Indicative Answers to the Take-home Exam

Theoretical and Empirical Foundations of DSGE Modeling

Summer School, 2015

Part 1

Question 1 **Proof under a perfectly competitive labor market**

As it is explained on page 244, the following equations need to be combined:

$$
\begin{aligned}
n_t^r &= \varphi^{-1}\left(w_t - c_t^r\right) && (1)\\
n_t^o &= \varphi^{-1}\left(w_t - c_t^o\right) && (2)\\
c_t^o &= E_t\left\{c_{t+1}^o\right\} - \left(r_t - E_t\left\{\pi_{t+1}\right\}\right) && (3)\\
c_t^r &= \frac{WN^r}{C^r}\left(w_t + n_t^r\right) - \frac{Y}{C^r}t_t^r && (4)\\
c_t &= \lambda c_t^r + (1-\lambda)\, c_t^o && (5)\\
n_t &= \lambda n_t^r + (1-\lambda)\, n_t^o && (6)
\end{aligned}
$$

Equations (1) and (2) can be combined with (5) and (6) to obtain:

$$w_t = c_t + \varphi n_t \tag{7}$$

We now plug (1) and (7) into (4):

$$c_t^r = \frac{WN^r}{C^r}\left(\underbrace{w_t}_{=c_t+\varphi n_t} + \underbrace{n_t^r}_{=\varphi^{-1}\left(\underbrace{w_t}_{=c_t+\varphi n_t} - c_t^r\right)}\right) - \frac{Y}{C^r}t_t^r$$

So as to get:

$$c_t^r = \frac{WN^r}{C^r}\left[\left(\varphi^{-1}+1\right)c_t + \left(\varphi^{-1}+1\right)\varphi n_t - \varphi^{-1}c_t^r\right] - \frac{Y}{C^r}t_t^r \tag{8}$$

Once we get to this stage, we need to recall some properties about the steady state of the model. First of all, Galí et al. (2007) impose $C^r = C^o = C$ and $N^r = N^o = N$. Recall also that the real marginal cost in the steady state is such that:

$$MC = \frac{WN}{(1-\alpha)\,Y} = \frac{1}{\mu^p}$$

so that

$$\frac{WN}{Y} = \frac{1-\alpha}{\mu^p} \text{ and } \frac{WN}{C} = \frac{1-\alpha}{\mu^p \gamma_c},$$

where $\gamma_c \equiv C/Y$, following Galí *et al.* (2007). (8) then becomes

$$c_t^r = \frac{1-\alpha}{\mu^p \gamma_c} \left[\left(\varphi^{-1}+1\right) c_t + \left(\varphi^{-1}+1\right) \varphi n_t - \varphi^{-1} c_t^r \right] - \frac{1}{\gamma_c} t_t^r$$

We can now transform the equation above so as to take c_t^r on the LHS:

$$\left[\mu^p \gamma_c \varphi + (1-\alpha)\right] c_t^r = (1-\alpha)(1+\varphi) c_t + (1-\alpha)(1+\varphi)\varphi n_t - \mu^p \varphi t_t^r \tag{9}$$

We now plug equation (5) into (3), so as to get rid of c_t^o:

$$c_t = \lambda c_t^r + (1-\lambda) \underbrace{\left[E_t\left\{c_{t+1}^o\right\} - \left(r_t - E_t\left\{\pi_{t+1}\right\}\right)\right]}_{=c_t^o}.$$

We then acknowledge that $E_t\left\{c_{t+1}^o\right\} = \frac{E_t\{c_{t+1}\} - \lambda E_t\left\{c_{t+1}^r\right\}}{1-\lambda}$, so that the equation above becomes:

$$c_t = E_t\left\{c_{t+1}\right\} - \lambda\left(E_t\left\{c_{t+1}^r\right\} - c_t^r\right) - (1-\lambda)\left(r_t - E_t\left\{\pi_{t+1}\right\}\right)$$

