Written Exam for the B.Sc. in Economics 2010-II

Econometrics A

2. year

June 23, 2010

(4 hours open book exam - calculators are not allowed)

Question 1

A couple has bought a new house and needs to sell their old house. The current market price of their old house is 5.0 million DKK. Their real estate agent tells them that at this price the probability of a sale is 10 per cent each month. The extra costs of having the old house is $30,000$ DKK every month. Assume that when a buyer arrives the house is immediately sold and the couple saves the $30,000$ DKK immediately. Furthermore, assume that time is discrete and that all values are net of sales costs.

1. How is the waiting time T_A until the house is sold distributed?

2. What is the probability that the old house is not sold within the first 3 months?

3.

   (a) What is the expected waiting time (in months) until house sale?

   (b) What is the variance of the waiting time until house sale?

The couple looses $30,000$ DKK each month the old house is not sold, so naturally they are concerned about the waiting time until sale. The real estate agent argues that at a price of 4.8 million DKK the probability of a sale is 20 per cent each month. Hence, the couple can pursue one of the two following strategies:

Strategy A:	Set the price at 5.0 million DKK and having a probability of sale of 10 per cent each month
Strategy B:	Set the price at 4.8 million DKK and having a probability of sale of 20 per cent each month

4. Taking into account that it costs the couple $30,000$ DKK each month the old house is not sold, what is the expected value $E(X_A)$ and $E(X_B)$ of respectively strategy A and strategy B? Based on the expected values which strategy is best?

The couple also fears that housing prices will fall. The real estate agent argues that the price will fall by $500,000$ DKK after 3 months if the house is not sold. You can assume that after the reduction after 3 months the price will stay constant. The implication of this is that the couple the first 3 months can choose between strategy A and B and after 3 months has two possible strategies:

Strategy C:	Set the price at 4.5 million DKK and having a probability of sale of 10 per cent each month
Strategy D:	Set the price at 4.3 million DKK and having a probability of sale of 20 per cent each month

5. Which of the four combinations {strategy A, strategy C}, {strategy A, strategy D}, {strategy B, strategy C}, or {strategy B, strategy D} gives the highest expected value [Hint: A good starting point is to use your answer in question 4 to determine whether strategy C or strategy D is best and then calculate $E(X_A|T_A \leq 3)$ and $E(X_B|T_B \leq 3)$, where T_B is the waiting time until sale when the probability of sale is 0.2 each month]?

Question 2

A firm is producing micro chips to, for example, computers. Ideally, the firm would test whether every micro chip is defective or nondefective before shipping them, but such testing is too expensive. Besides this, the firm's customers need different types of chips. Therefore, the firm needs to check each of the shipments separately. Hence, the quality control procedure in the firm works by selecting a sample of n chips from a given shipment of size N and count how many chips have errors. Let the number of errors found in the sample be denoted by X. For each shipment the firm precommits to having at most M errors.

1. Consider a given shipment of size N and suppose that a sample of size n is drawn from this shipment without replacement. How is the number of chips with errors X distributed?

2. For a shipment of size $N = 60$ a sample of $n = 5$ is drawn for quality control. The quality control shows that 1 of the micro chips has error. For this particular shipment the company has precommitted themselves to only having 5 per cent errors.

   (a) Suppose the quality control team uses binomial probabilities to calculate the probability of having at least 1 chip with error in a sample of size $n = 5$. Use the binomial distribution to calculate the probability of having at least 1 error.

   (b) If the found probability in question 2a) is less than 30 per cent, the firm will not ship the chips. Instead they will draw 5 extra micro chips such that a sample of $n = 10$ is drawn. None of the extra chips have errors so the total number of chips with errors is still 1. What is the probability of at least 1 chip with error out of a sample of size $n = 10$? Can the firm ship the 60 micro chips to its customer?

3. In reality, the quality control samples are drawn without replacement so using the binomial probabilities as the quality control team does, is not correct. Therefore, repeat the calculations of 2a) and 2b) taking into account that the sampling was done without replacement by assuming that $M = 60 \cdot 0.05 = 3$. Even though the probabilities are different, do they give rise to the same conclusion?

4. Under which conditions does it not matter much whether the quality control team calculates the probabilities assuming that the sample is drawn with replacement even though the sample is drawn without replacement. Provide some intuition for your answer.

5. The company has been approached by a large computer hardware company which considers using the firm's micro chips in the years to come. The first order is of $10,000$ chips and the quality control team wants to be very confident that a maximum of only 3 per cent of the chips have errors. For the first shipment a sample of 500 chips is selected. The quality control team finds errors in 12 of the micro chips and use the normal approximation of binomial probabilities to calculate the probability of having at least 12 errors out of 500 given that the probability of error is 3 per cent. If this probability is greater than 60 per cent, the firm will ship the micro chips. Calculate the probability similar to the quality control team. Will the $10,000$ chips be shipped to the large customer?

