Written Exam for the B.Sc. / M.Sc. in Economics 2009-II

Corporate Finance and Incentives

Elective Course/ Master’s Course

June 2009

(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different subquestions that do not necessarily have equal weight.
Please provide intermediate calculations.

Eksamen består af 4 opgaver. Alle opgaver skal besvares. For hver delopgave er den vejledende vægt i den samlede bedømmelse angivet. En opgave kan bestå af flere delspørgsmål, der ikke nødvendigvis har samme vægt.
Vis venligst mellemregninger.

Problem 1 (Fixed Income, 30%)

1)

The relation between the clean and dirty price is:

dirty price = clean price + accrued interest. Accrued interest is the interest payment owed to the bond seller. Even though the bond seller in a bond sale loses the right to receive the next coupon payment, he/she has still earned interest over a given period of "days" (depending on the applied day count convention) since last coupon was paid out.

Since the buyer of a bond receives the next coupon payment , he/she will pay the amount of accrued interest that is owed to the bond seller who is paid at the time of purchase. The dirty price of a bond therefore corresponds the the bond's clean (quoted) price + each bond's portion of accrued interest.

Accrued interest is calculated as:
Accrued interest = (No coupon "days"/No "days" in a year) * coupon

2)
Semiannual coupon = 8% / 2 = 4%
30/360 day count convention

Clean price	96,0000	on November 10
Days of accrued int.	15	june
	30	july
	30	august
	30	september
	30	october
	10	november
	145	**total**
accrued interest = 145/360 * 4% =	0,016	
Dirty price = clean price + acc. Int	96,016	

3)

Bond	Time to maturity	Coupon	Price	Note
Bullet	1 year	5,50%	501	
Serial	2 years	2,00%	485	
Annuity	3 years	5,75%	515	37,238% annual payments

Face value 500

1Y Bullet	**5,50%**	**1**
Interest		27,50
Principal reduction		500,00
Total payment		527,50

In general interest payment =
(Face value - cumulative principal reduction) *
Coupon rate

2Y Serial	**2,00%**	**1**	**2**
Interest		10,00	5,00
Principal reduction		250,00	250,00
Total payment		260,00	255,00

Principal reduction = 500/2

3Y Annuity	**5,75%**	**1**	**2**	**3**
Interest		28,75	19,70	10,12
Principal reduction		157,44	166,49	176,07
Total payment		186,19	186,19	186,19

Annual payments =
37.238% * 500

4)

$$\boldsymbol{\pi} = \mathbf{C}\mathbf{d} \quad \Updownarrow \quad \mathbf{d} = (\mathbf{C})^{-1}\boldsymbol{\pi}$$

$$y_t = \left(\frac{1}{d_t}\right)^{\frac{1}{t}} - 1 \qquad f_t = \left(\frac{d_t}{d_{t+1}}\right) - 1$$

π	*C*		
501	527,50	0,00	0,00
485	260,00	255,00	0,00
515	186,19	186,19	186,19

t	*d*	*y*	*f*
1	0,950	5,3%	1,7%
2	0,934	3,5%	5,8%
3	0,883	4,2%	

5)

Macaulay duration is based on the assumption of a flat term structure and
is only concerned with parallel shifts in the term structure.
It is defined as

(A) $$D = \frac{1}{PV(C,y)} \sum_{t=1}^{T} t \frac{c_t}{(1+y)^t}$$

or

(B) $$D = \sum_{t=1}^{T} t w_t, \text{ where } w_t = \frac{c_t}{(1+y)^t} \cdot \frac{1}{PV(C,y)}$$

Interpretation:
1: Duration (D) is a bond's interest rate risk. A 1% interest rate increase/decrease causes an approximately D% price decrease/increase.
2: Duration can also be interpreted as the length of time that a bond can ensure an average annual return equal to the bond's rate r.

6)
Yield to maturity (and term structure) has been assumed to be 2,15%

Macaulay duration:

			w	t
3Y ann PV =	535,39			
PV(c1)	182,27		34%	1
PV(c2)	178,44		33%	2
PV(c3)	174,68		33%	3

Macaulay duration is $\sum t \times w(t)$ = 1,986
Modified duration = Macaulay duration / (1 + y) = 1,944

7)

Convexity:

$$K = \sum_{t=1}^{T} t^2 \cdot w_t \quad = \quad 4{,}610$$

8)
A nonzero convexity measure indicates that the bond's PV is a decreasing, convex function of interest rates. I.e. increasing interest rates decrease the bond's PV by a decreasing amount. Decreasing interetest rates increase the bond's PV by an increasing amount.

Thus the first order approximation from using the modified duration measure will underestimate the effect of decreasing interest rates (and overestimate the effect of increasing interest rates).

Problem 2 (Real Investments, 20%)

1)
Risk free 5%

Remember that the value of each project must be evaluated by its <u>incremental</u> Cash flows
That is cash flows from including a project relative to those of not including the project.

	t	A	B
Ini. Outlay	0	-100	-50
Cost saving	1	50	25
	2	50	25
	3	50	25
	4	50	25
	5		25
	6		25
	7		25
	8		25
NPV		73,62	106,27

2)

There are two methods to comparing mutually exclusive projects with different horizons:
a) Least common multiple of lives method (LCML)
b) Equivalent annual annuity method (EAA)

LCML method assumes that some or all projects can be replicated such that each project or string of replicated identical projects has an investment horizon equal to the least common multiple of individual project lives.

As an example the least common multiple of lives between project A and B would be 8 years assuming that project A could be replicated at the end of the 4th year.

