Written Exam for the B.Sc. in Economics summer 2012

Macro C

Final Exam

May 31

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Problem A: Inflation bias, delegation and supply shocks

Consider an economy where the aggregate supply curve is given by:

$$\pi_t = \pi^e_{t,t-1} + y_t - \bar{y} + s_t \quad \text{(A.1)}$$

where s_t is a white noice process, satisfying:

$$E[s_t] = 0,\ E[s_t^2] = \sigma_s^2 > 0 \text{ and } E[s_t \cdot s_z] = 0 \text{ for } t \neq z \quad \text{(A.2)}$$

Social preferences are characterized by the social loss function:

$$SL_t = \kappa \cdot (\pi_t - \pi^*)^2 + (y_t - y^*)^2 \quad \text{(A.3)}$$

where we assume $\kappa > 0$ and:

$$y^* = \bar{y} + \omega \quad \text{(A.4)}$$

$\omega > 0$ reflects the presence of supply side distortions.

A fraction β (where $0 \leq \beta \leq 1$) of the responsibility for the conduct of monetary policy is delegated to a central bank with a loss function given by:

$$SL_t^{cb} = (\kappa + \varepsilon) \cdot (\pi_t - \pi^*)^2 + (y_t - y^*)^2 \quad \text{(A.5)}$$

where is it assumed that: $\varepsilon > 0$.

The game structure is as follows: *At time $t-1$ the agents in the private sector form their inflation expectations. At time t the value of s_t is realized and then afterwards the government and the central bank in cooperation chooses the short-term interest rate which has a direct effect on output and thereby on the inflation rate (according to equation (A.1)).* We will assume that all agents have rational expectations and we will for simplicity assume that the policy makers directly choose the inflation rate (through the control over the short-term interest rate).

1) Show that the choice of the inflation rate at time t (taking $\pi^e_{t,t-1}$ as given) is characterized by the first order condition:

$$(\kappa + \beta \cdot \varepsilon) \cdot (\pi_t - \pi^*) + \left(\pi_t - \pi^e_{t,t-1} - s_t - \omega\right) = 0$$

2) Show that the time consistent equilibrium is given by:

$$\pi_{t,t-1}^{e} = \pi^{*} + \frac{\omega}{\kappa + \beta \cdot \varepsilon}$$

$$\pi_t = \pi^{*} + \frac{\omega}{\kappa + \beta \cdot \varepsilon} + s_t \cdot \frac{1}{1 + \kappa + \beta \cdot \varepsilon}$$

$$y_t = \bar{y} - s_t \cdot \frac{\kappa + \beta \cdot \varepsilon}{1 + \kappa + \beta \cdot \varepsilon}$$

(*hint: start by deriving the expression for* $\pi_{t,t-1}^{e}$ *using among others the first order condition above, the assumption of rational expectations and the assumption of no auto-correlation in the supply shocks*).

3) Explain why there will in general exist an inflation bias and illustrate the inflation bias graphically in the case without supply shocks and delegation (i.e. with $s_t = 0$ and $\beta \cdot \varepsilon = 0$). Explain how the inflation bias depends on the effective amount of delegation ($\beta \cdot \varepsilon$).
4) Explain the cost of delegation (i.e. explain why an increase in $\beta \cdot \varepsilon$ will not necessarily reduce average social loss given by $E[SL_t]$).

Assume now that the game structure is changed to the following:

At time $t-1$ *the agents in the private sector form their inflation expectations. At time* t *the government and the central bank in cooperation chooses the short interest rate (which directly affects output and thereby inflation) and then afterwards the value of* s_t *is realized.* We still assume that all agents have rational expectations.

5) How will this change in the game structure change the costs and benefits of delegation?

Problem B: Public consumption in the Ramsey model

In the following a version of the Ramsey model with public consumption is considered. The government finances public consumption by lump-sum taxes. We ignore technological growth, and

define c_t and k_t as consumption and capital per worker and r_t as the real interest rate. The rest of the notation is as usual.

At first we consider the problem of the representative household which is to choose a consumption path $(c_t)_{t=0}^{\infty}$ in order to maximize the intertemporal utility function:

$$U_0 = \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} \cdot e^{-(\rho-n)\cdot t} dt$$

subject to the evolution of wealth per worker:

$$\dot{a}_t = (r_t - n) \cdot a_t + w_t - T_t - c_t$$

and a No Ponzi game condition, taking a_0 as given

1) Show that the solution to the problem of the representative household is characterized by the Keynes Ramsey rule in (B.1) (*Hint: the No Ponzi game condition can be ignored when deriving (B.1)).* Further, interpret the Keynes Ramsey rule.

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\theta} \qquad \text{(B.1)}$$

The general equilibrium can among others be described by equation (B.1) and the following three equations:

$$r_t = f'(k_t) - \delta \qquad \text{(B.2)}$$

$$\dot{k}_t = f(k_t) - c_t - g - k_t \cdot (n + \delta) \qquad \text{(B.3)}$$

$$g = T_t \qquad \text{(B.4)}$$

where $g = G_t/L_t$ is consumption per worker, which is assumed to be constant. (B.4) is the public budget constraint. In addition the relevant transversality is assumed to be satisfied. At each point in time k_t is predetermined while c_t is free to jump. The function $y_t = Y_t/L_t = f(k_t)$ satisfies the usual properties, i.e.: $f'(k_t) > 0$, $f''(k_t) < 0$, $f(0) = 0$ and the Inada conditions.

We assume: $g \geq 0$, $\rho > n$, $\rho > 0$, $\theta > 0$ and $n + \delta > 0$.

2) Interpret (B.2) and (B.3). Construct the phase diagram and comment.

Now we will consider a permanent increase in g. At first we will assume that the increase in g isn't announced in advance. We will assume that the economy is initially in steady state.

3) Use the phase diagram to analyze the consequences of the increase in g. Explain the economic intuition.

Now we assume instead that the increase in g is announced at time t_0 and implemented at time t_1, where $t_1 > t_0$. Up until time t_0 the economy is in steady state.

4) Use the phase diagram to analyze the effects from t_0 and onwards. Explain the economic intuition carefully.

5) Discuss how the result in question 3) changes if the increase in public consumption is instead financed by a tax on capital income.