Suggested solutions to the IO (BSc) exam on May 29, 2009
VERSION: June 21, 2009

Question 1

[To the external examiner: This question was taken from one of the problem sets, so the students had discussed it in class with the TA.]

a) The firm's problem: maximize its profits

$$\pi = (a - q_i)\, q_i - cq_i + sq_i = (a - c + s - q_i)\, q_i \tag{1}$$

with respect to q_i, subject to $q_i \geq 0$. Standard calculations yield

$$q^* = \frac{a - c + s}{2}. \tag{2}$$

This is positive, so the non-negativity constraint doesn't bind.

b) We can solve for the subgame-perfect Nash equilibrium by backward induction. We effectively solved the second-stage game in part a). Plugging (2) into (1) yields

$$\pi^* = (q^*)^2 = \frac{(a - c + s)^2}{4}.$$

Moreover, by standard arguments we have that consumer surplus given q^* is

$$CS = \frac{1}{2}\,(q^*)^2 = \frac{(a - c + s)^2}{8}.$$

Therefore, W given q^* equals

$$W = CS + n\pi^* - nsq^* = \frac{(a - c + s)^2}{8} + \frac{(a - c + s)^2}{4} - s\left[\frac{a - c + s}{2}\right]. \tag{3}$$

The government wants to maximize this expression w.r.t. s. Solving yields:

$$s^* = a - c.$$

This is positive, so the non-negativity constraint doesn't bind.

c) Modifying the expression in (3), we have

$$\begin{aligned} V &= CS + z\pi^* - sq^* = \frac{(1 + 2z)\,(10 + s)^2}{8} - \frac{4\,(10 + s)\, s}{8} \\ &= \frac{10 + s}{8}\,[(1 + 2z)\,(10 + s) - 4s] = \frac{(10 + s)\,[(1 + 2z)\,10 - (3 - 2z)\, s]}{8}. \end{aligned}$$

The government wants to maximize this expression w.r.t. s. The FOC:

$$[(1+2z)\,10-(3-2z)\,s]-(3-2z)\,(10+s)=0.$$

The SOC: $-2\,(3-2z)\,z<0$, which is always satisfied. Solving the FOC for s yields

$$s^{**}=\frac{10\,[(1+2z)-(3-2z)]}{2\,(3-2z)}=\frac{2z-1}{3-2z}10.$$

This is positive, so the non-negativity constraint doesn't bind.

- s^{**} is increasing in z; therefore, we have from (2) that

  $$q^{**}=\frac{10+s^{**}}{2}$$

  also in increasing in z. This in turn means that market price, $p^{**}=a-q^{**}$ is decreasing in z.
- The reason for this result:
  - If you care a lot about the firm's profit (full weight, $z=1$), then the subsidy doesn't cost you anything: what you pay out comes back with full weight in terms of profits for the firm. Therefore, you want to subsidize a lot in order to correct the monopolist's incentive to produce too little (this is an important point that was emphasized a lot in the course).
  - If the weight z is smaller, you still think the monopolist produces too little. However, now subsidizing is costly, as you don't get back as much as you pay out. Therefore you choose to subsidize less (i.e., s^{**} is increasing in z, which is consistent with the formula above).
  - Given that the subsidy is increasing in z, it is obvious that the market price is decreasing in z.
- The important thing with this question is that the students show that they can understand the logic of a model — that they are not just mechanically solving the first-order conditions etc without understanding what they're doing. It is also important that the students understand that the problem with a monopoly (or, more generally, with imperfect competition) is that there is too little production and trade, and that one therefore can improve welfare by subsidizing output — this relates to the discussion on page 68 in Tirole's text.

Question 2

a) One Nash equilibrium is that each firm charges a price that equals the marginal cost; that is $(p_1^*, p_2^*) = (8, 8)$. There are also a set of Nash equilibria $(p_1^*, p_2^*) = (k, k)$, for any $k > 10$. The students should of course prove their claims (the calculations that are needed are, at least for the first equilibrium, standard — see textbook or lecture notes).

