Written Exam for the B.Sc. or M.Sc. in Economics autumn 2012-2013

Operations Research

Elective Course

January 23rd, 2013

(3-hour open book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Part 1

Consider the simple Knapsack Problem (KP):

$$\begin{aligned}
&Max\;\; z = && c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \\
&s.t. && a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \leq b \\
& && x_i = 0 \text{ or } 1 \; (i = 1, 2, \ldots, n)
\end{aligned}$$

By the LP relaxation of the KP we refer to the model that results, when we replace the "$x_i = 0$ *or* 1" constraint with "$0 \leq x_i \leq 1$". This LP relaxation is easily solved by computing all c_i / a_i ratios and ordering the variables from best to worst. The optimal solution is then constructed by taking as much as desirable for the best variable, in turn, until all of the single resource expressed in b is consumed. In this way, the LP is solved without using Simplex.

Given is the following KP instance:

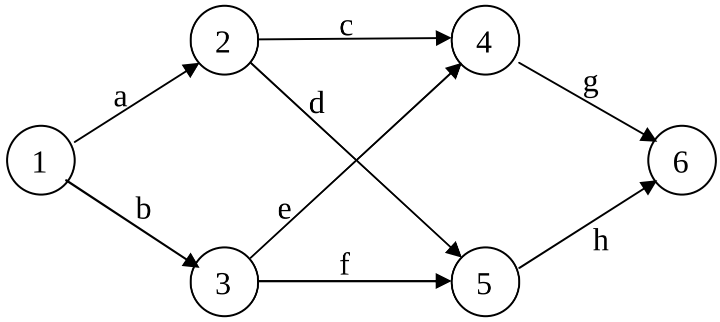
$$\begin{aligned}
&Max\;\; z = && 5 x_1 + 4 x_2 + 3 x_3 \\
&s.t. && 6 x_1 + 5 x_2 + 4 x_3 \leq 10 \\
& && x_i = 0 \text{ or } 1 \; (i = 1, 2, 3)
\end{aligned}$$

Q1.1: Solve the LP relaxed KP instance.

Q1.2: Use Branch & Bound to solve the KP instance to optimality.

Part 2

We consider a Minimum Cost Network Flow Problem (MCNFP) in the following network:



The arcs are labeled *a* through *h* and the nodes from *1* to *6*. The Minimum and Maximum flow in each arc as well at the unit flow costs are given in the table below. Also given in the rightmost column is the current solution:

Arc	From node	To node	Minimum flow	Maximum flow	Unit flow cost	Current solution
a	1	2	2	10	4	2
b	1	3	0	10	5	8
c	2	4	-5	5	7	1
d	2	5	0	5	6	1
e	3	4	5	10	5	8
f	3	5	0	5	6	0
g	4	6	5	9	4	9
h	5	6	0	5	4	1

Node *1* is the source node where 10 units are injected into the network. Node 6 is the sink node, where *10* units are drained.

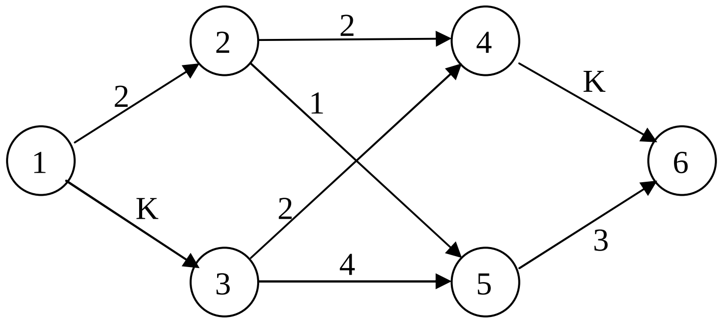
Q2.1: Verify that the current solution is a basic feasible solution for the Network Simplex Method.

Q2.2: Determine whether the current solution is optimal or not using the Network Simplex Method.

Q2.3: Now, disregard the current solution given above. Formulate the LP model that corresponds to the specific network flow problem above. You may use the letters a, b, …, h, *to denote the flows in the arcs (i.e. the decision variables).*

Part 3

We consider the same network structure as in the previous Part, but now we want to use Dynamic Programing (DP) to solve the *Longest Path Problem*. Here, the objective is to find the longest path from the starting point – Node *1* – to the ending pint – Node *6*. The length of each Arc is given in the network below:



The problem must be solved using DP backwards ("from right to left") in the Network. In the ending Stage, t=4, we can only be in State 6. In the previous Stage, t=3, we can be in State 4 or State 5. In Stage t=2, we can be in State 2 or State 3, and in the first Stage, t=1, we must be in State 1.

Q3.1: Let K = 7 *and use DP to find the Longest Path from Node* 1 *to Node* 6.

Q3.2: With K ≥ 0*, use DP to find the values of* K *for which the Longest Path from* 1 *to* 6 *includes the Arc from Node* 1 *to Node* 2.