Written Exam for the B.Sc. in Economics summer 2011

Macro A -- Solution

Reexamination

11. August 2011

(3-hour closed book exam)

1.1)

Social infrastructure can be defined as e.g. the quality of institutions and government policy. Factors influencing the social infrastructure of a country are e.g. the degree to which property rights are protected, the quality of the educational system, the extent of corruption and the openness to foreign trade and investment. The idea in Hall and Jones (1998) is that differences in social infrastructure across countries can account for a large part of cross-country differences in productivity and investment rates in physical and human capital, since social infrastructure influences the incentives to invest and accumulate skills. This hypothesis is supported by empirical evidence, even though it isn't possible to measure social infrastructure perfectly.

1.2)

Productive externalities can generate endogenous growth by creating increasing returns to scale at the social level. In chapter 8 is e.g. considering a model where the level of productivity depends positively on the aggregate capital stock, due to learning by doing (– or more precisely learning by investing) and fast knowledge spill-overs between firms. This learning by doing implies that when the capital stock and the labour force is increased by 1 percent the level of productivity will increase which further increases output, such that output increases by more than 1 percent. If the learning by doing effect is so strong that there are constant returns to scale with respect to capital at the social level truly endogenous growth can be sustained, i.e. positive long run growth in the absence of growth in the labour force. If however there are decreasing returns to capital at the social level (but increasing returns to capital and labour together at the social level) then steady state growth can only be sustained in the presence of growth in the labour force.

1.3)

The solution should explain the result in section 11.4. In the general equilibrium model of efficiency wages labour productivity depends positively on excess of the wage rate over the outside option (the expected income if a worker gets fired). The reason that the real wage isn't influenced by an increase in the unemployment benefit is that there are two opposing effects on the real wage which (by coincidence) exactly cancel each other. The first effect is that an increase in the unemployment benefit tends to increase the outside option and thereby decrease productivity. In order to counteract the decrease in productivity firms respond by setting higher wages. On the other hand firms respond by the tendency for a lower productivity (which is the same as an increase in real marginal costs per efficiency unit) by setting higher prices, since the price is set as a mark-up over the efficiency-adjusted wage rate. Higher goods prices decrease the real wage rate. These two effects exactly cancel each other leaving the real wage unaffected.

2.1)

Equation 1) is a Cobb-Douglas production function describing how output from the final goods sector depends on the aggregate capital stock and the amount of labour allocated to the final goods sector. The Cobb-Douglas production implies that the output elasticities with respect to capital and labour are constant.

Equation 2) is a production function for the research sector, describing how the output of the research sector (the creation of new ideas) depends on the amount of labour allocated to the research sector and the existing stock of ideas. In the case where $\lambda < 1$ there are decreasing returns to labour, capturing 'stepping-on-toes', i.e. duplication in the research process. The existing stock of ideas can affect the productivity of the research sector both positively and negatively. If $\phi > 0$ there is a positive externality, capturing 'standing-on-shoulders', i.e. the existing stock of knowledge makes it easier to come up with new inventions. If, on the other hand $\phi < 0$ there is a negative effect, which could capture 'fishing-out', reflecting that the most obvious ideas are discovered first, and that it get harder to come up with new ideas.

Equation 3) simply states that the total labour force must be allocated between production of final goods and the research sector.

Equation 4) states that the fraction of labour allocated to the research sector is assumed to be constant (and exogenous) and given by s_R.

Equation 5) is the capital accumulation equation stating that the source of capital accumulated is net investment, i.e. investment net of depreciation. Further investment equals savings (implying that the economy is closed) which is assumed to be a constant fraction of GDP.

Equation 6) states that the total labour force in the economy is assumed to grow at the constant and exogenous rate n each period.

The non-rival nature of ideas is reflected in the fact that the existing stock of ideas is productive in the production of final goods and the research sector simultaneously.

