

Answer Sheet to the Written Exam

Financial Markets

August 2009

In order to achieve the maximal grade 12 for the course, the student must excel in all three problems.

Problem 1:

This problem focuses on testing part 1 of the course's learning objectives, that the students show "The ability to readily explain and discuss key theoretical concepts and results from academic articles, as well as their interpretation." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

- (a) Draw on Harris chapters 14, 19 and 26, and possibly Malinova and Park (2009).
- (b) Draw on the chapter from Vives (2008), as well as Harris chapters 20 and 28.
- (c) Draw on Brunnermeier and Pedersen (2009), as well as Harris chapter 28.

Problem 2:

This problem focuses on testing part 2 of the course's learning objectives, that the students show "The ability to carefully derive and analyze results within an advanced, mathematically specified theoretical model." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) $\Pr(V = 1|\pi)$ is equal to $p\pi / (p\pi + (1 - p)(1 - \pi))$ by Bayes' rule. An informed trader with private belief π will expect the return $\hat{q}E\pi - \sum_{q=1}^{\hat{q}} \text{ask}_q$ if buying \hat{q} units. Going from $\hat{q} - 1$ to \hat{q} units will increase the return by $E\pi - \text{ask}_{\hat{q}}$ which is indeed a positive gain when $E\pi > \text{ask}_{\hat{q}}$. Since $0 < \text{ask}_1 < \dots < \text{ask}_Q < 1$, the trader will optimally buy q units when $E\pi \in [\text{ask}_q, \text{ask}_{q+1}]$. Since $E\pi$ is a strictly increasing function of π , this is equivalent to the desired result.

(b) Since buy orders walk up the book, the limit sell order for q units is hit by any trader (informed or uninformed) who buys q or more units.

(c) The expression for the ask price can be rewritten

$$\frac{\text{ask}_q}{1 - \text{ask}_q} = \frac{p\beta_q^1}{(1 - p)\beta_q^0}. \quad (3)$$

The Bayes rule expression from (a) can be similarly rewritten

$$\frac{E\pi_q}{1 - E\pi_q} = \frac{p\pi_q}{(1 - p)(1 - \pi_q)}. \quad (4)$$

It was noted in (a) that $E\pi_q = \text{ask}_q$, so the ratios in (3) and (4) are identical, giving the desired result (1).

(d) The left-hand side of (2) is a linear increasing function of π . It rises with constant slope $2(Q + 1 - q)\lambda$ and crosses the horizontal axis at $\pi = 1/2$. The right hand side a positive quadratic function of π . It is decreasing on the range $\pi \in [0, 1]$, reaching down to the horizontal axis at $\pi = 1$. Both functions are continuous, and they have a unique intersection at some $\pi \in (1/2, 1)$.

(e) While the right-hand side of (2) is a function that does not depend on q , the left-hand side has lower slope, the greater is q . It is evident from the drawing, that the lower slope implies that the intersection with the right-hand side occurs at a higher value of π_q . For comparative statics, an increase in μ will raise the right-hand side, while an increase in λ will depress the left-hand side. Again, the drawing reveals that the intersection π_q will rise with μ but fall with λ . Finally, we recall from Bayes' rule that $E\pi$ is a strictly increasing function of π that does not depend on μ or λ , so the same comparative static conclusion holds for $E\pi_q = \text{ask}_q$. While not being asked for it, the superb answer might here add some intuition: larger μ (and hence lower λ) means that there's relatively more informed trade in the market, increasing spreads in the limit order book due to adverse selection.

Problem 3:

This problem focuses on testing part 3 of the course's learning objectives, that the students show "The ability to apply the most relevant theoretical apparatus to analyze a given, new casebased problem." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

Below are some suggested applications of the course literature to this case. It is important to note that these applications have shortcomings which should be discussed.

- The fragmentation of liquidity caused by the rise of new exchanges is discussed at length in Harris chapter 26.
- More generally, Harris chapter 26 discusses advantages and disadvantages of exchange competition, and the forces that shape this.
- Malinova and Park (2009) and Harris chapter 27 discuss the co-existence of two market types, and also touch on the potential advantage of being able to conceal an order size.