Written Exam at the Department of Economics summer 2020

Mikro II

Final Exam

20th August 2020

(3-hour open book exam)

Answers in English or in Danish.

This exam question consists of 3 pages in total

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Notice that any communication with fellow students or others about the exam questions during the exam is considered to be cheating and will be reported. It is also considering cheating to let other students use your product.

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You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Question 1

The demand in the market for steel can be described by the following function:

$$D(p) = \max\{800 - 2p, 0\}$$

Imagine that the supply side of the market for steel can be described through a single representative firm, which has no market power. The production technology can be described by the following cost function:

$$C(y) = 300 + 0.25y^2$$

a) What are the equilibrium price and quantity in the market for steel?

b) What are the consumer surplus and the producer surplus in the market for steel?

Now assume that the production of steel causes pollution. The cost that occurs to society from pollution, $e(y)$, can be described by the following function:

$$e(y) = 100y + 0.25y^2$$

c) What is the socially optimal level of steel production?

d) What is the total net surplus in the market for steel, if you take the additional cost created through pollution into account? Explain the consequences of your results in words.

e) Is there a deadweight loss arising from pollution? Explain why it arises or not.

f) The government wants to introduce a Pigouvian tax to achieve the socially optimal level of steel production. The government proposes that the firm has to pay a constant tax on each unit of steel produced. How high would the Pigouvian tax have to be to achieve the socially optimal quantity?

g) Would you recommend the government to introduce the Pigouvian tax or rather to leave the market unregulated?

Solution Re-take exam question 1

a) Derive the inverse demand function, which is $p = 400 - 0.5q$, and the marginal cost curve, which is $p = 0.5q$. They intersect at $q = 400$ and $p = 200$, which are the equilibrium price and quantity.

b) This can be done using integrals or using the formula for the area of a triangle. Producer surplus will be $0.5 * (200 - 0) * 400 = 40,000$. Consumer surplus will be $0.5 * (400 - 200) * 400 = 40,000$.

c) Sum the private cost and the cost from pollution to get the total social cost arising from the production of steel as $SC(y) = 300 + 100y + 0.5y^2$. Taking the derivative w.r.t y gives the total social marginal cost curve for the production of steel as $SMC(y) = 100 + y$. This curve intersects with the inverse demand curve at $q = 200$.

d) Sum consumer and producer surplus from b) and subtract the area between total social marginal cost and private marginal cost curves. This area will just be equal to the total cost arising from pollution at the equilibrium quantity, which are given by $100 * 400 + 0.25 * 400^2 = 80,000$. The total net surplus will be $40,000 + 40,000 - 80,000 = 0$. This implies that closing down the market for steel or forbidding steel production would not reduce welfare.

e) Yes. The negative externality means that the firm will produce a suboptimally high level of steel. The most straightforward way of calculating the deadweight loss will be to compare total surplus under the efficient allocation and total surplus under the market allocation. Under the efficient allocation, total surplus will be given by the area between the inverse demand curve and the total social marginal cost curve. This area can be calculated using the formula for the area of a triangle: $0.5 * 300 * 200 = 30,000$ The deadweight loss will be given by the difference between $30,000$ and 0, so it will be equal to $30,000$. The deadweight loss arises because the firm does not take the costs occurring to society in the production of steel into account when making decisions.

f) The tax has to be equal to the difference between private marginal cost and total social marginal cost at the efficient level. This will be given by $100 + 1 * 200 - 0.5 * 200 = 200$. Thus, a Pigouvian tax of 200 on the production of steel will ensure production of the socially optimal quantity in the market for steel.

g) We compare total net surplus with and without the tax. Consumer surplus becomes $0.5 * (400 - 300) * 200 = 10,000$. Producer surplus becomes $0.5 * 100 * 200 = 10,000$. Tax revenue is $200 * 200 = 40,000$. The additional social cost from the externality is the pollution cost of steel in the new equilibrium, which is given by $100 * 200 + 0.25 * 200^2 = 30,000$. New net surplus is given by $CS + PS + T - E = 10,000 + 10,000 + 40,000 - 30,000 = 30,000$. This is higher than the 0 derived under d), so we can recommend the government to introduce the Pigouvian tax.

Question 2

You are the owner (principal) of a high-end bike shop. You have one employee, Asger (agent) working for you. Asger sells either 0 bikes or 1 bike in any given hour. When Asger puts in high effort ($e = 1$), i.e. he actively approaches potential clients in the shop, asks questions and makes a friendly face, the probability that he sells a bike in a given hour is $p_H = 0.9$. When Asger puts in only low effort ($e = 0$), however, he sells a bike in a given hour with a much lower probability, $p_L = 0.5$. Every sold bike brings you (the principal) a revenue of 1,000. Asger's utility function is $u(e, w) = w - 90e$, where w is the wage you pay him per hour. Due to the current recession, Asger's labor market prospects outside your bike shop look rather gloomy. His reservation utility is $\bar{u} = 0$.

