Macro III - exam solutions (February 8, 2016)

General remarks

Please grade each item of each question between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

1 False. In the Calvo model, only a fraction of firms are assumed to be able to change their prices in each period (or instant, in the original continuous-time version). This is the origin of aggregate price stickiness in this model.

2 False or uncertain. An aging population would require either that benefits be cut while contributions remain constant, or benefits remain constant and contributions increase, or a mix of benefit cuts and contribution increases. It is through the political process that society chooses the way in which a social security system adapts to aging. And if benefits are not to be reduced, it is not necessary to give workers incentives to increase private savings.

3 True. A distortionary capital income tax discourages saving by reducing its net rate of return. Thus after the introduction of the tax there will be dissaving (relative to the situation before the tax is introduced, since the argument follows even when initially capital in the economy is below the new steady state level). The desire to smooth consumption will make consumers to adjust at the announcement and not wait for the tax to be introduced.

4 a) In the budget constraint m appears multiplied by the nominal interest rate, $\pi + r$. Thus, this is the interest forgone by holding money instead of capital. It thus measures the implicit consumption of money services.

Control: c and m, state: a, costate, μ (I use current value Hamiltonian, H^c, obviously student gets full points if using present value Hamiltonian correctly). Hamiltonian is

$$H_t = e^{-\rho t} H_t^c = e^{-\rho t} \left[\ln c_t + \ln m_t - \gamma m_t + \mu_t \left(r_t a_t + w_t + z_t - (c_t + (\pi_t + r_t) m_t)\right)\right]$$

FOC (with no time indexes):

$$\begin{aligned}
\frac{1}{c} &= \mu \\
\frac{1}{m} - \gamma &= \mu(\pi + r) \\
\dot{\mu} - \rho\mu &= -r\mu \\
\lim_{t\to\infty} a_t \mu_t e^{-\rho t} &= 0
\end{aligned}$$

In equilibrium $z = \sigma m = 0$ (since money supply is constant). Steady state characterized by $\dot{a}_t = \dot{m}_t = \dot{\mu}_t = 0$. From FOC this gives $r = \alpha k^{\alpha-1} = \rho$ and thus steady state capital stock, $k^* = \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}}$. From $\dot{m}_t = 0$, $\pi = \sigma = 0$. From $\dot{a}_t = 0$ this gives $c^* = w^* + rk^* = (k^*)^\alpha$, which does not depend on m, these results are standard. m^* comes from $\frac{1}{m^*} - \gamma = \frac{1}{c^*}\rho$.

Yes, money is superneutral since if the government would contemplate changes in σ this would have no effect on k^* or c^*, since these do not depend on m^*. Optimal quantity of money is obtained when the marginal utility of real money balances is zero, i.e. $r + \pi = \sigma + \rho = 0$. Since $\rho > 0$ this would require $\sigma = -\rho < 0$. Thus the current steady state does not feature the optimal quantity of money.

b) The economy instantaneously goes from $M = H + D$ to $M' = D < M$. This shock has no effect on the dynamic budget constant, no effect on the Hamiltonian, and therefore no effect on the first order conditions.

Since money is superneutral we know that this shock has no effect on steady state real variables, c^*, k^*. Steady state real money balances are also unaffected. But since $m^* = \frac{M}{P} = \frac{M'}{P'}$, the shock has an effect on the price level (which in the original steady state was constant, since $\pi = 0$). The new price level with be given by

$$P' = P\frac{M'}{M} = P\frac{D}{H+D} < P.$$

There is no transition, the changes take place instantaneously. The intuition for this is that this is an economy with flexible prices and there is a sudden change in the level of money. Thus the price level instantaneously jumps to the new steady state level.

