Macro III - exam solutions (June 7, 2019)

General remarks

Please grade each item of each question between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

1 True. Basic real business cycle models assume perfect competitive markets, and no externalities. Thus, the first welfare theorem holds and the decentralized competitive equilibrium is Pareto optimal. Therefore, government intervention cannot increase welfare.

2 False. The real exchange rate is the price of a basket of goods in one country relative to the price of the *same* basket in another country. Thus, since the productivities of tradables and non-tradables respectively grow at the same rate across countries, real exchange rates are constant. Thus, the statement is false.

3 False. The extension of the voting franchise and technological improvements in tax collection took place mostly in the first half of the twentieth century. Thus they cannot explain the observed increase in redistributive transfers in the postwar period (at least in a static model, as e.g. Meltzer and Richard (1981)).

4 a) The wage and interest rate are determined by imposing equilibrium in factor markets where firms competitively demand labor and capital from households. Thus the student needs to maximize profit function for firms

$$\max_{L_t, K_t} {K_t}^{\alpha} {L_t}^{1-\alpha} - w_t L_t - r_t^L K_t$$

From FOC of firms' problem of maximizing profits we get

$$\begin{aligned} (1-\alpha){K_t}^{\alpha}{L_t}^{-\alpha} &= (1-\alpha)k_t^{\alpha} = w_t \\ \alpha {K_t}^{\alpha-1}{L_t}^{1-\alpha} &= \alpha k_t^{\alpha-1} = r_t^L \end{aligned}$$

where we impose that in equilibrium all firms work with the same capital labor ratio, k, which must be equal to the ratio of aggregate capital to labor.

Note the relations $d_t = a_t = \frac{k_t}{1-\gamma}$. Market wage, $(1-\alpha)k_t^\alpha = (1-\alpha)((1-\gamma)a_t)^\alpha$, and deposit rate r_t^D times saving per capita are payments to households (in per capita terms). Thus household income on saving is $a_t r_t^D = a_t \left(\alpha((1-\gamma)a_t)^{\alpha-1}(1-\gamma)\right) = \alpha((1-\gamma)a_t)^\alpha$. Partial credit if wrongly derived but intuition is correct.

b) Lagrangian (two forms are equivalent):

$$\begin{aligned}
\mathcal{L}_0 &= \sum_{t=0}^{\infty}\left[\beta^t(1+n)^t\frac{c_t^{1-\theta}}{1-\theta} + \lambda_t\left(w_t + (1+r_t^D)a_t - c_t - a_{t+1}(1+n)\right)\right] \\
\mathcal{L}_0 &= \sum_{t=0}^{\infty}\beta^t(1+n)^t\left[\frac{c_t^{1-\theta}}{1-\theta} + \mu_t\left(w_t + (1+r_t^D)a_t - c_t - a_{t+1}(1+n)\right)\right]
\end{aligned}$$

with $\beta^t(1+n)^t\mu_t \equiv \lambda_t$.

FOC:

$$\begin{aligned}
\frac{d\mathcal{L}_0}{dc_t} &= c_t^{-\theta} - \mu_t = 0 \\
\frac{d\mathcal{L}_0}{da_{t+1}} &= -(1+n)\mu_t + \beta(1+n)(1+r_{t+1}^D)\mu_{t+1} = 0
\end{aligned}$$

Combining both FOC we get the Euler equation, or Keynes-Ramsey condition. Note that r^D and not r or r^L or the marginal productivity of capital, is in the Euler equation:

$$c_t^{-\theta} = \beta c_{t+1}^{-\theta}(1+r_{t+1}^D).$$

Interpretation is that consumption (in per capita terms) is increasing/falling over time as long as interest rate is above/below rate at which future consumption is discounted, and that the elasticity of substitution (inverse of coefficient of relative risk aversion), measuring the response of consumption growth rate to a given level of $\beta(1+r_{t+1}^D)$ is $1/\theta$.

Steady state is characterized by $a_{t+1} = a_t$ and $c_{t+1} = c_t$. Thus $a_{t+1} = a_t$ implies that $c_t = w_t + r_t^D a_t - n a_t = (a_t(1-\gamma))^\alpha - n a_t$. $c_{t+1} = c_t$ implies that $r_t^D = (1-\gamma)\alpha\left(a_t(1-\gamma)\right)^{\alpha-1} = 1/\beta - 1$. This pins down the steady state saving per capita ($a^* = \frac{1}{1-\gamma}\left(\frac{\alpha\beta(1-\gamma)}{1-\beta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha\beta}{1-\beta}\right)^{\frac{1}{1-\alpha}}(1-\gamma)^{\frac{\alpha}{1-\alpha}}$).

