Exam

Econometrics II

December 2016

Introduction and Formal Requirements

The exam is a portfolio exam consisting of four separate parts. The first three parts are based on the assignments you have worked with during the semester. The last part is a theoretical part.

Each of the four parts is weighted by 25 percent in the overall assessment.

Formal Requirements:

(1) You are allowed to work in groups of up to three students. The formal requirements and the assessment criteria are the same for individuals, groups of two, and groups of three students.

(2) You are only assessed based on what you hand in for the final exam, so the assignments you have handed in during the semester are not included in the assessment.

(3) Your answers to each of Parts 1, 2, and 3 must be maximum of 12,000 characters including spaces, math, and formulas (corresponding to 5 normal pages), plus a maximum of two pages with figures and tables.

(4) For Part 4, there is no limit on the maximum number of characters.

(5) You must hand in the exam in one pdf-document, which includes a frontpage and your answer to each of the four parts of the exam.

(6) On the frontpage, you must provide a count of the characters (including spaces, math, and formulas, but excluding tables and figures) for each of Part 1, 2, and 3.

(7) If you hand in as a group, you must specify on the frontpage who is responsible for the individual parts of the exam.

(8) The exam must be handed in at the Digital Exam platform (`http://eksamen.ku.dk`).

(9) **The deadline is at 10:00 in the morning on Thursday, December 22.**

Part 1
What is Your Forecast of GDP Growth?

The Case: Forecasts of the future economic growth play an important role in economic policy. But, forecasting the growth rate in the gross domestic product (GDP) is difficult. In Denmark, major institutions, such as the Ministry of Finance, have been criticized for not being able to provide accurate forecasts for decision-makers. Recently, that led the Danish fact-checking television show *Detektor* to ask Assistant Professor Søren Hove Ravn the question: *Why is it so difficult to forecast economic growth?*[1]

Using quarterly data covering the period 1980:1–2015:3, the goal of this assignment is to estimate a univariate dynamic model and use the model to forecast the future growth in GDP.

The Data: The dataset Assignment2.in7 contains the following variable from Nationalbanken's MONA database (amounts in billion DKK):

Y Gross domestic product (GDP), nominal prices.

Consider the following transformations:

$$y_t = \log(\mathsf{Y}_t) \qquad\qquad \Delta y_t = y_t - y_{t-1}.$$

The Assignment: Construct a univariate time series model for the nominal growth in Danish GDP and forecast the growth rate over the period 2015:4–2025:4.

You are only required to assess stationarity of the variables based on the graphs of the data. You are allowed to consider only the class of autoregressive models, AR(p) models.

Hints:

(1) For the graphical analysis, use any transformations of the variables you find are relevant (among the transformations proposed above, or any other ones), and explain your choices.

[1] You can watch the clip here (in Danish): `https://www.dr.dk/tv/se/detektor-tv/detektor-2016-09-22#!/12:39`. See also this article in daily newspaper *Information* (also in Danish): `https://www.information.dk/indland/2013/12/hvorfor-oekonomer-saa-svaert-ved-forudsige-vaeksten`.

(2) When using the OLS or ML estimator, carefully discuss which assumptions are required and explain if they are fulfilled or not. If not, what are the consequences on your estimation results? Can you improve on this?

(3) If you come up with several model specifications, select the one that seems the most relevant to you, and justify your decision. Remember that in most cases, the simpler the better.

(4) If you perform some statistical tests, be precise about the hypotheses you test, why you test them, and what your conclusions are.

(5) Consider whether the same model should apply to the entire period.

Formal Requirements:

(1) You must hand in a report that (*i*) presents your graphical analysis, (*ii*) describes the econometric model, (*iii*) outlines the modeling progress (e.g., the approach you have taken, the alternative models you have tried, etc.), (*iv*) presents your preferred model including interpretation and statements on economic and statistical significance, and (*v*) discusses the potential weaknesses of the model.

Limitations:

(1) You are not required to perform formal unit root tests, such as the Dickey-Fuller unit root test.

