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Solutions to written exam for the M.Sc. in Economics International Monetary Economics

January 20, 2015

Number of questions: This exam consists of 2 questions.

1. **The Monetary Approach to the Exchange Rate**
   This question relates to the learning objective describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, and Lucas asset pricing model) and use these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models". The question focuses on the Mundell-Fleming model. All notation is standard and should therefore be known.

   The Mundell–Fleming model is:

$$\dot{s} = i - i^* \tag{1}$$

$$m = \sigma s + \kappa y - \theta i \tag{2}$$

   and

$$\dot{y} = \chi (\alpha + \mu s - y). \tag{3}$$

   (a) Give a brief interpretation of the main assumptions and economic mechanisms underlying the equations of the model. Why is s and not the price level in the money demand function?

   Equation (1) is the UIP relation formulated in continuous time where i is the domestic interest rate, i^* is the foreign interest rate and $\ldots s$ is the instantaneous change of the exchange rate s. Underlying assumption is that domestic and foreign bonds are perfect substitutes (or that agents are risk neutral) implying that there is no risk premium.

   Equation (2) is the standard money demand function where m is the money stock (demand for money is equal to the supply of money) and y is output. The parameter κ is the output elasticity of money demand, θ is the interest elasticity of money demand and σ is the exchange rate elasticity of money demand. In the standard money demand function, s is replaced by p the price level. The reason why the exchange rate determines the money demand is that the consumer price index is a weighted average of the price level on home goods and the price level of foreign goods (measured in home currency terms). The aggregate price level can be written as

$$p^c = \sigma (s + p^*) + (1 - \sigma) p$$

where the first part on the RHS is the price of imported goods measured in terms of the domestic currency and the second part is the price of domestically produced goods. The parameter σ measures the importance of foreign prices in the domestic consumer price index. If we assume that foreign and domestic prices are fixed, we can normalize such that $p = p^* = 0$ implying that $p^c = \sigma s$.

Equation (3) is the aggregate demand for goods. The first component on the RHS of this equation (α) is the autonomous component of aggregate demand, the next component (μs) represents foreign demand for domestic goods, the third component is output implying that the change in aggregate demand depends on output in the earlier period.

(b) Derive the IS– and LM–curves.

We would like to solve for $\dot{s}$ and $\dot{y}$ as functions of the other variables so we can illustrate the model in the $s - y$–plane. Start with the LM–curve: Assume that the monetary authority can set the money supply (it is given exogenously), then solving for i in equation (2) and insert this solution into (1) yields

$$\dot{s} = \frac{\sigma}{\theta}s + \frac{\kappa}{\theta}y - \frac{1}{\theta}m - i^*. \tag{4}$$

Note that the aggregate demand function as stated in equation (3) also is the IS–curve, it relates the change in output to the exchange rate and the previous level of output.

We can now rewrite the model as

$$\begin{bmatrix} \dot{s} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{\sigma}{\theta} & \frac{\kappa}{\theta} \\ \chi\mu & -\chi \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix} + \begin{bmatrix} -\frac{1}{\theta}m - i^* \\ \chi\alpha \end{bmatrix} \tag{5}$$

where the determinant of the coefficient matrix is

$$-\chi\frac{\sigma}{\theta} - \frac{\chi\mu\kappa}{\theta} < 0$$

such that the system is saddlepath stable.

The LM curve (in the s and y plane) is derived under the assumption that the exchange rate market is in equilibrium ($\dot{s} = 0$) and the IS curve when the goods market is in equilibrium ($\dot{y} = 0$). The slope of the LM curve is negative and the slope of the IS curve is positive. To show this, note that the LM curve (first row of the equation system above when $\dot{s} = 0$) is given by

$$s = -\frac{\kappa}{\sigma}y + \frac{\theta}{\sigma}\left(\frac{1}{\theta}m - i^*\right)$$

such that

$$\frac{\partial s}{\partial y} = -\frac{\kappa}{\sigma}$$

and the IS curve (second row of the equation system above when $\dot{y} = 0$) is given by

