## Suggested answers for written Exam for the B.Sc. in Economics autumn 2012-2013

Macro B

Final Exam

January 17 2013 (3 hours closed-book exam)

Academic Aim: The aim of the course is to describe and explain the macroeconomic fluctuations in the short run, i.e. the business cycles around the long run growth trend, as well as various issues related to this, and to teach the methodology used in formulating and solving formal models explaining these phenomena. Students are to learn the most important stylized facts about business cycles and to acquire knowledge about theoretical dynamic models aimed at explaining these facts. In connection with this, the aim is to make students familiar with the distinction between deterministic and stochastic models. Furthermore, students are to gain an understanding of the distinction between the impulses initiating a business cycle and the propagation mechanisms that give business cycles a systematic character. Finally students are to learn how to use the models for analyzing the effects of macroeconomic stabilization policy under various assumptions regarding the exchange rate regime. To obtain a top mark in the course students should at the end of the course be able to demonstrate full capability of using the techniques of analysis taught in the course as well as a thorough understanding of the mechanisms in the business cycle models for open and closed economies, including the ability to use relevant variants and extensions of the models in order to explain the effects of various shocks and the effects of macroeconomic stabilization policies under alternative monetary and exchange rate regimes.

## Problem A

1. Equation (A.1) is an arbitrage condition stating that the required return from investing in shares (the right-hand side) has to equal the return from investing in bonds plus a risk premium (the left hand side).

For simplicity the market rate of interest on bonds r is assume to be constant.  $rV_t$  is the interest income that the shareholder could have earned during period t had shares been sold at the initial market value  $V_t$  and invested the the amount in bonds. Thus,  $rV_t$  is the opportunity cost of investing in shares. The risk premium  $\varepsilon$  is included because investing in a bond is considered less risky than investing in shares. The risk premium reflects that stock prices in general are more volatile than bond prices and interest payments, so in the absence of a risk premium, the risk-averse investor would prefer to hold "low-risk" bonds rather than "high-risk" shares if expected returns from these investments were identical. The right-hand side of (A.1) reflects the expected return from investing in stocks, i.e. the expected real dividend from owning stocks during period t,  $D_t^e$  plus an expected capital gain  $V_{t+1}^e - V_t$  (the difference between the expected real market value at the beginning of period t+1 and the the actual real market value of the firm at the beginning of period t).

Showing (A.2) is straight forward.

$$(r+\varepsilon)V_t = D_t^e + V_{t+1}^e - V_t \Longrightarrow \tag{A.1}$$

$$(1+r+\varepsilon)V_t = D_t^e + V_{t+1}^e \Longrightarrow$$

$$V_t = \frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon}$$
(A.2)

Equation (A.2) states that the market valuation of stocks in the current period equals the present value of next periods expected valuation of the stock and the expected dividend payment. The discount factor is the market interest rate for investing in bonds plus a risk premium  $\varepsilon$  for investing in shares.

2. To derive (A.4), one may start with (A.2), lead it one period to obtain a measure of  $V_{t+1}^e$  and insert this into (A.2). Proceeding in this way for all

future periods gives

$$V_{t} = \frac{D_{t}^{e} + V_{t+1}^{e}}{1 + r + \varepsilon}$$

$$= \frac{D_{t}^{e} + (\frac{D_{t+1}^{e} + V_{t+2}^{e}}{1 + r + \varepsilon})}{1 + r + \varepsilon}$$

$$= \frac{D_{t}^{e}}{1 + r + \varepsilon} + \frac{D_{t+1}^{e}}{(1 + r + \varepsilon)^{2}} + \frac{V_{t+2}^{e}}{(1 + r + \varepsilon)^{2}}$$

$$= \frac{D_{t}^{e}}{1 + r + \varepsilon} + \frac{D_{t+1}^{e}}{(1 + r + \varepsilon)^{2}} + \frac{D_{t+2}^{e}}{(1 + r + \varepsilon)^{3}} + \dots + \frac{V_{t+n}^{e}}{(1 + r + \varepsilon)^{n}}$$

$$= \sum_{n=0}^{\infty} \frac{D_{t+n}^{e}}{(1 + r + \varepsilon)^{n+1}}$$
(A.4)

where (A.3) is used to obtain the last expression.

