Written Exam Economics Winter 2019-20

Econometrics II

Solution Guide

Part 1: Hand-In Assignment #2
Relationship Between Stock Prices and Economic Growth

The Case: The purpose of Part 1 is to analyze the empirical relevance of the PPP relation between Mozambique and South Africa based on monthly data for prices and exchange rates.

The Data: Graphs of the data and relevant transformations must be shown in the exam. It should be noted that there is a clear seasonal pattern in prices.

Econometric Theory: The analysis can be based on any tool from the co-integration tool-kit, i.e. the two-step Engle-Granger procedure, an analysis based on a single-equation ADL model or a co-integrated VAR. The good solution argues for the choice of approach and discusses advantages and drawbacks of the methods.

Empirical Results: The empirical results must include the following:

(1) A presentation and discussion of the relevant empirical results, so that the reader is able to understand the steps carried out in the process as well as the conclusions made.
(2) A description of the model selection process.
(3) A discussion of whether the assumptions for consistent estimation and valid inference are satisfied for the estimated models.
(4) A conclusion regarding the relevance of PPP.

Part 2: Hand-In Assignment #4
Stock market Return, Volatility and Trading Volume

The Case: The goal of Part 2 is to analyze the effect on return and volatility from unexpected movements in trading volumes using a GARCH model. The unexpected trading volume is constructed in a first step as the residual from a time series model.

The Data: Graphs of the data and relevant transformations must be shown. The graphs should emphasize the presence of volatility clustering to motivate the ARCH modelling framework.

Econometric Theory: The econometric theory must include the following:

(1) A discussion of the dynamic model used to filter out the predictable part of trading volume and discuss that the unexpected part has to have no autocorrelation.
(2) A precise definition and interpretation of the models considered and their properties. Specifically, some variants of the ARCH or GARCH model must be presented and interpreted. The interpretation of included explanatory variables should be discussed. Some remarks on asymmetry could be given.
(3) A description of the estimator used, e.g. the maximum likelihood (ML) estimation principle.
(4) The theory must be presented precisely and in a logical order with a consistent and correct notation.

Empirical Results: The empirical results must include the following:

(1) A presentation and discussion of the relevant empirical results, so that the reader is able to understand the steps carried out in the process as well as the conclusions made.
(2) A description of the model selection process based on a general-to-specific approach, information criteria, or both.
(3) A clear conclusion to the main question and a discussion of the limitations of the approach used to reach the conclusion.

Part 3: The Additional Theoretical and Empirical Assignments

#5.1 Two New Heteroskedastic Models

(1) It follows directly that

$$\begin{aligned} E(y_t|y_{t-1}) &= \rho y_{t-1} \\ V(y_t|y_{t-1}) &= \varpi + \alpha y_{t-1}^2 = h_t. \end{aligned}$$

(2) It follows from the conditional normality of z_t that

$$y_t \mid y_{t-1} \overset{d}{=} N(\rho y_{t-1}, \varpi + \alpha y_{t-1}^2),$$

and the contribution to the Gaussian likelihood function is given by

$$\ell(\theta \mid y_t, y_{t-1}) = \frac{1}{\sqrt{2\pi h_t}} \exp\left(-\frac{(y_t - \rho y_{t-1})^2}{2h_t}\right),$$

with $\theta = (\rho, \varpi, \alpha)'$, such that the contribution to the log-likelihood is

$$\log \ell(\theta \mid y_t, y_{t-1}) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\varpi + \alpha y_{t-1}^2) - \frac{(y_t - \rho y_{t-1})^2}{2(\varpi + \alpha y_{t-1}^2)}.$$

(3) We find by direct calculation

$$\begin{aligned} \frac{\partial \log \ell(\theta \mid y_t, y_{t-1})}{\partial \rho} &= \frac{(y_t - \rho y_{t-1}) y_{t-1}}{\varpi + \alpha y_{t-1}^2} \\ \frac{\partial \log \ell(\theta \mid y_t, y_{t-1})}{\partial \varpi} &= -\frac{1}{2\left(\varpi + \alpha y_{t-1}^2\right)} + \frac{(y_t - \rho y_{t-1})^2}{2\left(\varpi + \alpha y_{t-1}^2\right)^2} \\ \frac{\partial \log \ell(\theta \mid y_t, y_{t-1})}{\partial \alpha} &= -\frac{y_{t-1}^2}{2\left(\varpi + \alpha y_{t-1}^2\right)} + \frac{y_{t-1}^2 \left(y_t - \rho y_{t-1}\right)^2}{2\left(\varpi + \alpha y_{t-1}^2\right)^2}. \end{aligned}$$

(4) Assuming stationarity, $E(y_t) = E(y_{t-1})$, and we find

$$\begin{aligned} E(y_t) &= E(\rho y_{t-1} + \epsilon_t) \\ E(y_t) &= \rho E(y_{t-1}) + E(\epsilon_t) \\ E(y_t) &= \rho E(y_t) + 0, \end{aligned}$$

and

$$E(y_t) = 0.$$

For the variance, $V(y_t) = E(y_t^2) = E(y_{t-1}^2)$, we find

$$\begin{aligned} E(y_t^2) &= E((\rho y_{t-1} + \epsilon_t)^2) \\ &= E(\rho^2 y_{t-1}^2 + \epsilon_t^2 + 2\rho y_{t-1}\epsilon_t) \\ &= E(\rho^2 y_{t-1}^2 + \varpi z_t^2 + \alpha y_{t-1}^2 z_t^2 + 2\rho y_{t-1}\epsilon_t) \\ &= \rho^2 E(y_{t-1}^2) + \varpi E(z_t^2) + \alpha E(y_{t-1}^2 z_t^2) + 2\rho E(y_{t-1}\epsilon_t), \end{aligned}$$

where we have used that $\epsilon_t^2 = h_t z_t^2 = (\varpi + \alpha y_{t-1}^2) z_t^2$. Because z_t and y_{t-1} are independent, and $E(z_t^2) = 1$, we get

$$E(y_t^2) = \rho^2 E(y_t^2) + \varpi + \alpha E(y_{t-1}^2),$$

and

$$E(y_t^2) = \frac{\varpi}{1 - \rho^2 - \alpha}.$$

We note that $E(y_t^2) < \infty$ if $\rho^2 + \alpha < 1$.