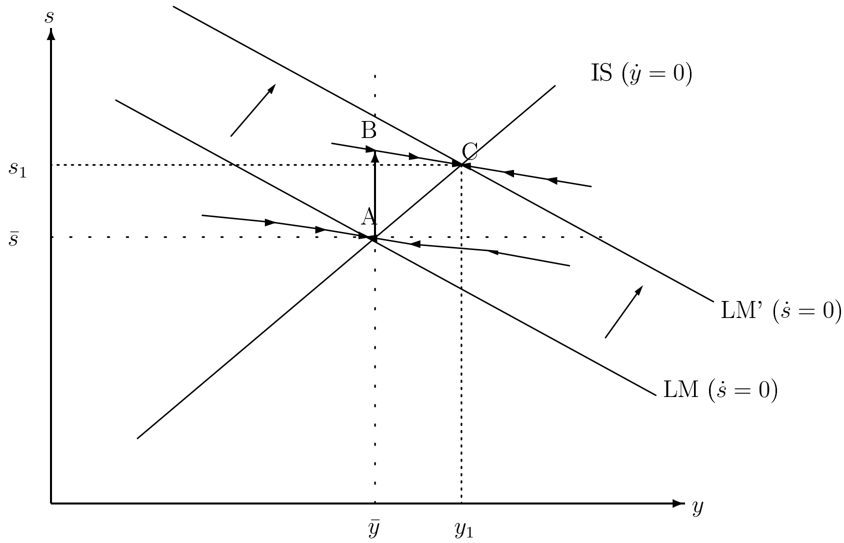
$$s = \frac{1}{\mu}y - \frac{\alpha}{\mu}$$

such that

$$\frac{\partial s}{\partial y} = \frac{1}{\mu}.$$

(c) Illustrate the model in a graph and explain the dynamics of the model. Show the effects of a monetary contraction.

We illustrate the model in a graph below where we show the effects of a monetary expansion.



An increase in m will lead to a shift in the LM–curve up to the right. For a given y, higher m implies a higher s (a depreciated exchange rate) which can be seen from the LM–equation. Since the economy is always on a saddlepath, the exchange rate jumps up to the new saddlepath, the exchange rate jumps from point A to point B. At point B, the economy is on a new saddlepath. Foreign demand will increase leading to higher output. Output increases and the economy moves along the saddlepath towards the new long–run equilibrium at point C. Total effect: Depreciated currency (an overshooting effect) and higher output (prices are fixed).

(d) Replace equation (3) with

$$\dot{p} = \gamma \left(\alpha + \mu \left(s - p\right) - \bar{y}\right) \tag{6}$$

and since output is assumed to be at its long–term equilibrium level, equation (2) becomes

$$m = \sigma s + \kappa \bar{y} - \theta i. \tag{7}$$

Explain the main assumptions underlying these equations and the differences between this model and the previous one.

Equation (6) (which is the Phillips-curve) describes how prices adjust to the real exchange rate (where we implicitly have normalized away foreign price level, it is assumed to be equal to 1) and the fixed output level. Note that we now assume that output is fixed at its equilibrium value whereas prices are flexible. This implies that the money demand function now is written as above. The main difference between this model and the Mundell-Fleming model above is the assumption concerning output and prices, output is fixed and prices flexible in the present model whereas prices are fixed and output is flexible in the Mundell-Fleming model.

(e) Show that the model can be written as

$$\begin{bmatrix} \dot{s} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1/\theta \\ \gamma\mu & -\gamma\mu \end{bmatrix} \begin{bmatrix} s-\bar{s} \\ p-\bar{p} \end{bmatrix} \tag{8}$$

where notation is standard.

