# Written Exam for the B.Sc. or M.Sc. in Economics winter 2014-15

# Microeconomics B

Final Exam

21/01/2015

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of xx pages in total

### Exercise 1

Consider the market for loanable funds. Denote by r the price of borrowing one dkk such that the demand for loans is D(r) = a - br, with a, b > 0 and the financial sector can provide loans at a constant cost of c > 0 per dkk.

The market for loanable funds is characterized by perfect competition and loans are always paid back by the borrower.

a) Derive the market equilibrium for loanable funds: the price,  $r^*$ , and quantity of loans,  $q^*$ .

Consider imposing a tax of t per borrowed dkk which the financial sector must pay to the government.

- b) What is the new equilibrium price the borrowers pay and the sellers receive? Furthermore find the change in the amount of loans issued as a consequence of the tax.
- c) Comment on the following proposition from a politician:

"The financial sector must pay for their part in the financial crisis, therefore we should tax loanable funds."

#### Solution

- a) We get that  $r^* = c$  and  $X^* = a bc$
- b) The borrowers pay  $r_b = c + t$ , the sellers receive  $r_s = c$ , if the price the seller receives differ from the marginal costs,  $r_s < c$ , the supply of loans would collapse since no financial institution would supply any loan. The new equilibrium loans are  $X' = a bc bt = X^* bt$  thus reducing the amount of loans by  $X^* X' = bt$ .
- c) Since the sellers' price is unchanged the tax incidence imposed on them is zero, while the borrowers pay the entire tax revenue  $(r_b r^*)X' = tX'$ . The entire tax burden is paid by the borrowers. This is due to the perfect elastic supply of loanable funds.

#### Exercise 2

Consider a small town on the country side in which a local farmer produce wheat and a paper mill produce paper.

The paper mill produces paper to a perfectly competitive market in which it can sell its output at a price  $p_x = 5$ , while the costs are  $c_p(x, e) = 4x + x^2 + e(e - 3)$  where e is the emission of waste water in the local river.

The river also provides water for irrigation for the farmer, but waste water increases the farmer's costs by lowering the wheat yields. The farmer can sell his output at the nearby city at a price  $p_v = 10$ , and the costs of the farmer is  $c_f(y, e) = 8y + y^2 + ey$ .

- a) Assume that each firm chooses his production to maximize his own profit, what will be the resulting production and profit levels.
- b) Are the production decisions efficient? Explain what generates the difference between your answer in a) and b).
- c) Can you offer a remedy that implemented the efficient production levels?

## Solution

- a) To maximize profit, each firm equals the price with marginal costs:  $p_x = \frac{dc}{dx_p}(x_p, e) \Leftrightarrow 5 =$ 4 + 2x and optimal choice of emission of waste water is  $\frac{dc}{de} = 0 \Leftrightarrow -3 + 2e = 0$ , such that the optimal production becomes  $e^* = \frac{3}{2}$  and  $x = \frac{1}{2}$ , such that for the farmer 10 = 8 + 2y + $e^* = 8 + 2y + \frac{3}{2}$  and thus  $y = \frac{1}{2}(2 - \frac{3}{2}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$ . Then the profits become  $\pi_p = \frac{1}{4} + \frac{9}{4} = \frac{1}{4}$  $\frac{10}{4}$  and  $\pi_f = \frac{1}{16} > 0$ .
- b) Efficiency requires that the aggregate profit is maximized. To maximize the aggregate profits,  $\pi_A = p_p x_p + p_w x_w - c(x_w, e) - c(x_p, e)$ , hence the profit function becomes  $5x + 10y - 4x - x^2 - e(e - 3) - 8y - y^2 - ye$

they must solve

$$5 = 4 + 2x$$

$$\frac{d}{de}(-e(e-3) - ye) = 0 \Leftrightarrow -2e + 3 = y$$

And

$$10 = 8 + 2v + \epsilon$$

10=8+2y+e Thus  $x=\frac{1}{2}$ , as before, and if 10=8-4e+6+e then  $e=\frac{4}{3}$  such that  $y=\frac{4}{3}$ .