Suppose you cannot directly observe Asger's effort level but you observe, at the end of each hour, whether he has sold a bike or not. You will pay a wage w_H in case Asger has sold a bike and a wage w_L in case he has not.

Assume, for now, that you want to incentivize Asger to put in the high level of effort. Moreover, there is no minimum wage in your country. You can therefore assume that $w_L = 0$

a) Write down the individual rationality (IR) constraint that makes sure that Asger works for you rather than taking his outside option, and solve it for w_H.

b) Write down the incentive compatibility (IC) constraint that makes sure that Asger puts in a high level of effort rather than a low level, and solve it for w_H.

c) Set up formally your (the principal's) profit maximization problem including constraints, assuming that you want to induce high effort. What wage will you pay Asger? What are your expected profits and what is Asger's expected utility? Note: What you maximize is your expected hourly profit.

d) We have assumed that you (the principal) want to induce high effort by Asger. Is this optimal for you?

e) Would your answer to d) change if we instead assumed $p_L = 0.3$? Why or why not?

Now imagine that you install a camera in the shop. This allows you to directly observe the effort level Asger puts in whereas before you only saw whether he sold a bike or not when you did the accounting. With the new camera in the shop you can now condition Asger's wage on his effort level e instead of making it dependent on his success which, by definition, involved some random component. You can continue to assume that you want to incentivize Asger to put in high effort. Assume that the wage will be zero in case of low effort $w_L = 0$. You will maximize your profit by choosing a wage w_H for high effort.

f) How will the camera affect your expected profit? And how will it affect Asger's expected utility compared to the case under c) when you did not have the camera? Calculate your profit and Asger's utility for the new scenario with the camera and compare it to your findings under c).

g) Does anybody receive information rents or pay information costs under the two situations in c) and f)? If yes, who?

Solution Re-take exam question 2

a) $(w_H - 90) * 0.9 + (w_L - 90) * 0.1 \geq 0$, so using $w_L = 0$ gives $0.9 * w_H - 90 \geq 0$ or $w_H >= 100$.

b) $(w_H - 90) * 0.9 + (w_L - 90) * 0.1 \geq (w_H - 0) * 0.5 + (w_L - 0) * 0.5$, so using $w_L = 0$ gives $w_H \geq 225$.

c) $\max_{w_H}(1000 - w_H) * 0.9 + (0 - w_L) * 0.1$ s.t. $w_H >= 100$ (IR) and $w_H >= 225$ (IC) (can also directly use $w_L = 0$). The IR constraint will not bind but the IC constraint will (otherwise lower wage and thereby increase profit), so the wage will be $w_H = 225$. This gives Asger an expected utility of $u = 0.9 * w_H - 90 * e = 0.9 * 225 - 90 = 112.5$. Your expected hourly profits will be $(1000 - 225) * 0.9 = 775 * 0.9 = 697.50$.

d) Alternatively, the principal could induce low effort ($e = 0$) by the agent. To do this, the principal would set $w_H = w_L = 0$. Under this contract, Asger would expect to receive just his outside utility of zero and never exert effort since effort is costly to Asger and Asger cannot increase his wage by exerting effort. The expected profit of the principal would be $0.5 * (1000 - 0) = 500$, which is smaller than the 697.50 from c). Inducing high effort is therefore optimal.

e) No. A lower probability of the good state without effort would decrease the expected profits from the low effort contract. At the same time, the incentive compatibility constraint from b) would become easier to fulfill, allowing for a lower wage in the good state, thereby increasing profits from inducing high effort.

f) The individual rationality constraint becomes $w_H - 90 \geq 0$, given that the wage will now always be paid when Asger applies high effort. Since high effort can be enforced, there is no longer any incentive compatibility constraint. Profit maximization will set the wage such that Asger just receives his outside utility of zero ($w_H = 90$), so the IR constraint will bind (otherwise increase profits by lowering the wage). Your expected profit becomes $0.9 * 1000 - w_H = 900 - 90 = 810$. Thus expected net utility of Asger will be lower than the 112.5 under c), and your expected profits will be higher than the 697.50 from c).

g) In the situation from c), there is an information cost arising to you from having to incentivize Asger, while Asger receives information rents, which enables him to earn more than his outside utility. Under the situation in f) there is no longer asymmetric information, so there are no information costs or information rents.

