Written Exam - Macroeconomics III

(suggested answers)

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Question 1

a Profit-maximization:

$$\alpha k_t^{\alpha-1} = \omega$$

b Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\theta}}{1-\theta} - \lambda_t \left[c_t + s_{t+1} q_t - w_t - s_t \left(q_t + d_t \right) \right] \right\}$$

Utility-maximization FOC's:

$$\begin{aligned} c_t^{-\theta} &= \lambda_t \\ \beta^t q_t \lambda_t &= \beta^{t+1} \left(q_{t+1} + d_{t+1} \right) \lambda_{t+1} \end{aligned}$$

Combined, they lead to the Euler equation:

$$c_t^{-\theta} = \beta \left(\frac{q_{t+1} + d_{t+1}}{q_t} \right) c_{t+1}^{-\theta}$$

c From the Euler:

$$d = \left(\frac{1-\beta}{\beta} \right) q$$

Steady-state capital is $k = \left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}$, so that $\pi = \left(\frac{\omega}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \omega \left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}$. Thus, combine the latter with the steady-state condition retrieved from the Euler equation:

$$q \left(\frac{1-\beta}{\beta} \right) = \left(\frac{\omega}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} - \omega \left(\frac{\omega}{\alpha} \right)^{\frac{1}{\alpha-1}} \Leftrightarrow q = \frac{\beta}{1-\beta} \left[\left(\frac{\omega}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} - \omega \left(\frac{\omega}{\alpha} \right)^{\frac{1}{\alpha-1}} \right]$$

Derive the first-order partial derivative of q w.r.t. ω:

$$\begin{aligned}
\frac{\partial q}{\partial \omega} &= \frac{\beta}{1-\beta}\left\{\frac{1}{\alpha}\frac{\alpha}{\alpha-1}\left(\frac{\omega}{\alpha}\right)^{\frac{\alpha}{\alpha-1}-1} - \left[\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}} + \frac{1}{\alpha}\frac{1}{\alpha-1}\omega\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}-1}\right]\right\} \\
&= \frac{\beta}{1-\beta}\left\{\frac{1}{\alpha-1}\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}} - \left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}} - \frac{1}{\alpha}\frac{1}{\alpha-1}\omega\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}-1}\right\} \\
&= \frac{\beta}{1-\beta}\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}\left\{\frac{2-\alpha}{\alpha-1} - \frac{1}{\alpha(\alpha-1)}\omega\left(\frac{\omega}{\alpha}\right)^{\frac{2-\alpha}{\alpha-1}-\frac{1}{\alpha-1}}\right\} \\
&= \frac{\beta}{1-\beta}\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}\left\{\frac{2-\alpha}{\alpha-1} - \frac{1}{\alpha(\alpha-1)}\omega\left(\frac{\omega}{\alpha}\right)^{\frac{1-\alpha}{\alpha-1}}\right\} \\
&= \frac{\beta}{1-\beta}\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}\left\{\frac{2-\alpha}{\alpha-1} - \frac{\alpha}{\alpha(\alpha-1)}\right\} \\
&= -\frac{\beta}{1-\beta}\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}
\end{aligned}$$

Increasing the marginal cost of production has a negative effect on the price of the share, as expected on a priori grounds, given that it makes the firm less profitable.

d Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\left\{\frac{c_t^{1-\theta}}{1-\theta} - \lambda_t\left[c_t + s_{t+1}q_t - w_t - \left[q_t + (1-\tau)\,d_t\right]\right]\right\}$$

Utility-maximization FOC's:

$$\begin{aligned}
c_t^{-\theta} &= \lambda_t \\
\beta^t q_t \lambda_t &= \beta^{t+1}\left(q_{t+1} + (1-\tau)\,d_{t+1}\right)\lambda_{t+1}
\end{aligned}$$

Combined, they lead to the Euler equation:

$$c_t^{-\theta} = \beta\left(\frac{q_{t+1} + (1-\tau)\,d_{t+1}}{q_t}\right)c_{t+1}^{-\theta}$$

From the Euler, in the steady state:

$$d = \frac{1-\beta}{(1-\tau)\,\beta}q$$

Steady-state capital is $k = \left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}$, so that $\pi = \left(\frac{\omega}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \omega\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}$. Thus, combine the latter with the steady-state condition retrieved from the Euler equation:

$$q\frac{1-\beta}{(1-\tau)\,\beta} = \left(\frac{\omega}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \omega\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}} \Leftrightarrow q = \frac{(1-\tau)\,\beta}{1-\beta}\left[\left(\frac{\omega}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \omega\left(\frac{\omega}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]$$

The steady-state stock price drops, as a result of households receiving lower dividends.