If the government preannouced this policy the analysis on real variables is the same as money is superneutral. But the desire to smooth the utility services from money holdings will drive down the price level on announcement (but not all the way down to P') and inflation will be negative from that moment until currency is retired. Negative inflation leads to real money holdings being larger than steady state during the transition. [The following is not required from students: Note that the initial jump in prices and the path of inflation are pinned down by the integral of inflation plus the initial jump being equal to P', and by the real money holdings satisfying the first order condition $\frac{1}{m} - \gamma = \mu(\pi + \rho)$ for initial money M during the transition. This requires inflation to be increasingly negative and then jumping to zero when currency is retired (since real money holdings will be increasing first and jump down to m^* when currency is retired).]

c) This shock still has no effect on the dynamic budget constant, but since it affects the utility of holding money, it will affect the Hamiltonian and the first order conditions. Since this is trivially done by having $\gamma' < \gamma$ I omit writing these. But the student is expected to point out that the FOC with respect to m changes.

Again, since money is superneutral, there are no effects on steady state real variables, c^*, k^*. Since the shock now affects the marginal utility of real money balances there will be an effect on m^*. This will now be determined by

$$\frac{1}{m^{*'}} = \gamma' + \frac{\rho}{c^*}.$$

Since $\gamma' < \gamma$, $m^{*'} > m^*$. The new price level is determined as in b) with the caveat that since now real money balances also change, we only can get an expression involving $m^{*'}$ (and not the original price level as in b)). Thus

$$P' = \frac{D}{m^{*'}}.$$

We can be sure that since real money holdings increase, the equilibrium price level will be smaller than the one found in b), i.e. $\frac{D}{m^{*'}} < \frac{D}{m^*} = P\frac{D}{H+D}$.

5 a) Since rigid-flex firms behave as rigid firms in some states of nature and as flexible firms in others, after the observation of m, we can treat this economy as one in which there are only two types of prices, rigid and flexible. Depending on the realization of m, the fraction of rigid prices will be α_1 (when $(m - E(m))^2 > \sigma^2$, we call these states H), or $\alpha_1 + \alpha_2$ (when $(m - E(m))^2 \leq \sigma^2$, we call these states L). To simplify notation we call α the ex post fraction of fixed prices in the economy. Thus the price level ex post will be

$$p = \alpha p^r + (1 - \alpha)p^f. \tag{1}$$

Importantly when making pricing decisions, rigid and rigid-flex firms expect α to take one of two values, and let's denote by θ the probability of states H, i.e. the probability that $\alpha = \alpha_1$. Prices are set according to the following equations

$$\begin{aligned} p^f &= E[p_i^*|m] = (1-\phi)p + \phi m && (2) \\ p^r &= E[p_i^*] = (1-\phi)E[p] + \phi E[m] && (3) \end{aligned}$$

Substituting (1) into (2)

$$p^f = (1-\phi)E[p|m] + \phi m = (1-\phi)\big(\alpha p^r + (1-\alpha)p^f\big) + \phi m \quad \Leftrightarrow$$

$$p^f\big(1-(1-\phi)(1-\alpha)\big) = (1-\phi)\alpha p^r + \phi m \quad \Leftrightarrow$$

$$p^f = \frac{(1-\phi)\alpha p^r + \phi m}{1-(1-\phi)(1-\alpha)} = \frac{(1-\phi)\alpha p^r + \phi m}{\phi + (1-\phi)\alpha} = \frac{\big[(1-\phi)\alpha + \phi\big]p^r - \phi p^r + \phi m}{\phi + (1-\phi)\alpha}$$

$$p^f = p^r + (m - p^r)\frac{\phi}{\phi + (1-\phi)\alpha} \qquad (4)$$

Substituting (1) into (3) and using above expression (4) for p^f

$$p^r = (1-\phi)E[p] + \phi E[m] = (1-\phi)E[\alpha p^r + (1-\alpha)p^f] + \phi E[m]$$

