Mikro B, August 2014
Guide to answers[1]

Problem 1

Consider the von Neumann-Morgenstern agent Arthur with Bernoulli utility function of income $u(x) = x^{½}$. There are two possible states of the world, each having probability 50 %. In state 1 he has an income of 25. In state 2, an accident with an income loss of 16 occurs, reducing his income to 9.

There is a market for insurance, and Arthur is considering buying insurance from Pressure Insurance who is risk-neutral. An insurance contract (K, Γ) has Pressure paying the insurance sum K to Arthur when state 2 occurs, and Arthur paying Pressure the amount Γ in insurance fees in both states.

Suppose that the insurance market is characterized by perfect competition: Arthur may choose any non-negative value of K he desires, and the insurance fee he has to pay then becomes $\Gamma = ½K$.

- a) Write the maximization problem that Arthur faces, solve it, and describe the income level Arthur will have in each state, as well as his expected utility, after having chosen a maximizing contract.

Now suppose that Pressure Insurance does in fact have monopoly power. It wants to maximize its expected profits, but has to make sure to design the insurance contract (K, Γ) such that Arthur receives his reservation level of expected utility (what he is able to obtain without insurance).

- b) Write the maximization problem that Pressure faces, solve it, and describe the income levels Arthur will have, as well as his expected utility, when having accepted the contract designed by Pressure.
- c) Which level of expected profits will Pressure earn in a) and in b), respectively, and which of the market outcomes will be efficient?

Answer:
a) In the perfect competition case, with the actuarially fair price, the consumer's maximization problem becomes Max $½(25–½K)^{½} + ½(9+½K)^{½}$ *wrt. K,*
with the solution $K = 16$, $\Gamma = 8$, *the income level becoming 17 in both states, and expected utility becoming* $17^{½} = 4.123$. *Not surprisingly, the agent chooses to become fully insured.*
b) In the monopoly case, where the expected utility level without insurance, and hence the reservation (exp) utility level, is 4 (= $½(25)^{½}+ ½(9)^{½}$*), the company's maximization problem is* $Max\ \Gamma – ½K$ *wrt.* Γ *and K s.t.* $½(25– \Gamma)^{½} + ½(9+K–\Gamma)^{½} = 4$.
Using the FOC one gets that Arthur must become fully insured, hence obtaining utility 4 in both states, so we must have $\Gamma = 9$ *and* $K = 16$; *income becomes 16 in both states, expected utility is 4.*
c) Pressure receives zero in expected profits in case of perfect competition, but 1 in the monopoly case. Both outcomes are efficient, albeit with very different distribution of welfare (or gains from trade). The monopoly case is actually an example of perfect price discrimination (first-order).

Problem 2

[1] Note that this guide is only indicative and does not provide full answers to the problems; this document merely outlines the correct mathematical results and the most important points to be made.

Consider the market for beer, characterized by perfect competition, the market demand function being $D(p) = \text{Max}\{a - b\cdot p, 0\}$ and the supply function being $S(p) = \text{Max}\{c+d\cdot p, 0\}$ with $a, b, c, d > 0$ and $a > c$.

The government has introduced a tax of t per beer. The Minister of Finance, worried about getting in trouble with the European Union for having too large public deficits, is keen to "increase the beer tax very significantly", in order to create a higher tax revenue, limiting the public deficit.

- Find an expression for the beer tax revenue as a function of t; furthermore, give the Minister some good advice.

Answer:
Equilibrium has (neglecting, for simplicity, the non-negativity constraints)
$p_S = (a-c-b\cdot t)/(b+d)$, $p_D = (a-c+d\cdot t)/(b+d)$, $q = (a\cdot d+b\cdot c-d\cdot b\cdot t)/(b+d)$, *so tax revenue becomes (still neglecting negativity issues)* $R(t) = (a\cdot d+b\cdot c)\cdot t/(b+d) - b\cdot d\cdot t^2/(b+d)$ *which will decrease when t is larger than* $(a\cdot d+b\cdot c)/(2\cdot d\cdot b)$; *the Laffer curve effect.*

Problem 3

Arthur and Bill each sell drinks on campus on Friday nights. The market demand for drinks is given by the function $D(p) = \text{Max}\ \{240 - 2\cdot p, 0\}$. They can sell with no fixed costs. Both of them have constant marginal costs of 30 (producing and selling one more drink).

