Suggestive solution for
Written Exam for the B.Sc. in Economics 2011-I-R
Macroeconomics C

Competence description: At the end of the course, the student should be able to demonstrate:

- Understanding of the main model frameworks for long-run macroeconomics. This includes the Diamond model with overlapping generations in discrete time and the Ramsey model in continuous time.
- Proficiency in the application of the concepts and methods from these frameworks, including competence in dynamic optimization and dynamic analysis in discrete and continuous time.
- Understanding of the role of expectations and basic knowledge of macroeconomic models with forwardlooking expectations under both perfect foresight and uncertainty and rational expectations.
- Proficiency in the application of the related concepts and methods.
- Competence in analyzing a macroeconomic problem, where the above-mentioned concepts and methodsare central, that is competence in solving such models and explaining in economic terms the results andimplications and how they derive from the assumptions of the model.

The particularly good performance, corresponding to the top mark, is characterized by a complete fulfilment of these learning objectives.

Problem A

1. **True.** In the long run, the economy in the Ramsey Model converges to a steady state where capital per capita, k_t, is constant. Consequently, also output per capita, y_t, is constant, since $y_t = f(k_t)$, and with output per capita being constant, aggregate output Y_t must grow at the rate of population growth, n, in a steady state where $Y_t = L_t y_t = L_t f(k^*)$.
   Also, the interest rate will not be equal to n in the long run . First of all there is no general economic argument that the interest rate should equal the growth of aggregate output, and more formally consumption per capita evolves according to
   $$\frac{\dot{c}_t}{c_t} = \frac{f'(k_t) - \delta - \rho}{\theta}$$
   Since also c_t is constant in the long run it follows that the interest rate which equals the marginal product of capital net of depreciation, will in the long run be $r^* = f'(k^*) - \delta = \rho$ where ρ is the rate of time preference which in the Ramsey Model is specifically assumed not to equal n.

2. **False.** In the Fischer model there are two groups of firms who take turns in setting, every second period, a price which has to be in effect for the current and the following period (i.e. the same price is chosen for both periods). Considering a change in nominal money supply in period t, the group of firms adjusting prices in period t (knowing the change in m) for period t and $t+1$ will not want to fully adjust w.r.t. period t because they care about the price relative to the average price level which in turn depends on prices chosen by both groups and the average price level cannot fully adjust because the other group will not have had a change to react. Since the price chosen for period t (and which will not fully adjust) also has to in effect in period $t+1$ this in turn implies that the other group, when being able to change wages will also not adjust fully w.r.t. period $t+1$ and when the first group realizes this, the adjustment will be even smaller. As a consequence, the price level never fully adjusts to changes in the money supply and thus there will be lasting real effects.

3. **False.** The Policy Ineffectiveness Proposition holds that active demand mangement policies will not affect real output (and employment). The intuition is that according to the SRAS curve, real output can only be affected if it is possible for economic authorities to create surprise inflation, i.e. to have actual inflation deviate from the expected inflation and this is impossible if there are rational expectations. However, this rests on an assumption that economic policy is conducted using only information that is also held by private agents in which case said agents are of course able to fully anticipate policy, but provided that policy is conducted using information that is not held by private agents, the policy will be able to cause surprise inflation. All this, however, has nothing to do with how the government social loss function looks

Problem B

1. There are various ways of deriving (B.6) but the easiest is probably to isolate s_t in (B.2) yielding

$$s_t = w_t - T_t - c_{1t}$$

and by inserting this expression into (B.3) we then obtain

$$\begin{aligned} c_{2t+1} &= (1 + r_{t+1})(w_t - T_t - c_{1t}) \Leftrightarrow \\ c_{1t} + \frac{1}{1+r_{t+1}} c_{2t+1} &= w_t - T_t \end{aligned} \tag{1}$$

which is the intertemporal budget constraint requiring that the present value of total consumption expenditure, i.e. the right-hand side, should equal after-tax wage income when young. The problem may now be thought of as one of maximizing (B.1) subject to (1), and the Lagrangian associated with this is

$$\mathcal{L} = \ln c_{1t} + (1+\rho)^{-1} \ln c_{2t+1} - \left(c_{1t} + \frac{1}{1+r_{t+1}} c_{2t+1} = w_t - T_t \right)$$

with first order conditions being

$$\frac{\partial \mathcal{L}}{\partial c_{1t}} = \frac{1}{c_{1t}} - \lambda = 0 \Rightarrow \frac{1}{c_{1t}} = \lambda \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial c_{1t}} = (1+\rho)^{-1} \frac{1}{c_{2t+1}} - \lambda \frac{1}{1+r_{t+1}} = 0 \Rightarrow (1+\rho)^{-1} \frac{1}{c_{2t+1}} = \lambda \frac{1}{1+r_{t+1}} \tag{3}$$

