Solutions for exam in Auctions, 18 August 2014

The following solutions manual gives thorough answers to all questions. This level of detail is not required to obtain full marks on each question.

Exercise 1: True or false statements (20%)

Question 1a: False.

A standard auction is an auction in which the person that submits the highest bid is awarded the object. In a lottery, the winner is drawn at random so the person that submitted the highest bid for the object is not necessarily awarded the object.

Question 1b: True.

The article by Roth and Ockenfels ("Late and multiple bidding in second price Internet auctions: Theory and evidence concerning different rules for ending an auction") shows both theoretically and empirically that the closing rule of the auction site and the features of the good for sale influences the amount of sniping. An auction site with a 'hard close' rule has more sniping than sites with a 'soft close' rule, partly because bidders with valuable insights about the good for sale can better avoid signaling this information by submitting a late bid in an auction with a hard close rule. Similarly, goods for which expert knowledge is key to correctly value the good are more prone to sniping than standard goods where bidders can easily find retail price comparisons on other sites. Antiques are used in the article as an example of a type of good where dealers and experts are better able to assess the correct value whereas computers are used as an example of a type of good where the retail price is readily available.

Question 1c: False.

Every undominated equilibrium of the uniform-price auction has the property that the bid on the first unit is equal to the value of the first unit whereas bids on other units are lower than the respective marginal values [Krishna, proposition 13.4].

The intuition for bidding below valuation on secondary units is similar to the intuition for bidding below valuation in single-unit first price auctions. There is an incentive to bid below valuation because each bid can determine the price paid and a bidder is then better off winning at a lower price. One difference between a single-unit first price auction and a multiple-unit uniform price auction is that the incentive to lower your bid in case it determines the price is reinforced by the fact that it determines not just the price paid for one unit but for all the units won.

The intuition for bidding equal to valuation for the first unit is similar to the intuition for bidding equal to valuation in single-unit second price auctions. If the bidder bids truthfully and wins the object it will be another bid (the first losing bid) that determines the price paid. If the bidder bids truthfully and does not win the object he could only have won the object at a price that he was not willing to pay (he would have to beat one of the existing winning bids in which case that bid would become the new highest losing bid that determines the price).

Exercise 2: Blueland (30%)

Question 2a: The proposed approach has serious information problems. The mining company that carries out the excavations, Berlin Mining, will have a large advantage over the other mining companies by getting first-hand information about the minerals in the subsoil. Moreover, Berlin Mining will have incentive to misinform the other mining companies in its excavation report since these companies are its competitors in the subsequent auction for the mining rights. The second-hand information in the report will not be very credible.

To minimize these information problems, the auctioneer can choose an open auction format that allows for price discovery. An open format, e.g. an English auction, will allow the other mining companies to observe how many bidders are (still) bidding in the auction and, notably, whether Berlin Mining is (still) bidding in the auction. A sealed-bid format will, on the other hand, not allow companies to observe the bids of, and thus learn from, Berlin Mining during the auction.

Question 2b: It is assumed that the excavations have been conducted, the report has been written, and all bidders are aware of this. In case the report shows large amount of minerals the field is more valuable than expected: In case the report shows a small amount it is less valuable than expected. The question is whether the government strategically should withhold a bad looking report. Regardless of the results of the report, publishing it or not will send a signal to the bidders. Publishing good results obviously sends a high signal, whereas not publishing will lead bidders to expect a bad report and thus sends a low signal.

It is assumed the government is a revenue-maximizing seller auction, and the excavation report is affiliated with the bidders own estimates on the amount of minerals in the fields. The linkage principle can help the assess the release of public information.

The linkage principle compares the statistical linkages between a bidders own signal and the price he would pay upon winning. The greater the linkage between a bidders own information and how he perceives others will bid, the greater the expected price paid upon winning.

In chapter 7.2 the linkage principle ranks the revenue of two auctions. A) *One* auction where public information **is** published and B) *one* where it **is not** published.

A) Publishing the report in a first-price auction. Suppose bidder 1's strategy is a function of the report, S, and his own signal, X_1. Suppose there exists a symmetric equilibrium strategy $\hat{\beta}(S, X_i)$ increasing in both variables. The expected price paid by the bidder if he is the *winning* bidder when he receives a signal x but bids as if this signal were z (that is, for all $S = s$, he bids $\hat{\beta}(s, z)$) is

$$\widehat{W}^I(z, x) = E\left[\hat{\beta}(S, X_i) | X_1 = x\right]$$

So $\widehat{W}_2{}^I(z, x) \geq 0$ because S and X_1 are affiliated.

B) Not publishing the report in a first-price auction. When the public information is not available, then we have that if equilibrium strategy $\beta \equiv \beta^I$, then the expected price paid by the bidder if he is the winning bidder when he receives a signal x but bids af is this signal were z

$$W^I(x, z) = \beta(z)$$

So $W_2^I(z, x) = 0$

We therefore have

$$\widehat{W}_2{}^I(z, x) \geq W_2^I(z, x)$$

Applying the linkage principle, we can see the expected revenue in a first price auction is higher, when the public information is made available than when it is not.

Similar arguments can be made for the second-price auction and the English auction.

A quick note on efficiency: Publishing the report does not change efficiency, as the signal is equally interpreted by all companies.

If mining companies have similar technology levels and cost functions the auction is always efficient, since it is equally valuable to all bidders. Oppositely, if some have spare capacity, lower costs, and better channels to sell the raw minerals, the auction is not always efficient as it allocates the right to mine for minerals to the bidder with the highest signal and not highest value.

We require the single crossing condition (proposition 6.7), such that *ex post* values of different bidders will be ordered in the same way as their signals.