First, we note from above that $p = \sigma s$ such that the money demand function is

$$m = p + \kappa\bar{y} - \theta i$$

and in the long-run equilibrium inflation is zero such that the price level is $\bar{p}$ and the money demand in equilibrium is then

$$m = \bar{p} + \kappa\bar{y} - \theta i$$

Take the difference between the money demand and the money demand in equilibrium we find that

$$p - \bar{p} = \theta(i - i^*)$$

and by using the UIP relation we find that

$$\dot{s} = (1/\theta)(p - \bar{p})$$

In equilibrium, $\dot{p} = 0$ such that the Phillips curve is

$$0 = \gamma(\alpha + \mu(\bar{s} - \bar{p}) - \bar{y})$$

which is subtracted from the Phillips-curve

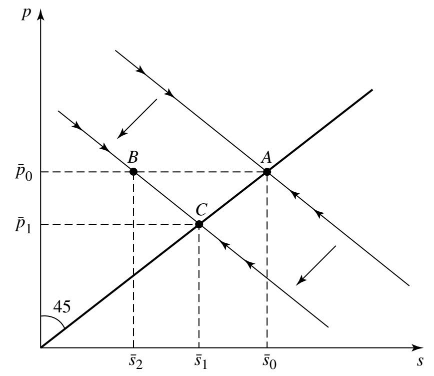
$$\dot{p} = \gamma\mu(s - \bar{s}) - \gamma\mu(p - \bar{p})$$

and then putting it together we find that

$$\begin{bmatrix} \dot{s} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1/\theta \\ \gamma\mu & -\gamma\mu \end{bmatrix} \begin{bmatrix} s-\bar{s} \\ p-\bar{p} \end{bmatrix}$$

(f) Illustrate the model in a graph and show the effects of a monetary contraction. Compare and contrast with the effects found in question (d).

First we note that the determinant is negative such that the system has a unique saddlepath, the saddlepath slopes down from left to right in (p,s)-space, see the graph below.



Consider now the effects of a reduction in the money supply, the money market curve (or saddlepath) shifts down. In the long run, the price level will fall and since we have assumed that PPP holds in the long-run and holding foreign prices constant, the long-run exchange rate will appreciate proportionately, s will also fall. The stable saddlepath, which originally went through point A must now go through the new long-run equilibrium C but since prices are sticky, the economy cannot jump directly from A to C. Instead, prices remain fixed and the exchange rate jumps to s_2 in order to get on to the new saddlepath. Prices then adjust slowly and the economy moves along the saddlepath from B to the new long-run equilibrium C . Therefore, the net effect of a reduction of the money supply is a long-run appreciation, s_0 to s_1 and an initial overshooting corresponding to $s_2 - s_1$.

Comparing the two models we note: In the Mundell–Fleming model output is demand determined whereas prices are fixed but in the second model, the sticky price monetary model, output is fixed at its long–run equilibrium and prices are flexible so that PPP holds in the long-run. There is an overshooting effect in both models but the mechanism is different.

(g) Summarize the empirical evidence on the sticky price and flexible price monetary model.

Empirical tests show that the estimated parameters usually have the wrong signs and are insignificant. The main conclusion from the literature is that the models are rejected. There are a few exceptions, Frenkel (1976) supports the model using high inflation data and some studies using data for flexible exchange rate regimes also support the flexible price monetary model. Using data for the 1980's, the model performs not so well. Usually we reject the FPMM. Same general results for the sticky price model.

Another approach is to use the theoretical models for forecasting and compare the forecasts with forecasts using a random walk model. In general the results are poor, there are models that could beat forecasts from a random walk model for certain exchange rates and periods, but the results are very unreliable, small changes to the sample affects the results.

However, when using cross-country data focusing on the long-run performance of the monetary models, we typically find quite strong empirical support. Both when using cross-sections and panel data analysis, the results suggest that the models can explain exchnage rate movements at longer-term horizons. Fundamentals (money, output, and interest rates) are important determinants of the exchange rate in the long-run but not in the short-run.