By dividing (2) with (3) we get

$$\frac{\frac{1}{c_{1t}}}{(1+\rho)^{-1} \frac{1}{c_{2t+1}}} = \frac{\lambda}{\lambda \frac{1}{1+r_{t+1}}} \Leftrightarrow$$

$$\frac{c_{2t+1}}{c_{1t}} = \frac{1+r_{t+1}}{1+\rho} \tag{B.6}$$

as required.

Eq. (B.6) is the so-called Euler equation which gives the optimal evolution of individual consumption over time. The equation says that:

- Consumption when old *relative* to consumption when young is an *increasing* function of the interest rate. It could be added that this result is not in conflict with changes in the interest rate having both a substitution and an income effect affecting the *level* of consumption when young in oppposite directions (and in the particular case cencelling out, cf. question 2).
- Consumption when old relative to consumption when young is a *decreasing* function of the rate of time preference, i.e. the rate at which instantaneous utility when old is discounted. A higher ρ may be thought of as reflecting a more impatient consumer and the resulting decrease in c_{2t+1} relative to c_{1t} is thus not surprising.

Equation (B.6) is the discrete time version of the Keynes Ramsey rule of the Ramsey Model which is stated in continues time. The Keynes Rasmey rule states that the optimal growht rate of individual consumption should be

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\varepsilon\left(c_t\right)} \tag{4}$$

with the same interpretation as eq. (B.6). In (3), $\varepsilon\left(c_t\right) \equiv -u''\left(c_t\right)\frac{c_t}{u'(c_t)}$ is the marginal elasticity of substitution which, however, is equal to 1 when instantaneous utility is logarithmic as is the case in this problem.
In discrete time the growth rate of individual consumption is $\frac{c_{2t+1}-c_{1t}}{c_{1t}}$ and the following (not required!) shows the 'equivalence' between eqs. (B.6) and (4) with $\varepsilon\left(c_t\right) = 1$. We use the facts that $\ln\frac{y}{x} \approx \frac{y-x}{x}$ and that $\ln\left(1+x\right) \approx x$:

$$\frac{c_{2t+1} - c_{1t}}{c_{1t}} \approx \ln\frac{c_{2t+1}}{c_{1t}} \underset{\text{Using eq. (B.6)}}{=} \ln\frac{1+r_{t+1}}{1+\rho} = \left(\ln\left(1+r_{t+1}\right) - \ln\left(1+\rho\right)\right)\left(r_{t+1} - \rho\right)$$

2. Isolating c_{1t} in (B.2) and inserting the resulting expression together with (B.3) into (B.1) the problem may be reduced to

$$\max_{s_t} U_t = \ln\left(w_t - T_t - s_t\right) + \left(1+\rho\right)^{-1}\ln\left(\left(1+r_{t+1}\right)s_t\right)$$

with first order condition

$$\frac{dU_t}{ds_t} = \frac{1}{w_t - T_t - s_t}\left(-1\right) + \left(1+\rho\right)^{-1}\frac{1}{\left(1+r_{t+1}\right)s_t}\left(1+r_{t+1}\right) = 0 \Rightarrow$$

$$\begin{aligned}
\frac{1}{w_t - T_t - s_t} &= \frac{1}{1+\rho}\frac{1}{s_t} \Leftrightarrow \\
\left(1+\rho\right)s_t &= w_t - T_t - s_t \Leftrightarrow \\
s_t &= \frac{1}{2+\rho}\left(w_t - T_t\right)
\end{aligned} \tag{B.7}$$

