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Solutions to written exam for the M.Sc. in Economics International Monetary Economics

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Number of questions: This exam consists of 3 questions.

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1. This question only requires short answers. The questions relate to the following learning objectives: describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model and Lucas asset pricing model) and use these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models; explain the theory of optimum currency area and apply this theory to the analysis of the European Monetary Union; describe the institutional features of the foreign exchange market products (spot and forward contracts) and be able to distinguish between speculation and arbitrage; and describe and explain Covered Interest Rate Parity (CIP), Uncovered Interest Rate Parity (UIP), and Purchasing Power Parity (PPP) and be able to summarize the empirical evidence on these parity conditions.

   (a) False. Both the CIP and the UIP relations are derived using arbitrage arguments. CIP equalizes foreign and domestic expected returns and thus implies that currency risk has been hedged (in other words, all rates of returns used to derive the relation are known beforehand). UIP, on the other hand, relates expected returns under either the assumption of risk neutrality or under the assumption that agents are risk averse. Risk averse agents will not arbitrage away all differences in expected returns if the underlying assets bear different risks. This implies that if CIP holds, it is not necessarily the case that UIP holds. This may also be the case if agents are not risk neutral.

   (b) Wrong. ECB should ensure price stability and also take into account general economic policies including unemployment.

   (c) Wrong. It does lead to a depreciating currency. If the domestic real interest rate exceeds the foreign real interest rate, then the nominal exchange rate depreciates.

   (d) Correct: Mundell argued that wage/price flexibility or factor mobility are alternative mechanisms capable to adjust when there are idiosyncratic shocks to aggregate demand. Therefore, such countries appear good candidates for a currency union.

2. This question relates to the learning objective: describe and explain Covered Interest Rate Parity (CIP), Uncovered Interest Rate Parity (UIP), and Purchasing Power Parity (PPP) and be able to summarize the empirical evidence on these parity conditions. The question focuses on the Harrod-Balassa-Samuelson hypothesis that deviations from PPP can be explained by different productivity levels. Moreover, this hypothesis also provides a possible answer to the question why rich countries tend to have higher general price levels than poor countries. In the question, the idea is to derive an expression for the nominal exchange rate using expressions for the aggregate price level and relations between prices, wages and productivity. The first part relates directly to the curriculum whereas the latter part represents an extension of the curriculum. Even though there are many sub-questions, the answers are very short. Often derivations can be done in just one or two lines.

   (a) The main underlying assumptions are: (1) Two countries and two sectors (tradable and non-tradable sectors). (2) Wages in the tradable and non-tradable sectors are assumed to be equal within each country. (3) Often we also assume that one country is rich implying that productivity is higher in the tradable sector in the rich country than in the poor country. For the non-tradable sector we assume that the productivity is the same across countries.

   Equation (1) simply states that the real wage is equal to productivity in each sector. The price level in the rich country in the non-traded goods sector is higher than in the poor country. The reason is that productivity is higher in the rich country, implying higher wages in the rich country and since wages are equal across sectors in the rich country, the price level in the non-traded sector is higher. This implies that PPP does not hold when using CPI's.

   (b) The following two-country model is given in the problem:

$$P_N = \frac{W_N}{Q_N} \quad \text{and} \quad P_T = \frac{W_T}{Q_T} \tag{1}$$

$$P_{N^*} = \frac{W_{N^*}}{Q_{N^*}} \quad \text{and} \quad P_{T^*} = \frac{W_{T^*}}{Q_{T^*}} \tag{2}$$

   where N denotes the non-traded sector and T the traded sector in each country. An asterisk denotes the foreign country. Assume that $W_N = W_T$ and $W_{N^*} = W_{T^*}$, $Q_N = Q_{N^*}$ and that $S = \frac{P_T}{P_{T^*}}$. In addition we assume that the aggregate price index in the two countries is equal to a weighted average of prices in the traded and non-traded sectors, i.e.,

$$P_1 = \alpha P_N + (1-\alpha) P_T \tag{3}$$

   and

$$P_{1^*} = \beta P_{N^*} + (1-\beta) P_{T^*}. \tag{4}$$

The first part is to show that the nominal exchange rate can be expressed as

$$S = \frac{P_1}{P_{1^*}} \frac{\beta \frac{P_{N^*}}{P_{T^*}} + (1-\beta)}{\alpha \frac{P_N}{P_T} + (1-\alpha)}. \tag{5}$$

