Eksamensopgave februar 2013, rettevejledning[1]

Problem 1

Ann consumes two goods, in continuous and strictly positive quantities, and has preferences which can be represented by the utility function $u_A(x_1,x_2) = \text{Min}\{x_1,x_2\}$.

Similarly, Bill has the utility function $u_B(x_1,x_2) = x_1 + 3\cdot x_2$.

Finally, Catie has the utility function $u_C(x_1,x_2) = x_1\cdot x_2$.

- 1a) Answer for each of the three consumers: Does he or she have preferences that are homothetic?
- 1b) Answer for each of the three consumers: Does he or she have preferences that are convex?
- 1c) Answer for each of the three consumers: Does he or she have preferences that are strictly convex?

Answer: All three have homothetic and convex preferences. However, only Catie has strictly convex preferences.

Problem 2

Consider a consumer who has the strictly quasi-concave, monotonically increasing, and twice differentiable utility function u, giving rise to the Marhallian demand function x(p,I), with p being the price system and I being an exogenously given income, and the Hicksian compensated demand function h(p,u).

- 2a) Is it possible, for a given good, m, that $\partial x_m(p,I)/\partial p_m > 0$?
- 2b) Is it possible, for a given good, m, that $\partial h_m(p,u)/\partial p_m > 0$?

Answer: The first is possible; for a Giffen good, the demand increases, even if the price increases, because the income effect for this inferior good (increasing demand, ceteris paribus) dominates the substitution effect (decreasing demand). The derivative of the Hicksian demand, however, must be non-positive.

Problem 3

Consider an industry with a market demand side (a downward sloping demand curve) which is the same in the short run and the long run. On the market's supply there is a (very large) number of potential producers all having access to identical productions technologies (with U-shaped average costs in the short run as well as the long run). Please explain how the following are determined

- equilibrium price level
- individual firm production quantities
- individual profits
- market output level
- the number of firms actually producing in the market

[1] Note that what is provided here is a merely a short-hand guide to the correct answers; it does not constitue a fully satisfactory hand-in

Do this for both the short run and the long run.

Answer: A good answer includes a description of how the supply curve of an individual firm is identified as the upward-sloping part of the MC-curve lying above the AC-curve, both in the short and the long run. Furthermore, the LRAC-curve is the lower envelope of the SRAC-curves (each of these corresponding to a certain amount of fixed capital). Equilibrium is found where the market supply curve intersects the market demand curve. In the long run, the individual firm adapts its fixed capital to minimize AC; furthermore, firms can enter and exit, both of which affect market supply and hence price and quantity. If market price is above the minimal AC, firms will tend to be attracted, increasing supply, putting a downward pressure on the market price until it has been lowered (approximately) to min-AC.

<u>Problem 4</u>

Consider a Koopmans economy with one consumer whose 24 hours can be used as labor in the manufacturing unit producing a consumption good (good 2) or enjoyed as leisure (good 1). The manufacturing unit has the production function $x = l$, with l being the number of labor hours (input), and x being the output quantity of the consumption good. The consumer's utility function is $u(f,x) = f^a \cdot x^{(1-a)}$, with f being leisure, and x being the quantity of the consumption good.

Find the efficient (Pareto Optimal) allocation and comment on the role the parameter a plays for the result.

Answer: The FOC for the efficient allocation is that the absolute value of MRS is 1, i.e. $[a \cdot q]/[(1-a)\cdot(24-q)] = 1$*, giving us* $x^* = (1-a)\cdot 24$*, and* $f^* = a \cdot 24$*. Obviously, the larger the value of a, the more the consumer appreciates leisure compared to consumption, and the less should be produced in the economy.*

<u>Problem 5</u>:

Consider a consumer who has the utility function $u(x_1,x_2) = x_1^{½} + x_2$, has the exogenously given money income I and meets the market price system (p_1,p_2).

