Exercise 1

State for each of the claims below whether they are false or true. Explain.

1) A consumer will always choose to consume such that the absolute value of her marginal rate of substitution between two goods equals the price ratio of the two goods, independently of her preferences/utility function.
2) The firm's supply curve will always be identical to the marginal cost curve.
3) In the long run, the price of the output from a competitive industry will have a tendency towards the minimal average costs.

Solution:

1) False, consider a consumer who has a solution on the boundary. The student should be able to explain why the two rates differ. A solution which mentions Leontief is also valid.
2) False, since it is the marginal cost curve above the average costs, because otherwise they would produce with negative profits and thus shut down the production
3) True, the student should be able to argue something like: a price which exceeds the minimal average costs would leave a positive profit and thus spurs entry by firms. Reversely, a price below the minimal average costs would leave some company with a negative profit, which would force these firms to exit the industry. This would tend to drive the supply up resp. down and thus the price down resp. up.

Exercise 2

Consider a consumer, Hanne, with a sweet tooth for chocolate (good 1) and then spends the rest of her income on other goods (good 2). Hanne has preferences for chocolate and other goods represented by a utility function given by

$$u(x_1, x_2) = 100ln(x_1) + x_2$$

for every positive amounts of each good. Hanne earns each 300 euros each month from her disability pension scheme.

The government plans to impose a 50 per cent tax on chocolate, which will increase the price of chocolate from 2 euros to 3 euros. Let the price of other goods be 1.

1) What quantity of chocolate is Hanne's choice before the planned tax on chocolate?
2) What quantity of chocolate is Hanne's choice after the tax has increased the chocolate price?
3) What amount of euros could the government tax Hanne as a lump-sum leaving her as well off as imposing the tax on chocolate?
4) What is the deadweight loss from the chocolate tax?

Solution:

1) The amount of chocolate is 50 (while consuming other goods of 200 euros)
2) The amount of chocolate is 100/3≈33.3 while other goods is 200 euros
3) Since it is quasi-linear preferences we can use the consumer surplus to calculate the EV (which is what we actually ask for), which is $100(\ln(50) - \ln(100/3)) - 100 + 100 = 100\ln(\frac{3}{2})$ = 40.5.
4) The deadweight loss is the difference between the tax revenue (33,3) and the EV, which hence becomes $EV - T = 100\ln\frac{3}{2} - \frac{100}{3} = \left(\ln\frac{3}{2} - \frac{1}{3}\right)100$ which is a loss of about 7 in monetary losses.

Exercise 3

Consider an economy in which butter (good 1) and bread (good 2) are the only goods available. The quantity amount of butter available in the economy is 10 pounds and bread is available in the quantity of 20 loafs. There are two consumers, Adam and Eve, who both enjoy bread and butter, but who have different preferences. Adam has preferences represented by a utility function $u_A(x_1, x_2) = x_1 + x_2$ and Eve has preference represented by $u_E(x_1, x_2) = \ln x_1 + x_2$.

Assume that there is private ownership in the economy, and that Eve owns 4 pounds of butter and 10 loafs of bread. Adam owns the rest of the total endowment.

1) Identify the Pareto efficient allocation of bread and butter in which Eve obtains the same utility level as she would obtain consuming her initial endowment.
2) Identify the Pareto efficient allocation of bread and butter in which Adam obtains the same utility level as he would obtain consuming his initial endowment.
3) Find the allocations from which Adam and Eve cannot find mutually agreeable improvements given their individual endowments.
4) Can any of the allocations in the core be obtained by a market equilibrium through a suitable lump-sum redistribution of endowments?

Solution:

1) At the interior we know that efficient allocations must have MRS_E = MRS_A or $\frac{1}{a} = 1$ where *a* is the pounds of butter allocated to Eve. Eve obtains a utility level of her endowment of 12. Letting *a=1* Eve must have 11 bread, and Adam obtain the consumption bundle given by 9 pounds of butter and 9 loafs of bread.
2) As above, but making sure that Adam obtain a utility level of 16, we let Eve get 1 pound of butter and 13 loafs of bread.
3) The Core is the Pareto efficient allocations which are individual rational in this two-consumer economy, i.e. such that no consumer is made worse off than his/her endowment. From 1) and 2) we see that Eve always receives 1 pound of butter, and the Core is then the set of allocations in the Edgeworth box where Adam receives 9 pounds of butter and varying bread between 9 and 7 loafs to Adam.
4) Since preferences are convex and strictly monotone, the fact that core allocations are Pareto efficient, implies that we can invoke the second fundamental theorem of welfare to obtain a market equilibrium.

Exercise 4

Consider the market for cigarettes. The inverse market demand is given by $P_D(x) = min\{0, a - bx\}$ where a,b>0 and x is the quantity of cigarettes, while the inverse market supply is given by $P_S(x) = c + dx$ where c,d>0.

1) Assume that *a-c>0*. Interprete this assumption.
2) Derive the equilibrium price of cigarettes and the quantity sold

The biannual wage bargains of cigarette workers have just concluded and resulted in an increase in the annual wage rate. Assume that no worker at the cigarette factory smokes.

3) How does the wage increase alter the market for cigarettes? What is the effect on the equilibrium price and quantity?

