Written Exam for M.Sc. in Economics 2010-I

Advanced Microeconomics

18. December 2009

Master course

3 hours written exam. Closed books. All questions should be clearly and briefly answered. Calculations and figures should be clear and understandable. Calculations and figures should be explained.

Exercise 1

Consider a pure exchange economy with private ownership $\mathcal{E}^P = (X_i, \omega_i, u_i)_{i=1}^I$. Suppose that $X_i = \mathbb{R}_+^L$, $\omega_i \in \mathbb{R}_{++}^L$ and $u_i : X_i \to \mathbb{R}$ is continuous representing a strongly monotone and strictly convex preference relation. Let $p \in \mathbb{R}_{++}^L$ denote the price vector.

1.1 State the utility maximization problem (UMP) of consumer i.

1.2 Show that (UMP) has at least one solution.

1.3 Define strict convexity for a preference relation and show that (UMP) has at most one solution.

Let $x_i : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to X_i$ be the demand function of consumer i so if the price vector is $p \in \mathbb{R}^L_{++}$, then $x_i(p, p \cdot \omega_i)$ is the solution to the problem of consumer i.

1.4 Define a Walrasian equilibrium for the economy and show that if

$$\sum_i x_i(\bar{p}, \bar{p} \cdot \omega_i) \;=\; \sum_i \omega_i,$$

then $(\bar{p}, \bar{x})$, where $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_I)$ and $\bar{x}_i = x_i(\bar{p}, \bar{p} \cdot \omega_i)$, is a Walrasian equilibrium.

1.5 Define Pareto optimality and show that if $(\bar{p}, \bar{x})$ is a Walrasian equilibrium, then $\bar{x}$ is Pareto optimal.

1.6 Suppose that $L = 2$ and $I = 1$. Suppose that the consumer has the endowment vector $\omega_1 = (1, 1)$ and the lexicographic preference relation so $x \succ y$ if and only if $x^1 > y^1$ or $x^1 = y^1$ and $x^2 > y^2$. Show that the economy has no equilibrium.

Exercise 2

Consider an economy with private ownership where there are two goods, one consumer and one firm. The consumer is described by $X = \mathbb{R}_+ \times \mathbb{R}_{++}$, $\omega = (6, 0)$ and $u(x^1, x^2) = x^1 + \ln(x^2)$. The firm is described by $Y = \{y \in \mathbb{R}^2 | y^1 \leq 0 \text{ and } y^2 \leq 5(-y^1)\}$. Let $p \in \mathbb{R}^2_{++}$ denote the price vector.

2.1 State the problem of the consumer and state the problem of the firm.

2.2 Find a Pareto optimal allocation.

2.3 Find a Walrasian equilibrium.

Exercise 3

Consider an overlapping generation economy. Time extends from $-\infty$ to ∞, there is one good at every date and there is one consumer, who is alive at two dates, in every generation. Consumers are described by their consumption identical consumption sets $X = \mathbb{R}^2_{++}$, endowment vectors $\omega_t = (\omega^y_t, \omega^o_t) \in X$ and utility functions $u_t : X \to \mathbb{R}$, which satisfies the standard assumptions.

3.1 Define an equilibrium with spot markets and a Walrasian equilibrium.

3.2 Show that if $((p_t)_{t\in\mathbb{Z}}, (x_t)_{t\in\mathbb{Z}})$ is an equilibrium with spot markets, then it is also a Walrasian equilibrium.

3.3 Define ordinary Pareto optimality and discuss other forms of Pareto optimality.

Suppose that $\omega_t = (7, 3)$ and $u_t(x) = ((x^y)^{1-a} + (x^o)^{1-a})/(1-a)$, where $a > 0$ and $a \neq 1$, for all t.

3.4 Find the demand functions of consumer t.

3.5 Show that $(x_t)_{t\in\mathbb{Z}}$, where $x_t = (7, 3)$ for all t, is an equilibrium allocation and show that the equilibrium allocation is not ordinarily Pareto optimal.

3.6 Find a monetary equilibrium.