- It suffices if a student has identified the equilibrium $(p_1^*, p_2^*) = (8, 8)$.

b) The **Bertrand Paradox**:

- According to the model, two firms are enough to eliminate all market power in a market.
- Our intuition and observation of real world markets would suggest otherwise.

c) *Three key assumptions of the Bertrand model are that (i) the game is played only once, (ii) the firms do not face any capacity constraints, and (iii) the product that the firms produce is homogeneous. Explain briefly, in words, how and why the result of the Bertrand model changes as we relax each one of the assumptions.* Each one of these assumptions can, if relaxed, restore market power to the firms.

(i) If the game is played repeatedly with an infinite horizon, then, provided the firms care sufficiently much about future profits, cooperation between the firms can be sustained as a subgame perfect Nash equilibrium. The idea is that if a firm were to deviate from the prescribed cooperative behavior (by cutting its price and thereby grab the whole market at the other firm's expense), it will be punished by the other firm in later periods (for example, the firms go back to playing the Nash equilibrium of the one-shot version of the game). If the firms care sufficiently much about the future, this threat of being punished will provide an incentive to stick to cooperation – the firms charge the same price and this is strictly above their marginal cost.

(ii) The proof of the result that the only Nash equilibrium involves marginal cost pricing exploits the fact that if some firm charged a price above marginal cost, then the other firm could undercut the first one slightly, thereby grabbing the whole market and a larger profit than if sharing the market. However, this argument requires that the firm that undercuts indeed is able to serve the whole market. If it were not, because it faces a capacity constraint, it is not clear whether it would payoff to slightly undercut the rival and it seems plausible that a Nash equilibrium in which the firms charge a price above marginal cost may exist. Indeed, one can show in simple examples that, at least for the case with relatively small capacities, such an equilibrium does exist (and looks exactly like a Cournot equilibrium, with the capacities being the quantities in the Cournot game).

(iii) If the firms produce goods that are not identical but at least somewhat differentiated, then a firm that undercuts its rival will not gain the whole market — the firm with the higher price will still sell at least something, as the consumers value that particular firm's good and therefore is willing to pay at least a little bit extra to get it. This behavior on the part of the consumers lower the firms' incentives to undercut each other, thereby making it possible to sustain a Nash equilibrium in which the firms charge a price above marginal cost.

d) *Explain briefly the conjectural-variations approach to modelling an oligopoly.*

- The idea is to assume that (in, say, a duopoly) the firms believe that a change in one firm's output leads to a change in the rival's output, even though the firms' choices are otherwise modelled as being simultaneous. The degree to which the rival's output changes is captured by a parameter, the conjectural variations parameter. This parameter is typically assumed to be constant (and often also identical across firms). As this parameter takes various values, the outcome of the model (the equilibrium quantities) can be made identical to, for example, the outcome under Cournot or Bertrand competition or the collusive outcome. The approach is therefore used as a reduced-form way of capturing a family of different models with different degrees of competition.

e) *Explain briefly what is meant by "double marginalization".* This refers to a situation where two vertically related firms interact, and this interaction leads to a consumer price that is too high from all parties point of view (both firms and all consumers). The reason for this is that there is an externality between the two firms: the retailer does not take into account the effect its choice of p has on the manufacturer's profit.

- Of course, this is true also for two horizontally related firms. But then the typical situation is that the goods are *substitutes*: firm 1's demand *drops* if firm 2 lowers its price, yielding an equilibrium price that is too low relative to the joint-profit maximizing price.
- In the vertical story, the input good and the final good are *complements*, so the externality works in the opposite direction.
- In a horizontal relationship with a demand complementarity, we can again get an equilibrium price that is too low also from the consumers' point of view.

f) *Give a brief verbal account of Rotemberg and Saloner's theory of price wars. What are the main ingredients of their model, what is their result, and what is the intuition for the result?*

