University of Copenhagen
Department of Economics
Michael Bergman

Solution to written exam for the M.Sc. in Economics International Monetary Economics

January 16, 2014

Number of questions: This exam consists of 2 questions.

This exam focuses on two main parts of the curriculum, exchange rate determination and more specifically the Dornbusch overshooting model and Frankel's real interest rate differential model and second generation currency crisis models.

1. **The Dornbusch model**

   This question relates to the following learning objective: describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model and Lucas asset pricing model) and use these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models.

   Consider the Dornbusch overshooting model with an exogenous risk premium comprised of the following equations

$$r - r^* - rp = E\dot{s}^e \tag{1}$$

$$E\dot{s} = \theta\left(\bar{s} - s\right) \tag{2}$$

$$m - p = \eta y - \sigma r \tag{3}$$

$$y^d = \beta + \alpha\left(s - p + p^*\right) + \phi y - \lambda r \tag{4}$$

$$\dot{p} = \pi\left(y^d - y\right). \tag{5}$$

   (a) Equation (1) is the UIP relation where we have assumed that domestic and foreign bonds are not perfect substitutes and therefore added a risk premium which is assumed to be constant. Equation (2) describes the expectations formation where θ is the speed of adjustment, $\bar{s}$ is the long–run equilibrium exchange rate and s is the current exchange rate. The equation states that the expected rate of depreciation is proportional to the deviation from the long–run equilibrium. If s is above $\bar{s}$, then the exchange rate is expected to appreciate. Equation (3) is a standard money demand function where real balance is a function of output and the interest rate. Equation (4) is a standard aggregate demand function where demand is given by output, interest rate and the real exchange rate (representing foreign demand for domestic goods). Finally, equation (5) states that the rate of inflation is determined by the gap between aggregate demand and aggregate supply. This model describes a small open economy, i.e., an economy facing a fixed world interest rate. We also assume that output is fixed and prices are sticky. PPP holds only in the long–run.

(b) Derive the money market and goods market equilibrium curves and illustrate the model in a graph [Hint: Start by deriving the equilibrium price level, then the expression for the equilibrium exchange rate.]

Consider first the money market schedule: Combine equations (1) and (2)

$$r - r^* - rp = \theta(\bar{s} - s)$$

and insert the solution of r from the money demand function (3) such that we find

$$s = \bar{s} - \frac{1}{\theta}\left[\frac{p - m + \eta y}{\sigma} - r^* - rp\right]$$

and then we solve for p

$$p = -\sigma\theta\left(\bar{s} - s\right) + m - \eta y + \sigma r^* + \sigma rp$$

and the slope is

$$\frac{dp}{dp} = -\sigma\theta.$$

In order to derive the goods market equilibrium schedule we first insert the expression for aggregate demand, equation (4), into the the expression for inflation, equation (5) and use the money demand function in (3) such that

$$\dot{p} = \pi\left[\beta + \alpha\left(s - p + p^*\right) + \left(\phi - 1\right)y - \frac{\lambda}{\sigma}\left(p - m + \eta y\right)\right].$$

