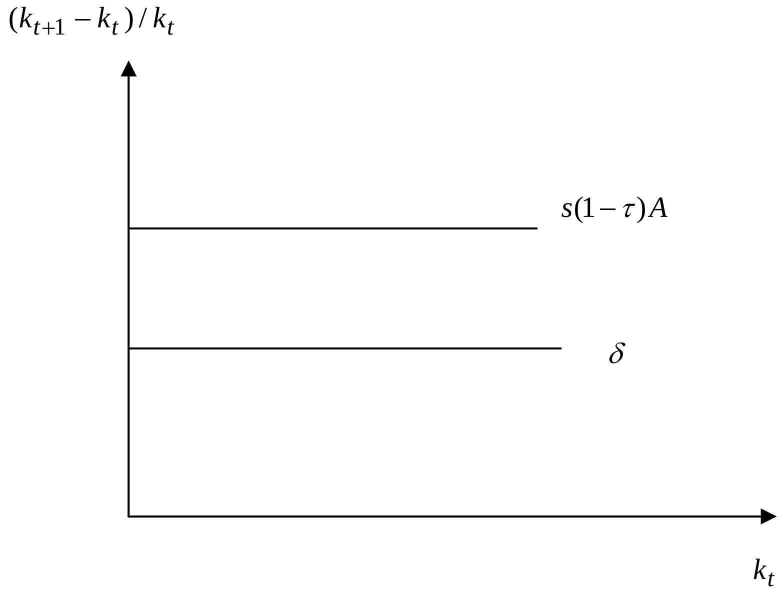
Subtracting k_t from both sides gives

$$k_{t+1} - k_t = s(1-\tau)k_t A - \delta k_t$$

The graphical illustration is as follows:



An alternative graphical illustration is



d.
Answer: The growth rate of GDP is equal to the growth rate of the capital intensity in this model. The latter can be computed from the equation

$$k_{t+1} - k_t = s(1-\tau)k_t A - \delta k_t$$

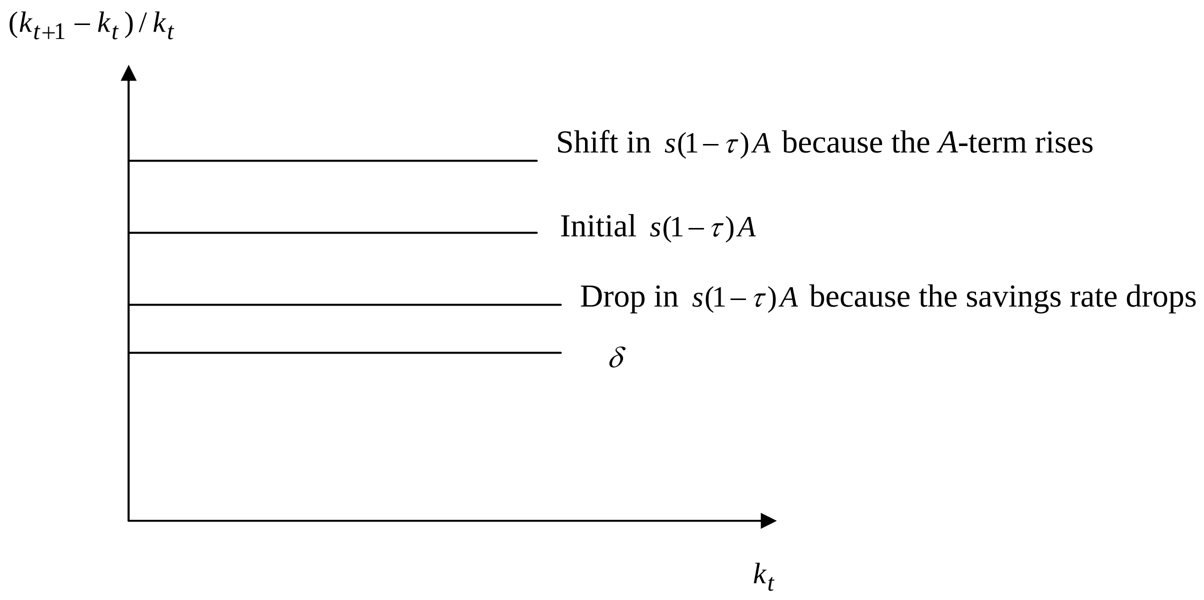
by dividing by k_t. This gives

$$\frac{k_{t+1} - k_t}{k_t} = s(1-\tau)A - \delta$$

Differentiating the growth rate w.r.t. the tax rate and noting the definition of $A = (\tau L)^{\frac{1-\alpha}{\alpha}}$ gives

$$-s(\tau L)^{\frac{1-\alpha}{\alpha}} + s(1-\tau)\frac{1-\alpha}{\alpha}(\tau L)^{\frac{1-\alpha}{\alpha}-1}L$$

The first term shows the drop in the savings rate and thus savings following a higher tax rate. The second term captures the induced rise in infrastructure spending and, thus, the rise in output and savings.



The graph shows the change in the growth rate. The rise in the tax rate increases the A-term and thereby shifts the $s(1-\tau)A$-line upwards. At the same time the rise in the tax rate lowers the savings rate and shifts the $s(1-\tau)A$-line downwards. The ultimate shift of the line depends on the relative strength of the two effects.

e.
Answer: In this model the growth rate of per-capita consumption equals the growth rate of GDP. This is given by

$$\frac{k_{t+1}-k_t}{k_t}=s(1-\tau)A-\delta$$

Differentiating the growth rate w.r.t. the tax rate gives (as shown above)

$$-s(\tau L)^{\frac{1-\alpha}{\alpha}}+s(1-\tau)\frac{1-\alpha}{\alpha}(\tau L)^{\frac{1-\alpha}{\alpha}-1}L$$

Setting the first-order condition equal to 0, expanding the last term by τ and rearranging gives

$$-1+\frac{1-\tau}{\tau}\frac{1-\alpha}{\alpha}=0$$

Solving for the tax rate yields $\tau=1-\alpha$.

f.
Answer:

in b. The production function now reads

$$Y_t=K_t^{\alpha}(\tau K_t)^{1-\alpha}L^{1-\alpha}$$

which simplifies to

$$Y_t=K_t(\tau)^{1-\alpha}L^{1-\alpha}$$

The production function still exhibits constant returns to scale w.r.t. capital.

in c. Following the analogue steps as in c. the law of motion reads

$$k_{t+1}-k_t=s(1-\tau)k_tA-\delta k_t\ ,$$

where the A term is now defined as $A=(\tau L)^{1-\alpha}$.

in d. A rise in the tax rate has the following effect on the growth rate

$$-s(\tau L)^{1-\alpha}+s(1-\tau)(1-\alpha)(\tau L)^{-\alpha}L$$

The interpretation of the two terms is the same as before.

in e. Setting the first-order condition equal to 0, expanding the last term by the tax rate and rearranging gives

$$-1+(1-\tau)(1-\alpha)=0$$

Solving for the tax rate gives $\tau = \frac{1-\alpha}{2-\alpha}$. Since $0 < \alpha < 1$, we get $\frac{1-\alpha}{2-\alpha} < 1-\alpha$. The tax rate is thus smaller than the one which was previously calculated.