Written Exam for the B.Sc. in Economics, Winter 2010/2011

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Second year

January 4, 2011

(3-hours closed book exam)

All questions, 1.1-1.3 and 2.1-2.8, to be answered, and all weighted equally.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e., if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Problem 1: Semi-endogenous versus endogenous growth

(In this problem the focus is on the verbal, intuitive explanations. Formal analysis can, however, be used in the explanations if wanted).

1.1 Define the concepts of semi-endogenous growth and (truly) endogenous growth, respectively.

1.2 Consider the R&D-based endogenous growth model (Chapter 9). Explain for this model conditions that can lead to semi-endogenous growth and (truly) endogenous growth, respectively.

1.3 Discuss whether semi-endogenous growth theory or endogenous growth theory delivers the most convincing explanation of long-run economic growth, that is, present some arguments in favour of each. Based on your discussion, express your own view.

Problem 2: The Solow model with scarce natural resources and the prospects for economic growth in the long run

(In this problem formal and computational elements are more important, but verbal, intuitive explanations are still important).

Equations (1) - (6), together with the parameter restrictions mentioned below, make up a Solow model with scarce natural resources for a closed economy. In all respects, including notation, the model is as known from the textbook's Chapter 7.

$$
\begin{aligned}
Y_t &= K_t^{\alpha} \left(A_t L_t\right)^{\beta} X^{\kappa} E_t^{\varepsilon} && (1)\\
K_{t+1} &= sY_t + (1-\delta) K_t && (2)\\
L_{t+1} &= (1+n) L_t && (3)\\
A_{t+1} &= (1+g) A_t && (4)\\
R_{t+1} &= R_t - E_t && (5)\\
E_t &= s_E R_t && (6)
\end{aligned}
$$

Equation (1) is the aggregate, Cobb-Douglas production function with the inputs of capital, K_t, effective labour, $A_t L_t$, "land", X (in fixed supply), and "energy", E_t (extracted "oil" in period t).

Equations (2), (3) and (4) describe capital accumulation, the evolution of the labour force and the evolution of technology, respectively.

Equation (5) is an identity saying that from period t to period $t+1$ the stock of oil is reduced by the amount used as energy input in period t. Finally, Equation (6) assumes that a given fraction s_E of the remaining stock of oil is used in each period.

The model's exogenous parameters are $\alpha,\ \beta,\ \kappa,\ \varepsilon,\ s,\ \delta,\ n,\ g,$ and s_E, all assumed to be strictly positive. Furthermore it is assumed that $\alpha+\beta+\kappa+\varepsilon=1$, that s, δ and s_E are strictly smaller than one, and that $s_E<\delta$. The state variables are K_t, L_t, A_t and R_t with given, strictly positive initial values K_0, L_0, A_0 and R_0.

2.1 Show that the production function exhibits constant returns to scale to the four inputs capital, labour, land and energy. Give an argument why this is plausible.

Define variables in per worker terms, $y_t \equiv Y_t/L_t,\ k_t \equiv K_t/L_t,\ x_t \equiv X/L_t$ and $e_t \equiv E_t/L_t$, and let approximate growth rates be $g_t^y \equiv \ln y_t - \ln y_{t-1},\ g_t^k \equiv \ln k_t - \ln k_{t-1},\ g_t^x \equiv \ln x_t - \ln x_{t-1}$ and $g_t^e \equiv \ln e_t - \ln e_{t-1}$.

2.2 Show that the "per capita" production function is:

$$y_t = k_t^\alpha A_t^\beta x_t^\kappa e_t^\varepsilon, \tag{7}$$

and that the following formula must hold for the approximate growth rates:

$$g_t^y = \alpha g_t^k + \beta g_t^A + \kappa g_t^x + \varepsilon g_t^e. \tag{8}$$

On a balanced growth path all variables considered have constant growth rates and the growth rates of capital and output are equal, so that the capital-output ratio, $z_t \equiv K_t/Y_t$, is constant.

2.3 Show that along a balanced growth path, g_t^y must be approximately a constant g^y (without subscript t) and that:

$$g^y \approx \frac{\beta}{\beta+\kappa+\varepsilon}g - \frac{\kappa}{\beta+\kappa+\varepsilon}n - \frac{\varepsilon}{\beta+\kappa+\varepsilon}\left(n+s_E\right). \tag{9}$$

Explain intuitively the influence on long run economic growth of the labour force growth rate and the "extraction rate" s_E, respectively

Figure 1 on page 5 plots the average annual growth rate of real GDP per worker between 1960 and 2003 against the average annual growth rate of the labour force over the same period across 83 countries.

2.4 Assuming (for now) that the model considered implies convergence to a balanced growth path in the long run, comment on Figure 1 in relation to Equation (9).

Let exact growth factors be $f_t^y \equiv y_t/y_{t-1}$, $f_t^k \equiv k_t/k_{t-1}$ etc.

2.5 Show that along a balanced growth path, the exact growth factor of income per worker must be a constant f^y and that:

$$f^y = (1+g)^{\frac{\beta}{\beta+\kappa+\varepsilon}} \left(\frac{1}{1+n}\right)^{\frac{\kappa}{\beta+\kappa+\varepsilon}} \left(\frac{1-s_E}{1+n}\right)^{\frac{\varepsilon}{\beta+\kappa+\varepsilon}}. \tag{10}$$

Show that equations (9) and (10) are in accordance with each other. (Hint: It can be of help to use repeatedly the approximation $\ln(1+a) \approx a$ for a close to zero).