The reason that saving is independent of the interest rate is that when instantaneous utility is logarithmic, the elasticity of substitution is equal to 1 in which case the substitution effect (according to which saving increases with the interest rate) and the income effect (causing lower saving following an increase in the interest rate) exactly cancel out.
Aggregate physical capital in period $t+1$ is given by the total saving of the young of period t, $K_{t+1} = L_t s_t$, which together with the definition $k_t \equiv \frac{K_t}{A_t L_t}$, $L_{t+1} = \left(1+n\right)L_t$ and $A_{t+1} = \left(1+g\right)A_t$ implies that

$$k_{t+1} \equiv \frac{K_{t+1}}{A_{t+1}L_{t+1}} = \frac{L_t s_t}{\left(1+g\right)A_t\left(1+n\right)L_t} = \frac{1}{\left(1+g\right)\left(1+n\right)A_t}s_t \tag{5}$$

Inserting into (5) the expression from (B.7), (B.5) and $T_t = \tau A_t$ we find

$$\begin{aligned}
k_{t+1} &= \frac{1}{(1+g)(1+n)A_t} s_t \\
&= \frac{1}{(1+g)(1+n)A_t} \frac{1}{2+\rho} (w_t - T_t) \\
&= \frac{1}{(1+g)(1+n)A_t} \frac{1}{2+\rho} (A_t (1-\alpha) k_t^\alpha - \tau A_t) \\
&= \frac{1}{(1+g)(1+n)(2+\rho)} ((1-\alpha) k_t^\alpha - \tau) \qquad \text{(B.8)}
\end{aligned}$$

as required.

The relation between k_t and k_{t+1} given in eq. (B.8) with $\tau = 0$ is illustrated in figure B.1 using that with $0 < \alpha < 1$, k_t^α passes through the origin, is increasing and strictly concave (and that $\lim_{k_t \to 0^+} \frac{d(k_t^\alpha)}{dk_t} = \infty$ and $\lim_{k_t \to \infty} \frac{d(k_t^\alpha)}{dk_t} = 0$)

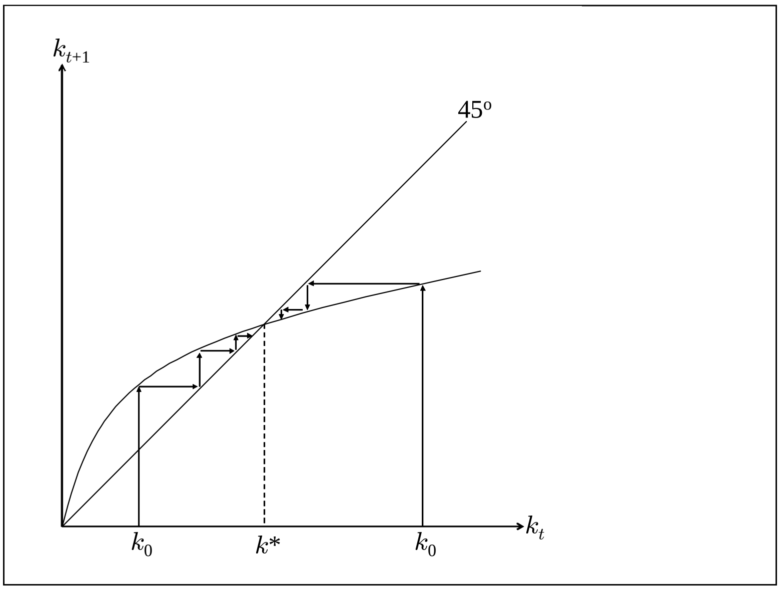

Figure B.1

The arrows indicating the evolution of the economy starting from an arbitrary $k_0 > 0$ shows the unique steday state to be stable, i.e. that $k_t \to k^*$ for $t \to \infty$.

3. As see from eq. (B.8) a positive value of τ will shift the curve relating k_t and k_{t+1} shown in figure B.2. Consequently the economy will experience a gradual decrease in capital per unit of labour from k_1^* to k_2^* (provided that we consider the realistic case where value of τ is not too large).