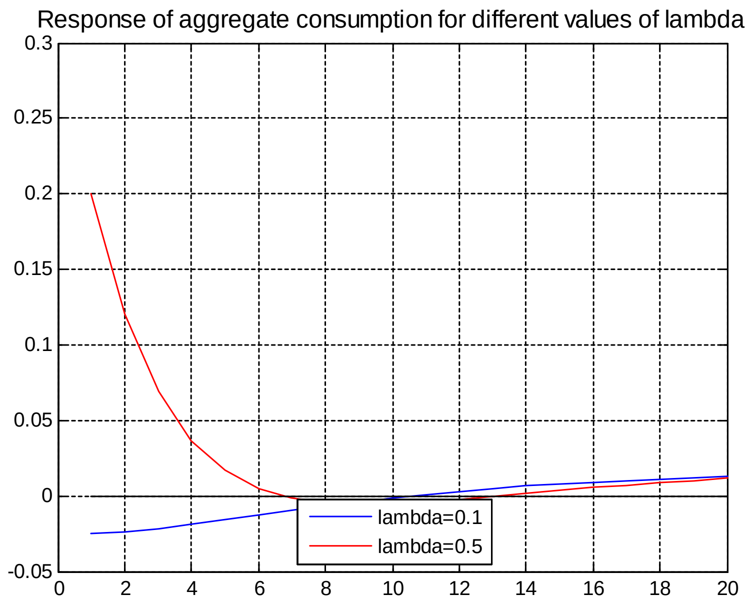
By first-differencing equation (9) we can the replace the term $\Delta E_t\left\{c_{t+1}^r\right\} = E_t\left\{c_{t+1}^r\right\} - c_t^r$ in the equation above, obtaining:

$$\begin{aligned} c_t \;=\; & E_t\left\{c_{t+1}\right\} - (1-\lambda)\left(r_t - E_t\left\{\pi_{t+1}\right\}\right) \\ & -\lambda \left(\underbrace{\frac{1}{\mu^p \gamma_c \varphi + 1 - \alpha} \left[(1-\alpha)(1+\varphi)\left[\Delta E_t\left\{c_{t+1}\right\} + \varphi \Delta E_t\left\{n_{t+1}\right\}\right] - \varphi \mu^p \Delta E_t\left\{t_{t+1}^r\right\} \right]}_{=\Delta E_t\left\{c_{t+1}^r\right\}} \right) \end{aligned}$$

After some reshuffling it is possible to obtain:

$$\begin{aligned} c_t \;=\; & E_t\left\{c_{t+1}\right\} - \frac{(1-\lambda)\left(\mu^p \gamma_c \varphi + 1 - \alpha\right)}{\mu^p \varphi \gamma_c + (1-\alpha)\left(1 - \lambda(1+\varphi)\right)} \left(r_t - E_t\left\{\pi_{t+1}\right\}\right) \\ & - \frac{(1-\alpha)(1+\varphi)\varphi\lambda}{\mu^p \varphi \gamma_c + (1-\alpha)\left(1 - \lambda(1+\varphi)\right)} \Delta E_t\left\{n_{t+1}\right\} \\ & + \frac{\mu^p \varphi \lambda}{\mu^p \varphi \gamma_c + (1-\alpha)\left(1 - \lambda(1+\varphi)\right)} \Delta E_t\left\{t_{t+1}^r\right\} \end{aligned}$$

Question 2 The impulse responses of aggregate private consumption for each of the two values of λ are displayed below. As can be seen, private consumption drops in response to an increase in government spending for $\lambda = 0.1$, while it increases when $\lambda = 0.5$. In both cases, the response of aggregate consumption is the sum of two components: consumption of optimizing households, and consumption of rule-of-thumb-households. Optimizing agents are standard, forward-looking households. In response to an increase in government spending, they realize that their current or future tax payments have increased, so that their permanent income has declined. As a result, they lower their current consumption. In contrast, rule-of-thumb households consume their current income in each period. After an increase in government spending, they will experience an increase in current income, as both the real wage and hours worked increase. As a result, these households will consume more after the shock. When rule-of-thumb households constitute only a small share of all households ($\lambda = 0.1$), the response of optimizing households dominates, and aggregate private consumption drops. However, if the share of rule-of-thumb households is sufficiently large, their response will dominate, and aggregate private consumption will go up, as seen when $\lambda = 0.5$. This result is one of the main insights of the paper by Galí *et al.* (2007).



