Written exam for the M.Sc. in Economics, Winter 2013/14

Game theory

Final Exam/Elective Course/Master's Course

21 January 2014

(3 hours, closed book exam)

The exam has 3 pages in total (including cover page).

Explain all your answers!

Question 1: *(rationalizability)* This exercise is a variation of the Bertrand game. Two firms compete by simultaneously setting prices. Prices have to be **natural numbers** (including 0), that is $A_i = \{0, 1, 2, \dots\}$. Consumers buy 100 units from the firm setting the lower price if this price is less or equal to 60. If all posted prices are above 60 consumers buy 0 units. If both firms set the same price (less or equal to 60), consumers buy 50 units from firm 1 and 50 units from firm 2. Each firm has costs of 2 if it sells 100 units and costs of 1 if it sells 50 units (and costs of 0 if it sells 0 units).
If we denote the price of firm i by $a_i \in \mathbb{N}$, the profits of firm i are therefore

$$\pi_i(a_i, a_j) = \begin{cases} 100a_i - 2 & \text{if } a_i < a_j \text{ and } a_i \leq 60 \\ 50a_i - 1 & \text{if } a_i = a_j \text{ and } a_i \leq 60 \\ 0 & \text{else.} \end{cases}$$

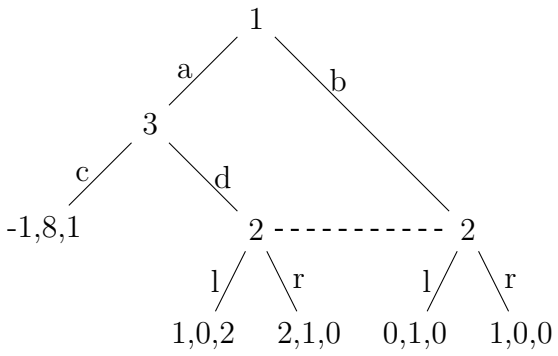
Each firm maximizes its own profit.

a) Is setting a price of 0 a rationalizable action?

b) Is setting a price of 2 a rationalizable action?

c) Which actions are rationalizable?

Question 2: *(extensive form games with incomplete information)* Consider the three-player, extensive form game below.
The dashed line indicates that the two nodes are in the same information set of player 2!



a) Find a *pure strategy* Nash equilibrium. Is the Nash equilibrium subgame perfect? Show that there is (at least) one player for which the Nash equilibrium strategy is not sequentially rational.

b) Derive a strategy profile that is sequentially rational and where beliefs satisfy Bayes' rule in every information set. (hint: consider mixed strategies)

Question 3: *(knowledge)* Both Alice and Bob receive an envelope with money. The amount (in \$) that Bob receives is an even number between 1 and 100. The amount Alice receives is either 1 more or 1 less than Bob's amount. Each player knows his own amount but not the amount of the other player. This setting is common knowledge among Bob and Alice.

Denoting Alice's amount by A and Bob's amount by B, (A, B) where $B \in \{2, 4, \ldots 100\}$ and $A \in \{B-1, B+1\}$ is a *state*.

Let E be the event "both players have more than 3\$", i.e. the set of states where $min\{A, B\} > 3$.

a) Suppose Bob has 6\$ and Alice has 5\$. Which states are considered possible by Bob? Do Bob and/or Alice know the event E? Is the event E common knowledge?

b) Now imagine there are 100 visitors who come one by one into the room in which Bob and Alice are. Each of these 100 visitors asks "Does one of you know that he has less money than the other?" and Bob an Alice answer simultaneously "yes" or "no".

   What is common knowledge among Bob and Alice (apart from the setting) if both answer "no" to the first visitor? In which states and after how many visitors is the event E common knowledge among Alice and Bob? After how many visitors will one of them answer "yes"?