Suggested answers for written Exam for the B.Sc. in Economics summer 2014

Macro B

Final Exam

August 19 2014
(3 hours closed-book exam)

Academic Aim: The aim of the course is to describe and explain the macroeconomic fluctuations in the short run, i.e. the business cycles around the long run growth trend, as well as various issues related to this, and to teach the methodology used in formulating and solving formal models explaining these phenomena. Students are to learn the most important stylized facts about business cycles and to acquire knowledge about theoretical dynamic models aimed at explaining these facts. In connection with this, the aim is to make students familiar with the distinction between deterministic and stochastic models. Furthermore, students are to gain an understanding of the distinction between the impulses initiating a business cycle and the propagation mechanisms that give business cycles a systematic character. Finally students are to learn how to use the models for analyzing the effects of macroeconomic stabilization policy under various assumptions regarding the exchange rate regime. To obtain a top mark in the course students should at the end of the course be able to demonstrate full capability of using the techniques of analysis taught in the course as well as a thorough understanding of the mechanisms in the business cycle models for open and closed economies, including the ability to use relevant variants and extensions of the models in order to explain the effects of various shocks and the effects of macroeconomic stabilization policies under alternative monetary and exchange rate regimes.

A. Problem A

1. Equation (A.1) states the goods market equilibrium value of real output. (Although not correct, it is acceptable if students refers to (A.1) as the goods market *equilibrium condition*). The model is log-linearized around the long run equilibrium in which output is given by natural output, $\bar{y}$. In equilibrium real output is equal to demand. Government consumption, g_t, is one part of total demand of goods. $\bar{g}$ represents normal or trend demand from the government sector. This explains the positive effect from $(g_t - \bar{g})$ on the output gap $(y_t - \bar{y})$. The other part of total demand of goods is private demand which depends on the real interest rate, r_t. In the textbook total private demand is assumed to decline when the real interest rate goes up. This is reflected in the assumption $\alpha_2 > 0$. This assumption is backed by the empirical evidence presented in the textbook. Theoretically the effect on total private demand from an increase in the real interest rate can not be determined. Private investments are declining in the real interest rate. This is due to a tougher discounting of future expected dividend payments which leads to a lower (stock) market valuation of firms. With a lower value of existing capital relative to the replacement cost (Tobins q) it is less attractive for the owners of existing capital to invest. However, the effect on private consumption from a higher real interest rate can not be determined theoretically. The substitution effect on private consumption from a higher real interest rate shifts consumption towards the future. The income effect on the other hand works in the opposite direction of the substitution effect and the income effect may dominate the substitution effect.
   Equation (A.2) defines the (ex ante) real interest rate.
   According to equation (A.3) the central bank follows a Taylor rule. The nominal interest rate is an increasing function of the inflation gap defined as the difference between actual inflation, π_t, and the target level of inflation, π^* and the output gap $y_t - \bar{y}$. Combining equation (A.3) with equation (A.2) it is seen that an increase in inflation will also increase the real interest rate since $h > 0$. This is known as the Taylor principle. As a result monetary policy will work as a stabilizing force which will curb private demand when activity is high relative to the structural level. As will be seen below this

will affect inflation negatively trough the supply side of the economy. $\bar{r}$ is the so-called equilibrium real interest rate, *i.e.* the real interest rate that is consistent with long-run equilibrium; $\pi^e = \pi = \pi^*$, $y_t = \bar{y}$.

Equation (A.4) is the short run aggregate supply curve (SRAS). Inflation is an increasing function of real output and expected inflation. A higher level of real output is associated with a higher level of employment. This in turn reduces the marginal product of labour. Therefore, a given increase in nominal wages leads to a stronger increase in marginal production costs. Firms react by increasing prices more and this leads to higher inflation. The expected inflation affects actual inflation positively. A labour union sets nominal wages based on the expected inflation (price level) so that an employment level consistent with $\overline{y}$ is realised if inflation expectations are fulfilled. A higher expected inflation will cause higher increases in nominal wages and thereby lead to higher marginal costs of firms and ultimately to higher inflation.
Equation (A.5) is the assumption of static expectations according to which the expectation about the value of a variable (here: inflation) is given as the actual value of the variable in the previous period.

