# Written Exam for the B.Sc. in Economics - Autumn 2013-14

## Macro C
### Final Exam

February 17, 2014

3-hour closed book exam

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. If you are in doubt about which title you registered, please see the print of your exam registration from the students' self-service system.

**Problem 1**

Consider the following version of the Ramsey model with population growing at rate $n$. Identical competitive firms maximize the following profit function:

$$\pi(K_t, L_t) = F(K_t, L_t) - w_t L_t - r_t K_t$$

where $r_t$ is the economy wide interest rate, and $w_t$ is the wage rate. Technology in the production of final goods is given by:

$$F(K_t, L_t) = \left(\frac{K_t^T}{L_t^T}\right)^{\gamma} K_t^{\alpha} L_t^{1-\alpha}$$

where $K_t^T$ and $L_t^T$ denote economy wide aggregate capital and labor, over which individual firms have no control. $K_t$ and $L_t$ denote the quantities of capital and labor employed by the firm. Assume $0 < \alpha < 1$, $0 \leq \gamma$ and $0 < \alpha + \gamma < 1$.

A large number of identical households maximize the following intertemporal utility function, that depends on per-capita consumption $c_t$:

$$U = \int_0^{\infty} \frac{c_t^{1-\theta}}{1-\theta} e^{-(\rho-n)t} dt$$

subject to their dynamic budget constraint:

$$c_t + \dot{a}_t + na_t = w_t + r_t a_t$$

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with $k_0 > 0$ given, and $a_t = k_t + b_{pt}$. (lower case variables represent quantities in per capita terms) $a$ is wealth, and $b_p$ lending to other households.

a) Find the first order conditions for the firms' maximization problem. Which must be the payment to households for the rent of their capital holdings? And for their labor services? Give an economic interpretation to the factor $\left(\frac{K_t^T}{L_t^T}\right)^\gamma$ in the production function.

b) Write the no Ponzi game condition. What is its economic meaning? What are the control and state variables in this problem? Write the Hamiltonian for the households' optimization problem, find the first order conditions that characterize the behavior of households, and from these the Euler equation (a.k.a. the Keynes Ramsey rule). Give an economic interpretation to this equation.

c) Why in equilibrium must aggregate private debt, $b_{pt}$ be equal to zero? Given this, and the first order conditions derived in b), draw the phase diagram and find the equations that characterize steady state as a function of control and state variables.

d) Assume that the economy is initially in the steady state, and parameters are such that $\frac{K^T}{L^T} > 1$. Now unexpectedly $\gamma$ is permanently reduced (i.e. there is an unexpected negative productivity shock for the aggregate economy). Find graphically the new steady state and the adjustment process for consumption and capital. Can you say what happens to consumption initially? Explain.

e) Consider now the optimization problem of a social planner that can control economy wide allocation of resources (you do not need to solve this problem to answer this question). Would the equilibrium be the same as in the decentralized economy? Is the decentralized economy equilibrium efficient? Discuss. Is there a tax/subsidy policy that implemented on the decentralized economy would eliminate efficiency problems? Explain.

## Problem 2

Assume that the monetary policy in the eurozone is determined according to credible rules, such that equilibrium inflation for the euro is given by

$$\pi_t^{eu} = \frac{1}{1 + \lambda^{eu}} \epsilon_t^{eu}$$

where $\epsilon^{eu}$ is a supply shock in Europe, characterized as white noise with variance $\sigma^{eu}$ (the parameter $\lambda^{eu}$ reflects preferences towards inflation and output fluctuations in Europe). Consider now the following equations that characterize the behavior of the Danish economy

$$\pi = m$$
$$x = \theta + (\pi - \pi^e) - \epsilon$$

where $\pi$ is the inflation rate, $\pi^e$ the expected inflation rate, $m$ the rate of growth of money, $x$ the rate of growth of output, $\theta$ the stochastic level for the potential growth rate of output, and $\epsilon$ is a supply shock. Assume that all shocks are independent from each other, and across time, with zero mean and known variances ($\sigma_\theta$, $\sigma_\epsilon$, $\sigma_\epsilon > \sigma^{eu}$). Sequential decision making implies that the private sector forms expectations on inflation after only observing $\theta$, while the monetary authority determines $m$ after observing $\theta$, $\pi^e$,

and $\epsilon$. Finally assume that society evaluates policy according to the following ex ante loss function

$$E[L(\pi, x)] = \frac{1}{2}(E(\pi - \overline{\pi})^2 + \lambda^D E(x - \overline{x})^2)$$

where $\lambda^D$ measures the relative importance of policy objectives with respect to output deviations from target level $\overline{x}$, and inflation deviations from target level $\overline{\pi}$. For all realizations of $\theta$, distortions in the economy imply that $\overline{x} - \theta > 0$. For simplicity assume that $\overline{\pi} = 0$

a) Find the optimal monetary policy and equilibrium inflation and growth rates in Denmark when there is commitment. Contrast this outcome with the one that results when policy is chosen under discretion. Explain the difference between the two outcomes.

b) Find inflation and growth rates under a simple rule that credibly pegs the krone to the euro (assume private inflation expectations are formed before observing $\epsilon_t^{eu}$). Is this outcome always preferable than policy under discretion? Why, o why not?

c) Find inflation and growth rates with a simple rule that sets money growth as a linear function of $\theta$: $m = a + b\theta$ with optimal choice of parameters $a$ and $b$. Is this rule preferable to the peg of b)? Does your answer depend on volatilities and other parameters? Explain.

d) Find inflation and growth rates under an independent central banker with preference parameter $\lambda^B$. Characterize the optimal choice for the preferences of the central banker and show that this implies $\lambda^B < \lambda$. Explain the economic intuition for this result.

e) Show that society's welfare is always higher with an independent central banker than under the simple rules of b) and c). Discuss why this is the case.