Written Exam for the B.Sc. in Economics autumn 2012-2013

Macro C

Final Exam

February 18, 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem A: A permanent productivity shock in the Diamond model

Consider the following version of the Diamond model. Competitive firms produce output according to a technology described by the following production function:

$$Y_t = \Psi \cdot F(K_t, L_t) = \Psi \cdot K_t^{\alpha} \cdot L_t^{1-\alpha} \tag{A.1}$$

Let's define output and capital in terms of per worker:

$$y_t = \frac{Y_t}{L_t}$$

$$k_t = \frac{Y_t}{L_t}$$

1) Show that output per worker is given by equation (A.2). Show also, that profit maximization (with respect to inputs of capital and labour) imply (A.3) and (A.4).

$$y_t = \Psi \cdot k_t^{\alpha} \tag{A.2}$$

$$w_t = (1-\alpha) \cdot \Psi \cdot k_t^{\alpha} \tag{A.3}$$

$$r_t = \alpha \cdot \Psi \cdot k_t^{\alpha-1} \tag{A.4}$$

In each generation the young households choose saving (s_t) and consumption as young and old (c_{1t} and c_{2t+1}) with the purpose of maximizing lifetime discounted utility given by:

$$U = \frac{c_{1t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \cdot \frac{c_{2t+1}^{1-\theta} - 1}{1-\theta}$$

Each household must allocate income as young between consumption in the young age and saving:

$$w_t = c_{1t} + s_t$$

while the old generation consumes their wealth:

$$c_{2t+1} = s_t \cdot (1 + r_{t+1})$$

2) Derive the intertertemporal budget constraint, and show that the optimal consumption profile is characterized by (A.5). Interpret the equation carefully.

$$\frac{c_{2t+1}}{c_{1t}} = \left(\frac{1 + r_{t+1}}{1+\rho}\right)^{1/\theta} \tag{A.5}$$

3) Show that the optimal consumption profile is characterized by (A.6) and (A.7). How will an increase in r_{t+1} affect consumption in the young age and saving?

$$c_{1t} = \frac{(1+\rho)^{1/\theta}}{(1+\rho)^{1/\theta} + (1+r_{t+1})^{(1-\theta)/\theta}} \cdot w_t \tag{A.6}$$

$$s_t = \frac{w_t}{1 + (1+\rho)^{1/\theta} \cdot (1+r_{t+1})^{(\theta-1)/\theta}} \equiv w_t \cdot s(r_{t+1}) \tag{A.7}$$

From now on we will only consider the case where: $\theta = 1$

Capital accumulation is given by:

$$K_{t+1} = L_t \cdot s_t \tag{A.8}$$

where L_t is the number of young households at time t. L_t is assumed to grow at the rate n, i.e.:

$$L_{t+1} = L_t \cdot (1+n) \tag{A.9}$$

4) Interpret (A.8) and show that capital per worker evolves according to:

$$k_{t+1} = \frac{1-\alpha}{(2+\rho)\cdot(1+n)} \cdot \Psi \cdot k_t^{\alpha} \tag{A.10}$$

5) Define the steady state and show that the steady state values of k_t and y_t are given by:

$$k^* = \left(\frac{1-\alpha}{(2+\rho)\cdot(1+n)} \cdot \Psi\right)^{1/(1-\alpha)}$$

$$y^* = \Psi^{1/(1-\alpha)} \cdot \left(\frac{1-\alpha}{(2+\rho)\cdot(1+n)}\right)^{\alpha/(1-\alpha)}$$

Now let’s consider an increase in Ψ. At first we assume that this is unanticipated.

6) Illustrate graphically the consequences and explain carefully the economic mechanisms.
7) Now assume that the increase is announced in advance. How will that change the results? What if the increase in Ψ is only temporary?

Problem B: Time inconsistency of monetary policy

Consider an economy where the short run aggregate supply curve is given by:

$$\pi_t = \pi_{t,t-1}^e + \gamma \cdot (y_t - \bar{y}) \tag{B.1}$$

The loss function of the government is given by:

$$SL_t = \kappa \cdot (\pi_t - \pi^*)^2 + (y_t - y^*)^2 \tag{B.2}$$

π^* is the inflation target of the monetary policy maker and y^* is the output target. We assume that:

$$y^* = \bar{y} + \omega \tag{B.3}$$

where $\omega > 0$.

Assume at first that the public believes in the central bank, i.e.: $\pi_{t,t-1}^e = \pi^*$

1) Interpret (B.3). Show that the temptation to cheat (i.e. the reduction in social loss) is given by the expression in (B.4). Explain this incentive and illustrate in an appropriate diagram.

$$Temptation\ to\ cheat = \frac{\omega^2}{1 + \kappa \cdot \gamma^2} \tag{B.4}$$

Consider now the time-consistent equilibrium where the agents in the private sector have rational expectations.

2) Show that equilibrium inflation exceeds π^*. Illustrate the time-consistent equilibrium in an appropriate diagram.
3) Explain how the monetary policy maker may rely on the reputation mechanism in order to alleviate the inflation bias problem.