Written Exam for the B.Sc. in Economics summer 2011

**Macro A - Solution**

Final Exam

24 June 2011

(3-hour closed book exam)

**Problem 1:**

**1.1)**

The answer should explain the result in chapter 12 (section 12.3), which shows that there will always exist unemployment in equilibrium in the considered general equilibrium model. *The reason for this result is briefly explained below:*

In each sector the trade union will act so as to maximize the total 'union rent' which consists of the difference between wages and the outside option multiplied by employment:

$$\Omega_i = L(w_i) \cdot (w_i - v)$$

where we should notice that $L(w_i)$ depends negatively on $w_i$. Further the outside option is specified as the expected income for a worker, who get fired in sector i:

$$v = (1 - u) \cdot w + u \cdot b$$

where $u$ is the unemployment rate, $w$ is the general wage level in the economy and $b$ is the unemployment benefit. *If initially the unemployment rate was equal to zero*, the outside option would be equal to the general wage rate and the union rent would be equal to zero (since all unions choose the same wage): $\Omega_i = L(w_i) \cdot (w_i - v) = 0$.

In this case each union would secure no rents for the members. Thus, unions would have a strong incentive to increase wages in order to secure a positive rent for the members. Accordingly unions increase wages, and firms respond by decreasing labour demand, which creates unemployment. In the new equilibrium there will exist unemployment, which implies that the wage rate in each sector will exceed the outside option, implying a positive rent for union members.

**1.2)**

In the basic Solow model positive growth in output per worker cannot be sustained due to the assumption of decreasing returns to capital. With a Cobb-Douglas production function with constant returns to scale output per worker is given by:

$$y_t = B \cdot k_t^{\alpha}$$

which implies that:

*the percentage increase in* $y_t \cong \alpha$ *multiplied by the percentage increase* $k_t$

since $B$ is constant over time. Notice that $\alpha < 1$ since $1 - \alpha$ is the labour income share. The source of increase in the capital stock is investment which is given by savings (assuming a closed economy) which further is assumed to be a constant fraction ($s$) of output. Now let's consider an example where the labour force increase at 1 per cent and assume that the capital stock also increase at 1 per cent. In this case output also increases by 1 per cent (due to the assumption of CRS) leaving output per worker constant → *no growth in output per worker*. Why can an increase in the capital stock above 1 per cent not be sustained? Let's consider a case where the capital stock increases by 2 per cent. In this case capital per worker will increase by 1 per cent, but due to the equation above output per worker will only increase by $\alpha$ per cent, which is less than 1 per cent. Accordingly savings per worker will increase by $\alpha$ per cent which is not sufficient to ensure an increase in the capital stock by 1 per cent. Thus, with decreasing returns to capital growth in output per worker can not be sustained in steady state in the absence of technological progress.

**1.3)**

Conditional convergence is defined by the condition that initial poor countries grow faster than richer countries *conditional on the countries being structurally alike* (such that the countries will reach the same steady state). This concept is not to be confused with absolute convergence stating that poor countries should grow faster than richer countries.

The Solow model implies conditional convergence. Also the model contains a convergence equation stating that economic growth outside steady state should depend negatively on the initial income once we control for the steady state position. We can use this equation to test conditional convergence even though the considered countries are not structurally alike. When this convergence equation is tested on actual cross country data the empirical results seem to support the idea of conditional convergence, since growth indeed seems to depend negatively on the initial income level once we control for the steady state position (this conclusion is stronger when human capital is included). The hypothesis of *absolute convergence* is however clearly rejected by the data.

**Problem 2: Social infrastructure and endogenous growth**

The model is for convenience restated below:

1) $Y_t = K_t^{\alpha} \cdot L_t^{1-\alpha} \cdot \Lambda_t^{\phi}$
2) $K_{t+1} = K_t \cdot (1-\delta) + I_t$
3) $I_t = S_t$
4) $S_t = s \cdot Y_t$
5) $L_{t+1} = L_t \cdot (1+n)$
6) $C_t = Y_t - S_t$
7) $\Lambda_t = Y_t^{\lambda}$

**2.1)**

Equation 1) is a Cobb-Douglas production function determining output as a function of the capital stock, the labour force and the level of productivity (which depends positively on social infrastructure). The Cobb-Douglas formulation implies that total labour income amounts to a constant fraction $(1-\alpha)$ of total income (GDP), which is supported by empirical evidence of a roughly constant wage share in most developed countries. The new element in this model (compared to the Solow model) is the assumption that productivity depends positively on the level of social infrastructure ($\phi > 0$). Taking the level of social infrastructure as given the production function exhibits constant returns to scale with respect to capital and labour (since $\alpha + 1 - \alpha = 1$).

