Written Exam for the B.Sc. in Economics - Autumn 2013-14

Macro C
Final Exam

January 7, 2014

3-hour closed book exam

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. If you are in doubt about which title you registered, please see the print of your exam registration from the students' self-service system.

Problem 1

Consider an economy where households maximize the following intertemporal utility:

$$U = \int_0^\infty [\ln c_t] e^{-\rho t} dt \tag{1}$$

where c is per capita consumption in the household, and $\rho > 0$ is a time discount factor. Population is constant.

Assume that there are many identical competitive firms that hire (from households) capital and labor to produce the consumption good with the following constant returns to scale technology:

$$Y_t^j = \left(K_t^j\right)^\alpha \left(L_t^j\right)^{1-\alpha}, \quad 0 < \alpha < 1,$$

where Y^j is firm j's output, K^j is firm j's demand for capital, and L^j is its demand for labor. Labor supply is inelastic and normalized at one unit per household member at each instant t. Government prints money at rate σ and distributes the proceeds to households in a lump sum way (if $\sigma < 0$ the government absorbs resources through lump sum taxes). Assume that households can save in capital or money, and that household i's budget constraint in nominal terms is given by,

$$P_t F_t^i(C_t^i) + P_t \dot{K^i}_t + \dot{M^i}_t = P_t w_t N_t^i + P_t r_t K_t^i + Z_t^i$$

where P is the price level, $F^i(C^i) \geq C^i$ tells the household the resources it needs to spend if it wants to buy a flow of C^i of consumption goods, K^i are capital holdings of the household, M^i their money holdings, w is the wage rate, N^i is the size of the household (equal to labor supply), r is the real interest rate, and Z^i are nominal lump sum transfers from the government (could be negative).

In this economy real money balances reduce the cost of searching for consumption goods. In order to consume flow C^i at time t, household i must spend resources $C^i(1+e^{-\phi m^i})$ where $\phi > 0$, and m^i are the household's real money holdings in per capita terms (thus $F^i(C^i) = C^i(1+e^{-\phi m^i})$).

a) Find the flow budget constraint in real and per capita terms. Interpret the terms related to money balances.

b) Set up the Hamiltonian for the household's optimization problem, stating which are control, state and costate variables. Write the first order conditions, including the transversality condition. Solve the first order conditions.

c) What are the equilibrium conditions that determine the wage, interest rate, and government transfers? Solve for the steady state of this economy. How does steady state money demand, m^*, depend on ρ, σ, and ϕ? Explain the economic intuition of your result. [Note: if households' assets are infinite in steady state, assume that the rate of growth of wealth during the transition does not violate the transversality condition.]

d) Is money superneutral? What is the optimal quantity of money, and does the Friedman rule apply? Interpret.

e) Now assume that the cost to acquire a flow C of consumption goods depends on aggregate real money balances, so household i must spend resources $C^i(1+e^{-\phi m})$ (i.e. the added cost is perceived by each household to be independent of their actions, in the notation of the budget constraint $F_t^i(\cdot)$ is now $F_t(\cdot)$). How does this affect your results in b), c)? How does m^* depend on ρ, σ, and ϕ? Explain.

Problem 2

Assume that the following demand and supply equations characterize the behavior of a given economy,

$$\begin{aligned}\pi_t &= m_t + \nu_t \\ x_t &= \theta_t + (\pi_t - \pi_t^e) - \epsilon_t\end{aligned}$$

where π is the inflation rate, π^e the expected inflation rate, m the rate of growth of money, x the rate of growth of output, θ the stochastic level for the potential growth rate of output, ν is a demand shock, and ϵ is a supply shock. Assume that all shocks are independent from each other, and across time, with zero mean and known variances (σ_θ, σ_ν, σ_ϵ). Sequential decision making implies that the private sector forms expectations on inflation each period after only observing θ, while the monetary authority determines m after observing θ, π^e, ν, and ϵ. Finally assume that there are two time periods and that society evaluates policy according to the following loss function

$$E_0[L(\pi,x)] = E_0\left[\frac{1}{2}\left((\pi_1-\overline{\pi})^2 + \lambda(x_1-\overline{x})^2 + (\pi_2-\overline{\pi})^2 + \lambda(x_2-\overline{x})^2\right)\right]$$

where λ measures the relative importance of policy objectives with respect to output deviations from target level $\overline{x}$, and inflation deviations from target level $\overline{\pi}$. For all realizations of θ, distortions in the economy imply that $\overline{x} - \theta > 0$. For simplicity assume that $\overline{\pi} = 0$.

a) Solve for the optimal monetary policy rule that if announced at time 0 maximizes social welfare. Does this rule prescribe that policy in period 2 depend on shocks observed in period 1? Comment. Solve also for equilibrium inflation and output growth.

b) Show that if monetary policy is sequential under discretion, there exists an inflation bias that reduces social welfare (this requires solving for equilibrium inflation and output growth). Explain why welfare is lower than under commitment.

c) Assume now that monetary policy can be delegated on an independent central banker whose preferences are different from average society's preferences (i.e. $\lambda^{CB} \neq \lambda$). Show that the optimal choice of central banker implies $\lambda^{CB} < \lambda$. Solve also for equilibrium inflation and output growth. Is output volatility higher or lower than in point b)? Interpret.

d) Now assume that strategic delegation as in point c) is not possible because there is no heterogeneity in society (i.e. everybody has the same preferences λ). An alternative is to delegate monetary policy on a central banker who is rewarded according to performance using a preannounced linear contract. Assume that payment, $P(\theta, \pi)$, takes the form:

$$P_t(\theta_t, \pi_t) = p_{c,t} + p_{\pi,t}(\theta_t)\pi_t$$

and the contract must satisfy that $E[P_t] \geq \bar{\omega} > 0$ where $\bar{\omega}$ is the alternative income that the central banker can make from working as a consultant in the private sector. Also assume that units are such that the central banker wants to maximize $-L(\pi, x) + P(\theta, \pi)$. Show that it is possible to attain the first best with a contract that is independent of time, i.e. for which $p_{c,1} = p_{c,2}$ and $p_{\pi,1}(\theta) = p_{\pi,2}(\theta)$. Find this contract and comment.

e) Now suppose that with probability $1 \geq 1 - (a - b\pi_1) \geq 0$ (with $a > 0$ and $b > 0$) the central banker is fired at the end of the first period (i.e. the higher is the inflation rate, the more likely that the central banker will be fired). Show that in this case it is still possible to attain the first best with a contract that satisfies $p_{\pi,1} \neq p_{\pi,2}$. Find this contract and comment.