Suggested answers for written Exam for the B.Sc. in Economics winter 2015
Macro B

Final Exam

February 17 2015
(3 hours closed-book exam)

Academic Aim: The aim of the course is to describe and explain the macroeconomic fluctuations in the short run, i.e. the business cycles around the long run growth trend, as well as various issues related to this, and to teach the methodology used in formulating and solving formal models explaining these phenomena. Students are to learn the most important stylized facts about business cycles and to acquire knowledge about theoretical dynamic models aimed at explaining these facts. In connection with this, the aim is to make students familiar with the distinction between deterministic and stochastic models. Furthermore, students are to gain an understanding of the distinction between the impulses initiating a business cycle and the propagation mechanisms that give business cycles a systematic character. Finally students are to learn how to use the models for analyzing the effects of macroeconomic stabilization policy under various assumptions regarding the exchange rate regime. To obtain a top mark in the course students should at the end of the course be able to demonstrate full capability of using the techniques of analysis taught in the course as well as a thorough understanding of the mechanisms in the business cycle models for open and closed economies, including the ability to use relevant variants and extensions of the models in order to explain the effects of various shocks and the effects of macroeconomic stabilization policies under alternative monetary and exchange rate regimes.

Problem A

1. By assumption, lifetime utility is additive separable meaning that lifetime utility is a (weighted) sum of utility obtained in each of the two periods separately. Preferences are assumed to remain unchanged over the entire life-span and described by the utility function u. In each period marginal utility of consumption is positive but diminishing. This provides an incentive for consumption smoothing as the consumer has an incentive to shift consumption between the periods in order to equalize marginal utility from consumption in the two periods and thereby increase total life-time utility. The parameter ϕ is the rate of time preference. It measures the consumer's impatience as utility stemming from consumption in period 2 is discounted using ϕ as a discount factor. The positive rate of time preference means that if $C_1 = C_2$ then $u(C_1) > u(C_2)/(1+\phi)$ if $\phi > 0$. Hence, an additional unit of consumption in the current period is valued more highly than an additional unit of consumption in the future. The higher the value of ϕ the more consumption is tilted towards the current period.

2. The household's budget constraint for period 1 is given by (A.2)

$$V_2 = (1+r)(Y_1 - T_1 + V_1 - C_1). \tag{A.2}$$

which states that total real savings brought into period 2 is determined as household savings (*i.e.* total real disposal income in period 1 plus initial real wealth minus consumption in the period) plus interests earned from these savings. The budget constraint for period 2 (equation (A.3)) holds with equalilty stating that it will not be optimal for the household to leave any savings for future consumption given that the household will not be around to enjoy consumption in period 3.

Substituting (A.2) into the budget constraint for period 2 (A.3) implies

$$\begin{aligned} C_2 &= V_2 + Y_2 - T_2 \qquad \text{(A.3)} \\ &= (1+r)(Y_1 - T_1 + V_1 - C_1) + Y_2 - T_2 \end{aligned}$$

Rearranging gives the households intertemporal budget constraint (IBC)

$$\begin{aligned} C_1 + \frac{C_2}{1+r} &= Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 = H_1 + V_1, \\ \text{where } H_1 &= Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} \end{aligned} \tag{A.4}$$

The IBC states that the present value of total consumption equals the present value of disposable lifetime income H_1 plus the initial stock of wealth. The numerical value of slope of the consumer's IBC is $(1+r)$ which is the market rate for shifting consumption across periods, *cf.* (A.2). By assumption, capital markets are perfect. The household can borrow and lend as it likes at the going (real) interest rate r. r is identical for borrowers and lenders. H_1 is the present value of the net income (income minus net taxes) earned by the consumer during his entire life span. Net income is discounted by the interest rate r. The theory does not describe how income come about, instead it is taken as given. Net-taxes are lump-sum and are taken as given.

The household chooses consumption in the two periods so that lifetime utility is maximized. The present value of lifetime consumption is limited by the present value of lifetime disposal income and initial wealth. Hence, the consumer has to solve the maximization problem (M.1).