Equation 2) is the basic capital accumulation equation stating that the change in the capital stock over time is given by investment net of depreciation.

Equation 3) states that investment must equal savings implying that the economy is closed. The equation can be seen as the equilibrium condition for the goods market.

Equation 4) states that savings is assumed to amount to a constant fraction ($s$) of income (GDP) (*the 'Solow assumption'*).

Equation 5) states that the labour force grows at a constant and exogenous rate ($n$).

Equation 6) states that private consumption is given by the part of income (GDP) which is not saved.

By using equation 1) and 7) we get:

$$Y_t = K_t^{\alpha} \cdot L_t^{1-\alpha} \cdot \Lambda_t^{\phi} = K_t^{\alpha} \cdot L_t^{1-\alpha} \cdot \left(Y_t^{\lambda}\right)^{\phi} = K_t^{\alpha} \cdot L_t^{1-\alpha} \cdot Y_t^{\lambda \cdot \phi} = K_t^{\alpha} \cdot L_t^{1-\alpha} \cdot Y_t^{\chi} \Rightarrow$$

$$Y_t^{1-\chi} = K_t^{\alpha} \cdot L_t^{1-\alpha} \Rightarrow Y_t = K_t^{\alpha/(1-\chi)} \cdot L_t^{(1-\alpha)/(1-\chi)}$$

where we have defined: $\chi = \lambda \cdot \phi$, which can be interpreted as the combined causal effect of prosperity (GDP) on productivity, working through the effect of prosperity on social infrastructure (whenever $\lambda > 0$) and the effect of social infrastructure on productivity (whenever $\phi > 0$).

We see that the sum of the exponents is:

$$\frac{\alpha}{1-\chi}+\frac{1-\alpha}{1-\chi}=\frac{1}{1-\chi}$$

which is larger than one whenever $\chi > 0$, such that $\chi > 0$ implies increasing returns to capital and labour together.

*The reason for increasing returns in this model* is that an increase in capital and labour by 1 per cent will at first also increase output by 1 per cent (since $\alpha + 1 - \alpha = 1$), but this increase in output (which can also be interpreted as an increase in prosperity) will increase social infrastructure (whenever $\lambda > 0$), which further increases output due to the higher productivity (whenever $\phi > 0$). Thus, through the effect on social infrastructure and productivity, an increase in capital and labour by 1 per cent will increase output by more than 1 per cent whenever $\chi > 0$. This is the reason why endogenous growth might be sustainable in this model. The condition for growth to be sustainable is explored below (in question 2.4). Also we see that the sum of the exponents (i.e. the strength of the increasing returns) depend positively on $\chi$, since

$$\frac{d\big(1/(1-\chi)\big)}{d\chi}=\frac{1}{(1-\chi)^2}>0$$

**2.2)**

By definition:

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t}=\frac{\dfrac{K_{t+1}}{A_{t+1}\cdot L_{t+1}}}{\dfrac{K_t}{A_t\cdot L_t}}=\frac{K_{t+1}}{K_t}\cdot\frac{A_t}{A_{t+1}}\cdot\frac{L_t}{L_{t+1}}$$

Now let’s use equation (11):

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t}=\frac{K_{t+1}}{K_t}\cdot\left(\frac{K_{t+1}}{K_t}\right)^{\frac{-\propto\cdot\chi}{(1-\alpha)\cdot(1-\chi)}}\cdot\left(\frac{L_t}{L_{t+1}}\right)^{\frac{\chi}{(1-\chi)}}\cdot\frac{L_t}{L_{t+1}}=$$

$$\left(\frac{K_{t+1}}{K_t}\right)^{\frac{-\alpha\cdot\chi}{(1-\alpha)\cdot(1-\chi)}+1}\cdot\left(\frac{L_t}{L_{t+1}}\right)^{\frac{1}{(1-\chi)}}=\left(\frac{K_{t+1}}{K_t}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}}\cdot\left(\frac{L_t}{L_{t+1}}\right)^{\frac{1}{(1-\chi)}}$$