Question 3

In the public debate in Denmark it is often fiercely discussed whether or not giving performance pay to teachers can increase the average attained learning outcomes for the students. A recent study in Israel attempted to address this question by a randomized trial[1]. This question is based on this experiment, which was conducted in the following way:

- In the beginning of year 2000 a random sample of Israeli schools is drawn.
- The attained math score for each senior class student at the final exam in the summer of 2000 is recorded.
- Shortly after the beginning of the academic year 2000/2001 the math teachers were informed that they were participating in a competition about who could improve their class average test score the most from the exam in the summer of 2000 to the exam in the summer of 2001. There would be awards ranging from \$1750 to \$7500 depending on their performance relative to the other teachers in the competition.
- At the end of the academic year 2000/2001 the math scores were collected and the awards payed out.

[1] V. Lavy (2009), American Economic Review, 99:5, 1979-2011

In the following questions we will attempt to measure the effect of the experiment by comparing the average test scores obtained for each school before the experiment (the exam in 2000) to after the experiment (the exam in 2001).

Let Z_{0i} be the average test score for school i before the experiment and let Z_{1i} be the average test score for school i after the experiment. We assume that Z_0 and Z_1 can be described as normal random variables:

$$Z_0 \sim N(\alpha_0, \sigma_0^2), \qquad Z_1 \sim N(\alpha_1, \sigma_1^2)$$

1. State an unbiased estimator for α_0 and the distribution of the estimator. State an unbiased estimator for σ_0^2.

2. The sample consists of data from 48 schools. Table 3 below presents some descriptive statistics.

Table 3: Average math scores

	2000 (Z_0)	2001 (Z_1)	Difference ($Z_1 - Z_0$)
Number of observations	48	48	48
Sample mean	75.03	76.39	1.36
Median	74.84	76.68	1.20
Q1	71.75	73.65	-0.37
Q3	77.35	78.32	3.44
Sample standard deviation	5.07	4.31	3.11
Min	64.55	68.79	-4.40
Max	91.63	91.89	8.31

Compute the estimators of α_0 and σ_0^2.

3. Test the hypothesis $H_0 : \alpha_0 = \alpha_1$ and state the p-value. Discuss the assumptions underlying the test procedure. Explain the conclusion of the test.

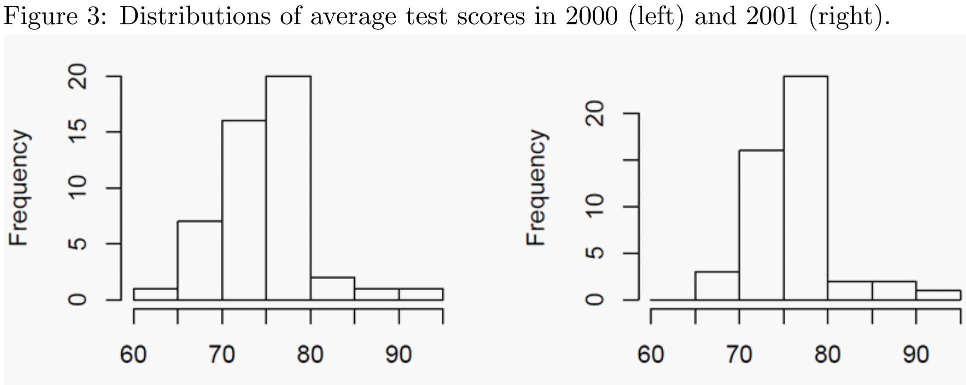
4.

   (a) Compute a 95% confidence interval for the effect of the experiment. Discuss how this interval should be interpreted.

   (b) Compute the interquartile range for the effect of the experiment and provide an interpretation of the interval.

5. Figure 3 below shows the distribution of Z_0 and Z_1. Is it justifiable to assumed that Z_0 and Z_1 are both normally distributed? Discuss how we could test if performance pay increases average test scores without imposing an assumption of normality.

Figure 3: Distributions of average test scores in 2000 (left) and 2001 (right).



Question 4

In this question we will again consider the experiment described in Question 3. Let X_i be a random variable which takes the value 1 if the average math score for school i in year 2001 is higher than in year

2000 and 0 otherwise. Hence the variable X_i indicates whether or not the experiment led to an increase in the average test score for school i.

We assume that X_i follows a Bernoulli distribution with probability parameter $p : P(X = 1) = p$.

1. State a consistent estimator for p and explain why it is a consistent estimator.

2. The sample consists of data from 48 schools. Table 4 below presents descriptive statistics. Compute an estimate of p.

3. State the likelihood function and draw a sketch of the function on the interval $p \in [0, 1]$. Which value of p maximizes the likelihood function?

4. In Israel some schools are Jewish while others are Arabic. It has been speculated that the effect of the experiment might depend on the type of school. Table 4 below presents a contingency table for the joint frequencies of X_i and school type. Discuss how independence of attained result and school type can be tested. Be specific about which assumptions you must make.

Table 4: Joint frequencies of school type and whether or not the school benefited from the experiment (X)

		— School type —		
		Jewish	Arabic	Total
Benefited	Yes	24	9	33
	No	11	4	15
	Total	35	13	48

5. Perform a χ^2-test for independence of attained result and school type. Write in words how the test should be interpreted.