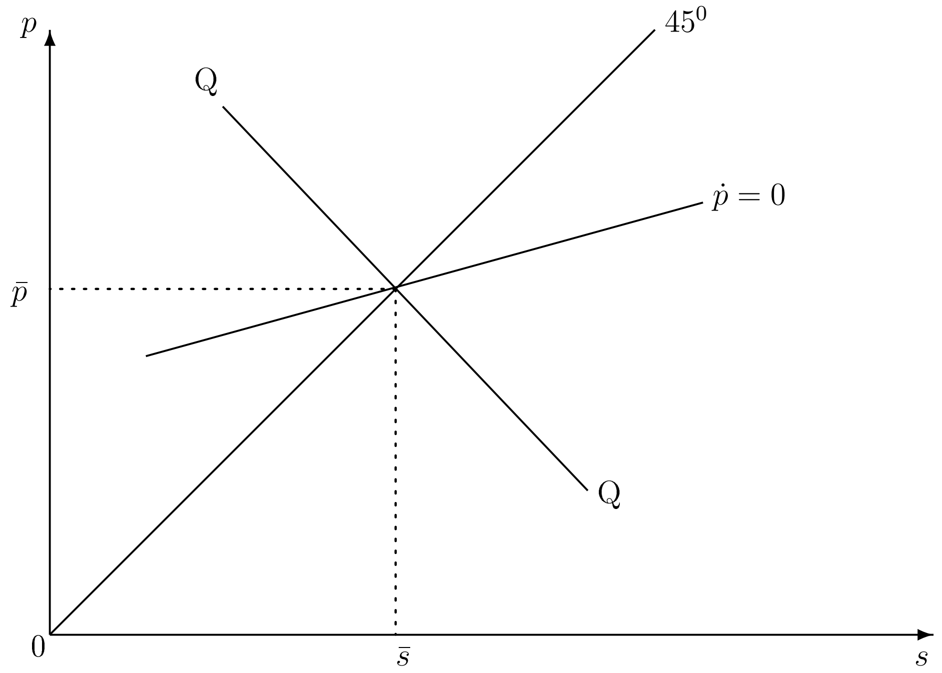
In equilibrium, $\dot{p} = 0$ giving us the following solution for the price level

$$p = \frac{\alpha}{\alpha + \frac{\lambda}{\sigma}}s + \beta + \alpha p^* + \frac{\lambda}{\sigma}m + \left(\phi - 1 - \frac{\lambda\eta}{\sigma}\right)y$$

such that the slope is

$$1 > \frac{\alpha}{\alpha + \frac{\lambda}{\sigma}} > 0.$$

We can now illustrate the model in the familiar graph below where we have added that PPP holds in the long–run.



where we note that points above the $\dot{p} = 0$–curve correspond to excess supply whereas points below correspond to excess demand. Reason is: for a given price level, an exchange rate appreciation reduces demand.

(c) How are the slopes of the money market and goods market equilibrium curves affected by the inclusion of an exogenous risk premium?

As can be seen above when determining the slopes of the money market and goods market equilibrium curves, the risk premium does not affect the slopes. The relative position of the curves, on the other hand, will be affected since the risk premium is assumed to be nonzero.

(d) Prove that the exchange rate overshoots in this model by deriving an expression for $\frac{ds}{dm}$. What factors determine the size of the overshooting effect? Is the relative size of the risk premium determining the extent of overshooting?

Use UIP to solve for r, insert into the money demand function, use equation (2) and rearrange such that

$$p - m = -\eta y + \sigma r^* + \sigma rp + \sigma\theta\left(\bar{s} - s\right).$$

Take the total differential of this equation such that

$$\mathrm{d}p - \mathrm{d}m = -\eta \mathrm{d}y + \sigma \mathrm{d}r^* + \sigma \mathrm{d}rp + \sigma\theta\left(\mathrm{d}\bar{s} - \mathrm{d}s\right)$$

and by noting that prices are sticky in the short–run $\mathrm{d}p = 0$, long–run homogeneity ensures that $\mathrm{d}\bar{s} = \mathrm{d}m$, and that y, r^* and rp are constant we obtain

$$-\mathrm{d}m = \sigma\theta\left(\mathrm{d}m - \mathrm{d}s\right)$$

implying that

$$\frac{\mathrm{d}s}{\mathrm{d}m} = 1 + \frac{1}{\sigma\theta}.$$

The risk premium does not affect the size of the overshooting effect.

(e) Replace equation (2) by the following expression

$$E\dot{s} = \theta\left(\bar{s} - s\right) + P\dot{e} - P\dot{e}^{*}$$

where $P\dot{e} - P\dot{e}^{*}$ is the long-run inflation differential between the domestic and foreign economies. Show that the solution for the short-run exchange rate is

$$s = \left(m - m^{*}\right) - \eta\left(y - y^{*}\right) + \sigma\left(P\dot{e} - P\dot{e}^{*} + rp\right) - \frac{1}{\theta}\left(r - r^{*} - rp - \left(P\dot{e} - P\dot{e}^{*}\right)\right).$$

[Hint: Assume that the money demand in the foreign country is given by $m^{*} = p^{*} + \eta y^{*} - \sigma r^{*}$.]