Now we use the capital accumulation equation (equation (2)) along with equations (3) and (4) and insert equation (5):

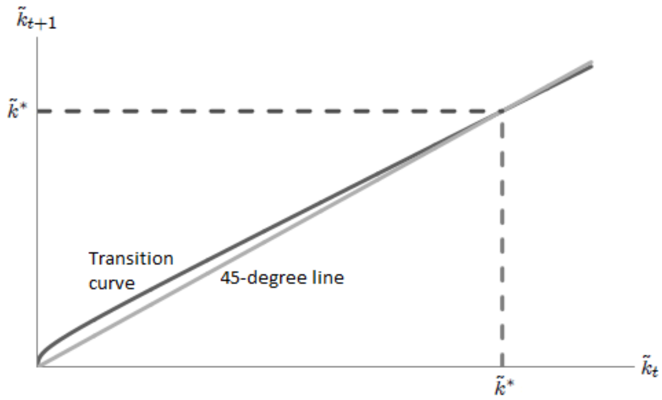
$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t}=\left(\frac{K_t\cdot(1-\delta)+s\cdot Y_t}{K_t}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}}\cdot\left(\frac{1}{1+n}\right)^{\frac{1}{(1-\chi)}}=\left(1-\delta+s\cdot\frac{Y_t}{K_t}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}}\cdot\left(\frac{1}{1+n}\right)^{\frac{1}{(1-\chi)}}$$

Finally, insert equation (12):

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t} = \left(1 - \delta + s \cdot \tilde{k}_t^{\alpha-1}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{1}{1+n}\right)^{\frac{1}{(1-\chi)}} \Rightarrow$$

$$\tilde{k}_{t+1} = \tilde{k}_t \cdot \left(1 - \delta + s \cdot \tilde{k}_t^{\alpha-1}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{1}{1+n}\right)^{\frac{1}{(1-\chi)}}$$

The transition curve (along with the 45-degree line and the steady state) is illustrated below:




When $\chi = 0$ we get the transition curve:

$$\tilde{k}_{t+1} = \tilde{k}_t \cdot \left(1 - \delta + s \cdot \tilde{k}_t^{\alpha-1}\right) \cdot \frac{1}{1+n} = \frac{(1-\delta) \cdot \tilde{k}_t + s \cdot \tilde{k}_t^{\alpha}}{1+n}$$

which should be recognized as the transition curve for the basic Solow model without technological progress (from chapter 3).

**2.3)**

The steady state is defined by the condition that $\tilde{k}_t$ is constant over time such that: $\tilde{k}_{t+1} = \tilde{k}_t$

This is exactly the point where the transition curve intersects the 45-degree line, as showed in the diagram above. Once the economy has reached this steady state it will stay there forever, in absence of changes in the structural parameters. By setting $\tilde{k}_{t+1} = \tilde{k}_t$ in equation (13) we can derive the steady state value of $\tilde{k}_t$:

$$\tilde{k}_{t+1} = \tilde{k}_t \Rightarrow \left(1 - \delta + s \cdot \left(\tilde{k}^*\right)^{\alpha-1}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{1}{1+n}\right)^{\frac{1}{(1-\chi)}} = 1 \Rightarrow$$

$$\left(1 - \delta + s \cdot \left(\tilde{k}^*\right)^{\alpha-1}\right)^{\frac{1-\alpha-\chi}{(1-\alpha)\cdot(1-\chi)}} = (1+n)^{\frac{1}{(1-\chi)}} \Rightarrow$$

$$1 - \delta + s \cdot \left(\tilde{k}^*\right)^{\alpha-1} = (1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} \Rightarrow \left(\tilde{k}^*\right)^{\alpha-1} = \frac{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}{s} \Rightarrow$$

$$\tilde{k}^* = \left(\frac{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}{s}\right)^{\frac{1}{\propto-1}} = \left(\frac{s}{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}\right)^{\frac{1}{1-\propto}}$$

Finally we can use $\tilde{y}_t = \tilde{k}_t^\alpha$ to express output per effective worker in steady state:

$$\tilde{y}^* = \left(\tilde{k}^*\right)^\propto = \left(\frac{s}{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}\right)^{\frac{\propto}{1-\propto}}$$

Both $\tilde{k}^*$ and $\tilde{y}^*$ depend positively on the savings rate and negatively on the growth rate of the labour force and the rate of depreciation of physical capital.

