Written exam for the M.Sc. in Economics, Winter 2013/14

Game theory

Final Exam (resit)/Elective Course/Master's Course

(3 hour, closed book exam)

21 February 2014

The exam has 3 pages in total (including cover page).

Explain each of your answers.

Question 1

In the model of knowledge we assumed that information functions are partitional which meant that they satisfied the properties

(P1) $\omega \in P(\omega)$ for every $\omega \in \Omega$,

(P2) if $\omega' \in P(\omega)$ then $P(\omega) = P(\omega')$.

We defined the knowledge function for an event $E \subseteq \Omega$ as

$$K(E) = \{\omega \in \Omega : P(\omega) \subseteq E\}.$$

which had the properties

(K4) (axiom of knowledge) $K(E) \subseteq E$

(K5) (axiom of transparency) $K(E) \subseteq K(K(E))$

(K6) (axiom of wisdom) $\Omega \backslash K(E) \subseteq K(\Omega \backslash K(E))$.

Take the following information function which is *not partitional*:
$\Omega = \{\omega_1, \omega_2, \omega_3\}$ and $P(\omega_1) = \{\omega_1\}$, $P(\omega_2) = \{\omega_2\}$ and $P(\omega_3) = \{\omega_2, \omega_3\}$

(a) Which of the properties P1 and P2 does P violate?
P2 is violated for $\omega' = \omega_2$ and $\omega = \omega_3$.

(b) Find an event E such that the knowledge function derived from P violates one of the properties K4-K6.
Take $E = \{\omega_2\}$. This violates K6 as $K(E) = \{\omega_2\}$ and $\Omega \setminus K(E) = \{\omega_1, \omega_3\}$ and $K(\Omega \setminus K(E)) = \{\omega_1\}$.

(c) Using this example, explain intuitively why an information function should be partitional.
In state ω_2, the player knows that this state occurred. If he finds himself in the information partition member $\{\omega_2, \omega_3\}$, he knows that he does not know that ω_2 occurred. From this, he should be able to infer that state ω_2 did not occur (because then he would know that ω_2 occurred). But this contradicts that he considers ω_2 to be possible in state ω_3. This problem cannot occur if an information function is partitional as P2 rules exactly this problem out.

Question 2

Consider the following strategic form game.

	L	M	R
T	-1,1	0,0	-1,1
B	0,0	-1,1	-1,1

a. Determine *all* (mixed) Nash equilibria of the game.

If P1 plays a completely mixed strategy, R is P2's best response. Also there is no equilibrium where P1 plays a pure strategy and P2 does not play R (e.g. L is a best response to T but then P1 wants to deviate to B). Hence, in every equilibrium P2 plays R. As R is a best response to any mix of P1 and P1 is indifferent given that P2 plays R, we get that the following mixed strategies are NE: P1 plays $(\alpha, 1-\alpha)$ with $\alpha \in [0,1]$ and P2 plays R

b. Show that each (mixed) Nash equilibrium is a perfect equilibrium.

First take an equilibrium with $\alpha \in (0,1)$. Let P2 tremble to L with probability ε and to M with probability ε. Then $P1$ is still indifferent between his two actions, i.e. it is still a best response to mix $(\alpha, 1-\alpha)$. Therefore, (for $\varepsilon \leq min(\alpha, 1-\alpha)$) this situation is a ε-perfect equilibrium. Letting ε go to zero gives the result.

If $\alpha = 0$ ($\alpha = 1$), take the same trembles for P2 as in the last paragraph and let P1 tremble with ε probability to B (T). Again this is a ε-perfect equilibrium. Taking the limit $\varepsilon \to 0$ gives the result.

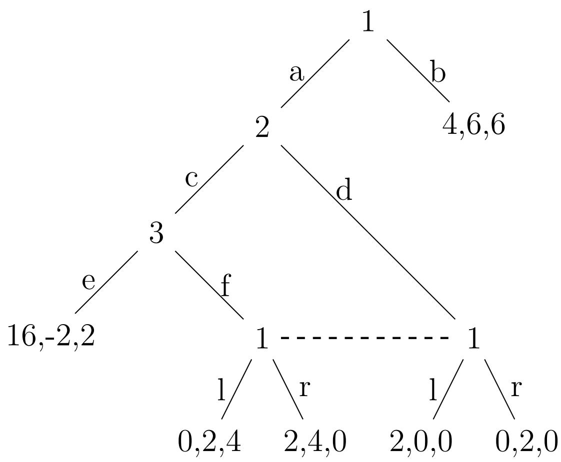
c. Assume now that L is not an available action for player 2, i.e. his action set is $\{M, R\}$ (and everything else is as above). Which Nash equilibria are perfect equilibria in the modified game?

B and M are weakly dominated actions in this game. As weakly dominated actions are not used in perfect equilibrium, the only perfect equilibrium is (T,R).

Question 3

Consider the three-player, extensive form game below.

The dashed line indicates that the two nodes are in the same information set of player 1!



a. Identify all subgames.

There are two subgames: The game itself and the game starting at the decision node of P2.

b. Find a *pure strategy* subgame perfect equilibrium. Show that this pure strategy subgame perfect equilibrium is not sequentially rational.
$((b, l), d, e)$ is the only pure strategy subgame perfect equilibrium. Playing e is, however, not sequentially rational for P3. He could gain by deviating to f (as P1 will play l giving 4 instead of 2 to P3)

c. Derive a strategy profile that is sequentially rational and where beliefs satisfy Bayes' rule in every information set. (hint: consider mixed strategies)
Given the previous question, a sequential equilibrium cannot be in pure strategies. P3 will mix only if he is indifferent and he will only be indifferent if P1 mixes 1/2-1/2 in his information set. P1 will only mix 1/2-1/2 if the two nodes in his information set are equally likely. Last but not least P2 will only mix if his expected payoff from c and d is the same. Denote by α (β) the probability that P2 (P3) plays d (f) and we get the following equations:

$$\begin{aligned} \alpha &= (1-\alpha) * \beta \\ -2(1-\beta) + \beta * 3 &= 1 \end{aligned}$$

This gives $\alpha = 3/8$ and $\beta = 3/5$. This implies that P1's expected payoff from a is $16 * 1/4 + 3/4 > 4$. Hence, P1 plays a at his initial decision node.