Combining UIP, equation (1), and the new expectations formation given in the question we find

$$s = \bar{s} - \frac{1}{\theta}\left[\left(r - P\dot{e}\right) - \left(r^{*} - P\dot{e}^{*}\right) - rp\right].$$

If we combine PPP and UIP we obtain an expression for the interest rate differential $r - r^{*}$. Usually we assume that the real interest rates in the two countries are equal, which is the so called real interest rate parity assumption. By combining UIP and PPP we find a relation between real interest rates in two countries. But in this model we have assumed that there is a risk premium implying that real interest rates in the two countries deviate with the risk premium. To see this, assume that PPP holds (it holds at least in the long–run) we have that $E\dot{s} = P\dot{e} - P\dot{e}^{*}$. Inserting into the UIP relation above we find that

$$P\dot{e} - P\dot{e}^{*} = r - r^{*} - rp.$$

Using the hint we can compute the difference between the money supply in the domestic and foreign economies

$$m - m^{*} = p - p^{*} + \eta\left(y - y^{*}\right) - \sigma\left(r - r^{*}\right)$$

which will give us an expression for the relative equilibrium price levels and since PPP holds in the long–run this relative price must be equal to the equilibrium exchange rate. We find that

$$\bar{s} = \bar{p} - \bar{p}^{*} = m - m^{*} - \eta\left(y - y^{*}\right) + \sigma\left(r - r^{*}\right).$$

Then we use the expression for the interest rates above (combination of PPP and UIP) such that

$$\bar{s} = (m - m^*) - \eta (y - y^*) + \sigma (P\dot{e} - P\dot{e}^* + rp).$$

Now we can insert this into the expression above and arrive at the result stated in the question

$$s = (m - m^*) - \eta (y - y^*) + \sigma (P\dot{e} - P\dot{e}^* + rp) - \frac{1}{\theta} (r - r^* - rp - (P\dot{e} - P\dot{e}^*)).$$

(f) Is there an exchange rate overshooting effect in this model also? Explain carefully the effects of a monetary expansion.

Yes! The argument is that while the asset market (the foreign exchange market) adjust immediately to shocks, prices on the goods market are sticky. In the model above this corresponds to assuming that θ is finite. If θ is infinite, the model reduces to the standard flexible price model. An unexpected increase in the money supply will lead to a fall in the domestic real interest rate. The price level is unchanged in the short–run but is expected to increase in the long–run. If the risk premium is constant, there is an expectation of a future appreciation of the exchange rate to compensate for the lower return on domestic bonds. As in the Dornbusch model, the only way agents can expect a future appreciation is if the short–run exchange rate depreciation exceeds the long–run depreciation, i.e., an overshooting effect. The mechanism is exactly the same in this real interest rate model as in the Dornbusch model.

2. **Second Generation Currency Crisis model** This question relates to the learning objective: describe, explain and compare first-, second- and third-generation models of currency crises and apply these models to analyze actual currency crises.

The question focuses around the second generation currency crisis model comprised of the following equations:

$$\mathcal{L} = \theta \dot{p}^2 + (y - \tilde{y})^2 + C(\dot{s}) \tag{6}$$

$$y = \bar{y} + \dot{p} - \dot{p}^e - v \tag{7}$$

$$\tilde{y} - \bar{y} = k > 0 \tag{8}$$

$$s = p - p^* \tag{9}$$

where notation is standard.

(a) Comment on the four equations above.

Equation (6) is the policy rule where θ is the weight on inflation, $\dot{p}$ is inflation, y is actual output whereas $\tilde{y}$ is the output target chosen by the policymaker. Equation (7) is a standard aggregate supply function where $\bar{y}$ is the natural level of output, $\dot{p}^e$ is expected inflation and υ is the output shock which is assumed to be a white noise sequence. Equation (8) defines whether there is a difference between target and natural output, this difference is k. It is assumed that k is positive such that there is an output target bias. There is a temptation to cheat in the model. The policymaker finds that by devaluating the currency such that inflation increases will raise output and the loss function will be minimized. Since the model is known, the market knows that speculative attacks may lead to devaluations. Finally, equation (9) is the PPP relation which is assumed to hold.

