Exam Microeconomics A – August 2015

Problem 1

True or false? In each case explain your answer.

1) If a consumer has rational preferences then he/she must prefer consumption bundles that contain more of every good.
2) Consider a rational consumer with strictly monotone and convex preferences representable by a differentiable utility function. Assume that the consumption bundle (x_1^*, x_2^*) maximizes utility given prices $p = (p_1, p_2) \gg 0$ and an exogenous income $I > 0$ and that $x_1^* = 0$. Then the (numeric value of the) marginal rate of substitution must exceed the price ratio, i.e. $|MRS(x_1^*, x_2^*)| > \frac{p_1}{p_2}$.
3) A firm that operates on a competitive market cannot earn positive (economic) profits in the long run.
4) A firm that uses labour and capital to produce one output, and which seeks to maximize profit, will, if faced with an increased wage rate, hire less labour input and employ more capital.

Answer:

1) False; if the good is a bad, then he will prefer less of this good
2) False; Since we are on the boundary, the consumer wishes to buy *less* of good 1; thus the marginal willingness to pay is less than the price ratio, i.e. $MRS < \frac{p_1}{p_2}$.
3) False; if there is decreasing returns to scale, such that marginal costs are increasing, then if the firm chooses to operate there must be a positive profit. Only if there is constant returns to scale or free entry into the industry, the profit on a competitive market is (approx.) zero. A partial correct answer is true if the answer conditions on free entry/exit, which must be explicitly stated.
4) False; if the production technology is a complement between capital and labour, then the output effect dominates the substitution effect;

Problem 2

A consumer has a Marshallian demand function $x(p, I) = \big(x_1(p_1, p_2, I), x_2(p_1, p_2, I)\big)$ and his Hicksian demand is denoted by $h(p, u_0) = \big(h_1(p_1, p_2, u_0), h_2(p_1, p_2, u_0)\big)$.

1) Show that

$$\frac{\partial x_1(p_1, p_2, I)}{\partial p_1} = \frac{\partial h_1(p_1, p_2, u_0)}{\partial p_1} - \frac{\partial x_1(p_1, p_2, I)}{\partial I} x_1(p_1, p_2, I)$$

where u_0 is the utility obtained when maximizing utility at prices (p_1, p_2) and income I.

2) Illustrate the two effects in a diagram with x_1 on the horizontal axis and x_2 on the vertical axis, when the price of good 1 increases.
3) Explain what this relation implies for the relationship between the Marshall demand curve and the Hicksian demand curve if good 1 is a normal good.

Answer:

1) We have that $\frac{\partial E}{\partial p_1} = h_1(p, u_0)$, and using the duality $x(p, E(p, u_0)) = h(p, u_0)$ and differentiating wrt. p_1 one obtains the Slutsky equation.
2) A standard illustration as figure 7.5 in Nechyba.
3) If the good is normal, the consumer increases his consumption of the good if the income increases, $\frac{\partial x_1}{\partial I} > 0$. Thus the Marshallian demand has a slope which in absolute value is greater than that of the Hicksian demand – thus the Marshall curve is graphically flatter (viewed from the quantity axis, as our usual diagram) than the Hicks curve.

Problem 3

Consider a competitive market with free entry where each firm has access to the same production technology and hence has the same cost function. The cost function implies that there is a value y_{MES}, that minimizes the average costs. The production technology uses labour and capital, and each year there is a reoccurring cost. Explain what happens with the following three:

- The equilibrium price,
- The quantity sold in equilibrium, and
- the number of firms active in the market.

And please do this for both the short run and the long run.

As you consider the following two changes in the economic environment

1) The reoccurring costs increases
2) The wage rate increases

Answer:

1) Short run: there is no effect, since marginal and average costs are unchanged in the short run. In the long run, the average costs increases while the marginal costs are still unchanged. The increased average costs yields a negative profit, which reduces the number of firms operating in the market. The price will increase, and the quantity sold will be reduced in the long run.
2) The short run marginal costs and average costs increases; this lowers the optimal production of each firm and thus the market supply, hence increasing price and lowering quantity sold. In the long run the capital stock must be adjusted to the new lower output, and hence a further decrease in production occurs. A negative profit lowers the number of firms, if the price has not risen above the new and higher minimal average cost.

Problem 4

Consider an Edgeworth economy, consisting of two consumers, Ib and Bo, where Ib has an utility function $u_I(x_1, x_2) = x_1 x_2$ and he owns the initial endowment $e_I = (1,5)$, while Bo has an utility function $u_B(x_1, x_2) = 3\ln(x_1) + x_2$ and his initial endowment is $e_B = (5,1)$.

