Written Exam for the B.Sc. or M.Sc. in Economics autumn 2012-2013

Advanced Microeconomics

Master's Course

22FEB2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

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Advanced Microeconomics, Autumn 2012-2013

3 hours closed book (re)exam

Anders Borglin, who is responsible for the exam problems, can be reached during the exam on +46 735 754176. There are, including the two pages with assumptions and the title page, altogether **6** pages.

There are 3 problems. The problems B and C have the same weight in the marking process and Problem A has half the weight of Problem B.

Below

$$\begin{aligned} \mathbb{R}^k_+ &= \left\{x \in \mathbb{R}^k \mid x_h \geq 0 \text{ for } h = 1, 2, \ldots, k\right\} \text{ and} \\ \mathbb{R}^k_{++} &= \left\{x \in \mathbb{R}^k \mid x_h > 0 \text{ for } h = 1, 2, \ldots, k\right\} \end{aligned}$$

for $k = 1, 2, \ldots$ and $]a, b[= \{z \in \mathbb{R} \mid a < z < b\}$

Problem A

(a) Let $\succsim$ be a rational preference relation on the consumption possibility set X. What does it mean that $u : X \longrightarrow \mathbb{R}$ represents $\succsim$?

(b) Give a graphic example of production possibility set $Y \subset \mathbb{R}^2$ which satisfies P1, but not P2, and where for some prices there is a continuum of solutions to the Producer Problem.

(c) Assume that a consumption possibility set, X, in $\mathbb{R}^2$ satisfies Assumption F1. Give an example of $p \in \mathbb{R}^2_+ \setminus \{0\}$ and wealth, $\mathrm{w} > 0$ such that the budget set is not a compact set.

(d) Assume that Arrow's assumptions for a SWF are satisfied and that Schedule 1 below is mapped to a ranking for society with a above b. Can we conclude something about society's ranking of a and b for Schedule 2?

Schedule 1			Schedule 2		
b	c	a	c	a	c
a	b	c	b	b	a
c	a	b	a	c	b

(e) Let $\mathcal{E} = \left\{(X^i, u^i)_{i\in\mathbb{I}}, (Y^j)_{j\in\mathbb{J}}, \omega\right\}$ be an economy (without private ownership). Let $\left((x^i)_{i\in\mathbb{I}}, (y^j)_{j\in\mathbb{J}}\right)$ be an allocation such that, for $i \in \mathbb{I}$, $x^i \in X^i$ and, for $j \in \mathbb{J}, y^j \in Y^j$. What further condition(s) must $\left((x^i)_{i\in\mathbb{I}}, (y^j)_{j\in\mathbb{J}}\right)$ satisfy to be a feasible allocation?

(f) Define what is meant by a homothetic preference relation $\succsim$ on $\mathbb{R}^L_+$ and draw a diagram $(L = 2)$ explaining the idea.

Problem B

(a) Let $X = \mathbb{R}^L_+$ be the consumption possibility set of a consumer with (continuous) utility function $u : X \longrightarrow \mathbb{R}$. Let $p \in \mathbb{R}^L_{++}$ and let $\mathrm{w} > 0$. Show that the budget set is upper bounded and that there is at least one solution to the Consumer (Utility Maximization) Problem.

(b) Let $\left((\bar{x}^i)_{i\in\{a,b,c\}}\right)$ be a Pareto optimal allocation for the economy $\mathcal{E} = \left((\mathbb{R}^L_+, u^i)_{i\in\{a,b,c\}}, \omega\right)$ where the consumers satisfy F1,F2 and F3 and $\omega \in \mathbb{R}^L_{++}$. Let, for $i \in \{a, b, c\}$, $u^i(\bar{x}^i) = \bar{u}^i$ and define

$$A^i = \left\{x \in \mathbb{R}^L_+ \mid u^i\left(x^i\right) \geq \bar{u}^i\right\}$$

Show that $\omega \in A^1 + A^2 + A^3$ but that ω is not an interior point of $A^1+A^2+A^3$. (**Hint:** To prove $\omega \notin \mathrm{int}(A^1 + A^2 + A^3)$ argue by contradiction.) Assume that it is known that there is $p \in \mathbb{R}^L_{++}$ such that

$$p\omega = p\left(\bar{x}^1 + \bar{x}^2 + \bar{x}^3\right) \leq pz \text{ for } z \in A^1 + A^2 + A^3$$

Show that then $p\bar{x}^1 \leq px^1$ for $x^1 \in A^1$. Thus $\bar{x}^1$ is an expenditure minimizer (at p and $\bar{u}^1$). Under what further condition will $\bar{x}^1$ be a solution to the Consumer (Utility Maximization) Problem at prices p with wealth $\mathrm{w} = p\bar{x}^1$?