Question 3 The first order derivative of σ wrt λ reads as:

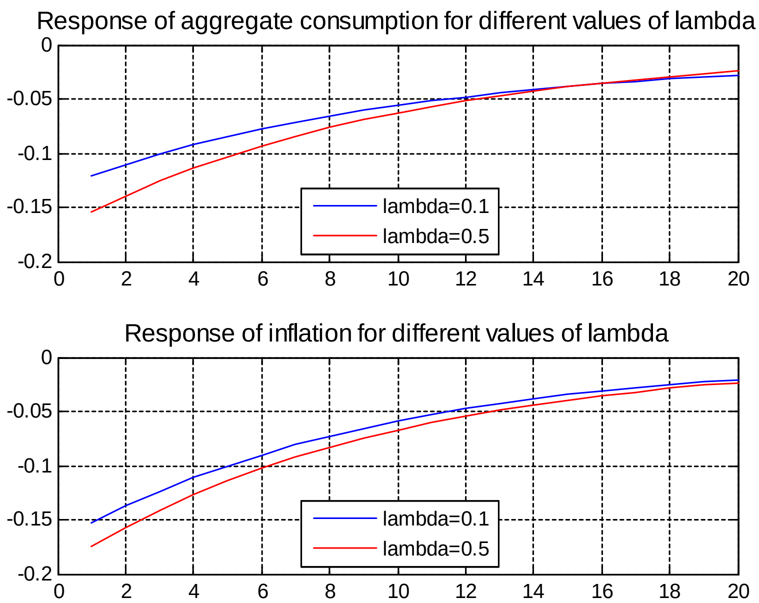
$$\frac{\partial \sigma}{\partial \lambda} = \frac{(1-\alpha)\,\gamma_c \mu^p - (\gamma_c \mu^p)^2}{[\gamma_c \mu^p - \lambda\,(1-\alpha)]^2}$$

which is greater than zero as long as

$$1 - \alpha > \gamma_c \mu^p$$

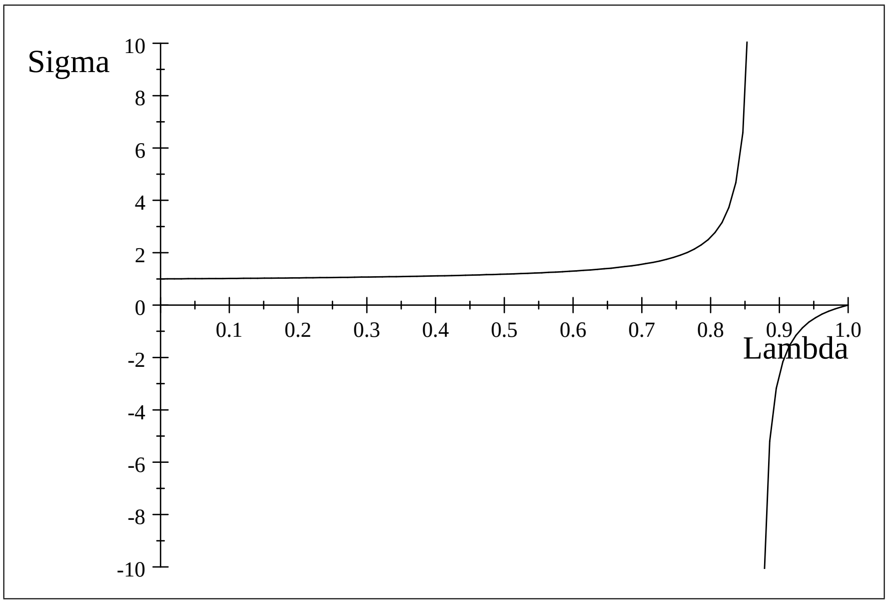
This turns out to be the case under the parameterization selected by Galí et al. (2007) ($\alpha = 0.4$, $\gamma_c \approx 0.45$, $\mu^p = 1.15$) and a wide range of alternative reasonable parameterizations. Therefore, decreasing the degree of participation in the financial market (i.e., increasing the share of rule-of-thumb households λ) increases the reactiveness of private aggregate consumption to changes in the monetary policy stance.

