Written Exam for the B.Sc. or M.Sc. in Economics winter 2012-2013

Family economics

Final Exam/ Elective Course/ Master's Course

January 8, 2013

(3-hour closed book exam)

Suggested answers

Important: Please note that ALL questions should be answered!

General note: The students do not necessarily have to answer the questions using the mathematical model formulations below, but they need to be familiar with and explain in words the main ideas in the mathematical formulations and the assumptions behind the models and their general results.

1. **Gains from marriage**

a) Consider a two-person model with person *a* and *b* having incomes y_a and y_b, respectively. There are two goods, a private good, *q*, and a public good, *Q*.
Outline the gains from marriage from the sharing of public goods. In doing so, you may compare the situation when the two persons live as singles versus the situation where they are married.

Answer:
Reference: Browning, Chiappori & Weiss (2011), ch. 2.1.
Let the felicity (private utility) functions be given by $u^s(Q, q^s)$ for $s = a, b$.
This formulation of the utility function implies no caring for the other partners welfare.
For convenience, normalize prices of q and Q to unity.
If the two agents live apart*:*
Each individual s solves

$$\max_{Q,q^s} u^s(Q, q^s)$$

$$\text{subject to } Q + q^s = y^s$$

Let the optimal choices be $(\hat{Q}^s, \hat{q}^s)$*, respectively.*
If the two agents live together:
They can pool their income, and their joint budget constraint is then:

$$Q + q^a + q^b = y^a + y^b$$

Under the (important) assumption that both partners' preferences are increasing in the level of the public good, they will always be better off by living together as one can find feasible allocations that Pareto dominate the case where both live separately.
For example: Suppose that person a has a higher preference for public consumption Q than person b, so person a prefers a higher level of Q than person b (in the single situation), i.e. $\hat{Q}^a > \hat{Q}^b$.
The couple can then simple choose a's preferred level of $Q = \hat{Q}^a$*, and give b* $q^b = \hat{q}^b$.
Now, person b is at least as content as when single (given increasing utility in all goods), and if a is given same level of consumption as before, $\hat{q}^a + \hat{Q}^a$*, a is just as content. The amount that b initially spent on public goods,* $\hat{Q}^b$*, is available to share among the two partners, a and b. No matter how this amount is spent and shared, both partners are at least as well off as in single situation, so marriage is a Pareto improvement to the couple compared to the situation of being singles.*
Thus a couple can always ***replicate*** *the private consumption of the two partners, purchase the maximal amount of each public good that the partners bought as singles and still have some income left over to spend.*
The assumption of increasing utility in the public good is crucial for this result to hold in general.

Examples where this may not be the case (one partner may like, the other dislike): Living close to high-way or music-club, having many (too many) cable-tv channels, heating.

b) Focus now on a situation where person a and b can consume two public goods, Q_1 and Q_2, and there are no private goods.
Explain why a possible gain from marriage from the sharing of public goods requires that there is some overlap in a's and b's preferences. You may illustrate graphically.

Answer:
Reference: Browning, Chiappori & Weiss (2011), ch. 2.1.
For convenience, we can assume that they have equal incomes, $y^a = y^b$.
Figure below illustrates two different cases. u^a and u^b reflect the indifference curves for person a and b, respectively, i.e. combinations of the two public goods Q_1 and Q_2 for which a and b are indifferent.

***If they are both single**, they will choose combinations of Q_1 and Q_2 where their budget line is tangent to their respective indifference curves.*

