Written Exam for the M.Sc. in Economics 2010

International Trade and Investment
Final Exam/ Elective Course/ Master's Course
Winter 2010/2011
16. February 2011
Answer Key

3-hour closed book exam

- There are pages in this exam paper, including this instruction page
- You need to answer all THREE questions, so manage your time accordingly.
- If a question asks you to list three things, please underline the list with preceding numbers as exampled below.

    1. Thing number 1
    2. Thing number 2
    3. Thing number 3

- Make your math legible and easily followed, with the final answer boxed.
- Partial credit may be given.

Good Luck!

1. Identify whether these statements are true or false. If false, rewrite the sentence to make it true, changing maximum 1 or 2 words.

   (a) In Melitz (2003), firms are vertically differentiated. A: False (vertically =horizontal)

   (b) The cross trade of very similar products exported and imported by trading partners seems to contradict both the Ricardian and Heckscher-Ohlin models. A: True

   (c) Leontief's Paradox was that US imports were more labor intensive than US exports. A::False labor =capital

   (d) A country is considered factor j abundant if it has more of factor j relative to its GDP than the USA: False: USA=world

   (e) Iceberg tariff rates include fixed shipping costs. A: False (do not)

2. Consider a CES utility function: $u(x) = \sum_{n=1}^{N} x_n^{\frac{\sigma-1}{\sigma}}$, where x_n denotes the quantity consumed of good n.

   (a) Given an income I, derive an individual consumer's demand $x_n(p, I)$ for good n, given a price vector $p \equiv (p_1, p_2, ..., p_N)$.
   A: $x_n(p, I) = \frac{p_n^{-\sigma} I}{\sum_{m=1}^{N} p_m^{1-\sigma}}$

   (b) What does Krugman assume about σ? Krugman (1980) (implicitly) assumes $\sigma > 1$.

   (c) We can define an indirect utility function $v(p, I) = [u(x(p, I))]^{\sigma}$. Show that $v(p, I)$ can be written as

$$v(p, I) = \left(\sum_{n=1}^{N} p_n^{1-\sigma} \right) I^{\sigma-1}$$

A:

$$
\begin{aligned}
u = \sum_{n=1}^{N} x_n^{\frac{\sigma-1}{\sigma}} &= \sum_{n=1}^{N} \left(\frac{p_n^{-\sigma} I}{\sum_{m=1}^{N} p_m^{1-\sigma}} \right)^{\frac{\sigma-1}{\sigma}} \\
&= \left(\sum_{m=1}^{N} p_m^{1-\sigma} \right)^{\frac{1}{\sigma}-1} \left(\sum_{n=1}^{N} \left(p_n^{-\sigma} I\right)^{\frac{\sigma-1}{\sigma}} \right) \\
&= \left(\sum_{m=1}^{N} p_m^{1-\sigma} \right)^{\frac{1}{\sigma}-1} I^{1-\frac{1}{\sigma}} \left(\sum_{n=1}^{N} p_n^{1-\sigma} \right) \\
&= \left(\sum_{m=1}^{N} p_m^{1-\sigma} \right)^{\frac{1}{\sigma}} I^{1-\frac{1}{\sigma}} \\
v = u^{\sigma} &= \left(\sum_{n=1}^{N} p_n^{1-\sigma} \right) I^{\sigma-1}
\end{aligned}
$$

(d) Suppose $p_n = p_1$ for all goods $n \in [1..N]$. Show that the consumer is better off if a new good $N+1$ is introduced to the market at any positive price p_{N+1}. A: Use ˆ to denote state with $N+1$ goods

$$
\begin{aligned}
\hat{v} &= \left(\sum_{n=1}^{N+1} p_n^{1-\sigma} \right) I^{\sigma-1} \\
&= \left(\sum_{n=1}^{N} p_n^{1-\sigma} + p_{N+1}^{1-\sigma} \right) I^{\sigma-1} \\
&= \sum_{n=1}^{N} p_n^{1-\sigma} I^{\sigma-1} + p_{N+1}^{1-\sigma} I^{\sigma-1} \\
&= v + p_{N+1}^{1-\sigma} I^{\sigma-1}
\end{aligned}
$$

Since $p_{N+1}^{1-\sigma} I^{\sigma-1} > 0$, $\hat{v} > v$.