2. **Temporary and permanent shocks in a small open production economy**

This question covers the learning objective Describe and use a standard small open economy model to analyze the current account.Ät a minimum an answer should find that capital accumulation (and thus profits and period 2 output) do not respond to a temporary shock in period 1, but *are* affected by a permanent shock. Agents smooth out a temporary increase in output, consumption responds less than one-for-one, and by more than that when the shock is permanent. The problem should be set up correctly and follow the steps that are outlined in part a) of the question. A good answer will do most of the corresponding math correctly. A very good answer will find that in response to a permanent shock output peaks *after* the initial shock, and as a result agents borrow anticipating the higher income, and the trade balance deteriorates on impact. The last part will also be addressed in a very good answer.

(a) Benchmark model

Household period budget constraints:

$$\begin{aligned} c_1 + W_1 &= y_1 \\ c_2 &= \Pi_2 + (1+r^*)W_1 \end{aligned}$$

where I have imposed that $W_0 = W_2 = 0$ (the first by assumption, the second by no Ponzi).

Lifetime budget constraint:

$$c_1 + \frac{c_2}{1+r^*} = y_1 + \frac{\Pi_2}{1+r^*}$$

Firms profits are

$$\Pi_2 = A_2 k_2^\alpha - (r^* + \delta)k_2$$

Substitute this into the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r^*} = y_1 + \frac{1}{1+r^*}\left(A_2 k_2^\alpha - (r^* + \delta)k_2\right)$$

The Lagrangian is

$$\mathcal{L} = \max_{c_1, c_2, k} u(c_1, c_2) + \lambda \left\{ y_1 + \frac{1}{1+r^*}\left(A_2 k_2^\alpha - (r^* + \delta)k_2\right) - \left(c_1 + \frac{c_2}{1+r^*}\right)\right\}$$

Necessary first order conditions are

$$\frac{1}{c_1} = \lambda \tag{9}$$

$$\frac{\beta}{c_2} = \frac{\lambda}{1+r^*} \tag{10}$$

$$\alpha A_2 k_2^{\alpha-1} = r^* + \delta \tag{11}$$

Equation (11) yields the solution for capital

$$k_2 = \left(\frac{r^* + \delta}{\alpha A_2}\right)^{\frac{1}{\alpha-1}} \tag{12}$$

which gives profits

$$\Pi_2 \quad = \quad A_2 \hat{k}_2^{\alpha} - (r^* + \delta)\hat{k}_2 \tag{13}$$

We can find optimal investment from the law of capital accumulation

$$I_1 = k_2 - (1 - \delta)k_1 = k_2 \tag{14}$$

Equations (9) and (10) yield the Euler equation

$$\frac{1}{c_1} = \beta(1 + r^*)\frac{1}{c_2}$$

Rearrange the Euler $c_1 = \frac{c_2}{\beta(1+r^*)}$, and substitute out c_1 from lifetime budget constraint to solve for c_2

$$c_2 \quad = \quad \frac{\beta}{1+\beta}(1+r)\left(y_1 + \frac{\Pi_2}{1+r^*}\right) \tag{15}$$

Subsitute out the expression for profits (13) and capital (12) to find c_2 as functions of parameters

$$\begin{aligned} c_2 &= \frac{\beta}{1+\beta}(1+r)\left(y_1 + \frac{A_2 k_2^{\alpha} - (r^* + \delta)k_2}{1+r^*}\right) & (16) \\ &= \frac{\beta}{1+\beta}(1+r)\left[y_1 + \frac{1}{1+r^*}\left(A_2\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{\alpha}{\alpha-1}} - (r^*+\delta)\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{1}{\alpha-1}}\right)\right] & (17) \end{aligned}$$

and plugging back into Euler to find c_1 as a function of parameters

$$\begin{aligned} c_1 &= \frac{1}{1+\beta}\left(y_1 + \frac{\Pi_2}{1+r^*}\right) & (18) \\ &= \frac{1}{1+\beta}\left[y_1 + \frac{1}{1+r^*}\left(A_2\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{\alpha}{\alpha-1}} - (r^*+\delta)\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{1}{\alpha-1}}\right)\right] & (19) \end{aligned}$$

Finally, $TB_1 = y_1 - c_1 - I_1$ implying that

$$\begin{aligned} TB_1 = y_1 - \frac{1}{1+\beta}\Bigg[y_1 + \frac{1}{1+r^*}\Bigg(A_2\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{\alpha}{\alpha-1}} - \\ (r^*+\delta)\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{1}{\alpha-1}}\Bigg)\Bigg] - \left[\left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{1}{\alpha-1}}\right] \end{aligned} \tag{20}$$

Note that we could back out implied net foreign assets using the open economy market clearing conditions $W_1 = b_1 + k_2$. They don't necessarily need to substitute all the way out provided they still get the right results in the next parts.

