ANSWERS

A1. Consider the following statement: "*Wealth Inequality is good for growth*". Do you agree or disagree. Explain.

Main readings: Galor and Zeira (1993).

Both agree and disagree can be defended. The nature of the argument is what matters for full credit.

Theoretically, things may well be more complicated that what is implied by the statement. It would therefore make sense to disagree with it.

In the theory developed by Galor and Zeira, for instance, the impact of (wealth) inequality on prosperity depends on the state of development. The economy under consideration is one where growth is driven by investments in skills, and structural change. If an individual invests in skills it is possible to be employed in the "advanced" (say, industrial) sector, which implies higher productivity and thus wages. However, attaining an education is costly, and credit markets are assumed imperfect. For the latter reason it is possible that certain segments of society (the poorest) are unable to invest in skills, due to its high cost. By way of contrast, sufficiently rich individuals will be able to invest. In between there is a critical level of income which acts as a barrier; below this critical level individuals do not invest, whereas they do if they have income above the boundary. The economy may well nest both types of households simultaneously.

Long run prosperity of society depends on the density of people in two possible long-run steady state equilibria; a non-skilled/low income equilibrium and a high skills/high income steady state. This is where the ambiguity comes into the picture.

Imagine a society where, on average, people have income below the critical level. Only few (the elite, of you wish) manage to pursue an education. In such a society, redistribution may be harmful, in that everyone in society may end up below the boundary. If so, long-run productivity suffers.

By contrast, in a richer society, redistribution may succeed in pulling more people above the boundary, to the benefit of long-run productivity.
At the lectures various other approaches (classical ("Keynesian savings"), socio-political instability, neighborhood choice etc) has been discussed. Full credit does not require the student to discuss these alternative theoretical approaches. Empirical evidence was also discussed. On balance the more compelling studies (from a methodological perspective; they deal with endogeniety of the distribution) tend to go against the statement in the question. Again, however, introducing such remarks would be great (and should make for a more positive over-all assessment of the exam), but is not required for full credit in the present question, taken in isolation.

A2. Discuss the problems associated with estimating the *impact* of fertility on growth, and how these problems may potentially be over come.

Readings: Li and Zhang for a relevant study of the impact of fertility on growth. The basic methodology (IV estimation) represent knowledge the students have acquired in previous courses. Their ability to use this knowledge, in the context of specific development issues, is what this question tests.

At the fundamental level the problem is that of disentangling the impact from fertility on prosperity, from the impact of prosperity on fertility.

Figure 1 illustrates the issue geometrically; the "solution" for a society is found in the intersection between the FF line (impact of income on fertility) and the YY (impact of fertility on growth) line. The case illustrated refers to a post-demographic transition society, in which fertility usually is decreasing in income per capita. The negative slope of the YY schedule can be motivated by the capital dilution effect of population growth, familiar from basic models of growth (Solow, Ramsey-Cass-Koopmans).

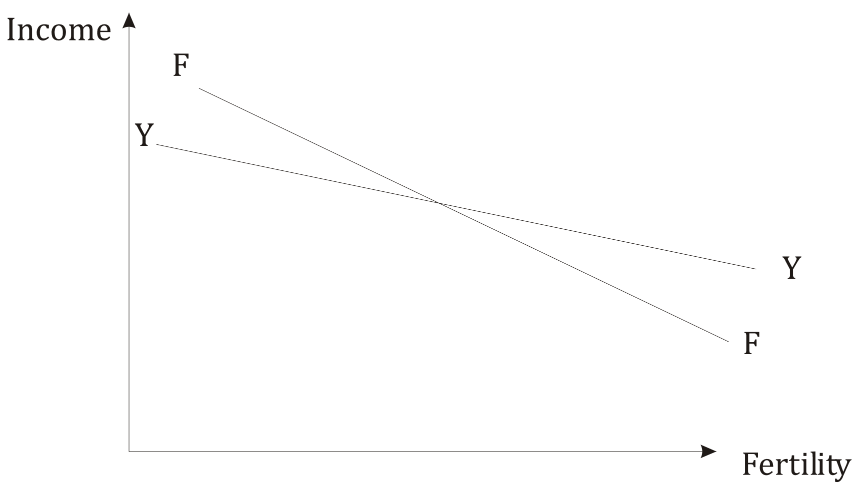


Figure 1. Income and Fertility within a country.

