Written Exam Economics Summer 2016

# Macroeconomics III

Final Exam

June 10, 2016

(3-hour closed book exam)

Please answer in English only.

**This exam question consists of 3 pages in total including this cover page.**

1. Consider the Diamond OLG model. Time is discrete and infinite, there is no population growth. As usual, small letters denote per capita units and large letters denote levels. The representative household in each generation lives for two periods and maximizes the following objective function

$$\log c_{1t} + \frac{1}{1+\rho} \log c_{2t+1}$$

where $c_{1t}$ denotes consumption when young in period $t$, and $c_{2t+1}$ denotes consumption when old in period $t+1$. He faces the following budget constraint when young

$$c_{1t} + s_t = w_t$$

where $w_t$ is the wage rate (and labor income since he inelastically supplies one unit of labor), and $s_t$ are savings. When old, his budget constraint is

$$c_{2t+1} = (1 + r_{t+1})s_t - \tau_{t+1}$$

where $r_{t+1}$ is the return he earns on his savings. The government imposes a lump sum tax $\tau_{t+1}$ on the old and uses the revenue to finance public expenditure. The government runs a balanced budget each period, so its budget constraint is

$$g_t = \tau_t$$

in all periods.

(a) Set up the household problem, derive the first order conditions and the Euler equation. Explain what behavior the Euler equation implies, and why it has to hold at the optimum.

(b) Derive the individual savings function. Does it depend on the interest rate? Why (not)?

The production side of the economy is standard: Competitive firms produce the consumption good with Cobb Douglas production technology

$$Y_t = K_t^\alpha L_t^{1-\alpha}$$

They hire capital and labor input from households. Assume in addition that government spending is proportional to output in the economy,

$$g_t = \gamma k_t^\alpha$$

and that capital fully depreciates each period, i.e. $\delta = 1$.

(c) Derive an expression for the law of motion for capital in this economy, that is $k_{t+1}$ as a (possibly implicit) function of $k_t$.

(d) Show that in steady state an increase in $\gamma$ increases capital and output in this economy. What is the intuition? How and why would this change if public expenditures were paid for with taxes on the young instead?

2. Consider the following model of monetary policy: the government controls inflation directly (i.e. $\pi_t = m_t$, where $\pi_t$ is the rate of inflation and $m_t$ is the rate of growth of money supply) and its instantaneous loss function is

$$L(\pi_t, x_t) = \frac{1}{2}\left[\pi_t^2 + \lambda\left(x_t - \bar{x}\right)^2\right]$$

where $x_t = \theta_t + \pi_t - \pi_t^e$. The following notation applies

| | |
|---|---|
| $\pi_t^e$: | expected rate of inflation |
| $x_t$: | output level |
| $\theta_t$: | potential output |
| $\bar{x}$: | policy output target |

We assume that potential output is stochastic and that its realizations are observed by both the public and the policy maker before expectations are formed by the private sector. Parameter $\lambda > 0$ measures the relative importance of output fluctuations around the target, $\bar{x}$, relative to inflation fluctuations.

(a) Show that the optimal policy under commitment implies $\pi_t^C = 0$ and $x_t^C = \theta_t$ [hint: i) recall that the loss function is quadratic, thus the optimal policy rule is linear and can be guessed to be of the form $\pi_t = \psi + \psi_\theta \theta_t$; ii) recall that the loss to be minimized is the unconditional one].

(b) Show that the optimal policy under discretion implies $\pi_t^D = -\lambda\left(\theta_t - \bar{x}\right)$ and $x_t^D = \theta_t$. The *inflation bias* increases in the target $\bar{x}$: explain why.

(c) Now set $\bar{x} = 0$ and assume there are two periods, i.e. $t = 1, 2$. Compute the optimal strategy at time $t = 1$ for a government that is expected to play $\pi_1^C$ but decides to deviate from the announced strategy (hint: the policy maker takes $\pi_1^e = 0$ as given when minimizing the loss function).

(d) Keep assuming $\bar{x} = 0$. What are the benefits and costs from deviating from commitment at time $t = 1$ and playing discretion at time $t = 2$ (hint: the benefit at $t = 1$ is the difference between the loss under commitment and the loss under the deviation strategy, while the cost at $t = 2$ is computed as the difference between the loss under discretion and the loss under commitment)? Show that for a (gross) rate of growth of potential output $\theta_2/\theta_1 > (1+\lambda)^{-\frac{1}{2}}$ it is ex-post optimal for the government to stick to the commitment rule announced at $t = 1$.