An increase in $s$ implies that a larger fraction of output is saved and thereby invested, which increases the steady state capital stock per effective worker. Also, output per effective worker increases due to the higher steady state capital stock.

An increase in $n$ or $\delta$ increases replacement investment, i.e. the amount of investment which is required just to keep the amount of capital per effective worker ($\tilde{k}^*$) constant. This decreases the steady state capital stock per effective worker.

**2.4)**

Since the steady state is defined by the requirement that $\tilde{k}_t = \frac{k_t}{A_t}$ is constant we get that $k_t^* = A_t^* \cdot \tilde{k}^*$ must increase at the same rate as $A_t^*$. Also, according to above $\tilde{y}_t$ is constant in steady state implying that $y_t^* = A_t^* \cdot \tilde{y}^*$ must also grow at the same rate as $A_t^*$. We denote this common growth rate $g_{se}$, such that in steady state:

$$1 + g_{se} = \frac{A_{t+1}}{A_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t}$$

Now let's use equation 11) and use the definitions: $k_t = \frac{K_t}{L_t}$ and $k_{t+1} = \frac{K_{t+1}}{L_{t+1}}$ implying that:

$$K_t = k_t \cdot L_t \text{ and } K_{t+1} = k_{t+1} \cdot L_{t+1}$$

Inserting these in equation (11) we get:

$$\frac{A_{t+1}}{A_t} = \left(\frac{k_{t+1} \cdot L_{t+1}}{k_t \cdot L_t}\right)^{\frac{\propto\cdot\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{(1-\chi)}} = \left(\frac{k_{t+1}}{k_t}\right)^{\frac{\propto\cdot\chi}{(1-\alpha)\cdot(1-\chi)}} \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\propto\cdot\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{(1-\chi)}} =$$

$$\left(\frac{k_{t+1}}{k_t}\right)^{\frac{\propto\cdot\chi}{(1-\alpha)\cdot(1-\chi)}} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{(1-\alpha)\cdot(1-\chi)}}$$

Now let's use that: $\frac{A_{t+1}}{A_t} = 1 + g_{se}$ and $\frac{k_{t+1}}{k_t} = 1 + g_{se}$

$$1 + g_{se} = (1 + g_{se})^{\propto \cdot \frac{\chi}{((1-\alpha)\cdot(1-\chi))}} \cdot \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{((1-\alpha)\cdot(1-\chi))}} \Rightarrow$$

$$(1 + g_{se})^{\frac{(1-\alpha-\chi)}{((1-\alpha)\cdot(1-\chi))}} = \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{((1-\alpha)\cdot(1-\chi))}} \Rightarrow 1 + g_{se} = \left(\frac{L_{t+1}}{L_t}\right)^{\frac{\chi}{1-\alpha-\chi}}$$

Finally insert equation (5):

$$1 + g_{se} = (1 + n)^{\frac{\chi}{1-\alpha-\chi}} \Rightarrow g_{se} = (1 + n)^{\frac{\chi}{1-\alpha-\chi}} - 1$$

which is the steady state growth rate of technology, capital per worker and output per worker.

We see that if $\chi = 0$ the steady state growth rate equals zero, which is simply the result from *the basic Solow model*, stating that in the absence of technological progress steady state growth in output per worker can not be sustained due to decreasing returns to capital.

Also, we see that if $n = 0$ the steady state growth rate equals zero, implying that long run growth is not sustainable in this case. This result is simply due to the fact that even though the production function exhibits increasing returns to scale with respect to labour and capital together at the social level *there are still decreasing returns to scale with respect to capital at the social level* (since we assume $\chi < 1 - \alpha$). With $n = 0$ capital is the only factor of production which is growing over time, and the increasing returns accruing to labour and capital together can not be exploited.