Question 2c: Their concerns are unwarranted. In a symmetric and increasing equilibrium then all units for sale will be allocated efficiently in both sequential first-price auctions and sequential second-price auctions [Krishna, chapter 15]. If all bidders apply the same bidding strategy (i.e. a symmetric strategy) and this strategy implies bidding more if you have a higher valuation (i.e. an increasing strategy) then the objects will be allocated according to valuations. The bidder with the highest valuation will be awarded the first object, the bidder with the second-highest valuation will be awarded the second object and so on.

<u>Exercise 3: Multiple object auctions (15%)</u>

Question 3a: The winning bids are the 8 highest bids. They are $\{95, 87, 86, 85, 82, 76, 55, 54\}$. The highest losing bid is 37 meaning that all winning bidders pay 37 for each object. That means that:

- Bidder 1 wins 3 objects and pays 3*37=111

- Bidder 2 wins 1 object and pays 1*37=37
- Bidder 3 wins 2 objects and pays 2*37=74
- Bidder 4 wins 2 objects and pays 2*37=74

Question 3b: The same 8 bids are the winning bids. In a Vickrey auction each bidder pays an amount equal to the sum of the bids that did not become winning bids because they were defeated by the given bidder's winning bids. In other words, each bidder pays for the externality he exerts on the other bidders. That means that:

- Bidder 1 wins 3 objects and pays 37+35+35=107
- Bidder 2 wins 1 object and pays 37
- Bidder 3 wins 2 objects and pays 35+31=66
- Bidder 4 wins 2 objects and pays 37+35=72

<u>Exercise 4: An auction for a house (35%)</u>

Question 4a: It is a weakly dominant strategy to bid truthfully, i.e. $\beta^{II}(x) = x$, in a second price auction [proposition 2.1 in Krishna].

To argue why, one can e.g. follow the reasoning in the proof of proposition 2.1 by considering the three cases for bidder 1 for given bids from its competitors, where the highest competing bidders bid is $p_1 = \max_{j \neq 1} b_j$:

1. Suppose bidder 1 bids his value, $b_1 = x_1$.
    a. He wins if $x_1 > p_1$, and gets profit $x_1 - p_1$
    b. He does not win if $x_1 < p_1$, and gets profit 0
    c. He is indifferent between winning or not when $x_1 = p_1$, as his surplus is 0
2. Suppose bidder 1 bids below his value, $z_1 < x_1$.
    a. If $x_1 > z_1 \geq p_1$, he still wins, but gets the same profit $x_1 - p_1$
    b. If $p_1 > x_1 > z_1$, he still loses, and gets profit 0
    c. If $x_1 > p_1 > z_1$, he loses whereas if he had bid x_1, he would have made a positive profit.
3. Suppose bidder 1 bids above his value, $z_1 > x_1$.
    a. If $z_1 > x_1 \geq p_1$, he still wins, but gets the same profit $x_1 - p_1$
    b. If $p_1 > z_1 > x_1$, he still loses, and gets profit 0
    c. If $z_1 > p_1 > x_1$, he wins, making a negative profit $x_1 - p_1 < 0$ whereas if he had bid x_1, he would have lost making zero profit.

In both deviating cases 2 and 3, the bidder is a best making the same profit, but can potentially forego profit by not winning or paying too much.

Question 4b: With uniform distributed values $X \sim U[a, b]$ where $a = 0, b = 8$ the cumulative distribution function and probability distribution functions are

$$F(x) = \frac{x-a}{b-a} = \frac{x-0}{8-0} = \frac{1}{8}x$$
$$f(x) = \frac{1}{b-a} = \frac{1}{8}$$

The value of the highest competing bid, Y_1 a highest order statistic distributed according to $G(x)$ [see appendix C]

$$G(x) = \left(F(x)\right)^{n-1} = \left(\frac{1}{8}x\right)^{3-1} = \frac{1}{64}x^2$$
$$g(x) = G'(x) = \frac{2}{64}x = \frac{1}{32}x$$

The equilibrium strategy bidding in a first price auction is given by

$$\beta^I(x) = \frac{1}{G(x)} \int_0^x y \cdot g(y) dy$$

Now insert the g(x) and G(x)

$$\beta^I(x) = \frac{1}{\left(\frac{x^2}{64}\right)} \int_0^x y \cdot \frac{y}{32} \cdot dy$$
$$= \frac{64}{32} \cdot \frac{1}{x^2} \cdot \left[\frac{1}{3} y^3\right]_0^x = \frac{2}{x^2} \cdot \frac{x^3}{3}$$
$$= \frac{2}{3} x$$

Question 4c: The expected payment of a bidder in a first price sealed bid auction is

$$m^I(x) = \int_0^x y g(y) dy$$

Now insert $g(x)$

$$m^I(x) = \int_0^x y \cdot \left(\frac{1}{32} y\right) dy$$
$$m^I(x) = \frac{1}{32} \left[\frac{1}{3} y^3\right]_0^x = \frac{1}{96}(x^3 - 0^3) = \frac{1}{96} x^3$$

Question 4d:

$$E[m^I(x_i)] = \int_0^\omega m^I(x_i) \cdot f(x) dx = \int_0^8 \frac{1}{96} x^3 \cdot \frac{1}{8}\, dx$$
$$E[m^I(x_i)] = \frac{1}{96} \cdot \frac{1}{8} \cdot \left[\frac{1}{4} x^4\right]_0^8 = \frac{4}{3}$$

Question 4e: The expected revenue to the seller is the *ex-ante* expected payment multiplied with the number of bidders

$$E[R^A] = N \cdot E[m^I(X)] = 3 \cdot \frac{4}{3} = 4$$

Hence, the couple is expected to make 4 million DKK on the sale of the house.