The objective is, in the present case, to identify the slope of the **YY line**. If we add "noise" to the system, we can gauge what the "data set" may look like if the "world" is described by the system depicted in figure 1. That is, many societies described by the same system, but where the "equilibrium" differs due to stochastic shocks (or, variation in other variables that impact either FF, YY or both). Figure 2 illustrates the situation

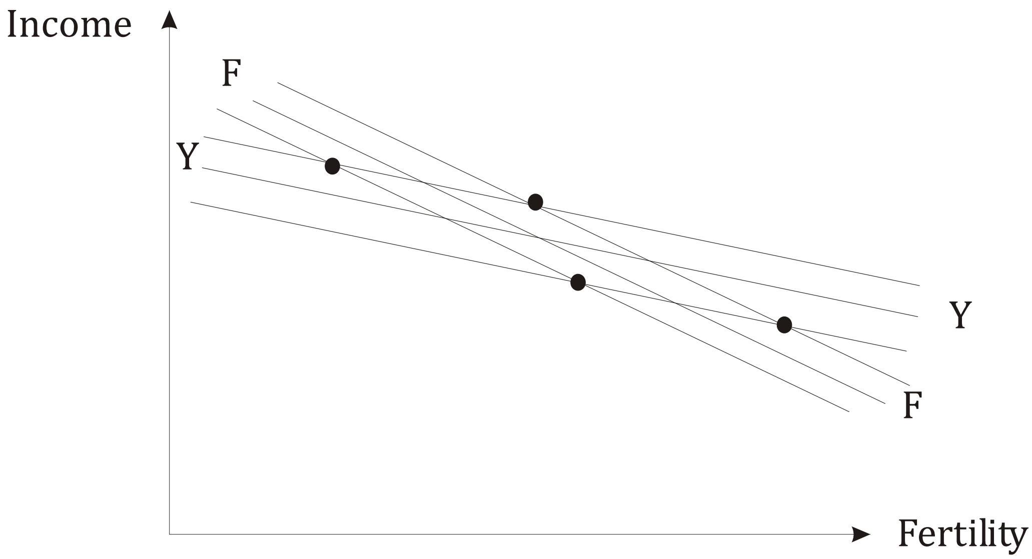


Figure 2. Income and Fertility: many countries.

If the “world” is characterized by many country specific equilibria, the data set visible to the research would fall within the diamond shaped area demarked by the black rings. If a researcher estimates the fertility/income nexus by OLS, the fitted line would tend to fall in between YY and FF. Accordingly, *OLS neither identifies the slope of YY or FF*.

A priori we don’t even know the direction of the bias, unless we a priori know that, say, YY is more steeply sloped than FF. In the figure above the OLS estimate would come out numerically smaller than the slope of the YY line; but that is just because the illustration assumes YY is less steep than FF.

A potential solution is the use of Instruments. Figure 3 illustrates the methodology.