2. Inserting equation (A.2) into equation (A.3) we obtain

$$\begin{aligned} & r = \bar{r} + \pi^e_{+1} + h\left(\pi - \pi^*\right) + b\left(y - \bar{y}\right) - \pi^e_{+1} \\ \Downarrow & \\ & r = \bar{r} + h\left(\pi - \pi^*\right) + b\left(y - \bar{y}\right) \end{aligned}$$

Inserting this expression into equation (A.1) it follows that

$$\begin{aligned} & y - \bar{y} = \alpha_1\left(g - \bar{g}\right) - \alpha_2\left(\bar{r} + h\left(\pi - \pi^*\right) + b\left(y - \bar{y}\right) - \bar{r}\right) + v \\ \Downarrow & \\ & y - \bar{y} = \alpha_1\left(g - \bar{g}\right) - \alpha_2\left(h\left(\pi - \pi^*\right) + b\left(y - \bar{y}\right)\right) + v \\ \Downarrow & \\ & \left(1 + \alpha_2 b\right)\left(y - \bar{y}\right) = \alpha_1\left(g - \bar{g}\right) - \alpha_2 h\left(\pi - \pi^*\right) + v \\ \Downarrow & \\ & y - \bar{y} = -\frac{\alpha_2 h}{1 + \alpha_2 b}\left(\pi - \pi^*\right) + \frac{1}{1 + \alpha_2 b}\left[\alpha_1\left(g - \bar{g}\right) + v\right] \end{aligned}$$

Using the definitions of α and z, equation (A.6) follows directly.

The SRAS curve (A.7) follows directly by inserting (A.5) into (A.4).

The AD curve has a negative slope because higher inflation induces the central bank to raise the policy interest rate, causing an increase in the real interest rate. This in turn induces a decline in total private demand as consumers and firms meet a higher real interest rate. See also above. The SRAS is upward sloping for the above mentioned reasons.

In the long run actual and expected inflation must coincide; $\pi_t = \pi_t^e = \pi_{t-1}$. It then follows from the SRAS in equation (A.7) that (in the absence of shocks, *i.e.* when $s_t = z_t = 0$) $y_t = \overline{y}$. From the AD curve in equation (A.6) it then follows that $\pi_t = \pi^*$. Thus, the long run equilibrium is given by the intersect between the vertical LRAS and the AD curve.

3. The formal analysis can be carried out by using the definition of output and inflation gap and assuming $z_t = s_t = 0, \forall\ t > 0$. Based on this, the AD curve (A.6) can be restated as

$$\widehat{y}_t = -\alpha\widehat{\pi}_t$$

and the AS curve (A.7) can be written as

$$\widehat{\pi}_t = \widehat{\pi}_{t-1} + \gamma\widehat{y}_t$$

Combining this gives

$$-\frac{1}{\alpha}\widehat{y}_t = -\frac{1}{\alpha}\widehat{y}_{t-1} + \gamma\widehat{y}_t$$

$$\Downarrow$$

$$\widehat{y}_t = \beta\widehat{y}_{t-1} \qquad \text{where} \qquad \beta \equiv \frac{1}{1+\alpha\gamma}$$