2.2)

From equation 2) we get:

$$g_t = \frac{A_{t+1} - A_t}{A_t} = \rho \cdot L_{At}^{\lambda} \cdot A_t^{\phi-1}$$

Forwarding this one period we get:

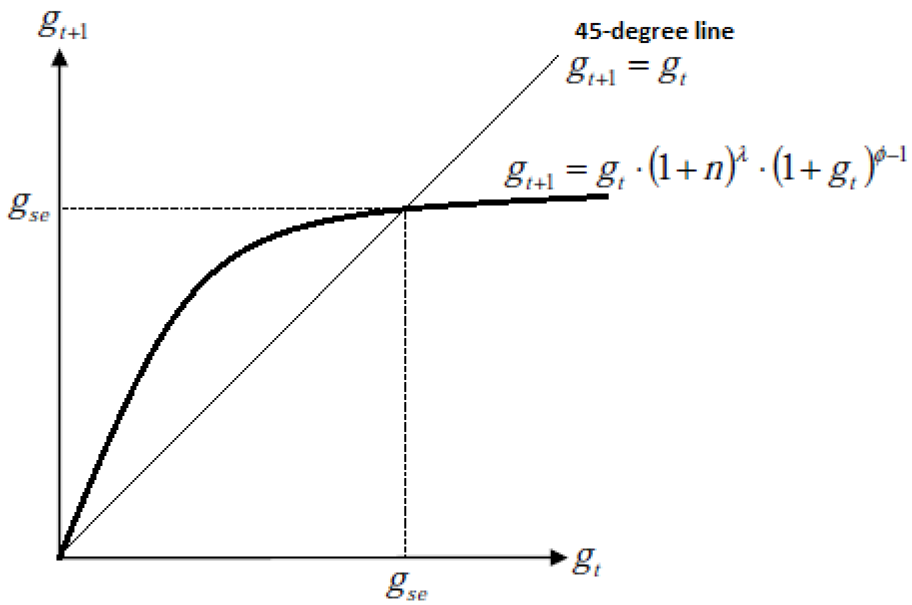
$$g_{t+1} = \rho \cdot L_{At+1}^{\lambda} \cdot A_{t+1}^{\phi-1} \Rightarrow$$

$$\frac{g_{t+1}}{g_t} = \frac{\rho \cdot L_{At+1}^{\lambda} \cdot A_{t+1}^{\phi-1}}{\rho \cdot L_{At}^{\lambda} \cdot A_t^{\phi-1}} = \left(\frac{L_{At+1}}{L_{At}}\right)^{\lambda} \cdot \left(\frac{A_{t+1}}{A_t}\right)^{\phi-1}$$

Now insert equation 4) and 6) and use that $\frac{A_{t+1}}{A_t} = 1 + g_t$:

$$\frac{g_{t+1}}{g_t} = \left(\frac{s_R \cdot L_{t+1}}{s_R \cdot L_t}\right)^{\lambda} \cdot (1 + g_t)^{\phi-1} = (1+n)^{\lambda} \cdot (1 + g_t)^{\phi-1} \Rightarrow$$

$$g_{t+1} = g_t \cdot (1+n)^{\lambda} \cdot (1 + g_t)^{\phi-1}$$



2.3)

In steady state the growth rate of A_t is constant over time implying that: $g_{t+1} = g_t$. From the transition curve this implies that:

$$(1+n)^{\lambda} \cdot (1+g^*)^{\phi-1} = 1 \Rightarrow 1 + g^* = (1+n)^{\frac{\lambda}{(1-\phi)}} \Rightarrow$$

$$g^* = (1+n)^{\frac{\lambda}{(1-\phi)}} - 1 \equiv g_{se}$$

This is illustrated in the diagram above as the point where the transition curve intersects the 45-degree line.

The steady state growth rate of knowledge depends positively on the growth rate of the labour force since a higher growth rate of the labour force implies that the labour input into the research sector grows faster (assuming a constant s_R). With a higher flow of scientists and engineers into the research sector more discoveries are made, and thus the stock of knowledge grows faster.

Steady state growth in stock of knowledge can not be sustained in the case $n = 0$ since there are decreasing returns to the existing stock of knowledge in the creating of new knowledge (since $\phi < 1$). From the expression derived in question 2.2:

$$g_t = \frac{A_{t+1} - A_t}{A_t} = \rho \cdot L_{At}^{\lambda} \cdot A_t^{\phi-1} = \frac{\rho \cdot (s_R \cdot L_t)^{\lambda}}{A_t^{1-\phi}}$$

we see that if the labour force employed in the research sector is constant (which is the case with $n = 0$) the growth rate of knowledge will converge towards zero since A_t is growing over time and $\phi < 1$.