PPP implies that $S = \frac{P_T}{P_{T^*}}$. Multiply both sides with P_{1^*}/P_1 and insert the expressions for P_{1^*} and P_1 such that

$$S\frac{P_{1^*}}{P_1} = \frac{P_T}{P_{T^*}} \frac{P_{1^*}}{P_1} = \frac{P_T}{P_{T^*}} \frac{\beta P_{N^*} + (1-\beta) P_{T^*}}{\alpha P_N + (1-\alpha) P_T}$$

which can be written as

$$S = \frac{P_1}{P_{1^*}} \frac{\beta \frac{P_{N^*}}{P_{T^*}} + (1-\beta)}{\alpha \frac{P_N}{P_T} + (1-\alpha)}.$$

The second part of the question relates to the interpretation of the equation above and its relation to tests of PPP. First, we note that the real exchange rate given by the expression above deviates from the real exchange rate given by the PPP relation. If prices in the non-traded goods sector in the home country (the rich country) exceeds the price in the non-traded goods sector in the foreign country (the poor country), then the real exchange rate implied by the expression above must be lower than unity. From the PPP relation and using the standard assumption that the real exchange rate given by PPP is equal to unity, we find that the real exchange rate must be lower than the nominal rate, the real exchange rate must have appreciated. Therefore, we should expect that tests based on broad baskets of goods tend to reject PPP. Second, if the shares of traded goods in the price index differs between the two countries, we will also find that the real exchange rate given by the expression above deviates from the PPP relation where only traded goods prices are equalized. This is another explanation to the failure of not rejecting PPP when using broad price index.

(c) Using the expressions above we note that the relative price ratios in the home and foreign countries can be written as

$$\frac{P_N}{P_T} = \frac{Q_T}{Q_N}$$

and

$$\frac{P_{N^*}}{P_{T^*}} = \frac{Q_{T^*}}{Q_N}$$

if we use that $Q_{N^*} = Q_N$. We then find that

$$S = \frac{P_1}{P_{1^*}} \frac{\beta \frac{Q_{T^*}}{Q_N} + (1-\beta)}{\alpha \frac{Q_T}{Q_N} + (1-\alpha)}$$

is increasing when foreign productivity in the traded sector increases such that the home exchange rate depreciates (the foreign exchange rate appreciates). One example is the very strong productivity growth in Japan since the Second World War leading to an appreciation of the real value of the Yen (and thus depreciation of the home currency). The reason for the real appreciation is that higher productivity growth in the traded sector leads to falling prices in this sector relative to the other country and therefore an appreciation.

(d) A change in the non-traded sector is reflected as a change in α or in β. An increase in the domestic non-traded sector implies an increase in α. The effect on the nominal exchange rate is zero which is easily seen in the equation for S above, i.e.,

$$S = \frac{P_1}{P_{1^*}} \frac{\beta \frac{P_{N^*}}{P_{T^*}} + (1-\beta)}{\alpha \frac{P_N}{P_T} + (1-\alpha)}$$

which can be written as the expression given in the problem, i.e., $S = \frac{P_T}{P_{T^*}}$ which is independent on the shares. The nominal exchange is unchanged.

The real exchange rate using the broad baskets of goods, on the other hand, may not be constant. The relative price level is

$$\frac{P_1}{P_{1^*}} = \frac{\alpha P_N + (1-\alpha) P_T}{\beta P_{N^*} + (1-\beta) P_{T^*}}$$

take the derivative with respect to α to obtain

$$\frac{P_N - P_T}{\beta P_{N^*} + (1-\beta) P_{T^*}}$$

where we note that there will be an effect on the real exchange rate from a change in α as long as $P_N \neq P_T$. The sign of the effect depends on this price difference. Similar results hold for changes in β. The derivative with respect to β is

$$-\left(P_{N^*} - P_{T^*}\right) \frac{\alpha P_N + (1-\alpha) P_T}{\left(\beta P_{N^*} + (1-\beta) P_{T^*}\right)^2}$$

where we note that the effect is zero unless $P_{N^*} \neq P_{T^*}$.

(e) The price level in the rich country in the non-traded goods sector is higher than in the poor country. The reason is that productivity is higher in the rich country, implying higher wages in the rich country and since wages are equal across sectors in the rich country, the price level in the non-traded sector is higher.

(f) Empirical results are mixed. There is no general consensus in the literature regarding this hypothesis. There are also attempts to extend the analysis by also including other variables such as government spending and income. However, there is no consensus in the literature on the question whether these additional variables could explain deviations from PPP. The best example of a case where the hypothesis holds is the higher productivity growth in Japan since the Second World War which has been accompanied with an appreciating Yen.

3. This question relates to the learning objective: describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model and Lucas asset pricing model) and use these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models.