1) If Ib and Bo exchange one unit of good 1 to one unit of good 2, such that Ib increases his consumption of good 2, will that constitute a Pareto improvement?
2) Find the Pareto optimal allocation in which Bo obtains 3 units of good 2.
3) Derive the Walrasian equilibrium for this economy

Answer:

1) The utility of each initially are $u_I(1,5) = 5$ and $u_B(5,1) = \ln 5 + 1 \approx 1.6989$; while after the exchange the utility becomes $u_I(0,6) = 0$ and $u_B(6,0) = \ln 6 \approx 0.77$. Hence Ib is worse off while Bo is worse off, and thus the exchange is not a Pareto improvement.
2) We should solve $\max 3\ln(x_{B1}) + x_{B2}$ such that $(6 - x_{B1})(6 - x_{B2}) = u_0$; or $\frac{3}{x_{B1}} = \frac{6-x_{B2}}{6-x_{B1}}$ and when $x_{B2} = 3$, then $x_{2I} = 3$, such that $\frac{3}{x_{B1}} = \frac{3}{6-x_{B1}}$ and hence $18 - 3x_{B1} = 3x_{B1}$, such that $x_{1B} = 3$, hence $x_I = (3,3)$, while $x_B = (3,3)$.
3) The Marshall demand functions are $x_I(p, I_I) = \frac{1}{2}\left(\frac{I}{p_1}, \frac{I}{p_2}\right)$ and $x_B(p, I) = \left(3\frac{p_2}{p_1}, \frac{I-3p_2}{p_2}\right)$, but then we have normalizing $p_2 = 1$ and market clearing $\frac{1}{2}\frac{p_1+5}{p_1} + \frac{1}{p_1} = 6 \Leftrightarrow \frac{5}{2}\frac{1}{p_1} + \frac{6}{2p_1} = \frac{11}{2}$ and thus $p_1^* = 1$; Then the incomes becomes $I_I = 1 * 1 + 5 = 6$ and $I_B = 5 * 1 + 1 = 6$, and the allocation is $x_I = (3,3)$ and $x_B = (3,3)$.

Problem 5

Consider a producer of concrete (cement) that both uses labour and capital in the production according to the production technology $f(l,k) = (l-1)^{\frac{1}{4}}k^{\frac{1}{4}}$ for every $l > 1$ and $f(l,k) = 0$ else.

The producer can acquire labour at the wage rate $w > 0$ and capital at the rental rate, $r > 0$. It can sell concrete at the price of $p > 0$ per unit output.

In the short run, the input of capital is fixed at $k_0 = 4$.

1) Derive the short run cost function, $c(w, r; y, k_0)$ given output y and capital k_0.
2) Derive the supply function in the short run, $y(w, r, p; k_0)$, and find the optimal supply if prices are $(p, w, r) = (2,1,4)$
3) Derive the long run supply function, $y(w, r, p)$ and find the optimal supply if prices are $(p, w, r) = (2,1,4)$
4) What happens in the short and long run if the output price changes to $p = 16$?

Answer

1) The economic costs are $c(w, r; y, k_0) = wl(w, r; y, k_0) = w + w\frac{y^4}{k_0}$ while the total costs are $c(w, r; y, k_0) + F = w + w\frac{y^4}{k_0} + rk_0$.

2) The optimal output satisfies $MC = p \Leftrightarrow \frac{4wy^3}{k_0} = p \Leftrightarrow y = \left(\frac{1}{4}\frac{p}{w}k_0\right)^{\frac{1}{3}}$, and it must satisfy that the marginal costs exceeds the average costs: $\frac{4wy^3}{k_0} \geq \frac{w}{y} + \frac{wy^3}{k_0} \Leftrightarrow \frac{3wy^3}{k_0} \geq \frac{w}{y} \Leftrightarrow y \geq \left(\frac{k_0}{3}\right)^{\frac{1}{3}}$. The solution to the FOC is then $y = (2)^{\frac{1}{3}}$, and thus also shutdown condition is satisfied.

3) To find the long run costs we solve the problem: $\min_{k_0} w + w\frac{y^4}{k_0} + rk_0$, which imply $4\frac{wy^4}{k^2} = r \Leftrightarrow k(y) = 2y^2\left(\frac{w}{r}\right)^{\frac{1}{2}}$, such that the long run costs $c_{LR}(w,r;y) = w + 2\sqrt{wr}y^2$; then the optimal supply will be $4\sqrt{wr}y^* = p \Leftrightarrow y^* = \frac{p}{4\sqrt{wr}}$ that solves the FOC. Using the prices we obtain $y^* = \frac{2}{4*2} = \frac{1}{4}$, and hence a profit of $\pi = \frac{1}{2} - 1 - 2\sqrt{1*4}\frac{1}{16} < 0$. Thus the firm should shut down.

4) The optimal choice according to the FOC is $y^* = 2$, where the profit is $\pi = 2 * 16 - 1 - 4 * 2^2 = 15 > 0$.