Also

$$\widehat{\pi}_t = \widehat{\pi}_{t-1} + \gamma\widehat{y}_t$$

$$\Downarrow$$

$$\widehat{\pi}_t = \widehat{\pi}_{t-1} - \alpha\gamma\widehat{\pi}_t$$

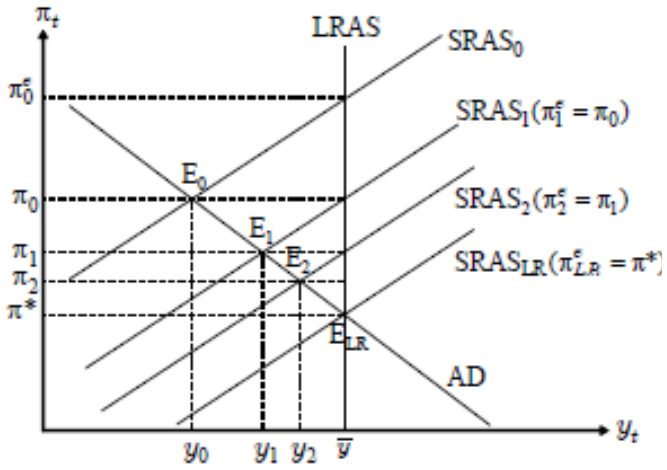
$$\Downarrow$$

$$\widehat{\pi}_t = \beta\widehat{\pi}_{t-1} \qquad \text{where} \qquad \beta \equiv \frac{1}{1+\alpha\gamma}$$

The long-run equilibrium is stable since $\beta^t \longrightarrow 0$ for $t \longrightarrow \infty$ as $0 < \beta \equiv \frac{1}{1+\alpha\gamma} < 1$ because $\alpha, \gamma > 0$. Hence, over time y_t and π_t will converge monotonically towards the long-run values $\bar{y}$ and π^* respectively, but deviations from the long-run equilibrium will show persistence. It could be noted that the convergence process will be faster the larger α and γ because β is declining in both parameters. γ is the slope on the SRAS curve. A larger value of γ means that domestic inflation reacts more to deviations in output from trend (or structural) output. Hence, a given deviation in activity from the structural level foster a larger central bank reaction. Accordingly, the pace at which the economy moves towards the long-run equilibrium is faster. A larger value of α corresponds to stronger central bank emphasis on inflation and/or a stronger sensitivity in private demand to the real interest rate.

The figure below illustrates the convergence process. As already noted the long run equilibrium is given by the intersection between the vertical LRAS curve and the AD curve where $y_t = \bar{y}$ and $\pi_t = \pi^*$.

Figure A.1: Convergence in the AD AS model



Initially the economy is in a recession at point E_0 which is the intersection between the AD and $SRAS_0$ curve. At E_0 inflation is π_0 and real output is $y_0 < \bar{y}$. Due to the assumption of static expectations it follows that in period 1 $\pi_1^e = \pi_0$. Assuming that the economy is not hit by any shocks the SRAS will shift down in period 1 to $SRAS_1$. Given static expectations the

SRAS will shift down by the full vertical distance $\pi_0 - \pi_0^e$ so that SRAS_1 goes through the point $(\bar{y}, \pi_0)$. This can be seen by using (A.7). The new short run equilibrium will then be given by point E_1 where inflation has decreased and real output increased compared to period 0.

The economic explanation is the following: Between period 0 and period 1 there is a decrease in expected inflation (from π_0^e to $\pi_1^e = \pi_0$). This will cause the labour union to reduce the growth rate of nominal wages which in turn makes the marginal costs of the firm increase at a slower pace. Consequently, firms will increase their prices at a slower pace, *i.e.* the rate of inflation will be reduced. According to the monetary policy rule the reduction in inflation will lead monetary authorities to lower the nominal interest rate so much that a lower real interest rate results. This in turn increases goods market demand which finally leads to increased real output.