   The intuition is the following: in period t_0 the pre-tax wage w_t is given, since it depends on the predetermined capital stock, k_1^*. Introducing the tax will decrease the after-tax wage income to $w_t - T_t$. This will lead to a decrease in consumption of the young in period t. However, since individuals want to smooth consumption over life, the reduction

in consumption of the young will be less than taxes paid. Consequently the savings of the young in period t will decrease (in order to also decrease consumption when old). The old in period t_0 will not be affected, since they do not earn wage income.
In period $t_0 + 1$ capital per unit of labour will be less than in period t_0 due to the decreased savings by the young in period t_0. This will decrease the wage in period $t_0 + 1$ and will further decrease consumption of the young. The interest rate will be higher since it equals the marginal product of capital, which is a decreasing function of capital per unit of effective labour labour. The lower savings of the young in period t_0 in combination with the higher interest rate in period $t_0 + 1$ affect consumption of the old in period $t_0 + 1$ in opposite directions. It is possible, however, to show that consumption of the old will be lower, but that is *not* required.
These effects with decreasing consumption, decreasing capital per unit of labour, decreasing wage and increasing interest rate will continue until the new steady state is reached.

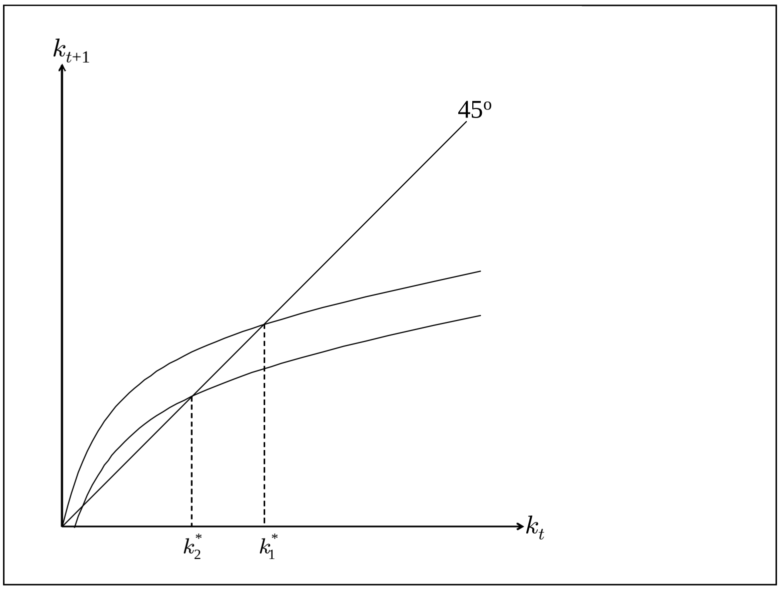

Figure B.2

4. In this case steady state capital and thus production would be unaffected and only consumption of the young and the old would decrease by an amount equivalent to VAT payments, leaving the expenditure including VAT unafffected. The intuition is that since the VAT is levied on consumption both when young and when old, individuals cannot avoid the VAT by changing the composition of consumption over life and therefore saving of the young is unaffected and this leaves capital, production, the real wage rate and the interest rate unaffected.
   To see this formally (not directly required), we note that is follwos from (B.9) and (B.10)

that

$$c_{1t} = \frac{1}{1+\tau^c}(w_t - s_t) \tag{C.9}$$

$$c_{2t+1} = \frac{1}{1+\tau^c}(1+r_{t+1})\, s_t \tag{C.10}$$

and by inserting these into (B.1) the problem of the individual is one of

$$\max_{s_t} U_t = \ln\left[\frac{1}{1+\tau^c}(w_t - s_t)\right] + (1+\rho)^{-1}\ln\left[\frac{1}{1+\tau^c}(1+r_{t+1})\, s_t\right]$$

with first order condition

$$\begin{aligned}
\frac{dU_t}{ds_t} &= \frac{1}{\frac{1}{1+\tau^c}(w_t - s_t)}\left(-\frac{1}{1+\tau^c}\right) + (1+\rho)^{-1}\frac{1}{\frac{1}{1+\tau^c}(1+r_{t+1})\, s_t}\frac{1}{1+\tau^c}(1+r_{t+1}) \\
&= -\frac{1}{(w_t - s_t)} + (1+\rho)^{-1}\frac{1}{s_t} = 0 \qquad (6)
\end{aligned}$$

and since the expression in (6) does not include τ^c, saving will be unaffected by τ^c.