Part 2
Money Demand and Interest Rates

The Case: Economic theory on money growth and inflation suggests the equilibrium relation in the money market,[2]

$$\frac{M}{P} = f(Y^R, R), \tag{2.1}$$

where M is the money stock, P is the price level, Y^R is real income, and R is the nominal interest rate. The demand for real money balances, given by $f(Y^R, R)$, is increasing in real income, $\partial f(Y^R, R)/\partial Y^R > 0$, and decreasing in the interest rate, $\partial f(Y^R, R)/\partial R < 0$.

Assuming log-linearity, the equilibrium relation can be written as,

$$m - p = a_1 y^r + a_2 R, \tag{2.2}$$

where lower-case variables denote the log of the variables in (2.1).

Finally, by assuming homogeneity between the real money stock and real income, $a_1 = 1$, and replacing the nominal interest rate with the opportunity cost of holding money relative to bonds gives an equilibrium relation for money velocity,[3]

$$v = -a_2(Rb - Rm), \tag{2.3}$$

where $v = y^r - (m - p)$ is the (log) money velocity, Rb is the bond rate, and Rm is the bank deposit rate.

Using Danish quarterly data covering the period 1980(1)—2005(4), the purpose of this assignment is to test the empirical validity of the theoretical relationship in equation (2.3).

The Data: The file `Assignment3.in7` contains the following variables:

V	Money velocity (log).
Rm	Bank deposit rate.
Rb	10-year bond rate.

The Assignment: Conduct an empirical analysis to determine if a long-run relationship exists between money velocity and the interest rate spread. If you find such a relationship, investigate how the corresponding equilibrium is sustained.

[2] See, for example, chapter 11 in Romer, P., *Advanced Macroeconomics*, 4th Edition. McGraw Hill.

[3] See section 2.2, pages 15-18 in Juselius, K., *The Cointegrated VAR Model: Methodolody and Applications.* Oxford University Press.

Hints:

(1) Conduct a graphical analysis to determine the stationarity of the time series variables, and to check if they seem to cointegrate.

(2) Implement unit root testing to formally confirm (or reject) the conclusions from your visual inspection.

(3) Perform a cointegration analysis based on the Engle-Granger two-step procedure and/or a dynamic model (ADL or ECM).

(4) Use an error-correction model to investigate the forces driving the system back to the equilibrium after a deviation from the long-run relationship.

Formal Requirements:

(1) You must hand in a report that (*i*) presents your graphical analysis, (*ii*) describes the econometric model, (*iii*) outlines the modeling progress (e.g., the approach you have taken, the alternative models you have tried, etc.), (*iv*) presents your preferred model including interpretation and statements on economic and statistical significance, and (*v*) discusses the potential weaknesses of the model.

Part 3
Is there Empirical Evidence of a Taylor Rule with Interest Smoothing under Different Fed Chairs?

The Case: Since the seminal paper by John B. Taylor published in 1993[4], many economists have suggested that monetary policy can be described by a simply *Taylor rule*,

$$r_t^* = r^* + \beta \cdot (E(\pi_{t+12}|\mathcal{I}_t) - \pi^*) + \gamma \cdot E(\widetilde{y}_t|\mathcal{I}_t), \tag{3.1}$$

where r_t is the Federal Funds rate, r_t^* is Federal Funds target rate, r^* is the target rate for r_t in equilibrium, π_t is the year-on-year inflation rate, $E(\pi_{t+12}|\mathcal{I}_t)$ is the forecast of inflation at time $t+12$ based on the information set $\mathcal{I}_t$, π^* is an inflation target, $\widetilde{y}_t = y_t - y_t^*$ is the output gap, and $E(\widetilde{y}_t|\mathcal{I}_t)$ is the forecast of the output gap at time t based on the information set $\mathcal{I}_t$.

However, it has been found difficult for estimated Taylor rules based on equation (3.1) to match the high persistence of the observed Federal Funds rate. Consequently, some economists have suggested to interpret the Taylor rule in (3.1) as a *target rate* (r_t^*) and allow for interest rate smoothing by specifying the relation for the *actual rate*, r_t:

$$r_t = \rho \cdot r_{t-1} + (1-\rho) \cdot r_t^*. \tag{3.2}$$

That implies that the Federal Funds rate only adjusts to eliminate a fraction $(1-\rho)$ of the gap between the actual rate and the target rate. Equations (3.2) and (3.1) can be combined and the parameters in the two equations can be estimated, for example, using *generalized method of moments* (GMM) estimation.