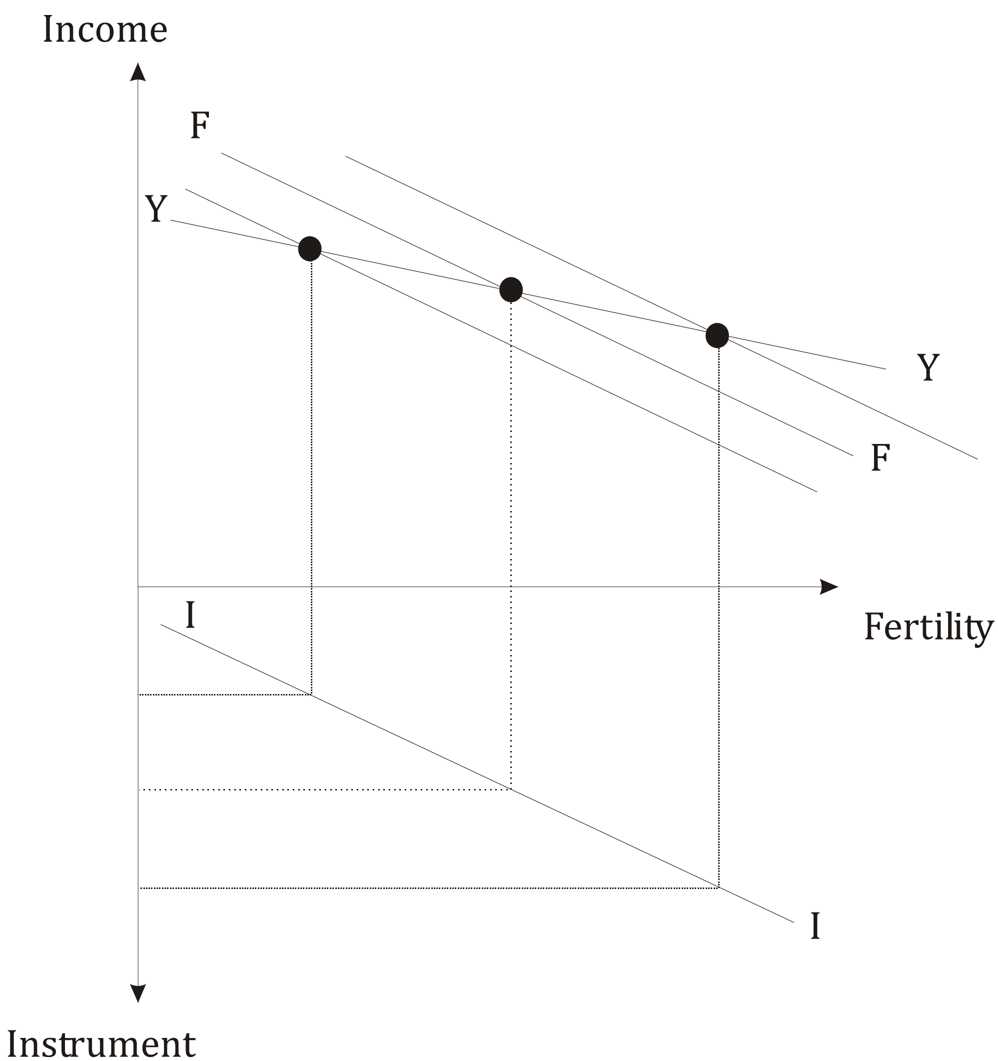


Figure 3. 2SLS estimation.

The approach, known as two stage least squares (2SLS) consists of first explaining the endogenous variable (here: fertility) by way of an alternative variable (here labeled "instrument"). Hence, stage one fits the II line in the figure above. Second stage uses the fitted values form the first stage to examine the hypothesis in question.

Under the assumption that our instrument *only* matters to fertility, but not YY, we can now identify the slope of the YY line, by employing the fitted values of fertility. In theory the researcher will be looking at data akin to that illustrated by the three dots., when examining the fitted values and prosperity. Again, *assuming* the instrument does not impact YY, these will all be ON the YY schedule. OLS estimates in the second stage therefore gives us the slope of YY.

In practice it is very difficult to obtain a valid instrument that fulfills the requirements outlined above; strongly correlated with the endogenous variable and ignorable in the second stage. Li and Zhang, examining China, does come up with a potentially valid

candidate. The one child policy in China only related to the so-called "Han Chinese"; the majority. Minority groups were exempt.
Now, if the one-child policy was a "binding constraint" the above fact may be useful in identifying the impact of fertility on growth.

Using data for Chinese administrative units, Li and Zhang show that areas with a larger share of the population made up of minority groups indeed has featured higher fertility over the last decades of the 20th century. This is consistent with non-Han Chinese having more children, because they were not "treated" by the one-child policy. Accordingly, the "minority share" is a potentially viable instrument for province level fertility; it shifts the FF curve around. The authors go on to estimate a substantial negative impact of fertility on growth.