Question 2

a Given the linear rule $\pi_t = \psi + \psi_\theta \theta_t$, as well as the fact that θ_t is observed by both the public and the policy maker before expectations are formed, output is determined as follows:

$$x_t = \theta_t + \pi_t - \pi_t^e = \theta_t + \psi + \psi_\theta \theta_t - (\psi + \psi_\theta \theta_t) = \theta_t$$

Thus, the expected loss reads as:

$$\begin{aligned}
E\left[L(\pi_t, x_t)\right] &= \frac{1}{2} E\left[\left(\underbrace{\psi + \psi_\theta \theta_t}_{=\pi_t}\right)^2 - \lambda\left(\underbrace{\theta_t}_{=x_t} - \bar{x}\right)\right] \\
&= \frac{1}{2} E\left[\psi^2 + 2\psi\psi_\theta\theta_t + \psi_\theta^2\theta_t^2 - \lambda\left(\theta_t - \bar{x}\right)\right] \\
&= \frac{1}{2}\left[\psi^2 + 2\psi\psi_\theta E\left[\theta_t\right] + \psi_\theta^2 E\left[\theta_t^2\right] - \lambda\left(E\left[\theta_t\right] - \bar{x}\right)\right] \\
&= \frac{1}{2}\left[\psi^2 + \psi_\theta^2\sigma_\theta^2 + \lambda\bar{x}\right]
\end{aligned}$$

Taking the first order conditions of $E\left[L(\pi_t, x_t)\right]$ with respect to ψ and ψ_θ we obtain:

$$\begin{aligned}
\frac{\partial E\left[L(\pi_t, x_t)\right]}{\partial \psi} &= 0 : 2\psi = 0 \\
\frac{\partial E\left[L(\pi_t, x_t)\right]}{\partial \psi_\theta} &= 0 : 2\psi_\theta\sigma_\theta^2 = 0
\end{aligned}$$

Thus, the expected loss is minimized by setting $\psi = \psi_\theta = 0$, which implies $\pi_t^C = 0$ and $x_t^C = \theta_t$.

b When the central bank conducts a discretionary policy, the inflation rate is chosen after expectations are formed. Hence, the goal of the central bank is to minimize the loss function, i.e. the monetary policy should be ex post optimal. Under this assumption, the problem reads as

$$\min_{\pi_t} \frac{1}{2}\left[\pi_t^2 - \lambda\left(\theta_t + \pi_t - \pi_t^e - \bar{x}\right)\right]$$

The first order condition for this problem reads as:

$$\frac{\partial L(\pi_t, x_t)}{\partial \pi_t} = 0 : \pi_t - \lambda = 0 \Leftrightarrow \pi_t^D = \lambda$$

Thus, the expected rate of inflation is:

$$E\left[\pi_t^D\right] = \lambda,$$

and

$$x_t^D = \theta_t$$

The excessively high equilibrium inflation associated with the inflation bias problem results from the combination of a lack of commitment and central bank's temptation to temporarily boost the economy beyond its potential level. This makes it clear why raising λ increases the temptation of the central bank to generate excess inflation in the vain attempt to stimulate real activity.

c The problem of the policy maker now reads as

$$\min_{\pi_1} \frac{1}{2}\left[\pi_1^2 - \lambda\left(\theta_1 + \pi_1\right)\right]$$

The first order condition for this problem is:

$$\frac{\partial L(\pi_1, x_1)}{\partial \pi_1} = 0 : \pi_1 - \lambda = 0 \Leftrightarrow \pi_1^* = \lambda$$

which implies

$$x_t^* = \theta_1 + \pi_1^* - \underbrace{\pi_1^e}_{=0} = \theta_1 + \lambda$$

d At time $t = 2$ the central bank plays discretion to accommodate the public's expectations. As suggested, the benefit at $t = 1$ is the difference between the loss under commitment and the loss under the deviation strategy:

$$B\left(\theta_1\right) = L\left(0, \theta_1\right) - L\left(\lambda, \theta_1 + \lambda\right) = \frac{1}{2}\left[\lambda^2 - \lambda\theta_1\right] - \frac{1}{2}\left[\lambda^2 - \lambda\left(\theta_1 + \lambda\right)\right] = \frac{\lambda^2}{2}$$

As for the cost at $t = 2$, this is computed as the difference between the loss under discretion and the loss under commitment:

$$C\left(\theta_2\right) = L\left(\lambda, \theta_2\right) - L\left(0, \theta_2\right) = \frac{1}{2}\left[\lambda^2 - \lambda\theta_2\right] - \frac{1}{2}\left[-\lambda\theta_2\right] = \frac{\lambda^2}{2}$$

Thus, deviating is irrelevant, as $B\left(\theta_1\right) = C\left(\theta_2\right)$.