(b) What are the main underlying assumptions of the model? Explain the assumed sequencing of events in the model and how currency crises are generated.

Sequencing: Private agents choose $\dot{s}^e$ before the shock υ hits the economy implying that the expectation is formed prior to observing $\dot{s}$. The monetary authority can choose $\dot{s}$ after observing both $\dot{s}^e$ and υ. This sequencing opens up for cheating.

Fundamentals, monetary authorities preferences and the shocks determine the equilibrium and whether there are multiple equilibria. A sudden shock to expectations trigger a speculative attack and we may end up in a currency crisis. There must be a temptation to abandon the fixed exchange rate or devalue the currency. To generate crises, shocks must lead to sudden jumps in expectations and even small shocks can lead to jumps from an initial equilibrium compatible with a fixed exchange rate to an equilibrium where the fixed exchange rate regime must fall.

(c) Define

$$\begin{aligned} C(\dot{s}) &= 0 \quad \text{if} \quad \dot{s} = 0 \\ C(\dot{s}) &= \bar{C} > 0 \quad \text{if} \quad \dot{s} > 0 \\ C(\dot{s}) &= \underline{C} > 0 \quad \text{if} \quad \dot{s} < 0. \end{aligned}$$

A devaluation or a revaluation will occur if

$$\frac{(\dot{s}^e + \upsilon + k)^2}{1+\theta} > C(\dot{s}).$$

Solve this equation for υ and explain under what conditions there will be a devaluation or revaluation.

We have that

$$\frac{(\dot{s}^e + \upsilon + k)^2}{1+\theta} > C(\dot{s})$$

and there will be a devaluation ($\dot{s} > 0$) if $C(\dot{s}) = \bar{C} > 0$. This implies that

$$(\dot{s}^e + \upsilon + k)^2 = (1+\theta)\,\bar{C}$$

which can be used to solve for υ. We find that

$$\upsilon = \sqrt{(1+\theta)\,\bar{C}} - k - \dot{s}^e$$

implying that if

$$\upsilon > \sqrt{(1+\theta)\,\bar{C}} - k - \dot{s}^e$$

then there will be a devaluation.

The negative square root determines whether there will be a revaluation. If

$$\upsilon < -\sqrt{(1+\theta)\,\underline{C}} - k - \dot{s}^e$$

then there will be a revaluation. The expression above implies that if there are negative output shocks (they add to output) then there will be a revaluation if the condition above holds.

(d) Under the assumption that the shocks are uniformly distributed we can derive the following expression for the expected exchange rate

$$\mathrm{E}(\dot{s}) = \frac{1}{1+\theta}\left\{\left[1 - \frac{\bar{\upsilon} - \underline{\upsilon}}{2V}\right](\dot{s}^e + k) - \frac{\bar{\upsilon}^2 - \underline{\upsilon}^2}{4V}\right\} \tag{10}$$

where $\bar{\upsilon}$ is the devaluation trigger, $\underline{\upsilon}$ is the revaluation trigger and V $(-V)$ is the largest (smallest) possible value of υ. Illustrate the model in the $\mathrm{E}(\dot{s})$–$\dot{s}^e$–plane. Provide a detailed discussion about how this graph is constructed.