In general the steady state growth rate of technology and output and capital per worker depends positively on $n$ since growth in the labour force allows the economy to exploit the increasing returns to capital and labour together (at the social level) due to the effect on social infrastructure. The result that the steady state growth rate of the economy depends positively on the growth rate of the labour force is known as a weak scale effect. These results are identical to the results in section 8.2.

**2.5)**

Question 2.4) derived *the steady state growth rate of output per worker*. In this question we derive the *steady state level of output per worker*.

By definition $y_t^* = A_t^* \cdot \tilde{y}^*$ where output per effective worker is constant in steady state (and given by the expression in question (2.3)). Now insert equation (9):

$$y_t^* = A_t^* \cdot \tilde{y}^* = (Y_t^*)^{\chi/(1-\alpha)} \cdot \tilde{y}^*$$

Also, let's use the definition of output per worker:

$$y_t^* = \frac{Y_t^*}{L_t} \Rightarrow Y_t^* = y_t^* \cdot L_t$$

Insert this in the equation above:

$$y_t^* = (Y_t^*)^{\chi/(1-\alpha)} \cdot \tilde{y}^* = (y_t^* \cdot L_t)^{\chi/(1-\alpha)} \cdot \tilde{y}^* = (y_t^*)^{\chi/(1-\alpha)} \cdot (L_t)^{\chi/(1-\alpha)} \cdot \tilde{y}^*$$

From this equation we can finally derive an expression for $y_t^*$:

$$(y_t^*)^{\frac{(1-\alpha-\chi)}{(1-\alpha)}} = (L_t)^{\frac{\chi}{(1-\alpha)}} \cdot \tilde{y}^* \Rightarrow y_t^* = (L_t)^{\frac{\chi}{(1-\alpha-\chi)}} \cdot (\tilde{y}^*)^{\frac{1-\alpha}{1-\alpha-\chi}}$$

Finally we can insert the result from question 2.3) and $L_t = L_0 \cdot (1+n)^t$

$$y_t^* = (L_0 \cdot (1+n)^t)^{\frac{\chi}{(1-\alpha-\chi)}} \cdot \left(\left(\frac{s}{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}\right)^{\frac{\propto}{1-\propto}}\right)^{\frac{1-\alpha}{1-\alpha-\chi}} =$$

$$(L_0 \cdot (1+n)^t)^{\frac{\chi}{(1-\alpha-\chi)}} \cdot \left(\frac{s}{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}\right)^{\frac{\propto}{1-\propto-\chi}}$$

Also we can use equation (4) and (6) to express steady state consumption per worker:

$$C_t = Y_t - S_t = (1-s) \cdot Y_t \Rightarrow$$

$$c_t^* = (1-s) \cdot y_t^* = (1-s) \cdot (L_0 \cdot (1+n)^t)^{\frac{\chi}{(1-\alpha-\chi)}} \cdot \left(\frac{s}{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}\right)^{\frac{\propto}{1-\propto-\chi}}$$

We see that there are two counteracting effects of an increase in $n$ on $y_t^*$ $and$ $c_t^*$. First of all an increase in $n$ will increase the steady state growth rate of output and consumption according to 2.4). However by increasing replacement investment an increase in $n$ will also decrease the level of output per effective worker through a decrease in the steady state capital stock per effective worker according to 2.3).

**2.6)**

We can write the log of steady state consumption per worker as:

$$lnc_t^* = ln(1-s) + \frac{\alpha}{1-\alpha-\chi} lns + lnX_t$$

where:

