Written Exam for the B.Sc. or M.Sc. in Economics winter 2013-2014

Advanced Microeconomics

Master's Course

21FEB2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

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This exam question consists of 7 pages in total including this page.

Advanced Microeconomics, Winter 2013-2014

3 hours closed book exam

Anders Borglin, who is responsible for the exam problems, can be reached by telephone during the exam. There are, including the front page and the two pages with assumptions, altogether 7 pages.

There are 3 problems. The Problems A and C have the same weight in the marking process and Problem B half the weight of Problem A

Below

$$\begin{aligned}\mathbb{R}^k_+ &= \left\{x \in \mathbb{R}^k \mid x_h \geq 0 \text{ for } h = 1, 2, \ldots, k\right\} \text{ and} \\ \mathbb{R}^k_{++} &= \left\{x \in \mathbb{R}^k \mid x_h > 0 \text{ for } h = 1, 2, \ldots, k\right\}\end{aligned}$$

for $k = 1, 2, \ldots$.

Problem A

(a) Give a graphic example of a production set $Y \subset \mathbb{R}^2$ satisfying P1, except the convexity part, and prices, $(p_1, p_2) \in \mathbb{R}^2_{++}$ such that there are precisely two solutions to the Producer Problem.

(b) Let a consumer have $\mathbb{R}^2_+$ as consumption set and lexicographic preferences. Define such preferences.

(c) For the consumer from (b) find the indifference class containing $x = (1, 1)$

(d) Assume that Arrow's assumptions A1 to A3 for a SWF are satisfied and that Schedule 1 below is mapped to a ranking for society with b above a. What can be concluded about society's ranking of a and b for Schedule 2?

Schedule 1				Schedule 2		
c	b	a		c	a	c
b	a	c		b	b	a
a	c	b		a	c	b

(e) Let $\mathcal{E} = (\mathbb{R}^2_+, u^i, \omega^i)_{i \in \{a,b\}}$ be a pure exchange economy with private ownership. Define what is meant by a Walras equilibrium for $\mathcal{E}$.

(f) Let the consumption set be $X = \mathbb{R}^2_+$ and consider a consumer with homothetic preferences $\succsim$. Assume that $x \sim \bar{x}$. Can it be the case that $2x \succ 2\bar{x}$? Illustrate in a diagram.

(g) Show by a graphic example, in $\mathbb{R}^2$, that when a consumer faces a price-wealth pair (p, w), $p \in \mathbb{R}^2_{++}$ such that $\mathrm{w} = \min \{px \mid x \in X\}$ then there might be $\hat{x} \in X$ such that $\hat{x}$ is a solution to the expenditure minimization problem (for utility level $\hat{u} = u(\hat{x})$ at prices p) but $\hat{x}$ does not solve the Utility Maximization Problem.

(h) In an economy the production sector has 2 producers, a and b, with production sets Y^a and Y^b. What is the (total, aggregate) production set for the production sector.

(i) Let $(\bar{x}, \bar{y}, p)$, $p = (1, 1)$, be a Walras equilibrium for an economy $\mathcal{E} = \{(u, \omega), Y, \alpha\}$ (one consumer, one producer) where $\alpha = 1$ (the consumer owns the producer). State the conditions that $(\bar{x}, \bar{y}, p)$ must satisfy. Be careful when defining the consumer´s wealth. Illustrate in a diagram.

(j) Assume that the preference relation $\succsim$ on $X = \mathbb{R}^L_+$ is represented by the non-negative strictly monotone utility function u with the property $u(\alpha x) = \alpha^2 u(x)$ for $\alpha \geq 0$ (homogenous of degree 2). Is $\succsim$ a homothetic preference relation?

Problem B

(a) Consider an economy with commodity space $\mathbb{R}^L$ having precisely two producers, a and b, with production sets $Y^a \subset \mathbb{R}^L$ and $Y^b \subset \mathbb{R}^L$. Let the price vector $p \in \mathbb{R}^L_{++}$ be given and let $\bar{y}^a$ and $\bar{y}^b$ be solutions to the individual producer problems. Show that $\bar{y}^a + \bar{y}^b$ solves the aggregate (total) profit maximization problem.

(b) Let $\mathcal{E} = (\mathbb{R}^2_+, u^i, \omega^i)_{i \in \{a,b\}}$ be an exchange economy with private ownership where $\omega^a = \omega^b$ and $(\hat{x}^a, \hat{x}^b, p)$ a Walras equilibrium for $\mathcal{E}$. Show that the equilibrium allocation is a fair allocation

Problem C

Consider a private ownership pure exchange economy with commodity space $\mathbb{R}^3$. The economy has two consumers, a and b, with consumption sets $X^a = X^b = \mathbb{R}^3_+$ initial endowments $\omega^a, \omega^b \in \mathbb{R}^3_{++}$ and preferences given by utility functions $U^a : \mathbb{R}^3_+ \longrightarrow \mathbb{R}$ and $U^b : \mathbb{R}^3_+ \longrightarrow \mathbb{R}$. where furthermore,

$$\begin{aligned} U^a(x_1, x_2, x_3) &= v^a(x_1) + v^a(x_2) + v^a(x_3) \\ U^b(x_1, x_2, x_3) &= v^b(x_1) + v^b(x_2) + v^b(x_3) \end{aligned}$$

and where, for $i = a, b$, the function $v^i : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ has positive first derivative (differentiably monotone), $Dv^i > 0$, and negative second derivative (differentiably concave), $D^2 v^i < 0$, on $\mathbb{R}_{++}$. Furthermore $\lim_{y \to 0} Dv^i(y) = \infty$.

