Written Exam for the B.Sc. in Economics winter 2012-2013

Macro C

Final Exam

January 4, 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem A: Tax reform in the Ramsey model

Consider the following version of the Ramsey model, augmented with a public sector. The government taxes capital income at the rate τ and uses the resulting revenue for transfers to households. At each point in time each person receives a (lump-sum) transfer from the government given by $v(t)$. For simplicity we ignore depreciation of physical capital and technological growth. All other notation is as usual.

The problem of the representative household is to choose a consumption path, $\left(c(t)\right)_{t=0}^{\infty}$, in order to maximize total discounted lifetime utility given by:

$$U(0) = \int_0^{\infty} ln\big(c(t)\big) \cdot e^{-(\rho-n)\cdot t} dt \tag{A.1}$$

subject to the intertemporal budget constraint:

$$\int_0^{\infty} c(t) \cdot e^{-\int_0^t (r(s)\cdot(1-\tau)-n)ds} dt \leq a(0) + h(0) \tag{A.2}$$

where:

$$h(0) = \int_0^{\infty} \big(w(t) + v(t)\big) \cdot e^{-\int_0^t (r(s)\cdot(1-\tau)-n)ds} dt$$

You are informed that (A.2) can be written as a combination of a differential equation:

$$\dot{a}(t) = (r(t) \cdot (1-\tau) - n) \cdot a(t) + w(t) + v(t) - c(t) \tag{A.3}$$

and a condition limiting the asymptotic evolution of $a(t)$ (the No Ponzi game condition).

1) Show that the optimal growth rate of consumption is given by the differential equation (A.4) and interpret this. What does the optimal consumption pattern imply with respect to (A.2)?

$$\frac{\dot{c}(t)}{c(t)} = r(t) \cdot (1-\tau) - \rho \tag{A.4}$$

You are informed that the solution to the differential equation (A.4) is given by:

$$c(t) = c(0) \cdot e^{\int_0^t (r(s)\cdot(1-\tau)-\rho)ds} \tag{A.5}$$

2) Show that the corresponding consumption *level* is given by (A.6). Further, interpret this equation. How will higher (after-tax) interest rates in the future affect the level of consumption?

$$c(0) = (\rho - n) \cdot \big(a(0) + h(0)\big) \tag{A.6}$$

Hint: In deriving (A.6) you may use the fact that: $\int e^{k\cdot x} dx = k^{-1} \cdot e^{k\cdot x}$.

Profit maximization by firms (with access to a constant returns production technology) implies:

$$R(t) = f'\big(k(t)\big) \tag{A.7}$$

$$w(t) = f\big(k(t)\big) - k(t) \cdot f'\big(k(t)\big) \tag{A.8}$$

where it is assumed that: $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$ and that the Inada conditions are satisfied.

Finally, the general equilibrium is also characterized by the following two relations:

$$R(t) = r(t) \tag{A.9}$$

$$a(t) = k(t) \tag{A.10}$$

and the (balanced) public budget constraint:

$$L(t) \cdot \tau \cdot r(t) \cdot k(t) = L(t) \cdot v(t) \tag{A.11}$$

At each point in time $k(t)$ is predetermined while $c(t)$ is free to jump.

It is assumed that $\rho > n > 0$ and that $0 \leq \tau < 1$.

3) Interpret each of equations (A.7)-(A.10). Show that they together with (A.3) and (A.11) imply (A.12) and interpret this equation:

$$\dot{k}(t) = f\big(k(t)\big) - c(t) - n \cdot k(t) \tag{A.12}$$

4) Construct the phase diagram. Comment.

Now let's consider a tax reform which lowers the tax burden on capital and reduces household transfers., i.e. a reduction in τ and corresponding changes in $v(t)$, according to equation (A.11). The economy is in steady state up until the tax reform.

5) Use the phase diagram to analyze the consequences of the tax reform. Interpret carefully.

Problem B: Over- or undershooting in the Dornbusch model

Consider the Dornbusch model for a small open economy with flexible exchange rates, described by equation (B.1) – (B.4).

$$y(t) - \bar{y} = z + \beta \cdot \big(e^n(t) - p(t)\big) - \epsilon \cdot \big(r(t) - r^f\big) \tag{B.1}$$

$$m - p(t) = y(t) - \bar{y} - \varepsilon \cdot \big(r(t) - r^f\big) \tag{B.2}$$

$$r(t) = r^f + \dot{e}^n(t) \tag{B.3}$$

$$\dot{p}(t) = \gamma \cdot (y(t) - \bar{y}) \tag{B.4}$$

You are informed that equation (B.1) and (B.2) can be solved to yield:

$$y(t) - \bar{y} = \frac{\epsilon \cdot m - (\beta \cdot \varepsilon + \epsilon) \cdot p(t) + \varepsilon \cdot z + \varepsilon \cdot \beta \cdot e^{n}(t)}{\varepsilon + \epsilon} \tag{B.5}$$

$$r(t) - r^{f} = \frac{z - m + (1 - \beta) \cdot p(t) + \beta \cdot e^{n}(t)}{\varepsilon + \epsilon} \tag{B.6}$$

It is assumed that: $\beta > 0$, $\varepsilon > 0$, $\epsilon > 0$ and $\gamma > 0$.

1) Interpret each of the equations (B.1) – (B.6). How do $y(t)$ and $r(t)$ respond to an increase in $p(t)$ (taking $e^{n}(t)$ as given) and an increase in m (taking $p(t)$ and $e^{n}(t)$ as given)

At each point in time $p(t)$ is predetermined while $e^{n}(t)$ is free to jump.

2) Construct the phase diagram. Consider both the case where $\beta < 1$ and where $\beta > 1$.

Now consider an unanticipated monetary expansion (an increase in m). The economy is in long run equilibrium up until the monetary expansion.

3) Consider the case where $\beta < 1$. Use the phase diagram to analyze the consequences of the monetary expansion. Does the exchange rate over- or undershot in the short run[1]? Interpret carefully.
4) Consider the case where $\beta > 1$. Use the phase diagram to analyze the consequences of the monetary expansion. Does the exchange rate over- or undershot in the short run? Interpret carefully.

Now, assume instead that the increase in m is announced at time t_0 and implemented at time $t_1 > t_0$. For simplicity we will only consider the case where $\beta < 1$. The economy is in long run equilibrium up until t_0.

5) Use the phase diagram to analyze the consequences from time t_0 and onwards. Interpret carefully.

[1] In order to avoid any confusion: Short-run overshooting refers to the case where the increase or fall in the short run is larger than in the long run, while undershooting refers to the case where the change is smaller in the short run than in the long run