Written Exam for the B.Sc. in Economics 2010-II-R

Econometrics A

2. year

August 23, 2010

(4 hours open book exam - calculators are not allowed)

Question 1

An investor is considering three different investment opportunities. He has $10,000$ DKK to invest in one of the assets. Asset A gives a risk free return of 2 per cent per year. Asset B is a bond and the value of asset B when investing the $10,000$ DKK after t years is $Y_B = \left(1.03^t + 0.03\sqrt{t} \cdot Z\right) \cdot 10,000$ where Z is standard normal distributed. Asset C is shares in a biotech company. The biotech company has only one product which is late in the research phase. However, it is unclear when the product can be launched onto the market. Assume that we know with certainty, that when the product is launched onto the market, the price of the biotech company's shares increases with 30 per cent. Assume that the Biotech company's stock price only changes when the product is launched and that the probability of the product being launched onto the market is 10 per cent each year until market launch. The investor's investment horizon is 5 years, and no matter the returns after 5 years, the investor will withdraw her money after 5 years. Assume that time is discrete.

1. What is the expected value and variance of investing in asset A after 5 years?
2. What is the expected value and variance of investing in asset B after 5 years?
3. What is the probability that the realized value of asset B is lower than the realized value of asset A after 5 years?
4.
   (a) How is the waiting time until market launch for the biotech firm's product distributed?
   (b) What is the expected waiting time until market launch for the biotech firm's product?
5. What is the expected value of asset C after 5 years? Which of the three assets has the highest expected value after 5 years?

Question 2

Consider a tele marketing company. Tele marketing is a method of direct marketing in which a salesperson, typically by phone, takes contact to prospective customers to persuade them to buy products or services. In many cases the approached customers are not interested in buying the product the tele marketing firm offers. Assume that the arrival of a sale follows a Poisson process with on average 2 sales per hour. For simplicity, we assume that each customer agreeing to buy the product only buys one item of the product.

1. What is the probability that a sales person sells at least 10 products within a day corresponding to 7.5 hours?
2. How is the waiting time of a sale of 10 products distributed?
3. What is the expected waiting time (in hours) of a sale of 10 products?

Suppose that there exist two types of salespersons; those who exert low effort such that the arrival of a sale follows a Poisson process with on average 1.8 sales per hour, and those who exert high effort such that the arrival of a sale follows a Poisson process with on average 2.4 sales per hour. From years of experience the manager knows that persons never change type (that is, no high effort persons become low effort persons or vice versa) and that only 1/3 of the newly hired salespersons exert high effort whereas 2/3 exert low effort. Therefore, the tele marketing company gives all newly hired salespersons a trial period of a day before offering them an employment contract.

4. Suppose the manager observes that a salesperson has sold 14 products within a day (7.5 hours). What is the probability that the salesperson has exerted low effort?

5. The manager only offers an employment contract to persons that she is at least 60 per cent sure are high effort types. However, the manager is concerned that by applying the 60 per cent decision rule based on only one days work she ends up employing too many low effort salespersons and also that she refuses to offer employment contracts to too many high effort types. Therefore, the manager considers extending the trial period from one (7.5 hours) to two days (15 hours). To check whether a trial period of two days is better, calculate the probability that a salesperson who sells $2 \cdot 14 = 28$ products in $2 \cdot 7.5 = 15$ hours, is a low effort type and compare the result to your answer in question 4. Will the manager be more sure that a given salesperson is a low effort type? Provide some intuition for your findings.

Question 3

In the public debate in Denmark it is often fiercely discussed whether or not giving performance pay to teachers can increase the average attained learning outcomes for the students. A recent study in Israel attempted to address this question by a randomized trial[1]. This question is based on this experiment, which was conducted in the following way:

- In the beginning of year 2000 a random sample of Israeli schools is drawn.
- The attained math score for each senior class student at the final exam in the summer of 2000 is recorded (the senior class is the class, which will graduate that year).
- Shortly after the beginning of the academic year 2000/2001 the math teachers were informed that they were participating in a competition about who could improve their class average test score the most from the exam in the summer of 2000 to the exam in the summer of 2001. There would be awards ranging from \$1,750 to \$7,500 depending on their performance relative to the other teachers in the competition.
- At the end of the academic year 2000/2001 the math scores were collected and the awards payed out.

