Solutions: Advanced Microeconomics, 22FEB2013

3 hours closed book exam

Anders Borglin, who is responsible for the exam problems, can be reached during the exam on +46 735 754176. There are, including the two pages with assumptions, altogether 5 pages.

There are 3 problems. The problems B and C have the same weight in the marking process and Problem A has half the weight of Problem B.

Below

$$\begin{aligned} \mathbb{R}^k_+ &= \left\{x \in \mathbb{R}^k \mid x_h \geq 0 \text{ for } h = 1, 2, \dots, k\right\} \text{ and} \\ \mathbb{R}^k_{++} &= \left\{x \in \mathbb{R}^k \mid x_h > 0 \text{ for } h = 1, 2, \dots, k\right\} \end{aligned}$$

for $k = 1, 2, \dots$ and $]a, b[= \{z \in \mathbb{R} \mid a < z < b\}$

Problem A

(a) Let $\succsim$ be a rational preference relation on the consumption possibility set X. What does it mean that $u : X \longrightarrow \mathbb{R}$ represents $\succsim$? **Solution:** See MWG

(b) Give a graphic example of production possibility set $Y \subset \mathbb{R}^2$ which satisfies P1, but not P2, and where for some prices there is a continuum of solutions to the Producer Problem.

Solution: See NotesProd

(c) Assume that a consumption possibility set, X, in $\mathbb{R}^2$ satisfies Assumption F1. Give an example of $p \in \mathbb{R}^2_+ \setminus \{0\}$ and wealth, $\mathrm{w} > 0$ such that the budget set is not a compact set.

Solution: If, for example, $p_2 = 0$ and $\mathrm{w} = p_1 x_1 + 1$ for some $x \in X$. Then the budget set will be unbounded and hence not compact.

(d) Assume that Arrow's assumptions for a SWF are satisfied and that Schedule 1 below is mapped to a ranking for society with a above

b. Can we conclude something about society's ranking of a and b for Schedule 2?

Schedule 1			Schedule 2		
b	c	a	c	a	c
a	b	c	b	b	a
c	a	b	a	c	b

Solution: (There are two possible answers to this question. Each of them should give maximum points.) The $a - b$ patterns are not the same so the Independence of Irrelevant Alteranatives can not be applied. But, on the other hand, since Schedule 1 is mapped to a ranking with with a above b individual 1 and 2 can not be dictators. Hence 3 is a dictator and thus also Schedule 2 should map to a ranking with a above b.

(e) Let $\mathcal{E} = \left\{(X^i, u^i)_{i\in\mathbb{I}}, (Y^j)_{j\in\mathbb{J}}, \omega\right\}$ be an economy (without private ownership). Let $\left((x^i)_{i\in\mathbb{I}}, (y^j)_{j\in\mathbb{J}}\right)$ be an allocation such that, for $i \in \mathbb{I}$, $x^i \in X^i$ and, for $j \in \mathbb{J}, y^j \in Y^j$. What further condition(s) must $\left((x^i)_{i\in\mathbb{I}}, (y^j)_{j\in\mathbb{J}}\right)$ satisfy to be a feasible allocation.

Solution: Balancedness: $\sum_{i\in\mathbb{I}} x^i = \sum_{j\in\mathbb{J}} y^j + \omega$

(f) Define what is meant by a homothetic preference relation $\succsim$ on $\mathbb{R}^L_+$ and draw a diagram ($L = 2$) explaining the idea.

Solution: See MWG

Problem B

(a) Let $X = \mathbb{R}^L_+$ be the consumption possibility set of a consumer with (continuous) utility function $u : X \longrightarrow \mathbb{R}$. Let $p \in \mathbb{R}^L_{++}$ and let $\mathrm{w} > 0$. Show that the budget set is upper bounded and that there is at least one solution to the Consumer (Utility Maximization) Problem.

Solution: See NotesCo&De or MWG

(b) Let $\left((\bar{x}^i)_{i\in\{a,b,c\}}\right)$ be a Pareto optimal allocation for the economy $\mathcal{E} = \left((\mathbb{R}^L_+, u^i)_{i\in\{a,b,c\}}, \omega\right)$ where the consumers satisfy F1,F2 and F3 and $\omega \in \mathbb{R}^L_{++}$. Let, for $i \in \{a,b,c\}$, $u^i(\bar{x}^i) = \bar{u}^i$ and define

$$A^i = \left\{x \in \mathbb{R}^L_+ \mid u^i\left(x^i\right) \geq \bar{u}\right\}$$

Show that $\omega \in A^1 + A^2 + A^3$ but that ω is not an interior point of $A^1+A^2+A^3$. (**Hint:** To prove $\omega \notin \text{int}(A^1 + A^2 + A^3)$ argue by contradiction.) Assume that it is known that there is $p \in \mathbb{R}^L_{++}$ such that

$$p\omega = p\left(\bar{x}^1 + \bar{x}^2 + \bar{x}^3\right) \leq pz \text{ for } z \in A^1 + A^2 + A^3$$

Show that then $p\bar{x}^1 \leq px^1$ for $x^1 \in A^1$. Thus $\bar{x}^1$ is an expenditure minimizer (at p and $\bar{u}^1$). Under what further condition will $\bar{x}^1$ be a solution to the Consumer (Utility Maximization) Problem at prices p with wealth $\mathrm{w} = p\bar{x}^1$?