Solution:

1) The parameters are interpreted as: recall that *a* is the marginal willingness to pay for the first cigarettes (for the consumer with the highest initial MWP), and c is the marginal costs of the first produced cigarette; thus a-c is the excess willingness to pay over the marginal cost of the first cigarette, enabling a market equilibrium in which a positive quantity is traded.
2) In equilibrium $P_D(x^*) = P_S(x^*)$ and solving this we obtain $x = \frac{a-c}{d+b}$ and $p = \frac{ad+bc}{b+d}$
3) The increase in the wage rate increases the (short run) marginal costs, which we know from our study of the firm and the market, affects the (inverse) supply function of each and thus all cigarettes producers. Thus, the increase shifts the supply curve upwards. The result is that the equilibrium price increases and the quantity of cigarettes sold and produced is reduced. This follows since $\frac{dx}{dd} = -\frac{a-c}{d+b} < 0$ and $\frac{dp}{dd} = x - d\frac{dx}{dd} > 0$.

<u>Exercise 5</u>

A producer of jelly beans, JB inc., uses labour (*l*) and capital (*k*) to produce, having the production function

$$f(l,k) = l^{\alpha}k^{\beta}$$

where *α,β>0* and *α+β≤1*. The firm can rent capital at the rate *r* and labour at the rate *w*. JB inc. is contractually bounded to the leasing arrangement in a period of 6 months.

The government suddenly increases the corporate tax rate. This increases the cost of capital and thus raises the rental rate of capital from *r* to *r**.

1) What is the effect on the short run costs of JB inc.?
2) What is the long run effect on the cost function of JB inc.?
3) Explain the effects on employment in JB inc. in the short resp. the long run?
4) How would a tax on profits affect the costs of the firm, as an alternative to the corporate tax?

Solution:

1) In the short run the (economic) costs are not affected by the increase in the capital rental rate since it is fixed in the short run, and thus the alternative costs is zero or sunk cost.

2) By solving the cost minimization problem we obtain the conditional demand functions: the FOC is $\frac{\alpha}{\beta}\frac{k}{l} = \frac{w}{r} \leftrightarrow l = \frac{\alpha}{\beta}\frac{r}{w}k$ and inserting this into $l^\alpha k^\beta = y$ we obtain $\left(\frac{\alpha}{\beta}\frac{r}{w}k\right)^\alpha k^\beta = \left(\frac{\alpha}{\beta}\frac{r}{w}\right)^\alpha k^{\alpha+\beta} = y$ and hence $k = \left(\frac{\alpha}{\beta}\frac{r}{w}\right)^{-\frac{\alpha}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$ and $l = \left(\frac{\beta}{\alpha}\frac{w}{r}\right)^{-\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$. Then we see that an increase in the capital user cost r we obtain a reduction in the capital stock and an increase in the labour demand. Then we can use the fact (Shepards lemma) that $\frac{\partial c(w,r,y)}{\partial r} = k(w,r,y)$ i.e. the effect of increase in the rental rate of capital increases the cost by the conditional demand for capital. Thus the costs increase.

3) In the short run, employment is unchanged since the capital stock is unchanged. In the long run, there are two effects: employment increases since JB inc wishes to substitute away from capital towards labour, but also due to the increase of costs the firm retracts production levels which pulls towards a lower employment. A diagram illustrating the cases is sufficient. Algebraic, we obtain the cost function $c(w,r,y) = \left(\left(\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{-\frac{\beta}{\alpha+\beta}}\right) w^{\frac{\alpha}{\alpha+\beta}} r^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$. Then the marginal costs are $\frac{\partial c(w,r;y)}{\partial y} = \frac{1}{\alpha+\beta}\left(\left(\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{-\frac{\beta}{\alpha+\beta}}\right) w^{\frac{\alpha}{\alpha+\beta}} r^{\frac{\beta}{\alpha+\beta}} y^{\frac{1-\alpha-\beta}{\alpha+\beta}}$ and the supply function is then

$y(p,w,r) = (\alpha+\beta)^{\frac{\alpha+\beta}{1-\alpha-\beta}}\left(\left(\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{-\frac{\beta}{\alpha+\beta}}\right)^{\frac{\alpha+\beta}{1-\alpha-\beta}} w^{-\frac{\alpha}{1-\alpha-\beta}} r^{-\frac{\beta}{1-\alpha-\beta}} p^{\frac{\alpha+\beta}{1-\alpha-\beta}}$. But then we obtain the effect of a change in the rental rate $\frac{\partial \ln y(w,r,p)}{\partial \ln r} = -\frac{\beta}{1-\alpha-\beta}$. We can then see that the effect of the rental rate on employment $\frac{\partial \ln l(w,r,p)}{\partial \ln r} = \frac{\beta}{\alpha+\beta} + \frac{1}{\alpha+\beta}\frac{d \ln y}{d \ln r} = \frac{\beta}{\alpha+\beta}\left(1 - \frac{1}{1-\alpha-\beta}\right) = \frac{\beta}{1-\alpha-\beta} > 0$. Thus, we find that employment increases.

4) Since the after-tax profit is maximized by maximizing the pre-tax profit, the cost minimization solution is equivalent to the original before the corporate tax increase.