- The key model ingredient: **demand fluctuates** stochastically (but is known when setting p). Otherwise it is a standard duopoly model with price-setting firms, interacting over an infinite horizon. One can, as in a standard repeated game, sustain a collusive equilibrium if the firms care sufficiently much about future profits. However, in this model, the requirement on the discount factor when having a high demand state is more stringent — the firms must be more patient than in the known-demand model for cooperation to be possible. The reason for this is that in the uncertainty model, in a high demand state, demand will be unusually high. The demand realization is by assumption independent over time, so the expected profits tomorrow and onwards are the same regardless of today's demand state. This means that when the demand is known to be high today, then the incentive to deviate from the equilibrium is higher than in the standard model, as the "one-period temptation" is unusually high whereas the "long-term reward of not deviating" is the same.

- The conclusion is that there is a tendency for collusion to break down in a high demand state (hence price war during booms and counter-cyclical prices).

Question 3

[To the external examiner: The students had not seen this particular question before, and we have not discussed the topic of strategic delegation specifically in the course. However, we have extensively discussed the importance of strategic actions (that need to be irreversible and observable) and the role of strategic substitutes and complements.]

a) The game consists of two stages. At the first stage the owners choose, independently and simultaneously, an instruction P_i or R_i. At the second stage we have four different possibilities, depending on what instructions the owners have chosen: both firms are profit maximizers, (P_1, P_2); both firms are revenue maximizers, (R_1, R_2); or one is a profit maximizer and the other is a revenue maximizer, (P_1, R_2) or (R_1, P_2). Given these objectives, the managers choose, independently and simultaneously, a quantity q_i.

- We can solve for the subgame-perfect Nash equilibria of the model by backward induction. We therefore start by solving the four second-stage subgames.
- **The case** (P_1, P_2)**.** Each firm maximizes

  $$\begin{aligned} & [45 - 9(q_1 + q_2)]\, q_i - 9q_i \\ = \; & [36 - 9(q_1 + q_2)]\, q_i. \end{aligned}$$

  The FOCs for the two firms are

  $$-9q_1 + [36 - 9(q_1 + q_2)] = 0$$

  and

  $$-9q_2 + [36 - 9(q_1 + q_2)] = 0.$$

  Solving these equations for q_1 and q_2 yields

  $$\left(q_1^{PP}, q_2^{PP}\right) = \left(\frac{4}{3}, \frac{4}{3}\right).$$

  The profit levels given these outputs are

  $$\pi_1^{PP} = \left[45 - 9\left(q_1^{PP} + q_2^{PP}\right)\right] q_1^{PP} - 9q_1^{PP} = 16$$

  and

  $$\pi_2^{PP} = \left[45 - 9\left(q_1^{PP} + q_2^{PP}\right)\right] q_2^{PP} - 9q_2^{PP} = 16.$$
- **The case** (R_1, R_2)**.** Each firm maximizes its revenues

  $$[45 - 9(q_1 + q_2)]\, q_i.$$

The FOCs for the two firms are

$$-9q_1 + [45 - 9(q_1 + q_2)] = 0$$

and

$$-9q_2 + [45 - 9(q_1 + q_2)] = 0.$$

Solving these equations for q_1 and q_2 yields

$$\left(q_1^{RR}, q_2^{RR}\right) = \left(\frac{5}{3}, \frac{5}{3}\right).$$

The profit levels given these outputs are

$$\pi_1^{RR} = \left[45 - 9\left(q_1^{RR} + q_2^{RR}\right)\right] q_1^{RR} - 9q_1^{RR} = 10$$

and

$$\pi_2^{RR} = \left[45 - 9\left(q_1^{RR} + q_2^{RR}\right)\right] q_2^{RR} - 9q_2^{RR} = 10.$$

- **The case (P_1, R_2).** Firm 1 maximizes its profit

$$\begin{aligned} & [45 - 9(q_1 + q_2)] q_i - 9q_i \\ = \quad & [36 - 9(q_1 + q_2)] q_i. \end{aligned}$$

