Macro III - exam solutions (August 14, 2017)

1 True. Basic real business cycle models assume perfect competitive markets, and no externalities. Thus, the first welfare theorem holds and the decentralized competitive equilibrium is Pareto optimal. Therefore, government intervention can not increase welfare.

2 False. Increasing the size of a pay-as-you-go social security system requires higher taxes that, on one hand reduce the young's available income, but at the same time increase their expected future income when retired. Thus, the initial young want to borrow against their future income and the only way to do this is from the rest of the world. Therefore, there is an initial current account deficit.

3 True or uncertain. If an economy is initially in a steady state, then increasing lump sum taxes permanently will not affect the Euler equation that characterizes the optimal choice between current and future consumption. Thus, the representative household would respond by reducing consumption in every period by the amount in which taxes increase. Resources left for investment are therefore unchanged, i.e. there is no crowding out of investment.

4 a) Characterizing individual saving behavior requires setting up the problem of workers.

$$
\begin{aligned}
\max_{s_t, c_{1t}, c_{2t+1}} \quad & \ln(c_{1t}) + \beta \ln(c_{2t+1}) \\
\text{s.t.} \quad & c_{1t} = w_t(1-\tau) - s_t \\
& c_{2t+1} = (s_t + w_t\tau) r_{t+1}
\end{aligned}
$$

Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \beta r_{t+1} c_{1t} \tag{1}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{\beta}{1+\beta} w_t - \tau w_t \tag{2}$$

To get capital accumulation we note that since the social security system is fully-funded, capital comes from private plus public saving. Since there is no population growth, $k_{t+1} = s_t + \tau w_t$. Using equilibrium expressions for wage, $w = (1-\alpha)Ak^{\alpha}$,

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)Ak_t^{\alpha}$$

From here imposing steady state we get the following

$$k^* = \left[\frac{\beta}{1+\beta}(1-\alpha)A\right]^{\frac{1}{1-\alpha}}.$$

From equation (2) we see that if $\tau > \frac{\beta}{1+\beta}$ desired private savings would be negative. Since this is not possible, then in that case we would have a corner solution with $s_t = 0$. To avoid this case is that we impose the restriction $\tau < \frac{\beta}{1+\beta}$. Note that capital accumulation is then independent of τ.

b) The shock is such that in the first period the ratio of workers to retirees is $1+n$, and in all subsequent periods is 1. What is different in the setup is that migrants do not contribute, nor benefit, from social security. But, since the return of social security is the same as the return from private saving, capital accumulation is independent of τ as seen in a) above.

Capital accumulation in the first period is then only affected by immigrants reducing initial capital per worker from the steady state level to $\frac{k^*}{1+n}$ (note that since from next period onwards the ratio of workers to retirees is 1 there is no $\frac{1}{1+n}$ term even in the first period). This leads to

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)A\left(\frac{k^*}{1+n}\right)^{\alpha} < k^*$$

It is clear that in the long run capital per worker is the same as we found in a) above. The reason for this is that the only effect of immigration is to change population size and a fully social security system is irrelevant for capital accumulation.

c) Since the presence of immigrants increases the workforce for a given level of capital in the first period, this increases the interest rate. This makes the old to be strictly better off since their income sources are all from capital income. The disposable income of the young residents in the first period is given by $w_t(1-\tau) = (1-\alpha)A\left(\frac{k^*}{1+n}\right)^{\alpha}(1-\tau) < w^*(1-\tau)$. Thus, the disposable income of the initial young generation of residents is lower than in the initial steady state.

5 a) Under discretion, monetary policy is chosen ex post to minimize social loses, i.e.

$$\min_{\pi} L(\pi, x) = \frac{1}{2}\left[\pi^2 + \lambda(\theta + \pi - \pi^e - \epsilon - \bar{x})^2\right]$$

The first order condition is, denoting π^D the solution under discretion,

$$\pi^D + \lambda(\theta + \pi^D - \pi^e - \epsilon - \bar{x}) = 0 \quad \longrightarrow \quad \pi^D = \frac{\lambda}{1+\lambda}(\epsilon - \theta + \pi^e + \bar{x}) \tag{3}$$

Next we need to compute expected inflation knowing that rational expectations imply the private sector is aware of the above ex post incentive for monetary policy. Thus π^e is the mathematical expectation of (3) given θ:

$$\pi^e = E[\pi^D|\theta] = \frac{\lambda}{1+\lambda}(-\theta + \pi^e + \bar{x}) \quad \longrightarrow \quad \pi^e = \lambda(\bar{x} - \theta)$$

