Written exam for the M.Sc. in Economics, Winter 2013/14

Game theory

Final Exam (resit)/Elective Course/Master's Course

(3 hour, closed book exam)

21 February 2014

The exam has 3 pages in total (including cover page).

Explain each of your answers.

Question 1

In the model of knowledge we assumed that information functions are partitional which meant that they satisfied the properties

(P1) $\omega \in P(\omega)$ for every $\omega \in \Omega$,

(P2) if $\omega' \in P(\omega)$ then $P(\omega) = P(\omega')$.

We defined the knowledge function for an event $E \subseteq \Omega$ as

$$K(E) = \{\omega \in \Omega : P(\omega) \subseteq E\}.$$

which had the properties

(K4) (axiom of knowledge) $K(E) \subseteq E$

(K5) (axiom of transparency) $K(E) \subseteq K(K(E))$

(K6) (axiom of wisdom) $\Omega \backslash K(E) \subseteq K(\Omega \backslash K(E))$.

Take the following information function which is *not partitional*:
$\Omega = \{\omega_1, \omega_2, \omega_3\}$ and $P(\omega_1) = \{\omega_1\}$, $P(\omega_2) = \{\omega_2\}$ and $P(\omega_3) = \{\omega_2, \omega_3\}$

(a) Which of the properties P1 and P2 does P violate?

(b) Find an event E such that the knowledge function derived from P violates one of the properties K4-K6.

(c) Using this example, explain intuitively why an information function should be partitional.

Question 2

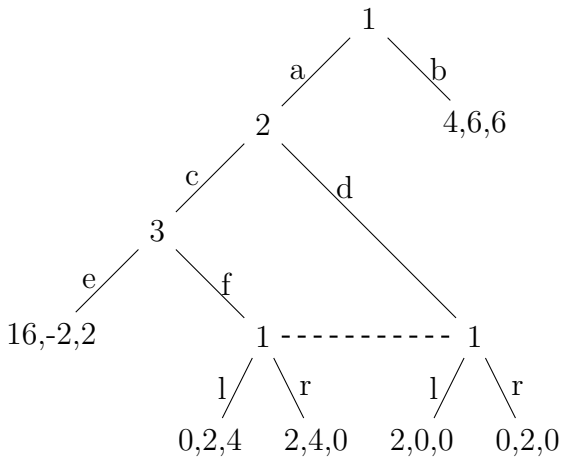
Consider the following strategic form game.

	L	M	R
T	-1,1	0,0	-1,1
B	0,0	-1,1	-1,1

a. Determine *all* (mixed) Nash equilibria of the game.

b. Show that each (mixed) Nash equilibrium is a perfect equilibrium.

c. Assume now that L is not an available action for player 2, i.e. his action set is $\{M, R\}$ (and everything else is as above). Which Nash equilibria are perfect equilibria in the modified game?



Question 3

Consider the three-player, extensive form game below.
The dashed line indicates that the two nodes are in the same information set of player 1!

a. Identify all subgames.

b. Find a *pure strategy* subgame perfect equilibrium. Show that this pure strategy subgame perfect equilibrium is not sequentially rational.

c. Derive a strategy profile that is sequentially rational and where beliefs satisfy Bayes' rule in every information set. (hint: consider mixed strategies)