Advanced Microeconomics, Fall 2012

3 hours closed book exam

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There are 3 problems. The problems B and C have the same weight in the marking process and Problem A has half the weight of Problem B.

Below

$$\begin{aligned}\mathbb{R}^k_+ &= \left\{x \in \mathbb{R}^k \mid x_h \geq 0 \text{ for } h = 1, 2, \ldots, k\right\} \text{ and} \\ \mathbb{R}^k_{++} &= \left\{x \in \mathbb{R}^k \mid x_h > 0 \text{ for } h = 1, 2, \ldots, k\right\}\end{aligned}$$

for $k = 1, 2, \ldots$.

Problem A

(a) What is meant by a rational preference relation? **Solution:** See MWG

(b) The production possibility set Y exhibits non-decreasing returns to scale. What does this mean? **Solution:** Seen NotesProd or MWG

(c) Give a graphic example of a consumption possibility set in $\mathbb{R}^2$ where commodity 1 is indivisible. Is your consumption possibility set a convex set. **Solution:** For example,

$$X = \left\{x \in \mathbb{R}^2 \mid x_1 \in \{0, 1, 2, \ldots\}, x_2 \geq 0\right\}$$

which is not a convex set since $(1, 5)$ and $(2, 5)$ belongs to X but $(1/2)(1, 5) + (1/2)(2, 5)$ does not belong to X.

(d) Assume that Arrow's assumptions for a SWF are satisfied and that Schedule 1 below is mapped to a ranking for society with a above b. What can be concluded about society's ranking of a and b for Schedule 2? **Solution:** Since the $a-b$ pattern is the same in Schedule 2 the SWF must, by Independence of Irrelevant Alternatives, map also Schedule 2 to a ranking with a above b.

Schedule 1			Schedule 2		
b	c	a	c	b	c
a	b	c	b	a	a
c	a	b	a	c	b

(e) Let $\mathcal{E} = (\mathbb{R}^2_+, u^i, \omega^i)_{i\in\{a,b\}}$ be a pure exchange economy where consumers satisfy assumptions F1,F2 and F3. How is the total (aggregate) excess demand defined for this economy? **Solution:** See NotesWa or MWG

(f) Let $\xi(p_1, p_2, \mathrm{w}) = \left(\frac{1}{4}\frac{\mathrm{w}}{p_1}, \frac{3}{4}\frac{\mathrm{w}}{p_2}\right)$ be the demand function of a consumer in a private ownership (pure exchange) economy with $\mathrm{w} = p_1\omega_1 + p_2\omega_2, \omega_1, \omega_2 > 0$. Will the consumer's excess demand function satisfy the Gross Substitutes assumption? Does you answer depend on the initial endowment ω of the consumer? **Solution:** See Example 17.F.2 in MWG.

Problem B

(a) Consider an economy $\mathcal{E} = \left((\mathbb{R}^L_+, u^i)_{i\in\{a,b\}}, Y, \omega\right)$ where the (only) producer satisfies P1 and the consumers satisfy F1 and F2. Let $\left(\left(\bar{x}^i\right)_{i\in\{a,b\}}, \bar{y}\right)$ be a Pareto optimal allocation. Show that $\bar{y}$ is an efficient production in Y. (**Hint**: Argue by contradiction.) **Solution:** See NotesOpt.

(b) State and prove The First Theorem of Welfare Economics for a pure exchange economy (without private ownership). (**Hint**: Argue by contradiction.) Where do you need Assumption F2? **Solution:**See NotesOpt

or MWG Proposition 16.C.1.

Problem C

Let $\mathcal{E} = \{(X, u), Y, \omega\}$ be a private ownership economy with a single consumer (who owns the single producer) and

$$\begin{aligned} X &= \left\{x \in \mathbb{R}^2 \mid x_1 \geq 2, x_2 \geq 0\right\} \\ Y &= \left\{y \in \mathbb{R}^2 \mid y_2 \leq 2\left(-y_1\right)^{1/2}, y_1 \leq 0\right\} \\ u\left(x_1, x_2\right) &= \left(x_1 - 2\right) x_2 \\ \omega &= (4, 0) \end{aligned}$$

(a) Does Y satisfy Assumption P1? State and solve the Producer Problem for prices $p = (p_1, p_2) \in \mathbb{R}^2_{++}$ and find the maximal profit.