The intuitive explanation is the following: After a positive shock to monetary policy, aggregate demand drops, so firms demand less labor. At the same time, optimizing households increase their labor supply, according to their consumption-leisure choice. With lower demand and higher supply of labor, the real wage drops. This affects the current income of rule-of-thumb households, who are therefore forced to reduce their consumption. As a result, the presence of rule-of-thumb households amplifies the drop in consumption. When the share of rule-of-thumb households is raised from 0.1 to 0.5, the drop in their consumption determines a larger drop in aggregate consumption, as confirmed by the impulse response functions. The impulse responses of aggregate private consumption and inflation to a positive shock to monetary policy for the two different values of λ are displayed below. As can be seen, the drop in private consumption as well as inflation is larger for $\lambda = 0.5$ than in the case of $\lambda = 0.1$. As consumption drops by more in the case of $\lambda = 0.5$, aggregate activity will also be lower, all else equal. Via the New Keynesian Phillips Curve, this implies that also inflation will be lower, i.e. it will drop by more, as seen from the graph above.



The effects described above are outlined in detail by Bilbiie (2008), although his model is not fully akin to the one of Galí et al. (2007). He concludes that "*If participation is restricted below a certain threshold, the predictions are strengthened: as the share of non-asset holders increases, the link between interest rates and aggregate demand becomes stronger, and monetary policy is more effective*".

Question 4 If we plot σ as a function of λ under the suggested parameterization, we obtain the following picture:



It is seen that when λ grows too large, the slope of the IS curve switches sign (i.e., σ becomes negative), so that the IS curve takes a positive slope, and eventually the Taylor principle is no longer satisfied (bear in mind that λ is not the only determinant of the Taylor principle, so that the model still displays determinacy even with a negative σ and for some values of λ on the RHS of the asymptote in the picture above). When the share of rule-of-thumb households grows too large, the economy experiences what Bilbiie (2008) calls "inverted aggregate demand logic": when the IS curve is upward-sloping, this implies a positive relationship between current output and the real interest rate, in contrast to the usual case. As a result of this, the standard Taylor principle no longer applies: if the central bank drives up the real interest rate in response to a positive shock to the economy, this will add even more fuel to the boom due to the upward-sloping IS curve. This means that the Taylor principle is not sufficient to produce a determinate equilibrium. In fact, Bilbiie (2008) demonstrates that the Taylor principle is inverted, so that the central bank needs to react *less* than 1-for-1 to an increase in inflation. The intuition is straightforward: in this way, the reaction of the central bank induces a drop in the real interest rate, which, by the inverted IS curve, leads to a drop in current output, and thus produces a stable equilibrium.

