Written Exam for the B.Sc. in Economics 2009-II

Micro Economics 1

Final Exam

June 8, 2009

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Question 1

Bill's preferences can be represented by the utility function $u(x_1, x_2) = \ln x_1 + 3\ln x_2$ the two goods have exogenous prices $p_1=3$ and $p_2=1$. Bill's budget is 500 Euro.

a) Find Bill's demand

Now consider a price increase in the price of good one to $p_1=5$.

b) What is Bill's demand now? Decompose the demand in substitution and income effects using the Slutsky compensation

Assume that the price increase is caused by a tax on good one. The government that imposed the tax has decided that no-one must be worse off as a consequence of the new tax.

c) What can the government do to ensure that Bill is just as well off? Find an exact compensation that satisfies this condition.
d) Explain why the answers and the utility Bill gets in question a), b) and c) can differ?
e) What if Bill's utility was quasi-linear, how would this influence the answers?

Answers

a) $x_1 = \frac{m}{4p_1} = \frac{500}{4*3} = 125/3, x_2 = \frac{3m}{4p_2} = \frac{1500}{4} = 375$

b) $x_1 = \frac{m}{4p_1} = \frac{500}{4*5} = 25, x_2 = \frac{3m}{4p_2} = \frac{1500}{4} = 375$. *The income needed to byu the old bundle with the new prices is m'=5*125/3+375=583,3. With this budget the compensated demand is* $x_1^S = \frac{m'}{4p_1} = \frac{583{,}3}{4*3} = 29{,}2 \quad x_2 = \frac{3m'}{4p_2} = \frac{3*583{,}3}{4} = 437{,}5$. *The substitution effect is thus (-12,4; 62,5) and the income effect is (-4,2; -62,5)*

c) *The government can give Bill an income compensation (Hicks) such that his utility is unchanged. This is found by minimising costs that ensure that his utility in question a) is kept unchanged. The FOC. Gives the following condition* $\ln\left(3^3 * \frac{(p_1^{'})^3}{p_2^3} x^4\right) = \ln(125/3 * 375^3)$ *which gives* $x_1 = \sqrt[4]{651{,}041\tfrac{2}{3}} \approx 28.405$ *and* $x_2=15x_1 \approx 426.08$. *The government thus have to give Bill an income with compensation equal to 5*28.405+1*426.08-500* $\approx$ *68.11*

d) *The tax is distortionary meaning that it affects the demand even though income is compensated to keep his utlity unchanged. The substitution effect is an effect that reduces the effect of the price increase. But the distortion is still present since the (Hicks) compensation is given a lump sum (negative) tax.*

e) *There is no income effect with a quasi-linear utility function, this implies that the Slutsky and the Hicks compensation are the same and thus also the choices made in b) and c). There is furthermore no distortion since demand of* x_1 *is the same irrespective of the compensation (Hicks or Slutsky).*

Question 2

a) Francis is considering an optimal investment of his earnings from a sale of his IT-company.

He has 1 million Euro to invest. Francis remembers that his teacher in Micro-economics talked about a *mean-variance model* and as he has not really learned about any other investment modes, he thinks that this may be a very good model to use to help him decide on finding an optimal porto-folio. Francs' preferences for return and risk can be represented by a Cobb-Douglas utility function. If you let *x* be the share of the 1 million Euro that he will invest in a risky asset, then explain how Francis can use the *mean variance model* to find his optimal porto-folio.

b) A government has a policy that it will increase its public infrastructure investments during recessions. This is very beneficial for private engineering consultancies. During periods with fast growing economies these consultancies with focus on public investments do not have very much to do, whereas consultancies focussing on housing constructions are very busy in such periods. How can we analyse the impacts of investments in such two firms (assests) and why are we in particular interested in this? You should relate your answer to the CAPM model.

a) *By investing the share x in a risky asset Francis' expected return of his proto folio is* $r_x = r_f + x\,(r_m - r_f)$*, where m represents the market porto folio, while f is a risk free asset, the r's represne expected returns. The risk by this porto folio is described the variance* $x^2\,\sigma_m^{\,2}$*. The relevant budget is* $r_x = \frac{\sigma_x}{\sigma_m}\left(r_m - r_f\right) + r_f$ *. He thus have to maximise his CD utility function subject to this budget to find the optimal share x. FOC are* $-\frac{\partial u(r_x,\sigma_x)/\partial r_x}{\partial u(r_x,\sigma_x)/\partial \sigma_x} = \frac{r_m - r_f}{\sigma_m}$ *where* $\partial u(r_x,\sigma_x)\big/\partial r_x = \alpha (r_x)^{\alpha-1}(\sigma_m)^{1-\alpha}$ *og* $\partial u(r_x,\sigma_x)\big/\partial \sigma_x = (1-\alpha)(r_x)^{\alpha}(\sigma_x)^{-\alpha}$ *so that FOC becomes* $-\frac{\alpha}{1-\alpha}\frac{x\sigma_m}{r_f + x\left(r_m - r_f\right)} = \frac{r_m - r_f}{\sigma_m}$ *hence MRS= price on return (mean) relative to the risk (variance).*

b) *The CAPM-model says that* $r_i = r_f + \beta_i \cdot (r_m - r_f)$*, where i is a risky asset, m is the market porto folio and f is a risk free assest, and* β_i *is an expression for the co-variance between the return on the asset and the market porto folio; i.e. with co-variance in the nominator for* β_i *and the market variance in the denominator. If* β_i*=1 then the asses is perfectly correlated with the market and the expected return has to correspond to the market return. If* β_i*>1 then the asset is more risky compared to the market and expected return must be larger, while*

