This set contains four pages (beginning with this page)
All questions must be answered
In the evaluation, the three main questions will be weighted equally

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MONETARY ECONOMICS: MACRO ASPECTS

SOLUTIONS TO AUGUST 18 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In Svensson's (1997, *European Economic Review*) model of inflation targeting, a more conservative central banker allows inflation to return to its target value at a slower rate.

A False. In the basic model, which portrays strict inflation targeting, policy is conducted so as to bring inflation back to target as fast as possible (i.e., at the implementation lag horizon). This would be a case where the central banker is infinitely conservative. In the extension where the central banker also have preferences for output gap stability, inflation is brought back to target slower. As this is a case of a less conservative central bank, the statement is false.

(ii) In Fuhrer and Moore's backward-looking IS/AD model, an interest-rate rule violating the Taylor principle implies real equilibrium indeterminacy.

A False. Indeterminacy builds on self-fulfilling expectations, and expectations play no role in the backward-looking model. An interest-rate rule violating the Taylor principle will in this model lead to instability of equilibrium; i.e., explosive inflation and output paths.

(iii) The difference between the domestic and foreign nominal interest rates on perfectly substitutable assets provides no information about the credibility of a fixed nominal exchange rate regime.

A False. From the uncovered interest rate parity condition, the domestic interest rate is given by the foreign rate, plus the *expected* rate of currency depreciation. Hence, a difference between interest rates may indicate expectations about exchange rate movements and thus indicate the market's lack of belief in the fixed exchange rate regime.

(iv) In the New-Keynesian model of Clarida, Galí and Gertler (1999, *Journal of Economic Literature*), a monetary policy based on past periods' policies is welfare improving.

A True. The full commitment policy is inertial, or history dependent, as this causes policy to affect expectations in the economy. This is beneficial as it improves the policy trade-off faced by the central bank.

QUESTION 2:

Monetary shocks and imperfect information

Consider a version of Lucas' flex-price "island model" where individuals live on local, isolated markets (or, islands), and after each period are randomly relocated to another market. Letting superscript "i" denote local/island variables, and no superscript denote economy-wide average variables, the four central equations describing the economy are

$$Y_t^i = \left(N_t^i\right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

$$C_t^i = Y_t^i, \tag{2}$$

$$u_{1-N}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = \left[(1-\alpha)\frac{Y_t^i}{N_t^i}\right] u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right), \tag{3}$$

$$\begin{aligned} u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) &= u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right), \qquad 0<\beta<1, \\ &\quad +\beta \mathrm{E}^i u_C\left(C_{t+1}, M_{t+1}/P_{t+1}, 1-N_{t+1}\right), \end{aligned} \tag{4}$$

where Y_t is output in period t, N_t is employment, C_t is consumption, M_t is the nominal money supply at the end of the period, and P_t is the price level. The function u is increasing and concave in all arguments, and u_j denotes the partial derivative of u with respect to variable j. E^i denotes expectations conditional on local information.

(i) Discuss equations (1)–(4) and explain how a change in the real money supply can have real effects in the model.

A (1) is the local production function; (2) is local market clearing; (3) is the condition for optimal labor supply; (4) is the condition for optimal money demand. In this money-in-the-utility function set up, a change in the real money stock can have real effects by affecting labor supply through (3). For example, if the marginal utility of consumption is increasing in the real money stock, the return from work increases and labor supply goes up when the real money stock rises. If the utility function is separable in real money, there will be no effects at all.

The stochastic process for the log of nominal money on island i is given by

$$m_t^i = \gamma m_{t-1}^i + u_t + u_t^i, \qquad 0<\gamma<1, \tag{5}$$

where u_t^i is a "local" nominal disturbance with mean zero and variance σ_i^2, and where u_t is an aggregate shock with mean zero and variance σ_u^2. The shocks u_t and u_t^i are assumed independent, and the informational assumption is the following. On island i, variables m_t^i and γm_{t-1}^i are known. The variables u_t and u_t^i, on the other hand, cannot be observed.

(ii) Discuss how this imperfect information about u_t and u_t^i can affect equilibrium real behavior towards a change in u_t.

