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Solutions to written exam for the M. Sc in Economics, April 18 2009
International Finance

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1. This question relates to several learning objectives including; describe the institutional features of the foreign exchange market products (spot, forward, swap and option contracts) and be able to distinguish between speculation and arbitrage; describe the relationships between four prices; spot prices, forward rates and money–market interest rates at home and in the foreign country and be able to predict any single price on the basis of information about the other three; and describe the channels by which central bank intervention can affect the exchange rate and summarize the empirical evidence on these channels;

   (a) True. The Central Bank buys and sells foreign exchange, there is a swap of foreign bonds (denominated in foreign currency) for domestic bonds (or the opposite) in the portfolios held by the Central Bank and domestic households.

   (b) False. In this case, the portfolio channel will not work since the portfolio balance model collapses to the standard monetary approach to the exchange rate. However, the signalling approach will still work. Sterilized interventions will affect the current exchange rate since the intervention signals future changes in monetary policy and since the current exchange rate is given by current and future monetary policy, there will be an immediate effect on the exchange rate.

   (c) False. This is the definition of speculation. Arbitrage is the simultaneous, or nearly simultaneous, purchase of securities in one market for sale in another market with the expectation of a risk–free profit.

   (d) True. Synthetic assets and liabilities can be created by using a spot contract combined with borrowing and lending. By applying Covered interest parity (which is an arbitrage relation)
$$F_{t,6} = S_t \frac{1 + i_{\$,6}/2}{1 + i_{€,6}/2}$$

we find that a combination of a spot contract with fixed–rate, n–period borrowing and lending in the two currencies, must be equivalent to an n–period forward or futures contract.

(e) False. An order flow is defined as the difference between purchase and sale orders initiated by customers during a certain trading period.

2. This question relates to three learning objectives: describe the institutional features of the foreign exchange market products (spot, forward, swap and option contracts) and be able to distinguish between speculation and arbitrage; graph the payoffs of a futures contract as a function of the price of the underlying asset, graph similar payoff profiles or combinations of futures contracts, and be able to show how futures contracts can be used to hedge an open risky position; and describe the types of risks that foreign exchange traders face and how these risks can be managed.

(a) The importer takes a short position, the importer has an outlay in foreign currency in three months.

(b) If the euro is \$1.30/€in three months, the importer will separately: settle the futures contracts, and buy the euros in the spot market to pay the exporter.

1. Gain on futures (\$1.30 - \$1.255) x €10m = \$450,000 profit.
2. Purchase €10 million at \$1.30 per euro = (\$13,000,000)
3. Net cost of buying the euros: -\$13m (spot) + \$.45m (futures) profit = \$12.55m (or \$1.255/€)

(c) The euro is \$1.20/Ă in 3 months. The importer will separately: settle the futures contracts, and buy the euros in the spot market as above.

1. Loss on futures (\$1.20 - \$1.255) x €10m = -\$550,000 loss.
2. Purchase €10m at \$1.20 = (\$12,000,000)
3. Net cost of buying the euros: -\$12m (spot) - \$.55m (futures) loss = \$12.55m (or \$1.255/€)

CONCLUSION: With futures contracts at \$1.255/€, GM guarantees an ex-rate of \$1.255/€and a total cost of \$12.55m, regardless of what happens to the euro. Even though the importer locks in an exchange rate of \$1.255, it doesn't actually buy euros at that rate. It will buy the 10m euros in the spot market in three months and at the same time, settle the futures contract in CASH.

(d) A minimum variance delta hedge in case the maturity dates differ can be constructed in the following way. Let S_1 be the spot exchange rate at the maturity date 1, the current period is 0, $F_{0,2}$ is the futures rate at time 0 for delivery at time 2, and $F_{1,2}$ is the futures rate at time 1 for delivery at time 2. The cost of converting home currency to foreign currency at the maturity date is

$$S_1 \times C$$

where C is the number of units of foreign currency needed at time 1. The difference between the futures prices at time 1 is

$$-N\left(F_{1,2} - F_{0,2}\right) \times Q$$

where N is the number of futures contracts and Q is the contract size. The value of the portfolio can then be written as

$$S_1 \times C - N\left(F_{1,2} - F_{0,2}\right) \times Q.$$

Define the hedge ratio

$$\beta = \frac{NQ}{C}$$

and insert above and compute the variance of the value of the portfolio (and remember that $F_{0,2}$ is known and therefore constant). We then obtain

$$\mathrm{Var}\left(S_1\right) + \beta^2 \mathrm{Var}\left(F_{1,2}\right) - 2\beta \mathrm{Cov}\left(S_1, F_{1,2}\right).$$

Minimizing this variance with respect to β we find that

$$2\beta \mathrm{Var}\left(F_{1,2}\right) - 2\mathrm{Cov}\left(S_1, F_{1,2}\right) = 0$$

implying that the optimal hedge ratio is

$$\beta = \frac{\mathrm{Cov}\left(S_1, F_{1,2}\right)}{\mathrm{Var}\left(F_{1,2}\right)}$$

such that the optimal number of contracts is $N = \beta \frac{C}{Q}$. To find β we note that it is the slope coefficient in a regression where S_1 is the dependent variable and $F_{1,2}$ the independent variable. To compute β we then make use of actual data on spot rates and future rates. Given the estimated β, we then compute the optimal number of contracts.

(e) Differences:

- Forwards: large range of delivery dates; Futures: limited range of delivery dates
- Forwards: no standardized amounts (large amounts); Futures: standardized amounts (smaller contracts)

- Forwards: no second hand market; Futures: second hand market
- Forwards: counterparty risk; Futures: contracts guaranteed by futures exchange or clearing house
- Forwards: covers over 50 currencies; Futures: only major currencies
- Forwards: profits/losses realized on maturity date; Futures: profits/losses realized prior to maturity (marking to market)

Note that it is not necessary to explain in detail how marking–to–market works.

