Suggested answers for written Exam for the B.Sc. in Economics summer 2014

Macro B

Final Exam

June 16 2014
(3 hours closed-book exam)

Academic Aim: The aim of the course is to describe and explain the macroeconomic fluctuations in the short run, i.e. the business cycles around the long run growth trend, as well as various issues related to this, and to teach the methodology used in formulating and solving formal models explaining these phenomena. Students are to learn the most important stylized facts about business cycles and to acquire knowledge about theoretical dynamic models aimed at explaining these facts. In connection with this, the aim is to make students familiar with the distinction between deterministic and stochastic models. Furthermore, students are to gain an understanding of the distinction between the impulses initiating a business cycle and the propagation mechanisms that give business cycles a systematic character. Finally students are to learn how to use the models for analyzing the effects of macroeconomic stabilization policy under various assumptions regarding the exchange rate regime. To obtain a top mark in the course students should at the end of the course be able to demonstrate full capability of using the techniques of analysis taught in the course as well as a thorough understanding of the mechanisms in the business cycle models for open and closed economies, including the ability to use relevant variants and extensions of the models in order to explain the effects of various shocks and the effects of macroeconomic stabilization policies under alternative monetary and exchange rate regimes.

Problem A

1. By assumption, lifetime utility is additive separable meaning that lifetime utility is a (weighted) sum of utility obtained in each of the two periods separately. Preferences are assumed to remain unchanged over the entire life-span and described by the utility function function u. In each period marginal utility of consumption is positive but diminishing. This provides an incentive for consumption smoothing as the consumer has an incentive to shift consumption between the periods in order to equalize marginal utility from consumption in the two periods and thereby increase total life-time utility. The parameter ϕ is the rate of time preference. It measures the consumer's impatience as utility stemming from consumption in period 2 is discounted using ϕ as a discount factor. The positive rate of time preference means that if $C_1 = C_2$ then $u(C_1) > u(C_2)/(1+\phi)$ if $\phi > 0$. Hence, an additional unit of consumption in the current period is valued more highly than an additional unit of consumption in the future.

2. The household's budget constraint for period 1

$$V_2 = (1+r)(Y_1 - T_1 + V_1 - C_1). \tag{A.2}$$

(A.2) states that total real savings brought into period 2 is determined as the sum of total real disposal income in period 1 (income net of taxes) and real initial wealth minus consumption in the period. Savings earn an interest rate of r.

Substituting (A.2) into the budget constraint for period 2 implies

$$\begin{aligned} C_2 &= V_2 + Y_2 - T_2 \\ &= (1+r)(Y_1 - T_1 + V_1 - C_1) + Y_2 - T_2 \end{aligned} \tag{A.3}$$

Rearranging gives the households intertemporal budget constraint (IBC)

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 = H_1 + V_1, \\ \text{where } H_1 &= Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} \end{aligned} \tag{A.4}$$

The IBC states that the present value of total consumption equals the present value of disposable lifetime income H_1 plus the initial stock of wealth. The numerical value of slope of the consumer's IBC is $(1+r)$ which is the market rate for shifting consumption across periods, *cf.* (A.2). By assumption, capital markets are perfect. The household can borrow and lend at a perfect capital market to the going (real) interest rate r. r is identical for borrowers and lenders. H_1 is the present value of the net income (income minus net taxes) earned by the consumer during his entire life span. Net income is discounted by the interest rate r. The theory does not describe how income come about, instead it is taken as given. Net-taxes are lump-sum and are taken as given.

3. Technically, the household's maximization problem may be solved by using either the Lagrange or the substitution method. By substitution the problem may be reformulated as

$$\max_{C_1} U = u(C_1) + \frac{u\left([1+r]\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 - C_1\right]\right)}{1+\phi}$$

When solving this problem the following first-order condition is found

$$\frac{dU}{dC_1} = 0 \Longrightarrow u'(C_1) = \frac{1+r}{1+\phi} u'\overbrace{\left([1+r]\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 - C_1\right]\right)}^{C_2}$$

from which we have

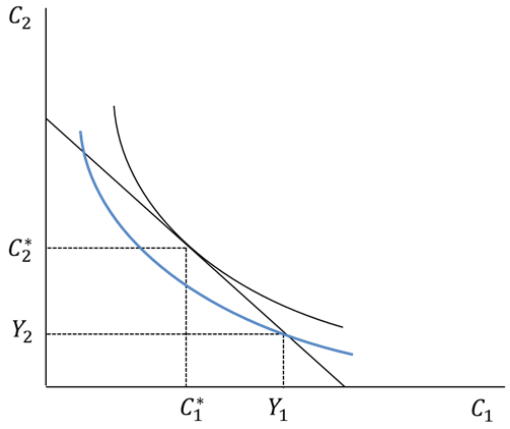
$$\frac{u'(C_1)}{u'(C_2)/(1+\phi)} = 1 + r. \tag{A.5}$$

