

Written Exam for the B.Sc. in Economics autumn 2012-2013

**Microeconomics B**

Final Exam

Re-exam Winter 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

### Question 1

We can use the Hotelling model of product differentiation to argue both for too much product differentiation and no product differentiation. We can moreover use the model to argue for a positive profit for firms even though we have competition. Explain how the model can be used for this and what are the main arguments to be used to reach these different conclusions. (argue using a model with only two firms).

*Answer:*

*The basic difference in the two outcomes is that in one of the models firms cannot compete on the price - i.e. the price is assumed fixed. In the model firms can only choose to differentiate products. The optimal choices for the two firms is to choose the same type of product. Formally in the model it is assumed that consumers' preferences for different product types can be modelled along a line (e.g. from 0 to 1 as an illustration of product type) the further a consumer is from his/her preferred type the smaller the utility. The firms can maximise their market share by locating as close to the other firms choice as possible and thus securing the market from the extreme end of the line until where it is located. Hence, since this is the case for both firms they choose to locate in the centre of the line capturing exactly 50% of the market each and offering almost exactly the same product type. The Hotelling model can also be used to argue for positive profit and maximal product differentiation if the firms can also compete on the price. Hence, the firms choose to differentiate as much as possible (being in either end of the product differentiation line) and can thus increase the price above their marginal costs without risking that the customers will shift to the other supplier (due to the costs of switching to a less preferred variant).*

### Question 2

Consider a situation where an employer wants to hire workers to produce an output. Assume that a worker can produce an output  $y_i$ , where  $i$  refers to two types of workers:  $H$  and  $L$  being high and low productive workers respectively. Assume that  $y_H > y_L$ . The price of output is  $p$ . We also assume that the two types of workers require a minimum (fixed) wage ( $r_i$ ) to take the job at the firm. We assume that  $r_H > r_L$ . Assume that the probability that a worker is of low productivity is  $q$ .

- Discuss how adverse selection can arise in a model like this. You should outline the four different cases that can arise and explain when and how adverse selection can arise.
- Explain how signalling can be used in this model to overcome the adverse selection problem. Can this always be done? What if the signal influence the productivity of the workers, how may this influence the outcome? (You are not supposed to derive the results, but should include mathematics to support your arguments).

*Answers:*

- This model follows the note on adverse selection. Of the four cases only the fourth case where only high productivity workers are profitable for the firm introduces adverse selection. Arguments about how a higher probability of low quality workers can cause the market to break down is part of an extraordinarily good answer.*
- The employer may decide to pay a higher wage to a worker that can demonstrate/signal special capabilities. This can be done by undertaking additional education. If this should have a chance then the costs of undertaking education must have a higher cost to low productivity workers such that we will have a possibility of reverting the reservation wages (such that the reservation wage for high productivity workers is smaller than for low productivity workers). However, if the reservation wage increase is too large then we may end in a situation where none of the worker*

types are profitable for the firm to hire. Hence, this will not be required and the low productivity workers are still hired despite their negative contribution to profits. If education increase productivity it increases the probability that it is worthwhile for the firm to ask for the additional effort. It may in addition be that this makes even low productivity workers profitable.

### Question 3

Suppose you have a supplier of a consumer good with a supply function  $s = \frac{B+p}{4}$ , where  $p$  is the price on the consumer good in the market. We also have a consumer with a demand function equal to  $d = \frac{A-p}{3}$ .  $A$  and  $B$  are parameters of the model.

a) What is the equilibrium price and quantity in this market?

When the consumer good is produced this adds costs for some other consumer in the market. We assume that this additional costs can be expressed as  $(2x)^2$ ; i.e. when the consumer good is produced we have costs (beyond those incurred for the producer) of  $(2x)^2$ .

b) What is the efficient level of production? Comment on this.

c) A social planner wants to regulate this economy. He is considering regulating using taxes. What tax should the planner introduce in this economy?

d) The planner is also considering setting up a scheme of polluter permits. Explain how this could work, why this is a solution and find the equilibrium permit price in this economy. (In relation to the assumed additional social costs mentioned above, we have assumed that each unit of output produces 2 units of external effect; i.e. the  $2x$  in the external cost function outlined above). Compare the tax and the equilibrium price.

*Answer:*

a) We set demand equal to supply, which gives us an equilibrium price of:  $p = \frac{4A-3B}{7}$  and quantity  $x = \frac{A+B}{7}$

b) We now have an externality with a marginal external cost of  $8x$ . Hence, this should be taken into consideration. The supply function is the inverse of the marginal cost curve. The firm's marginal cost are then  $p=MC=4x - B$ ; the total social marginal costs are thus:  $SMC=12x-B$ . Solving where  $SMC$  equals demand:  $12x-B=A-3x$  give us  $x=(A+B)/15$ . This level is lower than the market optimum, which do not take the externality into account.