One may worry about the identification strategy. The main worry concerns the exclusion restriction. If, for some reasons, provinces with more minority groups also grow more slowly, for reasons unrelated to fertility, the 2SLS procedure fails to give unbiased estimates. One example could be that the central government discriminates areas with more minority groups. If, by extension, this means fewer investments in (say) infrastructure, the minority share will tend to affect the location of the YY line. There is no way to test this issue, in the present case: there is only one instrument. Hence, it is a matter of faith whether one "believes" in the exclusion restriction.

Assignment B

(Based on Chakraborty, 2004**).**

B1 The profit maximization problem is

$$\max_{K,L} AK^{\alpha}L^{1-\alpha} - wL - \left(r+\delta\right)K \ ,$$

Straight forward differentiation yield the FOC: $r_t = \alpha A k_t^{\alpha-1} - \delta, w = \left(1-\alpha\right)Ak^{\alpha}{}_t$.

B2. Household's problem is

$\max_{c_t, c_{t+1}} u_t = \log\left(c_t\right) + \phi \log\left(c_{t+1}\right)$, s.t $w_t = s_t + c_t$, and $c_{t+1} = \left(1+\rho_{t+1}\right)s_t$.

Q i. The utility function reflects expected utility from life time consumption. ϕ reflects t
he survival probability; with probability 1- ϕ the agent dies, and therefore derives no utility from consumption.

Q ii. The simplest approach is to solve the problem via substitution; this leaves us with

$$\max_{s_t} u_t = \log\left(w_t - s_t\right) + \phi \log\left(\left(1+\rho_{t+1}\right)s_t\right)$$

The first order condition is

$$\frac{1}{w_t - s_t} = \phi \frac{1}{s_t} \Rightarrow s_t = \frac{\phi}{1+\phi} w_t .$$

Qiii. The parameter ϕ reflects the survival probability. Hence, the higher the probability of survival the more the individual saves. This captures in a simple way the basic idea that mortality patterns influences the incentive to save, as lower mortality increases the planning horizon.

B3. Law of motion for capital.

As $k_{t+1}=s_t$, and the labor force is normalized to one, it follows immediately upon substitution for savings that

$$k_{t+1}=\frac{\phi}{1+\phi}(1-\alpha)Ak_t^{\alpha}\equiv\psi(k_t),$$

where the FOC from the representative firm also has been used.

B4. Steady state analysis.

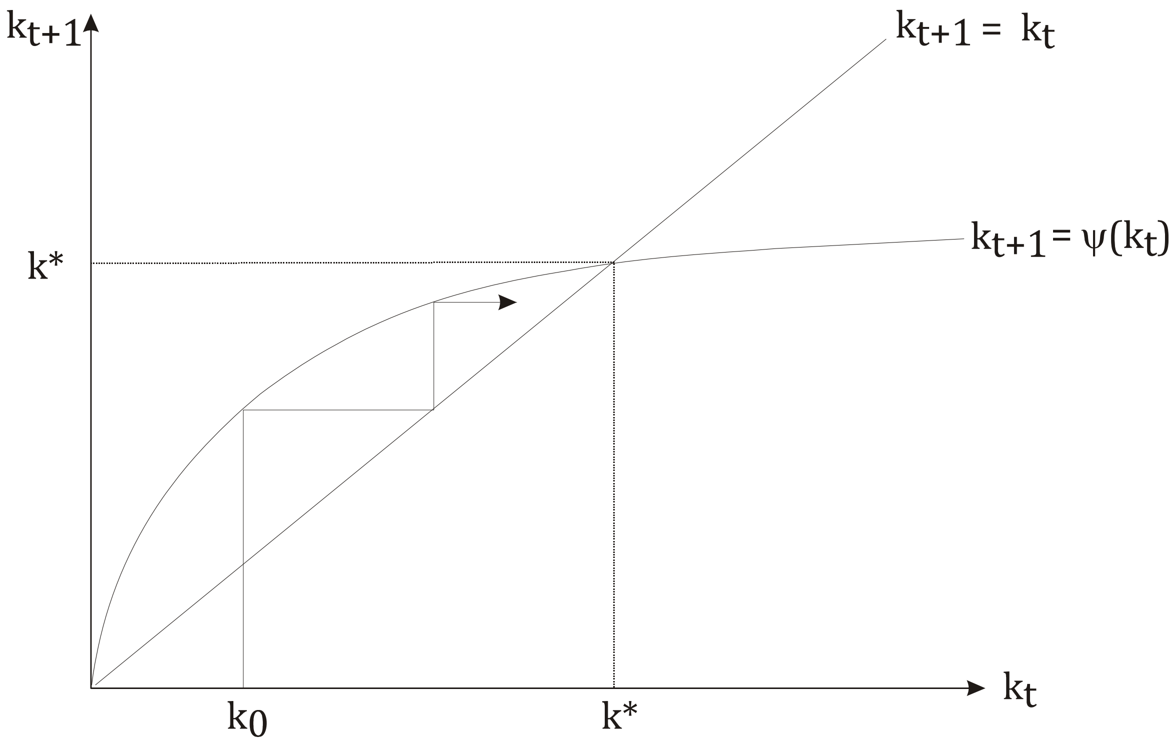
We find immediately that the following properties of Ψ(k) are fulfilled:

$$\psi(0)=0,\psi'(k)>0,\psi''(k)<0,$$
$$\lim_{k\to 0}\psi'=\infty$$
$$\lim_{k\to \infty}\psi'=0$$

These properties ensures that the law of motion only crosses the 45 degree line once. Hence, a unique steady state exist. Employing the steady state definition, $k_{t+1}=k_t=k^*$, in the law of motion and rearranging gives the steady state level of k.

$$k^*=\left[\frac{\phi(1-\alpha)A}{1+\phi}\right]^{\frac{1}{1-\alpha}}$$

The phase diagram of the model is illustrated in the figure below



It is plain to see from the figure that no matter where the economy starts out (as long as k0 is positive), the economy will converge toward k*. The steady state is globally stable.

(ii) Increased mortality is captured by a decrease in ϕ (lowered survival rate to period 2). It is clear from the steady state expression that this will work to lower k*. The intuition is simple: as mortality goes up consumers attach less weight on future consumption, effectively speaking. Consequently, savings are reduced, and thus capital accumulation leading to a lower k in the long run.

(iii) Here the student can discuss material broadly on health (Shastry and Weil) as well as econometric work by Acemoglu and Johnson. SW argue health capital is a substantial determinant of cross-country income differences. AJ, however, find that increases in life expectancy does not raise income per capita.

B5. Law of motion for capital.

The two new elements is that taxes are raised on wage income, and that the survival rate is increasing in health investments that are financed via taxation.

The period 1 budget constraint is therefore modified to:

$w_t\left(1-\tau\right)=s_t+c_t$, and so the solution for savings become

$$s_t = \frac{\phi_t}{1+\phi_t}(1-\tau)w_t.$$

Using the FOC from the firm, along with knowledge of ϕ(.), and h, we obtain

$$k_{t+1} = \sigma(k_t)(1-\tau)(1-\alpha)Ak_t^{\alpha} \equiv \psi(k_t),$$

$$\sigma(k_t) \equiv \frac{\phi(\tau(1-\alpha)Ak_t)}{1+\phi[\tau(1-\alpha)Ak_t]}$$

If α<1/2 the following properties of *ψ(k)* hold true:

$\psi(0) = 0, \psi'(k) > 0$ for all k,
$\lim_{k\to 0} \psi'(k) = \infty,$
$\lim_{k\to \infty} \psi'(k) < 1.$

B6. **Steady state analysis**. (i) α<1/2 is reasonable; under standard assumption the parameter maps into capitals' share in national accounts, which generally tend to fall in a fairly bounded interval between 1/3 and 0.4.

