LM Januar 19 – Vejl. løsn. skitse

① $$L = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

1) $L\underline{x} = \underline{0}$

$$L \Longleftrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (\text{gauss})$$

$x_4 = t$

$$N(L): \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \ t \in \mathbb{R}.$$

L es injektiv da $N(L) \neq \{\underline{0}\}$.

2) $R(L) = \text{span}\{\text{søjlerne}\} = \mathbb{R}^3$
da de tre første søjler er lin. uafh.

Dim. sætn.

$$4 - \dim(N(L)) = \dim(\mathbb{R}^3)$$
$$4 - 1 = 3 \quad \underline{\text{OK}}.$$

L er surjektiv, da $R(L) = \mathbb{R}^3$.

3)

$\mathcal{L}x = y$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & y_1 \\ 1 & 0 & 1 & 0 & y_2 \\ 0 & 0 & 1 & 1 & y_3 \end{array}\right] \longleftrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & y_2 - y_3 \\ 0 & 1 & 0 & 1 & y_1 - y_2 + y_3 \\ 0 & 0 & 1 & 1 & y_3 \end{array}\right]$$

$$\mathcal{L}: \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_2 - y_3 \\ y_1 - y_2 + y_3 \\ y_3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}.$$

4)

$$(L^T L)^T = L^T L^{TT} = L^T L, \quad \underline{\text{symmetrisk}}$$

5)

Da $N(\mathcal{L}) \neq \{\underline{0}\}$ findes $x \in \mathbb{R}^4 \setminus \{\underline{0}\}$ så $\mathcal{L}x = \underline{0}$ (f.ex $(1,-1,-1,1)$)

Men så er $\mathcal{L}^T(\mathcal{L}x) = \underline{0}$, dvs

$(\mathcal{L}^T\mathcal{L})x = \underline{0}$ har andre løsninger end $\underline{0}$.

Altså er $\mathcal{L}^T\mathcal{L}$ <u>ikke</u> regulær!

(Eller $N(\mathcal{L}) \subseteq N(L^T L)$, hvor $N(\mathcal{L}) \neq \{\underline{0}\}$)

3

2

1)

$v_1 \cdot v_3 = 0 \iff x_1 + x_2 + x_3 = 0$
$v_2 \cdot v_3 = 0 \iff -x_1 + x_3 = 0$

$v_3 = (1, -2, 1)$ er mulig egenvektor.

2) $A = QDQ^T$ med $D = \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix}$

Så er $A^2 = QD^2Q^T = QEQ^T = E$.

3) Da $AA = A^2 = E$ er A inv. og $A^{-1} = A$.

4) $Ax = v_1 + v_2$
$\Updownarrow$
$x = A^{-1}v_1 + A^{-1}v_2 = Av_1 + Av_2 = \underline{\underline{-v_1 - v_2}}$

$= (-1, -1, -1) - (-1, 0, 1) = \underline{\underline{(0, -1, -2)}}$

5) $E + A = Q \begin{bmatrix} 0 & & \\ & 0 & \\ & & 2 \end{bmatrix} Q^T$

$rg(E+A) = 1$, $\dim N(E+A) = 2$,

egenverdier $0, 2$, $rm(0) = em(0) = 2$,
$rm(2) = em(2) = 1$

6) $\frac{1}{2}(E+A)(v_1+v_2+v_3) = 0v_1 + 0v_2 + \frac{1}{2}2v_3$

$= \underline{\underline{v_3}}$.

3

1) $P_A(\lambda) = (a-\lambda)(c-\lambda) - b^2$

$= \lambda^2 - (a+c)\lambda + ac - b^2$

Diskriminanten er

$(a+c)^2 - 4(ac-b^2) = a^2+c^2+2ac-4ac+4b^2$

$= a^2+c^2-2ac+4b^2$

$= (a-c)^2+4b^2 \geq 0$, hvorfor

rødderne = egenværdierne er reelle.

2) $w^2 = -2-i4$

$w = x+iy$, $x,y \in \mathbb{R}$. $w^2 = x^2-y^2+i2xy$

$\begin{cases} x^2-y^2 = -2 \\ 2xy = -4 \end{cases}$ (Dvs x,y begge $\neq 0$.)

