Written Exam for M.Sc. in Economics

Investment Theory

20. August 2012

Master Course

3 hours written exam. Closed Books. All questions should be clearly and briefly answered. Calculations and figures should be clear and understandable. Calculations and figures should be explained.

Exercise 1.

Consider an investment project. The investment cost is $I > 0$. It is possible to alter scale of the project. Indeed, the dividend of the project is either $K(P - C)$ or $L(P - C)$, where

$$dP = \alpha P dt + \sigma P dz$$

with $\sigma > 0$, $L > K > 0$ and $C > 0$.

The investor decides whether the start dividend is $K(P-C)$ or $L(P-C)$. The dividend can be altered from $K(P - C)$ to $L(P - C)$ at cost $X > 0$. The dividend can be altered from $L(P-C)$ to $K(P-C)$ at cost $Y > 0$. The scale can be altered repeatedly.

Once the project is started, the probability that the project dies in the small time interval dt is λdt with $\lambda > 0$.

The discount rate of the investor is $\rho > 0$ with $\rho > \alpha$.

Let $F(P)$ be the value of the option to invest. Let $V_K(P)$ be the value of the project when the dividend is $K(P - C)$. Let $V_L(P)$ be the value of the project when the dividend is $L(P - C)$.

(a) Interpret the project. Give an example of an investment project that fits the above project.

(b) State possible strategies for starting the project, altering the scale from $K(P-C)$ to $L(P-C)$ and altering the scale from $L(P-C)$ to $K(P-C)$. Use the strategies to relate $F(P)$, $V_K(P)$ and $V_L(P)$ and discuss their properties.

(c) State the Bellman equations for $V_K(P)$ and $V_L(P)$ and use the Bellman equations to find differential equations in $V_K(P)$ and $V_L(P)$.

(d) Find $V_K(P)$ and $V_L(P)$ up to undetermined constants.

(e) Interpret your expressions for $V_K(P)$ and $V_L(P)$.

(f) Explain how the optimal strategies of alterning scales can be found. Compare the strategies.

(g) State the Bellman equation for $F(P)$. Find $F(P)$ up to undetermined constants.

(h) Discuss whether the investor will start the project with dividend $K(P-C)$ or $L(P-C)$. Explain how the optimal entry strategy can be found.