Written Exam for the B.Sc. in Economics autumn 2011-2012

Macro C

Final Exam

4 January 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem A: Taxation of capital income in the Ramsey model

Consider the following version of the Ramsey model where capital income is taxed at the rate τ. There is no public consumption in the model so the government revenue from taxation is transferred back to households as lump-sum transfers (given by v_t per worker) (this is captured in the public budget constraint in equation (A.5)). We ignore technological growth, and define c_t and k_t as consumption and capital per worker. r_t is the real interest rate and R_t the real rental rate. The rest of the notation is as usual.

$$\frac{\dot{c}_t}{c_t} = \frac{(1-\tau)\cdot r_t - \rho}{\theta} \tag{A.1}$$

$$(1-\tau)\cdot r_t = (1-\tau)\cdot(R_t - \delta) \tag{A.2}$$

$$R_t = f'(k_t) \tag{A.3}$$

$$\dot{k}_t = f(k_t) - c_t - k_t\cdot(n+\delta) \tag{A.4}$$

$$\tau\cdot r_t\cdot k_t = v_t \tag{A.5}$$

where: $\rho > 0$, $\theta > 0$, $0 \leq \tau < 1$ and we assume that: $\rho > n$.

In addition we assume that the relevant transversality condition is fulfilled.

1) Show that (A.1) is the solution to the problem of the representative household, which is to choose consumption at each date in order to maximize the intertemporal utility function:

$$U_0 = \int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta}\cdot e^{-(\rho-n)\cdot t}dt$$

subject to the evolution of wealth per worker:

$$\dot{a}_t = (r_t\cdot(1-\tau) - n)\cdot a_t + w_t + v_t - c_t$$

and a No Ponzi game condition, taking a_0 as given

(*Hint: the No Ponzi game condition can be ignored when deriving (A.1)).*

2) Interpret each of the equations (A.1) – (A.4). Explain in particular equation (A.1). Show that (A.1) – (A.4) can be boiled down to the two equations:

$$\frac{\dot{c}_t}{c_t} = \frac{(1-\tau)\cdot(f'(k_t)-\delta)-\rho}{\theta}$$

$$\dot{k}_t = f(k_t) - c_t - k_t \cdot (n+\delta)$$

3) Construct the phase diagram. Comment.
4) Analyze the consequences of an unexpected increase in τ at time t_0 when it is assumed that the economy is initially in steady state. Explain the effects carefully.

Problem B: Taxation of interest income in the Blanchard model

Consider the following version of the Blanchard model (describing a small open economy with perfect international capital mobility and fixed exchange rates) where the return to both domestic and foreign bonds are taxed at the rate τ. All variables follow the usual notation.

$$y_t = z + \eta \cdot Q_t - \beta \cdot p_t \qquad \text{(B.1)}$$

$$r_t \cdot (1-\tau) = r^f \cdot (1-\tau) \qquad \text{(B.2)}$$

$$\frac{D_t + \dot{Q}_t}{Q_t} = r_t \cdot (1-\tau) \qquad \text{(B.3)}$$

$$D_t = \alpha \cdot y_t \qquad \text{(B.4)}$$

$$\dot{p}_t = \gamma \cdot (y_t - \bar{y}) \qquad \text{(B.5)}$$

where: $\eta > 0$, $\beta > 0$, $0 \leq \tau < 1$, $r^f > 0$, $0 < \alpha < 1$ and $\gamma > 0$.

You are informed that equation (B.2) and (B.3) in combination with the relevant transversality condition implies that:

$$Q_t = \int_t^{\infty} D_s \cdot e^{-r^f\cdot(1-\tau)\cdot(s-t)} ds \qquad \text{(B.6)}$$

Finally, we assume in *all* the following that:

$$r^f \cdot (1-\tau) - \alpha \cdot \eta > 0 \tag{B.7}$$

1) Interpret briefly each of the equations (B.1) - (B.6). Show that the dynamic evolution of the economy can be described by the two differential equations:

$$\dot{p}_t = \gamma \cdot \eta \cdot Q_t - \gamma \cdot \beta \cdot p_t + \gamma \cdot (z - \bar{y})$$

$$\dot{Q}_t = Q_t \cdot (r^f \cdot (1-\tau) - \alpha \cdot \eta) + \alpha \cdot \beta \cdot p_t - \alpha \cdot z$$

2) Define the long run equilibrium in this model (the steady state) and show that the long run values of y_t, Q_t and p_t (marked with a *) are given by:

$$y^* = \bar{y}$$

$$Q^* = \frac{\alpha \cdot \bar{y}}{r^f \cdot (1-\tau)}$$

$$p^* = \frac{1}{\beta} \cdot \left(z + \eta \cdot \frac{\alpha \cdot \bar{y}}{r^f \cdot (1-\tau)} - \bar{y} \right)$$

Interpret the expression for Q^*.

3) Construct the phase diagram. Comment.

Assume that the economy is initially in a long run equilibrium when, at time t_0, there is an unexpected and permanent increase in τ.

4) Use the phase diagram to analyze the effects. Explain the economic effects carefully.

Now, assume that the increase in τ is announced at time t_0 and implemented at time t_1 (where $t_1 > t_0$). Again we assume that the economy is initially (until time t_0) in long run equilibrium.

5) Use the phase diagram to analyze the effects from time t_0 and onwards. Explain the economic effects carefully.

Now assume that dividends are also taxed at the rate τ.

6) Use the phase diagram to analyze the effects of an (unexpected) increase in τ in this case (assume again that the economy is initially in a long run equilibrium)