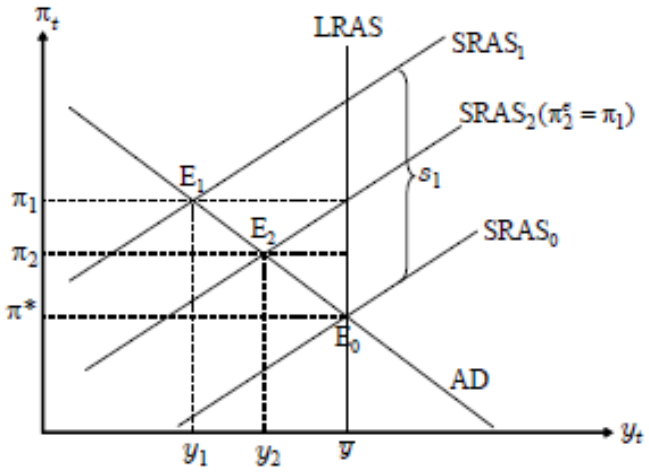
In period 2 we then have that $\pi_2^e = \pi_1 < \pi_1^e$ and consequently this process of downward shift in the SRAS curve will continue with gradually lower inflation and higher activity and gradually smaller changes in inflation and activity from one period to the next until the economy reach the long run equilibrium. Hence the economy is stable but deviations from long-run equilibrium shows persistency.

4. According to the SRAS curve in equation (A.7) a negative supply shock will have to *either* affect inflation positively, affect real output negatively or a combination of both. Hence, the conclusion is that in case of a supply shock policy makers have to choose between stabilizing real output *or* inflation.

   An illustration of the effect of the supply shock is could look like the figure below. The economy is in a long run equilibrium at E_0 in period 0. Then in period 1 a negative supply shock occur. The SRAS shifts up by the vertical distance s_1 to SRAS_1. This moves the economy from the long run equilibrium at point E_0 to point E_1 causing inflation to rise and real output to decrease.

   The economic explanation is the following: A negative supply shock directly leads to higher inflation through, *e.g.* lower productivity or higher prices of raw materials etc. which causes firms' costs to increase thereby leading them to increase prices. The central bank responds to this by increasing the policy rate. The response is determined by the monetary policy rule according to

Figure A.2: The response to a supply shock

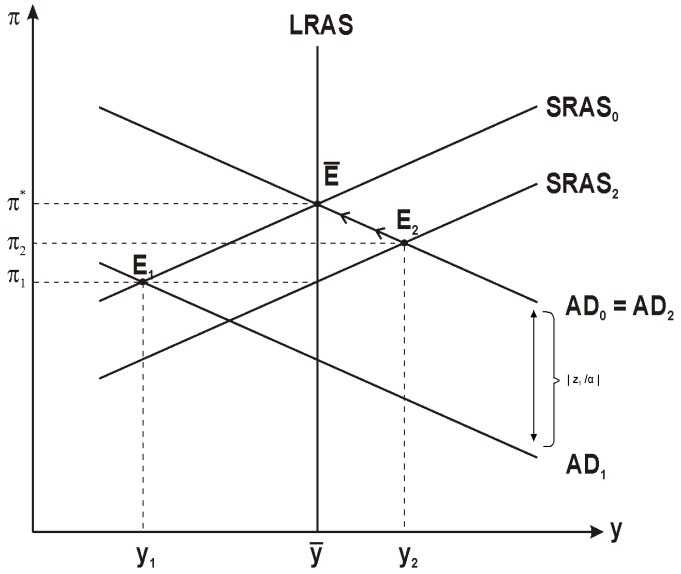


which the nominal rate is increased so much that the real interest rate is higher (the Taylor principle). As a result private demand is curbed. Hence the economy moves along the AD curve.

In period 2 the negative supply shock has disappeared. However compared to the initial situation inflation expectations have increased because inflation was higher in period 1 than in period 0 and inflation expectations are static. Therefore in period 2, the SRAS only shifts down to $SRAS_2$. In period 2 $\pi_2^e = \pi_1$ due to the static inflation expectations. As a result the $SRAS_2$ curve intersects the LRAS curve at the point $(\bar{y}, \pi_1)$. The economy moves from E_1 to E_2 whereby inflation decreases and real output increases. Monetary policy responds to the decrease in inflation (the mechanism already described) by lowering the nominal interest rate by so much that the real interest rate is reduced. Accordingly private demand is spurred. From E_2 the economy then moves back to long run equilibrium as described in question 3.