Question 3

Consider the following payoff matrix:

Player 1\Player 2	Left	Right
High	6;0	5;5
Low	10;0	1;1

a) Determine all Nash equilibria in pure strategies of this simultaneous one-period game. Explain your reasoning in words.

The two players now decide to make a contract. Under this contract, Player 1 is obliged to pay the amount X to Player 2 whenever Player 2 chooses the action “Left”. Put differently, the contract states that whenever Player 2 plays “Left”, the payoff of Player 1 is reduced by X whereas the payoff of Player 2 increases by X.

b) Write down the new payoff matrix arising from this contract.

c) What is the minimum value of X such that (Low; Left) is a Nash equilibrium under the described contract?

d) What condition on X must hold such that (Low; Left) is the only Nash equilibrium in this game?

e) Imagine that $X = 7$. Is there a strictly dominant strategy for any of the two players?

Solution Re-take question 3

a) In a Nash Equilibrium all players will play strategies that are best responses to each other. For Player 2, the best response to Player 1 playing High is Right, while the best response to Player 1 playing Low is also Right. For Player 1, the best response to Player 2 playing Left is Low, and the best response to Player 2 playing Right is High. Thus, the only combination of strategies where both players play strategies that are best responses to each other is (High; Right).

b)

Player 1\Player 2	Left	Right
High	6-X;0+X	5;5
Low	10-X;0+X	1;1

c) It has to become a best response for Player 2 to play Left when Player 1 plays Low. This means that the payoff to Player 2 from playing Left has to be at least as high as the payoff from playing Right, which is 1. If Player 2 plays left, it will always be a best response of Player 1 to play Low, regardless of the value of X. Thus, we need that $X \geq 1$.

d) From b) we know that we need that $X \geq 1$ to have (Low; Left) being a NE. The other NE is [High, Right] (from a)). We need to set X such that one player is no longer playing a best response under that NE. Playing High will always be a best response of Player 1 to Player 2 playing Right. Thus, we need to set X such that it is no longer a best response for Player 2 to play Right when Player 1 plays High. Thus we need to have that the payoff to Player 2 of playing Left when Player 1 plays High is greater than 5. We therefore need $X > 5$.

e) A strictly dominant strategy will strictly dominate all other strategies for a player regardless of what the other player does (i.e. it will give a strictly higher payoff). Player 1 will always be better off with strategy Low if Player 2 plays Left, and will always be better off with strategy High if Player 2 plays Right, regardless of the level of X, so it can only be Player 2 who might have a strictly dominant strategy available. When $X = 7$, Left becomes a strictly dominant strategy for Player 2. (The only Nash equilibrium in this game will be (Low; Left), and player 2 is playing a strictly dominant strategy in it.)

Question 4

Do you agree or disagree with the following statements? Explain your answers.

a) "Second-degree price discrimination will typically involve lower profits for a monopolist than third-degree price discrimination."

b) "The standard model of labor supply tells us that a universal basic income is distortionary. In the model a universal basic income can be viewed as an hourly tax on labor and will lead to a deadweight loss." (Note: A universal basic income is a fixed amount being paid to everybody by the government. You can neglect how the basic income is financed in your answer.)

Solution Re-take exam question 4

a) This statement is true. Second-degree price discrimination describes a situation where the monopolist knows that there are two groups of consumers, but he cannot tell them apart from each other (there is asymmetric information). Third-degree price discrimination describes a situation where there are two groups of consumers whom the monopolist can tell apart. In the latter case, the monopolist can simply charge different monopoly prices from each group, which maximizes his profits. Under second-degree price discrimination the monopolist in addition has to make sure that the packages that he offers to consumers fulfill the consumers' incentive compatibility constraints, i.e. that no consumer has an incentive to lie about her type. This will involve information cost to the monopolist and information rents to some consumers, so profits to the monopolist will tend to be lower. Alternatively, the monopolist could accept that the IC constraints will not be fulfilled and offer the same package to everybody. However, this will also tend to give lower profits than third-degree price discrimination, as now the monopolist is not doing any price discrimination anymore.

b) This statement is not true. A universal basic income is a fixed amount being paid to everybody. Since it is independent of the choices of the agent, and in particular independent of the amount of labor supplied, it will not be distortionary. In the standard model of labor supply, a universal basic income will shift the budget line outward, as for any level of leisure the agent can now afford consumption that is higher by a fixed amount. This will in most cases increase both consumption and leisure, the latter meaning there will be a decrease in labor supply. However, there will not be an inward rotation of the budget line as in the case of an hourly tax on labor. If we removed the universal basic income and wanted to keep the utility of the agent constant, we would need to compensate the agent by the exact same amount of the universal basic income. This means that there is no deadweight loss.