3. Technically, the household's maximization problem may be solved by using either the Lagrange or the substitution method. Using the latter method we may reformulate the problem as

$$\max_{C_1} U = u(C_1) + \frac{u\left([1+r]\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 - C_1\right]\right)}{1+\phi}$$

When solving this problem the following first-order condition is found

$$\frac{dU}{dC_1} = 0 \Longrightarrow u'(C_1) = \frac{1+r}{1+\phi} u' \overbrace{\left([1+r]\left[Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} + V_1 - C_1\right]\right)}^{C_2}$$

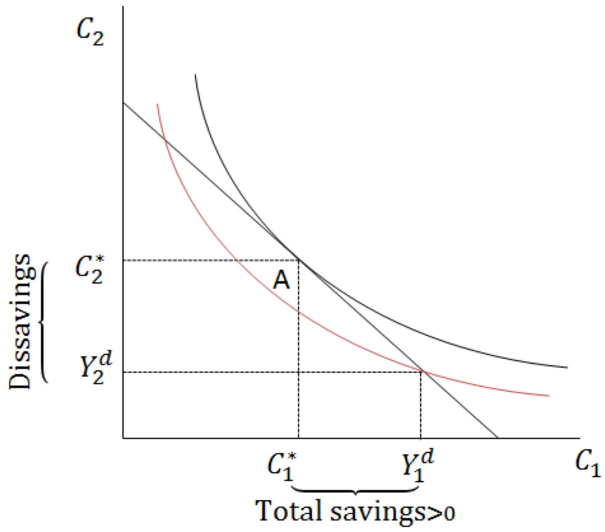
from which we have

$$\frac{u'(C_1)}{u'(C_2)/(1+\phi)} = 1 + r. \tag{A.5}$$

Equation (A.5) is the Keynes-Ramsey rule known from the text book. In optimum the consumer is indifferent between consuming an extra unit today and saving an extra unit today and thereby consume more in the future. The left-hand side is the marginal rate of substitution, MRS between consuming in the two periods. MRS expresses the consumers willingness to shift consumption between the two periods. If MRS is "low" the consumer requires only a small amount of additional consumption in the future in exchange of giving up an unit of current consumption. In an optimun MRS has to equal the relative price of present consumption (the right-hand side) at the capital market. If MRS is greater than $1+r$ the consumer is willing to give up more consumption in period 1 in order to get an additional unit of consumption in period 2 than required at the going real interest rate. This mechanism also explains why a change in current income, Y_1, leads to only a less than proportional change in optimal consumption so that $0 < \partial C_1/\partial Y_1 < 1$. Consumption is smoothed across time in oprder to obtain a higher level for life-time utility. Also, it follows that optimal savings are likely to be positive if Y_2 is "small" compared to Y_1. From (A.5) and the characteristics of the utility function u it follows directly that the ratio C_2/C_1 is increasing in r and declining in ϕ. If r is "high" it is optimal to postpone consumption to the future as current consumption is relatively expensive. If ϕ is "high" the consumer brings forward consumption as current consumption is valued relatively highly. For $r = \phi$ optimization implies $C_1 = C_2$ so that consumption is smoothed completely.

The existence of capital markets makes it possible for the consumer to shift consumption between different phases in life. Consumption in a given period is not restricted completely by income in this period. Instead the consumer may trade consumption between the periods at the given market rate. Thereby, capital markets makes it possible for the consumer to obtain a higher utility. An illustration may look like the following where it is seen that the optimal consumption (C_1^*, C_2^*) is found at point A at an indifference curve to the "north east" of the indifference curve going through the income endowment (Y_1, Y_2).

Under the assumption of $V_1 = 0$ total savings in period 1 is the difference



between total disposal income and optimal consumption.

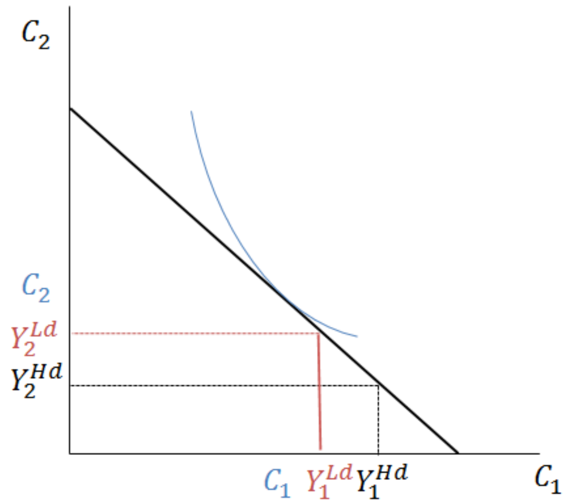
4. If capital markets are perfect the IBC faced by the two groups of consumers are identical. The groups have the same total lifetime wealth and face the same interest rate at financial markets. Consumers who currently have a relative high income will have a relatively low income in the next period and vice versa. The only difference between the two income groups is related to the timing of high vs. low incomes. Formally, the relationship between incomes may be backed out by assuming that $Y_1^H = Y_1^L + \Delta$ so that the high income group currently earn an income which is $\Delta > 0$ larger than the low income group. By assumption both groups have the same human capital $(H^H = H^L)$ and it follows that