A If agents know that a shock is global, they know that all prices on all Islands will increase proportionally leaving real money unchanged. No agent would change behavior. If the shock is known to be local, on the other hand, the agents know that all prices will not adjust proportionally, and the real value of carrying money increases. Labor supply will then be affected as discussed in (i). When there is imperfect information, agents will rationally guess that the shock has some global and some local component leading to some response.

(iii) Derive $\mathrm{E}^i\left[u_t|u_t+u_t^i\right]$ under the assumption that expectations about u_t are formed by use of a linear least squares projection. (Hint: agents derive an estimate of u_t, which is a linear function of what is observed, $\widehat{u}_t=\kappa\left(u_t+u_t^i\right)$, where κ is the estimation coefficient minimizing the squared forecast error.) Discuss how σ_i^2 and σ_u^2 affect expectations.

A Using the hint, we derive κ as the solution to

$$\min_{\kappa}\ \mathrm{E}\left[\widehat{u}_t-u_t\right]^2,$$

$$\min_{\kappa}\ \mathrm{E}\left[\kappa\left(u_t+u_t^i\right)-u_t\right]^2,$$

$$\min_{\kappa}\ \mathrm{E}\left[\kappa^2\left(u_t+u_t^i\right)^2+u_t^2-2\kappa\left(u_t+u_t^i\right)u_t\right],$$

As shocks are independent and have zero means, this becomes

$$\min_{\kappa}\ \left(\kappa^2\mathrm{Var}\left[u_t+u_t^i\right]+\mathrm{Var}\left[u_t\right]-2\kappa\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]\right).$$

The first-order condition is

$$\kappa\mathrm{Var}\left[u_t+u_t^i\right]-\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]=0,$$

leading to

$$\kappa=\frac{\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]}{\mathrm{Var}\left[u_t+u_t^i\right]}=\frac{\mathrm{Var}\left[u_t\right]}{\mathrm{Var}\left[u_t\right]+\mathrm{Var}\left[u_t^i\right]}=\frac{\sigma_u^2}{\sigma_u^2+\sigma_i^2}<1,$$

and thus

$$\mathrm{E}^i\left[u_t|u_t+u_t^i\right]=\kappa\left(u_t+u_t^i\right)=\frac{\sigma_u^2}{\sigma_u^2+\sigma_i^2}\left(u_t+u_t^i\right).$$

One notes that the higher is σ_u^2 relative to σ_i^2 the more likely it is that the change in $u_t+u_t^i$ are caused by changes in u_t rather than in u_t^i. In consequence, κ increases.

It can be shown that (log of) employment is given by

$$n_t = \Phi(1-\kappa)\,u_t, \qquad \Phi > 0, \qquad 0 < \kappa < 1, \tag{6}$$

where κ is the estimation coefficient mentioned in the hint to Subquestion (iii).

(iv) The coefficient Φ is a function of the underlying parameters of the model. Explain intuitively what happens to Φ as

- agents become infinitely impatient, i.e., when $\beta \to 0$;
- agents' utility function becomes separable in consumption and real money balances;
- the stochastic process for nominal money becomes more persistent, i.e., γ increases.

A When $\beta \to 0$, the model turns into a static one-period model where agents ignore that they are relocated to another island next period. Any confusion about local and aggregate shocks becomes irrelevant (aggregate conditions will not play any role to the agents), and a local shock to nominal money will have no effects on labor supply as local prices will adjust proportionally. When agent's utility function is separable in consumption and money, there will only be real effects of aggregate shocks if the change in the real money stock changes the marginal utility of leisure directly. In the standard model in Walsh this is not the case. Concerning γ, this represents a predictable component of the nominal money supply process, and it has therefore no effect on output determination of the model. So, Φ is independent of γ.

QUESTION 3:

The instrument choice in monetary policymaking

Consider the following static, log-linear IS/LM-style model:

$$y = -\alpha i + u, \qquad \alpha > 0 \tag{1}$$

$$m = -ci + y + v, \qquad c > 0, \tag{2}$$

where y is output, i is the nominal interest rate, m is the nominal money supply, and u and v are mean-zero, independent shocks with variances σ_u^2 and σ_v^2, respectively. The objective of monetary policy is to minimize output variance, and policy is conducted before the shocks hit the economy.

