Answers
Final exam in Public Finance - Fall 2018
3-hour closed book exam

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Part 1: Effective marginal tax rates

Consider individuals facing the budget constraint

$$(1 + t_x)x = z - t_L + b, \tag{1}$$

where x is consumption, z is labor earnings, b is a public transfer, t_x is a tax rate on consumption and t_L is a lump sum tax. The public transfer is phased out with earnings at a rate q so that

$$b = \bar{b} - qz, \tag{2}$$

where $\bar{b}$ is fixed.

(1A) *Show that a marginal increase in labor earnings increases individual consumption by*

$$\frac{dx}{dz} = \frac{1-q}{1+t_x}, \tag{3}$$

and compute the effective marginal tax rate (m) by using the formula $\frac{dx}{dz} = 1 - m$.
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Substituting equation (2) into equation (3), isolating x and differentiating wrt. z yield

$$(1 + t_x)x = z - t_L + \bar{b} - qz \Leftrightarrow x = \frac{(1-q)z - t_L + \bar{b}}{1+t_x} \Rightarrow \frac{dx}{dz} = \frac{1-q}{1+t_x}.$$

Exploiting the definition of the effective marginal tax rate yields

$$\frac{dx}{dz} = 1 - m \Leftrightarrow \frac{1-q}{1+t_x} = 1 - m \Leftrightarrow m = \frac{q+t_x}{1+t_x}.$$

(1B) *Provide an interpretation of equation (3). How do t_x, t_L and q affect the incentive to earn income.*

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Equation (3) describes how much the individual can increase her consumption by increasing her labor earnings (marginally) and is often refered to as the *after-tax rate*. The after-tax rate depends negatively on both q and t_x: q as the public transfer is phased out with higher earnings and t_x as the tax on consumption decreases the purchasing power of (earned) income. The after-tax rate does not depend on t_L as it is a lump sum tax independent of earnings.

A larger after-tax rate implies a larger incentive to earn income and hence to supply labor through the substitution effect and both q and t_x hence reduce the incentive to earn income through this channel. Pulling in the opposite direction t_x, t_L and q all mechanically reduce the real income of the individuals, which give an incentive to increase labor supply through the income effect (assuming that leasure is a normal good).

The government considers a reform that changes the public transfer into a universal basic income that pays out the same benefits to everybody irrespectively of their income ($b = \bar{b}$). The reform is budget neutral and in order to finance the expansion of benefits, the goverment plans to introduce a proportional tax on labor earnings (t_z), so that the new budget constraint will be

$$(1 + t_x)x = z - t_z z - t_L + b. \tag{4}$$

(1C) *Discuss the effect of the proposed reform on the incentives to earn income and the likely effects on labor supply.*

#

To assess the effect of the proposed reform, we can compare the after-tax rate created by the new system to the after-tax rate in the old system. From equation (4), we can derive the after-tax rate as

$$\frac{dx}{dz} = \frac{1 - t_z}{1 + t_x}.$$

This can be bigger or smaller than the after-tax rate in equation (3) depending on t_z and q, however given that the proposed reform is budget neutral, we should expect $t_z = q$. With $t_z = q$ the effective budget set under the new system is exactly the same as under the old system, hence there are no real changes in the incentives to earn income and we should expect no effects on labor supply.

Part 2: The socially optimal top tax rate

Consider an economy with N high income individuals. Their preferences are represented by the utility function

$$u(c_i, z_i) = c_i - \frac{1}{1+\frac{1}{\varepsilon}} z_i^{1+\frac{1}{\varepsilon}}, \tag{5}$$

where c_i is consumption, z_i is labor income and ε is a preference parameter. The budget constraint is given by

$$c_i = z_i - T(z_i), \tag{6}$$

where $T(z_i)$ is a tax function. Assume that the tax function is described by

$$T(z_i) = m_L K + m_H(z_i - K) \tag{7}$$

where K is a threshold, which is below the individuals' optimal labor income z_i^*, while m_L and $m_H > m_L$ are marginal tax rates.