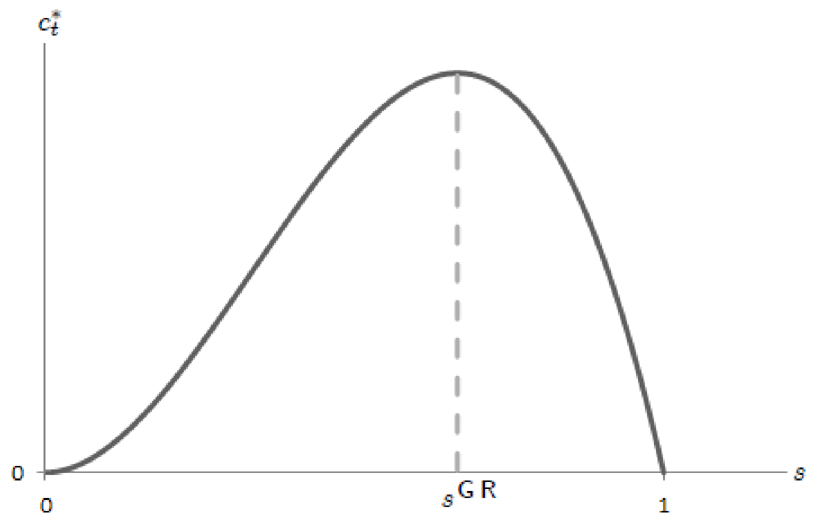
$$X_t = (L_0 \cdot (1+n)^t)^{\frac{\chi}{(1-\alpha-\chi)}} \cdot \left(\frac{1}{(1+n)^{\frac{1-\alpha}{(1-\alpha-\chi)}} - (1-\delta)}\right)^{\frac{\propto}{1-\propto-\chi}}$$

is independent of $s$. By differentiating the expression for $lnc_t^*$ with respect to $s$ we get:

$$\frac{\partial lnc_t^*}{\partial s} = -\frac{1}{1-s} + \frac{\alpha}{1-\alpha-\chi} \cdot \frac{1}{s}$$

We see that there are two counteracting effects of an increase in $s$. The first term reflects that an increase in the savings rate will increase savings and thereby decrease consumption *for a given income level*.

However the second term reflects that an increase in $s$ will also increase income (GDP) per worker since a larger fraction of income is saved and invested, which increases the steady state capital stock and thereby steady state output and income. For low values of $s$ the second effect will dominate while the first effect will dominate for high values of $s$ as showed below.




The golden rule value of $s$ is by *definition the value of $s$ which maximizes steady state consumption per worker*. By setting the derivative of $lnc_t^*$ with respect to $s$ equal to zero we get that the golden rule value of $s$ is characterized by:

$$\frac{\partial lnc_t^*}{\partial s} = 0 \Rightarrow \frac{1}{1-s^{GR}} = \frac{\alpha}{1-\alpha-\chi} \cdot \frac{1}{s^{GR}} \Rightarrow$$

$$(1-\alpha-\chi) \cdot s^{GR} = \alpha \cdot (1-s^{GR}) \Rightarrow s^{GR} = \frac{\alpha}{1-\chi}$$

(see the figure above for a graphical interpretation of $s^{GR}$)

We see that if $\chi = 0$ then $s^{GR} = \alpha$ (which is the result from the basic Solow model). When $\chi > 0$ the golden rule level of $s$ is larger, since in this case there is a further positive effect from an increase in $s$ on steady state output (compared to the Solow model). An increase in $s$ will increase the steady state capital stock, but thereby also increase the level of social infrastructure (since $\lambda > 0$) and thus increase steady state output further (since $\phi > 0$).

**2.7)**

Now we assume that: $\chi = 1-\alpha$ and $n = 0$. Going back to the production function in equation (1) we get:

$$Y_t = K_t^\alpha \cdot L^{1-\alpha} \cdot Y_t^\chi = K_t^\alpha \cdot L^{1-\alpha} \cdot Y_t^{1-\alpha} \Rightarrow$$

$$Y_t^\alpha = K_t^\alpha \cdot L^{1-\alpha} \Rightarrow Y_t = K_t \cdot L^{\frac{(1-\alpha)}{\alpha}} = K_t \cdot \theta$$

where we have defined the constant $\theta = L^{\frac{(1-\alpha)}{\alpha}}$. This is the *AK-model with constant returns to capital at the social level*. It is easy to derive the growth rate of capital, output and technology since this common growth rate is constant at all times. By dividing the production function by the labour force we get an expression for output per worker:

$$y_t = \frac{Y_t}{L} = \frac{K_t}{L} \cdot \theta = k_t \cdot \theta$$

Now use the capital accumulation equation, insert equation (3) and (4) and divide by L:

$$k_{t+1} = \frac{K_{t+1}}{L} = \frac{K_t}{L} \cdot (1-\delta) + s \cdot \frac{Y_t}{L} = k_t \cdot (1-\delta) + s \cdot y_t$$