Solution: See NotesOpt or MWG

Problem C

Below we want to study a pure exchange economy with a continuum of Walras equilibria. Consider a pure exchange economy $\mathcal{E} = (X^i, u^i, \omega^i)_{i\in\{a,b\}}$ where

$$\begin{aligned}
X^a &= \mathbb{R}_+ \times \mathbb{R}_{++}, X^b = \mathbb{R}_{++} \times \mathbb{R}_+ \text{ and} \\
u^a &: \mathbb{R}_+ \times \mathbb{R}_{++} \longrightarrow \mathbb{R} \text{ with } u^a(x_1,x_2) = x_1 - \delta\frac{1}{x_2} \text{ and } \omega^a = (1,0) \\
u^b &: \mathbb{R}_{++} \times \mathbb{R}_+ \longrightarrow \mathbb{R} \text{ with } u^b(x_1,x_2) = x_2 - \delta\frac{1}{x_1} \text{ and } \omega^b = (0,1)
\end{aligned}$$

for some $\delta \in]0,1[$. Consider normalized prices $p = (p_1, 1)$ with $p_1 \in]\delta, 1/\delta[$ (to avoid boundary solutions)

(a) Show that $u^a : \mathbb{R}_+ \times \mathbb{R}_{++} \longrightarrow \mathbb{R}$ is a concave function. (**Hint:** u^a is the sum of $(x_1,x_2) \longrightarrow x_1$ and $(x_1,x_2) \longrightarrow -\delta\frac{1}{x_2}$. Use that the sum of concave functions is a concave function.) Is Assumption F2' satified?

Solution: u^a is a concave function but not strictly concave. It is, however, strictly quasi-concave. Assumption F2' is satisfied

(b) State consumer $a's$ problem.

Solution:

$$\text{Max } \left(x_1 - \delta\frac{1}{x_2}\right) \text{ subject to } p_1x_1 + x_2 \leq p_1$$

(c) Find consumer $a's$ demand for good 1 as $p_1 \in \left]\delta, 1/\delta\right[$.

Solution: In a solution the budget restriction will be satisfied with equality and so $x_2 = p_1(1 - x_1)$. Consider

$$x_1 - \delta\frac{1}{p_1(1 - x_1)}$$

If the maximum is attained for a positive value of x_1 then the derivative is 0. Thus

$$1 - \delta\frac{1}{p_1}\frac{1}{(1 - x_1)^2} = 0$$

which has the solution $x_1 = 1 - (\delta/p_1)^{1/2}$. But then $x_1 > 0$ only if $p_1 > \delta$. Thus

$$\xi_1^a(p_1, 1, p\omega^a) = \xi_1^a(p_1, 1, p_1) = \ 1 - (\delta/p_1)^{1/2} \quad \text{if} \quad p_1 > \delta$$

Hence the demand for good 1 increases as p_1 increases.

(d) Find consumer $b's$ demand for good 1 as $p_1 \in \left]\delta, 1/\delta\right[$.

Solution: Consumer $b's$ problem

$$x_2 - \delta\frac{1}{x_1} \text{ subject to } p_1x_1 + x_2 \leq 1$$

We have, from the budget restriction, $x_2 = 1 - p_1x_1$. Consider

$$1 - p_1x_1 - \delta\frac{1}{x_1}$$

with derivative

$$-p_1 + \delta\frac{1}{x_1^2}$$

This derivative is 0 for $x_1 = \left(\frac{\delta}{p_1}\right)^{1/2}$ and thus

$$\xi_1^b(p_1, 1, p\omega^b) = \xi_1^b(p_1, 1, 1) = \left(\frac{\delta}{p_1}\right)^{1/2}$$

(e) Find the total (aggregate) excess demand for good 1 as a function of p_1,for $\delta < p_1 < (1/\delta)$

Solution:

$$\xi_1^a(p_1, 1, p_1)+\xi_1^b(p_1, 1, 1)-\left(\omega_1^a + \omega_1^b\right) = \begin{cases} \left(\left(1 - \left(\dfrac{\delta}{p_1}\right)^{1/2}\right) + \left(\dfrac{\delta}{p_1}\right)^{1/2} - 1\right) = 0 \\ \text{if } \delta < p_1 < (1/\delta) \end{cases}$$

(f) Thus we have found a continuum of equilibrium price systems. Will the equilibrium allocations all be different?

Solution: Consumer a´s consumption of good 1 is different for each $p_1 \in \,]\delta, 1/\delta[$ and so are the equilibrium allocation.