(2A) *Illustrate the budget set created by the tax system and the optimum of one individual in a diagram with labor income (z) on the x-axis and consumption (c) on the y-axis. Show that the individuals' optimum is characterized by $z_i^* = (1 - m_H)^{\varepsilon}$.*

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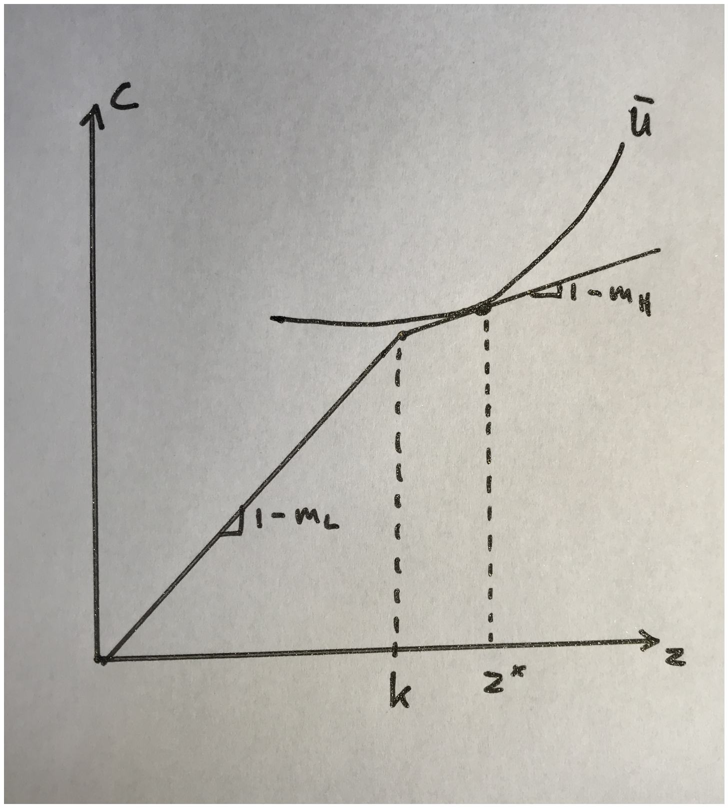
The budget set and optimum is illustrated in the figure below.



To derive the individual optimum, we substitute the equations (6) and (7) into (5) and differentiate wrt. z_i

$$u(c_i, z_i) = z_i - [m_L K + m_H(z_i - K)] - \frac{1}{1+\frac{1}{\varepsilon}} z_i^{1+\frac{1}{\varepsilon}}$$

$$\Rightarrow \frac{du(c_i, z_i)}{dz_i} = 0 \Leftrightarrow 1 - m_H = z_i^{\frac{1}{\varepsilon}} \Leftrightarrow z_i^* = (1 - m_H)^{\varepsilon},$$

which is what we were asked to show.

The total tax revenue (R) from the high income individuals can be written as

$$R = \sum_i T(z_i) = \sum_i [m_L K + m_H(z_i - K)] \tag{8}$$

(2B) *Show that the effect of a change in m_H on the total tax revenue can be written as*

$$\frac{dR}{dm_H} = N(\bar{z} - K) - N\frac{m_H}{1 - m_H}\varepsilon \bar{z}, \tag{9}$$

where $\bar{z} = \frac{1}{N}\sum_i z_i$ and $\varepsilon = \frac{dz_i}{d(1-m_H)}\frac{1-m_h}{z_i}$ is the labor supply elasticity, which is assumed constant for all individuals. Describe the result in equation (9). How does it relate to the mechanical (dM) and behavioral (dB) effects of the change in m_H?

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Differentiating equation (8) wrt. m_H yields

$$\frac{dR}{dm_H} = \sum_i \left[(z_i - K) + m_H \left(\frac{dz_i}{d(1-m_H)} \underbrace{\frac{d(1-m_H)}{dm_H}}_{=-1} \right) \right],$$

which using the definition of the labor supply elasticity can be written to

$$\frac{dR}{dm_H} = \sum_i \left[(z_i - K) - \varepsilon \frac{m_H}{1-m_H} z_i \right] = \underbrace{N(\bar{z} - K)}_{dM} \underbrace{-N \frac{m_H}{1-m_H} \varepsilon \bar{z}}_{dB}.$$

The expression for $\frac{dR}{dm_H}$ consists of two terms. The first is the mechanical effect of the tax change (dM), which describe the increase in revenue holding fixed labor supply. The tax base for m_H is given by the labor income above K, that is $N(\bar{z} - K)$.