Problem C

1. From (C.5) we obtain

$$\dot{p} = 0 \Rightarrow \frac{\gamma\beta\delta}{\eta+\delta}e - \gamma\frac{\beta\delta+\eta}{\eta+\delta}p + \frac{\gamma\eta}{\eta+\delta}m + \frac{\gamma\delta}{\eta+\delta}z - \gamma\bar{y} = 0 \Leftrightarrow$$

$$\begin{aligned}
\frac{\gamma\beta\delta}{\eta+\delta}e &= \gamma\frac{\beta\delta+\eta}{\eta+\delta}p - \frac{\gamma\eta}{\eta+\delta}m - \frac{\gamma\delta}{\eta+\delta}z + \gamma\bar{y} \Leftrightarrow \\
e &= \gamma\frac{\beta\delta+\eta}{\gamma\beta\delta}p - \frac{\gamma\eta}{\gamma\beta\delta}m - \frac{\gamma\delta}{\gamma\beta\delta}z + \frac{\gamma}{\gamma\beta\delta}\bar{y} \Leftrightarrow
\end{aligned}$$

$$\dot{p} = 0 : e = \frac{\beta\delta+\eta}{\beta\delta}p - \frac{\eta}{\beta\delta}m - \frac{1}{\beta}z + \frac{\eta+\delta}{\beta\delta}\bar{y} \tag{1}$$

Eq. (1) shows the $\dot{p} = 0$ locus to be a straight line with a slope (in a (p, e)-diagram) equal to $\frac{\beta\varepsilon+\delta}{\beta\varepsilon} > 1$.
From eq. (C.6) we then get

$$\dot{e} = 0 \Rightarrow \frac{\beta}{\eta+\delta}e + \frac{1-\beta}{\eta+\delta}p - \frac{1}{\eta+\delta}m + \frac{1}{\eta+\delta}z - r^f = 0 \Leftrightarrow$$

$$\frac{\beta}{\eta+\delta}e = -\frac{1-\beta}{\eta+\delta}p + \frac{1}{\eta+\delta}m - \frac{1}{\eta+\delta}z + r^f \Leftrightarrow$$

$$\dot{e} = 0 : e = -\frac{1-\beta}{\beta}p + \frac{1}{\beta}m - \frac{1}{\beta}z + \frac{\eta+\delta}{\beta}r^f \Leftrightarrow \tag{2}$$

showing the $\dot{e} = 0$ locus to be a straight line with a slope equal to $-\frac{1-\beta}{\beta} = 1 - \frac{1}{\beta}$. Since $0 < 1 - \frac{1}{\beta} < 1$ when $\beta > 1$ the $\dot{e} = 0$ locus is thus positively sloped but flatter than the

$\dot{p} = 0$ locus.

Figure C.1 shows the $\dot{p} = 0$ and $\dot{e} = 0$ loci when, as assumed in the text, they intersect in the positive orthant.

The directions of motion indicated by the arrows in figure 1 are found from eqs. (C.5) and (C.6) in the following way: beginning at any point on the $\dot{p} = 0$ locus moving either vertically up (increasing e with p unchanged) or horizontally to the left (decreasing p with e unchanged), it may be concluded from eq. (C.5) that at the new point it is the case that $\dot{p} > 0$, i.e. that p will be increasing over time. This is indicated in figure C.1 by the horizontal and rightward-pointing arrows to the left of (above) the $\dot{p} = 0$ locus. In a similar manner it may be concluded that $\dot{p} < 0$ to the right of (below) the $\dot{p} = 0$ locus explaining the horizontal leftward-pointing arrows there.

