Written Exam for the B.Sc. in Economics winter 2015-16

Microeconomics B (II)

Final Exam

18. February 2016

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total

Problem 1

Consider Henning who has a wealth of W dkk to be invested in either a government bond yielding an interest rate of $r > 0$ or a mutual fund yielding either a yield rate of $a > r$ with a probability π or a rate $b < r$ with a probability of $1 - \pi$.

Henning decides upon uncertain alternatives using expected utility with a bernoulli function u(x), which is a strictly increasing and concave function.

a) If $r > \pi a + (1 - \pi)b$ will Henning invest any amount in the mutual fund?
b) Show that if $r < \pi a + (1 - \pi)b$ Henning will always invest a positive amount in the mutual fund.

Solution:

a) Even a risk-neutral person will not invest in the mutual fund, such that a risk-adverse person will never invest. More specifically, $u\big((1 + r)W\big) > u\big((1 + \pi a + (1 - \pi)b)W\big) = u(\pi(1 + a)W + (1 - \pi)(1 + b)W) > \pi u\big((1 + a)W\big) + (1 - \pi)u\big((1 + b)W\big)$.
b) The expected utility from investing $\alpha \geq 0$ in the mutual fund: $\pi u(r(1 - \alpha)W + a\alpha W + W) + (1 - \pi)u(r(1 - \alpha)W + b\alpha W + W)$ then the first order condition $\pi(a - r)u'(c_1) + (1 - \pi)(b - r)u'(c_2) = 0$, but if $\alpha = 0$, then $c_1 = c_2 = (1 + r)W$ and thus the FOC cannot be satisfied since $r < \pi a + (1 - \pi)b$.

Problem 2

The population consists of two types of labour: L-types and H-types, that among many differences differ in their probability of being unemployed. The L-types have an annual probability of being unemployed of π_L, while the H-types have a larger probability $\pi_H > \pi_L$. Assume, however, that they are similar in their income, their bernoulli utility function and their loss in income when being unemployed.

The share of L-types in the population is $\alpha > 0$. An insurance contract is a specification of: K the insurance amount and $P = \gamma K$ the insurance premium.

a) Define an actuarial fair insurance contract. How should the premium rate γ be set if an actuarial fair contract should cover both types?

The government offers an unemployment insurance that covers both types: both pays the same premium and receives the same amount in the event of unemployment. The insurance benefit offered is the optimal amount that the L-types would choose with the rate $\gamma = \alpha\pi_L + (1 - \alpha)\pi_H$.

Assume that neither the government nor any private company can observe the type of a labour type.

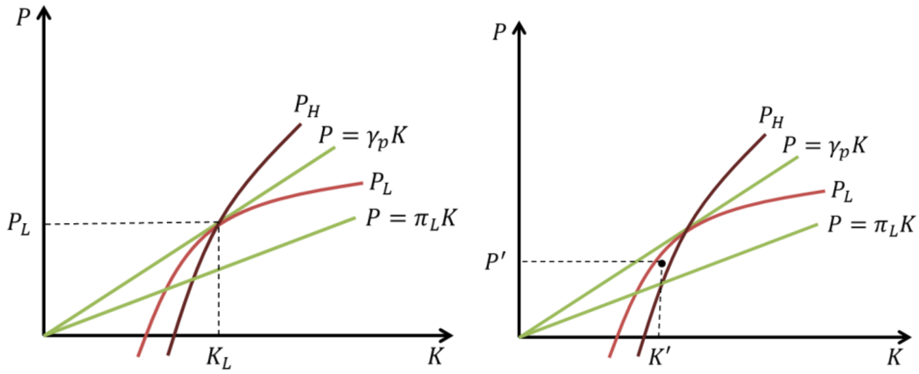
b) Illustrate the actuarial fair pooled insurance contracts, the government offered insurance contract and the types' indifference curves through this contract in a (P, K)-diagram

c) Could a private insurance company offer an insurance contract that makes a positive profit and only attract the L-types? What would this imply for the government sponsored program?

Solution:

a) Actuarial fair is an insurance contract that yields a zero expected profit: If P is the insurance premium and K the insurance, and π is the probability of unemployment $P - \pi K = 0$; when covering both types the rate $\gamma \equiv \frac{P}{K}$ must be $\gamma = \alpha\pi_L + (1-\alpha)\pi_H$ since the expected payment of insurance is $\alpha E[K_L] + (1-\alpha)E[K_H]$ and $E[K_i] = \pi_i K$ where α is the share of L-types in the population

b) This should be something like the graph below (left)



c) See above for a positive profit contract as described (P', K'). This will imply that the government insurance company runs a deficit, since the expected profit is $(1-\alpha)P_L - (1-\alpha)\pi_H K_L = (1-\alpha)(\alpha\pi_L + (1-\alpha)\pi_H)K_L - (1-\alpha)\pi_H K_L = \alpha(\pi_L - \pi_H)K_L < 0$.

Problem 3

Consider two car manufacturers producing the same quality car and targeting the same consumers. The consumer demand for the car is represented by the inverse demand function

$$P(Q) = \max\{900 - 2Q, 0\}$$

where Q is the total production of cars.

The car manufacturers produce with a constant marginal cost of 36 dkk. They choose their production levels and then sell at the price that clears the market.

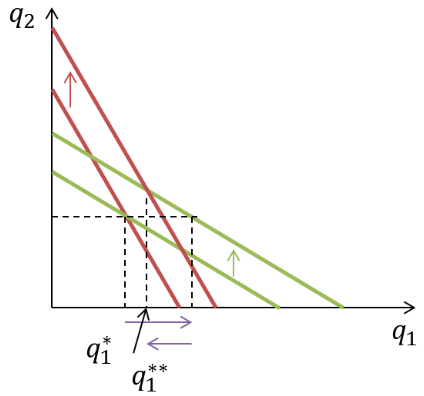
a) Find the number of cars produced and the price of a car in equilibrium.