The second term is the behavioral effect (dB), which describe the change (drop) in revenue from the change in labor supply following the tax increase. The size of the behavioral effect is govern by two parameters. First, the labor supply elasticity (ε), which determine how strongly individuals react to a change in the after-tax rate $(1 - m_H)$. A larger elasticity implies a larger loss of tax revenue. Second, the marginal tax rate (m_H), which determines the effect on revenue of a given change in labor supply. With $m_H = 0$, the behavioral effect is zero regardless the size of the labor supply elasticity. In a more general case, the behavioral effect could also depend on $\bar{z} - K$ as the income effect of a tax change depends on the share of income affected by the tax. However, at the utility function in (5) is quasi-linear, the income effects are zero in the case at hand.

The government considers a reform that increases m_H marginally. The extra tax revenue is paid back to everyone in the economy lump sum.

(2C) *Argue why the effect of an increase in m_H on aggregate social welfare (W) can be written as*

$$dW = dM + dB - g_H dM, \tag{10}$$

where $g_H < 1$ is the marginal social welfare weight on individuals with labor income above K relative to everyone in the economy.

#

The considered tax reform has two effects on aggregate welfare. First, it collects revenue from the individuals with labor income above K, which reduces their utility. From the envelope theorem, we know that behavioral changes to a small change in the tax rate has no first order effect on private utility (this can be shown by differentiating equation (5) wrt. m_H and using the first order condition) and the effect of a small increase in m_H on their utility is therefore simply given by the mechanical effect, which the government values g_H (on the margin). Hence, the collection of revenue from the individuals with labor income above K reduces aggregate social welfare by $g_H dM$.

Second, the reform redistribute the additional tax revenue to everyone in the economy lump sum. The revenue available to redistribution is given by $dM + dB$ and because it is distributed to everyone the implied welfare weight is (by denifition) 1. Hence, the distribution of the collected tax revenue increases aggregate welfare by $dM + dB$.

Adding the two effects together gives equation (10).

(2D) *Use the equations (9) and (10) to show that the socially optimal top tax rate* (m_H^*) *is given by*

$$m_H^* = \frac{1 - g_H}{1 - g_H + \varepsilon\alpha}, \tag{11}$$

where $\alpha \equiv \frac{\bar{z}}{\bar{z} - K}$.

#

Inserting (9) into (10) yields

$$dW = (1 - g_H)N\,(\bar{z} - K) - N\frac{m_H}{1 - m_H}\varepsilon\bar{z}.$$

Using that at optimum $dW = 0$, the above equation implies

$$(1 - g_H)N\,(\bar{z} - K) - N\frac{m_H}{1 - m_H}\varepsilon\bar{z} = 0$$

$$\Leftrightarrow m_H^* = \frac{1 - g_H}{1 - g_H + \varepsilon\frac{\bar{z}}{\bar{z} - K}} = \frac{1 - g_H}{1 - g_H + \varepsilon\alpha},$$

which is what we were asked to show.

The table below reports the size of m_H^* *for different values of* ε, α *and* g_H.

The socially optimal top tax rate

α:	3.4	3.4	1.8	1.8
ε:	0.1	0.3	0.1	0.3
$g_H = 0$	74.6%	49.5%	84.7%	64.9%
$g_H = 0.6$	54.1%	28.2%	69.0%	42.6%

(2E) *Provide a thorough discussion of the importance of ε, α and g_H for the socially optimal top tax rate.*

#

We see from equation (11) and the table above that the socially optimal top tax rate is decreasing in ε, α and g_H.

The intuition for the effect of α is the following: the revenue of an increase in m_H depends negatively on α, which is a measure of the income constration of the top taxpayers. When the average income of the top taxpayers ($\bar{z}$) is close to the top tax threshold (K), α is high and vice versa. As only the income above K is taxed at the top tax rate (m_H), a high α implies that only a small portion of the top taxpayers' income is affected by the higher tax rate and the mechanical revenue effect is therefore small.

Similarly, the intuition for the negative effect of ε comes from the fact that a higher labor supply elasticity increase the (negative) behavioral effect. Both a higher α and a higher ε therefore reduce the socially optimal top tax rate as they reduce the amount of additional revenue from a given increase in m_H that can be redistributed to everyone.

Finally, g_H decreases the socially optimal top tax rate by increasing the negative effect of the utility loss of top taxpayers on aggregate welfare. If $g_H = 0$, we put no (marginal) social value on the consumption of the top taxpayers and the socially optimal top tax rate is therefore equal to the revenue maximizing top tax rate. If $g_H = 1$, we put the same welfare weight on top taxpayers as on everybody else and hence there are nolonger any reasons to redistribute using distortionary taxation. The socially optimal top tax rate therefore becomes zero for any $\varepsilon > 0$ and α.