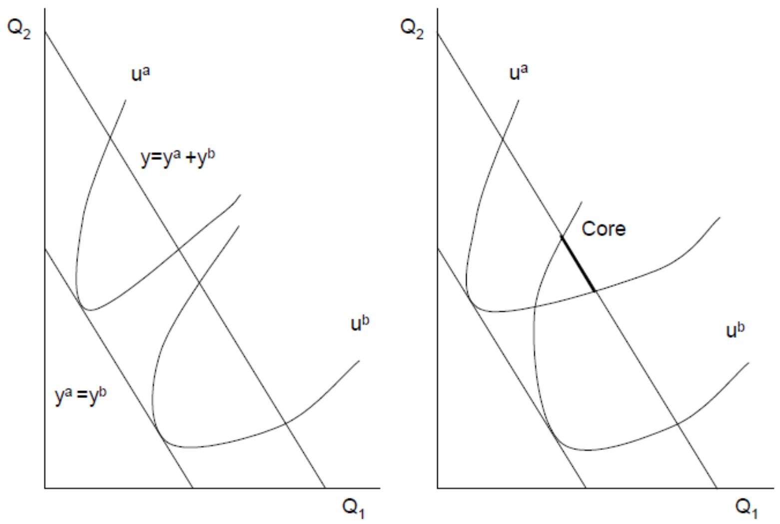


FIGURE 2.1. Preferences over two public goods

***If they marry**, they pool their resources and have total income $y^a + y^b$.*
*In the situation in the **left panel** in the figure above, a's and b's preferences over the two public goods are very different, and the two indifference curves do not intersect anywhere within their combined budget constraint. Thus marriage does not lead to higher welfare for any of the partners.*
*In the **right panel**, the two indifference curves intersect within the joint budget constraint. Thus combinations of Q_1 and Q_2 in the area where the interiors of the two indifference curves overlap (and below the joint budget line) represent Pareto improvements compared to the single situation. Combinations of Q_1 and Q_2 along the budget line above that area (i.e.*

along the "core") reflect optimal combinations of Q_1 and Q_2 *given the joint budget of person a and b.*
Hence, the example illustrates that, in general, some concordance of preferences (for public goods) is required to generate gains from marriage.
Examples of public goods in marriage where overlap in preferences could be important are demand for children and demand for e.g. housing. If the two partners have very different preferences for such public goods, their indifference curves may not intersect below their joint budget line.

c) Discuss the concept of public goods. Give examples of public goods in marriage. Discuss in what way public goods enter individual utility functions of *a* and *b*.

Answer:
Reference: Browning, Chiappori & Weiss (2011), ch. 2.1.
Some consumption goods of a family are public (non-rival), and both partners (and potentially other family members, e.g. children) can consume them equally. That they are non-rival implies that the fact that one person consumes the good does exclude another household member from enjoying the good also. Family members can have different preferences over public goods (as in the examples above).
Obvious examples of public goods include children, housing, heating, electricity, telephone, tv, internet etc.
Private goods include food, clothing etc.
Many goods are partly private, partly public. For example, the family may share a car, but each member's use of the car may exclude another family members simultaneous use of the car.
Housing may also be partly private. For example, more household members implies extra demand for space, and some members may e.g. request a room of their own.

In some cases, the literature discusses the concept of assignable goods. The consumption of such goods can be explicitly assigned to a person using them in the household. Assignable goods are private goods, but not all private goods in the household may be assignable. For example we may know total consumption of food in the household. Food is a private good, but we may not be able to assign the consumption of food to specific family members.

d) In a two person context, outline the gains from specialisation from living together.

Answer:
Textual background: Browning, Chiappori & Weiss (2011) ch. 2.2 and Becker (1993), ch. 2. The students can choose ***either*** *of these textbook sources when explaining specialisation.*

<u>Becker</u> *considers optimal investment in two types of human capital,* H^1 and H^2, *which raise market wages and household productivity, respectively. Each person allocates time between the market and household sectors given a time constraint where total time is divided between time spent in market work* t_w *and time spent in household production* t_h*:*

$$t_w + t_h = t'$$

The model allows that there is an investment period for accumulating human capital, and after this period, individual consumption is stationary.

A single person uses fixed amount of time to maintain capital stock, remaining time between household and market to maximize aggregate consumption Z. In optimum, the marginal productivity is the same across all time uses.
In a two-person household*, the household takes into account the skills of the different household members. The theory of comparative advantage implies that the resources of members of household should be allocated to various activities according to their comparative advantage or relative efficiencies.*
After the investment period*, we have that* ***if*** *all accumulate* ***same capital****, household product (in a wider sense) does not depend on distribution of hours between members. However,* ***if*** *capital differs across members, some household members are more productive than others, so output depends on* ***distribution*** *of hours.*
Marginal utilities of time spent in market and household, respectively, should be equaled only if both partners supply time to both sectors. However, it may be optimal that one partner uses all his/her time in one sector (is completely specialized), while the other may work in both sectors. For members who supply all their time to, e.g., the household sector, their marginal product in housework may exceed their marginal product in the market.

