Written Exam - Macroeconomics III

(suggested answers)

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Question 1

a We first set up the Lagrangian for households' optimization:

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \frac{h_t^{1+\nu}}{1+\nu} - \lambda_t \left[c_t + Rb_{t-1} - w_t h_t - b_t - d_t \right] - \phi_t \left[b_t - b \right] \right\} \tag{1}$$

The first order conditions with respect to the choice variables (c_t, h_t, b_t) are:

$$\frac{\partial \mathcal{L}_t}{\partial c_t} = 0 \Longrightarrow \frac{1}{c_t} - \lambda_t = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}_t}{\partial h_t} = 0 \Longrightarrow -h_t^{\nu} + \lambda_t w_t = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}_t}{\partial b_t} = 0 \Longrightarrow \lambda_t - \phi_t - \beta R \lambda_{t+1} = 0 \tag{4}$$

b The Euler equation reads as:

$$\frac{1}{c_t} = \phi_t + \beta R \frac{1}{c_{t+1}}$$

Assuming $\phi_t = \psi_t \lambda_t$:

$$\frac{1}{c_t} = \frac{\beta R}{1-\psi_t} \frac{1}{c_{t+1}} \tag{5}$$

Thus, ceteris paribus, an increase in ψ_t is equivalent to a tightening of the financial constraint, so that current consumption decreases.

c Firms' objective is to maximize lifetime discounted profits. However, since there are no dynamic links across periods, this is the same as maximizing profits in every periods. Specifically, the representative firm chooses labor hours so that

$$\max_{h_t} z_t h_t^{\alpha} - w_t h_t \tag{6}$$

which leads to the necessary condition:

$$\alpha z_t h_t^{\alpha-1} = w_t \tag{7}$$

d To characterize the labor market equilibrium, we take (7) as the labor demand function. As to the supply side of the labor market, combine (2) and (3) so as to get:

$$h_t^\nu = \frac{w_t}{c_t} \tag{8}$$

We then employ the market clearing condition $y_t = c_t$ and the technology constraint $y = z_t h_t^\alpha$ to substitute for h_t and c_t in (8):

$$h_t = \left(\frac{w_t}{z_t}\right)^{\frac{1}{\nu+\alpha}} \tag{9}$$

To find equilibrium wage and hours, we equalize (7) and (9), obtaining:

$$\begin{aligned} h_t &= \alpha^{\frac{1}{1+\nu}} && (10)\\ w_t &= \alpha^{\frac{\alpha+\nu}{1+\nu}} z_t && (11) \end{aligned}$$

e From (10), we notice that, as $\nu \to \infty$, $h_t \to 1$. Thus, as the elasticity of labor supply to the wage rate decreases (recall this is inversely related to ν), households allocate inelastically their entire time endowment to working.

Question 2

a Given the linear rule $\pi_t = \psi + \psi_\theta \theta_t$, as well as the fact that θ_t is observed by both the public and the policy maker before expectations are formed, output is determined as follows:

$$x_t = \theta_t + \pi_t - \pi_t^e = \theta_t + \psi + \psi_\theta\theta_t - (\psi + \psi_\theta\theta_t) = \theta_t$$

Thus, the expected loss reads as:

$$\begin{aligned} E\left[L(\pi_t, x_t)\right] &= \frac{1}{2}E\left[\left(\underbrace{\psi + \psi_\theta\theta_t}_{=\pi_t}\right)^2 + \lambda\left(\underbrace{\theta_t}_{=x_t} - \bar{x}\right)^2\right] \\ &= \frac{1}{2}E\left[\psi^2 + 2\psi\psi_\theta\theta_t + \psi_\theta^2\theta_t^2 + \lambda\left(\theta_t^2 - 2\bar{x}\theta_t + \bar{x}^2\right)\right] \\ &= \frac{1}{2}\left[\psi^2 + 2\psi\psi_\theta \underbrace{E\left[\theta_t\right]}_{=0} + \psi_\theta^2 \underbrace{E\left[\theta_t^2\right]}_{=\sigma_\theta^2} + \lambda\left(\underbrace{E\left[\theta_t^2\right]}_{=\sigma_\theta^2} - 2\bar{x}\underbrace{E\left[\theta_t\right]}_{=0} + \bar{x}^2\right)\right] \end{aligned}$$

Taking the first order conditions of $E\left[L(\pi_t, x_t)\right]$ with respect to ψ and ψ_θ we obtain:

$$\begin{aligned} \frac{\partial E\left[L(\pi_t, x_t)\right]}{\partial\psi} &= 0 : \psi = 0 \\ \frac{\partial E\left[L(\pi_t, x_t)\right]}{\partial\psi_\theta} &= 0 : \psi_\theta\sigma_\theta^2 = 0 \end{aligned}$$