Solution: The Producer Problem is

$$Max_{y\in Y}\; p_1 y_1 + p_2 y_2$$

A production solving the Producer Problem is be an efficient production. Thus we can consider the problem

$$Max\; p_1 y_1 + p_2 y_2 \text{ subject to } y_2 - 2\left(-y_1\right)^{1/2} = 0$$

and derive the following marginal conditions

$$\begin{aligned} p_1 &- \lambda\left(-y_1\right)^{-(1/2)} &= 0 \\ p_2 &- \lambda &= 0 \end{aligned}$$

which gives $(y_1, y_2) = \left(-\frac{p_2^2}{p_1^2}, 2\frac{p_2}{p_1}\right)$ and the profits are $-p_1\frac{p_2^2}{p_1^2} + 2p_2\frac{p_2}{p_1} = -\frac{p_2^2}{p_1} + 2\frac{p_2^2}{p_1} = \frac{p_2^2}{p_1}$.

(b) Solve the Consumer Problem as $p = (p_1, p_2) \in \mathbb{R}^2_{++}$ and wealth is given by $\mathrm{w} \geq 2p_1$.**Hint:** You may consider rewriting the budget restriction as $p_1(x_1 - 2) + p_2x_2 \leq \mathrm{w} - 2p_1$ if you recall the solution with a Cobb-Douglas utility function. **Solution:**

Using the solution for the Cobb-Douglas function we get

$$\begin{aligned}(x_1 - 2, x_2) &= \left(\frac{1}{2}\frac{\mathrm{w} - 2p_1}{p_1}, \frac{1}{2}\frac{\mathrm{w} - 2p_1}{p_2}\right) \text{ which implies} \\ (x_1, x_2) &= \left(\frac{1}{2}\frac{\mathrm{w} - 2p_1}{p_1} + 2, \frac{1}{2}\frac{\mathrm{w} - 2p_1}{p_2}\right)\end{aligned}$$

(c) Assume that wealth is now given by the value of initial endowment and profits. Derive the market balance condition for good 1. If $p = (p_1, p_2)$ satisfies this condition is then (p_1, p_2) an equilibrium price system? **Solution:**

The market balance condition for good 1 is $x_1 = y_1 + \omega_1$, or using the results from (a), (b) and $\mathrm{w} = 4p_1 + \frac{p_2^2}{p_1}$

$$\frac{1}{2}\frac{4p_1 + \frac{p_2^2}{p_1} - 2p_1}{p_1} + 2 = -\frac{p_2^2}{p_1^2} + 4$$

(d) Put $p_1 = 1$ and find p_2 from the market balance condition in (c). **Solution:**

$$\begin{aligned}\frac{1}{2}\left(2 + p_2^2\right) &= -p_2^2 + 2 \iff \\ \frac{3}{2}p_2^2 &= 1 \iff \\ p_2 &= \left(\frac{2}{3}\right)^{1/2}\end{aligned}$$

(e) Find the Walras equilibrium for $\mathcal{E}$. Check that both markets balance.

Solution:

The equilibrium prices are $(p_1, p_2) = \left(1, \left(\frac{2}{3}\right)^{1/2}\right)$

$$(x_1, x_2) = \left(\frac{1}{2}\frac{\mathrm{w} - 2p_1}{p_1} + 2, \frac{1}{2}\frac{\mathrm{w} - 2p_1}{p_2}\right) = \left(\frac{1}{2}(2 + p_2^2) + 2, \frac{1}{2}\frac{2 + p_2^2}{p_2}\right) =$$

$$\left(\frac{10}{3}, \frac{\frac{4}{3}}{\left(\frac{2}{3}\right)^{1/2}}\right) = \left(\frac{10}{3}, 2\left(\frac{2}{3}\right)^{1/2}\right)$$

$$(y_1, y_2) = \left(-\frac{p_2^2}{p_1^2}, 2\frac{p_2}{p_1}\right) = \left(-\frac{2}{3}, 2\left(\frac{2}{3}\right)^{1/2}\right)$$

Obviously market 2 balances and for market 1: $\frac{10}{3} = -\frac{2}{3} + 4$ so also that market balances