Written Exam - Macroeconomics III

(suggested answers)

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Question 1

a The first part is trivial: $w_t = A$ and $r_t = B$. Moreover, since $n = 0$ and $B > 0$, the economy is dynamically efficient: $1 + r_t > 1 + n$.

b When $\sigma > 1$ (note: we assume this throughout, not clear in question). The marginal utility of consumption is

$$c^{-\sigma} C^{\alpha(\sigma-1)},$$

and its derivative with respect to aggregate consumption is

$$\alpha(\sigma - 1) c^{-\sigma} C^{\alpha(\sigma-1)-1} > 0.$$

What this shows is that there are externalities in consumption. Aggregate consumption affects the individual's incentives to consume. When $\sigma > 1$ the effect is such that an increase in aggregate consumption increases the value of individual consumption. This is known as "keeping up with the Joneses", because the observation that others consume gives incentives to consume oneself. Also note that with this formulation and increase in aggregate consumption makes an individual worse off (the level of his/her utility decreases).

c The Euler equation for individual savings is

$$c_{1t}^{-\sigma} C_{1t}^{\alpha(\sigma-1)} = \frac{1+B}{1+\rho} c_{2t+1}^{-\sigma} C_{2t+1}^{\alpha(\sigma-1)}$$

Replacing from period budget constraints

$$(A - s_t)^{-\sigma} C_{1t}^{\alpha(\sigma-1)} = \frac{1+B}{1+\rho} (s_t(1+B))^{-\sigma} C_{2t+1}^{\alpha(\sigma-1)}$$

impose $c = C$ and solve

$$(A - s)^{-\sigma+\alpha(\sigma-1)} = \frac{1+B}{1+\rho} (s(1+B))^{-\sigma+\alpha(\sigma-1)} \tag{1}$$

Thus, solve for savings

$$s = k = \frac{A}{1 + (1+B)\left[\frac{1+B}{1+\rho}\right]^{\frac{1}{-\sigma+\alpha(\sigma-1)}}}$$

Note that for the log case ($\sigma = 1$), $s_t = \frac{1}{2+\rho} A$.

d To characterize the optimal consumption and saving decisions we need to solve for a social planner that internalizes that $c = C$. Thus the planner, wants to maximize

$$\frac{C_{1t}^{(1-\alpha)(1-\sigma)}}{1-\sigma} + \frac{1}{1+\rho}\frac{C_{2t+1}^{(1-\alpha)(1-\sigma)}}{1-\sigma}$$

FOC gives Euler equation

$$C_{1t}^{-\sigma+\alpha(\sigma-1)} = \frac{1+B}{1+\rho} C_{2t+1}^{-\sigma+\alpha(\sigma-1)}$$

But after replacing for budget constraints this is exactly equation (1). Therefore the private decisions were optimal. Although the externality distorts the consumption incentives of individuals, because the externality is present both when young and old, its aggregate effects on savings compensate (externality when young leads to lower saving since it gives incentive to increase consumption when young, but externality when old leads to higher saving since it gives incentive to increase consumption when old). In a richer model where there are other choices, such as labor decision only when young, the decentralized equilibrium is in general inefficient.

e Note that with a linear production function saving will be the same in periods t_0 and $t_0 + 1$. Introducing social security in equation (1) gives

$$(A - \tau - s_{t_0})^{-\sigma+\alpha(\sigma-1)} = \frac{1+B}{1+\rho}(s_{t_0}(1+B) + \tau)^{-\sigma+\alpha(\sigma-1)}$$

solving gives

$$s_{t_0} = k_{t_0+1} = \frac{A - \tau - \tau\left[\frac{1+B}{1+\rho}\right]^{\frac{1}{-\sigma+\alpha(\sigma-1)}}}{1 + (1+B)\left[\frac{1+B}{1+\rho}\right]^{\frac{1}{-\sigma+\alpha(\sigma-1)}}},$$

so savings and capital accumulation are depressed by the presence of social security. Thus, the reform will be supported by the old, and rejected by the young.