Hence the data set consists of two student waves; those who graduated in the summer of 2000 and those who graduated in the summer of 2001.

In the following questions we will examine whether the experiment had the same effect on boys and girls. We will restrict our attention to a single (randomly selected) school among the included schools. However, first we will examine the sex ratio on the selected school for the class who graduated in the summer of 2001. Let Z_i be a random variable indicating if person i is a boy:

$$Z_i = \begin{cases} 1 \text{ if person } i \text{ is a boy} \\ 0 \text{ if person } i \text{ is a girl} \end{cases}$$

We assume that Z_i follows a Bernoulli distribution with probability parameter $p : P(Z = 1) = p$.

1. State a consistent estimator of p. Explain the concept of confidence intervals and write how to obtain a confidence interval for the estimator of p.
2. Based on the Table 3b below compute the estimator of p along with its 95% confidence interval.
3. Test the hypothesis $H_0 : p = 0.5$ against the alternative $H_A : p \neq 0$. Discuss your finding in the light of your answer to question 2.

In the rest of the questions we will examine how the attained test scores depend on the experiment and the sex of the student. Define the following random variables

$X_{bb} \sim N(\mu_{bb}, \sigma^2_{bb})$ the score for a boy before the experiment
$X_{gb} \sim N(\mu_{gb}, \sigma^2_{gb})$ the score for a girl before the experiment
$X_{ba} \sim N(\mu_{ba}, \sigma^2_{ba})$ the score for a boy after the experiment
$X_{ga} \sim N(\mu_{ga}, \sigma^2_{ga})$ the score for a girl after the experiment

The random sample consists of a total of 227 students. Tables 3a and 3b below presents descriptive statistics for the attained scores.

[1] V. Lavy (2009), American Economic Review, 99:5, 1979-2011

Table 3a: Student math scores in year 2000 (the class graduating before the experiment)			
	Girls (X_{gb})	Boys (X_{bb})	Total
Number of observations	58	48	106
Sample mean	69.94	66.73	68.49
Median	70.44	68.62	69.24
Sample standard deviation	16.46	9.92	13.92

Table 3b: Student math scores in year 2001 (the class graduating after the experiment)			
	Girls (X_{ga})	Boys (X_{ba})	Total
Number of observations	66	55	121
Sample mean	80.15	71.16	76.06
Median	81.65	70.29	77.04
Sample standard deviation	9.72	12.38	11.85

4. Discuss if it is reasonable to assume that the attained scores are independent realizations from the four normal distributions.

In the follwing assume that the scores are independent (irrespectively of your answer to question 4)

5. Test the hypothesis $H_0 : \sigma^2_{ba} = \sigma^2_{ga}$ against the alternative $H_A : \sigma^2_{ba} \neq \sigma^2_{ga}$ at the 1% significance level. Explain in words the conclusion of the test. Hint: The 0.5% and 99.5% percentiles in the F-distribution $F(65, 54)$ are 0.51 and 1.99, respectively.

6. Irrespectively of your answer to the previous question assume that $\sigma^2_{ba} = \sigma^2_{ga}$. State an estimator of the common variance.

7. Test whether girls benefited from the experiment, that is, test the hypothesis $H_0 : \mu_{gb} = \mu_{ga}$ against the alternative $H_A : \mu_{gb} \neq \mu_{ga}$. Explain in words the conclusion of the test.

8. State the asymptotic distribution of $\hat{\delta}_b = \hat{\mu}_{ba} - \hat{\mu}_{bb}$ and of $\hat{\delta} = \hat{\delta}_g - \hat{\delta}_b$, where $\hat{\delta}_g = \hat{\mu}_{ga} - \hat{\mu}_{gb}$.

9. Compute a 90% confidence interval for δ. Discuss if boys and girls benefited equally from the experiment.

10. On the basis of Question 5 and 7 define and discuss the concepts "Type I errors" and "the power of a test".