Written Exam Economics Summer 2016

Macroeconomics III

Final Exam

August 22, 2016

(3-hour closed book exam)

Please answer in English only.

This exam question consists of 3 pages in total including this cover page.

1. Consider the following Ramsey model. There is no technological growth, population grows at rate n, capital does not depreciate. As usual, small letters denote per capita variables, large letters variables in levels. Firms operate in perfectly competitive markets and produce the final good using the production technology

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$$

where $K(t)$ is physical capital, $L(t)$ is labor and $\alpha \in (0, 1)$. Firms rent both inputs from households. The infinitely-lived, representative household derives utility from consumption $c(t)$ and discounts the future at rate ρ. His objective is to maximize the discounted present value of lifetime consumption:

$$\int_t^{\infty} e^{-(\rho-n)t} u(c(t)) dt$$

Assume

$$u(c(t)) = \frac{c(t)^{1-\sigma}}{1-\sigma}$$

The government in this economy levies a proportional capital income tax on households, and transfers the revenues lump sum back to them. It runs a balanced budget each period. Households thus face the following budget constraint:

$$\dot{a}(t) = w(t) + (1-\tau)r(t)a(t) - na(t) - c(t) + s(t)$$

where $a(t)$ denote the household asset holdings, $w(t)$ is labor income, $r(t)$ is the interest rate earned on assets, τ is the capital income tax rate and $s(t)$ the lump-sum transfer. $\dot{a}(t)$ denotes the time derivative of $a(t)$. The government budget constraint is given by

$$s(t) = \tau r(t) a(t)$$

Assume that initial assets in the economy are given, and that the no Ponzi condition holds.

(a) Set up and solve the household problem using optimal control theory. (What are the state, control, and co-state variables? Write down the Hamiltonian, derive the maximum principle conditions.)

(b) Derive the Euler condition. Explain in words the what the condition states and why it has to hold at the optimum. How does the capital income tax affect it? It is not sufficient to describe the equations, explain the economics of the effects of the capital tax.

(c) Derive the equilibrium $\dot{c} = 0$ and $\dot{k} = 0$ curves and plot them in the phase diagram. Explain the diagram. How are the curves different from an economy without a capital tax? Again, it is not sufficient to describe the equations, explain the economics.

(d) Suppose the economy is in steady state with $\tau = 0$. In period t_0, the government unexpectedly and permanently raises the capital income tax rate to $\tau > 0$. What happens in the economy on impact, in the transition, in the new steady state, and why? Plot the changes in the phase diagram.

(e) Suppose the economy is in steady state with $\tau = 0$. In period t_0, the government unexpectedly and *temporarily* raises the capital income tax rate to $\tau > 0$ until t_1. From t_1 onwards, $\tau = 0$ again. What happens in the economy on impact, in the transition, in the new steady state, and why? Plot the changes in the phase diagram. Compare to the permanent shock case from the previous part.

2. The representative agent i maximizes utility

$$U_i = C_i - \frac{1}{\beta} L_i^\beta, \qquad \beta > 0$$

subject to the budget constraint

$$PC_i = P_i Y_i$$

where C_i is consumption, L_i labor supply, P the aggregate price level, P_i the price of good i and Y_i the quantity of good i. The production function equals

$$Y_i = L_i^\alpha, \quad 0 < \alpha < 1$$

There is monopolistic competition in the goods market. The demand for good i is

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$$

where Y denotes aggregate output and $\eta > 1$ is the elasticity of substitution in the demand for differentiated goods. The aggregate demand equation is

$$Y = \frac{M}{P}$$

where M denotes money supply. Agents have rational expectations. Employ the following notation: $x \equiv \ln X$.

(a) After linearizing the first order condition from the utility maximization problem, derive the optimal production y_i^* as a function of the relative price $p_i - p$. Show that the equilibrium (log) aggregate price level equals

$$p = \mu + m$$

where μ is a constant to be found. Show analytically that y increases in η and provide adequate interpretation to this result.

(b) Suppose now that p_i is fixed for 3 periods and that price-setting is staggered, such that 1/3 of the prices are set in period t at the level x_t, 1/3 were set in period $t-1$ at the level x_{t-1}, while a remaining 1/3 were set in $t-2$ at the level x_{t-2}. Thus, the aggregate price level equals

$$p_t = \frac{1}{3}\left(x_t + x_{t-1} + x_{t-2}\right)$$

Assuming certainty equivalence (i.e., $x_t = \frac{1}{3}\left(p_{i,t}^* + \mathbf{E}_t\left[p_{i,t+1}^*\right] + \mathbf{E}_t\left[p_{i,t+2}^*\right]\right)$), show that the equilibrium reset price, x_t, depends on m_t, $\mathbf{E}_t\left[m_{t+1}\right]$ and $\mathbf{E}_t\left[m_{t+2}\right]$.

(c) Suppose that the (log) money supply follows a random walk: $m_t = m_{t-1} + \varepsilon_t$. Show that aggregate price inflation, $\pi_t = p_t - p_{t-1}$, is an MA(2) process taking the following form:

$$\pi_t = \frac{1}{3}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2}\right)$$