Written Exam - Macroeconomics III

(suggested answers)

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Question 1

a Factors are being remunerated as their marginal products, so that:

$$\begin{aligned} 1 + r_t &= B \text{ (in general } 1 + r_t = 1 + F_K - \delta) \\ w_t &= A \end{aligned}$$

Since $n = 0$ and $B > 0$, the economy is dynamically efficient.

b The Euler equation for individual savings is

$$c_{1t}^{-\sigma} = \frac{B}{1+\rho} c_{2t+1}^{-\sigma}$$

Replacing from the period budget constraints

$$(A - s_t)^{-\sigma} = \frac{B}{1+\rho} (B s_t)^{-\sigma} \tag{1}$$

Thus, solve for savings and remove the time subscript

$$s_t = k_{t+1} (= s = k) = \frac{A}{1 + B \left(\frac{B}{1+\rho}\right)^{-\frac{1}{\sigma}}}$$

Note that for the log case ($\sigma = 1$), $s = \frac{1}{2+\rho} A$.

c Note that with a linear production function saving will be the same in periods t_0 and $t_0 + 1$. Introducing social security modifies (1) as

$$(A - \tau - s_{t_0})^{-\sigma} = \frac{1}{1+\rho} (B s_{t_0} + \tau)^{-\sigma},$$

solving which gives

$$s_{t_0} = k_{t_0+1} = \frac{A - \left[1 + \tau \left(\frac{B}{1+\rho}\right)^{-\frac{1}{\sigma}}\right]}{1 + B \left(\frac{B}{1+\rho}\right)^{-\frac{1}{\sigma}}},$$

so savings and capital accumulation are depressed by the presence of social security. Thus, the reform will be supported by the old, but rejected by the young, as the rate of return is below the avalable rate of interest.

Question 2

a Substitute the budget constraint, the technology constraint and the demand function into the utility function, so as to get:

$$U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\lambda} Y_i^{\frac{\lambda}{\alpha}}$$

Maximizing w.r.t. Y_i:

$$\frac{\partial U_i}{\partial Y_i} = 0 \Rightarrow -\frac{1}{\eta}\left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} (Y_i)^{-\frac{1}{\eta}-1} Y_i + \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} - \frac{1}{\alpha} Y_i^{\frac{\lambda}{\alpha}-1} = 0$$

After some manipulation we obtain

$$\left(1 - \frac{1}{\eta}\right)\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} = \frac{1}{\alpha} Y_i^{\frac{\lambda}{\alpha}-1}$$

Which translates into

$$\frac{P_i}{P} = \frac{\eta}{\alpha(\eta - 1)} Y_i^{\frac{\lambda}{\alpha}-1}$$

b Taking logs and rearranging, we obtain the desired level of production:

$$p_i^* - p = \frac{\lambda - \alpha}{\alpha} y_i + \ln\left[\frac{\eta}{\alpha(\eta - 1)}\right]$$

Imposing homogeneity and exploiting the fact that households are the same and they have unit size:

$$p^* - p = \frac{\lambda - \alpha}{\alpha} y + \ln\left[\frac{\eta}{\alpha(\eta - 1)}\right]$$

Since $y = m - p$:

$$p^* - p = \frac{\lambda - \alpha}{\alpha}(m - p) + \ln\left[\frac{\eta}{\alpha(\eta - 1)}\right]$$

So that

$$p^* = \frac{\lambda - \alpha}{\lambda} m + \frac{\alpha}{\lambda} p + \ln\left[\frac{\eta}{\alpha(\eta - 1)}\right]$$

Imposing the notation $\phi \equiv \frac{\lambda - \alpha}{\alpha}$, we obtain:

$$p^* = \phi m + (1 - \phi) p + c \tag{2}$$

where $c = \ln\left[\frac{\eta}{\alpha(\eta-1)}\right]$.

c The model economy can be summarized by the following equations:

$$p^f = (1-\phi)p + \phi m \tag{3}$$

$$p^r = (1-\phi)E[p] + \phi E[m] \tag{4}$$

$$p = qp^r + (1-q)\,p^f \tag{5}$$

$$y = m - p \tag{6}$$

where $0 \leq \phi \leq 1$ and $0 \leq q \leq 1$.

Substituting (5) into (3):

$$p^f = (1-\phi)\left(qp^r + (1-q)\,p^f\right) + \phi m$$

and rearranging so as to bring p^f on the left-hand side of the equality:

$$p^f = p^r + \frac{\phi}{\phi + (1-\phi)\,q}\,(m - p^r) \tag{7}$$

Now, recall that $p^r = E[p^r]$. Substituting (7) into (5) we obtain

$$p = p^r + \frac{\phi\,(1-q)}{\phi + (1-\phi)\,q}\,(m - p^r)$$

Now substitute the latter into (4), so as to get:

$$p^r = E[m] \tag{8}$$

d Substituting (7) and (8) into (5):

$$p = E[m] + (m - E[m])\,\frac{\phi\,(1-q)}{\phi + (1-\phi)\,q}$$

Substituting the latter into (6):

$$y = (m - E[m])\,\frac{q}{\phi + (1-\phi)\,q}$$

e As ϕ lowers we observe greater real rigidity. This is what happens as α increases. Under these circumstances, as demand increases, proportionally less labor is required to increase production, given the attenuation of decreasing returns to scale. Thus, a lower cost is passed into prices and quantities vary by relatively more. The effect of a marginal increase in ϕ on the pass-through of $m - E[m]$ on p is:

$$\frac{\partial\left[\frac{\phi(1-q)}{\phi+(1-\phi)q}\right]}{\partial\phi} = \frac{\phi\,(1-q)}{\left[\phi + (1-\phi)\,q\right]^2} > 0$$

as for y:

$$\frac{\partial \left[\frac{q}{\phi+(1-\phi)q}\right]}{\partial \phi} = -\frac{q\,(1-q)}{\left[\phi+(1-\phi)\,q\right]^2} < 0$$

As expected, higher (lower) real rigidity the pass-through on prices decreases (increases), and the opposite holds true for y.