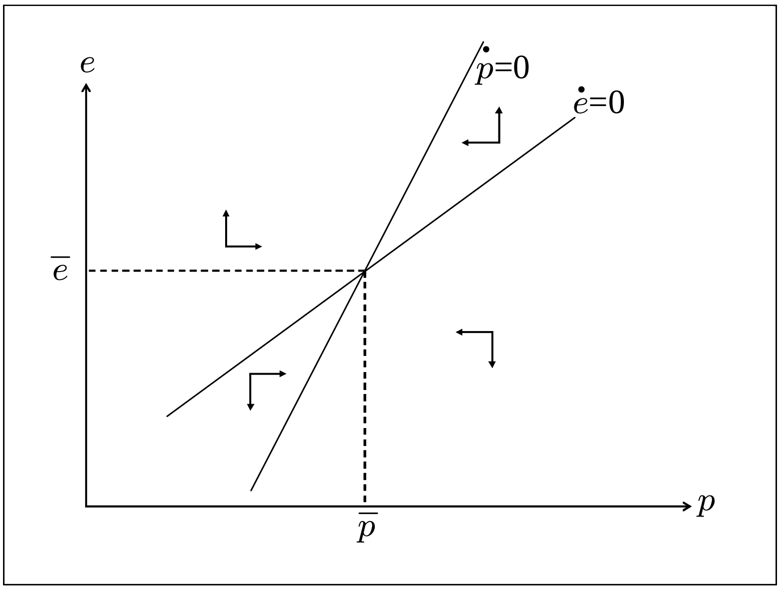

Figure C.1

Beginning at any point on the $\dot{e} = 0$ locus moving either vertically up or horizontally to the left, it follows from eq. (C.6) that at these points it is the case that $\dot{e} > 0$ (moving horizontally to the left, i.e. decreasing p with e unchanged, one should use the assumption that $\beta > 1$). Consequently e is increasing over time above (to the left of) the $\dot{e} = 0$ locus which explains the upward-pointing vertical arrows there. In a similar manner one can explain the downward-pointing vertical arrows to the right of (below) the $\dot{e} = 0$ locus indicating that at these points e will be decreasing over time.

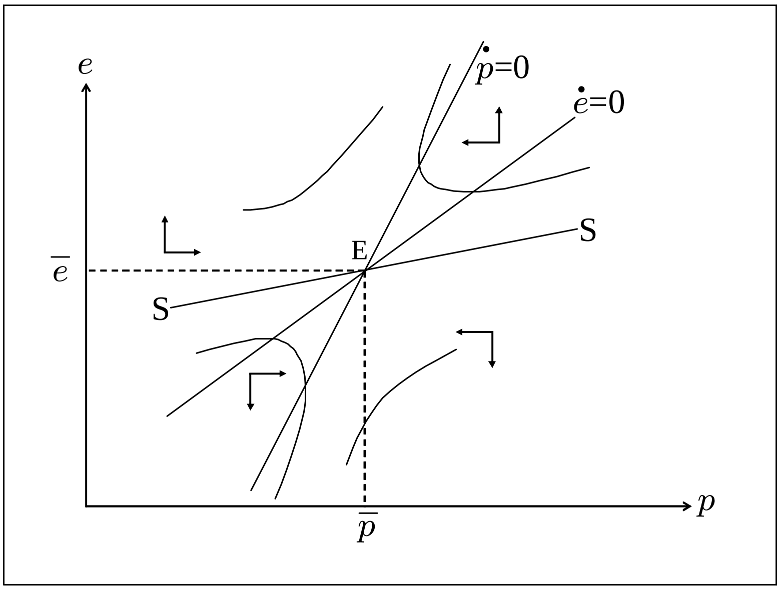

Figure C.2

The directions of motion shown in figure C.1 reveal that the economy is saddle point stable meaning that there is a unique path, the positively sloped saddle path SS in figure C.2, approaching (from either side) the long run equlibrium at E, where p and e are constant, while all other paths are diverging from the long run equilibrium at E. The diverging paths may be thought of as bubbles in the exchange rate where the exchange rate is eventually ever increasing or decresing due to self-fullfilling expectations. However, since there is rational expectations and no uncertainty which together imply perfect foresight, such evolutions may be ruled out on the argument that rational agents will not believe the exchange rate to be forever increasing or decreasing.[1] Since at any point in time p is predetermined, while e is free to jump/adjust, we can conclude that for any initial value of p, e will be chosen such that the economy is on the saddle path and evolves along this to the long run equilibrium at point E.