$$H_1^H = Y_1^H + \frac{Y_2^H}{1+r} = Y_1^L + \frac{Y_2^L}{1+r} = H_1^L$$
$$\Downarrow$$
$$Y_1^L + \Delta + \frac{Y_2^H}{1+r} = Y_1^L + \frac{Y_2^L}{1+r}$$
$$\Downarrow$$
$$\Delta = Y_1^H - Y_1^L = \frac{1}{1+r}\left(Y_2^L - Y_2^H\right)$$

As both income groups have identical preferences they have identical indifference curves. Accordingly, the optimal consumption path (C_1, C_2) is the same for both income groups assuming capital markets are perfect so that consumers may borrow or lend as much as they like at a given real interest rate no matter the current income. It follows that the high-income group has the lowest average propensity to consume whereas the low income group has the highest average propensity to consume in the current period. (Same consumption but lower income). This is in line with empirical observations from microeconomic cross section data.. (Though not asked for it could be noted this result is slightly more general as it also holds if capital markets are imperfect however only if these imperfections do not distort consumption). Hence, in general the high-income group has the lowest average propensity to consume).

Also both income groups choose to smooth consumption between the two periods. Hence, an increase in current income only in part gives rise to higher consumption in the current period. Hence the marginal propensity is below 1 for both groups.

An illustration could look like the following.

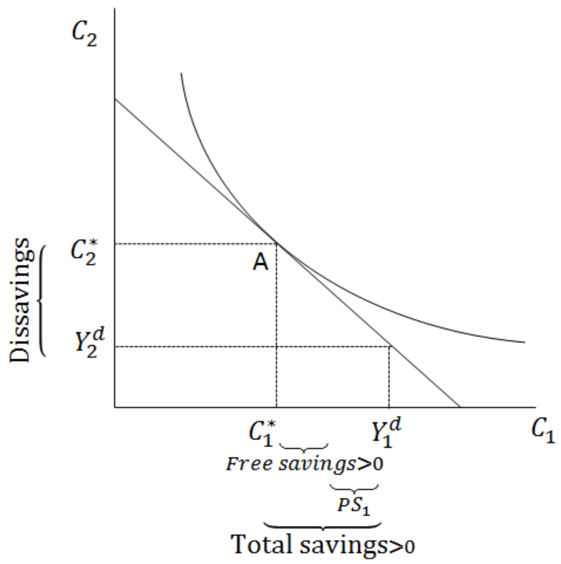


5. For household A there is no substantial difference between pension savings and free savings. The interest rate earned on pension savings is equal to the return on free savings, *i.e.* the going market interest rate. In addition, pension savings may be borrowed against at the going real interest rate. Hence, in essence optimal wealth brought into period 2 is found as the solution to

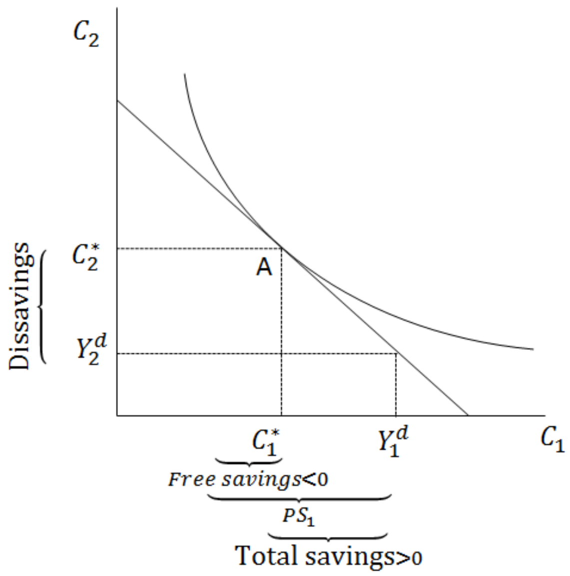
the household's maximization problem $(M.1)$. For a given income Y_1 pension savings are determined by the contribution rate a according to $(A.6)$ and free savings accommodates so that the sum mandatory pension savings and free savings equals optimal total savings. If optimal savings are greater than (or equal to) pensions savings, then free savings are positive (or equal to zero). On the other hand, if optimal savings are smaller than pension savings then the household borrows against pension savings; optimal free savings are negative.