(A.5) is the Keynes-Ramsey rule. In optimum the consumer is indifferent between consuming an extra unit today or saving an extra unit today. The left-hand side is the marginal rate of substitution, MRS, which expresses the consumers willingness to shift consumption between the two periods. If MRS is "low" the consumer requires only a small amount of additional consumption in the future in exchange of giving up an unit of current consumption. The marginal rate of substitution between present and future consumption (the left-hand side) must equal the relative price of present consumption (the right-hand side). Also, it follows directly from the characteristics of

the utility function u that C_2/C_1 is increasing in r and declining in ϕ. If r is "high" the it is optimal to postpone consumption to the future as current consumption is relatively expensive. If ϕ is "high" the consumer brings forward consumption as current consumption is valued relatively highly.

For $r = \phi$ optimization implies $C_1 = C_2$. In this situation the consumer will smooth consumption completely.

The existence of capital markets makes it possible for the consumer to shift consumption between different phases in life. Consumption in a given period is not restricted completely by income in this period. Instead the consumer may trade consumption between the periods at the given market rate. Thereby it possible for the consumer to obtain higher utility. An illustration may be the following where it is seen that the optimal consumption (C_1^*, C_2^*) is found at an indifference curve to the "north east" of the indifference curve going through the income endowment (Y_1, Y_2).



4. The consumer's problem can be solved by combining the Keynes-Ramsey rule (A.6) and the intertemporal budget constraint (A.4). We have

$$\begin{aligned} C_2 &= \beta\,(1+r)\,C_1 && \text{(from (A.6))} \\ C_2 &= (1+r)\left(Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 - C_1\right) && \text{(from (A.4))} \end{aligned}$$

Hence

$$(1+r)\left(Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 - C_1\right) = \beta(1+r)C_1$$

$$\Downarrow$$

$$\underbrace{Y_1 - T_1 + \frac{Y_2 - T_2}{1+r}}_{H_1} + V_1 = (1+\beta)C_1$$

$$\Downarrow$$

$$C_1^* = \frac{H_1 + V_1}{1+\beta} \tag{S.1}$$

so that

$$C_2^* = \beta(1+r)C_1^* = \frac{\beta}{1+\beta}(1+r)(H_1 + V_1) \tag{S.2}$$

Now V_2^* can be found by using (A.3)

$$\begin{aligned} V_2^* &= C_2 - Y_2 - T_2 \qquad (A.3)\\ &= \frac{\beta}{1+\beta}(1+r)\left(Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1\right) - Y_2 - T_2 \\ &= \frac{\beta}{1+\beta}(1+r)(Y_1 - T_1 + V_1) + \left[\frac{\beta}{1+\beta} - 1\right](Y_2 - T_2) \\ &= \frac{\beta}{1+\beta}(1+r)(Y_1 - T_1 + V_1) - \frac{1}{1+\beta}(Y_2 - T_2) \qquad (S.3) \end{aligned}$$

Savings brought into period 2 are declining in period 2 disposal income $Y_2^d = Y_2 - T_2$ and increasing in period 1 disposal income $Y_1^d = Y_1 - T_1$.This is in accordance with consumption smoothing. Also, V_2^* is increasing in r. When the reward for postponing consumption is high so are savings (this result is not general but depends crucially on the assumed functional form for u).

The ratio between optimal consumption in period 2 and optimal consumption in period 1 is given by the Keynes-Ramsey rule

$$\frac{C_2^*}{C_1^*} = \beta(1+r) = \frac{1+r}{1+\phi}$$

An increase in the impatiens parameter ϕ relative to r will increase current consumption relative to future consumption.

5. If V_1 is reduced C_1^* and C_2^* are reduced proportionally (the ratio C_2^*/C_1^* is unaffected) as the consumer smooth consumption. A decrease in initial wealth shifts the budget constraint inwards in the diagram as total wealth is reduced. This lowers consumption in both periods as the consumer smooth consumption. Thus, the decrease in current consumption will be smaller than the decrease in initial wealth. This is also the case if Y_1 is reduced. Due to the consumption smoothing the lowering of current income only partially reflected in current consumption. Hence, the reduction in C_1 is smaller than the reduction in Y_1; $0 < \partial C_1^*/\partial Y_1 < 1$. In case c, income is reduced by the same amount in both periods and in both periods by the same amount as income is reduced in period 1 in case b. Hence, life income is lower in scenario c than in scenario b. Accordingly the effect on current consumption is stronger in scenario c.

