Written Exam for the B.Sc. or M.Sc. in Economics, Winter 2010/2011

Operations Research

Elective Course

January 21st 2011

3-hour open book exam

CORRECTION GUIDE
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RETTEVEJLEDNING

Part 1

<u>Question 1.1:</u>

z	x_1	x_2	s_1	s_2	RHS
	1	1	1		3
	3	1		1	6
1	-6	-1			0
		2/3	1	-1/3	1
	1	1/3		1/3	2
1		1		2	12

Pivot brings x_1 into basis in the second row (s_2 exits basis)

<u>Question 1.2:</u>

The solution after one simplex iteration is optimal since we maximize and all reduced cost are non-negative. We also notice that $x_1 = 2$ and $x_2 = 0$.

Sensitivity analysis on b_1: $1 + \theta \geq 0$ or $\theta \geq -1$ or $b_1 \geq 2$
OBJ = 12

Sensitivity analysis on b_2: $1 - 1/3\ \ \theta \geq 0$ or $-6 \leq \theta \leq 3$ or $0 \leq b_2 \leq 9$
$2 + 1/3\ \ \theta \geq 0$
OBJ = $12 + 2\ \theta$

<u>Question 1.3:</u>

$$
\begin{array}{ll}
\text{Min} & w = 3\ y_1 + 6\ y_2 \\
\text{S.t.} & 1\ y_1 + 3\ y_2 \geq 6 \\
& 1\ y_1 + 1\ y_2 \geq 1 \\
& y_1, y_2 \geq 0
\end{array}
$$

Question 1.4:

w	y_1	y_2	s_1	s_2	RHS
	-1	-3	1		-6
	-1	-1		1	-1
1	-3	-6			0
	-1/3	1	-1/3		2
	-1 1/3		-1/3	1	1
1	-1		-2		12

We are asked to set up the problem in a simplex tableau so that it can be solved using the dual simplex method. This implies that the solution is dual feasible (i.e. all reduced costs are non-positive for a minimization problem) and possibly primal infeasible (i.e. some negative RHS's). This is obtained from multiplying the two restrictions in the model by -1 and adding slack variables.

Comments on the results: We see that the solution is optimal (after one dual simplex iteration, the solution is now also primal feasible). The solution is $y_1=0$ and $y_2=2$ which was the same dual values that we saw in the z-line for the primal LP in Question 1.1.

Another way of concluding that we have reached an optimal solution is to notice that the objective function value of 12 is the same as the optimal solution for the primal LP and that the solution of the dual LP is feasible. This is due to that fact that for all feasible solutions to LP and dual LP we have that $z \leq w$.

Part 2

<u>Question 2.1:</u>

We use the Hungarian method:

4	3	1	2	2	-1 →	3	2	0	1	1
5	3	2	2	2	-2 →	3	1	0	0	0
2	2	5	3	3	-2 →	0	0	3	1	1
4	3	1	2	2	-1 →	3	2	0	1	1
4	3	1	2	2	-1 →	3	2	0	1	1
						↓	↓	↓	↓	↓
1	0	$\underline{\bar{0}}$	0	0	←	2	1	0	0	0
2	$\underline{\bar{0}}$	1	0	0	←	3	1	1	0	0
$\underline{\bar{0}}$	0	5	2	2	←	0	0	4	1	1
1	0	0	0	$\underline{\bar{0}}$	←	2	1	0	0	0
1	0	0	$\underline{\bar{0}}$	0	←	2	1	0	0	0

As we see in the last table there are many zero assignments – one is highlighted.
The weight of this assignment is 10 (1+3+2+2+2)

Question 2.2a:
The Assignment problem is a special case of the transportation problem in which all demands and supplies are equal to one unit.

We notice, however, that rows 1, 4 and 5 are identical just like columns 4 and 5 are identical. The 3 identical rows can therefore be modelled as a supplier with the supply of 3 units just like the 2 identical columns can be modelled as a demander with a demands of 2 units.

	D1	D2	D3	D4	Supply
S1	4	3	1	2	3
S2	5	3	2	2	1
S3	2	2	5	3	1
Demand	1	1	1	2	

Question 2.2b:

Fundamentally, both VM (Vogel's Method) and NCM (Northwest Corner Method) are heuristics that yield feasible but not necessarily optimal solutions.

One can imagine a case where NCM in fact finds the optimal solution (because the problem is designed so that the optimal solution revolves around the diagonal of the cost matrix) but where VM does not find the optimal solution. In such a case the NCM works better than VM even if the opposite is the more general case.

Part 3

<u>Question 3.1</u>

We will omit a state variable and use only the stage variable, t, describing an hour.
Let f_t be the number of shows Mr. Frenzy can see from the beginning of hour t and until the end.
Obviously, we have that $f_{49} = 0$.
For an earlier hour, he can either find a show that starts on that hour, or wait until the next hour to find a show to see. So the recursion function can be written as:

$$f_t = Max\{ f_{t+1} \,;\, Max(1 + f_{e(j)+1}) \text{ where } s(j)=t \}$$

This can be calculated from t=48 down to t=1, with f_1 being the answer to Mr. Frenzys problem.

<u>Question 3.2</u>

We now need a state variable to describe the remains on the budget and subtract the amount $w(j)$ when show number j is chosen.

Let $f_t(w)$ be the number of shows Mr. Frenzy can see from the beginning of hour t and until the end, when he has the amount w left. We now have that $f_{49}(w) = 0$ for all w.

Then we can expand the recursion function to:

$$f_t(w) = Max\{ f_{t+1}(w);\, Max(1 + f_{e(j)+1}(w-w(j))) \text{ where } s(j)=t \text{ and } w(j) \leq w \}$$

Part 4

Question 4.1:
This is in fact the News Vendor Problem in disguise.
The energy company has to decide an mount, *q*, to sell in advance.
The production (the "demand"), *d*, will not be realized until later, but we know the cumulative distribution function for the production, ***D***.

In the overstocked situation, we have that q>d and in this case we need to buy back the power at the unit price of c_o=5.
In the understocked situation, we have that q<d and we need to sell additional power. This we can do at the unit cost of c_u=2.

According to Winston, the optimal size of q is then the $c_o/(c_o+c_u)$ percentile of ***D***.

We have that $c_o/(c_o+c_u)$ = 5/(5+2)=5/7 so we see that <u>q* = a + (5/7)(b – a)</u>

Question 4.2:

This is a queing problem of type M/M/1/GDI/∞/∞

The windmills break down (arrives to the system) with the intensity of $\lambda = 20/365$ and the service process finishes repairs with the intensity of $\mu=1/3$.

We are asked about the size L, the average number of "people" in system, and we have that
$L = \lambda/ (\mu-\lambda) = (20/365)/(1/3 - 20/365) = 0.1967$ which is the average number of wind mills waiting for service or being serviced at a given moment in time.