Assumptions: *At the beginning, everyone is identical. Differences in efficiency are not determined by biological or other intrinsic differences, but are the result of different experiences and investment in human capital. Similarly, goods supplied are perfect substitutes, even though the partners accumulate different types of human capital. The output of a multi-person household depends only on aggregate inputs of goods and effective time.*

Becker goes on to formulate a number of theorems which explain that:

1. *If different comparative advantages in household, no more than one member allocates time to both sectors.*
2. *If difference comparative advantages in household, no more than one member invests in both types of capital.*
3. *No more than one member would invest in both types of capital and allocate time to both types of time use.*
   "Proof": Assume contrary. E.g. both spent half their time in market.
   Output would not change if one spouse spent all the time in market, the other in household. But if both members also concentrated their human capital investments in each one sector, they could both improve output. ➔ Contradiction of assumption.

   Important: The above three theorems are completely independent on whether there are scale economies in the household:
4. *If there are* ***also*** *constant or increasing returns to scale, then in optimum,* ***all*** *members specialize completely in the market or household sectors, and invest accordingly.*
   "Proof": Assume that one member of an n-person household spends time in both sectors. If two of these households combine resources, one member can supply total time of one product and also invest more in one sector, hence increasing output. So two households can pool resources and then obtain more than before, due to specialization.

Students may discuss the implications of specialization for male and female wages and labor market attachment. One may also discuss the role of institutions as marriage contracts for *men's and women's willingness to specialize. Moreover, one may reflect on the life cycle development in the value of time for men and women given specialization in market work and housework, respectively.*

Browning et al. *(chapter 2.2) present a simple model to illustrate the gains from marriage due to specialization and increasing returns to scale.*
For a single person the household production function is:

$$z = x \cdot t$$

Where x: purchased good, t: time spent on production. This production function displays increasing returns to scale (IRTS). Expenditure on the market good is given by $x = w(1-t)$ *of a two-person household with two members, a and b.*
It is assumed that agents only derive utility from the amount of z consumed, so agents are indifferent between spending time in market work and housework.
If a person, s, is single, he/she chooses to divide time equally between market and household work, so $t^s = ½,\ z^s = w^s / 4$.
If two people, a and b, live together, we assume that household production is given by:

$$z = x(t^a + t^b)$$

The household budget constraint is then:

$$x = w^a(1-t^a) + w^b(1-t^b)$$

And aggregate output z in marriage is:

$$z = (t^a + t^b)(w^a(1 - t^a) + w^b(1 - t^b))$$

We assume that the output z is a private good to be shared in the household, and the household maximizes z under the budget constraint and the time constraints.
Replicating the time allocation situation from the singles state, $t^s = ½$*, aggregate output is* $z = (w^a + w^b)/2$ *which is larger (double) than simply adding output of the two singles.*
Thus, the marriage produces an additional output due to IRTS.
If wages differ in the marriage, e.g. $w^a > w^b$*, and the two partners specialize (completely) each in one sector so that* $t^a = 0, t^b = 1$ *, we find that total output is* $z = w^a$*, which is larger than total output in any of the two situations above. Thus specialization due to intra-household differences in productivities produces additional gains in marriage. Gains in marriage thus accrue from two sources:*

- *IRTS*
- *Specialization due to differences in productivities*

This result confirms Becker's theorems discussed above. Becker further shows that there are gains to specialization in marriage due to differential investment in human capital, also in situations without IRTS.

e) Discuss possible gains from marriage related to risk sharing, imperfect credit markets and coordination of child care.

Answer:
Textbook: Browning, Chiappori and Weiss (2011), ch. 2.3-2.5.

