Written Exam for the B.Sc. in Economics 2009-II

Micro 3

Final Exam

August 20th, 2009

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

PLEASE ANSWER ALL QUESTIONS BELOW. PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Find *all* Nash equilibria in the following game

	L	R
T	$3,3$	$1,6$
B	$2,4$	$5,1$

(b) Solve the following game by eliminating strictly dominated strategies. Could there be a Nash equilibrium in this game, in which Player 1 mixes between all three strategies s_1, s_2 and s_2 with positive weights? If yes, find one. If no, explain why not.

	t_1	t_2	t_3
s_1	1,4	4,3	5,2
s_2	1,3	2,2	4,4
s_3	2,0	3,4	5,3

(c) Consider the extensive-form game represented by the game tree on Figure 1:

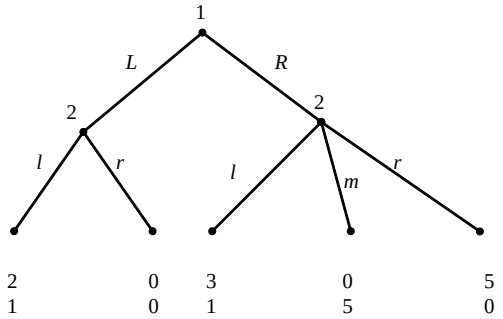


Figure 1.

i. How many subgames are in this game? Find all subgame perfect Nash equilibria.

ii. Rewrite this game in normal form and find all pure-strategy Nash equilibria.

iii. Could you have a game which has more SPNE than NE? If yes, provide an example. If no, explain.

2. Two students are working together on a project. When students 1 and 2 choose effort levels e_1 and e_2, $e_i \in [0,1]$, the probability that the project is successfully completed is equal to

$$\frac{e_1 e_2 + e_1 + e_2}{3}.$$

The disutility of effort for student i, $i = 1,2$, is given by $\frac{1}{2}e_i^2$. Further, student i values the completed project at A_i utils. That is, student 1's payoff in the game is

$$U_1(e_1, e_2) = A_1 \frac{e_1 e_2 + e_1 + e_2}{3} - \frac{1}{2}(e_1)^2,$$

and student 2's payoff is

$$U_2(e_1, e_2) = A_2 \frac{e_1 e_2 + e_1 + e_2}{3} - \frac{1}{2}(e_2)^2.$$

Assume that students choose their effort levels simultaneously and non-cooperatively.

(a) Assume that each student values the project at $A_i = 1$, $i = 1,2$, and this is common knowledge. Find the best response functions of both students and determine the Nash equilibrium effort levels e_1^*, e_2^*.

(b) Assume that student 1 values the project at $A_1 = 1$, while student 2 does not value the project at all, so that $A_2 = 0$. Again, this is common knowledge. Find the best response functions of both students and determine the Nash equilibrium effort levels e_1^{**}, e_2^{**}. (HINT: remember that all efforts should be non-negative).

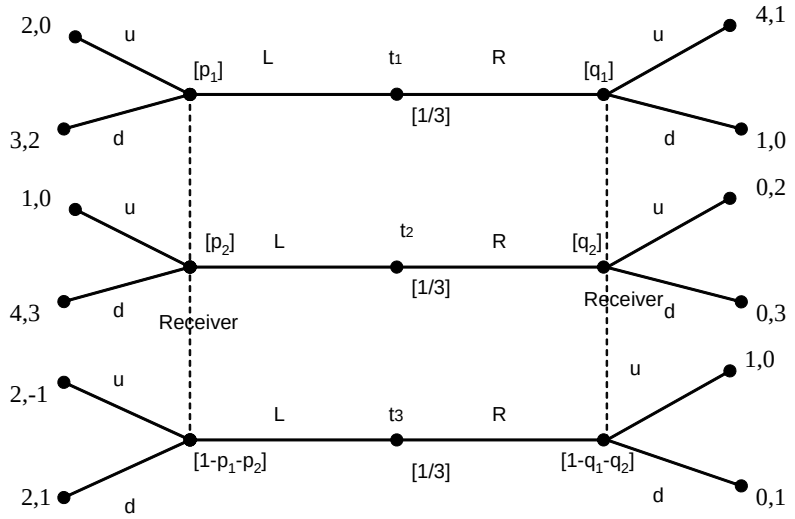
(c) Finally, assume that student 1 still values the project at $A_1 = 1$ and this is common knowledge. The value of the project to student 2, A_2, is however known only to student 2 herself. The only thing student 1 knows about A_2 is that $A_2 = 1$ with probability p, and 0 with probability $(1 - p)$.

i. What is the best response function of the student 2 with $A_2 = 0$, $e_2^L(e_1)$? What is the best response function of the student 2 with $A_2 = 1$, $e_2^H(e_1)$?

ii. What is the best response function of student 1 $e_1(e_2^L, e_2^H)$?

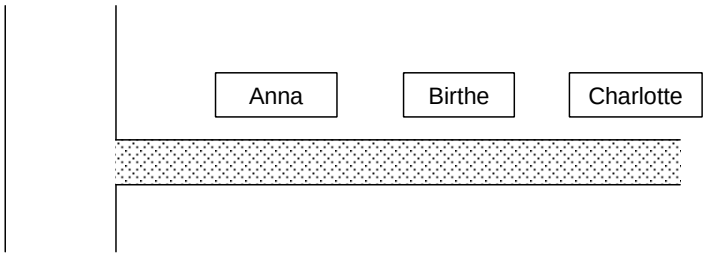
iii. Find the Bayes-Nash equilibrium of this game. How does it depend on p? Interpret (a comparison to your results in (a) and (b) may be useful).

3. Consider the following signalling game with three types of sender, which are equally probable (the nature move is not shown on the picture):



Find a pooling Perfect Bayesian equilibrium in which all senders choose L.

4. Anna, Birthe and Charlotte have houses located along a small road. They need to repair the road from the intersection with a bigger road to their houses. Anna's house is the closest to the intersection, and Charlotte's house is the furthest away. More precisely, Anna's house is 100 m away from the intersection, Birthe's house is 200 m away from the intersection and Charlotte's house is 300 m away. To repair 100 m of road costs DKK 1000. They are discussing how to divide the repair costs.



(a) Think of this situation as of coalitional game with transferable utilities. Write down the value of each coalition.

(b) Assume that they decided to divide the costs according to the Shapley value. What will each of them pay?