University of Copenhagen
Department of Economics
Emiliano Santoro, Laura Sunder-Plassmann

Written exam
Macroeconomics C

June 2, 2015

Closed book exam, 3 hours
Number of questions: This exam consists of 2 questions.

---

1. Consider the following Ramsey model. The production function of the representative firm is given by
$$Y(t) = AK(t)^{\alpha}L(t)^{1-\alpha}E(t)$$
where the externality $E(t) = K^{AG}(t)^{1-\alpha}$ and K^{AG} is the total amount of capital in the economy. Consumers have utility function
$$u(c(t)) = \frac{c(t)^{1-\sigma}}{1-\sigma}$$
Assume no population growth ($n = 0$), no depreciation of capital ($\delta = 0$), and normalize population to 1. Initial household assets are given by a_0. Households supply 1 unit of labor inelastically and earn wage $w(t)$.
Suppose there is a tax policy that subsidizes savings by households: For each unit of assets that the household own at time t, the government gives the household a payment of ϕ (this is in addition to the rental rate $r(t)$ that the household receives). Assume ϕ does not vary over time. The government finances this expenditure with a labor income tax $\tau(t)$.
   (a) Write the household's maximization problem, set up the Hamiltonian,write down the Maximum Principle conditions and transversality condition.
   (b) Use your results from the previous part to show that the differential equations describing the dynamics of $c(t)$ and $a(t)$ are given by
$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}(r(t) + \phi - \rho)$$
$$\dot{a}(t) = (1 - \tau(t))w(t) + (r(t) + \phi)a(t) - c(t)$$
   Give a *brief* interpretation of both equations.
   (c) Write the maximization problem of the firm and solve this problem to obtain solutions for the rental rate $r(t)$ and wage rate $w(t)$.
   Assume the government runs a balanced budget each period. What is its budget constraint?

(d) Use the information from the previous parts to show that the equilibrium differential equations for $c(t)$ and $k(t)$ are given by

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}(\alpha A + \phi - \rho)$$
$$\dot{k}(t) = Ak(t) - c(t)$$

The social planner problem of this economy is

$$\max \int e^{-\rho t} u(c(t))dt$$
$$\dot{k}(t) = Ak(t) - c(t)$$

(g) Solve the social planner problem and derive the equilibrium differential equations for $c(t)$ and $k(t)$. Can the subsidy ϕ be chosen such that the equilibrium is optimal? If so, what is it, and what is the implied tax rate $\tau(t)$?

2. Consider a competitive economy with an infinite number of identical firms and households. The representative firm maximizes its profits, remunerating labor hours, h_t, at the wage rate, w_t. Production is carried out by means of the following technology:

$$y_t = z_t h_t^{\alpha} \tag{1}$$

where y_t denotes output and z_t is a technology shock. Households' income is allocated between consumption and equity (i.e., stocks of the representative firm). The representative household maximizes the discounted stream of expected utility:

$$\max_{c_t, h_t, \mu_t} \mathbf{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \frac{h_t^{1+\nu}}{1+\nu} \right] \right\}, \tag{2}$$

subject to the following constraint:

$$w_t h_t + \mu_{t-1}(d_t + q_t) = c_t + \mu_t q_t. \tag{3}$$

where μ_t denotes the amount of equity holdings at time t (i.e., the amount of shares), $d_t \equiv y_t - w_t h_t$ are the dividends (profits) rebated by firms to households at time t (these are taken as given by households) and q_t denotes the stock price at time t. The parameters (β, α and ν) are all positive, with $\beta \in [0, 1)$ and $\alpha \in [0, 1]$. Furthermore, households' time endowment is normalized to 1 and the aggregate resource constraint is such that $y_t = c_t$. Given this environment, address the following questions, providing adequate comment to the derivation of each and every result:

(a) Set up the representative firm's and household's optimization problems and derive the necessary first order conditions, respectively.

(b) Characterize the labor demand and supply schedules and prove that the equilibrium wage and hours are $w_t = \alpha^{\frac{a+v}{1+v}} z_t$ and $h_t = \alpha^{\frac{1}{1+v}}$, respectively.

(c) Find the equilibrium value of dividends. Following a positive realization of the technology shock (i.e., $z_t > 0$), equilibrium dividends are negative. True or false? Why?

(d) Starting from the first order condition with respect to μ_t, prove that

$$q_t = \beta \mathbf{E}_t \left[\frac{z_t \alpha^{\frac{\alpha}{1+v}}}{z_{t+1} \alpha^{\frac{\alpha}{1+v}}} \left(z_t \alpha^{\frac{\alpha}{1+v}} \left(1 - \alpha^{1+v}\right) + q_{t+1} \right) \right] \tag{4}$$

Under which value for α does equation (4) reduce to $\frac{q_t}{z_t} = \beta \mathbf{E}_t \left[\frac{q_{t+1}}{z_{t+1}} \right]$?