Thus, and using $E[\alpha] = \alpha_1 + (1-\theta)\alpha_2$, and $E[m|L] = E[m|H] = E[m]$,

$$\begin{aligned} p^r &= (1-\phi)p^r(\alpha_1 + (1-\theta)\alpha_2) + (1-\phi)\left((1-\theta)(1-\alpha_1-\alpha_2)\left[p^r + \frac{\phi(E[m|L]-p^r)}{\phi + (1-\phi)(\alpha_1+\alpha_2)}\right]\right. \\ &+ \left.\theta(1-\alpha_1)\left[p^r + \frac{\phi(E[m|H]-p^r)}{\phi+(1-\phi)\alpha_1}\right]\right) + \phi E[m] \\ &= (1-\phi)p^r(\alpha_1 + (1-\theta)\alpha_2) + (1-\phi)\left((1-\theta)(1-\alpha_1-\alpha_2)\left[p^r + \frac{\phi(E[m]-p^r)}{\phi + (1-\phi)(\alpha_1+\alpha_2)}\right]\right. \\ &+ \left.\theta(1-\alpha_1)\left[p^r + \frac{\phi(E[m]-p^r)}{\phi+(1-\phi)\alpha_1}\right]\right) + \phi E[m] \\ &= (1-\phi)p^r + (1-\phi)(E[m]-p^r)\left(\frac{(1-\theta)(1-\alpha_1-\alpha_2)\phi}{\phi+(1-\phi)(\alpha_1+\alpha_2)} + \frac{\theta(1-\alpha_1)\phi}{\phi+(1-\phi)\alpha_1}\right) + \phi E[m] \\ \Leftrightarrow & \\ &- \phi(E[m]-p^r) = (1-\phi)(E[m]-p^r)\left(\frac{(1-\theta)(1-\alpha)\phi}{\phi+(1-\phi)\alpha} + \frac{\theta(1-\alpha(1-\alpha))\phi}{\phi+(1-\phi)\alpha(1-\alpha)}\right) \end{aligned}$$

This can only be satisfied if

$$p^r = E[m]. \qquad (5)$$

Note that an alternative, more straightforward solution strategy starts by assuming $p^r =$

$E[m]$, replacing this in (4), and then verifying the assumption (which in this case boils down to proving that $E[p^f] = E[m]$.

b) Substituting (4) and (5) into (1)

$$p = \alpha p^r + (1-\alpha)p^f = \alpha p^r + (1-\alpha)p^r + (m - p^r)\frac{\phi(1-\alpha)}{\phi + (1-\phi)\alpha} = p^r + (m - p^r)\frac{\phi(1-\alpha)}{\phi + (1-\phi)\alpha}$$
$$= E[m] + (m - E[m])\frac{\phi(1-\alpha)}{\phi + (1-\phi)\alpha} \tag{6}$$

Since $y = m - p$, substituting (6) into this relation

$$\begin{aligned} y &= m - E[m] - (m - E[m])\frac{\phi(1-\alpha)}{\phi + (1-\phi)\alpha} = (m - E[m])\left(1 - \frac{\phi(1-\alpha)}{\phi + (1-\phi)\alpha}\right) \\ &= (m - E[m])\left(\frac{\phi + (1-\phi)\alpha - \phi(1-\alpha)}{\phi + (1-\phi)\alpha}\right) = (m - E[m])\left(\frac{\alpha}{\phi + (1-\phi)\alpha}\right) \end{aligned} \tag{7}$$

From this equation we see that only unanticipated changes in m have an effect on output. The reason for this is that output is given by the difference between m and the price level, and the latter fully reflects anticipated changes in m (changes in $E[m]$). This is expected in a static model where money is neutral.

c) This requires looking at (7), in particular

$$\frac{dy}{dm} = \frac{\alpha}{\phi + (1-\phi)\alpha}$$

This is higher the higher α, which happens when the economy is in an L state, such that $\alpha = \alpha_1 + \alpha_2$.

To see the effect of an increase in α_2 note that this will only impact $\frac{dy}{dm}$ in L states. As we saw, in that case $\alpha = \alpha_1 + \alpha_2$, thus an increase in α_2 makes output more responsive to unanticipated changes in m.

Since the response of output to demand shocks depends on the degree of nominal rigidities, then a higher rigidity (higher α_2), or situations when less prices change (when shocks are small, such that rigid-flex firms do not adjust their prices), lead to a stronger response of output to unanticipated shocks.