(5) We find that

$$\begin{aligned} E(y_t | y_{t-1}) &= E(\rho_t y_{t-1} | y_{t-1}) + E(\eta_t | y_{t-1}) \\ &= E(\rho_t | y_{t-1}) y_{t-1} + 0 \\ &= a y_{t-1}, \end{aligned}$$

where we have used that $E(\rho_t | y_{t-1}) = a$.
For the variance, we get

$$\begin{aligned} V(y_t | y_{t-1}) &= E\left((y_t - E(y_t | y_{t-1}))^2 | y_{t-1} \right) \\ &= E\left((\rho_t y_{t-1} + \eta_t - a y_{t-1})^2 | y_{t-1} \right) \\ &= E\left(((a + \xi_t) y_{t-1} + \eta_t - a y_{t-1})^2 | y_{t-1} \right) \\ &= E\left((\xi_t y_{t-1} + \eta_t)^2 | y_{t-1} \right) \\ &= E\left(\eta_t^2 + \xi_t^2 y_{t-1}^2 + 2 \xi_t \eta_t y_{t-1} | y_{t-1} \right) \\ &= E\left(\eta_t^2 | y_{t-1} \right) + E\left(\xi_t^2 | y_{t-1} \right) y_{t-1}^2 + E\left(\xi_t \eta_t | y_{t-1} \right) 2 y_{t-1} \\ &= b + c y_{t-1}^2 + 0, \end{aligned}$$

where we have used that $E\left(\eta_t^2 | y_{t-1}\right) = b$, $E\left(\xi_t^2 | y_{t-1}\right) = c$, and $E\left(\xi_t \eta_t | y_{t-1}\right) = 0$.

(6) The two models have identical first and second conditional moments if

$$a = \rho, \quad b = \varpi, \quad \text{and} \quad c = \alpha.$$

Under the normality assumption in (5.7) the models have the same likelihood function and are equivalent.

#5.2 Co-integrated Vector Autoregression

(1) The model implies that

$$\begin{aligned} \Delta y_t &= -0.5 y_{t-1} + a_t \\ y_t &= 0.5 y_{t-1} + a_t. \end{aligned}$$

By recursive substitution it holds that

$$y_t = a_t + 0.5 a_{t-1} + 0.5^2 a_{t-2} + ... + 0.5^{t-1} a_1 + 0.5^t y_0.$$

It also holds that

$$\Delta x_t = b_t,$$

and the solution is given by

$$x_t = x_0 + \sum_{i=1}^{t} b_i.$$

The solutions show that y_t is stationary, I(0), while x_t is a random walk, I(1). In this case, we say that the vector Z_t is I(1).

(2) In this case,

$$\Pi = \alpha\beta' = \begin{pmatrix} -0.5 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -0.5 \\ 0 & 0 \end{pmatrix},$$

and the model implies

$$\begin{pmatrix} \Delta y_t \\ \Delta x_t \end{pmatrix} = \begin{pmatrix} 0 & -0.5 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} a_t \\ b_t \end{pmatrix}.$$

The equation for x_t is unchanged, and it still holds that

$$x_t = x_0 + \sum_{i=1}^{t} b_i,$$

such that x_t is I(1). Now, however,

$$\Delta y_t = -0.5 x_{t-1} + a_t,$$

such that

$$\begin{aligned} y_t &= y_0 + \sum_{i=1}^{t} \Delta y_i \\ &= y_0 + \sum_{i=1}^{t} (-0.5 x_{i-1} + a_i) \end{aligned}$$

Using that $x_{t-1} = x_t - b_t$, we get

$$\begin{aligned} y_t &= y_0 + \sum_{i=1}^{t} (-0.5 x_i + 0.5 b_i + a_i) \\ &= y_0 - 0.5 \sum_{i=1}^{t} x_i + \sum_{i=1}^{t} (0.5 b_i + a_i). \end{aligned}$$

From the solution to x_t we find

$$\sum_{i=1}^{t} x_i = \sum_{i=1}^{t} \left(x_0 + \sum_{s=1}^{i} b_s \right) = x_0 t + \sum_{i=1}^{t} \sum_{s=1}^{i} b_s.$$

Collecting terms, we find

$$y_t = y_0 - 0.5 x_0 t - 0.5 \sum_{i=1}^{t} \sum_{s=1}^{i} b_s + \sum_{i=1}^{t} (0.5 b_i + a_i).$$

The process, y_t, has a linear trend, a random walk component, and a cumulated random walk component.

Observe that Δy_t still has a random walk component and is therefore non-stationary, I(1). We say that y_t is integrated of second order, I(2).

(3) The solution should perform an empirical analysis using the co-integrated VAR. Most likely, the solution finds one stationary relation. The co-integration vector is probably close to $\beta = (0, 1)'$, with a weak effect from consumption.

(4) The solution should comment on the error correction, short-run parameters as well as the residual correlation as a measure of the contemporaneous effect.