A temporary demand shock give rise to business cycle movements in the economy. This is illustrated in the figure below. Due to the temporary negative demand shock in period 1 both inflation and output is below the long-run equilibrium values. In the next period the shock disappear. Consequently the AD curve shifts back to the original position. However, the short-run AS curve shifts down as inflation expectations are static and hence

Figure A.3: The response to a demand shock



wage setting in period 2 is affected by the low inflation in period 1. As a consequence inflation is below it's long run equilibrium value and monetary policy is stimulating demand; $y_2 > \bar{y}$.

It could be noted that policy makers do not face a trade off between stabilizing output and inflation in case of a temporary demand shock.

5. The new parameters for monetary policy $h_{New} > h_{Old} > 0$, $b_{New} = b_{Old}$ only influences the AD curve. From (A.6) it follows that the AD curve may be written as

$$\pi_t = \pi^* - \frac{1}{\alpha}\left(y_t - \bar{y} - z_t\right)$$

Hence the slope of the AD curve is given by

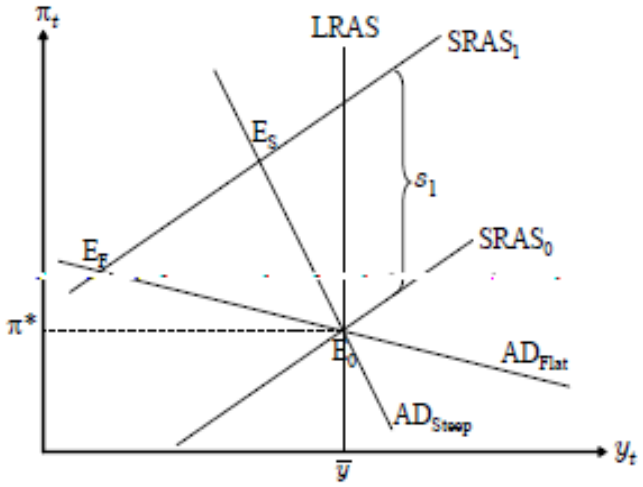
$$-\frac{1}{\alpha} = -\frac{1}{\frac{\alpha_2 h}{1+\alpha_2 b}} = -\frac{1+\alpha_2 b}{\alpha_2 h}$$

and it can be concluded, that a higher value of h (relative to b) makes the AD curve flatter. Therefore, the AD curve is flatter under the new parameters for monetary policy where more weight is attached to the inflation gap compared to output stability; $h_{New} > h_{Old} > 0$, $b_{New} = b_{Old}$.

The effect of a negative supply shock when the AD curve is either flat or steep is shown in the figure below. It is seen that the more monetary policy

is focused on stabilizing activity around the trend output the steeper is the AD curve (b high relative to h) and the smaller is the effect on real output and the larger is the effect on inflation. Hence, the conclusion remains that in case of a supply shock policy makers have to choose between stabilizing real output *or* inflation.

Figure A.4: The effect of a supply shock and the importance of monetary policy parameters



This is not the case when shocks occur on the demand side of the economy. A graphical analysis is somewhat complicated because the new value of h affects the slope of the AD curve (the vertical shift given $z_t \neq 0$ will not be changed since b remain unchanged). This can be seen directly from (A.6). However, the economic intuition is the following: A demand shock will directly affect goods market demand and thereby equilibrium real output. Real output in turn affects inflation through the supply side of the economy. Consequently, by stabilizing real output around its long run level, inflation will also be stabilized (and by stabilizing inflation, real output will be stabilized through monetary policy).

The speed at which the convergence process takes place is determined by β. It is seen that the time needed to half a given gap is shorter under the new set of policy parameters. This follows from $\beta \equiv \frac{1}{1+\alpha\gamma}$. The "old" and "new" values of α are given by $\alpha_{Old} \equiv \frac{\alpha_2 h_{Old}}{1+\alpha_1 b_{Old}} < \frac{\alpha_2 h_{New}}{1+\alpha_1 b_{New}} = \alpha_{New}$, so

that $\beta_{Old} \equiv \frac{1}{1+\alpha_{Old}\gamma} > \frac{1}{1+\alpha_{New}\gamma} = \beta_{New}$. The economic reason is that actual inflation is closer to expectations. Therefore actual real wages are closer to the unions target real wages and hence employment is closer to target/the natural employment which is consistent with structural activity.

