Written Exam for the B.Sc. in Economics summer 2013

Macro C

Final Exam

6 August

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total including this page.

The exam consists of the parts: A, B and C. All three parts should be answered.

A) **Short question:** Explain whether or not the following statement is true:

Following a monetary expansion the exchange rate always overshoots in the short run in the Dornbusch model

B) **The Blanchard model**

In the following we will consider a slightly modified version of the Blanchard model. Compared to the usual formulation aggregate demand is also assumed to depend on the interest rate and a constant term has been included in equation (B.4). All variables follow usual notation:

$$y(t) = z + \eta \cdot Q(t) - \beta \cdot p(t) - \epsilon \cdot r(t) \tag{B.1}$$

$$\frac{D(t) + \dot{Q}(t)}{Q(t)} = r(t) \tag{B.2}$$

$$r(t) = r^f \tag{B.3}$$

$$D(t) = \bar{\phi} + \alpha \cdot y(t) \tag{B.4}$$

$$\dot{p}(t) = \gamma \cdot (y(t) - \bar{y}) \tag{B.5}$$

At each point in time $p(t)$ is predetermined while $Q(t)$ is free to jump. You are informed that (B.2) and (B.3) can be combined with the transversality condition (which states that: $\lim_{T\to\infty} Q(T) \cdot e^{-r^f \cdot T} = 0$) to yield:

$$Q(t) = \int_{s=t}^{\infty} D(s) \cdot e^{-r^f \cdot (s-t)} ds \tag{B.6}$$

All parameters are positive, α is less than one and we assume in all the following that:

$$r^f - \alpha \cdot \eta > 0 \tag{B.7}$$

1) Interpret briefly each of equations (B.1) – (B.6), and explain in particular why it is reasonable to assume that $\epsilon > 0$. Show that the dynamics can be described by the two differential equations (B.8) and (B.9) (in addition to the transversality condition).

$$\dot{p}(t) = \gamma \cdot \left(z + \eta \cdot Q(t) - \beta \cdot p(t) - \epsilon \cdot r^f - \bar{y}\right) \quad \text{(B.8)}$$

$$\dot{Q}(t) = \left(r^f - \alpha \cdot \eta\right) \cdot Q(t) + \alpha \cdot \beta \cdot p(t) - \bar{\phi} - \alpha \cdot \left(z - \epsilon \cdot r^f\right) \quad \text{(B.9)}$$

2) Construct the phase diagram and comment. *Hint:* Start by showing that:

$$\dot{p}(t) = 0 \Rightarrow Q(t) = \frac{\epsilon \cdot r^f + \bar{y} - z + \beta \cdot p(t)}{\eta}$$

$$\dot{Q}(t) = 0 \Rightarrow Q(t) = \frac{\bar{\phi} + \alpha \cdot \left(z - \epsilon \cdot r^f\right) - \alpha \cdot \beta \cdot p(t)}{r^f - \alpha \cdot \eta}$$

3) Consider now a permanent fall in $\bar{\phi}$ (e.g. due to a political legislation which reduces the ability of firms to pay out dividends to shareholders). Use the phase diagram to analyze the consequences (when it is assumed that the economy starts out in the initial steady state). Explain the effects carefully.

Now let's instead consider an increase in r^f. For convenience let's assume that the magnitude of the effect on the long run (steady state) value of $Q(t)$ is the same as in 3). Once again the economy starts out in the initial steady state.

4) Use the phase diagram to analyze the consequences. Compare the short run effects with the short run effects in 3).

C) Higher population growth in the Ramsey model

Consider the following version of the Ramsey model, without a public sector. For simplicity we ignore technological growth. All notation is as usual.

The problem of the representative household is to choose a consumption path, $\left(c(t)\right)_{t=0}^{\infty}$, in order to maximize total discounted lifetime utility given by:

$$U(0) = \int_0^{\infty} \frac{c(t)^{1-\theta} - 1}{1 - \theta} \cdot e^{-(\rho - n) \cdot t} dt \quad \text{(C.1)}$$

subject to the intertemporal budget constraint, which can be written as a combination of a differential equation:

$$\dot{a}(t) = (r(t) - n) \cdot a(t) + w(t) - c(t) \quad \text{(C.2)}$$

and a condition limiting the asymptotic evolution of $a(t)$ (the No Ponzi game condition).

1) Show that the optimal growth rate of consumption is given by the differential equation (C.3) and interpret this.

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\theta} \tag{C.3}$$

Profit maximization by firms (with access to a constant returns production technology) implies:

$$R(t) = f'\big(k(t)\big) \tag{C.4}$$

$$w(t) = f\big(k(t)\big) - k(t) \cdot f'\big(k(t)\big) \tag{C.5}$$

where it is assumed that: $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$ and that the Inada conditions are satisfied.

Finally, the general equilibrium is also characterized by the following two relations:

$$R(t) - \delta = r(t) \tag{C.6}$$

$$a(t) = k(t) \tag{C.7}$$

At each point in time $k(t)$ is predetermined while $c(t)$ is free to jump. It is assumed that $\rho > n > 0$.

2) Interpret each of equations (C.4)-(C.7). Show that they together with (C.2) imply (C.8) and interpret this equation:

$$\dot{k}(t) = f\big(k(t)\big) - c(t) - (n + \delta) \cdot k(t) \tag{C.8}$$

3) Construct the phase diagram. Comment.

Now let's consider an increase in the population growth rate (n). Assume at first that the increase is unanticipated (and that the economy starts out in the initial steady state).

4) Use the phase diagram to analyze the consequences. Interpret carefully.

Now, assume instead that all the agents in the economy become aware at time t_0 that n will increase at time $t_1 > t_0$. The economy is in steady state up until t_0.

5) Use the phase diagram to analyze the consequences. Interpret carefully.