Phase diagram should show the $a_{t+1} = a_t$ and $c_{t+1} = c_t$ curves and the local dynamics of the variables in the four quadrants they define. The phase diagram should also have the saddle path of convergent dynamics to the steady state (and this correctly identified as the intersection of the $a_{t+1} = a_t$ and $c_{t+1} = c_t$ curves).

c) The unexpected permanent increase in γ will shift down the $a_{t+1} = a_t$ curve, and inwards (to the left) the $c_{t+1} = c_t$ curve (this is immediate from the derivations done in b)). Thus, unambiguously the new steady state will feature lower savings per capita, and lower consumption per capita. Consumption jumps on impact as households respond to the change in the economic environment. Note that it is not possible to say if consumption initially jumps upwards or downwards. But, from the new initial level, consumption will embark on a downward trajectory consistent with the Euler equation, up to reaching the new steady state with lower capital and lower consumption.

5 a) This is the Calvo (1983) model of price setting in discrete time. If households own firms, then these will maximize profits taking into account households' preferences, in particular the fact that utility is discounted at rate β. Thus, the price they will choose in period t, if they are able to change prices, will be a weighted average of current and future expected optimal prices p^*_{t+j}. Weights will reflect both time preferences (less weight will be given to future prices) and the likelihood that the firm will not be able to change prices in periods from t to $t+j$. The latter is given by $(1-\alpha)^j$, because each period the probability of not being able to change prices is $1-\alpha$ and this probability is i.i.d. over time (and across firms). Thus the chosen price will be proportional to:

$$\sum_{j=0}^{\infty} \beta^j (1-\alpha)^j E_t[p^*_{t+j}],$$

where $E[p^*_t] = p^*_t$ as there is no uncertainty about the desired optimal price in period t. What remains to be done is to divide the previous expression by $\sum_{k=0}^{\infty} \beta^k (1-\alpha)^k$ such that weights add up to one (otherwise a proportional change in all present and future p^*_{t+j} would not lead to a proportional change in x_t and the model will be misspecified). Thus, we arrive at

$$x_t = \sum_{j=0}^{\infty} \frac{\beta^j (1-\alpha)^j}{\sum_{k=0}^{\infty} \beta^k (1-\alpha)^k} E_t[p^*_{t+j}].$$

Manipulating this equation we can get the following

$$\begin{aligned}
x_t &= (1-\beta(1-\alpha)) \left[p^*_t + \beta(1-\alpha) \sum_{j=1}^{\infty} \beta^{j-1} (1-\alpha)^{j-1} E_t[p^*_{t+j}] \right] \\
&= (1-\beta(1-\alpha)) \left[p^*_t + \beta(1-\alpha) \sum_{k=0}^{\infty} \beta^{k} (1-\alpha)^{k} E_t[p^*_{t+1+k}] \right] \\
&= (1-\beta(1-\alpha)) p^*_t + \beta(1-\alpha) E_t[x_{t+1}].
\end{aligned}$$

b) Substracting p_t from previous result

$$\begin{aligned}
x_t - p_t = (x_t - p_{t-1}) - (p_t - p_{t-1}) &= (1-\beta(1-\alpha))(p_t^* - p_t) + \beta(1-\alpha)(E_t[x_{t+1}] - p_t) \\
\frac{\pi_t}{\alpha} - \pi_t &= (1-\beta(1-\alpha))(\phi m_t - \phi p_t) + \beta(1-\alpha)(E_t[x_{t+1}] - p_t) \\
&= (1-\beta(1-\alpha))\phi y_t + \beta(1-\alpha)(E_t[\frac{\pi_{t+1}}{\alpha}])
\end{aligned}$$

Thus, we arrive at the new Keynesian Phillips curve:

$$\pi_t = \frac{\alpha}{1-\alpha}(1-\beta(1-\alpha))\phi y_t + \beta E_t[\pi_{t+1}].$$

This equation states that inflation is a function of two factors: i) next period's expected inflation rate (because prices are set for, in principle, several periods), ii) the output gap (because prices depend positively on real marginal cost —through a mark-up— with monopolistic competition price setting firms, and ϕ measures degree of real rigidities).

c) To solve we start by looking at new Keynesian Phillips curve in period $t+1$:

$$\pi_{t+1} = \frac{\alpha}{1-\alpha}(1-\beta(1-\alpha))\phi y_{t+1} + \beta E_{t+1}[\pi_{t+2}].$$

Replacing this in the original equation and using the law of iterative expectations:

$$\pi_t = \frac{\alpha}{1-\alpha}(1-\beta(1-\alpha))\phi y_t + \beta E_t[\frac{\alpha}{1-\alpha}(1-\beta(1-\alpha))\phi y_{t+1}] + \beta^2 E_t[\pi_{t+2}].$$

Repeating this procedure, under the assumption that $\lim_{j\to\infty} \beta^j E_t[\pi_{t+j}] = 0$ and that the rate of growth of $E_t[y_{t+j}]$ in the limit is lower than $1/\beta$ we get:

$$\pi_t = \frac{\alpha\phi}{1-\alpha}(1-\beta(1-\alpha))\sum_{j=0}^{\infty} \beta^j E_t[y_{t+j}].$$

Inflation today reflects expected future output realizations in excess of output potential (in these models "normal" output in logs is zero). So even if output is at potential level today, if output is expected to be above potential in the following periods we get inflation today.