Written Exam for the B.Sc. / M.Sc. in Economics 2009-II

Corporate Finance and Incentives

Elective Course/ Master's Course

June 17, 2009

(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different subquestions that do not necessarily have equal weight.
Please provide intermediate calculations.

Eksamen består af 4 opgaver. Alle opgaver skal besvares. For hver delopgave er den vejledende vægt i den samlede bedømmelse angivet. En opgave kan bestå af flere delspørgsmål, der ikke nødvendigvis har samme vægt.
Vis venligst mellemregninger.

Problem 1 (Fixed Income, 30%)
Assume a 10 year 8.00% bullet bond with semi annual coupons paid out (settled) on June 15 and December 15. Use a 30/360 day count convention for computing interest.

1. Describe briefly the relation between the "clean price" and the "dirty price" of a coupon paying bond.

2. The clean price in a bond trade which is settled on November 10 is 96.000 (therefore interest is also owed for this day). What is the (full) dirty price?

Assume now a bond market consisting of only 3 risk free bonds, all with annual coupon payments and a face value of DKK 500. A 2 year 2.00% serial bond priced at 485, a 1 year 5.50% bullet bond priced at 501 and a 3 year 5.75% annuity priced at 515. The annuity has annual cash flows of 37.238% of face value.

3. Compute the annual total cash flow payments for each bond.

4. Calculate the corresponding 1, 2 and 3 year zero-coupon bond discount rates and zero coupon bond yields.

(Hint: we can express the entire bond market as an equation system $\pi = Cd$, where π is a bond price vector, C is the cash flow matrix and d is the zero-coupon bond discount rate.)

Calculate the two 1 year zero-coupon forward rates.

5. How is Macaulay duration defined and what are the two interpretations of the duration measure?

Assume from now on a flat term structure and a yield to maturity on the 3 year annuity bond of 2.15%.

6. Calculate the Macaulay duration and the modified duration on the 3 year annuity bond.

7. Calculate the convexity of the 3 year annuity bond

8. What does the found convexity measure indicate about the annuity bond's interest rate sensitivity? What does it tell us about the estimation error that will occur from simply using the modified duration measure as an estimator of bond price changes from interest rate movements?

Problem 2 (Real Investments, 20%)
Assume a firm producing soft drinks at an annual production cost of 100 million. The risk free rate is 5%. The firm may choose between investing two mutually exclusive types of projects that may serve to temporarily reduce production costs.

Project	**Lifetime**	**Initial investment outlay**	**Subsequent annual production costs**
A	4	100	50
B	8	50	75

1. What is the net present value (NPV) to the firm of investing in project A and B respectively?

2. What two methods may be used by an investor with long term horizon to compare the attractiveness of two mutually exclusive types of investment projects? Describe each method briefly.

3. Assume the firm has an 8-year investment horizon and that project A can be replicated immediately at the end of the 4th year. How should a profit maximizing firm invest if it has no budget restrictions? Calculate the corresponding net present value (NPV). (brug LCM metode)

Assume an investor considering the following list of investment projects:

Project	**Cost**	**Profitability Index (PI)**
C	-1200	4,17
D	-450	3,34
E	-1000	4,80
F	-150	2,68
G	-800	3,76

4. Calculate the present value (PV) of each project.

5. Assume you have a budget constraint of 2150. What investments would maximize the investors profit (NPV) given his budget constraint?

6. Why would you not necessarily choose the investments with the top Profitability Index scores?

Problem 3 (Options, 20%)
Consider an economy where a non-dividend paying stock is trading at €50 and the continuous compounded risk-free rate is 10%. A zero-cost forward with the stock as the underlying asset is traded in the market. The forward is settled in one year.

1. Estimate the zero-cost forward price of the forward (hint: use continuous compounding)

Assume a European call option on the stock is traded in the market and that the stock satisfies the Black-Scholes assumptions. The option expires in one year and has a strike price of €40 and the volatility of the stock's log return is 20%.

2. Estimate the price of the European call option.
3. Estimate the price of a comparable European put option.
4. Comment briefly on how the price of the European call would be related to the price of the European put if their strike price was similar to the zero-cost forward price in question 1.
5. Comment on the possibility of arbitrage if another European call option with identical expiration is trading with another implied volatility.

Now assume that the underlying asset with certainty pays a dividend of €10 such that the ex-dividend date is just prior to expiration of the European call option.

6. Estimate the price of the European call option on the dividend paying stock.

Problem 4 (Essay questions, 30%)

1. Discuss the delta and gamma of options.
2. What are the implications of the Modigliani-Miller theorem on the firm's capital structure?
3. Discuss whether a firm's cost of capital is identical to the required rate of return of a tax exempt investor's portfolio which only consists of the firm's securities. Assume that the mix of the portfolio is identical to the firm's debt-to-equity ratio.