3. In the Heckscher Ohlin model, labor and capital are presumed to move freely from sector to sector. Consider a model where that is not true. We have two sectors (Agriculture and Manufacturing) which uses capital and labor. The total (exogenous) Labor endowment is L. The total (exogenous) agricultural capital is K_A. The total (exogenous) manufacturing capital is K_M. Labor is

free to move between the two sectors, but agricultural capital cannot be used in the manufacturing sector and vice versa. For simplicity, let's assume there is a single firm in each sector takes prices and wages and rents as given and makes zero profit. The production function for agricultural firm is $y_A = L_A^{\alpha} K_A^{1-\alpha}$ and the production function for manufacturing is $y_M = L_M^{1-\alpha} K_M^{\alpha}$. Suppose $0 < \alpha < 1$. Firms take output prices p_M and p_A determined on the world market. They pay wages w to labor and sector specific rents r_A and r_M to capital, all three of which are determined by the market.

(a) Write down the individual firm's maximization problem for both sectors.

$$\max_{l_i, k_i} \left(p_i y_i - w L_i - r_i K_i\right)$$

(b) The unit labor demand in Agriculture can be written as $L_A^* = K_A \left(\frac{\alpha p_A}{w}\right)^{1-\alpha}$. Derive the unit labor demand $L_M^*\left(p, w, K_A, K_M\right)$ for Manufacturing as a function of prices, wages, and capital use.

$$\max_{l_M} \left(p_M L_M^{1-\alpha} K_M^{\alpha} - w L_M - r_M K_M\right)$$

$$0 = p_M \left(1-\alpha\right) L_M^{-\alpha} K_M^{\alpha} - w$$

$$L_M^* = K_M \left(\frac{\left(1-\alpha\right) p_M}{w}\right)^{\frac{1}{\alpha}}$$

The answer "$L_M^* = K_M \left(\frac{(1-\alpha) p_M}{w}\right)^{\alpha}$ due to symmetry conditions" is also accepted.

(c) An increase in the price of agriculture p_A increases both L_A and the wage w. Is the increase in w more or less than the relative wage $\frac{w}{p_A}$? Show it.

$$\frac{dw}{dp_A} = \alpha L_A^{\alpha-1} K_A^{1-\alpha} - \left(1-\alpha\right) p_A \alpha L_A^{\alpha-2} K_A^{1-\alpha} \frac{dL_A}{dp_A} = \frac{w}{p_A} - \left(1-\alpha\right) p_A \alpha L_A^{\alpha-2} K_A^{1-\alpha} \frac{dL_A}{dp_A} < \frac{w}{p_A}$$

(d) From the zero profit condition for each sector, derive the rents $r_A^*\left(p_A, w\right)$ and $r_M^*\left(p_M, w\right)$ as a function of the price and wage and capital usage.
For manufacturing (agriculture by symmetry):

$$\pi_M = 0 = p_M L_M^{*1-\alpha} K_M^{\alpha} - w L_M^* - r_M^* K_M$$

$$r_M^* K_M = p_M \left(\left(\frac{\left(1-\alpha\right) p_M}{w}\right)^{\frac{(1-\alpha)}{\alpha}}\right) K_M - w K_M \left(\frac{\left(1-\alpha\right) p_M}{w}\right)^{\frac{1}{\alpha}}$$

$$r_M = \alpha \left(1-\alpha\right)^{\frac{(1-\alpha)}{\alpha}} p_M^{\frac{1}{\alpha}} w^{\frac{-\alpha+1}{\alpha}}$$

(e) Is the change in r_M due to an increase in p_A positive or negative? Show it.

$$\frac{dr_M}{dp_A} = -(1-\alpha)(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} p_M^{\frac{1}{\alpha}} w^{\frac{-\alpha+1}{\alpha}} \frac{dw_A}{dp_A} < 0$$

(f) Do owners of manufacturing capital better off or worse off when the world price of agricultural goods increases? Explain

A: Worse off. They have lower income and face higher prices.