From the expression:

$$g_t = \frac{A_{t+1} - A_t}{A_t} = \rho \cdot L_{At}^{\lambda} \cdot A_t^{\phi-1}$$

we can derive an expression for the steady state stock of knowledge:

$$\rho \cdot L_{At}^{\lambda} \cdot (A_t^*)^{\phi-1} = g_{se} \Rightarrow A_t^* = \left(\frac{\rho \cdot L_{At}^{\lambda}}{g_{se}}\right)^{1/(1-\phi)}$$

Now insert equation 4):

$$A_t^* = \left(\frac{\rho \cdot (s_R \cdot L_t)^{\lambda}}{g_{se}}\right)^{1/(1-\phi)}$$

The steady state stock of knowledge depends positively on the productivity of the research sector (ρ), since more discoveries can be made for a given labour force when ρ is higher. Also the steady state stock of knowledge depends positively on s_R, since a higher value of s_R implies more scientist and engineers and thus more discoveries (*this is a level effect – the steady state growth rate of knowledge is not influenced by s_R due to the assumption of decreasing returns to the existing stock of knowledge*).

2.4)

From equation 1), 3) and 4) we get:

$$\tilde{y}_t = \frac{K_t^\alpha \cdot (A_t \cdot (1-s_R) \cdot L_t)^{1-\alpha}}{A_t \cdot L_t} = (1-s_R)^{1-\alpha} \cdot \frac{K_t^\alpha \cdot (A_t \cdot L_t)^{1-\alpha}}{A_t \cdot L_t} =$$

$$(1-s_R)^{1-\alpha} \cdot K_t^\alpha \cdot (A_t \cdot L_t)^{-\alpha} = (1-s_R)^{1-\alpha} \cdot \left(\frac{K_t}{A_t \cdot L_t}\right)^\alpha = (1-s_R)^{1-\alpha} \cdot \tilde{k}_t^\alpha$$

From equation 5), 6) and the fact that: $A_{t+1} = A_t \cdot (1+g_t)$ we get:

$$\tilde{k}_{t+1} = \frac{K_{t+1}}{A_{t+1} \cdot L_{t+1}} = \frac{K_t \cdot (1-\delta) + s \cdot Y_t}{A_t \cdot (1+g_t) \cdot L_t \cdot (1+n)} =$$

$$\frac{1}{(1+g_t) \cdot (1+n)} \cdot \frac{K_t \cdot (1-\delta) + s \cdot Y_t}{A_t \cdot L_t} = \frac{\tilde{k}_t \cdot (1-\delta) + s \cdot \tilde{y}_t}{(1+g_t) \cdot (1+n)}$$

Finally inserting the expression for $\tilde{y}_t$ we get:

$$\tilde{k}_{t+1} = \frac{\tilde{k}_t \cdot (1-\delta) + s \cdot (1-s_R)^{1-\alpha} \cdot \tilde{k}_t^\alpha}{(1+g_t) \cdot (1+n)}$$

By subtracting $\tilde{k}_t$ on both sides we get:

$$\tilde{k}_{t+1} - \tilde{k}_t = \frac{\tilde{k}_t \cdot (1-\delta) + s \cdot (1-s_R)^{1-\alpha} \cdot \tilde{k}_t^\alpha}{(1+g_t) \cdot (1+n)} - \tilde{k}_t \cdot \frac{(1+g_t) \cdot (1+n)}{(1+g_t) \cdot (1+n)} =$$

$$\frac{\tilde{k}_t \cdot (1-\delta) + s \cdot (1-s_R)^{1-\alpha} \cdot \tilde{k}_t^\alpha}{(1+g_t) \cdot (1+n)} - \tilde{k}_t \cdot \frac{1+n+g_t+n \cdot g_t}{(1+g_t) \cdot (1+n)} =$$

$$\frac{s \cdot (1-s_R)^{1-\alpha} \cdot \tilde{k}_t^\alpha - \tilde{k}_t \cdot (\delta + n + g_t + n \cdot g_t)}{(1+g_t) \cdot (1+n)}$$

These equations simply states that the source of growth in the capital stock is savings (investment – but we consider a closed economy such that investment = savings) which is assumed to be a constant fraction of

GDP. On the other hand depreciation, growth in the labour force and technological progress tends to decrease the amount of physical capital per effective worker.

The evolution of $\tilde{k}_t$ differs from the evolution in the general Solow model since the growth rate of technology isn't constant outside steady state and also, only a part of the labour force is engaged in the production of the final good.