Imperfect credit markets
The family (including marriage) can be seen as a way to offer (informal) credit within the family. One example is investment in schooling. With perfect credit markets, both partners

can borrow for schooling. However, if credit markets are imperfect (one may not be able to borrow for schooling), couples can extend credit within marriage, so that one person goes to school, and the other person works.
Students may explain using the two-period model in BCW section 2.3 in which two potential partners, a and b, who both live in two periods, consider marriage.
If capital markets are perfect, people can borrow for education and thus smooth consumption over time. Any person can then either work or go to school in period 1 and can work in period 2. The cost of education is earnings lost in period 1. Investment in education is profitable if the increase in wages in period 2 is sufficient to compensate for lost earnings in period 1.
If there are imperfect capital markets, the two people can marry and choose to support education of one of the spouses while the other partner is working. One partner works in period 1, while the other partner goes to school. Both partners work in period 2. The couple shares total income among them.
BCW choose a specific functional form for the utility function as e.g. $u(c_t) = lnc_t$*, assuming a wage per period of 1 if just working without having attended school, and a second-period wage w if a person has attended school in period 1. They further assume a discount factor of unity and a real rate of zero.*
One can then show that:
- *With perfect capital markets, a person will choose schooling in period 1 if w>2, irrespective of being married or not.*
- *With imperfect capital markets, being married, the couple will find it efficient to send one of the persons to school in period 1 if w>3.*

Important points:
- *Marriage can - partially - overcome credit constraints and hence imply higher demands for returns to schooling.*
- *Disadvantages: Only one person goes to school (at a time), so there will be less schooling than when capital markets are perfect. Individuals who are ex-ante identical may voluntarily agree to pursue different careers. Also, the requirement for choosing schooling with imperfect capital markets is higher, as the wage required to choose schooling for one of the spouses is higher than in the situation with perfect capital markets.*
- *Commitments are crucial for this solution due to risk of marriage break-up. Therefore, one has previously observed divorce settlements influenced by arrangements where one spouse supported the other spouse through education.*

<u>*Risk sharing*</u>
The family can be seen as a mechanism for providing mutual insurance between family members. The idea is that when family members pool resources, they compensate each other for welfare losses from idiosyncratic (transient) income shocks due to e.g. unemployment or sickness. In order to experience gains from this risk sharing facility, it is important that their incomes are not perfectly correlated – or that risk aversions differ.
Examples of such risk sharing are found in the empirical literature, e.g. in Rosenzweig and Stark (1989) who show that marriages in rural India are arranged between partners who come from distant regions to reduce the correlation in rainfall.

Children

Students can answer this question in various ways, and reference to a specific model is not required.

Production and rearing of children is a central gain from marriage, see both Becker (e.g. chapter 5) and BCW (e.g. chapter 2).

One option is to discuss parental transfers of human capital and other capital to children, taking e.g. Becker as a starting point.

Another option is to refer to the model in BCW chapter 2.5 in which children derive utility from time spent with their parents and from consumption. Both parents' utility depends on the utility of their child. In the model, parents only spend time with the children if they live together, and the two parents have different productivities in childcare and market work. These model features have implications for:

- *The allocation of the parents' time between market work and childcare, depending on parental productivities in market work and childcare*
- *The utility of the child and its parents after a possible divorce, depending also on who gets custody to the child*

2. Family decision making

a) Consider a model of a household with a woman (*a*) and a man (*b*). The two partners consume a public good, *Q*, and a private good, *q*. Present the household utility function. Outline the main features of the collective model of household decision making.

Answer:

Reference: Browning, Chiappori & Weiss (2011), ch. 3.5.