Part 3: Social insurance

Consider an unemployed individual, who has to decide how hard to search for a new job. If the individual chooses a search effort of e, she finds a job with probability $p(e) = e$. Searching for a new job has the disutility cost of $v(e)$ with $v'(e) > 0$ and $v''(e) > 0$. Once employed, the individual earns an income of z and pays taxes tz. If the individual remains unemployed, she *receives the benefits b. The individual's expected utility is given by:*

$$U = e \cdot u\left(z(1-t)\right) + (1-e) \cdot u(b) - v(e), \tag{12}$$

where $u(\cdot)$ is the utility of consumption with $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The government's budget constraint is given by $e \cdot t \cdot z = (1-e)b$.

(3A) *Show that the first best insurance scheme (where the government can control e directly) implies that individuals have full insurance ($z(1-t) = b$). Explain why full insurance is optimal in this case.*

#

To show this we need to maximize (12) wrt. e, t and b subject to the government's budget constraint.

Using the government's budget constraint we can first elimate t in the expected utility function by rewriting and inserting:

$$e \cdot t \cdot z = (1-e)b \Leftrightarrow t = \frac{1-e}{e}\frac{b}{z} \Rightarrow U = e \cdot u\left(z\left(1 - \frac{1-e}{e}\frac{b}{z}\right)\right) + (1-e) \cdot u(b) - v(e).$$

Maximizing wrt. b (while holding e fixed) yields:

$$\frac{\partial U}{\partial b} = -e \cdot u'\left(z - \frac{1-e}{e}b\right)\frac{1-e}{e} + (1-e) \cdot u'(b) = 0$$

$$\Leftrightarrow u'\left(z - \frac{1-e}{e}b\right) = u'(b) \Leftrightarrow u'\left(z(1-t)\right) = u'(b).$$

Given the properties of the utility function, $u'\left(z(1-t)\right) = u'(b)$ implies that also $z(1-t) = b$. That is, if the government can fully control the individuals search effort (e), it is optimal to fully smooth consumption over the two states (employed and unemployed) when individuals are risk adverse (decreasing marginal utility of consumption $u''(\cdot) < 0$).

The answer may also show the optimal e, but this is not necessary.

(3B) *Consider instead the situation, where the government sets b and t without being able to observe (or control) e. Show that the individual optimization, when b and t are taken as given, implies $v'(e) = u(z(1-t)) - u(b)$. What would be the consequence if the individual had full insurance in this case?*

#

To find the individual optimum, we maximize (12) wrt. e while taking t and b as given.

$$\frac{\partial U}{\partial e} = u(z(1-t)) - u(b) - v'(e) = 0$$

$$\Leftrightarrow u(z(1-t)) - u(b) = v'(e).$$

If the government provided full unemployment insurance in this case the left hand side of the equation above would be zero, and hence search effort would likely be zero too, which is too low compared to the first best (moral hazard).

Given that the government cannot control e, it is very difficult to overcome this problem without (further) government intervention, and the government will have to lower b in order to secure sufficient search effort from individuals. Hence the government faces a trade-off between efficiency (sufficient search effort) and social insurance (coverage of income loss).

The article "Cash-on-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market" in the Quarterly Journal of Economics (2007) by Card, Chetty and Weber studies the effects of unemployment benefits (and severance payments) on unemployment using data from Austria. Below (next page) is a copy of Figure 2, Figure 8 and Figure 10 from the article.

(3C) *Describe the empirical analysis. What do the figures imply in terms of the effect of unemployment benefits (and severance payments) on the duration of unemployment and the match quality of the subsequent jobs.*

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Card et al. (2007) use a regression discontinuity method to estimate the effect of unemployment insurance on the duration of unemployment. They exploit that the unemployment insurance (UI) benefit period in Austria depends on the employment history of the individual with a jump in the period (from 20 weeks to 30 weeks) when a person has been employed for more than a certain threshold (36 months during the past 5 years). Under some assumptions (discussed in question 3D), it is possible to obtain a casual estimate of the effect of extending the UI benefit period by comparing individuals with past employment just below and just above the threshold. In contrast, simply comparing all individuals with long UI benefit periods to all individuals with short periods would in general lead to biased estimates, as those with long UI periods have a stronger employment history and hence are more likely to find a job quickly.