To sum up: In case of a supply shock policy makers have to choose between stabilizing real output *or* inflation. Demand shocks on the other hand can be eliminated completely and there is no trade off between stabilising y and π.

6. According to the social welfare analysis fluctuations in activity should be minimised whereas inflation should be allowed to fluctuate. In case economic shocks are entirely a supply shock phenomenon the above analysis point out that policy makers should choose a low but positive value for $h; h \longrightarrow 0_+$. The analysis also show that demand shocks can be eliminated completely and that there is no trade off between stabilising y and π.

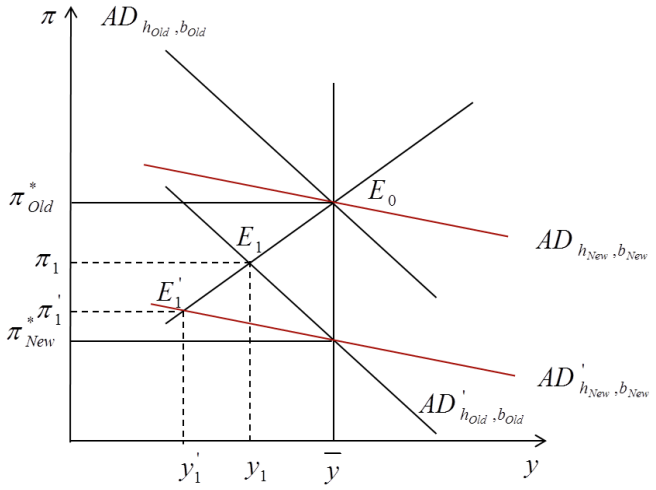
   In case there is a significant social loss associated with both fluctuations in inflation and activity the trade off between output and inflation stability comes into play. In this scenario h should take a positive number and should not be too close to zero.

7. In a situation where a new board also introcuce a lower target for inflation ($0 < \pi^*_{New} < \pi^*_{Old}$) the AD curve shifts down. This is seen directly from (A.6). It is also seen that the downward vertical shift in the AD curve is of the magnitude $|\pi^*_{Old} - \pi^*_{New}|$. Also, the AD curve is flatter because of the change $h_{New} > h_{Old} > 0$, $b_{New} = b_{Old}$, see above. The AS curve is unaffected given inflation expectations are static. Due to the lowering of the inflation target the economy is forced into a recession. Activity is below $\overline{y}$ and inflation is $\pi^*_{New} < \pi_1 < \pi^*_{Old}$. Convergence towards the new long-run equilibrium $(\overline{y}, \pi^*_{New})$ is secured via the gradual lowering of interest rates as inflation is reduced due to the downward shifts in the AS curve as inflation expectations are aligned with the new lower inflation target for the central bank. During this (very long) convergence period activity gradually increase and inflation gradually decrease. The economic intuition is as described when answering question 3.

Compared to a situation where monetary policy was described by the old parameters h_{Old} and b_{Old} the AD curve is flatter because of the new boards stronger commitment to the inflation target objective. Therefore, the downward shift in policy is associated with a deeper recession compared to a situation where the old parameters where maintained.

An illustration could look like the following where the red AD curves are associated with the new board.

Figure A.5: A lower inflation target and new monetary polict parameters



If inflation expectations on beforehand are firmly anchored to the new central bank inflation target is possible to mitigate (and possibly eliminate completely) the deeper recession. In this situation wage formation is aligned with monetary policy and hence actual inflation corresponds to actual inflation and real wages are at the targeted level. Hence, employment is at the natural level consistent with an output of $\overline{y}$. This is in principle possible by announcing the new inflation target before it is implemented. The student is not expected to discuss credibility issues related to such an announcement.