The household's problem can be formulated as below:

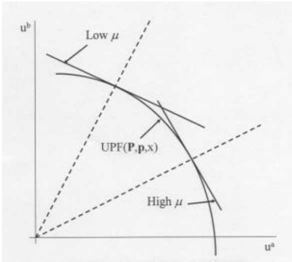
$$\max_{Q,q^a,q^b} \mu u^a(Q,q^a,q^b) + u^b(Q,q^a,q^b)$$

s.t. budget constraint

μ: Pareto weight for *a*

$\mu = \mu(P,p,x,z)$

So households maximize a weighted sum of the two partners' utilities, u^a *and* u^b. *In this very general formulation, both spouses have utility over their own private good, their partner's privately consumed good, and a public good, Q. Individual utility functions may also be egotistic, e.g.* $U^a(Q,q^a)$ *and* $U^b(Q,q^b)$*The Pareto weight μ attaches a weight to person a's utility relative to b's utility in the household's total utility. The Pareto weight is a function of prices P and p of public and private goods, respectively. One may characterize* μ *as a measure of a's "power" in the household. A high value of* μ *implies that the household attaches a high weight to person a's utility and vice-versa, see figure below. The utility possibility frontier (UPF) shows combinations of utilities for person a and b for a set of prices and income (P,p,x). The couple chooses combinations of* u^a *and* u^b *in order to maximize household utility. This is achieved where the Pareto weight is tangent to the UPF.*



If μ does indeed vary with z, the socalled "distribution factors", we can rule out unitary decision making in the family. Thus the family does not act as one unit.

b) Discuss the characteristics of the collective model compared with two other main types of household models, namely the unitary model and non-cooperative models.

Answer:
Reference: Browning, Chiappori & Weiss (2011), ch. 3.
There are three main types of models describing household decision making: 1) the unitary model, 2) non-cooperative models, and 3) the collective model.

1) *The unitary model* *basically assumes that the household acts as one decision maker. We assume that choices are made according to a unitary household utility function which may be formulated in the following way:*

$$U(Q,q^a,q^b)=W(U^a(Q,q^a,q^b),U^b(Q,q^a,q^b))$$

W is a utility weighting function which is strictly increasing in individual utilities. Thus conflicting interests among the members of the family are ruled out.
Implications:

a. *Demand functions satisfy Slutsky conditions: adding-up, homogeneity, symmetry and negativity (students may shortly explain Slutsky conditions which are 3rd year microeconomics curriculum).*
b. *Only depend on prices and total income, but independent on distribution of income within couple.*

➔ *Demand in a unitary setting displays income pooling.*

2) *Non-cooperative models: Over the last couple of decades, there has been a growing emphasis on the perception that household members may have conflicting interests and that households may not always act as one unit. Alternatively, one may specify the family decision process through a non-cooperative model which assumes:*

a. *No binding agreements*
b. *Optimal decisions may not necessarily be Pareto efficient.*

Demand in a non-cooperative model does not necessarily satisfy income pooling or the Slutsky conditions. Income pooling can be the case when:

- *There are no public goods and the two partners have linear Engel curves with the same slope (equal budget shares) for any good, see BCW 3.4 (students do not have to explain this special case).*
- *There is one public good, and both partners contribute to the public good.*

Thus the cooperative model does not necessarily imply income pooling, but income pooling is not ruled out in a non-cooperative setting.

3) *<u>The collective model</u> suggests an alternative approach to household decision making when compared with both the unitary model and non-cooperative models.*
   In contrast to the unitary model, the collective model does not imply that the household acts as one decision maker. Household members may have different preferences and conflicting interests.
   The collective model belongs to a class of cooperative models. It thus contrasts non-cooperative models which generally often lead to inefficient outcomes.
   In a household context, one may, however, expect that household members act in a cooperative manner. The partners know each other's preferences, and as marriages are usually expected to last for a longer period, they will be interested in finding cooperative solutions and exploit possibilities of Pareto improvements. In the collective model, household decisions are Pareto efficient, so no other feasible choice would have been preferred by all household members.

There are a number of possibilities for testing for the unitary model versus alternative specifications of household decision making. First, one may test whether distribution factors (see question 2.c) affect the intra-household allocation. One important distribution factor is relative income in the household. A common test is the test for income pooling (see question 2.e). Secondly, one may test whether the Slutsky conditions (see question 2.b) hold. In the collective model setup, the Slutsky conditions are not fulfilled. Students may explain how the Slutsky matrix can be decomposed into two parts containing the conventional Slutsky matrix (for the unitary model) and a second term which reflects that individual demand may be affected by relative price changes due to in the intra-household distribution of resources.

c) Discuss the role of distribution factors in the collective model. Give examples of distribution factors.