Figure 8 and Figure 10 are so-called bin-scatter plots, which shows the average outcome for individuals within each bin (months of past employment) as dots and a fitted polynomium as a line.

Figure 8 shows that individuals just above the threshold are without a job 7 days longer and have a lower average job finding rate than those just below the threshold. Figure 8 therefore points to a positive casual effect of UI benefits on unemployment (longer UI benefit period increases unemployment duration). Figure 10 examines the effect of extending the UI benefit period on the match quality of the subsequent jobs proxied by either subsequent wage growth (good match) or subsequent job ending rate (bad match). Here we see no (discontinuous) difference between individuals just above and just below the threshold. Hence, the results do not point to any casual effect of UI benefits on the match quality of the subsequent jobs.

(3D) *Provide an argument for whether or not the results in Card, Chetty and Weber (2007) are likely to be causal estimates of the effects of unemployment benefits. Is there anything in the graphs that validates or invalidates a causal interpretation?*

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The main threat to identification when using a regression discontinuity method/design, is that the distribution of individuals just around the threshold is not fully random. This may for example not be the case if individuals somehow can manipulate on what side of the threshold they end up. In the analysis of Card et al. (2007), firms may for example fire the least productive workers just before the 36 month threshold to avoid paying severance payments, while more productive workers might be able to persuade the firm to postpone the job termination to after the 36 months of employment. In this case, the regression discontinuity method would give biased results, as we effectively would compare more productive workers just above the threshold to less productive workers just below.

To assess the possibility of such manipulation/sorting, Card et al. (2007) perform a number of placebo tests, which examines whether there are discontinuous differences in variables prior to entry into the UI system. If for example more productive workers were able to move above the threshold, we should expect to see a discontinuous jump in the average past wages at the threshold. Card et al. (2007) do not find (strong) evidence for this type of behavior.

Another test, is to examine the distribution of layoffs around the threshold. If individuals were able to manipulate on what side of the threshold they end up, we should except to see an excess mass of layoffs just above the threshold. This does not appear to be the case as illustrated in Figure 2. Hence, Figure 2 (together with the other tests in their paper) overall validates causal interpretation of the results in Card et al. (2007).

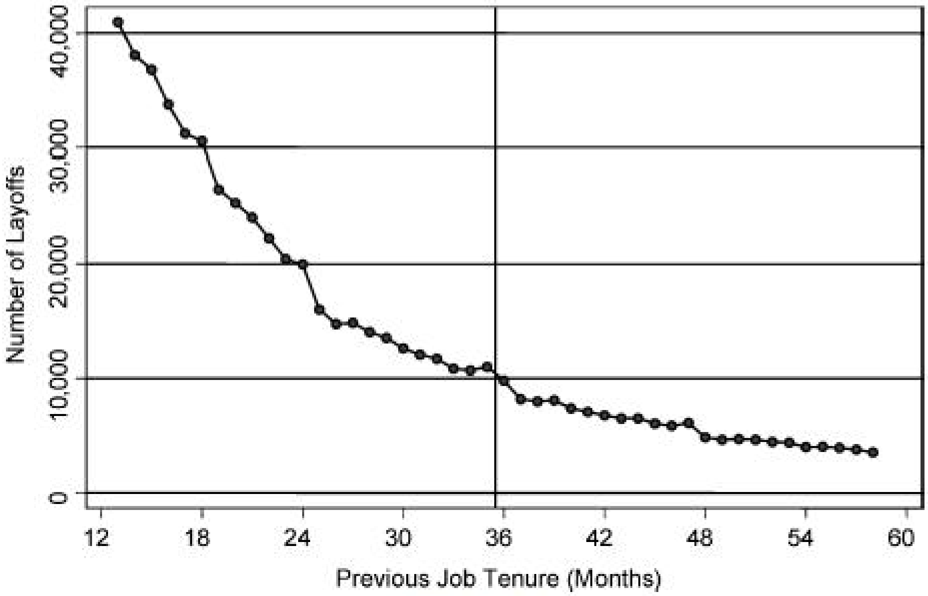

FIGURE II

Frequency of Layoffs by Job Tenure

Note. In this figure, individuals in the analysis sample are grouped into "tenure-month" categories based on the number of whole months they worked at the firm from which they were laid off. The figure plots the frequency of layoffs by tenure-month category, that is, the total number of individuals in the sample within each tenure-month category. The vertical line denotes the cutoff for severance pay eligibility.