(b) Temporary shock to TFP

Period 1 output increases directly because of the TFP increase, that is $\frac{\partial y_1}{\partial A_1} = q$. Investment, capital accumulation, profits and period 2 output are not affected because period 2 TFP is unchanged (see (12) and (14)). Households smooth out the temporary increase in income by saving in period 1: c_1 increases by less than output $\frac{q}{1+\beta} < q$, while c_2 increases by $\frac{\beta(1+r)q}{1+\beta} > 0$. The trade balance improves by $\frac{\beta}{1+\beta}q > 0$.

(c) Permanent shock to TFP

As before, period 1 output increases directly because of the TFP increase. From (12) and (14), in anticipation of higher productivity tomorrow, agents optimally invest more in period 1 and have a higher capital stock in period 2:

$$\begin{aligned}
\frac{\partial I_1}{\partial A_2} = \frac{\partial k_2}{\partial A_2} &= -\frac{1}{\alpha-1} A_2^{-\frac{1}{\alpha-1}-1} \left(\frac{r^*+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \\
&= -\frac{1}{\alpha-1} A_2^{-\frac{\alpha}{\alpha-1}} \left(\frac{r^*+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \\
&= -\frac{1}{\alpha-1} \frac{1}{A_2} \left(\frac{r^*+\delta}{\alpha A_2}\right)^{\frac{1}{\alpha-1}} \\
&= -\frac{1}{\alpha-1} \frac{k_2(A_2)}{A_2} > 0
\end{aligned}$$

Profits rise:

$$\begin{aligned}
\frac{\partial \Pi_2}{\partial A_2} &= \frac{\partial [A_2 k_2^\alpha - (r^*+\delta) k_2]}{\partial A_2} \\
&= k_2^\alpha + \left(\alpha A_2 k_2^{\alpha-1} - (r^*+\delta)\right) \frac{\partial k_2}{\partial A_2} \\
&= f(k_2(A_2))
\end{aligned}$$

where the last line follows from the fact that profit maximization implies equalization of the marginal product of capital with its marginal user cost. Period 2 output is higher:

$$\begin{aligned}
\frac{\partial y_2}{\partial A_2} &= k_2^\alpha + A_2 \frac{\partial k_2}{\partial A_2} \\
&= f(k_2(A_2)) - \frac{1}{\alpha-1} k_2(A_2) > 0
\end{aligned}$$

Consumption increases (by the same amount in periods 1 and 2 if $\beta(1+r^*) = 1$):

$$\begin{aligned}
\frac{\partial c_1}{\partial A_1} + \frac{\partial c_1}{\partial A_2} &= \frac{1}{1+\beta} \left(f(k_2(\bar{A})) + \frac{1}{1+r} \frac{\partial \Pi_2}{\partial A_2} \right) \\
&= \frac{1}{1+\beta} \left(f(k_2(\bar{A})) + \frac{f(k_2(\tilde{A}))}{1+r} \right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial c_2}{\partial A_1} + \frac{\partial c_2}{\partial A_2} &= \frac{\beta(1+r^*)}{1+\beta}\left(q + \frac{1}{1+r}\frac{\partial \Pi_2}{\partial A_2}\right) \\
&= \frac{\beta(1+r^*)}{1+\beta}\left(f(k_2(\bar{A})) + \frac{f(k_2(\tilde{A}))}{1+r}\right)
\end{aligned}$$