We have to consider three cases, small, intermediate and large values of $\dot{s}^e$! The reason is that the slope of the expression above depends on the relative size of $\dot{s}^e$. Consider first small values of $\dot{s}^e$. According to UIP: $\dot{s}^e = i - i^*$ so that the minimum value must be $\dot{s}^e = -i^*$ (expected revaluation). In this case $\underline{\upsilon} > -V$ and $\bar{\upsilon} < V$ implying that $-V < \underline{\upsilon} \leq \bar{\upsilon} < V$. From above we have that

$$\bar{\upsilon} = \sqrt{\bar{C}(1+\theta)} - k - \dot{s}^e$$

and

$$\underline{\upsilon} = -\sqrt{\underline{C}(1+\theta)} - k - \dot{s}^e.$$

The slope of the $\mathrm{E}(\dot{s})$–curve is therefore

$$\frac{\partial \mathrm{E}(\dot{s})}{\partial \dot{s}^e} = \frac{1}{1+\theta}$$

since the partial derivatives of $\bar{\upsilon}$ and $\underline{\upsilon}$ with respect to $\dot{s}^e$ are both equal to -1.

Intermediate values of $\dot{s}^e$: Increase the value of $\dot{s}^e$. This implies that $\underline{\upsilon}$ falls until it is equal to $-V$. An increase in $\dot{s}^e$ will reduce $\bar{\upsilon}$ so that it will depart from V. We assume that $\dot{s}^e$ increases but not such that $\bar{\upsilon} \to -V$. This implies that

$-V < \bar{\upsilon} < V$. In this case $\frac{\partial \underline{\upsilon}}{\partial \dot{s}^e} = 0$ since $\underline{\upsilon}$ cannot fall any further (the limit is $-V$). But $\frac{\partial \bar{\upsilon}}{\partial \dot{s}^e} = -1$. We then find that the slope for this range of $\dot{s}^e$ is given by

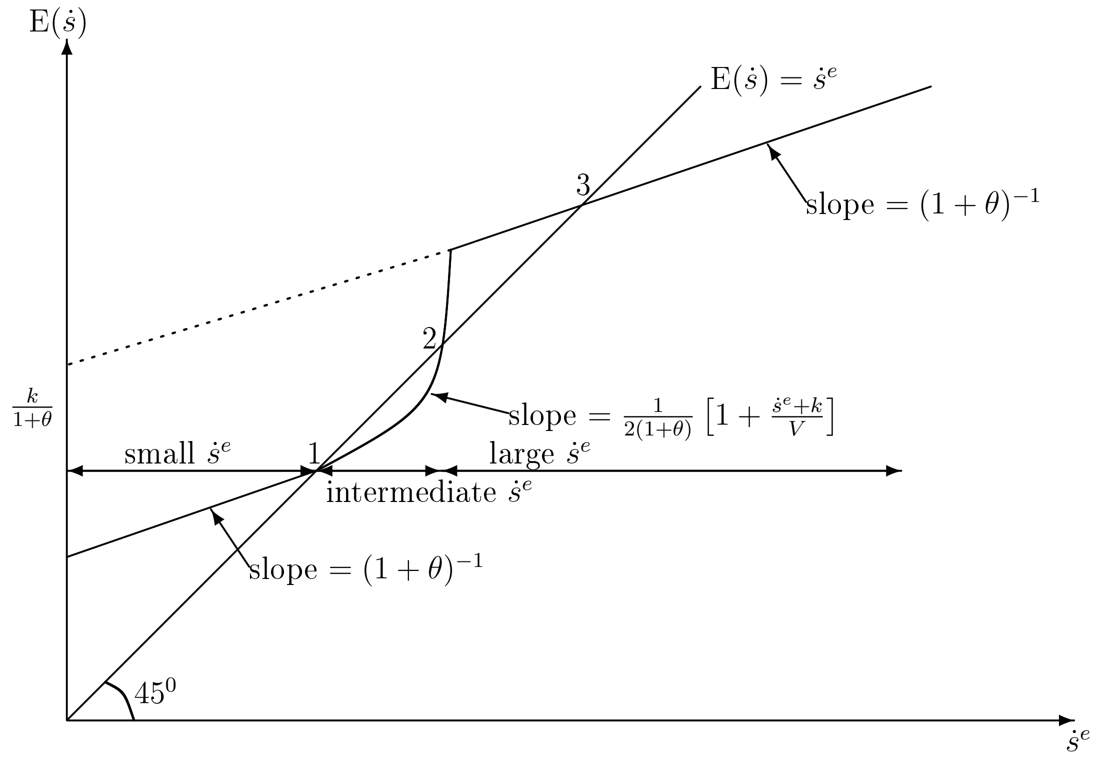
$$\frac{\partial \mathrm{E}(\dot{s})}{\partial \dot{s}^e} = \frac{1}{2(1+\theta)}\left(1 + \frac{\dot{s}^e + k}{V}\right)$$

implying that the slope is increasing in $\dot{s}^e$.