Så fås $y = \frac{-2}{x}$, så

$$x^2 - \left(\frac{-2}{x}\right)^2 = -2$$

$$\Updownarrow$$

$$x^4 + 2x^2 - 4 = 0. \quad \text{Med } u = x^2 > 0$$

fås $u = \frac{-2 \pm \sqrt{20}}{2} = -1 + \sqrt{5}$ (– forkast

så $x = \pm\sqrt{-1+\sqrt{5}}$.

Da $y = \frac{-2}{x}$ fås så

$$W = x + iy = \pm\left(\sqrt{-1+\sqrt{5}} - i\frac{2}{\sqrt{-1+\sqrt{5}}}\right).$$

4

$$\sum_{n=0}^{\infty}\left(\frac{1}{x^2-4}\right)^n.$$

1) $g(x) = \frac{1}{x^2-4}$. $|g(x)| < 1$ løses.

$$\left|\frac{1}{x^2-4}\right| < 1$$

Vi løser $\frac{1}{x^2-4} = 1$ og $\frac{1}{x^2-4} = -1$

$$\frac{1}{x^2-4} = 1 \Leftrightarrow x^2-4 = 1 \Leftrightarrow x = \pm\sqrt{5}$$

$$\frac{1}{x^2-4} = -1 \Leftrightarrow x^2-4 = -1 \Leftrightarrow x = \pm\sqrt{3}$$

Heraf ses at f er veldefineret

for $\quad x \in M =]-\infty;-\sqrt{5}[\cup]-\sqrt{3},\sqrt{3}[$
$\cup]\sqrt{5},\infty[$,

med sum

2) $$f(x) = \frac{1}{1-\frac{1}{x^2-4}} .$$

3) f og g samme monotoni-forhold

$$g(x) = (x^2-4)^{-1}$$

$$g'(x) = -(x^2-4)^{-2} \cdot 2x = 0 \Leftrightarrow x = 0$$

x		$-\sqrt{5}$		$-\sqrt{3}$		0		$\sqrt{3}$		$\sqrt{5}$	
f'	$+$				$+$	0	$-$				$-$
f	↗				↗	lok max	↘				↘

4) $f(x)=f(-x)$, ej inj. (flere mulige forklaringer)

$Vm(f)$.

For $x \to \pm\infty$ vil $f(x) \to 1$

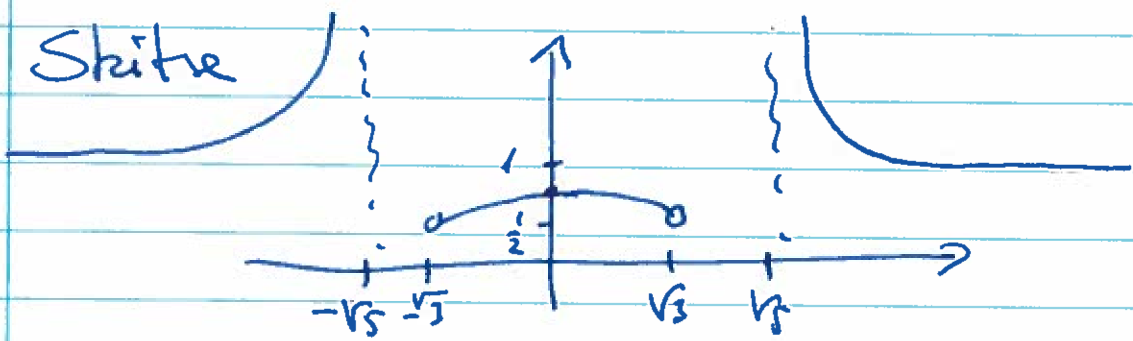
$x \to \pm\sqrt{5}^{\mp}$ vil $f(x) \to \infty$

$x \to \pm\sqrt{3}^{\mp}$ vil $f(x) \to \frac{1}{2}$

$f(0) = \frac{4}{5}$, lok max.

$Vm(f) = \,]\frac{1}{2}, \frac{4}{5}] \cup]1, \infty[$

Skitse



5) $f(x) = y$, $y \in Vm(f)$

$$\frac{1}{x^2-4} = \frac{y-1}{y}$$

$$x^2 = 4 + \frac{y}{y-1}$$

$$x = \pm\sqrt{4 + \frac{y}{y-1}}, \quad y \in Vm(f)$$