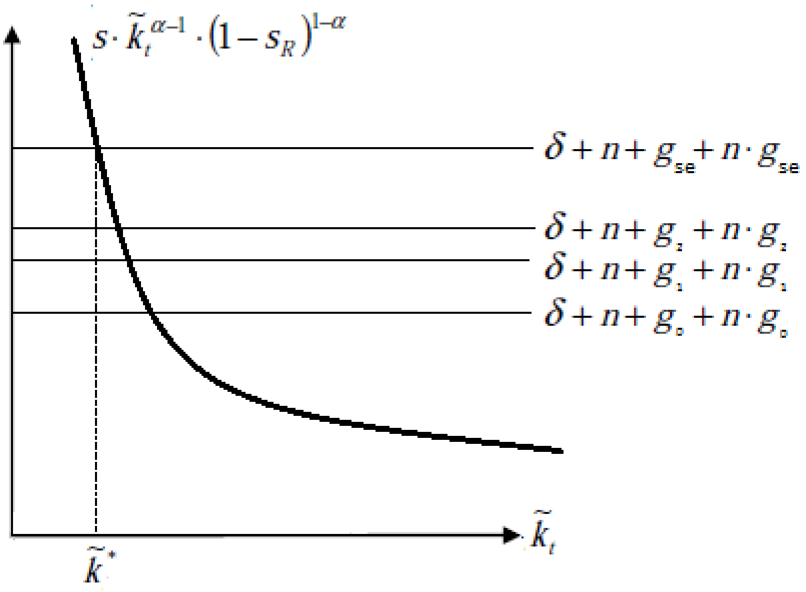
2.5)

From the equation for $\tilde{k}_{t+1} - \tilde{k}_t$ derived above we get:

$$\frac{\tilde{k}_{t+1} - \tilde{k}_t}{\tilde{k}_t} = \frac{1}{\tilde{k}_t} \cdot \frac{s \cdot (1 - s_R)^{1-\alpha} \cdot \tilde{k}_t^{\alpha} - \tilde{k}_t \cdot (\delta + n + g_t + n \cdot g_t)}{(1 + g_t) \cdot (1 + n)} =$$

$$\frac{s \cdot (1 - s_R)^{1-\alpha} \cdot \tilde{k}_t^{\alpha-1} - (\delta + n + g_t + n \cdot g_t)}{(1 + g_t) \cdot (1 + n)}$$

Compared to the Solow model the new element is that the growth rate of technology isn't constant outside steady state. Thus the equilibrium towards which the economy is converging is a 'moving target'. This further implies that the convergence is slower, compared to the Solow model, since the economy cannot settle down at steady state before the growth rate of technology has settled down at its steady state level. In the diagram below (a modified Solow diagram) the economy will at first (in period 0) converge towards the intersection between the downward sloping curve (given by $s \cdot (1 - s_R)^{1-\alpha} \cdot \tilde{k}_t^{\alpha-1}$) and the $(\delta + n + g_0 + n \cdot g_0)$-curve. However in the next period the growth rate of technology increases to g_1 and the economy will now converge towards the intersection between the downward sloping curve and the $(\delta + n + g_1 + n \cdot g_1)$-curve. This process will continue until the growth rate of technology has settled down at the steady state value (g_{se}). When the growth rate of technology has reached its steady state level the capital stock will start converging towards the steady state value $\tilde{k}^*$ where capital per effective worker is constant over time (this is characterized by the intersection between the $s \cdot (1 - s_R)^{1-\alpha} \cdot \tilde{k}_t^{\alpha-1}$-curve and the $(\delta + n + g_{se} + n \cdot g_{se})$-curve in the diagram below).



2.6)

By definition the steady state is given by g_t and $\tilde{k}_t$ being constant over time.

The condition for $\tilde{k}_t$ being constant is that: $\tilde{k}_{t+1} - \tilde{k}_t = 0$ from which we get:

$$\frac{s \cdot (1-s_R)^{1-\alpha} \cdot \left(\tilde{k}^*\right)^{\alpha} - \tilde{k}^* \cdot (\delta + n + g^* + n \cdot g^*)}{(1+g_t) \cdot (1+n)} = 0 \Rightarrow$$

$$\left(\tilde{k}^*\right)^{1-\alpha} = \frac{s \cdot (1-s_R)^{1-\alpha}}{\delta + n + g^* + n \cdot g^*} \Rightarrow$$

$$\tilde{k}^* = \left(\frac{s}{\delta + n + g^* + n \cdot g^*}\right)^{1/(1-\alpha)} \cdot (1-s_R)$$

which further implies that:

$$\tilde{y}^* = (1-s_R)^{1-\alpha} \cdot \left(\tilde{k}^*\right)^{\alpha} = (1-s_R)^{1-\alpha} \cdot \left(\frac{s}{\delta + n + g^* + n \cdot g^*}\right)^{\frac{\alpha}{(1-\alpha)}} \cdot (1-s_R)^{\alpha} =$$

$$(1-s_R) \cdot \left(\frac{s}{\delta + n + g^* + n \cdot g^*}\right)^{\frac{\alpha}{(1-\alpha)}}$$