(a) Does $f : \mathbb{R}_+ \longrightarrow \mathbb{R}$ with values $f(t) = 2t^{\frac{1}{2}}$ satisfy the conditions assumed for v^a?

(b) Is U^a a quasi-concave function? **Hint:** U^a is the sum of the functions $(x_1, x_2, x_3) \longrightarrow v^a(x_1), (x_1, x_2, x_3) \longrightarrow v^a(x_2)$ and $(x_1, x_2, x_3) \longrightarrow v^a(x_3)$.

(c) Let $p \in \mathbb{R}^3_{++}$ and income (wealth) $\mathrm{w}^a > 0$. State consumer a´s problem (the UMP). Does it have a solution?

(d) Assume that $\bar{x}^a$ is a solution to the problem from (c). Derive the marginal conditions that $\bar{x}^a$ will satisfy.

(e) Assume that $p_1 < p_2 < p_3$ and let λ be the multiplier from (d). Make a qualitatively correct plot of the first derivative of $(1/\lambda) v^a$. Indicate in your diagram the location of pairs $\left(\frac{1}{\lambda} Dv^a(\bar{x}^a_h), \bar{x}^a_h\right) = (p_h, \bar{x}^a_h)$, $h = 1, 2, 3$ where λ is the multiplier from (d).

(f) Assume in the sequel that total initial endowment $\omega = \omega^a + \omega^b$ is such that $\omega_1 > \omega_2 > \omega_3$. Let $(\hat{x}^a, \hat{x}^b, \hat{p})$ be a Walras equilibrium. State the market balance conditions for commodity 1 and 2. Using these show that $\hat{x}^a_1 > \hat{x}^a_2$ or $\hat{x}^b_1 > \hat{x}^b_2$.

(g) Show that the result in (f) implies $p_1 < p_2$. **Hint:** Apply the reasoning from (e).

(h) Thus in an exchange economy with preferences like U^a and U^b (separable preferences) there is a relation between the ordering of the total endowment and Walras equilibrium prices. Does this extend also to the ordering of the equilibrium consumptions?

Assumptions on Producers

Assumption P1: *The production set $Y \subset \mathbb{R}^L$ satisfies*

(a) $0 \in Y$ (Possibility of inaction)

(b) Y *is a closed subset of* $\mathbb{R}^L$ (Closedness)

(c) Y *is a convex set* (Convexity)

(d) $Y \cap (-Y) = \{0\}$ (Irreversibility)

(e) *If* $\bar{y} \in Y$, $y \in \mathbb{R}^L$ *and* $y \leq \bar{y}$ *then* $y \in Y$ (Free disposal, downward comprehensive)

Assumption P2:*(constant returns to scale) If the vector* $y \in Y$ *and* $\lambda \in [0, +\infty[$ *then* $\lambda y \in Y$.

Assumptions on Consumers

Assumption F1.
The consumption set $X \subset \mathbb{R}^L$ satisfies:

(a) X is a non-empty set.

(b) X is a closed set

(c) X is a convex set

(d) X is a lower bounded set (in the vector ordering)

(e) X is upward comprehensive ($x \in X$ and $\nabla \in \mathbb{R}^L_+$ implies $x + \nabla \in X$)

Monotonicity assumptions

Assumption of **weak monotonicity**
$\mathbf{F2}^0 : x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow x^1 \succsim x^2$

$\mathbf{F2}^0 : x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow u(x^1) \geq u(x^2)$

In the interpretation: "at least as much of each commodity is at least as good".

Assumption of **monotonicity** (MWG Def. 3.B.2)

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow x^1 \succ x^2$

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow u(x^1) > u(x^2)$

In the interpretation: "more of each commodity is better".

Assumption of **strict (or strong) monotonicity** (MWG Def. 3.B.2)

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow x^1 \succ x^2$

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow u(x^1) > u(x^2)$

The preference relation $\succsim$ is **locally non-satiated** if: Given $x \in X$ and $\varepsilon > 0$ there is $x' \in X$ such that $x' \succ x$ and $\| x' - x \| < \varepsilon$. (Definition 3.B.3, MWG)

A preference relation, $\succsim$, is a **convex preference relation** if and only if, for $x \in X$, the set $\{x' \in X \mid x' \succsim x\}$ is a convex set. (MWG Def. 3.B.2).

If u represents $\succsim$ then $\succsim$ is a convex preference relation if and only if u is a quasi-concave function.

We want to consider also a stronger convexity assumptions

F3: A preference relation, $\succsim$, is a **strictly convex** preference relation if: $x^1, x^2, x^3 \in X$, $x^1 \succsim x^2$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)x^2$ for some $t \in]0,1[$ implies $x^3 \succ x^2$.

F3: The utility function is **strictly quasi-concave** if: $x^1, x^2, x^3 \in X$, $u(x^1) \geq u(x^2)$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)x^2$ for some $t \in]0,1[$ implies $u(x^3) > u(x^2)$.