6. In a market equilibrium total net savings equal zero. Since the N consumers are identical it follows that $V_2^* = 0$ for all consumers. Then from (S.3) we have that

$$\frac{\beta}{1+\beta}(1+r)(Y_1 - T_1 + V_1) = \frac{1}{1+\beta}(Y_2 - T_2)$$

$$\Downarrow$$

$$r^* = \frac{Y_2 - T_2}{\beta(Y_1 - T_1 + V_1)} - 1 \tag{A.7}$$

This equation makes clear that the equilibrium interest rate is determined by the disposal income in each period $Y_t - T_t, t = 1, 2$, initial wealth, V_1 and by β. If the households becomes increasingly impatient; ϕ is "higher" and accordingly β is "lower". Hence, the equilibrium interest rate is "higher". This is due to the consumption smoothing. Households will try to bring consumption forward, (reduce first-period savings and hence investments of first-period income in bonds). In equilibrium this cannot happen, since net bond holdings must be zero, so the equilibrium interest rate must increase in order to provide an incentive to save, exactly counteracting households' desire to smooth consumption.

$\partial r^*/\partial V_1 < 0$; in scenario a) the equilibrium interest rate is increased. This is due to the consumption smoothing. A reduction in initial wealth will in part

lower savings. Since net bond holdings must be zero, the equilibrium interest rate has to increase so that consumers's incentive to save is stronger. In scenario b current income is reduced. In this situation the equilibrium interest rate is increased in order to make consumers increase savings; $\partial r^*/\partial Y_1 < 0$. In scenario c disposal income is reduced by the same amount in both periods; $d(Y_1 - T_1) = d(Y_2 - T_2) \equiv d(Y - T) < 0$. Given that $V_1 = 0$ it follows from (A.7) that

$$\begin{aligned} dr^* &= -\frac{Y_2 - T_2}{\beta} \frac{1}{(Y_1 - T_1)^2} d(Y_1 - T_1) + \frac{1}{\beta (Y_1 - T_1)} d(Y_2 - T_2) \\ &= \frac{1}{\beta (Y_1 - T_1)^2} [(Y_1 - T_1) - (Y_2 - T_2)] d(Y - T) \end{aligned}$$

Hence,

$$\frac{dr^*}{d(Y - T)} = \frac{1}{\beta (Y_1 - T_1)^2} [(Y_1 - T_1) - (Y_2 - T_2)]$$

It is seen that the impact an the equilibrium interest rate depends on the distribution of income over the life span. If disposal income is relatively high in period 2 then an equally sized reduction in income in both periods will increase the ratio between disposal income in period 2 and period 1. That is, though poorer in absolute terms the consumer becomes even more well of in period two in relative terms. This induces a reduction in period 1 savings in order to smooth consumption. In order to counter this the interest rate has to increase so that a bond market equilibrium is established. In the specific situation where disposal income in period 1 is equal to period 2 disposal income the equilibrium interest rate is unaffected. Again this is due to consumption smoothing. Disposal income is reduced proportionally in both periods leaving relative consumption and hence relative savings unchanged.

7. Assuming $V_1 = 0$ and $Y_1 - T_1 = Y_2 - T_2$ it follows from (A.7) that

$$r^* = \frac{Y_2 - T_2}{\beta (Y_1 - T_1 + V_1)} - 1 = \frac{1}{\beta} - 1 = 1 + \phi - 1 = \phi.$$

In equilibrium total net savings are equal to zero and when $\phi = r$ it is optimal to smooth consumption completely $C_1^* = C_2^*$.

Ignoring any Ricardian equivalence considerations a tax relief in period 1 that implies an increase in period 1 disposal income so that $Y_1 - \widetilde{T}_1 > Y_1 - T_1 = Y_2 - T_2$ leads to an increase in savings. However given that the bonds market has to clear a decline in the interest rate results.