Now using that $y_t = A_t \cdot \tilde{y}_t$ and $k_t = A_t \cdot \tilde{k}_t$ and inserting equation 9) we get:

$$k_t^* = \tilde{k}^* \cdot A_t^* = \left(\frac{s}{\delta + n + g^* + n \cdot g^*}\right)^{1/(1-\alpha)} \cdot (1 - s_R) \cdot \left(\frac{\rho \cdot (s_R \cdot L_t)^{\lambda}}{g_{se}}\right)^{1/(1-\phi)}$$

$$y_t^* = \tilde{y}^* \cdot A_t^* = \left(\frac{s}{\delta + n + g^* + n \cdot g^*}\right)^{\alpha/(1-\alpha)} \cdot (1 - s_R) \cdot \left(\frac{\rho \cdot (s_R \cdot L_t)^{\lambda}}{g_{se}}\right)^{1/(1-\phi)}$$

Since $\tilde{y}^*$ and $\tilde{k}^*$ are constant in steady state $y_t^* = \tilde{y}^* \cdot A_t^*$ and $k_t^* = \tilde{k}^* \cdot A_t^*$ must be growing at the same rate as A_t^*, such that in steady state:

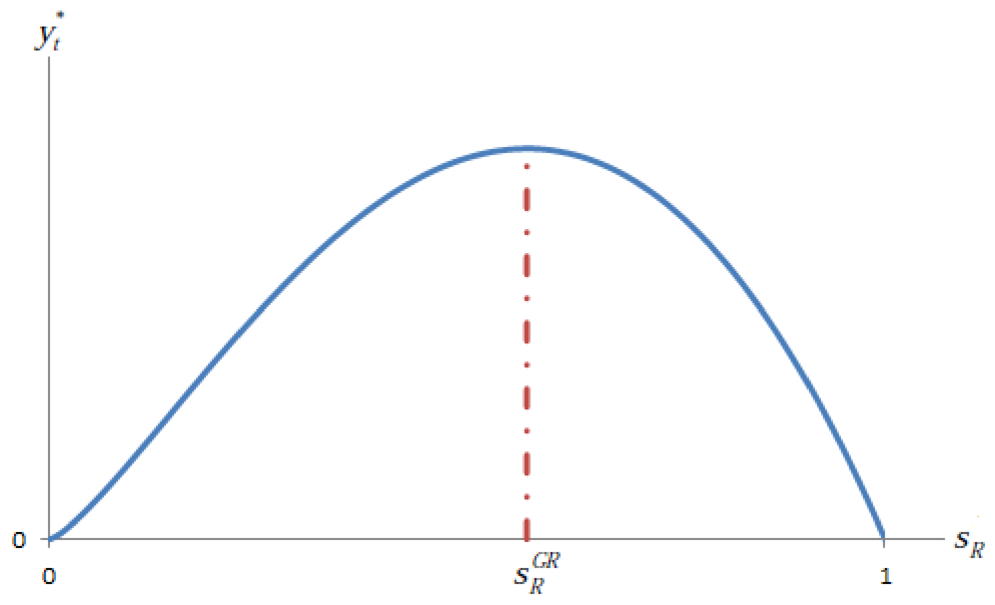
$$\frac{k_{t+1} - k_t}{k_t} = \frac{y_{t+1} - y_t}{y_t} = \frac{A_{t+1} - A_t}{A_t} = (1+n)^{\frac{\lambda}{(1-\phi)}} - 1$$

An increase in s_R will decrease the amount of labour allocated to production of final goods tending to decrease steady state production of final goods and capital accumulation, for a given level of knowledge. However the steady state level of knowledge also increases (since more scientists and engineers implies that more ideas are discovered), which tends to increase steady state production of final goods and capital accumulation due to the increased productivity (knowledge).

When L_t increases more labour can be allocated to both the production of final goods and knowledge simultaneously. Thus, the first effect described above vanishes and steady state production of final goods and capital accumulation increases due to the higher steady state level of knowledge.

2.7)

The relationship between s_R and y_t^* is u-inverted as illustrated below. For low values of s_R the second effect mentioned above will dominate while the first effect will dominate for high values of s_R.