   The question is a straightforward application of the continuous time Mundell-Fleming and Dornbusch models discussed in the curriculum. Also here there are many subquestions but they only require short answers in general. There is one issue that is not required, the proof that there exists a saddlepath. It is sufficient to include the saddlepath in the graph.

   (a) We assume a small open economy, output is demand determined and prices are fixed. Equation (6) is the UIP relation. We assume that UIP holds and we assume rational expectations such that the change of the exchange rate $\dot{s}$ is equal to the interest rate differential between the two countries $i - i^*$.

   Equation (7) is the money market equilibrium condition where m is money supply (which is equal to money demand), y is output. The parameter κ is the output elasticity of money demand, θ is the interest elasticity of money demand and σ is the exchange rate elasticity of money demand. Note that this expression differs from the standard money demand function since we include s instead of p on the RHS. The reason for this is that we have an underlying assumption in the M-F model that prices are fixed (and we then normalize such that the log of the price levels in the domestic and in the foreign countries are both equal to zero). Then, the overall price level (comprised of the price on domestic goods and of imported goods measured in home currency) is a function of the exchange rate. The parameter σ measures the importance of foreign prices in the domestic consumer price index.

   Equation (8) is the aggregate demand for goods. The first component on the RHS of this equation (α) is the autonomous component of aggregate demand, the next component (μs) represents foreign demand for domestic goods, and the last component is output such that the change in aggregate demand also depends on output in the previous period.

   (b) To solve for the LM–curve: Assume that the monetary authority can set the money supply (it is given exogenously), then solving for i in equation (7) and insert this solution into (6) yields

$$\dot{s} = \frac{\sigma}{\theta}s + \frac{\kappa}{\theta}y - \frac{1}{\theta}m - i^*. \tag{6}$$

The IS–curve: Use equation (7) to solve for i and insert into (8) such that

$$\dot{y} = \chi\left(\alpha + \mu s - y\right). \tag{7}$$

The system can now be rewritten in matrix form

$$\begin{bmatrix} \dot{s} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{\sigma}{\theta} & \frac{\kappa}{\theta} \\ \chi\mu & -\chi \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix} + \begin{bmatrix} -\frac{1}{\theta}m - i^* \\ \chi\alpha \end{bmatrix} \tag{8}$$

where the determinant of the coefficient matrix is

$$-\chi\frac{\sigma}{\theta} - \frac{\chi\mu\kappa}{\theta} < 0$$

such that the system is saddlepath stable.

We can now obtain the LM curve (in the s and y plane) where the exchange rate market is in equilibrium ($\dot{s} = 0$) and the IS curve where the goods market is in equilibrium ($\dot{y} = 0$). The slope of the LM curve is negative and the slope of the IS curve is positive. To show this, note that the LM curve (first row of (8) and let $\dot{s} = 0$) is given by

$$s = -\frac{\kappa}{\sigma}y + \frac{\theta}{\sigma}\left(\frac{1}{\theta}m - i^*\right)$$

such that

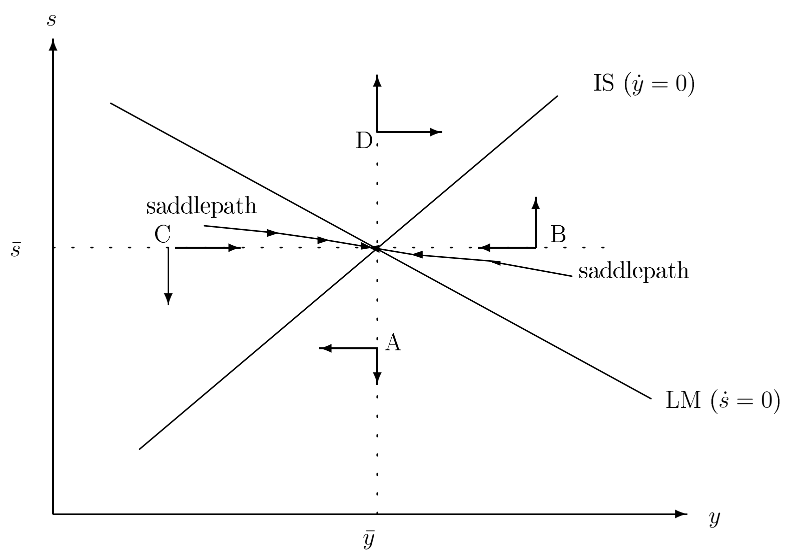
$$\frac{\partial s}{\partial y} = -\frac{\kappa}{\sigma}$$