Substituting this into (3) we find equilibrium inflation

$$\pi^D = \frac{\lambda}{1+\lambda}\epsilon + \lambda(\bar{x} - \theta)$$

And equilibrium output is given by

$$x^D = \theta - \frac{1}{1+\lambda}\epsilon$$

Equilibrium does not depend on supply shocks in Europe since neither ϵ^{eu} nor π^{eu} affect the equations that characterize the Danish economy.

b) Under a credible peg of the krone to the euro, $\pi^e = 0$ and $\pi = \pi^{eu} = \frac{\lambda^{eu}}{1+\lambda^{eu}}\epsilon^{eu}$. Thus, equilibrium output is given by

$$x = \theta + \frac{\lambda^{eu}}{1+\lambda^{eu}}\epsilon^{eu} - \epsilon$$

To determine the conditions under which a peg is preferable one needs to calculate expected loses and compare them. When they are lower under a peg, it is preferable. Note that

$$\begin{aligned} E[L(\pi^D, x^D) &= \frac{1}{2}\left(E(\frac{\lambda}{1+\lambda}\epsilon + \lambda(\bar{x} - \theta))^2 + \lambda E(\theta - \frac{1}{1+\lambda}\epsilon - \bar{x})^2\right) \\ E[L(\pi^{eu}, x)] &= \frac{1}{2}\left(E(\frac{\lambda^{eu}}{1+\lambda^{eu}}\epsilon^{eu})^2 + \lambda E(\theta + \frac{\lambda^{eu}}{1+\lambda^{eu}}\epsilon^{eu} - \epsilon - \bar{x})^2\right) \end{aligned}$$

A peg is preferable when the loss from the inflation bias, $\lambda E(\bar{x}-\theta)^2$, is high, while discretion is preferable when volatility of supply shock are high, Europe inflation is very volatile, and λ is high.

c) When delegating monetary policy on a central banker with preferences given by parameter λ^B we know, from a) above, that equilibrium policy is given by

$$\begin{aligned}
\pi^B &= \frac{\lambda^B}{1+\lambda^B}\epsilon + \lambda^B(\bar{x}-\theta) \\
x^B &= \theta - \frac{1}{1+\lambda^B}\epsilon
\end{aligned}$$

The optimal choice of λ^B si the one that minimizes social expected loses

$$\begin{aligned}
\min_{\lambda^B} E[L(\pi^B, x^B) &= \frac{1}{2}\left(E\left(\frac{\lambda^B}{1+\lambda^B}\epsilon + \lambda^B(\bar{x}-\theta)\right)^2 + \lambda E\left(\theta - \frac{1}{1+\lambda^B}\epsilon - \bar{x}\right)^2\right) \\
&= \frac{1}{2}\left(\lambda^{B^2}(\bar{x}^2+\sigma_\theta^2) + \left(\frac{\lambda^B}{1+\lambda^B}\right)^2\sigma_\epsilon^2 + \lambda\left(\sigma_\theta^2 + \frac{1}{(1+\lambda^B)^2}\sigma_\epsilon^2 + \bar{x}^2\right)\right)
\end{aligned}$$

The first order condition is given by

$$\lambda^B(\bar{x}^2+\sigma_\theta^2) + \frac{\lambda^B}{1+\lambda^B}\frac{1}{(1+\lambda^B)^2}\sigma_\epsilon^2 - \lambda\frac{1}{(1+\lambda^B)^3}\sigma_\epsilon^2 = 0$$

Rewriting this gives

$$\lambda^B(\bar{x}^2+\sigma_\theta^2) + \frac{\lambda^B-\lambda}{(1+\lambda^B)^3}\sigma_\epsilon^2 = 0$$

First we note that $\lambda^B > 0$ since if $\lambda^B = 0$ the first order condition would be violated (it would be negative). This then implies that $\lambda^B(\bar{x}^2+\sigma_\theta^2) > 0$, which requires that $\frac{\lambda^B-\lambda}{(1+\lambda^B)^3}\sigma_\epsilon^2 < 0$. Therefore, $0 < \lambda^B < \lambda$.

It is never preferable to have a peg, since a peg with zero inflation would be equivalent to an independent central bank with $\lambda^B = 0$ and we found that $\lambda^B > 0$. [Furthermore, a peg would expose Denmark to fluctuations from supply shocks in Europe that would lead to a higher loss function than under a peg with zero inflation.]