Written Exam for the B.Sc. in Economics 2009-II

Micro Economics 1

Final Exam

12. August 2009

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Question 1

Jessica has preferences for lipstick and fashion magazines, which can both be consumed in continuous amounts. Her preferences for these two goods can be represented by the utility function $u(x_1, x_2) = \frac{1}{4}x_1 + \ln(x_2)$, where good 1 is lipstick and good 2 is magazines.

a) Find the income elasticities for each of the two goods

Jessica's budget for these two goods is 5000 kr. each year. A fashion magazine can be bought for 30 kr. And lipstick can be bought for 75 kr. The government (which is male dominated!) has realised that fashion magazines are an un-necessary luxury good. Hence it has decided that this should be taxed by 20 kr.

b) Find the Equivalent Variation (EV) and the Compensated Variation (CV) following this change in prices.
c) Illustrate the change in Jessica's Consumer's Surplus graphically and find what it is?
d) Discuss the statement: *If the government returns the revenue from the imposed tax to the consumers as a lump sum subsidy, then the impact on Jessica's welfare is 0.*

Answers:

a) Jessica's preferences are quasi-linear. Hence, we know that the income elasticity for x_2 is 0. To find the elasticity for good one we first find the demand function $x_1=m/p_1-4$, from which we can derive the income elasticity $\eta_1 = \frac{\partial x_1}{\partial m}\frac{m}{x_1} = \frac{1}{p_1}\frac{m}{m/p_1 - 4} = \frac{m}{m-4p_1}$

b) We use that the income elasticity is 0, which implies that EV and CV are the same. If the formula ($EV = v(x_2^*) - v(x_2) + p_2 \cdot x_2 - p_2^* \cdot x_2^* = CV$)can be remembered, this can be used. The alternative approach is to find the minimal expenditure necessary to maintain the original utility at new prices (CV) and calculate the difference between this and the original budget. First the demand for good 2 has to be found in the two price regimes. The demand function is $x_2=4p_1/p_2$ and consumption before and after the price change is $x_2^*=10$ and $x_2^*=6$. $CV=\ln(10)-\ln(6) + 50*6 - 30*10 = \ln(5/3)$

c) We know that EV=CV and CS=CV=EV. However, we could also find CS by the area under the partial demand curve, which is what the illustration should show us.

d) No, there is a reduction in demand as the illustration in c) also indicates. This implies that the revenue is not from the demand before the price change. The tax induces a dead weight loss.

Question 2

Explain how the *Mean-Variance* model can be used to choose optimal portfolio's.

Answer

By investing the share x in a risky asset the expected return of a portfolio is $r_x = r_f + x\,(r_m - r_f)$, where m represents the market portfolio, while f is a risk free asset, the r's represent expected returns. The risk by this portfolio is described the variance $x^2\,\sigma_m^2$. The relevant budget is $r_x = \frac{\sigma_x}{\sigma_m}(r_m - r_f) + r_f$*. He thus have to maximise his utility function subject to this budget to find the optimal share x. FOC are* $-\frac{\partial u(r_x, \sigma_x)/\partial r_x}{\partial u(r_x, \sigma_x)/\partial \sigma_x} = \frac{r_m - r_f}{\sigma_m}$ *or MRS= price on return (mean) relative to the risk (variance).*

Question 3

Consider an economy with two consumer goods. We also have two consumers Adrian and Benjamin. Consumption possibilities for both is the positive domain $\mathbf{R}_+^2$ and their preferences can both be represented by the utility function $u(x_1, x_2) = \ln x_1 + \ln x_2$.
The endowments in this economy are $\omega^A = (30, 8)$, and $\omega^B = (12, 6)$.

a) Find the Walras equilibrium in this economy using the price on good 2 as numeraire.
b) Can the allocation where Adrain and Benjamin have equal amounts of each of the two goods be implemented as a Walras equilibrium with transfers? If yes, then explain why and find such an equilibrium; if not, then argue why not.

There is also a firm producing good 2 from good 1, which can be described by the production function $y = q$, where q is the amount of good 1 (input) and y is the produced amount of good 2 (output). We assume that Adrian and Benjamin each own 50% of the shares in the firm.

c) Find the Walras equilibrium in this new economy with production. You must provide equilibrium prices, allocations and production. Comment on the consumers' income and utilities and compare with the situation without production
d) Is the equilibrium in c) Pareto optimal? Explain why/why not. Can we ensure an equilibrium as the one in question b) by changing the ownership between Adrian and Benjamin?

a) *Find Adrian's and Benjamin's demand functions, which in this case are the same. Note that the utility functions are a positive monotone transformation of the Cobb-Douglas utility function. Equilibrium in the markets gives us the price $p_1=1/3$. This gives $(x_{1A},x_{2A})=(27, 9)$ og $(x_{1E},x_{2E})=(15,5)$*
b) *Yes by 2. welfare theorem then all Pareto Optimal allocations can be implemented as a Walsras equilibrium with transfers. The allocation where Adrian and Benjamin have equal amounts of the each of the two goods is PO as they have identical preferences and thus that their MRS are equal. This can be obtained by simply transferring endowments to an equal distribution, which can be sustained by a pricefound by $x_{2A}=14/2=p_1x_{1A}=p_1 42/2$,giving $p_1=1/3$.*
c) *The price is now determined from the supply side. Maximising profits gives $p_1=1$, and thus a profit of 0. The income for Adrian and Benjamin are now38 and 18. Te price further implies that both Adrian and Benjamin should each have equal amounts of the two goods in equilibrium. By their incomes this gives $(x_{1A},x_{2A})=(19, 19)$ og $(x_{1E},x_{2E})=(9,9)$which implies that $q=y=14$.Utility is increased for Adrian, but fallen for Benjamin. Adrian who is a net buyer of good 2 benefits from the firm which can produce good 2 relatively cheap, while Benjamin is worse off due to "competition" in consumption of good 1 from the firm and thus no longer can benefit from his endowment of the scarce ressource (good 2) and thus his wealth.*
d) *The equilibrium in c) is PO since the MRS are equal to the relative price, which is further equal to the MRT of the firm. We cannot ensure an equal distribution by shifting profit shares since profits are always 0.*

Question 4

Describe the principles of the Weak and Strong Axiom of Revealed preferences and give examples, where these are not satisfied.

Answer
It should be explained that preferences may not be reversed. This can be done in several ways. One is to explain that a consumer must not choose a bundle in one case(with one set of prices) over another that can be afforded and then choose this other bundle in another case (with another set of prices) if both can be afforded. The strong axiom also includes the indirectly revealed preferences. Examples of violations of both axioms are presented in Varian, but many examples are possible. It is essential that it is explained why they are a violation.

Question 5

A consumer has preferences that can be represented by the utility function $u(x_1, x_2) = x_1^{3/2} + x_2$. Let the optimal choice before the price change be x^*. Consider a Hicks Compensation. Explain why this compensation is equal to $\boldsymbol{px}^*$. Is this always the case? Explain

Answers
This is a special case, where there is no income effect – we have a quasi-linear utility function – this means that the income needed to ensure the original utility is the same as the income the consumer has before the price change namely px^**. However, this is not always the case since utility functions with income effects have a Hicks compensation ensuring the same utility level as before the price change. This compensation income is not the same as in the Slutsky compensation*