Fixed Income

$\pi = Cd$

$$y(0,t) = \left(\frac{1}{d_t}\right)^{\frac{1}{t}} - 1 = r_t$$

$$yield\ to\ maturity, y\ solves\colon\ \ \pi = \sum_{t=1}^{T} \frac{c_t}{(1+y)^t}$$

	c_t	i_t	δ_t
	payment	interest	deduction of principal
Annuity	$F\alpha^{-1}_{\tau\rceil R}$	$R\frac{F}{\alpha_{\tau\rceil R}}\alpha_{\tau-t+1\rceil R}$	$\frac{F}{\alpha_{\tau\rceil R}}(1 - R\alpha_{\tau-t+1\rceil R})$
Bullet	RF for $t < \tau$ $(1+R)F$ for $t = \tau$	RF	0 for $t < \tau$ F for $t = \tau$
Serial	$\frac{F}{\tau} + R\left(F - \frac{t-1}{\tau}F\right)$	$R\left(F - \frac{t-1}{\tau}F\right)$	$\frac{F}{\tau}$

$$D(c;r) = \sum_{t=1}^{T} t w_t$$

$$K(c;r) = \sum_{t=1}^{T} t^2 w_t$$

$$w_t = \frac{c_t}{(1+r)^t}\frac{1}{PV(c;r)}$$

Mean-Variance Optimization, CAPM, APT and Factor Models

$$E(\tilde{r}_i) = \sum_{s=1}^{S} q_s \times \tilde{r}_{i,s} = \bar{r}_i$$

$$Var(\tilde{r}_i) = E[(\tilde{r}_i - E(\tilde{r}_i))^2] = \sum_{s=1}^{S} q_s \times (\tilde{r}_{i,s} - \bar{r}_i)^2 = \sigma_i^2$$

$$Cov(\tilde{r}_1, \tilde{r}_2) = E\big[\big(\tilde{r}_1 - E(\tilde{r}_1)\big)(\tilde{r}_2 - E(\tilde{r}_2))\big] = \sigma_{12}$$

$$\rho_{12} = \frac{Cov(\tilde{r}_1, \tilde{r}_2)}{\sigma_1\sigma_2}$$

$$Var\big(\widetilde{R_P}\big) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j\, \sigma_{ij}$$

$$\beta_i = \frac{Cov(\widetilde{r_i}, \widetilde{R_T})}{Var(\widetilde{R_T})}$$

$$\widetilde{r_i} = \alpha_i + \beta_{i1}\widetilde{F_1} + \beta_{i2}\widetilde{F_2} + \cdots + \beta_{iK}\widetilde{F_K} + \tilde{\varepsilon}_i, \quad Factor\ Model$$

$$\widetilde{r_i} = r_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \cdots + \beta_{iK}\lambda_K + \tilde{\varepsilon}_i, \quad APT\ Model$$

Derivatives

$$F_0 = S_0(1 + r_f)^T$$

$$f = (F_0 - K)(1 + r_f)^{-T}$$

$$c_0 - p_0 = S_0 - PV(K)$$

$$S_0 - K \leq C_0 - P_0 \leq S_0 - PV(K)$$

$$u = e^{\sigma\sqrt{(T/N)}}$$

$$d = \frac{1}{u}$$

$$\pi = \frac{1 + r_f - d}{u - d}$$

$$c_0 = S_0 N(d_1) - PV(K)N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Real Investments:

$$E = P \times n$$

$$EV = D + E$$

$$NI = EBT(1 - T_C)$$

$$CE(\tilde{C}) = E(\tilde{C}) - b(\bar{R}_T - r_f)$$

Cumulative Normal Distribution											
d	*N(d)*	*d*	*N(d)*	*d*	*N(d)*	*d*	*N(d)*	*d*	*N(d)*	*d*	*N(d)*
-3,00	0,0013	-1,58	0,0571	-0,76	0,2236	0,06	0,5239	0,86	0,8051	1,66	0,9515
-2,95	0,0016	-1,56	0,0594	-0,74	0,2296	0,08	0,5319	0,88	0,8106	1,68	0,9535
-2,90	0,0019	-1,54	0,0618	-0,72	0,2358	0,10	0,5398	0,90	0,8159	1,70	0,9554
-2,85	0,0022	-1,52	0,0643	-0,70	0,2420	0,12	0,5478	0,92	0,8212	1,72	0,9573
-2,80	0,0026	-1,50	0,0668	-0,68	0,2483	0,14	0,5557	0,94	0,8264	1,74	0,9591
-2,75	0,0030	-1,48	0,0694	-0,66	0,2546	0,16	0,5636	0,96	0,8315	1,76	0,9608
-2,70	0,0035	-1,46	0,0721	-0,64	0,2611	0,18	0,5714	0,98	0,8365	1,78	0,9625
-2,65	0,0040	-1,44	0,0749	-0,62	0,2676	0,20	0,5793	1,00	0,8413	1,80	0,9641
-2,60	0,0047	-1,42	0,0778	-0,60	0,2743	0,22	0,5871	1,02	0,8461	1,82	0,9656
-2,55	0,0054	-1,40	0,0808	-0,58	0,2810	0,24	0,5948	1,04	0,8508	1,84	0,9671
-2,50	0,0062	-1,38	0,0838	-0,56	0,2877	0,26	0,6026	1,06	0,8554	1,86	0,9686
-2,45	0,0071	-1,36	0,0869	-0,54	0,2946	0,28	0,6103	1,08	0,8599	1,88	0,9699
-2,40	0,0082	-1,34	0,0901	-0,52	0,3015	0,30	0,6179	1,10	0,8643	1,90	0,9713
-2,35	0,0094	-1,32	0,0934	-0,50	0,3085	0,32	0,6255	1,12	0,8686	1,92	0,9726
-2,30	0,0107	-1,30	0,0968	-0,48	0,3156	0,34	0,6331	1,14	0,8729	1,94	0,9738
-2,25	0,0122	-1,28	0,1003	-0,46	0,3228	0,36	0,6406	1,16	0,8770	1,96	0,9750
-2,20	0,0139	-1,26	0,1038	-0,44	0,3300	0,38	0,6480	1,18	0,8810	1,98	0,9761
-2,15	0,0158	-1,24	0,1075	-0,42	0,3372	0,40	0,6554	1,20	0,8849	2,00	0,9772
-2,10	0,0179	-1,22	0,1112	-0,40	0,3446	0,42	0,6628	1,22	0,8888	2,05	0,9798
-2,05	0,0202	-1,20	0,1151	-0,38	0,3520	0,44	0,6700	1,24	0,8925	2,10	0,9821
-2,00	0,0228	-1,18	0,1190	-0,36	0,3594	0,46	0,6772	1,26	0,8962	2,15	0,9842
-1,98	0,0239	-1,16	0,1230	-0,34	0,3669	0,48	0,6844	1,28	0,8997	2,20	0,9861
-1,96	0,0250	-1,14	0,1271	-0,32	0,3745	0,50	0,6915	1,30	0,9032	2,25	0,9878
-1,94	0,0262	-1,12	0,1314	-0,30	0,3821	0,52	0,6985	1,32	0,9066	2,30	0,9893
-1,92	0,0274	-1,10	0,1357	-0,28	0,3897	0,54	0,7054	1,34	0,9099	2,35	0,9906
-1,90	0,0287	-1,08	0,1401	-0,26	0,3974	0,56	0,7123	1,36	0,9131	2,40	0,9918
-1,88	0,0301	-1,06	0,1446	-0,24	0,4052	0,58	0,7190	1,38	0,9162	2,45	0,9929
-1,86	0,0314	-1,04	0,1492	-0,22	0,4129	0,60	0,7257	1,40	0,9192	2,50	0,9938
-1,84	0,0329	-1,02	0,1539	-0,20	0,4207	0,62	0,7324	1,42	0,9222	2,55	0,9946
-1,82	0,0344	-1,00	0,1587	-0,18	0,4286	0,64	0,7389	1,44	0,9251	2,60	0,9953
-1,80	0,0359	-0,98	0,1635	-0,16	0,4364	0,66	0,7454	1,46	0,9279	2,65	0,9960
-1,78	0,0375	-0,96	0,1685	-0,14	0,4443	0,68	0,7517	1,48	0,9306	2,70	0,9965
-1,76	0,0392	-0,94	0,1736	-0,12	0,4522	0,70	0,7580	1,50	0,9332	2,75	0,9970
-1,74	0,0409	-0,92	0,1788	-0,10	0,4602	0,72	0,7642	1,52	0,9357	2,80	0,9974
-1,72	0,0427	-0,90	0,1841	-0,08	0,4681	0,74	0,7704	1,54	0,9382	2,85	0,9978
-1,70	0,0446	-0,88	0,1894	-0,06	0,4761	0,76	0,7764	1,56	0,9406	2,90	0,9981
-1,68	0,0465	-0,86	0,1949	-0,04	0,4840	0,78	0,7823	1,58	0,9429	2,95	0,9984
-1,66	0,0485	-0,84	0,2005	-0,02	0,4920	0,80	0,7881	1,60	0,9452	3,00	0,9987
-1,64	0,0505	-0,82	0,2061	0,00	0,5000	0,82	0,7939	1,62	0,9474	3,05	0,9989
-1,62	0,0526	-0,80	0,2119	0,02	0,5080	0,84	0,7995	1,64	0,9495		
-1,60	0,0548	-0,78	0,2177	0,04	0,5160						