Written Exam at the Department of Economics winter 2017-18

Macroeconomics III

Final Exam

February 14, 2018

(3-hour closed book exam)

This exam question consists of 3 pages in total

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Written Exam - Macroeconomics III

University of Copenhagen
February 14, 2018

Question 1

Consider a small open economy with an infinite number of identical firms and households. The representative household, whose time endowment is normalized to one, maximizes the discounted stream of expected utility under perfect foresight:

$$\max_{c_t, h_t, b_t} \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \frac{h_t^{1+\nu}}{1+\nu} \right] \right\}, \tag{1}$$

subject to the following budget constraint:

$$c_t + Rb_{t-1} = w_t h_t + b_t + d_t \tag{2}$$

where c_t denotes households' consumption, h_t is the amount of labor hours, w_t is the real wage, b_t denotes the amount of borrowing at time t, $d_t \equiv y_t - w_t h_t$ are the dividends (profits) rebated by firms to households at time t (these are taken as given by the households), y_t is the total production, $Rb_{t\;1}$ accounts for the debt service, which depends on R — the gross rate of interest (taken as exogenous) — and the level of debt inherited from the previous period, b_{t-1}. The parameters β and ν are both positive, with $\beta \in [0, 1)$. Households can borrow from abroad in accordance with the following financial constraint:

$$b_t \leq b, \quad b > 0.$$

Production at the representative firm is carried out by means of the following technology:

$$y_t = z_t h_t^{\alpha}, \quad \alpha \in [0, 1] \tag{3}$$

where z_t is a technology shock.

The aggregate resource constraint is such that $y_t = c_t$. Given this environment, address the following questions, providing adequate comments to the derivation of each and every result:

a Take the following Lagrangian for households' optimization with respect to c_t, h_t, and b_t:

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \frac{h_t^{1+\nu}}{1+\nu} + \lambda_t \left[w_t h_t + b_t + d_t - c_t - Rb_{t-1} \right] + \phi_t \left[b - b_t \right] \right\}$$

where $\lambda_t \geq 0$ and $\phi_t \geq 0$ are the Lagrange multipliers associated with the budget and the financial constraint, respectively. Report the necessary first-order conditions.

b Derive households' Euler equation [*hint: set* $\phi_t = \lambda_t \psi_t$, so as to normalize the multiplier applying to the financial constraint]. What is the effect of a tightening of the financial constraint (i.e., an increase in ψ_t) on current consumption, *ceteris paribus*?

c Set up the representative firm's optimization problem and derive the necessary first order conditions.

d Characterize the labor demand and supply schedules and prove that the equilibrium wage and hours are $w_t = \alpha^{\frac{\alpha+\nu}{1+\nu}} z_t$ and $h_t = \alpha^{\frac{1}{1+\nu}}$, respectively.

e What happens to equilibrium labor hours as $\nu \to \infty$? Explain.

Question 2

Consider the following model of monetary policy: the government controls inflation directly (i.e. $\pi_t = m_t$, where π_t is the rate of inflation and m_t is the rate of growth of money supply) and its instantaneous loss function is

$$L(\pi_t, x_t) = \frac{1}{2}\left[\pi_t^2 + \lambda\left(x_t - \bar{x}\right)^2\right]$$

where $x_t = \theta_t + \pi_t - \pi_t^e$. The following notation applies

π_t^e:	expected rate of inflation
x_t:	output level
θ_t:	potential output
$\bar{x}$:	policy output target

We assume that potential output is stochastic (with mean zero and variance σ_θ^2) and that its realizations are observed by both the public and the policy maker before expectations are formed by the private sector. The parameter $\lambda > 0$ measures the importance of output fluctuations around the target ($\bar{x}$) relative to inflation fluctuations.

a Show that the optimal policy under commitment implies $\pi_t^C = 0$ and $x_t^C = \theta_t$ [*hint: i) recall that the loss function is quadratic, thus the optimal policy rule is linear and can be guessed to be of the form* $\pi_t = \psi + \psi_\theta \theta_t$*; ii) recall that the loss to be minimized is the unconditional one*].

b Show that the optimal policy under discretion implies $\pi_t^D = -\lambda\left(\theta_t - \bar{x}\right)$ and $x_t^D = \theta_t$. The *inflation bias* increases in the target $\bar{x}$: explain why.

c Assume that potential output cannot be observed before expectations are formed. The goal of the central bank is still to minimize the loss function. However, the monetary policy stance should now result as ex-post optimal given both π_t^e and θ_t (as the latter is not observed until after expectations are formed). Show that the optimal policy under discretion now implies $\pi_t^{D*} = \frac{\lambda}{1+\lambda}\left[\bar{x}\left(1+\lambda\right) - \theta_t\right]$ and $x_t^{D*} = \frac{1}{1+\lambda}\theta_t$. Under $\lambda = 0$ it is possible to ensure that $\pi_t^{D*} = \pi_t^C$ and $x_t^{D*} = x_t^C$. Explain why this is the case.