Now insert the expression for output per worker above:

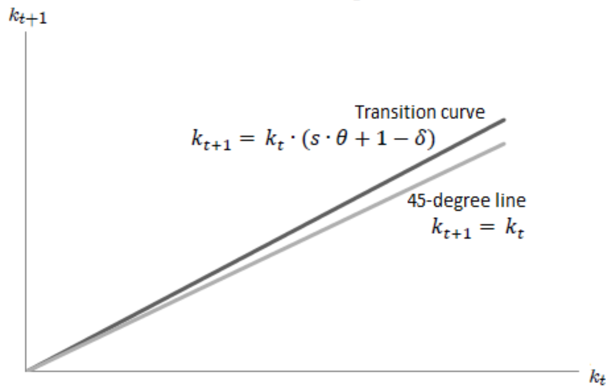
$$k_{t+1} = k_t \cdot (1-\delta) + s \cdot \overbrace{k_t \cdot \theta}^{y_t} = k_t \cdot (s \cdot \theta + 1 - \delta) \Rightarrow$$

$$k_{t+1} - k_t = k_t \cdot (s \cdot \theta - \delta) \Rightarrow \frac{k_{t+1} - k_t}{k_t} = s \cdot \theta - \delta \equiv g_e$$

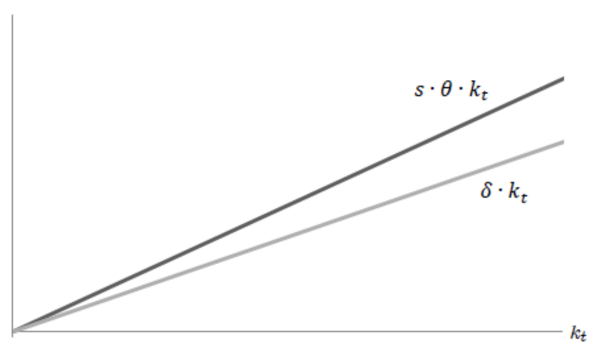
This is also the growth rate of output per worker since $y_t = k_t \cdot \theta$. Finally, this is also the growth rate of consumption per worker (since this is given by: $c_t = (1-s) \cdot y_t$) and technology (since the level of labour augmenting technology is now given by $A_t = Y_t$)

We see that an increase in $s$ will now increase the long run growth rate. This is simply due the fact that in this model there are constant returns to capital at the social level. An increase in $s$ will increase the capital stock (due to the increase in savings and investments) which increases output. *In this case* output will increase just as much (in percentage terms) as the capital stock increases (since there are constant returns to capital at the social level). Thus, savings will increase by the same amount as the capital stock (in percentage terms) implying that the increase in the capital stock can now be sustained. This is also implies that there isn't any transitional dynamics. At any point in time the economy grows at the rate $g_e$. This is showed in the diagrams below, assuming that the structural parameters are such that: $g_e = s \cdot \theta - \delta > 0$

**Transition diagram**




**Solow diagram**



**2.8)**

First of all the model have some unattractive features. In particular the growth rate of the economy depends positively on the size of the labour force (since $\theta = L^{\frac{(1-\alpha)}{\alpha}}$ depends positively on the labour force). This is a strong scale effect and implies that larger economies should grow faster than smaller economies, which certainly do not occur in reality. Further the strong scale effect implies that growth will be explosive in the presence of positive population growth. Also, the model does not predict any sort of convergence (neither absolute nor conditional). However, as showed in chapter 5 and 6 there are some empirical evidence speaking in favor of conditional convergence. These features are speaks in favor of discarding the model with $\chi = 1 - \alpha$.

However as discussed in chapter 8 there are also some attractive features regarding the model. First of all we are able to actually explain the growth rate (this was also for the semi-endogenous case), and growth is now sustainable even in case of no population growth (which was not the case for the semi-endogenous case). Also the prediction that long run growth in output per worker and technology depends positively on the savings rate (investment rate) is consistent with empirical correlations (according to chapter 8), which speaks in favor of this type of model with truly endogenous growth.