Written exam for the M.Sc. in Economics, Winter 2012/13

Game theory

Final Exam/Elective Course/Master's Course

(3 hours, closed book exam)

22 January 2013

Question 1

In this exercise we model penalty shooting in soccer. There are two players: The goalkeeper G and the penalty taker P. We neglect the possibility of shooting (or standing) in the middle and assume that both have to choose a side: either "right" (R) or "left" (L). If both choose opposite sides, a goal results with probability 1 if P chooses R and with probability 0.9 if P chooses L. If both choose L, the scoring probability is 0.3. If both choose R, the scoring probability is 0.5.
The objective of P is to maximize the expected probability of a goal and the objective of G is to minimize the expected probability of a goal.

a. Model this situation in two ways:

   - Assume that both players move simultaneously. Model the situation as a strategic game and determine its Nash equilibrium/equilibria.
   - Now G is very quick. Assume that he can observe which side P chooses and moves *after* P. Model this situation as extensive form game and determine a subgame perfect Nash equilibrium of this game.

b. For the extensive form game of the previous subquestion:

   - Write down the corresponding normal form game.
   - Is there a Nash equilibrium which is not subgame perfect? Explain your answer.

c. Now we go back to the simultaneous move game. G knows that P has shot all penalties in his career to the left. G suspects that P might be "crazy", i.e. that P's objective might not be maximizing the scoring probability but simply to shoot left all the time.

   Assume G assigns probability 1/5 to the possibility that P is "crazy" and 4/5 to the possibility that P is a scoring-probability-maximizer as above. This belief is common knowledge. Model this situation as a Bayesian game and derive a Bayesian Nash equilibrium.

   Briefly explain how the Bayesian Nash equilibrium of this game depends on the probability that P is "crazy", i.e. what happens for values different from 1/5.

Question 2

Consider the "hat game" we had in the lecture on knowledge. Here is a brief summary of the setting as a reminder:

> There are N players in a room and each wears a hat. The color of the hat is either black or white. Each player can see the color of the other hats but does not see the color of his own hat. An outside observer asks: "Do you know the color of your hat?" If a player knows, he raises his hand (and all

other players can see who raises his hand). Then the outside observer asks the same question again and again players who know raise their hand. This is repeated several rounds.

Deviating from the case we had in the lecture, the outside observer announces the following before the first round: "At least one of you wears a black hat and at least one of you wears a white hat." This announcement is true and it is common knowledge among the players that the outside observer tells the truth.

a. Now assume that $N = 5$ and assume that three players wear white hats and two wear black hats. Consider the event E: "at least two players wear a white hat". Is E known by all players? Is E common knowledge? Briefly explain the answer to these two questions at the following two points in the game:

- immediately after the announcement of the outside observer
- after the first round.

b. How many rounds will it at most take until the first player(s) raise their hand? (this question is for general N and general distributions of white/black hats)

   Does your answer differ from the solution we obtained in the setting of the lecture (where the outside observer only announced: "At least one of you wears a black hat.")? Why (not)?

Question 3

There are three players (1,2,3) on a committee. They have to decide between three alternatives (a,b,c). There are only two possible preference profiles (R and R'):

$$
\begin{aligned}
R_1 &= R'_1 = a \succ b \succ c \\
R_2 &= b \succ c \succ a \\
R'_2 &= b \succ a \succ c \\
R_3 &= c \succ a \succ b \\
R'_3 &= a \succ c \succ b
\end{aligned}
$$

The social planner wants to implement the following choice function f:

$$f(R) = b \qquad f(R') = a$$

a. Is the choice function f Nash equilibrium implementable? Either give an example game implementing it or show that it is not Nash equilibrium implementable.

b. Is the choice function f subgame perfect Nash equilibrium implementable? Either give an example game or show that it is not.