$$V_2 = (1+r)(Y_1 - T_1 + V_1 - C_1) = (1+r)[PS_1 + \text{Free savings}]$$

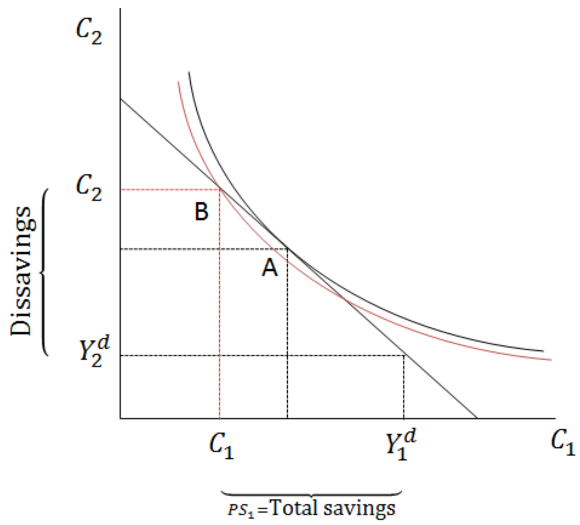
This also explains why optimal consumption in this situation is unaffected by the introduction of a mandatory pension scheme and by an increase in the pension contribution rate. The illustrations asked for could look like the following. First in case mandatory pension savings are lower than total savings and subsequently in case mandatory pension savings are higher than total savings. Again optimum is at the point labelled A.



6. If pension savings are illiquid and cannot be borrowed against the assumption of perfect capital markets is no longer valid. Now consumers may find themselves in a situation where they would have preferred to have negative free

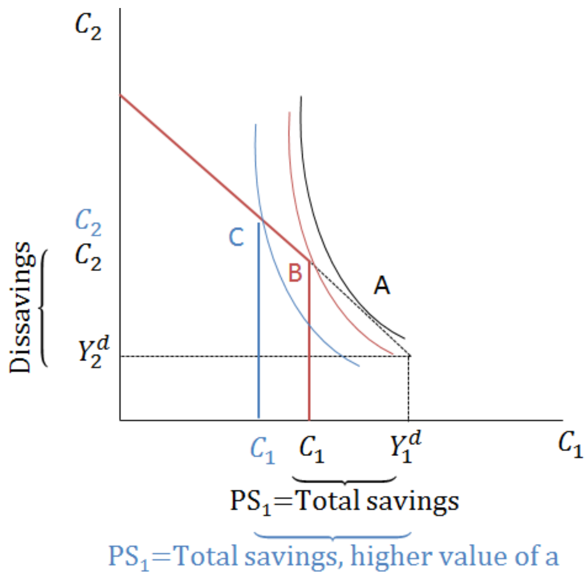


savings but are unable to. As a result total savings equals pension savings and in effect the houshold is credit constrained and is forced into choosing point B in the chart below. This is in contrast to the situation described in