The change in the trade balance is given by:

$$\begin{aligned}
\Delta TB_1 &= \frac{\partial TB_1}{\partial A_1} + \frac{\partial TB_1}{\partial A_2} \\
&= \frac{\partial y_1}{\partial A_1} - \frac{\partial c_1}{\partial A_1} - \frac{\partial c_1}{\partial A_2} - \frac{\partial I_1}{\partial A_2} \\
&= f(k_2(\bar{A})) - \frac{1}{1+\beta}\left(f(k_2(\bar{A})) + \frac{f(k_2(\tilde{A}))}{1+r}\right) + \frac{1}{\alpha-1}A_2^{-\frac{\alpha}{\alpha-1}}\left(\frac{r^*+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}}
\end{aligned}$$

Note that $y_1 = A_1 f(k_2(\bar{A}))$ so output is the same in both periods under the baseline $\bar{A}$.

Output in period 2 is higher than in period 1: Since $k_2(\bar{A}) < k_2(\tilde{A})$, the increase in period 2 output in response to the permanent shock is larger than the increase in period 1 output, and since $y_1(\bar{A}) = y_2(\bar{A})$, we have, after the permanent shock,

$$y_2(\tilde{A}) > y_1(\tilde{A})$$

Intuitively, we started from the same level before the shock, period 1 output rises directly because of higher TFP, but period 2 output rises for two reasons: directly because of higher TFP and indirectly because of higher capital.

Consumption in period 1 increases by more than output in period 1:

$$\begin{aligned}
\frac{1}{1+\beta}\left(f(k_2(\bar{A})) + \frac{f(k_2(\tilde{A}))}{1+r}\right) &> f(k_2(\bar{A})) \\
f(k_2(\bar{A})) + \frac{f(k_2(\tilde{A}))}{1+r} &> (1+\beta)f(k_2(\bar{A})) \\
\frac{f(k_2(\tilde{A}))}{1+r} &> \beta f(k_2\bar{A})) \\
f(k_2(\tilde{A})) &> f(k_2(\bar{A})) \\
\tilde{A} &> \bar{A}
\end{aligned}$$

where we have used that the production function is strictly increasing and that $\beta(1+r) = 1$. From the perspective of period 1, agents see an increase in future income so they borrow against that in period 1 and increase c_1 by more than the increase in y_1.

It then follows that **the trade balance deteriorates**.

(d) Comparison of adjustment to temporary versus permanent shock

The key in terms of comparisons is that (i) consumption responds less than one-for-one to temporary shock - smooth via the trade balance -, (ii) consumption responds by annuity value to permanent shocks - no point in smoothing -, (iii) output responds by more via investment channel to permanent shock and as a result procyclical trade balance under temporary shock, countercyclical trade balance under permanent shock.

(e) Permanent shock in endowment economy

Now period 2 output rises by exactly as much as period 1 output because there is no amplification of the TFP shock to production via capital accumulation. Consumption responds in the same way (equal increase), and now consumption increases exactly by as much as output:

$$\begin{aligned} c_1 &= \frac{1}{1+\beta}\left(y_1(A_1) + \frac{y_2(A_2)}{1+r^*}\right) \\ c_2 &= \frac{\beta}{1+\beta}(1+r^*)\left(y_1(A_1) + \frac{y_2(A_2)}{1+r^*}\right) \end{aligned}$$

Intuitively, there is no point in smoothing anything since the increase is permanent and equal across periods. This would change if $\beta(1+r^*) \neq 1$, for example. The trade balance is unchanged:

$$\begin{aligned} TB_1 &= y_1(A_1) - c_1 \\ &= y_1(A_1) - \frac{1}{1+\beta}\left(y_1(A_1) + \frac{y_2(A_2)}{1+r^*}\right) \end{aligned}$$

$$\begin{aligned} \frac{\partial TB_1}{\partial A_1} + \frac{\partial TB_1}{\partial A_2} &= q\left(1 - \frac{1}{1+\beta}\right) - \frac{1}{1+\beta}\frac{q}{1+r^*} \\ &= 0 \end{aligned}$$