Guide[1] to answers, Written Exam for the B.Sc. or M.Sc. in Economics

Microeconomics A, 2nd Year

February 2014

Problem 1

Consider the following claim:

"Assume we have a consumer who consumes two goods and whose preferences can be represented by the utility function $u(x_1,x_2)$. Also, assume that the price system is (p_1,p_2), and the consumer has the exogenous money income I.

Now, the solution to the consumer's utility maximization problem can be found by simply solving the following system of two equations with two unknowns (x_1 and x_2):

$$[\partial u(x_1,x_2)/\partial x_1]/\,[\partial u(x_1,x_2)/\partial x_2] = p_1/p_2$$
$$p_1 \cdot x_1 + p_2 \cdot x_2 = I$$

Solve these, and we have found the unique consumption plan which maximizes utility".

- Please comment on the claim above.

Answer: It is true that one often needs to find a consumption plan on the budget line that has numerical MRS equal to relative prices, which is the economic content of the two equations. The approach above, however, overlooks some caveats:

- *Non-continuous goods*
- *non-monotonic preferences*
- *non-differentiability of u*
- *non-uniqueness of solutions*
- *corner solutions*
- *second-order-conditions (non-convexity).*

A good answer need not list all of these, but the major part of them.

Problem 2

Peter consumes beer (commodity 1) and sandwiches (commodity 2), both in continuous quantities. Peter has preferences which can be represented by the utility function $u(x_1,x_2) = x_1^{½} \cdot x_2^{½}$. It can be shown that Peter has the Marshall demand function $x(p,I) = (½I/p_1, ½I/p_2)$ and the Hicksian (compensated) demand function $h(p,u) = (up_1^{-½}p_2^{½}, up_1^{½}p_2^{-½})$.

Peter is on a stipend, giving him an exogenous money income of 120. Currently, the price of beer is 1, and the price of sandwiches is 1.

[1] What is presented here is not a full, satisfactory answer to the problems, but indicates the correct results and important points to be made. Important in most answers is that the student shows command of mathematical expressions, provides a clear graphical illustration, and sheds light on the economic intuition behind the results.

Consider the following two Slutsky equations, $u = u(x(p,I))$:

$$\partial x_1(p,I)/\partial p_1 = \partial h_1(p,u)/\partial p_1 - [\partial x_1(p,I)/\partial I]\cdot x_1(p,I)$$

$$\partial x_2(p,I)/\partial p_1 = \partial h_2(p,u)/\partial p_1 - [\partial x_2(p,I)/\partial I]\cdot x_1(p,I)$$

- Verify that these two equations to hold true at price system $p = (1,1)$ and income $I = 120$.

Answer: For beer, the derivative of Marshallian demand wrt. changes in beer price, $-\frac{1}{2}I/p_1^2$ *, has value* -60*, the substitution part,* $-\frac{1}{2}up_1^{-3/2}p_2^{1/2}$ *, takes on value* -30*, and the income part is* -30 *(first term is* $\frac{1}{2}/p_1$*, i.e.* $\frac{1}{2}$*, and beer quantity demanded is 60). For sandwiches, the Marshallian derivative is 0, the substitution part,* $\frac{1}{2}up_1^{-1/2}p_2^{-1/2}$*, takes on value +30 (u is 60), and the income part is* -30 *(similar to above).*

<u>Problem 3</u>

Anne consumes food (commodity 1) and clothing (commodity 2), both in continuous quantities. She has the utility function $u(x_1,x_2) = x_1\cdot x_2^2$. She is on a stipend, giving her the income $I = 90$, and currently the price system is (1,1).

- a) Identity Anne's utility maximizing consumption plan

The government levies a unit tax of 1 on clothing, hence changing the price system to (1,2)

- b) Identity Anne's consumption after the unit tax has been introduced
- c) How much tax revenue is raised from Anne?
- d) What would happen if the government, instead of introducing the unit tax, and keeping the price system at (1,1), asked Anne to pay a lump-sum tax corresponding to the revenue amount found in c?