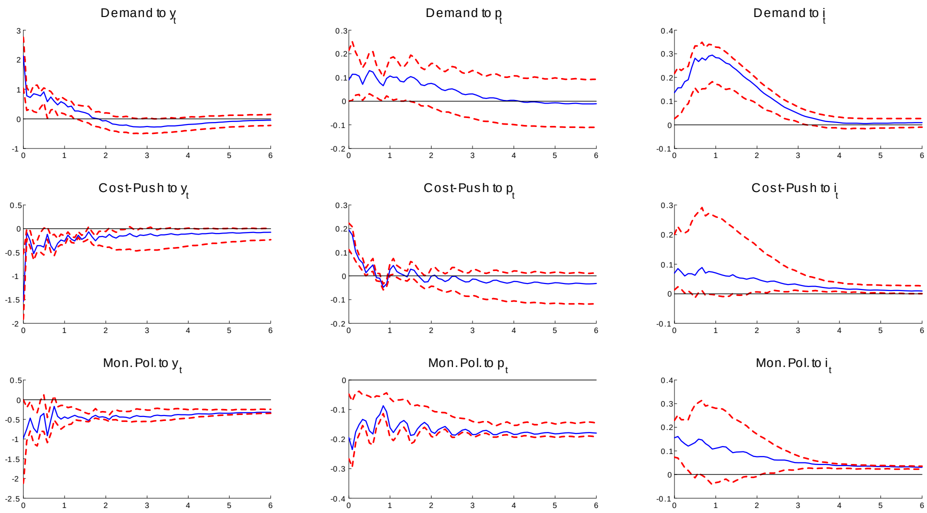
Part 2

Question 1 See e.g. Jordi Gali' "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework" MIT Press. Chapter 5.

Question 2 The sign restrictions that can be derived from a standard New Keynesian DSGE model are

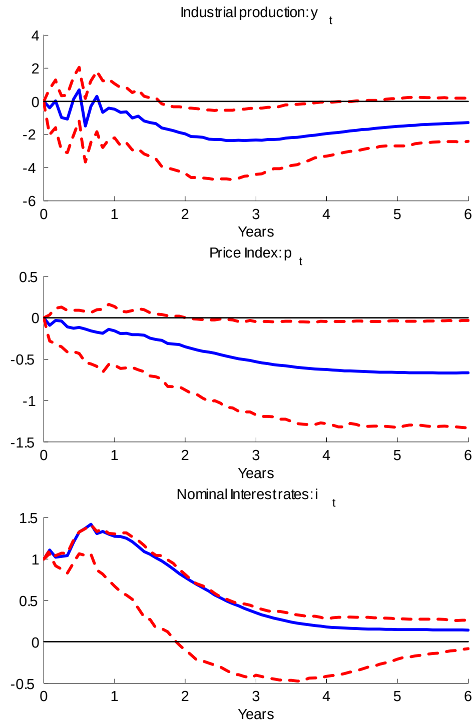
	Demand	*Cost-Push*	*Monetary Policy*
y_t	+	-	-
p_t	+	+	-
i_t	+	+	+

Imposing the restrictions to the Danish data you get



The figure above plots the entire coverage of random 1000 rotations that satisfy the sign restrictions (red lines) and the median IRF (blue line). I am not considering estimation uncertainty, this is the reason why the confidence intervals typically remain thin even at long horizons.

Question 3 One widely used strategy for estimating the effects of a monetary policy shock is based on the recursiveness assumption (see e.g. Christiano et al., 1999) according to this monetary policy shocks are orthogonal to the information set of the monetary authority. In practice this implies that the variables in the monetary authority information set, i.e. output and prices, are (contemporaneously) uncorrelated to the MP shock. Conversely, output and prices respond with a delay to a monetary policy shock. Imposing the restrictions to the data



Question 4 With the recursive identification of the MP shock (as in CEE) prices are slow to adjust to a tightening of the monetary policy stance, therefore monetary policy has a transitory contractionary effect on output. The response of output displays the typical hump shape. The IRFs identified trough sign restrictions instead display the response of output without 'hump-shape', furthermore in these IRFs there is less evidence of a slow price adjustment, with most of the adjustment in prices happening on impact.

References

Bilbiie, F., 2008, Limited Asset Markets Participation, Monetary Policy, and (Inverted) Aggregate Demand Logic, *Journal of Economic Theory* 140, p. 162-196.