Written Resit Exam for M.Sc. in Economics

Winter 2010/2011

Investment Theory

4. January 2011

Master course

3 hours written exam with closed books

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Exercise 1.

Consider an investment project. The investment cost is $I > 0$. If the project is started at date t, the investment cost is paid at date t and dividends are received from date t and forward. The dividend of the project at date t is either D_H or D_L where $D_H > 0 > D_L$. If the dividend is D_s at date $t-1$, then the dividend is D_s at date $t+1$ with probability π_s, where $1 > \pi_s > 0$, and D^r probability $1 - \pi_s$ for $r \neq s$ and $s \in \{H, L\}$. Suppose that the discount rate is $\rho > 0$

(a) Suppose the project is active. Calculate NPVs of the active project.

(b) Find the optimal investment strategy of the project. Find the value of the option to invest.

Suppose that the project can be temporary suspended for a cost $E > 0$ and reactived for a cost $R > 0$. If the project is suspended at date t, the suspension cost is paid at date t and no dividend is received from date t and forward. If the project is reactivated at date t, the reactivation cost is paid at date t and the dividend is received from date t and forward.

(c) Suppose the project is active. Consider a strategy where the project is suspended when the dividend is D_L and reactivated when the dividend is D_H. Calculate NPVs of the active project for the strategy.

(d) Suppose the project is active. Consider a strategy where the project is suspended when the dividend is D_L and never reactivated. Calculate NPVs of the active project for the strategy.

(e) Find the optimal suspension and reactivation strategies of the project.

(f) Find the optimal investment strategy of the project. Find the values of the option to invest.

(g) Suppose that $I = 500$, $D_H = 100$, $D_L = -25$, $\pi_H = 0.9$, $\pi_L = 0.8$, $\rho = 0.2$, $E = 25$ and $R = 50$. Find the optimal strategies for the project.

Exercise 2.

Consider an investment project. The investment cost is $I > 0$. The dividend of an active project follows a Geometric Brownian Motion

$$dP = \alpha P dt + \sigma P dz$$

An active project can die. Indeed with probability λdt, where $\lambda > 0$, the project dies in a time interval of length dt. There is a cost $T > 0$ of cleaning up the mess after a dead project. The discount rate is $\rho > 0$.

Let $F(P)$ be the value of the option to invest and $V(P)$ the value of an active project.

(a) Give an example of an investment project which fits the above project.

(b) State the possible cut-off strategy for the project.

(c) Show how to find differential equations in $F(P)$ and $V(P)$.

(d) Solve the differential equations in $F(P)$ and $V(P)$.

(e) Find $V(P)$. Give a short interpretation of your expression for $V(P)$.

(f) Find $F(P)$. Give a short interpretation of your expression for $F(P)$.

(g) Find the optimal strategy for the project.