Written Exam for the B.Sc. in Economics Summer 2010

Macro A

Final Exam

Date: 2 June 2010

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Exercise 1

Consider the Solow model of a small open economy which consists of the following equations:

$$V_t = K_t + F_t$$
$$V_{t+1} - V_t = S_t, \quad V_0 \text{ given}$$
$$Y_t^n = Y_t + \bar{r}F_t$$
$$Y_t = BK_t^{\alpha}L_t^{1-\alpha}$$
$$L_{t+1} = (1+n)L_t, \quad L_0 \text{ given}$$
$$S_t = sY_t^n$$

V_t denotes household wealth, K_t is the capital stock, F_t is the net foreign asset position, S_t is total savings, Y_t^n is household income and Y_t is national output. B is an exogenous productivity parameter, n is the population growth rate and s is the savings rate. The interest rate $\bar{r}$ is given from the perspective of the small open economy. Capital does not depreciate, i.e $\delta = 0$.

a. Derive the first-order conditions which determine the firm’s demand for labor and capital. Explain why or why not capital demand depends on the savings rate *s*.

b. Show analytically that the law of motion for per-capita wealth $v_t = V_t / L_t$, is given by

$$v_{t+1} = \frac{1+s\bar{r}}{1+n} v_t + \frac{sw^*}{1+n},$$

where w^* denotes the wage rate in the economy. Provide a graphical illustration of the law of motion assuming that $s\bar{r} < n$.

c. Compute the steady-state value of v_t. Explain how the steady state value is affected by the savings rate *s*? Explain why the slope as well as intercept of the accumulation equation changes when the savings rate increases.

Consider now that the population growth rate *n* increases.

d. How does the rise in *n* affect the accumulation of wealth? Provide a graphical illustration. Also, provide an economic interpretation of why the steady state value of per-capita wealth changes.

e. Show analytically how per-capita GDP, i.e. $y_t = Y_t / L_t$, is affected by the higher population growth rate *n*. Provide an economic interpretation of your result. How important is capital mobility for your finding?

Consider now that the parameter *B* in the production function increases.

f. Show graphically whether this change influences the accumulation of per-capita wealth. Explain your finding.

Exercise 2

Consider the following model of a closed economy:

$$Y_t = \left(K_t\right)^{\alpha} \left(A_t L_t\right)^{1-\alpha}, \quad 0 < \alpha < 1$$

$$S_t = s(1-\tau)Y_t,$$

$$K_{t+1} = S_t + \left(1-\delta\right) K_t,$$

$$L_t = L.$$

The notation is as in exercise 1. The only exception is that the productivity term now reads A_t. It is taken as given by all individual firms. τ is the tax rate the government levies on income Y_t.

a. For the moment assume $A_t = 1$. Derive analytically the law of motion for the capital intensity $k_t = K_t / L_t$. Provide a graphical illustration of the law of motion. Show how a higher tax rate affects the steady state value of k_t. Explain!

Now assume that the government does not waste tax revenues, but uses them to finance infrastructure which all firms can use. In the model this implies that the public budget constraint reads

$$G_t = \tau Y_t .$$

Infrastructure spending affects the productivity term A_t in the following way: $A_t = G_t$.

b. Rewrite the production function when the productivity effect of public spending $A_t = G_t$ is taken into account. Which returns to scale does the production exhibit with respect to physical capital?

c. Compute the law of motion for the capital intensity $k_t = K_t / L$ and provide a graphical illustration.

d. Which effect does a higher tax rate have on the growth rate of GDP? Show it analytically and graphically. When the effect is ambiguous, explain the counteracting forces.

e. The government would like to choose a tax rate which maximizes the growth rate of per-capita consumption. Derive the optimal tax rate when the law of motion of the capital intensity is given by

$$k_{t+1} = s(1-\tau)k_t A + (1-\delta)k_t \text{, where } A = (\tau L)^{\frac{1-\alpha}{\alpha}} .$$

f. Would your answers to questions b. – e. change if the government would not tax income but the capital stock? That is tax revenues would be

$$G_t = \tau K_t .$$

In particular, would the optimal tax rate be higher or lower than the one you have computed in e.? (Hint: There is no need to re-compute all answers. A ‘sophisticated’ look at the answers to the previous questions suffices in some of the cases.)