Guide[1] to answers, Written Exam for the B.Sc. or M.Sc. in Economics

Microeconomics A, 2nd Year

January 2014

Problem 1

Consider a consumer who consumes two goods in continuous quantities and whose preferences can be represented by a differentiable, quasi-linear utility function.

- a) Show that the marginal rate of substitution (MRS) will depend on the quantity of only one of the goods (equivalently, will be independent of the quantity of the other good).
- b) How will the map of indifference curves look for such a consumer?

Answer: The utility function will have the form $u(x_1,x_2) = v(x_1) + x_2$, hence MRS becomes $-v'(x_1)$, clearly independent of x_2, and indifference curves are parallel, seen vertically, MRS being identical along a vertical line.

Problem 2

Peter consumes beer (commodity 1) and sandwiches (commodity 2), both in continuous quantities. Peter has preferences which can be represented by the utility function $u(x_1,x_2) = x_1^{½} \cdot x_2^{½}$.

- a) Please find the expression for Peter's Marshall demand function x(p,I).
- b) Please find the expression for Peter's Hicksian (compensated) demand function h(p,u)

Peter is on a stipend, giving him an exogenous money income of 120. Currently, the price of beer is 1, and the price of sandwiches is 1.

- c) Which consumption plan is optimal for Peter?

However, to reduce students' drinking, the minister of health, Ms. Crowe, levies a heavy tax on beer, increasing the price of beer from 1 to 4.

- d) How does Peter's consumption plan change after this price increase for drinks?
- e) Please divide the changes into a substitution effect and an income effect, respectively.

Answer: $x(p,I) = (½I/p_1, ½I/p_2)$, $h(p,u) = (up_1^{-½}p_2^{½}, up_1^{½}p_2^{-½})$. Initial consumption plan is (60,60) which changes to (15,60), the substitution effect being (–30,+60), the income effect being (–15, –60). adding up to (–45,0).

Problem 3:

[1] What is presented here is not a full, satisfactory answer to the problems, but indicates the correct results and important points to be made.

Charlie consumes health services (commodity 1) and a composite consumption good (commodity 2), both in continuous quantities, and has the utility function $u(x_1,x_2) = 10\cdot\ln(x_1) + x_2$.

The price system is $p = (p_1, 1)$, with commodity 2 being numeraire, and Charlie has the exogenous income I.

In the following, consider only interior solutions (i.e. consumption plans having strictly positive quantities of both commodities).

- a) Please find the expression for Charlie's Marshall demand function, x(p,I).
- b) Please find the expression for his Hicksian (compensated) demand function, h(p,u)
- c) Consider the following Slutsky equation describing what happens when the price of health services increases marginally, and verify that the following two equations hold true, with u = u(x(p,I)):

$$\partial x_1(p,I)/\partial p_1 = \partial h_1(p,u)/\partial p_1 - [\partial x_1(p,I)/\partial I]\cdot x_1(p,I)$$

$$\partial x_2(p,I)/\partial p_1 = \partial h_2(p,u)/\partial p_1 - [\partial x_2(p,I)/\partial I]\cdot x_1(p,I)$$

Answer: Marshall demand is $(10/p_1, I-10)$, Hicksian demand is $(10/p_1, u-10\cdot\ln(10)+10\cdot\ln(p_1))$.
Hence, for commodity 1 the LHS is $-10/p_1^2$, the RHS is $-10/p_1^2 + 0\cdot 10/p_1$.
For commodity 2 LHS is 0 and RHS is $-10/p_1 + 1\cdot 10/p_1$.

Problem 4:

Please define and explain, for a firm producing an output by using labor and capital as inputs, what is meant by the firm's "conditional factor demand".

Answer: Those demand functions, $l_b(w,r,x)$ and $k_b(w,r,x)$, are derived by minimizing production costs, wl+rk, being conditional on producing output x, hence the conditional demand depends on both the prices of inputs and the output requested, but does not depend on output price p.

Problem 5:

In a Koopmans economy, the consumer Robinson has utility function $u(x_1,x_2) = x_1\cdot x_2$, commodity 1 being time which can be enjoyed as leisure or allocated in production, giving the output food (commodity 2), both goods being continuous.

The production technology is described by the following production function, with y being the quantity of food output, and z being the number of hours worked in food production:
$y(z) = \max\{ z - 2, 0 \}$.

The initial endowment of time is 24 hours, while there is no initial endowment of food.

- a) Identify the efficient (Pareto Optimal) allocation
- b) Is it possible to implement this allocation as belonging to a Market Equilibrium with transfers?

Answer: The efficient allocation with absolute value of MRS equal to MP in production (=1 for input larger than 2) solves (z–2)/(24–z) = 1, i.e. a labor input of 13, giving Robinson the consumption (11,11). A potential price system implementing this as a market equilibrium would be (1,1), supporting (11,11) as a utility maximizing consumption plan, but yielding negative profits. What goes wrong here is the non-convexity of the production technology.

Problem 6:

Please present The First Welfare Theorem. What does it state, which are the assumptions behind it, what are the implications of it, and what are the limitations?

Answer: Provided consumers have monotonously increasing preferences, any allocation belonging to a Walrasian equilibrium will be efficient. This is the mathematical/Arrow-Debreu version of Adam Smith's "invisible hand" story: Prices function as signals reflecting, at the same time, consumers' preferences (marginal benefits) and production technologies (marginal costs). This means that no redistribution of resources can make any consumer better off without hurting other consumers. One might mention that there will be efficiency in the way consumer goods are allocated between consumers, in the way production factors are allocated between firms, and in consumption vis-à-vis production. Some limitations may be mentioned (we have assumed away externalities, market power, public goods, asymmetric information etc.). Also, efficiency does not necessarily entail an equal distribution of welfare. An advanced answer may show how satiation may result in an equilibrium allocation "wasting" consumption which could be redistributed, increasing other consumers' welfare without hurting the satiated consumer.

Mtn, 17 January 2014