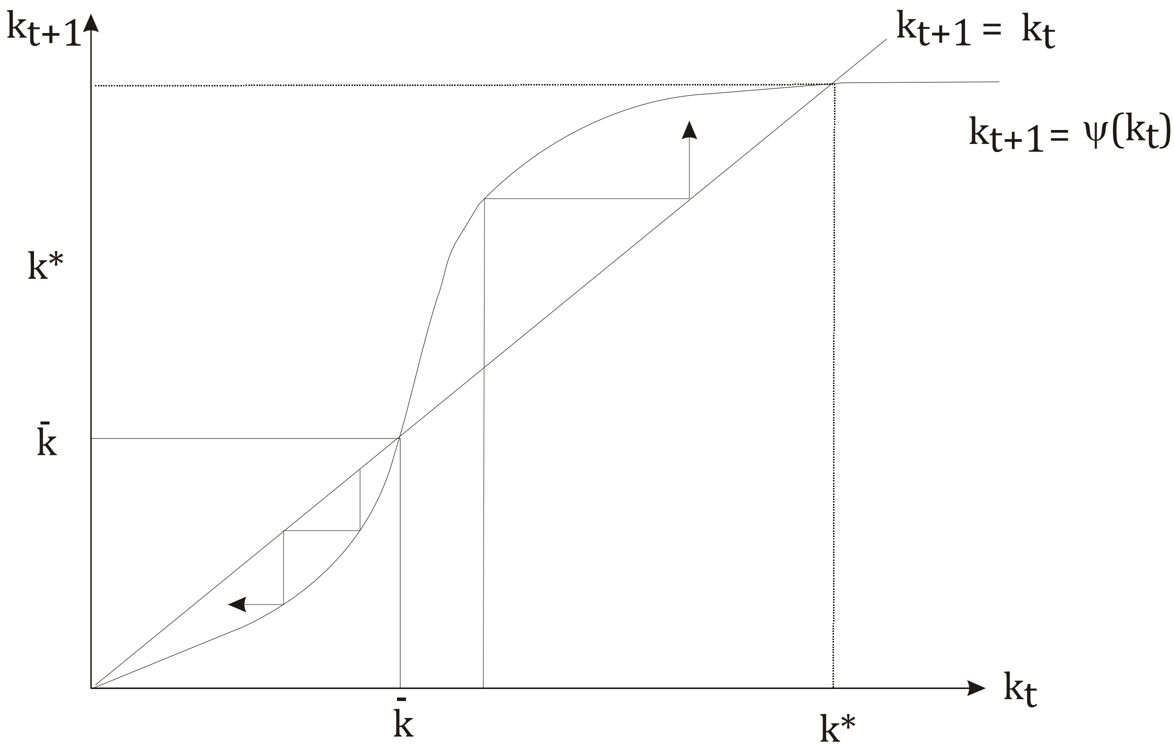
Given the information listed above, we can draw the phase diagram. Visually it is identical to the one illustrated above: a unique steady state exists and it is clearly stable.

B7. Discussion.

The mortality transition refers to the secular decline in mortality which tends to precede the fertility transition (which involves a declining fertility rate). Due to the timing (which seems to hold true at each repetition of the transitions - together referred to as the demographic transition), the decline in mortality is the cause of the "hump shaped" path taken by population growth during the demographic transition

The model is broadly consistent with the mortality transition in the sense that during transition to steady state the mortality rate clearly declines, insofar as the capital stock is converging towards the steady state from below. This is becomes evident if one examines ϕ(.), which is an increasing function of h, and thereby k.

B8. In the event α>1/2 it can be shown that $\lim_{k\to 0} \psi'(k) = 0$. As a result the model may allow for multiple equilibria, as illustrated below



Hence, $k > \bar{k}$ the economy will converge towards k*. If $k < \bar{k}$ the economy will shrink over time.

(ii) The key difference in terms of policy implications is this. With multiple steady state equilibria a temporary capital transfer may have permanent effects. If, for instance, the economy initially is located in the region $k < \bar{k}$, but then receives a transfer (e.g., foreign aid) so that $k > \bar{k}$ the level of income will be permanently higher.

In the case with a unique steady state only permanent policy changes can instigate permanent effects.