- a) Determine the Cournot equilibrium, i.e. the quantities sold, the price, and the profits made.
- b) What will happen to the equilibrium, if Arthur would be able to find a better supplier, enabling him to sell drinks at marginal costs below 30?

Answer:
a) Inverse demand is $p(x) = 120 - \frac{1}{2}x$, *so both of the producers have best-response function* $R_i(x_j) = 120 - 30 - \frac{1}{2}\, x_j$.. *The NE has both of them producing quantity 60, price becoming 60, both of the earning profits of 1,800.*
b) With c_A *lower than* $c_B = 30$, *A will gain a competitive advantage, being able to become "more aggressive" in his response function. In the new NE, A will produce more, B will produce less, total quantity will increase, the price goes down, A will earn more profits, B less (but B will not be blown completely out of the market, as would be the case with Bertrand competition).*

Problem 4

Consider an economy with two agents both of whom have an initial endowment of a private good (money). One unit of the private good can be transformed into one unit of a public good. The two agents have preferences which can be represented by utility functions $u_A(x_A,G) = v_A(G) + x_A$ and $u_B(x_B,G) = v_B(G) + x_B$, respectively. We assume that both v-functions are strictly increasing, strictly concave, and continuously differentiable.

- a) Show, mathematically and/or in a clear diagram, how voluntary individual donations to finance production of the public good will typically result in an inefficient outcome

- b) Show, by a mathematical example and/or in a clear diagram, how, in the case of voluntary individual donations, one agent may free-ride on the other agent's donation

Answer:
a) When both agents fulfill their FOC for private donation in a NE (i.e. both choose a positive donation), each of them have $v_i'(G) = 1$, the sum of their marginal utilities (willingness to pay) hence adding to 2, whereas the FOC for efficiency is this sum, $v_A'(G) + v_B'(G)$, being 1, so G will become inefficiently low.
b) If A has donated g_A and B has donated nothing, resulting in $G = g_A$, this will be a NE, if A has $v_A'(G) = 1$, whereas $v_B'(G) < 1$.

<u>Problem 5:</u>

Consider a monopolist who faces a downward-sloping and differentiable demand curve and who has chosen to produce a positive quantity.

- Please derive mathematically how, in the profit-maximizing situation, the price charged by the monopolist depends on the marginal costs and the elasticity of demand, respectively.

Answer:
When production is positive, we know that this FOC is satisfied: $MR(x) = MC(x)$. LHS can be expressed $p'(x)\cdot x + p(x) = p(x)\cdot[1+p'(x)\cdot x/p(x)]$. The last expression in the bracket is the elasticity of the inverse demand function, $p(x)$, and this is equal to the inverse of the elasticity of the demand function, $D'(p)\cdot p/D(p)$. Call this expression $e(x)$, with $x = D(p)$.
With $D'(p) < 0$, we can express the LHS as $p(x)\cdot[1-1/|e(x)|]$, so $p^* = MC(x)/[1-1/|e(x)|]$. So the price is the MC with an added mark-up, this mark-up being larger, the smaller the absolute value of the elasticity (with this approaching infinity, the mark-up approaches zero).

<u>Problem 6</u>

Please define, explain and comment on the concept "Lindahl Equilibrium"

Answer:
The Lindahl Equilibrium is a way of implementing the efficient outcome in an economy with a public good, doing it in a market compatible fashion. The authorities identify the efficient level of the public good G for which the sum of the individuals' marginal willingness to pay for the public good equals the (constant) MC of the public good. Each individual is asked to "order" his or her quantity of the public good, but at individualized prices (in fact the agent's marginal willingness to pay at G*). The result is that every agent orders the quantity G*. The concept is a somewhat artificial construction; in the real world, there are huge informational problems, each agent having an incentive to free-ride by under-stating his or her preferences for the public good.*

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