and the IS curve (second row of (8) and let $\dot{y} = 0$) is given by

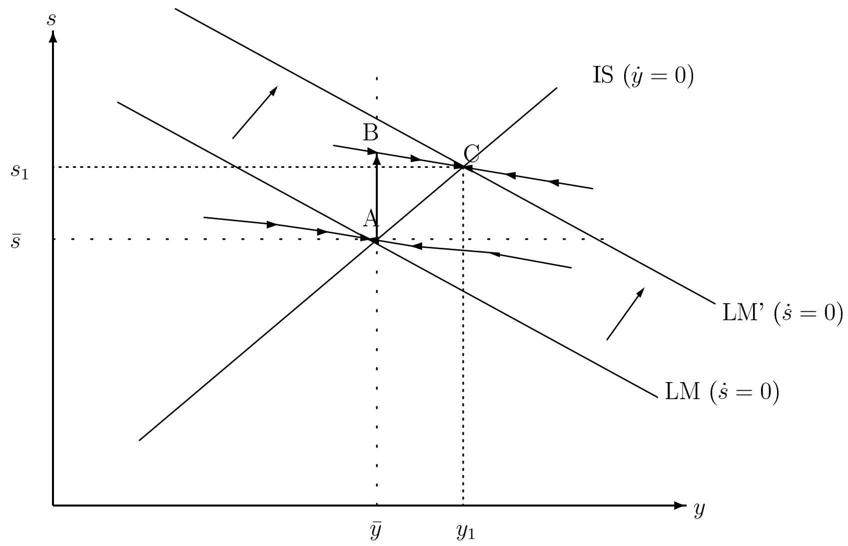
$$s = \frac{1}{\mu}y - \frac{\alpha}{\mu}$$

such that

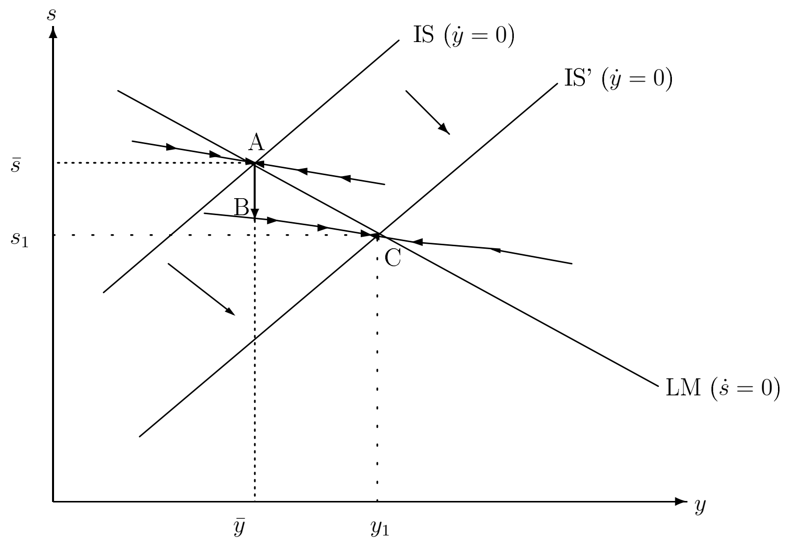
$$\frac{\partial s}{\partial y} = \frac{1}{\mu}.$$



(c) Expansionary monetary policy in the perfect–foresight Mundell–Fleming model. An increase in m will lead to a shift in the LM–curve up to the right. For a given y, higher m implies a higher s (a depreciated exchange rate) which can be seen from the LM–equation. Since the economy is always on a saddlepath, the exchange rate jumps up to the new saddlepath, the exchange rate jumps from point A to point B. At point B, the economy is on a new saddlepath. What happens next? Foreign demand will increase leading to higher output. Output increases and the economy moves along the saddlepath towards the new long–run equilibrium at point C. Total effect: Depreciated currency (an overshooting effect) and higher output (prices are fixed).



(d) Expansionary fiscal policy (a change in α). For given exchange rate, output must increase if $\alpha \uparrow$. The IS–curve shifts down to the right. Output is unchanged initially and the exchange rate jumps down to the new saddlepath. Exchange rate is at point B on the new saddlepath at point B. The lower s reduces foreign demand (but less than α such that there is still excess demand $\mu s < \alpha$). Output will increase along the new saddlepath to new equilibrium. Total effect: An appreciated currency with no overshooting and an increase in output.



