Written Exam at the Department of Economics winter 2019-20

Macroeconomics III

Final Exam

January 7, 2020

(3-hour closed book exam)

Answers only in English.

This exam question consists of 4 pages in total

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- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

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You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Written Exam - Macroeconomics III

University of Copenhagen
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Question 1

Consider an economy where individuals live for two periods, and the population grows at a constant rate $n > 0$. Identical competitive firms maximize their profits employing a Cobb-Douglas technology that combines labor, L_t, and capital, K_t, so that $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, with $\alpha \in (0,1)$. Assume full capital depreciation (i.e., $\delta = 1$). Under these assumptions, profit maximization leads to:

$$\begin{aligned} 1 + r_t &= \alpha A k_t^{\alpha-1}, \\ w_t &= (1-\alpha) A k_t^{\alpha}, \end{aligned}$$

where r_t is the (net) rental rate of capital, w_t is the wage rate, and k_t denotes capital in per-worker units.

Utility for young individuals born in period t is

$$U_t = \ln c_{1t} + \frac{1}{1+\rho} \ln c_{2t+1},$$

with $\rho > -1$. c_{1t} denotes consumption when young, c_{2t+1} consumption when old. Young agents spend their entire time endowment, which is normalized to one, working. Suppose the government runs an unfunded (pay-as-you-go) social security system, according to which the young pay a contribution d_t that amounts to a fraction $\tau \in (0,1)$ of their wages. Thus, the contributions are paid out in the same period to the current old. The latter do not work, and sustain their consumption though their savings and the social security benefits. Thus, the budget constraints in each period of life read as:

$$\begin{aligned} c_{1t} + s_t &= (1-\tau) w_t, \\ c_{2t+1} &= s_t (1 + r_{t+1}) + (1+n) d_{t+1}. \end{aligned}$$

a Set up and solve the individual's problem of optimal intertemporal allocation of resources. Derive the Euler equation. Show that individual saving behavior is characterized by

$$s_t = \frac{1}{2+\rho}(1-\tau) w_t - \tau \frac{1+\rho}{2+\rho} \frac{1+n}{1+r_{t+1}} w_{t+1}.$$

b Show that the capital accumulation equation that gives k_{t+1}, as a function of k_t, is given by

$$k_{t+1} = \frac{1}{1 + \frac{1+\rho}{2+\rho} \frac{(1-\alpha)}{\alpha} \tau} \left[\frac{(1-\alpha)(1-\tau)}{(1+n)(2+\rho)} A k_t^{\alpha} \right].$$

Show also that, in the steady state, the amount of capital-per-worker is

$$\bar{k} = \left[\frac{1}{1 + \frac{1+\rho}{2+\rho} \frac{(1-\alpha)}{\alpha} \tau} \frac{(1-\alpha)(1-\tau)A}{(1+n)(2+\rho)} \right]^{\frac{1}{1-\alpha}}.$$

c Suppose that, at time T, before saving decisions are made, the government decides to switch to a fully funded social security system according to which the young pay a contribution d_T that amounts to a fraction $\tau \in (0,1)$ of their wages. These contributions are then paid out in the next period, together with the accrued interest rate. The budget constraints in each period of life now read as:

$$\begin{aligned} c_{1t} + s_t &= (1-\tau) w_t, \\ c_{2t+1} &= (s_t + \tau w_t)(1 + r_{t+1}), \quad \text{for } t \geq T. \end{aligned}$$

Show that the new steady-state capital-per-worker, which is denoted by $\bar{k}'$, is such that

$$\bar{k}' = \left[\frac{(1-\alpha)A}{(1+n)(2+\rho)} \right]^{\frac{1}{1-\alpha}}.$$

d In the absence of any compensation from the government, the old generation at time T is worse off, after the social security system is changed. Explain why. How could the government intervene to compensate them, without imposing any burden on the current generation of workers?

Question 2

Consider the following model of monetary policy: the government controls inflation directly (i.e. $\pi_t = m_t$, where π_t is the rate of inflation and m_t is the rate of growth of money supply) and its instantaneous loss function is

$$L(\pi_t, x_t) = \frac{1}{2} \left[\pi_t^2 + \lambda (x_t - \bar{x})^2 \right]$$

where $x_t = \theta_t + \pi_t - \pi_t^e$. The following notation applies

π_t^e:	expected rate of inflation
x_t:	output level
θ_t:	potential output
$\bar{x}$:	policy output target

We assume that potential output is stochastic (with mean zero and variance σ_θ^2) and that its realizations are observed by both the public and the policy maker before expectations are formed by the private sector. Moreover, assume $\bar{x} > \theta_t$. The parameter $\lambda > 0$ measures the importance of output fluctuations around the target ($\bar{x}$) relative to inflation fluctuations.

a Show that the optimal policy under commitment implies $\pi_t^C = 0$ and $x_t^C = \theta_t$ [*hint: i) recall that the loss function is quadratic, thus the optimal policy rule is linear and can be guessed to be of the form* $\pi_t = \psi + \psi_\theta \theta_t$*; ii) recall to take the unconditional expectation of the loss function, prior to tackle its minimization*].

b Show that the optimal policy under discretion implies $\pi_t^D = -\lambda(\theta_t - \bar{x})$ and $x_t^D = \theta_t$. The *inflation bias* increases in the target $\bar{x}$: explain why.

c Assume that potential output cannot be observed before expectations are formed. The goal of the central bank is still to minimize the loss function. However, the monetary policy stance should now result as ex-post optimal given both π_t^e and θ_t (as the latter is not observed until after expectations are formed). Show that the optimal policy under discretion now implies $\pi_t^{D*} = \frac{\lambda}{1+\lambda}[\bar{x}(1+\lambda) - \theta_t]$ and $x_t^{D*} = \frac{1}{1+\lambda}\theta_t$. Under $\lambda = 0$ it is possible to ensure that $\pi_t^{D*} = \pi_t^C$ and $x_t^{D*} = x_t^C$. Explain why this is the case.

d Does this model face the policymaker with a meaningful inflation/output stabilization trade-off? Explain.