Firm 1's FOC is

$$-9q_1 + [36 - 9(q_1 + q_2)] = 0. \tag{4}$$

Firm 2 maximizes its revenues

$$[45 - 9(q_1 + q_2)] q_i.$$

Firm 2's FOC is

$$-9q_1 + [45 - 9(q_1 + q_2)] = 0. \tag{5}$$

Solving equations (4) and (5) for q_1 and q_2 yields

$$\left(q_1^{PR}, q_2^{PR}\right) = (1, 2).$$

The profit levels given these outputs are

$$\pi_1^{PR} = \left[45 - 9\left(q_1^{PR} + q_2^{PR}\right)\right] q_1^{PR} - 9q_1^{PR} = 9$$

and

$$\pi_2^{PR} = \left[45 - 9\left(q_1^{PR} + q_2^{PR}\right)\right] q_2^{PR} - 9q_2^{PR} = 18.$$

- **The case (R_1, P_2).** This is symmetric to the case (P_1, R_2). Therefore, $\left(q_1^{RP}, q_2^{RP}\right) = (2, 1)$,

$$\pi_1^{RP} = 18,$$

and

$$\pi_2^{RP} = 9.$$

- We have now solved all the stage 2 subgames and derived expressions for the equilibrium profit levels in all of these. Using these profit levels we can illustrate the stage 1 interaction between O_1 and O_2 in a game matrix (where O_1 is the row player and O_2 is the column player):

	P_2	R_2
P_1	16, 16	9, 18
R_1	18, 9	10, 10

  We see that each player has a strictly dominant strategy and that, in particular, the unique Nash equilibrium of the stage 1 game is that both owners choose revenue maximization, (R_1, R_2).
- **Conclusion:** the game has a unique SPNE. In this equilibrium, both owners choose revenue maximization, (R_1, R_2). In the stage 2 equilibrium path subgame, the managers choose $\left(q_1^{RR}, q_2^{RR}\right) = \left(\frac{5}{3}, \frac{5}{3}\right)$. In the three off-the-equilibrium path subgames, the managers choose $\left(q_1^{PP}, q_2^{PP}\right) = \left(\frac{4}{3}, \frac{4}{3}\right)$, $\left(q_1^{PR}, q_2^{PR}\right) = (1, 2)$, and $\left(q_1^{RP}, q_2^{RP}\right) = (2, 1)$.

b) **Interpretation:** The owners would be better off if they both chose to instruct their manager to maximize profit. The reason why this cannot be part of an equilibrium is that each firm can gain by unilaterally instruct its own manager to maximize revenues instead. Why is this the case? First, a manager who maximizes revenues will be more aggressive (i.e., produce more) than a profit maximizing manager. Second, the rival manager, expecting this behavior, will respond by producing less (since the firms' outputs are strategic substitutes). This will increase the first firm's market share and profit.

- If the managers' choice variables had been strategic complements instead we should expect the opposite result: each firm would like to make the rival behave in a way that is good for the own profits (i.e., charge a high price or choose a small quantity). If the choice variables are strategic complements, this means that to induce the rival to behave like that a firm should behave in the same way itself (i.e., charge a high price or choose a small quantity). Therefore, an owner could gain by instructing its manager to be relatively non-aggressive (i.e., to have a strong incentive to charge a high price or choose a small quantity) — this can be achieved by instructing the manager to maximize profits rather than revenues.
- The assumption that the instruction is observable for the rival firm is crucial. Without that assumption, an owner would always want the own manager to maximize profits (but maybe still be *telling* the rival manager that the instruction was R). The point with choosing R is that then the rival *knows* this (and knows that this choice is irreversible), which will (in the model with strategic substitutes) have a beneficial effect on the rival manager's optimal choice at the second stage.

- In one of the problem sets we looked at Bagwell's example and his argument about noisy commitment — a student may want to mention this to show that he/she has understood the point.