## Written Exam for the B.Sc. in Economics - Fall 2015

## Macroeconomics III Final Exam

January 4, 2016

## 3-hour closed book exam

Please note that the language for this exam is English.

The points for each question should guide you in allocating time to answering them (they add up to 180, thus proportional to the total time you have for the exam).

1 (20 points) Answer true, false, or uncertain. Justify your answer.

In the Lucas and Calvo models, even if there is a positive relation between output and inflation (a Philips curve), there is no scope for policymakers to increase output in the short run by announcing an increase in the rate of money growth.

2 (20 points) Answer true, false, or uncertain. Justify your answer.

In models that have real money balances in the households' utility function, hyperinflations can be ruled out when money is essential (i.e. when the marginal utility of money increases faster than the rate at which real money balances go to zero).

**3** (20 points) Answer true, false, or uncertain. Justify your answer.

According to the Meltzer and Richard model all increases in inequality, as measured by the Gini coefficient, lead to an increase in redistribution.

 $\mathbf{4}$  (60 points) Consider the following version of the Ramsey model with population growing at rate n. Identical competitive firms maximize the following profit function:

$$\pi^{F}(K_{t}^{i}, L_{t}^{i}) = K_{t}^{i\alpha} L_{t}^{i1-\alpha} - w_{t} L_{t}^{i} - r_{t}^{L} K_{t}^{i},$$

where  $r_t^L$  is the interest rate at which firms can borrow funds,  $w_t$  is the wage rate,  $K_t^i$  and  $L_t^i$  denote the quantities of capital and labor employed by the firm. Assume  $0 < \alpha < 1$ . There is no capital depreciation.

A large number of identical households maximize the following intertemporal utility function, that depends on per-capita consumption  $c_t$ :

$$U = \int_0^\infty \ln(c_t) e^{-(\rho - n)t} dt,$$

subject to their dynamic budget constraint:

$$c_t + \dot{a}_t + na_t = w_t + r_t^D a_t.$$

Take  $a_0 > 0$  as given, a is wealth (lower case variables represent quantities in per capita terms),  $r_t^D$  is the interest rate that the household gets for its savings, and assume  $\rho > n$ . For simplicity we rule out private lending between households.

In this economy there are financial intermediaries that take deposits  $d_t$  from households paying them the rate  $r_t^D$  for this (households cannot borrow from intermediaries, i.e.  $d_t \geq 0$ ). Intermediaries are regulated and are thus required to store the fraction  $\gamma$  of deposits as liquid assets on which they receive no return. The remaining fraction  $1 - \gamma$  they lend to firms at rate  $r_t^L$ . Competition between intermediaries implies that they make zero profits, i.e.

$$\pi_t^I = r_t^L (1 - \gamma) d_t - r_t^D d_t = 0.$$

Thus the following relation must be satisfied at every point in time:  $r_t^L(1-\gamma) = r_t^D$ .

Note: Think of the fraction  $\gamma$  of deposits that the intermediaries must store as "reserves" that make the economy stable for reasons exogenous to our capital accumulation model. We take this fact as given.

For points a) and b) you will receive partial credit if you assume  $\gamma = 0$ .

- a) Find the first order conditions for the firms' maximization problem that characterize how much capital and labor a firm demands at given factor prices. As a function of saving per capita, a, what is the income that the representative household member receives on his/her saving? (Hint: note the equilibrium relation between a, d and k) And for his/her labor services? What are the control and state variables in the households' optimization problem?
- b) Write the Hamiltonian, find the first order conditions that characterize the behavior of households, and from these the Euler equation (also known as the Keynes Ramsey rule). Give an economic interpretation to this equation. Find the equations that characterize steady state as a function of control and state variables. Draw the phase diagram that describes the dynamics in this economy with a in the horizontal axis, and c in the vertical axis.
- c) Assume that the economy is initially in the steady state. Unexpectedly the government starts to use distortionary capital income taxes, and rebates the revenue to households in a lump sum way (to rationalize this, you might think that this is a redistributive policy implemented by a new government). How does this shock affect the  $\dot{c}=0$  and  $\dot{a}=0$  curves? Characterize analytically the new steady state capital per capita, k. What happens initially with consumption? Explain.

**5** (60 points) Consider an economy where individuals live for two periods, and population is initially constant. Identical competitive firms maximize the following profit function:

$$\pi^F(K_t^i, L_t^i) = AK_t^{i\alpha} L_t^{i1-\alpha} - w_t L_t^i - r_t K_t^i,$$

where  $r_t$  is the interest rate at which firms can borrow capital,  $w_t$  is the wage rate,  $K_t^i$  and  $L_t^i$  denote the quantities of capital and labor employed by the firm, and A > 0 is total productivity. Assume  $0 < \alpha < 1$ . There is no capital depreciation. Utility for young individuals born in period t is

$$U_t = \ln(c_{1t}) + \frac{1}{1+\rho} \ln(c_{2t+1}), \quad \rho > -1$$

where  $c_{1t}$  is consumption when young, and  $c_{2t+1}$  consumption when old. Young agents work a unit of time (i.e. their labor income is equal to the wage they receive). Old agents do not work and provide consumption through saving and social security benefits. The old get gross return  $1 + r_{t+1}$  for their savings.

Suppose that the government runs a balanced pay-as-you-go social security system in which the young contribute a fraction  $0 < \tau < 1$  of their wages that is received by the old  $(\tau w_t)$  are then the benefits received by the old in period t).

a) Characterize individual saving behavior by solving the individual's problem of optimal intertemporal allocation of resources. Find the capital accumulation equation that gives  $k_{t+1}$  as a function of  $k_t$ . Find the level of capital in steady state.

Assume that the economy is initially in the steady state. Now unexpectedly there is a permanent flow of immigrants at rate n per period (i.e. the economy moves to a regime of constant population growth at rate n driven by immigration). Immigrants are young, have same preferences as residents, and are assumed to get employment. They stay in the country when they get old and thus receive social security benefits. Both immigrants and residents receive the same wage, make the same contributions, and receive same benefits from social security.

The government decides to reduce the size of social security such that the initial old generation receives the same benefits that they would have received in the absence of immigration. Denote by  $\tau'$  the new contribution rate. Assume that parameters are such that the economy is always dynamically efficient.

Note that to solve what follows you have to consider the general equilibrium effects that the flow of immigrants has on wages and interest rate.

- b) Characterize  $\tau'$  as a function of  $\tau$ , n, and  $\alpha$ . How does this shock affect the economy? What are the effects of the shock on consumption and capital accumulation in the first period (compared to consumption and capital accumulation in the previous steady state)? And on the new steady state? Explain.
- c) Show that the initial old are strictly better off with immigration, even though they receive the same benefits. Show that for some parameters, the disposable income of the first young generation is lower, despite the reduction in contributions. Explain.