B. Problem B

1. When the economy is small anything that happens in the economy is without any substantial effect on the foreign economy.

   When the economy is specialized domestic and foreign produced goods are imperfect substitutes. So the price on domestic produced goods may deviate from the price on foreign produced goods. The endogenous adjustment of relative prices on domestic goods is a mechanism for macroeconomic adjustment to long run equilibrium.

2. The uncovered interest parity (UIP) is a financial arbitrage condition (or rather absence of arbitrage). When capital mobility is perfect domestic and foreign assets are perfect substitutes. Investors can reallocate their portfolios instantaneously and without cost. Assuming investors are risk neutral the arbitrage condition whereby the return on domestically denominated financial assets is tied to the return on foreign denominated assets may be written

$$1 + i = \left(1 + i^f\right) \times \frac{E^e_{+1}}{E}, \tag{B.2.1}$$

   where E is the current exchange rate and $E^e_{+1}$is the expected exchange rate, i is the domestic interest and i^f is the interest rate abroad. If the value of the domestic currency is expected to be reduced (and increase in E) the domestic interest rate has to be larger than the foreign interest rate. Using the approximation $\ln(1+x) \approx x$ when x is small the uncovered interest parity (UIP)

$$i = i^f + e^e_{+1} - e, \tag{B.2.2}$$

   follows directly. $e^e_{+1} - e$ is the expected reduction in the value of the domestic currency. If the value of the domestic currency is expected to be reduced (and increase in e) the domestic interest rate has to be larger than the foreign interest rate so that the expected investment return is the same when measured in the same currency. If investors are risk averse a risk premium is added.

   The "impossible trinity" may be shown by assuming that two of the three policy goals are upheld. Then the third can not be obtained.

If cross-border capital flows are free and the exchange rate is fixed then UIP holds it follows directly from (B.2.2) that $i = i^f$, hence monetary policy can not be independent. This can be explained as follows: In a fixed exchange rate system the central bank has promised to convert domestic and foreign exchange at a given rate and in unlimited volume. If $i < i^f$ capital will flow out of the domestic economy. Investors sell domestic denominated assets (and thereby domestic currency) and buy foreign denominated assets (and thereby foreign currency) the the central bank has to sell foreign currency and buy domestic currency the maintain the fixed exchange rate. However eventually the foreign exchange reserves are exhausted and the central bank can not defend the exchange rate any more. If $i > i^f$ capital will flow into of the domestic economy. This creates a pressure for an appreciation of the domestic currency.

If cross-border capital flows are free and monetary policy is independent, $i \neq i^f$, then $e^e_{+1} \neq e$ which is at odds with a fixed exchange rate.

If monetary policy is independent $i \neq i^f$ and the exchange rate is fixed $e^e_{+1} = e$ then UIP stresses that capital can not move freely across borders as it would take an infinite foreign exchange reserve to uphold a fixed exchange rate.

3. Equation (B.1) states that in optimum the consumer is indifferent between consuming an extra unit today and saving an extra unit today. The left-hand side is the marginal rate of substitution, MRS, which expresses how much utility decreases today relative to the increase in future utility if an infinite small part of today's consumption is postponed to the future. The marginal rate of substitution between present and future consumption (the left-hand side) must equal the relative price of present consumption (the right-hand side). Also, it follows directly from the characteristics of the utility function u that C_2 is increasing in r and declining in ϕ.

   For $r = \phi$ optimization implies $C_1 = C_2$. In this situation the consumer will smooth consumption completely.

   Capital markets makes it possible for the consumer to shift consumption between different phases in life. Hence, consumption in a given period is

not restricted completely by income in this period. Instead the consumer may trade consumption between the periods at a given market rate. This makes it possible for the consumer to obtain higher utility (illustrated in the figure below where the optimal consumption stream is at a higher indifference curve than the indifference curve going through the income stream (Y_1, Y_2). An illustration may be the following.

Figure B.1: The importance of capital markets for welfare