Problem B

1. In the union model labor, market imperfections results from the wage setting structure in the economy. Wages are set by a monopoly union striving to maximize utility given that firms then determine labor demand taking the wage level as given This is called a Right to Manage model. When setting the wage level the union cares about the total rent accruing to the workers/members, *i.e.* through total employment and the gain from having a member employed earning a wage w which is higher than the alternative unemployment benefit, b. The real wage is set as a mark-up over the real rate of unemployment benefit. The size of the mark-up depends on how much the union cares about employment (lower wages if the union cares much about employment). Also, the size of the mark-up depends on how much employment reacts to changes in wages. This in turn depends on the price setting in the economy. If product markets are highly competitive a higher wage claim will have a large negative impact on employment/labor demand (as higher marginal costs in production and hence higher selling prices leads to a large decline in the amount sold). These labor and product market imperfections imply a structural unemployment which also depends positively on the generosity of the unemployment benefits.

   In the efficient wage model imperfections arise as firms are not able to monitor perfectly and without costs the effort put into work by the employees As a result, workers may shirk without being caught. In order to obtain the effort needed firms pay higher wages. Thereby it becomes more costly to get sacked in case you are caught shirking. Hence, the incentive to shrirk is reduced which increase efficiency. Again the outside option plays a crucial role for the wage setting and ultimately employment. If the outside option which includes the possibility of having unemployment benefits instead of a wage income is attractive firms have to pay higher wage rate to make people work instead of shirk. This in turn affects labor demand. Hence again structural unemployment results.

   It could be noted that both models results in AS curves witch have the same functional form.

2. It is socially desirable to stabilize inflation around a constant target value

because it makes it easier for wage setters, consumers and firms to forecast inflation. A fluctuating inflation rate typically leads to unanticipated inflation, causing the ex post real interest rate and the ex post real wage to deviate from their ex ante expected levels. Because of these expectation errors, economic agents will make suboptimal decisions and hence experience lower welfare relative to a situation where they had anticipated inflation correctly. Unanticipated inflation leads to an arbitrary redistribution of real incomes between lenders and borrowers and real wages deviate from target.

Even a stable positive inflation rate is socially costly due to so-called 'shoe-leather costs', 'menu costs', relative price distortions due to staggered price setting. Also the tax system may fail to distinguish between nominal and real income from capital. As long as the rate of inflation is kept low these cost are likely to be small.

The target rate should be above zero as policy makers should take into account that a very low inflation rate and hence a low level of nominal interest rates reduces the scope for a cut in the real interest rate when the economy is hit by a negative shock, since the nominal interest rate cannot fall below zero. A very low inflation rate may also reduce the scope for a downward adjustment of real wages if nominal wages are downward sticky.

To sum up there is a trade of between on the one hand the cost of inflation and on the other hand the risk of ending up in a liquidity strap and a limited scope for reducing real wages through (unexpected) high inflation.

3. The uncovered interest parity (UIP) is essential when explaining the "impossible trinity". UIP which is a financial arbitrage condition (or rather absence of arbitrage) whereby the return on domestically denominated financial assets is tied to the return on foreign denominated assets may be stated as

$$i = i^f + e^e_{+1} - e,$$

where i is the domestic interest, i^f is the interest rate abroad and $e^e_{+1} - e$ is the expected reduction in the value of the domestic currency. If investors are risk averse a risk premium is added. If the value of the domestic currency is expected to be reduced (and increase in e) the domestic interest rate has

to be larger than the foreign interest rate so that the expected investment return is the same when measured in the same currency.

The "impossible trinity" may be shown by assuming that two of the three policy goals are upheld. Then the third can not be obtained.

If cross-border capital flows are free and the exchange rate is fixed then from the UIP it follows directly that $i = i^f$. Hence, monetary policy can not be independent. If $i < i^f$ capital will flow out of the domestic economy. Investors sell domestic denominated assets (and thereby domestic currency) and buy foreign denominated assets (and thereby foreign currency). The central bank has to sell foreign currency and buy domestic currency to maintain a fixed exchange rate. However eventually the foreign exchange reserves are exhausted and the central bank can not defend the exchange rate any more. If $i > i^f$ capital will flow into of the domestic economy. This creates a pressure for an appreciation of the domestic currency as the foreign reserve keep growing.

If cross-border capital flows are free and monetary policy is independent, $i \neq i^f$, then $e^e_{+1} \neq e$ which is at odds with a fixed exchange rate.

If monetary policy is independent, *i.e.* $i \neq i^f$ and the exchange rate is fixed $e^e_{+1} = e$ then UIP stresses that capital can not move freely across borders as it would take an infinite foreign exchange reserve to uphold a fixed exchange rate.