Taking logs in the expression for y_t^* we get:

$$lny_t^* = \tilde{y}^* \cdot A_t^* = ln(1 - s_R) + \frac{\lambda}{1 - \phi} \cdot lns_R + lnX_t$$

where X_t is independent of s_R. We can find the value of s_R which maximizes y_t^* by setting the derivative of lny_t^* with respect to s_R equal to zero:

$$\frac{\partial lny_t^*}{\partial s_R} = -\frac{1}{1 - s_R} + \frac{\lambda}{1 - \phi} \cdot \frac{1}{s_R} = 0 \Rightarrow$$

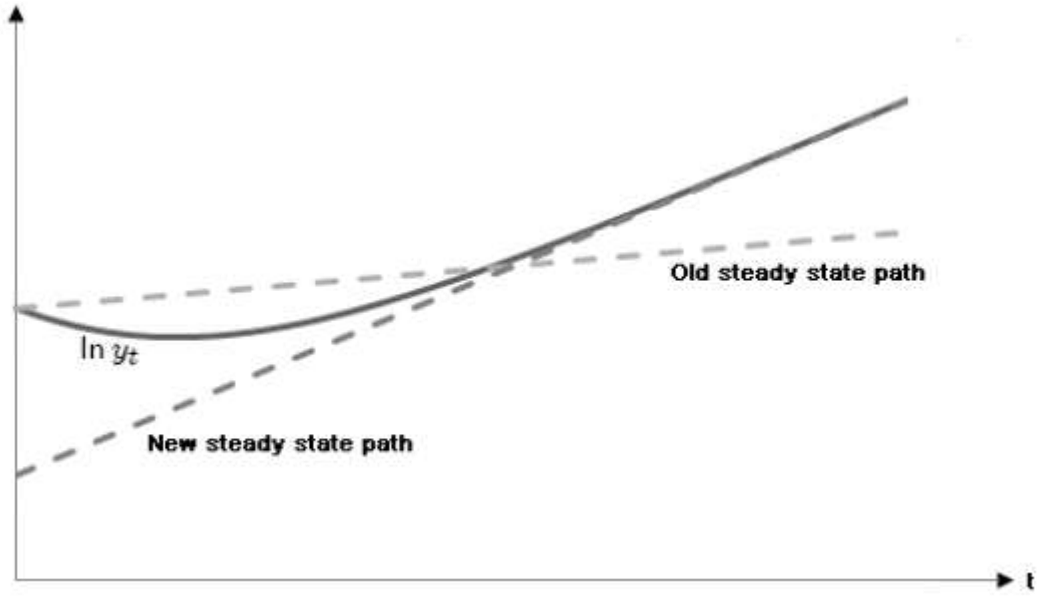
$$(1 - \phi) \cdot s_R = \lambda \cdot (1 - s_R) \Rightarrow$$

$$(1 - \phi + \lambda) \cdot s_R = \lambda \Rightarrow s_R = \frac{\lambda}{1 - \phi + \lambda}$$

We see that the golden rule value of s_R depends positively on ϕ, since a high value of ϕ implies a large increase in the level of knowledge when more labour is allocated to the research sector. Conversely, a lower value of λ will decrease the golden rule value of s_R since this implies more duplication (stepping-on-toes) in the research process, which lowers the increase in knowledge resulting from an increase in the amount of labour allocated to the research sector.

2.8)

The consequence of an increase in n on output per worker is illustrated in the diagram below.



An increase in n will increase the steady state growth rate of technology and output per worker (see question 2.3 and 2.6). In the diagram the new steady state growth path has a higher slope. However while converging towards the new steady state output per worker will be lower as a result of the increase in n. This is due to the fact that a higher growth rate of the labour force imply a 'thinning-out' effect, since the increase in output cannot keep up with the higher increase in the labour force (due to diminishing returns to labour). At first output per worker will fall (as showed in the diagram) while output will eventually be higher (compared to the old steady state path).

The steady state effect of an increase in n isn't very plausible since empirical cross-country evidence tends to indicate a negative relationship between labour force (population) growth and prosperity (both in terms of the level and the growth rate of GDP per worker). Thus, the prediction that steady state growth in output per worker (and ultimately also the level of output per worker) should depend positively on the growth rate of the labour force. However, one may object to this conclusion for different reason. First of all, one can object towards evaluating the model in terms of cross-country evidence if knowledge flows between countries. In this case the model considered above should be seen as covering the region of developed countries or maybe even the entire world. Also one can argue that maybe the empirical evidence should contain longer periods since we are actually comparing steady state, and convergence towards steady state might be very long-lasting.