Question 2

a Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\lambda} Y_i^{\frac{\lambda}{\alpha}}$$

Maximizing w.r.t. Y_i:

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta}\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} - \frac{1}{\alpha} Y_i^{\frac{\lambda}{\alpha}-1} = 0$$

After some manipulation we obtain

$$\left(1-\frac{1}{\eta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}}=\frac{1}{\alpha}Y_i^{\frac{\lambda}{\alpha}-1}$$

Which translates into

$$\frac{P_i}{P}=\frac{\eta}{\alpha\,(\eta-1)}Y_i^{\frac{\lambda}{\alpha}-1}$$

b Taking logs and rearranging, we obtain the desired level of production:

$$p_i^*-p=\frac{\lambda-\alpha}{\alpha}y_i+\ln\left[\frac{\eta}{\alpha\,(\eta-1)}\right]$$

Imposing homogeneity and exploiting the fact that households are the same and they have unit size:

$$p^*-p=\frac{\lambda-\alpha}{\alpha}y+\ln\left[\frac{\alpha\,(\eta-1)}{\eta}\right]$$

Since $y=m-p$:

$$p^*-p=\frac{\lambda-\alpha}{\alpha}\,(m-p)+\ln\left[\frac{\alpha\,(\eta-1)}{\eta}\right]$$

So that

$$p^*=\frac{\lambda-\alpha}{\lambda}m+\frac{\alpha}{\lambda}p+\ln\left[\frac{\alpha\,(\eta-1)}{\eta}\right]$$

Imposing the notation $\phi\equiv\frac{\lambda-\alpha}{\alpha}$, we obtain:

$$p^*=\phi m+(1-\phi)\,p+c \tag{2}$$

where $c=\ln\left[\frac{\alpha(\eta-1)}{\eta}\right]$.

c By rearranging (2):

$$p^*-p=\phi y+c.$$

As ϕ lowers we observe greater real rigidity. This is what happens following a marginal increase in α. Under these circumstances, as demand increases, proportionally less labor is required to increase production, given the attenuation of decreasing returns to scale. Thus, a lower cost is passed into prices and quantities vary by relatively more.

d The model economy can be summarized by the following equations:

$$p^f=(1-\phi)p+\phi m \tag{3}$$

$$p^r=(1-\phi)E[p]+\phi E[m] \tag{4}$$

$$p = qp^r + (1-q)\,p^f \tag{5}$$

$$y = m - p \tag{6}$$

where $0 \leq \phi \leq 1$ and $0 \leq q \leq 1$.

Substituting (5) into (3):

$$p^f = (1-\phi)\left(qp^r + (1-q)\,p^f\right) + \phi m$$

and rearranging so as to bring p^f on the left-hand side of the equality:

$$p^f = p^r + \frac{\phi}{\phi + (1-\phi)\,q}\,(m - p^r) \tag{7}$$

Now, recall that $p^r = E[p^r]$. Substituting (7) into (5) we obtain

$$p = p^r + \frac{\phi\,(1-q)}{\phi + (1-\phi)\,q}\,(m - p^r)$$

Now substitute the latter into (4), so as to get:

$$p^r = E[m] \tag{8}$$

e Substituting (7) and (8) into (5):

$$p = E[m] + (m - E[m])\,\frac{\phi\,(1-q)}{\phi + (1-\phi)\,q}$$

Substituting the latter into (6):

$$y = (m - E[m])\,\frac{q}{\phi + (1-\phi)\,q}$$

f As q tends to zero, rigid-price firms vanish, and prices are only set in a flexible manner, so that $p = m$ and $y = 0$. As a result, the unanticipated component of aggregate demand $(m - E[m])$ is no longer relevant, as all firms are able to set prices after observing the actual m. Therefore, output deviations from potential are null.