There are several different options: a pigou-tax, a pollution-tax or the Coassian approach. A pigou-tax however will not restore the optimal pollution level, since a Pigou-tax will only affect the production level and not the pollution level. A pollution tax however will make the profit of the paper-mill  $p_x x - te - c(x, e)$  then one should set  $t = \frac{dc_f}{de}(e^*) = 2 * \frac{4}{3} = \frac{8}{3}$ . The Coassian approach would argue that the government should either set up a market for tradable quotas or that they should make the legal system robust in order to enforce any agreement between the two parties, and if the transaction costs are sufficiently low then there will be no externality since they could negotiate a mutual beneficial contract.

## Exercise 3

Consider an unemployment insurance policy provided by a governmental sponsored program, which is the sole provider of unemployment insurance. Each worker can voluntarily sign up for the program.

When employed there is a risk of being fired, and this risk depends on the effort the worker exerts in the daily working process. Thus, if the worker exerts a low effort there is a high risk of being fired,  $\pi_L$ , while there is a lower risk of being fired if she exerts a high effort,  $\pi_H < \pi_L$ .

If the worker is fired she has an income of  $w_1$ , while being on the job pays a wage of  $w_2$ . An insurance contract is characterized by the payment in each circumstance:  $(c_1, c_2)$  where  $c_1$  is the consumption of the insured if the worker is unemployed and  $c_2$  is the consumption if employed. The expected profit of an insurance contract where the effort is e, is  $\pi_e(w_1 - c_1) + (1 - \pi_e)(w_2 - c_2)$ .

The worker is risk adverse with a Bernoulli function v(c) - e, where c is the consumption and e the effort. The effort levels can be either low  $e = e_L = 0$  or high  $e = e_H > 0$ . If the worker does not get employment he or she can obtain a utility level of  $\bar{u}$ .

- a) Assume that the insurance program can observe the effort of each worker on the job. What contract will it offer if a high effort will be exerted? For a low effort?
- b) If the insurance program cannot observe the effort, will it be optimal to offer the contracts in a)?
- c) How should the insurance contract be designed to maximize expected profits?

#### **Solution**

- a) The problem is to solve  $\max \pi_e(w_1 c_1) + (1 \pi_e)(w_2 c_2)$  such that  $\pi_e v(c_1) + (1 \pi_e)v(c_2) e \ge \bar{r}$ , where  $\bar{r}$  is the maximum utility achievable if not insured. This implies that  $c_1 = c_2 = v^{-1}(\bar{r} + e)$ , hence the worker is fully insured and the workers with high effort are paid a higher income  $c^H = v^{-1}(r + e_H) > v^{-1}(r) = c^L$ . The contract is however NOT actuarial fair, due to monopoly.
- b) If effort is unobservable, then the workers will choose the contract with high income and effort, but will exert a low effort. The insurance company will be faced with a moral hazard issue. This lowers the expected profit of the insurance program, and thus it will yield a higher profit to only offer the low effort insurance contract. Any contract with full insurance will have that the worker will exert a low effort.
- c) To maximize profit, the insurance program must ensure *incentive compatible contracts*, if the insurance company wants to make sure that a high effort is exerted, such that  $\pi_H v(c_1) + (1 \pi_L)v(c_2) e_H \ge \pi_L v(c_1) + (1 \pi_L)v(c_2)$ , or  $(\pi_L \pi_H)(v(c_2) v(c_1)) \ge e_H$  in addition to the individual rationality condition. The restrictions determine the optimal contract, and one obtain that  $c_1 < c_2$ , i.e. that the consumption in the event of unemployment should be less than the working income. The contract needs to provide incentives for high effort, and thus not provide full insurance. However, it may not be optimal to actually induce a high effort, in which case it is optimal to only provide the low effort contract.

## **Exercise 4**

Consider an industry, producing microchips to PC's, consisting of two firms, Intel and AMD, with the total demand for microchips is given by D(p) = 200 - 2p. The marginal costs in each firm <u>are</u> constant, but different, as Intel has a cost advantage due to a patented production technology, such that  $MC_I = 10$  and  $MC_{AMD} = 15$ .