If $\sigma \to 0$, then the LM–curve is vertical. In this case, fiscal policy will be ineffective. Thus, fiscal policy is ineffective under floating exchange rates. σ is the weight of foreign prices in the domestic aggregate price level. If foreign prices do not affect the domestic price level, then an increase in α is completely offset by a reduction of foreign demand, there will be no net effect. The inefficiency of fiscal policy in MF–model is a limiting case, not a general case.

(e) The Dornbusch model is also in continuous time. There are two main differences between this model and the M-F model above. In the Dornbusch model output is fixed but the price level is not. In the M-F model it is the opposite. Prices are sticky in the Dornbusch model but the asset market (the exchange rate market) will react immediately to monetary policy. There is overshooting in both models but the mechanisms are different.

(f) The first two model equations are the same as above (noting that output is constant and remember to replace s with p in the money demand function).

$$\dot{s} = i - i^* \tag{9}$$

$$m = \sigma p + \kappa y - \theta i \tag{10}$$

and then we have equation (9)

$$\dot{p} = \gamma\left(\alpha + \mu\left(s - p\right) - \bar{y}\right) \tag{11}$$

In the long-run equilibrium we have that $\dot{s} = 0$ implying that $i = i^*$ and $p = \bar{p}$. This further implies that

$$m - \bar{p} = \kappa\bar{y} - \theta i^*$$

Subtract this from the money demand function such that

$$p - \bar{p} = \theta(i - i^*)$$

or if we use the UIP relation

$$\dot{s} = \frac{1}{\theta}(p - \bar{p})$$

In equilibrium we know that $\dot{p} = 0$ implying that

$$0 = \gamma\left[\alpha + \mu(\bar{s} - \bar{p}) - \bar{y}\right]$$

which is subtracted from equation (9)

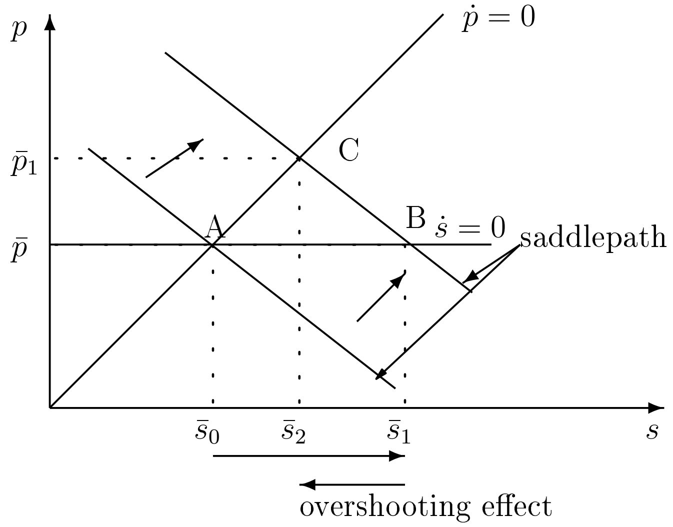
$$\dot{p} = \gamma\mu(s - \bar{s}) - \gamma\mu(p - \bar{p})$$

Now we can summarize the model in matrix form

$$\begin{bmatrix} \dot{s} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\theta} \\ \gamma\mu & -\gamma\mu \end{bmatrix} \begin{bmatrix} s - \bar{s} \\ p - \bar{p} \end{bmatrix}$$

from which we can calculate the determinant which is $-\frac{\gamma\mu}{\theta}$ indicating that the system has a unique saddlepath. The model can then be illustrated as in the graph below.

(g) Next we are asked to show the effects of expansionary monetary policy. In the long run, the price level will be higher and so will the exchange rate, the exchange rate will depreciate. PPP will ensure that, holding foreign prices constant, the long-run exchange rate will depreciate proportionately. Assume that the economy is in the long-run equilibrium at point A. Expansionary monetary policy will lead to a shift in the saddlepath up and to the right, the saddlepath must go through the new long-run equilibrium point C where the price level has increased and the exchange rate depreciated. Because prices are sticky the economy cannot jump directly from A to C. Since prices remain fixed in the short-run, the exchange rate jumps to s_2 in order to get on to the new saddlepath. Prices then adjust slowly and the economy moves along the saddlepath from B to the new long-run equilibrium C. The net effect of the money supply increase is a long-run depreciation corresponding to the distance s_0 to s_1, with an initial overshooting effect corresponding to s_2 to s_1. We note here that we also obtain an overshooting effect in the Dornbusch model as well.



p
$\dot{p} = 0$
$\bar{p}_1$
C
B $\dot{s} = 0$
saddlepath
A
$\bar{p}$
$\bar{s}_0$
$\bar{s}_2$
$\bar{s}_1$
s
overshooting effect