2. The phase diagram is shown in figure C.3. As is seen from eqs. (1) and (2) the $\dot{p} = 0$ locus is unaffected, while the $\dot{e} = 0$ shifts up (slope unchanged). We can thus conclude that the long run equlibrium changes from E_1 to E_2 implying that both the price level and the exchange rate increase in the long run.(In figure C.3 the original $\dot{e} = 0$ locus is dotted while the new is denoted $\dot{e} = 0_{new}$. Figure C.3 only shows the directions of motion associated with the *new* loci.)

[1]It could be added that an evolution where the economy follows a diverging path for a while and then jumps to the saddle path is also not consistent with rational behaviour, since with perfect foresight, the time of the jump would be known to all agents. If therefore, e.g., the exchange rate were to increase discretely (jump) all agents would want to purchase foreign currency the instant before the jump. This, however, would drive up the exchange rate the instant before, making agents want to purchase foreign currency even earlier making the exchange rate increase even earlier and so forth. By continuing to drive this argument backwards in time, it may be concluded that such an evolution could never get started.

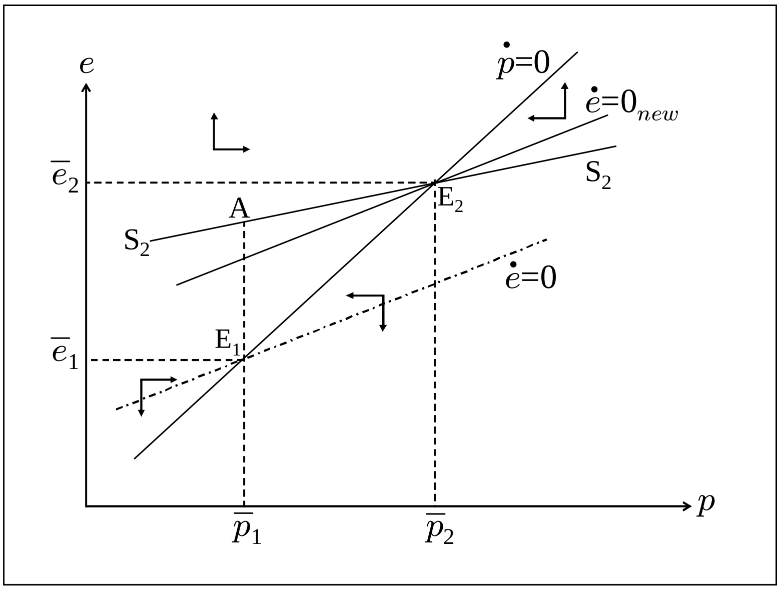

Figure C.3

We now make the following considerations:

a) Before time t_0 the economy is at point E_1 and thus at time t_0 p is predetermined at $\bar{p}_1$, while e is free to jump.

b) Sooner or later the economy must reach the new saddle path, S_2S_2, since otherwise it will forever follow one of the diverging paths which we have ruled out.

c) If e is to jump at any point in time it must be at time t_0, since otherwise the jump would be expected/known meaning that agents would sit around waiting for capital gains or losses from holding foreign currency and this is not compatible with rational behaviour.

Applying these considerations to figure C.3 it is seen that is must be the case that exactly at time t_0 when the foreign interest rate is increased, the economy jumps from E_1 to A and then moves continously along the new saddle path to the new long run equlibrium at E_2 where both e and p are higher.
The economic intuition is the following: The increase in the foreign interest rate is seen to cause the exchange rate to increase in the long run. Knowing this, private agents will therefore immediately following the announcement want to purchase foreign currency in order to enjoy a capital gain. The effect will, *ceteris paribus*, be to immediately drive the exchange rate up to its new long run equlibrium level, $\bar{e}_2$.
At the same time, however, two other effects are taking place. One is the actual increase in the foreign interest rate which makes foreign bonds more attractive relative to domestic bonds and which therefore tends to further increase the exchange rate, since investors will want to purchase fopreign currency in order to purchase foreign bonds. The other is the fact that the increase in the exchange rate will cause the real exchange rate to

increase thus increasing demand for domestic goods. Consequently real output will increase and this will lead to an increase in the demand for money which will drive up the domestic interest rate. The increase in the domestic interest rate will have the exact opposite effect on the exchange rate as the increase in the foreign exchange rate. In the case where $\beta > 1$, however, the effect from the exchange rate onto net exports and eventually onto the domestic interest rate will be of a magnitude such that the increase in the domestic interest rate is greater than the increase in the foreign interest rate. This explains why initially the exchange rate undershoots its new long run level by jumping initially to a value lower than $\bar{e}_2$.