Answer:
Distribution factors are variables that affect the decision process through their effect on the Pareto weight. Distributions factors do not directly affect preferences or the budget constraint. These include:

- *Divorce legislation, factors affecting the marriage market. Potential effect as risk of divorce may affect decision process.*
- *Individuals' income, wealth, education, age etc. For any given level of Y, the individual income of a relative to total income, can only affect the outcome through its impact on the decision process*

Examples (which are often used in the literature) are:

- *Relative income,* Y_a/Y_b
- *Relative wage rate,* w_a/w_b
- *Relative unearned income,* nl_a/nl_b
- *Relative age,* age_a/age_b
- *Relative education,* $educ_a/educ_b$
- *Local sex ratio (proportion of men to women or opposite)*
- *Background family factors*
- *Control of land*
- *Divorce laws*
- *Alimonies*
- *Single parent benefits*

d) The two partners in the household can have egotistic preferences over their own goods or they may have some form of altruistic preferences over their partner's consumption, e.g. *caring* preferences.
Discuss the concept of caring preferences and how such preferences may be reflected in individual utilities and in household utility in a collective model setting.

Answer:
Egotistic preferences implies that the two partners only care about their own consumption of private goods and public goods.

$$U^a(Q,q^a) \text{ and } U^b(Q,q^b)$$

The partner's private consumption may also enter directly into a person's utility function:

$$U^a(Q,q^a,q^b) \text{ and } U^b(Q,q^a,q^b)$$

This very general formulation may include situation in which a and b are altruistic towards each other – in this case, q^b enters with a positive sign in U^a and vice versa. Or private consumption goods of one spouse may affect utility of the other spouse in a negative manner, i.e. in the form of externalities as e.g. smoking.
A popular form taken in the literature is the situation where utility functions include "caring" for the spouse's welfare. For this form, the two spouses do not care directly about their partner's consumption, but only indirectly as they care for whether goods consumed by the partner gives their partner pleasure. Assume that the two partners have felicity functions $u^a(Q,q^a)$ and $u^b(Q,q^b)$ over their own private good and the public good in marriage. Each partner can then also enjoy utility over the partner's utility:

$$U^a(Q,q^a,q^b) = W^a(u^a(Q,q^a),u^b(Q,q^b))$$
$$U^b(Q,q^a,q^b) = W^b(u^a(Q,q^a),u^b(Q,q^b))$$

where W^a, W^b monotone and increasing.

So partner a does not care about b's allocation between Q and q^b, she only care about the utility b enjoys from his preferred mix of goods. This rules out paternalism (maternalism) and externalities.

$$U^a(Q,q^a,q^b) = u^a(Q,q^a) + \delta^a u^b(Q,q^b)$$
$$U^b(Q,q^a,q^b) = u^b(Q,q^b) + \delta^b u^a(Q,q^a))$$
$$\delta^a, \delta^b \geq 0$$

With caring preferences, a higher value of e.g. δ^b may look like person a has more power in the household (see BCW section 3.5.7).

e) Explain in what way the test for income pooling can serve as a test of the unitary model versus alternative models of household decision making.

Answer:
This question extends question 2.b).
The test for income pooling may serve as a test for or against the unitary model. Households display income pooling if the purchase and allocation of goods in the household is only affected by total income, but not affected by the intra-household distribution of income. Thus

sharing of goods (and leisure) within the household is not affected by whom of the partners earned the income.
On the one hand, if income pooling is <u>not</u> fulfilled, we can reject the unitary model. This suggests that another (non-unitary) model for household decision making would be more appropriate.
A rejection of the unitary model based on the income pooling test does not point to a specific alternative model as e.g. a non-cooperative model or the collective model.
On the other hand, if income pooling is fulfilled, the unitary model can be accepted as an appropriate description of household decision making. But income pooling is also possible in other settings as e.g. non-cooperative models under certain circumstances.