c) The tax is a Pigouvian tax, which is equal to the marginal external costs in the social optimum. We can find it comparing the prices for the consumer in the social optimum and subtract the supply price in the social optimum:  $p^s=4*(A+B)/15-B$  and  $p^d=A-3*(A+B)/15$ . Giving  $t=8(A+B)/15$

d) For each unit of external effect produced the producers must hold a permit. This means that for each unit of consumer good the producer should hold 2 permits. Hence the producers marginal cost change to  $4x - B - 2r$ , where  $r$  is the price of a permit. Now setting this new MC equal to the demand price give us an equilibrium quantity depending on the price of permits:  $x(r) = \frac{A+B-2r}{7}$ . So all the planner has to do is to set the right quantity of permits, such that the social optimum is reached. Since each unit of production requires 2 permits we can see that if there is  $v$  permits issued output is reduced by  $v/2$ . Hence, the price on the voucher can be expressed as:  $r(v) = \frac{2(A+B)-7v}{4}$ . The optimal number of permits will be  $v^{opt}=2x^{opt}=2*(A+B)/15$  giving a price of  $r(v^{opt})=4(A+B)/15$ . Since

the relation between the output and pollution is 2x we can see that the price of the permits and the tax on output corresponds to each other.

#### Question 4

Assume that a monopoly firm faces two types of customers with different willingness to pay for the firm's product, but the firm is not able to distinguish between them. The firm can offer a quantity discount.

What are the aspects that the firm must consider in a case like this?

*Answer:*

*The firm has a possibility of offering a second-degree price differentiation, where high wtp customers are offered a larger quantity at a higher price and low wtp customers a smaller quantity at a lower price. The firm has to consider both individual rationality constraints (that the two customers both find it worthwhile to buy one of the packages offered) and incentive compatibility constraints where each customer will only choose the package aimed for them. The 4 constraints that arise from this problem are not all binding. E.g. the IR constraint for the low wtp is often enough also to ensure that the high wtp customer's IR constraint is satisfied and it is most often the case that it is only the high wtp customer that has an incentive to choose another package not intended for this type. To avoid this the IC constraint must be such that the high wtp type gets a higher CS from choosing the package aimed for him compared to the other package. This means that the monopoly can increase profits by reducing the package aimed for the low wtp type and it also means that the price for the high wtp type contains an element of premium if the high wtp type behaves according to what the IC constraint intends to ensure.*

#### Question 5

Nick and Jill are considering designing/buying a new common garden, which they can both enjoy from the windows in their respective apartments. Their preferences over 'garden' ( $G_i$ ) and other consumption ( $x_i$ ) can be represented by the utility functions  $u_i(G_i, x_i) = G_i^{a_i} x_i^{1-a_i}$ . Moreover, one unit of garden can be bought by paying one unit of the consumption good. Nick and Jill have exogenous incomes  $I_i$ . The income can be used to contribute to purchase of the common garden and the remaining part is used for the general consumption good.

- How much garden will be bought by Nick and Jill (the total garden is the sum of their individual contribution to the common garden)? It is not possible to contribute with a negative amount.
- Assume that  $a_i=0.5$  for both Nick and Jill. What is the efficient quantity of garden? It differs from the quantity in coming from the answer in a) using these parameter values. Why?
- What could be done to solve this problem - at least theoretically?

*Answer:*

*a) This is classical public good example. Each of Nick and Jill will maximise individual utility constrained by income and the contribution of the other:*

$$\begin{aligned} \max u_i(G, x_i) &= G^{a_i} x_i^{1-a_i} \\ \text{s. t. } I_i &= x_i + G_i \text{ and } G = G_1 + G_2 \end{aligned}$$

*This is solved for each of them to find the "best response functions".  $G_i(G_j) = a_i I_i - (1 - a_j) G_j$ .*

*Solving for the (Nash) equilibrium gives  $G_i = \frac{a_i - (1-a_i)a_j I_j}{a_i + a_j - a_i a_j}$*

b) This level is where the sum of their MRS is equal to the marginal cost of providing the garden. In this case this is constantly equal to 1. Since we have Cobb-Douglas preferences we know that demand is equal to  $G = a_i I_i / p$  which can be inverted to give  $p = a_i I_i / G$ . This is the marginal willingness to pay for the public good. We then know that in optimum  $MB_i + MB_j = 1$ . Hence in our model we have that  $a_i I_i / G + a_j I_j / G = 1$  or that  $G = a_i I_i + a_j I_j$  and with the parameter values we get  $G = \frac{1}{2}(I_i + I_j)$ . Inserting the parameter values in the solution from a) give us  $G = (4 - (I_i + I_j)) / 3$ , which is less than the efficient level. This is because the agents have incentives to free-ride on each other's contributions and they will thus both reduce the level of private contribution to the garden.

c) A possibility is to introduce Lindahl (individual) prices another is to set up a Clarke-Groves Mechanism, but neither of these are without problems - e.g. information about all individuals' preferences and that we risk up overpaying for the provision of the public good due to the Clarke Tax

### Question 6

Explain why a voting procedure may not be a clever way to make joint decisions in a society.

Answer:

Pairwise voting can be manipulated such that any of the outcomes that are candidates can be the outcome - depending on the order of pairwise voting decisions i.e. it is intransitive.