At point A we have that $y > \bar{y}$, since the depreciation (of the domestic currency) has increased demand for domestic goods through increased net exports. Over time this will induce the price level to increase. This will erode the competitiveness of domestic goods making real output decline over time. At the same time the increasing price level will cause the domestic interest rate to decrease (since $\beta > 1$, as seen from eq. (C.8)) which will gradually make domestic bonds less attractive thereby putting upward pressure on the demand for foreign currency. The reason behind the increase in the interest rate is that the increasing price level directly decreases the real money supply which tends to increase the interest rate but the increasing price level also decreases real output which in turn lowers real demand for money and this cuases the interest rate to decrease. With $\beta > 1$, the latter effect dominates the former. With the domestic interest rate decreasing, the exchange rate will be incrasing. This explains the effects along the saddle path from A to E_2, where y is back at its natural level and the domestic interest rate has decreased until it equals the foreign interest rate.

3. The phase diagram is shown in figure C.4 where now only the directions of motion (dotted) associated with the *original* $\dot{e} = 0$ locus are shown.

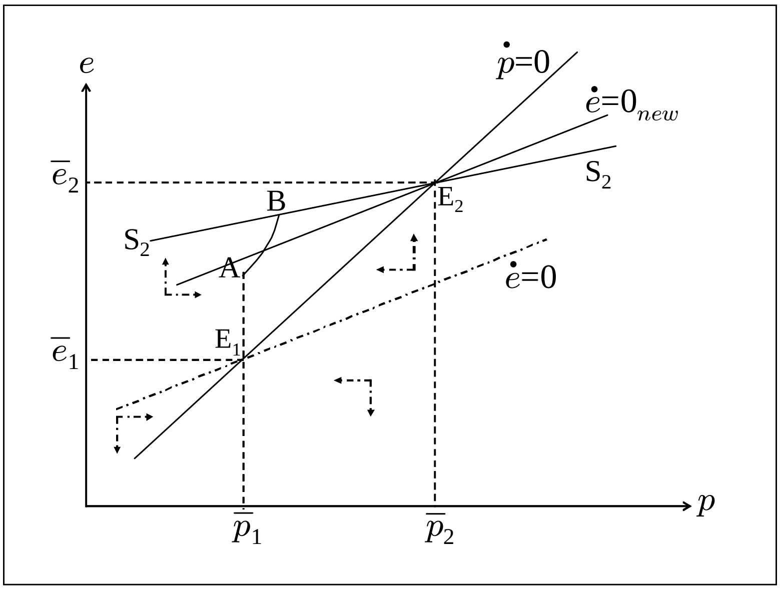

Figure C.4

In addition to considerations a)-c) i question 2 above we now also have:

d) Between time t_0 and time t_1 the economy is governed by the original directions of motion and from time t_1 by the new directions of motion.

Applying these considerations to figure C.4 we conclude that the economy must at time t_0 jump from E_1 to A which is below the new saddle path. Between time t_0 and time t_1 the economy then moves according to the original directions of motion from A to B which is reached exactly at time t_1 after which point in time the economy moves along the new saddle path to the new long run equlibrium at E_2.
The economic intuition is the following: At time t_0 when it is learned that at the future time t_1 the foreign interest rate will increase, we have the same effects as in the unannounced case of question 2, *except* that the foreign interest rate has not yet increased. This makes foreign bonds less attractive compared with the case in question 2 and this explains why on impact the depreciation of the domestic currency is smaller. Still, the depreciation causes real output to increase such that at point A we have $y > \bar{y}$. Consequently p will begin to increase. From A to B the exchange rate is increasing due to the fact that the increasing price level causes the domestic interest rate to be decreasing and the fact that foreign currency is becoming increasingly attractive the closer one gets to the point in time where the foreign interest rate increases. This happens at point B where, however, there is no jump in the exchange rate, since the change has been fully anticipated. From point B to point E_2 the mechanisms are the same as in question 2 from A to E_2.