Written Exam for M.Sc. in Economics

Winter 2010/2011

Advanced Microeconomics

20. December 2010

Master course

3 hours written exam with closed books

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Exercise 1:

Consider an economy with two goods, two consumers and one firm. Consumers have identical consumption sets $X_i = \mathbb{R}^2_+ = \{v \in \mathbb{R}^2 | v_1, v_2 \geq 0\}$. Consumers are described by their endowment vectors $\omega_i \in \mathbb{R}^2_{++} = \{v \in \mathbb{R}^2 | v_1, v_2 > 0\}$, shares in the firm $\delta_i \in [0, 1]$ with $\delta_1 + \delta_2 = 1$ and utility functions $u_i(x_i^1, x_i^2) = (x_i^1)^{a_i}(x_i^2)^{b_i}$ where $a_i, b_i > 0$. The firm has a production set $Y \subset \mathbb{R}^2$ where

$$Y = \{(y^1, y^2) \in \mathbb{R}^2 | y^1 \leq 0 \text{ and } y^2 \leq -y^1\}.$$

1.1 State the utility maximization problems of the consumers (UMP). Find the demand functions of the consumers.

1.2 Find the supply correspondence of the firm.

1.3 Define Walrasian equilibria for the economy.

1.4 Find a condition on ω_1 and ω_2 such that the economy has a Walrasian equilibrium $(\bar{p}, \bar{x}_1, \bar{x}_2, \bar{y})$ where $\bar{y} = 0$.

Exercise 2:

Consider a pure-exchange economy with $L \geq 1$ goods and $I \geq 1$ consumers. Consumers have identical consumption sets $X = \mathbb{R}^L_+ = \{v \in \mathbb{R}^L | v^\ell \geq 0 \text{ for all } \ell\}$. Consumers are described by their initial endowment vectors $\omega_i \in \mathbb{R}^L_{++} = \{v \in \mathbb{R}^L | v^\ell > 0 \text{ for all } \ell\}$, and preference relations $\succeq_i$, where $\succeq_i$ is rational, strongly monotone, strictly convex and continuous for all i.

2.1 State the utility maximization problem of a consumer (UMP). Show that for every $p \in \mathbb{R}^L_{++}$ there exists a unique solution to (UMP).

2.2 Show that for every $p \in \mathbb{R}^L_+$ with $p_\ell = 0$ for some ℓ there exists no solution to (UMP).

2.3 Define allocations and Walrasian equilibria for the economy.

2.4 Let $(\bar{p}, (\bar{x}_i)_{i=1}^I)$ be a Walrasian equilibrium and $C \subset \{1, \ldots, I\}$ a group of consumers. Show that there exists no allocation $(x_i)_{i=1}^I$ such that $x_i \succ_i \bar{x}_i$ for all $i \in C$ and $\sum_{i \in C} x_i = \sum_{i \in C} \omega_i$.

Exercise 3:

Consider a stationary overlapping generations economy with time going from $-\infty$ to ∞, one good per date and one consumer, who lives for two dates, per generation. Consumers are described by their identical consumption sets $X = \mathbb{R}^2_+$, endowment vectors $\omega \in X$ and utility functions $u(x^y, x^o) = (\bar{v} - x^y)^2 + (\bar{v} - x^o)^2$, where $\bar{v} > 0$. It is assumed that $\omega^y, \omega^o \leq \bar{v}$ and $(\bar{v} - \omega^y)^2 + (\bar{v} - \omega^o)^2 \leq \bar{v}^2$. The market structure is spot markets and money where $p_t > 0$ is the price of the good at date t.

3.1 Illustrate the utility function. Illustrate the assumptions $\omega^y, \omega^o \leq \bar{v}$ and $(\bar{v} - \omega^y)^2 + (\bar{v} - \omega^o)^2 \leq \bar{v}^2$.

3.2 State the utility maximization problem of consumer t. Find the solution.

3.3 Define ordinary Pareto optimality for the economy. Find an allocation that is not ordinarily Pareto optimal. Explain you answer.

3.4 Define equilibria for the economy and show that there exists an equilibrium $((\bar{p}_t)_{t \in \mathbb{Z}}, (\bar{x}_t)_{t \in \mathbb{Z}})$ where $\bar{p}_t = 1$ for all t.