Written Exam for the B.Sc. or M.Sc. in Economics winter 2013-14

Operations Research

Elective Course

Friday, January 17th

(3-hour open book exam)

The language used in your exam paper must be English or Danish.

This exam question consists of 4 pages in total (including this front cover)

Part 1

Consider the following model, P:

$$
\begin{aligned}
\text{Max } z = \; & 3x_1 + 4x_2 + 3x_3 \\
\text{s.t. } \; & 1x_1 + 2x_2 + 3x_3 \leq 8 \\
& 3x_1 + 2x_2 + 1x_3 \leq 10 \\
& 1x_1 + 2x_2 + 1x_3 \leq 7 \\
& x_i \geq 0 \qquad (i = 1, 2, 3)
\end{aligned}
$$

Q1.1: The model P is an LP model. What characterizes an LP model?

The model P has been approached with the Simplex algorithm and the following tableau has appeared (where all non-integer values are shown as simple fractions):

z	x1	x2	x3	s1	s2	s3	RHS
0	0	0	2	1	0	-1	1
0	1	0	0	0	1/2	-1/2	3/2
0	0	1	1/2	0	-1/4	3/4	11/4
1	0	0	-1	0	1/2	3/2	31/2

Q1.2: Is the presented solution optimal? If it is not optimal, then please continue the Simplex algorithm until optimality is reached.

Q1.3: Set up the dual model to model P. Call it D. What is the optimal solution to model D?

Consider to following model P2, which is an expansion of P where one new variable is added:

$$
\begin{aligned}
\text{Max } z = \; & 3x_1 + 4x_2 + 3x_3 + 2x_4 \\
\text{s.t. } \; & 1x_1 + 2x_2 + 3x_3 + 1x_4 \leq 8 \\
& 3x_1 + 2x_2 + 1x_3 + 1x_4 \leq 10 \\
& 1x_1 + 2x_2 + 1x_3 + 2x_4 \leq 7 \\
& x_i \geq 0 \qquad (i = 1, 2, 3, 4)
\end{aligned}
$$

Q1.4: Without solving this new model P2, determine whether the solution for P (with x_4=0) is still optimal. Use the results from Question 1.2 and 1.3

Part 2

Consider an Assignment Problem, AP, with the following cost matrix:

1	1	2	2	2	4
1	1	2	2	2	4
1	1	2	2	2	4
4	4	1	1	1	2
4	4	1	1	1	2
4	4	1	1	1	2

Q2.1: Find a minimum cost assignment in AP

Consider now a transportation problem, TP with the following cost matrix and supply/demands:

				supply:
	1	2	4	3
	4	1	2	3
demand:	2	3	1	

Q2.2: Describe how the TP above can be transformed into the AP above.

Q2.3: Find a feasible solution to the TP by using the minimum cost heuristic.
Is it (in this case) an optimal solution?

Part 3

Consider the following IP problem instance:

$$\begin{aligned} Max\ \ z = \ & 15\,x_1 + 8\,x_2 \\ s.t.\ \ & 4\,x_1 + 1\,x_2 \leq 20 \\ & 8\,x_1 + 6\,x_2 \leq 48 \\ & x_i \geq 0 \text{ and integers } (i = 1, 2) \end{aligned}$$

By relaxation of the integer constraints, the resulting LP has been solved using the Simplex algorithm and the following optimal Simplex tableau was found:

z	x_1	x_2	s_1	s_2	RHS
	1		3/8	-1/16	4.5
		1	-1/2	1/4	2
1			13/8	17/16	83.5

Q3.1: Find the first additional constraint that results from the Cutting Plane Algorithm (do not solve the resulting model)

Q3.2: Find the additional constraints that results from the Branch and Bound Algorithm (do not solve the resulting models)

Q3.3: The LP relaxed problem has an optimal objective function value of 83.5. What do we know about the optimal objective function value of the IP problem?