(i) Discuss briefly (1) and (2), and derive optimal monetary policy when m is the instrument and when i is the instrument.

A Equations (1) and (2) are IS and LM curves, respectively, in a fixed price model. Aggregate demand decreases with the nominal interest rate, as does money demand. When m is the policy instrument, y can be solved by (1) and (2) as a function of m:

$$y = \frac{\alpha m + cu - \alpha v}{\alpha + c}$$

As the shocks are unknown when setting m, the output-variance minimizing policy is $m = 0$. When the interest rate is the instrument, the IS curve gives output immediately as

$$y = -\alpha i + u$$

and optimal policy follows as $i = 0$.

(ii) Show that i is preferable as the monetary policy instrument when

$$\left(1 + \frac{2c}{\alpha}\right) \sigma_u^2 < \sigma_v^2. \tag{3}$$

Discuss the intuition behind condition (3).

A With optimal policy under money supply as an instrument, $m = 0$, output variance is

$$\mathrm{E}_m [y_t]^2 = \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}.$$

With optimal policy when the interest rate is the instrument, $i = 0$ leads to

$$\mathrm{E}_i [y_t]^2 = \sigma_u^2.$$

The interest rate is thus the preferred instrument if the output variance is lowest under that policy procedure. This requires

$$\begin{aligned}
\mathrm{E}_i [y_t]^2 &< \mathrm{E}_m [y_t]^2, \\
\sigma_u^2 &< \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}, \\
\sigma_u^2 \left[(\alpha + c)^2 - c^2\right] &< \alpha^2\sigma_v^2, \\
\sigma_u^2 \left[\alpha (\alpha + 2c)\right] &< \alpha^2\sigma_v^2,
\end{aligned}$$

and thus

$$\left(1 + \frac{2c}{\alpha}\right)\sigma_u^2 < \sigma_v^2.$$

The condition shows that high money market volatility (σ_v^2) relative to high goods demand volatility (σ_u^2) is conducive for choosing the nominal interest rate as instrument. In effect, the economy will be perfectly insulated from money market shocks. On the other hand, if demand shocks are predominant, the inequality will fail, as having the interest rate endogenously increase when a positive goods demand occurs, will have a stabilizing effect not present under an interest rate targeting procedure.

(iii) Consider an extension where

$$m = b + hi + \omega, \qquad h > 0, \tag{4}$$

is added to the model. In equation (4), b is the money base, which is now a possible monetary policy instrument, and ω is a mean-zero shock with variance σ_ω^2. The variable m is now interpreted as a broad measure of money. Is the case for using a nominal interest rate operating procedure strengthened or weakened relative to condition (3) in this extended version of the model? A thorough verbal discussion is sufficient.

A The case is strengthened as the introduction of the shock ω essentially increases economic volatility stemming from the money market. One can show that under a monetary base operating procedure it is optimal to set $b = 0$, which yields output as:

$$y_t = \frac{(c+h)\,u_t - \alpha v_t + \alpha \omega_t}{\alpha + c + h}.$$

Output variance becomes

$$\mathrm{E}_b\left[y_t\right]^2 = \frac{(c+h)^2\,\sigma_u^2 + \alpha^2\sigma_v^2 + \alpha^2\sigma_\omega^2}{(\alpha+c+h)^2},$$

emphasizing the additional volatility induced by σ_ω^2. On the other hand, the solution under an interest rate operating procedure is unchanged.

(iv) Assume that monetary policymaking takes the form of a money base rule of the form $b = \mu i$. If there are no shocks to the monetary side of the model, $\sigma_v^2 = \sigma_\omega^2 = 0$, will a "pure" money base rule, $\mu = 0$, be optimal? Explain.

A The answer is no. Even though there are no shocks from the monetary side of the model, it is not optimal to chose a constant b. In that case, a positive goods demand shock would increase output and the nominal interest rate in the conventional IS/LM fashion. But letting $\mu < 0$, b would fall in response to the increasing nominal rate, thereby further dampening the increase in output.