2.6 Show that the model considered implies the following transition equation for the capital-output ratio:

$$z_{t+1} = \left(\frac{1}{(1+n)(1+g)}\right)^{\beta} (1-s_E)^{-\varepsilon} \left[s + (1-\delta) z_t\right]^{1-\alpha} z_t^{\alpha}. \tag{11}$$

2.7 Verify that under the assumptions made here, starting from any $z_0 > 0$, the transition equation (11) does indeed imply convergence of the capital-output ratio to a constant level z^*, and that:

$$z^* = \frac{s}{((1+n)(1+g))^{\frac{\beta}{\beta+\kappa+\varepsilon}} (1-s_E)^{\frac{\varepsilon}{\beta+\kappa+\varepsilon}} - (1-\delta)} \tag{12}$$

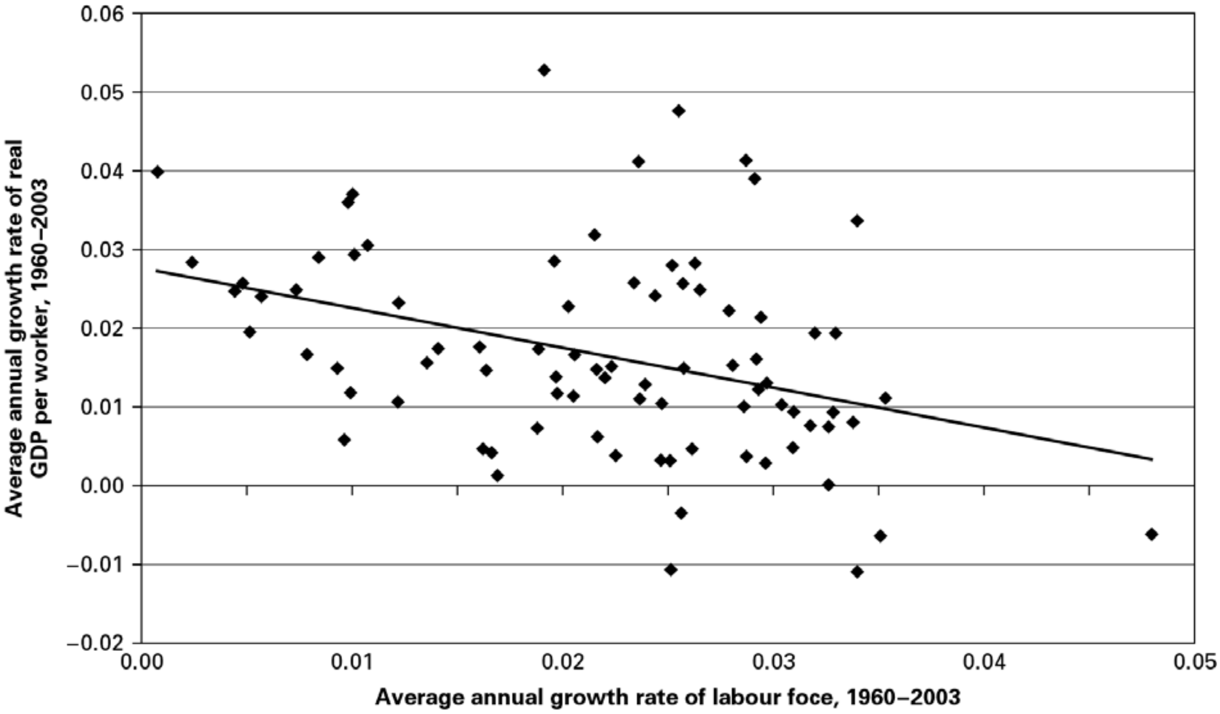
(Information: It follows from our assumptions, particularly $s_E < \delta$, that $z^* > 0$).

2.8 In view of the model considered here and the importance of scarce natural resources for aggregate production, discuss long-run growth optimism versus long-run growth pessimism. In particular, bring the importance of the substitution possibilities between technology (effective labour) and other inputs into the discussion. For the latter it could be relevant to derive the long run growth factor of income per worker under balanced growth (corresponding to the f^y of Question 2.5 above) if the aggregate production function were of the more general CES (constant elasticity of substitution) type:

$$Y_t = \left[\alpha K_t^{\frac{\sigma-1}{\sigma}} + \beta (A_t L_t)^{\frac{\sigma-1}{\sigma}} + \kappa X^{\frac{\sigma-1}{\sigma}} + \varepsilon E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0,\ \sigma \neq 1. \tag{1'}$$

(The Cobb-Douglas production function (1) corresponds to the case $\sigma = 1$). It is OK here to concentrate on the case of a relatively low degree of substitution between the inputs, $0 < \sigma < 1$.

Figure 1. Average annual growth rate of real GDP per worker against average annual growth rate of labour force across 83 countries



Note: The indicated line of best fit is estimated by OLS. The slope estimate is −0.5 with a standard error of 0.14.