Large values of $\dot{s}^e$: If $\dot{s}^e$ is very large, then $\underline{\upsilon} = -V$ but $\bar{\upsilon} = -V$. This implies that revaluations are precluded with certainty $\Pr\{\upsilon \leq \underline{\upsilon}\} = 0$ and $\upsilon \geq -V$ or in other words $\Pr\{\upsilon \geq \bar{\upsilon}\} = 1$. Insert $\bar{\upsilon} = \underline{\upsilon}$ into the equation above and compute the slope

$$\frac{\partial \mathrm{E}(\dot{s})}{\partial \dot{s}^e} = \frac{1}{1+\theta}.$$

We can now put all this together and illustrate the model in the following graph.



In the graph, there are three equilibria, points 1, 2 and 3! Equilibrium 3 is the zero commitment solution $\mathrm{E}(\dot{s}) = \frac{k}{\theta}$ which must be located above $\frac{k}{1+\theta}$. Note also that equilibrium 1 implies a certain devaluation. Expected devaluation is equal to zero only if there is an equilibrium at origo. If agents devaluation expectations suddenly increase (given that we are in equilibrium 1), we will move along the curve towards equilibrium 2, which is unstable. Depending on the shock υ we may end up at point 3, the insight is that speculation can move the economy to equilibrium 3. Note that it is not necessarily the case that we have multiple equilibria. If $k \downarrow$ or $\theta \uparrow$, then $\frac{k}{1+\theta} \downarrow$. The obvious solution is to set $\tilde{y} = \bar{y} \Rightarrow k = 0$ or to increase the weight on inflation in the objective function (appoint a more conservative central bank).

(e) Can this model be used to explain the ERM crisis? If so, why?

The model is often used to illustrate what happened during the ERM crisis. The reason is that most fundamentals in ERM countries were consistent with the exchange rate but sudden shocks or unexpected events made market participants revise their expectations. Even small or minor events had large effects on expectations and therefore triggered speculation. Among explanations to why the crisis broke out are: The German unification in 1990, large and persistent inflation differences, and the result of the Danish referendum on the Maastricht Treaty. These events, and in particular the result of the Danish referendum, were considered as negative news. There was an immediate reaction on the foreign exchange market. Speculative attacks initially were targeted against the lira and the British Pound and without strong support from the Bundesbank, Italy and the UK left ERM I. Speculative attacks then targeted the Irish punt, the Portuguese escudo and the Spanish peseta and spread to the Belgian franc, the Danish krone and the French franc. Finally, ERM I broke down in August 2, 1993. These event illustrate that sudden events can have a tremendous effect of expectations and trigger speculation even in cases when fundamentals are in line with the exchange rate.

Moreover, the ERM crisis provides an example of the impossible trinity, fixed exchange rate, monetary policy independence, and full capital mobility cannot be had at the same time. The liberalization of financial markets in EU countries together with fix exchange rates was incompatible with divergent monetary policies. At the same time, Germany was going through its unification. This may also have contributed to the breakdown of the ERM system.

(f) What are the main differences between ERM I and ERM II?

ERM I was symmetric with no anchor currency where central parities were defined on a grid of bilateral parities. ERM II asymmetric, all central rates defined vis–à–vis the euro. ERM I margin of fluctuations was explicitly set. In ERM II it was less precisely defined (standard (+/- 15%) but narrower bands were also allowed). In ERM I there were automatic unlimited interventions whereas in ERM II they were also automatic and unlimited but ECB was allowed to suspend intervention if there was a conflict between the exchange rate goal and other objectives. It is usually argued that ERM II was more flexible and less committal.