Written Exam for the B.Sc. in Economics autumn 2011-2012

Macro C

Final Exam

20 February 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem A: Time inconsistency of monetary policy and delegation

Consider an economy where the short run aggregate supply curve is given by:

$$\pi_t = \pi_t^e + \gamma \cdot (y_t - \bar{y}) \tag{A.1}$$

where π_t^e is expected inflation and $\bar{y}$ is the natural (long run) output level.

The loss function of the government is given by:

$$SL_t = (y_t - y^*)^2 + \kappa \cdot (\pi_t - \pi^*)^2 \tag{A.2}$$

π^* is the inflation target and y^* is determined by:

$$y^* = \bar{y} + \omega \tag{A.3}$$

where $\omega > 0$ can be interpreted as reflecting supply side distortions or 'political pressure' for pushing output (and thereby employment) above the long run level.

The central bank announces that it will implement the inflation rate π^*. At first we assume that the public trust the central bank such that: $\pi_t^e = \pi^*$

1) What is the implied value of y_t if the central bank actually implements π^*? Show that this equilibrium isn't time-consistent and explain the intuition behind this result.

Now, let's instead consider the time-consistent equilibrium.

2) Show that the time-consistent equilibrium is given by:

$$y_t = \bar{y}$$
$$\pi_t = \pi_t^e = \pi^* + \frac{\omega}{\kappa \cdot \gamma}$$

(*Hint: use (A.1), (A.2) and (A.3) to derive the optimal value of* π_t *for a given value of* π_t^e *and then use the fact that agents have perfect foresight).*

3) Explain why there is an inflation bias in the time-consistent equilibrium and illustrate the equilibrium graphically. In what special cases does the inflation bias vanish?

Now assume that a fraction β of the responsibility for the conduct of monetary policy is delegated to an independent central bank with the loss function given by:

$$SL_t^{cb} = (y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2$$

(i.e. that the independent central bank is insulated from political pressure).

4) Show that the loss function of the 'combined policy maker' is given by:

$$SL_t^* = (y_t - \bar{y})^2 + \kappa \cdot (\pi_t - \pi^*)^2 + (1 - \beta) \cdot \omega \cdot \big(\omega - 2 \cdot (y_t - \bar{y})\big)$$

What does this combined social loss reduce to in the cases where i) $\beta = 0$ and ii) $\beta = 1$?

5) Show that the time-consistent equilibrium in this case is given by:

$$y_t = \bar{y}$$

$$\pi_t = \pi^* + (1 - \beta) \cdot \frac{\omega}{\kappa \cdot \gamma}$$

(*hint: use the same procedure as in question 2) but now applied to the new loss function*). Which value of β minimizes the social loss (in (A.2))? Are there any modifications of the model which can potentially change this result?

Problem B: A public sector in the Diamond model

Consider the following version of the Diamond model, augmented with a public sector. For simplicity we ignore technological growth. The consumer preferences of each household are characterized by the intertemporal utility function:

$$U_t = ln(c_{1t}) + \frac{1}{1 + \rho} \cdot ln(c_{2t+1}) \qquad \text{(B.1)}$$

In their young age each household is constrained by the budget constraint given by:

$$c_{1t} + s_t = w_t - T \qquad \text{(B.2)}$$

Consumption in the old age is given by:

$$c_{2t+1} = s_t \cdot \big(1 + r_{t+1} \cdot (1 - \tau)\big) \tag{B.3}$$

T is a lump-sum tax on the young households, and τ is the tax rate on capital income. The tax revenue finances public consumption. You are informed that (A.2) and (A.3) implies the following intertemporal budget constraint of each household:

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1} \cdot (1 - \tau)} = w_t - T$$

1) Show that the optimal consumption profile is characterized by the following:

$$\frac{c_{2t+1}}{c_{1t}} = \frac{1 + r_{t+1} \cdot (1 - \tau)}{1 + \rho} \tag{B.4}$$

$$c_{1t} = \frac{1 + \rho}{2 + \rho} \cdot (w_t - T) \tag{B.5}$$

$$s_t = w_t - T - c_{1t} = \frac{1}{2 + \rho} \cdot (w_t - T) \tag{B.6}$$

Further, interpret (B.4) and (B.5).

We assume Cobb-Douglas technology, such that $y_t = k_t^{\alpha}$ where $k_t = K_t/L_t$ and $y_t = Y_t/L_t$.

The real interest rate and the wage rate is given by:

$$r_t = \alpha \cdot k_t^{\alpha-1}$$

$$w_t = (1 - \alpha) \cdot k_t^{\alpha}$$

The labour force L_t grows at the rate n such that:

$$L_{t+1} = L_t \cdot (1 + n)$$

Finally, capital accumulation is given by:

$$K_{t+1} = L_t \cdot s_t$$

2) Show that k_t evolves according to the difference equation (the transition curve):

$$k_{t+1} = \frac{1}{1+n} \cdot \frac{1}{2+\rho} \cdot \left((1-\alpha) \cdot k_t^{\alpha} - T\right)$$

Illustrate the relationship between k_t and k_{t+1} graphically and explain in economic terms why k_{t+1} depends positively on k_t (according to this transition curve).

3) Show that in the case where $T = 0$ the steady state is given by:

$$k^* = \left(\frac{1}{1+n} \cdot \frac{1-\alpha}{2+\rho}\right)^{1/(1-\alpha)}$$

$$y^* = \left(\frac{1}{1+n} \cdot \frac{1-\alpha}{2+\rho}\right)^{\alpha/(1-\alpha)}$$

Explain the consequences of a fall in ρ and illustrate graphically.

4) Explain now the consequences of an increase in T (with the tax revenue being used to finance an increase in public consumption). Illustrate graphically.
5) Explain the consequences on an increase in τ (with the tax revenue again being used to finance an increase in public consumption). Compare with the result in the previous question. Is this a general result?