β_i<1 menas that we have a less risky and expected return has to be smaller. β_i<0 is especially interesting since the asset varies opposite to the market. This means that we can use this asset to insure against recessions as this asset gives a higher return in such periods. The two different consultancies have such opposite returns and these assets satisfy the β_i<0 in the CAPM model

Question 3

Consider the following two investments projects that the Government can consider as a solution to the large delays occurring daily at the Vejle fiord bridge. The first project is one additional bridge across the fiord. The other project is a Kattegat link. The two projects have very different cost and benefit structures, which you can find in the table below.

Project	Year 1		Year 2		Year 3		Year 4		Year 5	
(in millon Kr)	Costs	Benefit	Costs	Benefit	Costs	Benefit	Costs	Benefit	Costs	Benefit
Vejle fiord	500		500		250	750	250	750	250	750
Kattegat	1.000		1.000		1.000		250	2.000	250	2.000

a) If the market rent is r=5% and we disregard any other costs and benefits that may occur as consequences of the two projects, which of the two projects should the Government prefer? and why? Explain the importance of the market rent and relate this to the two current projects.

b) What must (net) benefits in year 6 of the non-chosen project be for this project to be the preferred one?

a) We have to consider the discounted value of the net-benefits (benefits minus costs). The series with the highest discounted value is the better project. NPV(Vejle)=305,33 and NPV(Kattegat)=87,65. Hence, the government should choose Vejle. A hiher market rent means that the future is valued less compared to earlier values. This works against Kattegat, where the net benefits are higher at the later stage.

b) We now have to consider the extra discounted amount in year 6 that would make the NPV of the two series equal. $\frac{Net\,benefit_{Kattegat}}{(1+r)^6} = NPV_{Vejle} - NPV_{kattegat}$, which gives a net difference of the two net benefits of 291, 71 in year 6, which discounted gives us a net difference in year 0 of 305,33-87,65

Question 4

A farmer has the possibility of renting out his land or grow it himself. Land is a normal good. The rent on land has decreased due to a huge technological efficiency increase, which increases the output from the growing the land.
Can we be certain that this rent decrease will mean that the farmer will decrease the amount of land he rents out? Why/why not?

Answer
No this is not certain. The farmer's (opportunity) cost of renting out the land has decreased, which induces a decrease in the amount he rents out. On the other hand the value of his land endowment has also decreased and he has to rent out more land to earn the same income. The relevant issue is to consider the Slutsky equation with endowment income effects. If the endowment income effect is larger than the ordinary income effect (assuming land is a normal good) then we can have that the decrease in rent could induce a decrease in the amount he rents out (an increase in his own consumption/use of land).

Question 5
Consider an economy with two agents (Allen and Bank). There are only two goods in the economy: money today and money tomorrow. Allen will inherit 400 (thousand) kr. in period 2 and Bank has 1000 (thousand) kr. in period 1. Allen's preferences over money today and tomorrow can be represented by a utility function $u_A(x_{1,A}, x_{2,A}) = 0.8\ln x_{1,A} + 0.2\ln x_{2,A}$ and the Bank's preferences by $u_B(m_{1,B}, m_{2,B}) = m_{1,B}^{½} m_{2,B}^{½}$

a) Find the Walras equilibrium in this economy and explain why this is Pareto optimal?

b) We now want Allen to have a consumption in period 2 corresponding to half of his original heritage (i.e. x_{2B}=200). Can we ensure this as a market equilibrium? If you agree then show how, if not, then argue why not.

Answers

a) *the WE relative price is* $\frac{p_1}{p_2} = \frac{16}{25}$ *is* x_{1A}*=320*25/16=500 and* x_{2A}*=80,* m_{1B}*=500 and* m_{2B}*=320. This is PO since the MRS of the two are the same. It means that we cannot make either Allen or bank better off by changing the allocation between the two periods, without the other one being worse off.*
b) *If the allocation is a Pareto optimum, then we can implement this as a market equilibrium with transfers (the second welfare theorem). It is PO if his consumption in period 1 is 800, because* MRS_A*=*MRS_B *by this allocation. This could be the outcome of the redistribution (transfer) of incomes (or endowments, but there are other possible transfers that would also satisfy the conditions). The allocation can be sustained as a market equilibrium if the relative price is 1.*