Problem C

Below we want to study a pure exchange economy with a continuum of Walras equilibria. Consider a pure exchange economy $\mathcal{E} = (X^i, u^i, \omega^i)_{i \in \{a,b\}}$ where $X^a = \mathbb{R}_+ \times \mathbb{R}_{++}, X^b = \mathbb{R}_{++} \times \mathbb{R}_+$ and

$$\begin{aligned} u^a &: \mathbb{R}_+ \times \mathbb{R}_{++} \longrightarrow \mathbb{R} \text{ with } u^a(x_1, x_2) = x_1 - \delta \frac{1}{x_2} \text{ and } \omega^a = (1, 0) \\ u^b &: \mathbb{R}_{++} \times \mathbb{R}_+ \longrightarrow \mathbb{R} \text{ with } u^b(x_1, x_2) = x_2 - \delta \frac{1}{x_1} \text{ and } \omega^b = (0, 1) \end{aligned}$$

for some $\delta \in]0, 1[$. Consider normalized prices $p = (p_1, 1)$ with $p_1 \in \,]\delta, 1/\delta[$ (to avoid boundary solutions)

(a) Show that $u^a : \mathbb{R}_+ \times \mathbb{R}_{++} \longrightarrow \mathbb{R}$ is a concave function. (**Hint:** u^a is the sum of $(x_1, x_2) \longrightarrow x_1$ and $(x_1, x_2) \longrightarrow -\delta \frac{1}{x_2}$. Use that the sum of concave functions is a concave function.) Is Assumption F2' satisfied?

(b) State consumer $a's$ problem.

(c) Find consumer $a's$ demand for good 1 as $p_1 \in \,]\delta, 1/\delta[$.

(d) Find consumer $b's$ demand for good 1 as $p_1 \in \,]\delta, 1/\delta[$.

(e) Find the total (aggregate) excess demand for good 1 as a function of p_1, for $\delta < p_1 < (1/\delta)$

(f) Thus we have found a continuum of equilibrium price systems. Will the equilibrium allocations all be different?

Assumptions on Producers

Assumption P1: *The production set $Y \subset \mathbb{R}^L$ satisfies*

(a) $0 \in Y$ (Possibility of inaction)

(b) *Y is a closed subset of $\mathbb{R}^L$* (Closedness)

(c) *Y is a convex set* (Convexity)

(d) $Y \cap (-Y) = \{0\}$ (Irreversibility)

(e) *If $\bar{y} \in Y$, $y \in \mathbb{R}^L$ and $y \leq \bar{y}$ then $y \in Y$* (Free disposal, downward comprehensive)

Assumption P2:*(constant returns to scale) If the vector $y \in Y$ and $\lambda \in [0, +\infty[$ then $\lambda y \in Y$.*

Assumptions on Consumers

Assumption F1.
The consumption set $X \subset \mathbb{R}^L$ satisfies:

(a) X is a non-empty set.

(b) X is a closed set

(c) X is a convex set

(d) X is a lower bounded set (in the vector ordering)

(e) X is upward comprehensive ($x \in X$ and $\nabla \in \mathbb{R}^L_+$ implies $x + \nabla \in X$)

Monotonicity assumptions

Assumption of **weak monotonicity**
$\mathbf{F2}^0 : x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow x^1 \succsim x^2$

$\mathbf{F2}^0 : x^1, x^2 \in X$ and $x^1 \geq x^2 \Longrightarrow u(x^1) \geq u(x^2)$

In the interpretation: "at least as much of each commodity is at least as good".

Assumption of **monotonicity** (MWG Def. 3.B.2)

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow x^1 \succ x^2$

F2: $x^1, x^2 \in X$ and $x^1 >> x^2 \Longrightarrow u(x^1) > u(x^2)$

In the interpretation: "more of each commodity is better".

Assumption of **strict (or strong) monotonicity** (MWG Def. 3.B.2)

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow x^1 \succ x^2$

F2': $x^1, x^2 \in X$ and $x^1 > x^2 \Longrightarrow u(x^1) > u(x^2)$

The preference relation $\succsim$ is **locally non-satiated** if: Given $x \in X$ and $\varepsilon > 0$ there is $x' \in X$ such that $x' \succ x$ and $\parallel x' - x \parallel < \varepsilon$. (Definition 3.B.3, MWG)

A preference relation, $\succsim$, is a **convex preference relation** if and only if, for $x \in X$, the set $\{x' \in X \mid x' \succsim x\}$ is a convex set. (MWG Def. 3.B.2).

If u represents $\succsim$ then $\succsim$ is a convex preference relation if and only if u is a quasi-concave function.

We want to consider also a stronger convexity assumptions

F3: A preference relation, $\succsim$, is a **strictly convex** preference relation if: $x^1, x^2, x^3 \in X$, $x^1 \succsim x^2$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)x^2$ for some $t \in]0,1[$ implies $x^3 \succ x^2$.

F3: The utility function is **strictly quasi-concave** if: $x^1, x^2, x^3 \in X$, $u(x^1) \geq u(x^2)$, $x^1 \neq x^2$and $x^3 = tx^1 + (1-t)x^2$ for some $t \in]0,1[$ implies $u(x^3) > u(x^2)$.