Such a consumer will have the following expression for his or her Marshall demand function (looking solely at interior solutions to the consumer's problem):

- $x_1(p,I) = [p_2^2/(4\cdot p_1^2)]$
- $x_2(p,I) =[(I/p_2) - p_2/(4\cdot p_1)]$

He or she will have the following expression for his or her Hicksian compensated demand function (looking solely at interior solutions to the consumer's problem):

- $h_1(p,u) = [p_2^2/(4\cdot p_1^2)]$
- $h_2(p,u)=[u - p_2/(2\cdot p_1)]$

- 5a) Show that when $I^* = 9$ and the price system is $p^* = (p_1^*, p_2^*) = (1,2)$, the consumer chooses $(x_1^*, x_2^*) = (1,4)$ obtaining utility level $u^* = 5$.
- 5b) Consider the Slutsky expressions for the impact of a marginal price increase for good one on demand for both goods (evaluated at price system p* and income I*):
$\partial x_1(p^*,I^*)/\partial p_1 = \partial h_1(p^*,u^*)/\partial p_1 - [\partial x_1(p^*,I^*)/\partial I]\cdot x_1^*$
$\partial x_2(p^*,I^*)/\partial p_1 = \partial h_2(p^*,u^*)/\partial p_1 - [\partial x_2(p^*,I^*)/\partial I]\cdot x_1^*$

and show, by the calculating the value of these derivatives etc. that these two Slutsky equations are true, and comment on the results.

Answer:

For good 1, $\partial x_1(p,I)/\partial p_1 = -½\cdot p_1^{-3}\cdot p_2^2$, which at (p,I*) becomes: −2; $\partial h_1(p,u)/\partial p_1 = -½\cdot p_1^{-3}\cdot p_2^2$, which at (p*,u*) becomes: −2. [$\partial x_1(p^*,I^*)/\partial I$] = 0. Hence, not surprisingly, preferences being quasi-linear, the impact from a price increase for good one on that good itself is merely the substitution effect.*

For good 2, $\partial x_2(p,I)/\partial p_1 = p_2\cdot p_1^{-2}/4$, which at (p,I*) becomes: +½. $\partial h_2(p,u)/\partial p_1 = ½\cdot p_2\cdot p_1^{-2}$, which at (p*,u*) becomes: 1. [$\partial x_2(p^*,I^*)/\partial I$] = $1/p_2$, which at (p*,u*) becomes: ½. With $x_1^* = 1$, we get the result that the marginal substitution effect is +1, whereas the marginal income effect is −½ (good 2 being normal), giving the total marginal effect of +½.*

<u>Problem 6:</u>

Consider some statements which concern production units within the milk industry. Alra Milk's production unit can be described by the production function x = f(l, k), where l is the quantity of labour input and k is the quantity of capital, x is the quantity of milk output, and f is a differentiable function.

Now, consider the following statements. If you think a claim is true, please prove it, or at least explain why you think it is true. If you think it is not true, please provide a counter-example.

- 6a) If Unity Milk similarly can be described by the production function x = g(l, k), and g is a monotonically increasing transformation of f (g = φ∘f, where φ is the transformation function, φ'(t) > 0), then Unity and Alra will have identical cost functions
- 6b) If (l, k, x) is a feasible production plan (i.e. x = f(l, k)), and (l, k) minimizes production costs at input prices (w,r) and output price p, then this production plan is profit-maximizing.
- 6c) If (l, k, x) is a feasible production plan and maximizes profits at input prices (w,r) and output price p, and if both q_1 and q_2 are strictly positive, then the absolute value of the marginal rate of transformation between the two inputs will be identical to the relative input price, w/r.

Answer: First claim is false. Let $y = f(q_1, q_2) = q_1^{1/4}\cdot q_2^{1/4}$, which clearly has decreasing returns to scale and will have increasing marginal costs, and let $\varphi(t) = t^2$. Then g will have constant returns to scale and constant MC. Second claim is false. Cost minimization is a necessary, not a sufficient, condition for profit maximization. Third claim is true and follows from cost minimization-FOC (iso-cost curve tangent to the iso-quant curve).