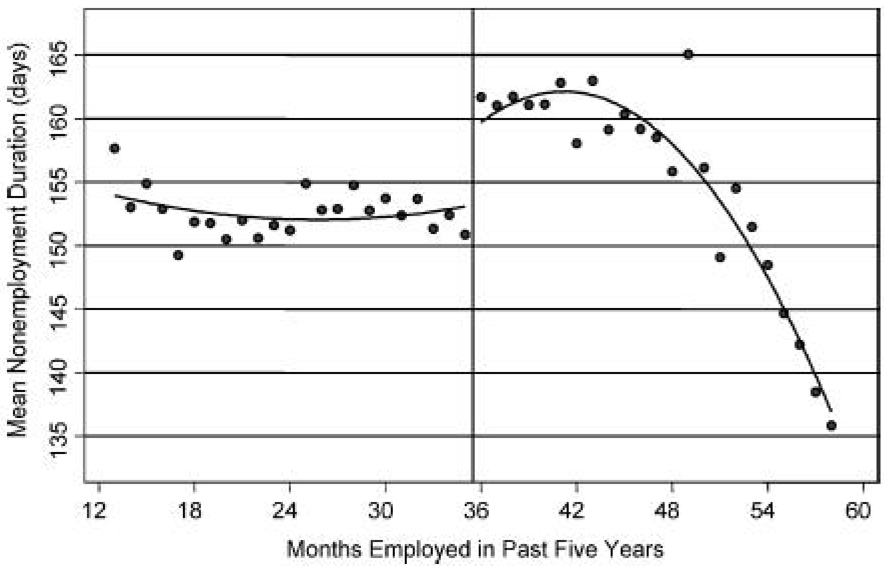

FIGURE VIIIa
Effect of Benefit Extension on Nonemployment Durations

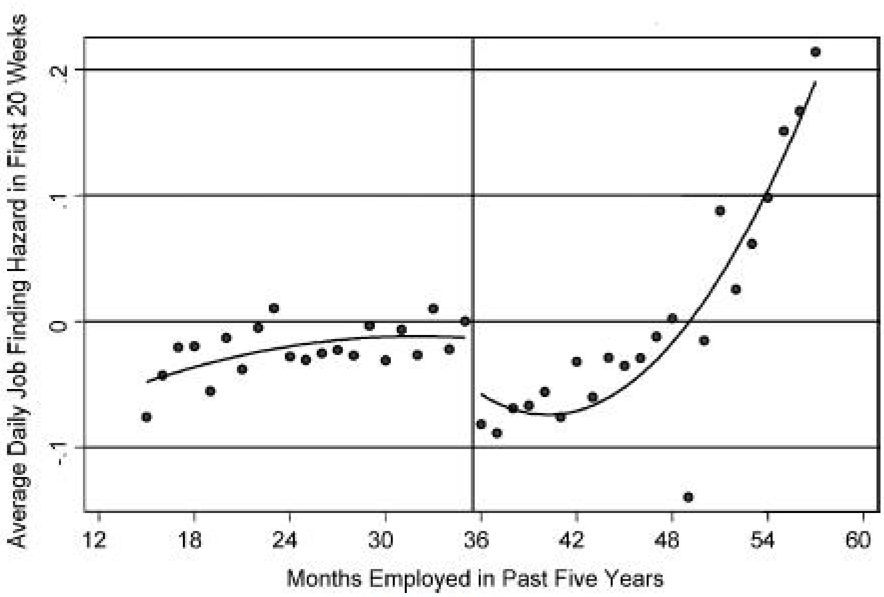

FIGURE VIIIb
Effect of Extended Benefits on Job-Finding Hazards

Note. In these figures, individuals are grouped into "months-employed" categories based on the number of whole months they worked at any firm within the past five years. Figure VIIIa plots mean nonemployment durations, excluding observations with nonemployment durations of more than two years. Figure VIIIb plots coefficients from a Cox model analogous to that used in Figure VI, controlling for the severance pay effect using a cubic polynomial. The values plotted can be interpreted as the percentage difference in the average job finding hazard during the first twenty weeks of the spell between each months-worked group and the group with months-worked equal to 35.