Thus, the expected loss is minimized by setting $\psi = \psi_\theta = 0$, which implies $\pi_t^C = 0$ and $x_t^C = \theta_t$.

b When the central bank conducts a discretionary policy, the inflation rate is chosen after expectations are formed. Hence, the goal of the central bank is to minimize the loss function, i.e. the monetary policy should be ex post optimal, given π_t^e. Under this assumption, the problem reads as

$$\min_{\pi_t} \frac{1}{2}\left[\pi_t^2 + \lambda\left(\theta_t + \pi_t - \pi_t^e - \bar{x}\right)^2\right]$$

The first order condition for this problem reads as:

$$\frac{\partial L(\pi_t, x_t)}{\partial \pi_t} = 0 : \pi_t + \lambda\left(\theta_t + \pi_t - \pi_t^e - \bar{x}\right) = 0 \Leftrightarrow \pi_t^D = \frac{\lambda}{1+\lambda}\left(\pi_t^e - \theta_t + \bar{x}\right)$$

Thus, the expected rate of inflation is found by taking expectations:

$$E\left[\pi_t^D \middle| \theta_t\right] = \frac{\lambda}{1+\lambda} E_t\left[\pi_t^e - \theta_t + \bar{x}\right] = \frac{\lambda}{1+\lambda}\left(E_t\left[\pi_t^D \middle| \theta_t\right] - \theta_t + \bar{x}\right)$$

which implies $E\left[\pi_t^D \middle| \theta_t\right] = -\lambda\left(\theta_t - \bar{x}\right)$. Therefore:

$$\begin{aligned} \pi_t^D &= \frac{\lambda}{1+\lambda}\left(\underbrace{-\lambda\left(\theta_t - \bar{x}\right)}_{=\pi_t^e} - \theta_t + \bar{x}\right) = -\lambda\left(\theta_t - \bar{x}\right) \\ x_t^D &= \theta_t \end{aligned}$$

The excessively high equilibrium inflation associated with the inflation bias problem results from the combination of a lack of commitment and central bank's temptation to temporarily boost the economy beyond its potential level. The latter incentive is embodied by the condition $\bar{x} > \theta$. This makes it clear why raising $\bar{x}$ increases the temptation of the central bank to generate excess inflation in the vain attempt to stimulate real activity.

c Once again, when the central bank conducts a discretionary policy, the inflation rate is chosen after expectations are formed. Hence, the goal of the central bank is to minimize the loss function, i.e. the monetary policy should be ex post optimal, now given π_t^e and θ_t, as the latter is not observed. Under this assumption, the problem reads as

$$\min_{\pi_t} \frac{1}{2}\left[\pi_t^2 + \lambda\left(\theta_t + \pi_t - \pi_t^e - \bar{x}\right)^2\right]$$

The first order condition for this problem reads as:

$$\frac{\partial L(\pi_t, x_t)}{\partial \pi_t} = 0 : \pi_t + \lambda\left(\theta_t + \pi_t - \pi_t^e - \bar{x}\right) = 0 \Leftrightarrow \pi_t^D = \frac{\lambda}{1+\lambda}\left(\pi_t^e - \theta_t + \bar{x}\right)$$

Now, the expected rate of inflation is found by taking unconditional expectations (as θ_t is not observed before expectations are formed):

$$E\left[\pi_t^D\right] = \frac{\lambda}{1+\lambda} E\left[\pi_t^e - \theta_t + \bar{x}\right] = \frac{\lambda}{1+\lambda}\left(E\left[\pi_t^D\right] + \bar{x}\right)$$

which implies $E\left[\pi_t^D\right] = \lambda\bar{x}$. Therefore:

$$\begin{aligned}
\pi_t^{D*} &= \frac{\lambda}{1+\lambda}\left(\underbrace{\lambda\bar{x}}_{=\pi_t^e} - \theta_t + \bar{x}\right) = \frac{\lambda}{1+\lambda}\left[\bar{x}\left(1+\lambda\right) - \theta_t\right] \\
x_t^{D*} &= \theta_t + \frac{\lambda}{1+\lambda}\left[\bar{x}\left(1+\lambda\right) - \theta_t\right] - \lambda\bar{x} = \frac{1}{1+\lambda}\theta_t
\end{aligned}$$

As we set $\lambda = 0$, the policy maker does not face a real activity stabilization objective, so that there is no temptation to inflate the economy to raise output above the target. Thus, no matter the information structure, output will always be equal to θ_t, and thus to its solution under commitment. The same holds true for the rate of inflation.