Each firm chooses its production levels, and then the price is determined when both firms output has been chosen.

a) Find the Nash equilibrium when Intel and AMD simultaneously choose their production levels

When the patent right of Intel's production technology expires, AMD can adopt the new technology without costs, and the marginal costs reduces to  $MC_{AMD} = 10$ .

- b) Derive the new equilibrium. How does each firm react to this adoption of the new technology? Explain.
- c) If the adoption of the new technology had a (fixed) cost, how high should it be in order to prevent AMD from adopting the new technology?

### Solution

- a) Since each producer chooses the quantity simultaneously the relevant equilibrium is the Cournot-Nash equilibrium. The reaction functions are  $R_I(x_A) = \frac{1}{2}(180 x_A)$  and  $R_A(x_I) = \frac{1}{2}(170 x_I)$ , and in equilibrium  $x_I = R_I(x_A)$  and likewise for A, but then inserting  $\frac{3}{2}x_I = 95$  which then gives a Cournot equilibrium  $x_A = \frac{160}{3} \approx 63$  and  $x_I = \frac{190}{3} \approx 53$ , such that the total production is  $X = x_I + x_A = \frac{350}{3} \approx 116$  with a price of  $p = 100 \frac{1}{2} \frac{350}{3} = \frac{125}{3} \approx 41$ .
- b) With the new technology the equilibrium becomes symmetric:  $x_A = x_I = \frac{200 2*10}{3} = \frac{180}{3}$ , such that Intel cut back its production while AMD increases its production. Intel's reaction is due to the increased competitiveness of AMD that Intel takes into account when deciding its production level. The price of microchips decreases to  $p' = \frac{200 + 2*2*10}{3*2} = \frac{120}{3} = 40$
- c) The profit of AMD prior to the adoption is  $\pi^* = \left(\frac{125}{3} 15\right) * \frac{160}{3} \cong 1422$  while the profit after the adoption is  $\pi' = (40 10) * \frac{180}{3} = 1800$ , so if the adoption costs exceed 378 then AMD will not adopt the new production technology.

#### Exercise 5

A local homeowner's association is considering if they should replace an old sewer system to reduce leaked water and prevent increased heavy rain fall from resulting in costly floods.

The homeowner's association has 15 members: 9 members each have a willingness to pay for the sewer system of 450dkk while 6 members each have a willingness to pay of 1000dkk.

The total cost of investing in a new sewer system is 7500dkk.

- a) Is it efficient for the homeowner's association to invest in the new sewer system?
- b) To choose if they should make the investment the board has decided to cast a majority vote in which they each pay an equal share of the costs. Would the vote implement the efficient decision?
- c) Could you design a mechanism that implements the efficient decision? Explain why the mechanism will implement the efficient decision.

#### Solution

- a) Yes, since the total willingness to pay is 9 \* 450 + 6 \* 1000 = 10050 which exceeds the cost of the project, 7500.
- b) If each member pays 1/15 of the total costs, each of the nine individuals has a net benefit from the project being carried through of  $450 \frac{7500}{15} = -50$ , and thus will vote against the project.
- c) A mechanism that would only ask each to state their true (net) value of the project would not be successful in implementing the project: those with a negative value would have an incentive to overstate their negative value. Instead, a Vickrey-Clarks-Groves mechanism can be used: ask each homeowner to state his (net) valuation of the project,  $n_i$ , and invest iff  $\sum_{i=1}^{15} n_i \geq 0$ . Given an equal share of cost the net value is either  $n_L = -50$  and  $n_H = 500$ . If  $S_i \cdot (S_i + n_i) < 0$  when  $S_i = \sum_{j \neq i} n_i$  then agent i should pay a tax of  $|S_i|$ . This will implement the efficient decision: it is a Nash equilibrium to reveal his true value, hence it is a truth-telling mechanism. Consider a L-type, then  $S_i = 2600$  when everyone else tells the their true value, such that telling the truth will give him a utility of -50, while the only relevant alternative is  $n_i < -2600$  and submitting such a signal would leave him with a net gain of -2600 < -50.