Due to an increase in the households' income, the demand changes to $P(Q) = \max\{1200 - 2Q, 0\}$.

b) How does the increased demand affect the equilibrium and the profits of the car manufacturers?
c) Illustrate the impact and a possible adjustment process to the new equilibrium.

Solution:

a) The reaction function $R_i(q_j) = \frac{900-c}{4} - \frac{1}{2}q_j$ and the equilibrium quantities are $q_1^* = q_2^* = \frac{900-c}{6} = 144$, and the price $p^* = 324$
b) The equilibrium quantities changes to $q_1^{**} = q_2^{**} = \frac{1200-c}{6} = 194$ and the price $p^{**} = 424$; the change in profit is $(424 - 36) * 194 - (324 - 36) * 144 = 33800$
c) The reaction function of both firms shift outwards, such that the direct effect is that firms increase quantities from an increase in the MR-curve; the indirect effect is negative, and thus dampening since an increase in the competitor's quantity lowers the optimal quantity



Problem 4

Consider a community of individual fishermen who during the year catch fish from the same fishing area. Each kilo of fish can be sold at a price of 20 dkk while the fisherman must endure a cost of 50 dkk in terms of gasoline, wear and tear etc., if he sets out to sea. If n fishermen went to the sea the total amount of fish all n fishermen catch is $25\sqrt{n}$.

a) If 25 fishermen have sent out their boats, what will the profit of the 26th fisherman be? What will the effect on the fishermen who have already sent out their boat be?
b) Find the number of fishermen that go to the sea. (Hint: Assume that the number of fishermen is continuous and there are sufficiently many potential fishermen, no matter what solution you find)
c) What is the socially optimal number of boats?
d) Explain why the inefficiency in the number of boats arises.

Solution:

a) If $X = 25$ then the total profit of adding one more is the profit of letting 26 is $20 * 25 * \sqrt{26} - 50 * 26 \approx 1249$ and thus the next fisherman can obtain a profit of $1249/26 \approx 48$ by sending out his boat. Before the boat is sent out, each fisherman earns $\frac{1250}{25} = 50$ dkk, and thus by sending out his boat he lowers the other fishermens' profit by $50 - 48 \approx 2$ dkk.
b) we have that $p\frac{f(n)}{n} = a$ such that $n' = \left(20 * \frac{25}{50}\right)^2 = 100$ is the number of boats if each individual take his decision individually.
c) we have that $pf'(n^*) = a$ such that $n^* = \left(\frac{20*12,5}{50}\right)^2 = 25$.
d) there is an externality that arises since each boat put out to sea decreases the yield to each boat already at sea.

Problem 5

Consider a small community that has decided to purchase a new statue to be located in front of the local Netto. However, they have not yet decided upon the amount to use on purchasing the statue. Denote by S the amount to be spent on the statue.

There are two individuals: A and B. The individual A has an utility of the statue and own money given by $u_A(x, G) = x + \ln G$, where G is denotes the size and/or the quality of the statue. and he owns a money stock of 10, while individual B has an utility $u_B(x, G) = x + 3\ln G$, and own a money stock of 50.

The costs of the statue are $C(G) = G$ such that G is the amount to be spent on the statue.

a) Find the Pareto efficient amount to be spent on the statue.
b) What is the amount actually spent if each individual voluntarily submits contributions?
c) Explain why the total amount of the public collected with voluntary contributions differs from the efficient amount.
d) Derive the Lindahl equilibrium of this economy

Solution:

a) The Samuelson condition states that the sum of willingness to pay must equal the marginal costs: $\frac{1}{G} + \frac{3}{G} = 1$ hence $G^* = 4$
b) For consumer A he solves $\max_{g_A} 10 - g_A + \ln(g_A + g_B)$ and thus the reaction function becomes $\max\{1 - g_B, 0\} = g_A = R_A(g^A, g_B)$ where g_B is the contribution of individual B. For the B the reaction function $\max\{3 - g_A, 0\} = R_B(g_A, g^B)$. In equilibrium, both maximize their individual utility by taking the other's contribution as given. Now we note that we cannot have an interior solution: if $g_A^* > 0$ and $g_B^* > 0$, then we must have that $1 - g_B^* = g_A^*$ and $3 - g_A^* = g_B^*$; which cannot be the solution. But then we have that considering $(g_A^*, g_B^*) = (1,0)$, which is not an equilibrium, since $R_B(1) = 2 \neq 0$; while $(g_A^*, g_B^*) = (0,3)$, which is an equilibrium, since $R_A(3) = 0$ and $R_B(0) = 3$. Both being the best response given the other's choice of contribution.
c) Free riding is when each contributor does not take into account the non-rival nature of the public good: by contributing he/she provide benefits to the other individuals in the community, but he/she only considers his/her own benefits. In other words, when considering how much to contribute, he/she takes as given the contributions of the others and those benefits to him or herself from these contributions. Hence, they benefit but they do not sufficiently contribute.
d) The Lindahl equilibrium is implemented by setting the price equal to the marginal benefits of the efficient amount of the public good: $t_A = \frac{1}{4}$ and $t_B = \frac{3}{4}$, and the total payment is $t_A G^* + t_B G^* = \frac{1}{4} * 4 + * \frac{3}{4} 4 = 4 = c * G^*$.