An empirical question which has attracted a lot of attention, is whether the monetary policy reaction function has changed with the Chair of the Fed.[5] Using monthly data for the US economy covering the period from 1983(1) to 2014(1), the aim of this assignment is to analyze if there is empirical evidence of a Taylor rule with interest rate smoothing

[4] Taylor, John B., "Discretion versus policy rules in practice", *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, pp. 195-214. `http://dx.doi.org/10.1016/0167-2231(93)90009-L`

[5] See, for example, the empirical analysis in Clarida, R., J. Galí, and M. Gertler (2000), "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory", *The Quarterly Journal of Economics*, 115 (1), pp. 147-180. `http://dx.doi.org/10.1162/003355300554692`

during the periods where Paul Volcker, Alan Greenspan, and Ben Bernanke were Chair of the Fed.

The Data: The file `Assignment5.in7` contains the variables:

ff	Federal Funds rate, monthly average.
infl	Inflation rate, year-on-year.
inflex	Inflation rate excluding food and energy, year-on-year.
caputil	Capacity utilization, total industry. Percent of capacity.
capgap	Measure of the output gap. Deviation of caputil from its mean.
unr	Unemployment rate, percent.
unrgap	Measure of the output gap. Deviation of unr from its mean.
bond	10-year Treasury constant maturity rate.

The data is for the US economy and the data source is the FRED Database maintained by the Federal Reserve Bank of St. Louis.

The Assignment: Conduct an empirical analysis based on *generalized method of moments* estimation to analyze if there is empirical evidence of a Taylor rule with interest smoothing during the periods where Paul Volcker, Alan Greenspan, and Ben Bernanke were Chair of the Fed.

Hints:

(1) You must explain how the *generalized method of moments* estimator can be derived from the equations (3.1) and (3.2). In particular, explain how the moment conditions can be derived and how you choose your instruments for GMM estimation.

(2) You should explain the requirements for consistent estimation and valid statistical inference of the GMM estimator, and you should discuss if these requirements are fulfilled in your empirical analysis.

(3) You should report some robustness analysis for your empirical results. Hints (4) to (7) below gives some suggestions for robustness analyses along different dimensions.

(4) The data set contains two inflation rates, one based on the personal consumption expenditures and one for the personal consumption expenditures excluding food and energy (including oil), and two proxies for the output gap, one based on the capacity utilization and one based on the unemployment rate. You can choose which variables to work with, and you can see if your results are robust to the choice of variables.

(5) You can try different choices of instruments. Are your results robust for your choice of instruments? Argue for your preferred choice of instruments and report the Hansen test for overidentification.

(6) You can estimate your model with different weight matrices. What seems like a reasonable assumption about the moments, and what does it imply for the choice of the weight matrix? Are your results robust for different weight matrices?

(7) You can try estimate the Taylor rule without interest rate smoothing by setting $\rho = 0$. Does your estimated Taylor rule depends on whether you allow for interest rate smoothing?

(8) Conduct a graphical analysis to detect if there is evidence for a Taylor rule. You can plot the actual Federal Funds rate together with the rate predicted by the estimated reaction function if it can be interpreted as a Taylor rule.

Formal Requirements:

(1) You must hand in a report that (*i*) presents your graphical analysis, (*ii*) describes the econometric model, (*iii*) outlines the modeling progress (e.g., the approach you have taken, the alternative models you have tried, etc.), (*iv*) presents your preferred model including interpretation and statements on economic and statistical significance, and (*v*) discusses the potential weaknesses of the model.

Limitations:

(1) You are not required to report robustness analyses along all possible dimensions discussed under the hints above. Choose the dimensions you find most interesting and focus your robustness analysis on those dimensions.