Answer: Anne's Marshall demand function is $x(p,I) = ((1/3)I/p_1,(2/3)I/p_2)$*. Consumption starts out being (30,60) and becomes (30,30) after the introduction of the unit tax. The tax revenue raised is 30. At prices (1,1) and income 60 (=90–30), i.e. the lump sum tax alternative, consumption is (20,40). Utility is higher in this case, as it becomes 32000, whereas utility is 27000 in the unit tax case. This illustrates how a unit tax, distorting relative prices, implies a dead-weight loss.*

<u>Problem 4:</u>

Consider an exchange economy with the two consumers, Arnie and Bernie. There are two commodities: Commodity 1 is food, commodity 2 is drinks, and both commodities can be consumed in continuous quantities.

Arnie has the utility function $u_A(x_{1A},x_{2A}) = \ln(x_{1A}) + x_{2A}$, and Bernie has $u_B(x_{1B},x_{2B}) = \ln(x_{1B}) + x_{2B}$.

The economy has the initial endowment (e_1, e_2), with both e_1 and e_2 being strictly positive.

- a) Identify the efficient (Pareto Optimal) allocations in this Edgeworth economy and illustrate these in an Edgeworth Box
- b) If you have identified an efficient allocation in which Arnie has a strictly positive consumption of food but zero consumption of drinks, identify a price system that can implement this allocation in a market equilibrium with income transfers.

Answer: The two agents have identical MRS's at $x_{1A} = x_{1B} = ½e_1$. There are, however, also corner solutions, with A having consumption $(x_{1A}, 0)$, with $0 \leq x_{1A} \leq ½e_1$, and B having the residual consumption, and with (x_{1A}, e_2), with $½e_1 \leq x_{1A} \leq e_1$, and B having the residual consumption. In the first part of these, the equilibrating relative price is determined by B's absolute value of MRS, which is $(e_1 - x_{1A})^{-1}$, in the second part it is $(x_{1A})^{-1}$

<u>Problem 5</u>:

Comment on the following claims:

- a) "For a firm operating in a market characterized by perfect competition, the firm's short-run supply curve will be equal to its short-run marginal-cost curve"
- b) "For a firm operating in a market characterized by perfect competition, the firm's demand for labor will increase in the long run, if the price of capital increases (all other prices and costs remaining the same as before)"

Answer: a) is wrong in two senses: It is only the part of the MC-curve that has a positive slope and lies above the short-term AVC-curve (Nechyba dubs it the AC-curve) that constitutes the supply curve. b) Not necessarily; while it is true that there will be a substitution effect increasing demand for labor, the increased costs will also reduce output in the long run, which will tend to lower demand for labor, so labor demand may decrease in the long run.

<u>Problem 6</u>:

Comment on the following claim:

"When the interest rate decreases, the reward for saving is weakened, so every rational consumer will want to decrease his or her savings".

Answer: The claim is false, as it neglects the wealth effect. While it is true that the incentive to save is weakened (the substitution effect), the wealth effect for a consumer with positive savings works in the opposite direction: The wealth decreases, decreasing consumption in youth, hence increasing savings. If the latter effect dominates, savings may go up. Formally, the Slutsky equation drives home this point, as a lower interest rate corresponds to a decrease in the price of today's consumption, p_1. $\partial z_1(p)/\partial p_l = \partial h_1(p,u)/\partial p_l - [\partial x_1(p,I)/\partial I] \cdot [z_l(p)]$, where z_1 is excess demand as young, i.e. the negative value of savings, so the first term on the RHS is negative, the second derivative is positive, and with z_1 being negative and the minus sign in front, this adds up to the two parts on the RHS having opposite signs, and if the wealth effect dominates, the expression becomes positive, such that a higher p_1 means lower savings, such that a lower p_1 means higher savings.

Mtn, 16 February 2014