The project or string of replicated identical projects with an investment horizon equal to the LCML that has the highest NPV identifies the profit maximizing investment opportunity for a long term investor.

EAA method calculates the annuity (incremental) cash flow that corresponds to a project with a principal value equal a project's NPV over its lifetime at an interest rate equal to the discount rate. The project that is found to have the highest annuity (incremental) cash flow is the most attractive to a long term investor.

3)

New incremental cash flow profile of projects:

(Old annual cost - New annual cost) = (100 - 75)

	t	A x 2	B
Ini. Outlay	0	-100	-50
Cost saving	1	50	25
	2	50	25
	3	50	25
	4	-50	25
	5	50	25
	6	50	25
	7	50	25
	8	50	25
NPV		134,18	106,27

(Old annual cost - New annual cost) - Initial Investment outlay= (100 - 50) - 100

A profit maximizing investor with an 8 year investment horizon would optimally invest in project A and replicate the same project at the end of the 4th year to an increase in the firm's net present value of 134,18

4)
Profitability Index (PI) = PV/-Cost <=> PV = PI x (-Cost)

Project	**Cost**	**PI**		**PV**
C	-1200	4,17		5004
D	-450	3,34		1503
E	-1000	4,80	=>	4800
F	-150	2,68		402
G	-800	3,76		3008

In decreasing order the most profitable projects are:

Project	**Cost**	**PV**	**PI**
E	-1000	4800	4,80
C	-1200	5004	4,17
G	-800	3008	3,75
D	-450	1503	3,33
F	-150	402	2,67

5)
Constraint 2150

Projects C + G + F have a total cost of 2150 and yield a total PV of 8414

6)
An investor would not necessarily always choose projects with the highest PI if they have a budget constraint. This is because a budget constraint may sometimes prevent him/her from choosing freely between projects. For each project the investor accepts to invest in, his set of feasible additional investments is reduced due to a lowered amount of free capital. A profit maximizing investor with a given budget constraint should seek to maximize the sum of his projects' total present value while keeping his expenses within his budget.

Problem 3 (Options, 20%)
Consider an economy where a non-dividend paying stock is trading at €50 and the risk-free rate is 10%. A zero-cost forward with the stock as the underlying asset is traded in the market. The forward matures in 1 year.

1. Estimate the zero-cost forward price of the forward

$F_0 = S_0 e^{r_f T} = 55{,}26$

Assume a European call option on the stock is traded in the market and that the stock fulfills the Black-Scholes assumptions. The option expires in one year and has a strike price of €40 and volatility of the stock's log return is 20%.

2. Estimate the price of the European call option.

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = 1{,}716$$
$d_2 = d_1 - \sigma\sqrt{T} = 1{,}516$
$N(d_1) = 0{,}957$
$N(d_2) = 0{,}935$
$c_0 = S_0 N(d_1) - K e^{-r_f T} N(d_2) = 14{,}00$

3. Estimate the price of a comparable European put option.

Use put-call parity in continous time:
$p_0 = c_0 - S_0 + K e^{-r_f T} = 0{,}19$

4. Comment briefly on how the price of the European call would be related to the price of the European put if the strike price was similar to zero-cost forward price in question 1.

Based on the put-call parity they should be identical.

5. Comment on the possibility of arbitrage if another European call option with identical maturity is trading with another implied volatility.

Arbitrage is possible as at least one of the options can be synthetically created by a tracking portfolio with a different price.

Now assume that the underlying asset with certainty pays a dividend of €15 such that the ex-dividend date is just prior to expiration of the European call option.

6. Estimate the price of the European call option on the dividend paying stock.

Strip the share price of dividends.
$S_0^* = S_0 - Div \times e^{-r_f T} = 36{,}43$
$$d_1^* = \frac{\ln\left(\frac{S_0^*}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = 0{,}132$$

$d_2^* = d_1^* - \sigma\sqrt{T} = -0{,}068$
$N(d_1^*) = 0{,}553$
$N(d_2^*) = 0{,}473$

$c_0^* = S_0^* N(d_1^*) - Ke^{-r_f T} N(d_2^*) = 3{,}01$

The price of the call is lower than in question 1 as expected.

Problem 4 (Essay questions, 30%)

1. Describe delta and gamma hedging?

- Delta hedging
  - Make a portfolio's value neutral to changes in the price of the underlying asset
  - Delta denotes the relative number of shares needed in order to hedge against movements in the share prices
  - Assumes that option price is linearly dependent on the price of the underlying asset
- Gamma hedging:
  - Corrects for the option's price not necessarily being linearly dependent on the price of the underlying assets
  - The purpose is to make the portfolio remaining delta hedged against subsequent share price movements
  - Thus, gamma hedging reflects the relative number of shares the portfolio should be adjusted with when the price of the underlying assets changes in order to remain delta-neutral

2. What are the implications of the Modigliani-Miller theorem on the firm's capital structure?

- Show it
- The value of the firm is unaffected by the mix of debt and equity financing

3. Discuss whether the cost of capital of a firm is identical to the required rate of return of a tax exempt investor's portfolio which only consists of the firm's securities. The mix of the portfolio is identical to the firm's debt-to-equity ratio.

The investors required rate of return is defined as:

$$r_A = \frac{D}{D+E} r_D + \frac{E}{D+E} r_E$$

The cost of capital is defined as:

$$WACC = \frac{D}{D+E} r_D (1 - T_C) + \frac{E}{D+E} r_E$$

Thus, in a world with corporate taxes the cost of capital is lower than the required rate of return on the investor's portfolio. This is caused by the debt being tax deductible.