Written Exam for the M.Sc. in Economics 2009-II
Advanced Development Economics - macro aspects
Master's Course
June 10, 2009, 9-13.
(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. That is, if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The percentage weights assigned to each question should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to all questions in the exam in their totality.

Assignment A is "verbal discussion". This means that full credit does not *require* the use of formal arguments. Naturally, though, formal arguments are admissible. But in general geometric arguments and plain text will do.

A. Verbal Discussion: Various Topics (30%)

A1. Consider the following statement: "*Wealth Inequality is good for growth*". Do you agree or disagree. Explain.

A2. Discuss the problems associated with estimating the *impact* of fertility on growth, and how these problems may potentially be overcome.

B. Analytical Discussion: Longevity and Growth (70%)

Consider an economy in the process of development. The economy is inhabited by an infinite sequence of overlapping generations. The size of the labor force is constant, and of measure one. Time is discrete and extents into the infinite future, t=0,1,2 …

The economy is closed, and markets are fully competitive. People live for two periods. In period one they work and consume. In period two they retire, and life off their savings.

At the end of the first period individuals face a risk of dying prematurely. With probability ϕ they survive until the end of period 2 (where they die for sure); with probability 1-ϕ they die right after period 1 without being able to consume their savings.

To solve the problem of "unused savings" we introduce a non-profit life insurance company. All consumers provide it with their savings, which it in turn invests in the firms. Since all consumers (infinitely many) use the same insurance company it faces no uncertainty. No profits therefore dictates that the return obtained from savings by the survivors, ρ, is simply the real rate of return, r, divided by the survival probability, $\rho = r/\phi$.

B1. **The problem of firms**. We assume the representative firm uses the following Cobb-Douglas production technology, $Y_t = AK_t^{\alpha}L^{1-\alpha}$, where the notation is standard, and implicitly L=1 by assumption. The factor prices of labor and capital are w and r+δ, respectively. Show that the first order conditions from the profit maximization problem are

$$r_t = \alpha A k_t^{\alpha-1} - \delta, w = (1-\alpha)Ak^{\alpha}_t$$

where δ is the rate of capital depreciation, and $k_t \equiv K_t / L$.

B2. **Household problem**. Households have preferences over consumption in the two phases in life, (c_t,c_{t+1}). The utility function is logarithmic: $u_t = \log(c_t) + \phi \log(c_{t+1})$. In period 1 (youth) the household is faced by the restriction $w_t = s_t + c_t$, and in period 2 (old age) the households are subject to $c_{t+1} = (1+\rho_{t+1})s_t$. (*i*) Comment on the utility function. (*ii*) Solve the maximization problem, and show that savings of the young are given by $s_t = [\phi/(1+\phi)]w_t$. (*iii*) Explain why the parameters enters into the savings function the way they do.

B3. **Law of motion for capital**. Show that the capital stock, per unit of labor, evolves in accordance with

$$k_{t+1} = \frac{\phi}{1+\phi}(1-\alpha)Ak_t^{\alpha} \equiv \psi(k_t)$$

B4. **Steady state analysis**. A steady state fulfills $k_{t+1} = k_t = k^*$, and the law of motion for capital per worker. (*i*) Show that a unique steady state exist, and that it is stable. (*ii*) What is the impact from reduced mortality on long-run productivity? Provide intuition. (*iii*) Discuss whether the result in (B4,ii) is consistent with available empirical evidence.

We now extend the model by an government. The government levy taxes on labor income at the rate τ. All revenue is spend on health investments, h_t, so that $\tau w_t = h_t$. The tax rate is constant through time. Furthermore, assume health investments affects the chances of survival into period 2. Specifically, we assume $\phi_t = \beta h_t / (1+h_t) \equiv \phi(h_t)$.

B5. Law of motion for capital. Show that the law of motion for capital now becomes

$$k_{t+1} = \sigma(k_t)(1-\tau)(1-\alpha)Ak_t^{\alpha} \equiv \psi(k_t),$$

$$\sigma(k_t) \equiv \frac{\phi(\tau(1-\alpha)Ak_t)}{1+\phi[\tau(1-\alpha)Ak_t]}$$

where $\phi(\bullet)$ is defined above.

If $\alpha<1/2$ the following properties of $\psi(k)$ hold true:

$\psi(0)=0, \psi'(k)>0$ for all k,
$\lim_{k\to 0} \psi'(k) = \infty$,
$\lim_{k\to\infty} \psi'(k) < 1$.

B6. Steady state analysis. (i) Is $\alpha<1/2$ reasonable? (ii) Draw the phase diagram for the augmented model given $\alpha<1/2$; is the steady state unique and stable?

B7. Empirical Implications. What is the "mortality transition", and is the model consistent with this phenomenon?

B8. Alternative parameter values and policy. (i) How would the analysis from B6 change if instead $\alpha>1/2$ is admitted? (ii) Does the policy implications differ depending on whether $\alpha>1/2$ or $\alpha<1/2$?