3. This question relates to the following learning objectives: list the determinants of put and call options prior to maturity and at maturity; describe the relationships between put prices, call prices and forward rates and be able to apply the put-call-forward parity formula; and show that a simple option can be replicated by borrowing/lending and holding of a fractional position in the underlying asset and that this replicating portfolio can be used to construct an option, price an option and hedge an option position.

   (a) To derive the price of a put option using the binomial approach we make use of the idea of a replicating portfolio. An option can be replicated by lending, spot transaction and borrowing. For arbitrage reasons the price of the cash flow from this replicating portfolio and the put option must be the same. Using this idea we then can compute the price of the put option within a one–period binomial framework.

   A put option gives the right to sell foreign currency in the future at a specified price. The put option generates cash flows in the second period, either zero in case the future spot rate goes up or a positive cash flow if the future spot rate falls, see the first row of the table below.

   The alternative to a put option (and to generate cash flow in period 2) is to buy the underlying, in our case the foreign exchange, spot and borrow the amount Δ_p and then lend the amount L which will give a certain payoff in period 2 depending on whether the future exchange rate goes up or falls, see the second and third row of the table below.

Portfolio	Cash flow period 1	Cash flow period 2	
		$S_2 \leq X$	$S_2 > X$
Buy put	P_1	0	P_{2d}
Buy spot and borrow Δ_p units	$\Delta_p S_1$	$-\Delta_p S_{2u} \exp r_f$	$-\Delta_p S_{2d} \exp r_f$
Lend L units	$-L$	$L \exp r_d$	$L \exp r_d$

Compare the cash flows in period 1 and 2 to find that $P_1 = \Delta_p S_1 - L$. To solve for L and Δ_p we make use of the cash flows in period 2. We find that

$$\Delta_p = \frac{P_{2d}}{S_1(u-d)e^{r_f}}$$

and

$$L = frac{-}uP_{2d}(u-d)e^{r_d}$$

where we have used the fact that $S_{2u} = S_1 u$ and $S_{2d} = S_1 d$. Inserting this into the expression for P_1 we then find the price on the put option. We obtain

$$P_1 = \frac{P_{2d}}{u-d}\left(\frac{1}{e^{r_f}} + \frac{u}{e^{r_d}}\right).$$

(b) Consider two different portfolios, one with options and one using borrowing/lending at home and abroad. If the payoff from these two portfolios are the same, then the price must be the same otherwise there will be arbitrage opportunities.

Assume that we buy one call option and sell one put option at the same strike price K. The cash flow in period 1 and period 2 ar shown in the table below.

Portfolio	Cash flow period 1	Cash flow period 2	
		$S_2 \leq K$	$S_2 > K$
Buy call	-C	0	$S_2 - K$
Sell put	P	$S_2 - K$	0
Total	P-C	$S_2 - K$	$S_2 - K$

To receive the cash flow $S_2 - K$ in the next period, we have to pay the price $P - C$. How can we construct another portfolio giving us the same cash flow in period 2? We can borrow in the domestic currency, buy foreign exchange on the spot exchange market and invest abroad. It is important that we construct the portfolios so that we obtain the same cash flow in period 2 as when using options. First, we borrow at home generating a cash outflow in period 2 equal to K and buy spot exchange to invest abroad which should give us the cash inflow S_2 in the next period. The present value of the cash outflow K in period 2 is $K\exp(-r_d)$ (the amount we must borrow in period 1 to generate the required cash flow in period 2) where r_d is the domestic interest rate. Let the spot exchange rate in the first period be equal to S_1 and the foreign interest rate is r_f, then the cash flow in period 1 when investing abroad is $S_1 \exp(-r_f)$. The implied cash flows for the forward contract are shown in the table below.

Portfolio	Cash flow period 1	Cash flow period 2	
		$S_2 \leq K$	$S_2 > K$
Borrow at home	$K\exp(-r_d)$	$-K$	$-K$
Buy spot and invest abroad	$-S_1\exp(-r_f)$	S_2	S_2
Total	$K\exp(-r_d) - S_1\exp(-r_f)$	$S_2 - K$	$S_2 - K$

Comparing the two tables we find that identical cash flows in period 1 implies that

$$P - C = K\exp(-r_d) - S_1\exp(-r_f).$$

This is the put–call parity condition. From this equation we note that buying a call option is equivalent to buying a put option plus a forward contract. Similarly, buying a put option is identical to buying a call option and selling a forward contract. Therefore, buying a call is essentially identical to buying a put. This is the reason why the two option strategies above produce the same payoff structure.

(c) The two equations given in the problem says that

$$C = S\exp(-r^*T)N(d_1) - X\exp(-rT)N(d_2)$$

and

$$P = X\exp(-rT)N(-d_2) - S\exp(-r^*T)N(-d_1).$$

We know that $N(-d_2) = 1 - N(d_2)$. This allows us to write the difference between put and call option prices as

$$P - C = X\exp(-rT)(1 - N(d_2)) - S\exp(-r^*T)(1 - N(d_1)) -$$

$$S\exp(-r^*T)N(d_1) + X\exp(-rT)N(d_2) =$$

$$X\exp(-rT) - S\exp(-r^*T)$$

which is the put–call parity condition.

(d) For S/X approaching zero, $\ln(S/X)$ approaches minus infinity; thus both N(.) factors go to zero. The price on the call option then approaches zero. When S/X approaches plus infinity, $ln(S/X)$ approaches plus infinity so that both N(.) factors approach 1. The call option price is then given by $C = S\exp(-r^*T) - X\exp(-rT)$ which is the same as a forward purchase contract.