Part 4
Theoretical Problems

#4.1 Cointegration and Error-Correction

Consider the system for $x_t = (x_{1t}, x_{2t}, x_{3t})'$, given by,

$$x_{1t} = \rho x_{1t-1} + \delta + \epsilon_{1t} \tag{4.1}$$

$$x_{2t} = b_1 x_{1t-1} + b_3 x_{3t-1} + \epsilon_{2t} \tag{4.2}$$

$$x_{3t} = x_{3t-1} + \epsilon_{3t}, \tag{4.3}$$

for $t = 1, 2, ..., T$, where $|\rho| < 1$, the error terms are uncorrelated, independent over time and normally distributed, i.e. $\epsilon_{it} \sim N(0, \sigma_i^2)$ for $i = 1, 2, 3$, and the initial values $x_0 = (x_{10}, x_{20}, x_{30})'$ are given.

(1) Derive the moving-average representation for each of the three variables in x_t.
Explain the dynamic properties of the three variables based on the moving-average representations.

(2) Use the moving-average representations of the variables in x_t to show that a cointegration vector, β, exists, so that the linear combination $\beta' x_t$ is a stationary process. Give an interpretation of the concept of cointegration.

(3) Show that at least one of the variables in x_t are error-correcting.
Derive the relevant error-correction model (ECM) and explain how it is related to the concept of cointegration.

(4) Now, consider the autoregressive distributed lag (ADL) model for x_{2t}, given by,

$$x_{2t} = \delta_2 + \theta_1 x_{2t-1} + \theta_2 x_{2t-2} + \phi_0 x_{1t} + \phi_1 x_{1t-1} + \phi_2 x_{1t-2} + \psi_0 x_{3t} + \psi_1 x_{3t-1} + \psi_2 x_{3t-2} + \varepsilon_t, \tag{4.4}$$

for $t = 1, 2, ..., T$, where the error term is assumed independent over time and normally distributed, i.e. $\varepsilon_t \sim N(0, \sigma^2)$, and the initial values x_0 and x_{-1} are given.
Derive the corresponding error-correction model for x_{2t}, and explain how it is related to the model in (4.2).

(5) Explain how one can test for cointegration using the *PcGive test for no error-correction* in the ADL or ECM model for x_{2t}.

(6) Explain briefly how one can test the hypotheses, $\mathcal{H}_0 : \phi_1 = 0$ and $\mathcal{H}_0 : \psi_2 = 0$, in the ADL model (4.4).

(7) Explain briefly the limitations of using a single-equation cointegration approach based on the ADL model in (4.4) and the corresponding error-correction model.

#4.2 Forecasting Volatility

Consider the GARCH-X model for y_t given the stationary exogenous variables x_t,

$$y_t = \delta + \epsilon_t \tag{4.5}$$

$$\epsilon_t = \sigma_t z_t \tag{4.6}$$

$$\sigma_t^2 = \varpi + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \phi x_{t-1}^2, \tag{4.7}$$

for $t = 1, 2, ..., T$, where $\varpi > 0$, $\alpha \geq 0$, and $\beta \geq 0$, the innovation z_t is assumed independent over time and standard normally distributed, i.e. $z_t \sim N(0, 1)$, and the initial values are given. The exogenous variable x_t influences the conditional variance of y_t in (4.7). Assume that x_t is given by a stationary first-order autoregressive process,

$$x_t = \rho x_{t-1} + \eta_t, \tag{4.8}$$

where $|\rho| < 1$ and the error term η_t is independent of z_t, independent over time and normally distributed, i.e. $\eta_t \sim N(0, \sigma_x^2)$.

(1) State the condition for (weak) stationarity of the process for y_t.
Derive the unconditional variance of the innovations ϵ_t, $\sigma^2 = E(\epsilon_t^2)$, under the assumption that the stationarity condition is fulfilled.

(2) Derive the volatility forecasts of σ_{T+1}^2 and σ_{T+2}^2 conditional on the information set at time T, $\mathcal{I}_T$, i.e. derive $\sigma_{T+1|T}^2 = E(\epsilon_{T+1}^2|\mathcal{I}_T)$ and $\sigma_{T+2|T}^2 = E(\epsilon_{T+2}^2|\mathcal{I}_T)$.

(3) Explain briefly what happens to the volatility forecasts, $\sigma_{T+k|T}^2$, as the forecasting horizon, k, increases.

(4) Explain how you would estimate the parameters of the model in (4.5)-(4.7) using maximum likelihood.