Written Exam for the B.Sc. or M.Sc. in Economics autumn 2012-2013

Mikroøkonomi A

Final Exam

22 February 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem 1

Ann consumes two goods, in continuous and strictly positive quantities, and has preferences which can be represented by the utility function $u_A(x_1,x_2) = \text{Min}\{x_1,x_2\}$.
Similarly, Bill has the utility function $u_B(x_1,x_2) = x_1 + 3\cdot x_2$.
Finally, Catie has the utility function $u_C(x_1,x_2) = x_1\cdot x_2$.

- 1a) Answer for each of the three consumers: Does he or she have preferences that are homothetic?

- 1b) Answer for each of the three consumers: Does he or she have preferences that are convex?

- 1c) Answer for each of the three consumers: Does he or she have preferences that are strictly convex?

Problem 2

Consider a consumer who has the strictly quasi-concave, monotonically increasing, and twice differentiable utility function u, giving rise to the Marhallian demand function x(p,I), with p being the price system and I being an exogenously given income, and the Hicksian compensated demand function h(p,u).

- 2a) Is it possible, for a given good, m, that $\partial x_m(p,I)/\partial p_m > 0$?

- 2b) Is it possible, for a given good, m, that $\partial h_m(p,u)/\partial p_m > 0$?

Problem 3

Consider an industry with a market demand side (a downward sloping demand curve) which is the same in the short run and the long run. On the market's supply there is a (very large) number of potential producers all having access to identical productions technologies (with U-shaped average costs in the short run as well as the long run). Please explain how the following are determined

- equilibrium price level
- individual firm production quantities
- individual profits
- market output level
- the number of firms actually producing in the market

Do this for both the short run and the long run.

Problem 4

Consider a Koopmans economy with one consumer whose 24 hours can be used as labor in the manufacturing unit producing a consumption good (good 2) or enjoyed as leisure (good 1). The manufacturing unit has the production function $x = l$, with l being the number of labor hours

(input), and x being the output quantity of the consumption good. The consumer's utility function is $u(f,x) = f^a \cdot x^{(1-a)}$, with f being leisure, and x being the quantity of the consumption good.
Find the efficient (Pareto Optimal) allocation and comment on the role the parameter a plays for the result.

Problem 5:

Consider a consumer who has the utility function $u(x_1,x_2) = x_1^{½} + x_2$, has the exogenously given money income I and meets the market price system (p_1,p_2).

Such a consumer will have the following expression for his or her Marshall demand function (looking solely at interior solutions to the consumer's problem):

- $x_1(p,I) = [p_2^2/(4 \cdot p_1^2)]$
- $x_2(p,I) =[(I/p_2) – p_2/(4 \cdot p_1)]$

He or she will have the following expression for his or her Hicksian compensated demand function (looking solely at interior solutions to the consumer's problem):

- $h_1(p,u) = [p_2^2/(4 \cdot p_1^2)]$
- $h_2(p,u)=[u – p_2/(2 \cdot p_1)]$

- 5a) Show that when $I^* = 9$ and the price system is $p^* = (p_1^*, p_2^*) = (1,2)$, the consumer chooses $(x_1^*, x_2^*) = (1,4)$ obtaining utility level $u^* = 5$.

- 5b) Consider the Slutsky expressions for the impact of a marginal price increase for good one on demand for both goods (evaluated at price system p* and income I*):
$\partial x_1(p^*,I^*)/\partial p_1 = \partial h_1(p^*,u^*)/\partial p_1 – [\partial x_1(p^*,I^*)/\partial I] \cdot x_1^*$
$\partial x_2(p^*,I^*)/\partial p_1 = \partial h_2(p^*,u^*)/\partial p_1 – [\partial x_2(p^*,I^*)/\partial I] \cdot x_1^*$
and show, by the calculating the value of these derivatives etc. that these two Slutsky equations are true, and comment on the results.

Problem 6:

Consider some statements which concern production units within the milk industry. Alra Milk's production unit can be described by the production function $x = f(l, k)$, where l is the quantity of labour input and k is the quantity of capital, x is the quantity of milk output, and f is a differentiable function.

Now, consider the following statements. If you think a claim is true, please prove it, or at least explain why you think it is true. If you think it is not true, please provide a counter-example.

- 6a) If Unity Milk similarly can be described by the production function $x = g(l, k)$, and g is a monotonically increasing transformation of f ($g = \varphi \circ f$, where φ is the transformation function, $\varphi'(t) > 0$), then Unity and Alra will have identical cost functions

- 6b) If (l, k, x) is a feasible production plan (i.e. $x = f(l, k)$), and (l, k) minimizes production costs at input prices (w,r) and output price p, then this production plan is profit-maximizing.

- 6c) If (l, k, x) is a feasible production plan and maximizes profits at input prices (w,r) and output price p, and if both q_1 and q_2 are strictly positive, then the absolute value of the marginal rate of transformation between the two inputs will be identical to the relative input price, w/r.

Ref.: mtn 25. november 2012