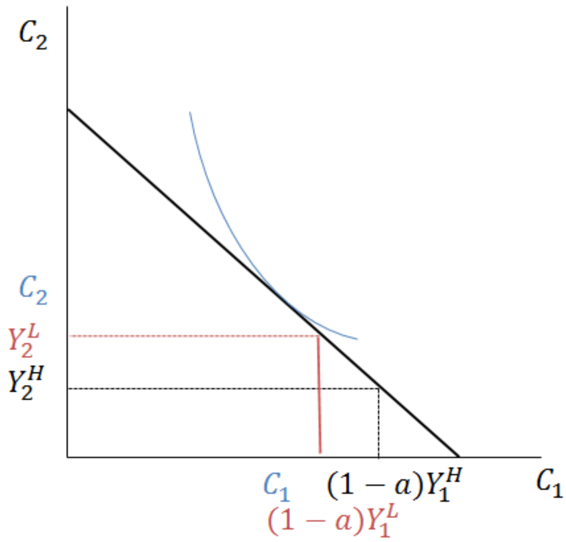
question 4 where mandatory pension savings was higher than total savings. In question 4 the household was able to borrow against pension savings and (due to the simplifying assumption regarding return) intertemporal budget constraint was unaffected. Accordingly, the household was able to choose the optimal consumption flow given by point A in the chart. Thus, as a consequence when pension savings cannot be borrowed against, the household may be forced into saving too much in period 1 so that consumption is too low in period 1 and too high in period 2 compared to the optimal situation (point A in the chart above). As a result life–time utility is lower.

It should be noted, that mandatory pension savings may be so small that the liquidity constraint is not binding. This could be the situation if pension savings allow the household to choose point A in the chart below. Also, free savings may be positive. If the household's free savings are strictly positive, then a marginal increase in mandatory pension savings (through a marginal increase in a) has no effect on the optimal consumption pattern. The household may reduce free savings savings so that total savings in period 1 and hence consumption remains unchanged. However, as free savings "dry up" with an increase in a it is increasingly likely that the household end up in a situation where total savings are larger than the optimal savings. This happens when pension savings are larger than optimal free savings (point A) and has happened in point B.



7. The introduction of a pension system where pension savings can not be borrowed against affects the IBC as described above. As a consequence consumers who have a realtive small current income is more likely to end up in a corner solution where current consumption is determined by income

after pension savings. This is especially so if the pension contribution rate a is raised. In the figure below it is optimal for both income groups to have positive free savings. However, consider a gradual increase in a. When a is increased a gradual reduction in free savings occour leving total savings unchanged. However, ultimately free savings dissapears as a is increased. The complete erosion of free savings first (mainly) affect consumption of the low income group as they do not have free savings large enough to absorb the effect from higher mandatory savings. Hence, they cannot cancel out the effect on consumption (entirely).



Problem B

1. The Taylor principle is to adjust interest rates on a more than one-for-one basis with inflation expectations so that real interest rates rise and fall with inflation expectations.

2. The real exchange rate it is defined as

$$\varepsilon = \frac{eP^f}{P}$$

where e is the nominal exchange rate (the number of domestic currency units needed to aquire one unit of foreign currency), P^f is the foreign price for the foreign good in terms of foreign currency units and P is the price of the domestic goods in terms of domestic currency units.

The real exchange rate is the number of domestic (Danish) goods exchanged for a unit of foreign good. There is a real appreciation if ε decreases, that is, fewer domestic goods are exchanged for a unit of foreign good, *i.e.*, you get more foreign goods in exchange for a Danish good. This mean that the terms of trade is improved.

3. False. Under perfect capital mobility the domestic nominal interest rate must equal the sum of the foreign nominal interest rate and expected depreciation of the domestic currency. This follows from the uncovered interest parity (UIP).

Though not asked for the student may add that the UIP is a financial arbitrage condition (or rather absence of arbitrage). When capital mobility is perfect domestic and foreign assets are perfect substitutes. Investors can reallocate their portfolios instantaneously and without cost. Assuming investors are risk neutral the arbitrage condition whereby the return on domestically denominated financial assets is tied to the return on foreign denominated assets may be written

$$1 + i = \left(1 + i^f\right) \times \frac{E^e_{+1}}{E},$$

where E is the current exchange rate and $E^e_{+1}$is the expected exchange rate, i is the domestic interest and i^f is the interest rate abroad. Using the approximation $\ln(1 + x) \approx x$ when x is small the uncovered interest parity

(UIP)

$$i = i^f + e^e_{+1} - e,$$

follows directly. $e^e_{+1} - e$ is the expected reduction in the value of the domestic currency. If the level of the domestic interest rate is above the foreign interest rate then an increase in E) the value of the domestic currency is expected to be reduced (an increase in e) so that the expected investment return is the same when measured in the same currency. If investors are risk averse a risk premium is added.