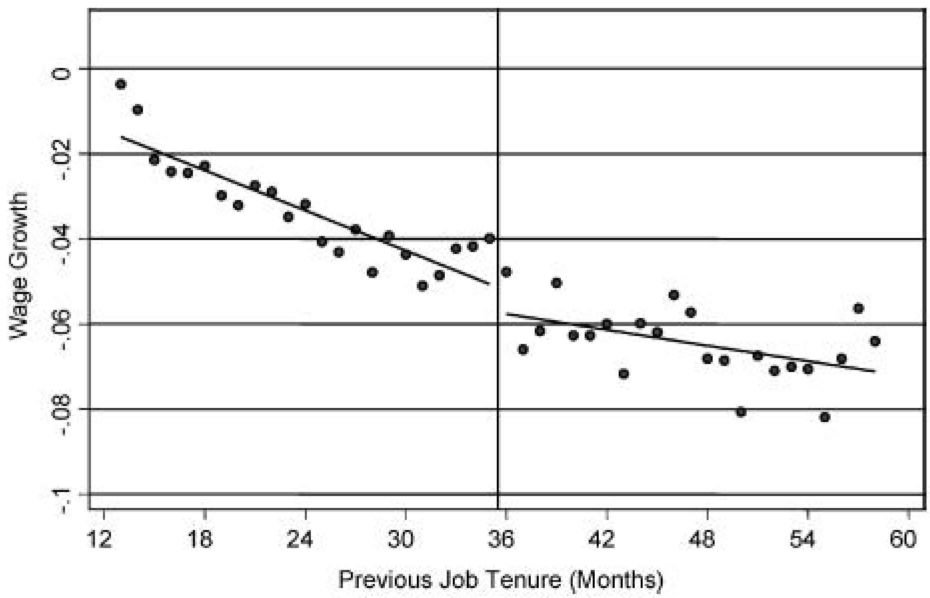

FIGURE Xa
Effect of Severace Pay on Subsequent Wages

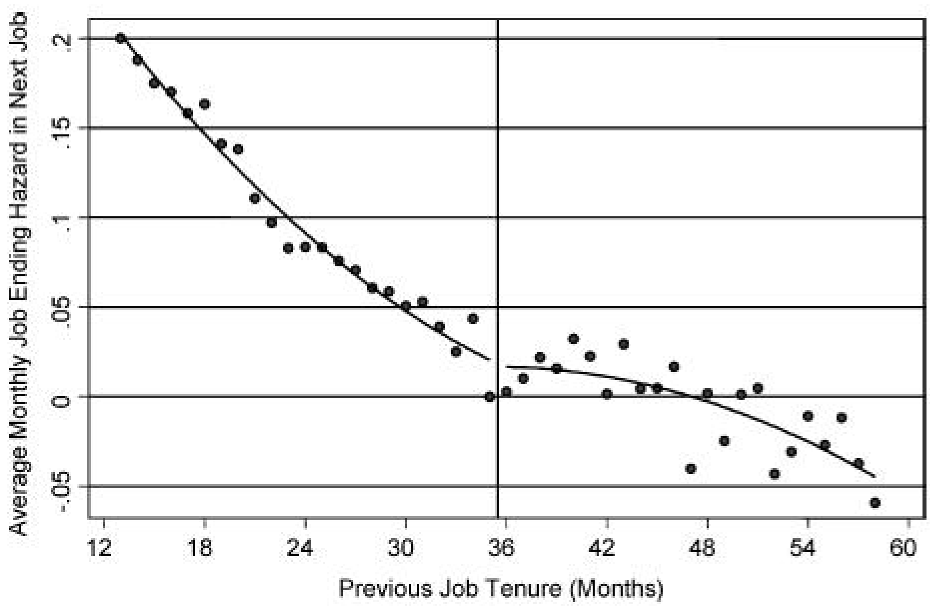

FIGURE Xb
Effect of Severance Pay on Subsequent Job Duration

Note. Figure Xa plots average wage growth (difference in log annual wage between next job and the job from which the individual was laid off) in each tenure-month group. Figure Xb plots coefficients from a Cox proportional hazards model for the duration of the next job with dummies for each job tenure category. The values can be interpreted as the percentage difference in the average job leaving hazard during the first five years of the next job between each job tenure group and the group with job tenure equal to 35. The sample for both figures includes all individuals observed in a new job.