Equation (A.4) states that the market value of the firm equals the present discounted value of the expected future dividends paid out by the firm. This is called "fundamental" because the valuation of the firm reflects the firm's fundamental ability to generate future cash flows to its owners.

- 3. In the figure the real stock price and the present value of real dividends is shown using either a constant discount rate or a market interest rate. Hence, the figure takes equation (A.4) to the data although by its very nature actual dividends are only an indicator for all future expected dividends. The figure shows that stock prices fluctuations are much larger than fluctuations in discounted dividends. Equation (A.4) offers three possible explanations for the observed stock market volatility and why this volatility may be perfectly rational:
  - Fluctuations in the (growth rate) of expected future real dividends  $D_t^e$ .
  - Fluctuations in the (expected) real interest rate r.
  - Fluctuations in the required risk premium on shares,  $\varepsilon$ .

The theory is compatible with rational behavior although nothing is said about how expectations are formed. All being said is how stock prices relate to future dividends. The model says nothing about how expectations on r, D and  $\varepsilon$  are formed.

4. Equation (A.5) defines  $q_t$  as the ratio between the market value of the firm and the replacement cost of the firm's capital stock where the price of capital has been normalized to 1. Hence, if the market price of shares in a company goes up, the value of  $q_t$  increases correspondingly.  $q_t$  offers a link between  $V_t$  which shareholders seek to maximize and the decision variable  $I_t$ .

Equation (A.6) postulates that investors believe the current share price per unit of capital one period from now is equal to the value of today, e.g. investors form static expectations on  $q_t$ .

Equation (A.7) states that next periods capital stock is equal to what is left of the current period's capital stock after depreciation plus what was being invested in the current period. Hence,  $K_{t+1}$  is predetermined.

Equation (A.8) states that expected dividends are equal to profits minus what is spend in total on investing in the firm's capital stock. Total investment costs are determined by the acquisition cost  $(I_t)$  plus the costs associated with installing new capital  $c(I_t)$ . Installation costs are assumed to increase in investments.

To find equation (A.9) we lead (A.5) by one period and note that since  $K_{t+1}$  is a predetermined variable we may drop expectations on this variable. Using (A.6) and (A.7) this gives us that

$$q_t = q_{t+1}^e = \frac{V_{t+1}^e}{K_{t+1}} \Longrightarrow V_{t+1}^e = q_t K_{t+1} = q_t [K_t(1-\delta) + I_t].$$

Substituting this and (A.8) into (A.2) gives (A.9)

$$V_{t} = \frac{D_{t}^{e} + V_{t+1}^{e}}{1 + r + \varepsilon}$$

$$= \frac{\prod_{t}^{e} - I_{t} - c(I_{t}) + q_{t} [K_{t}(1 - \delta) + I_{t}]}{1 + r + \varepsilon}.$$
(A.9)

5. In order to maximize the shareholder value companies are solving the problem

$$\max_{I_t} V_t = \frac{\prod_{t=0}^{e} -I_t - c(I_t) + q_t \left[ K_t (1 - \delta) + I_t \right]}{1 + r + \varepsilon}.$$

The first order condition for maximizing the shareholder value if found by

taking the derivative of  $V_t$ :

$$\frac{\partial V_t}{\partial I_t} = \frac{-1 - c'(I_t) + q_t}{1 + r + \varepsilon} = 0$$

$$\downarrow \qquad \qquad \underbrace{q_t} \qquad = \underbrace{1 + c'(I_t)}_{\text{foregone}}.$$
expected foregone capital gain dividend

From this expression it follows that the firm will invest until the point where the increase in its market value induced by the marginal investment is exactly equal to the sum of the acquisition and installation cost of buying and installing an additional unit of capital. The higher the valuation of the firm, the higher is  $q_t$  and the more the firm will invest. The slower installation costs rise, the more will be invested. Without installation costs it would be optimal to invest without any upper limit assuming that  $q_t > 1$ . For illustrative purposes we may follow the textbook and assume that installation costs take the form  $c(I_t) = \frac{a}{2}I_t^2$ . Then the first order condition reads  $q_t = 1 + aI_t$  which can be reformulated as an investment function  $I_t = \frac{q_t-1}{a}$ . Investments depend on  $q_t$  and investments in period t are smaller the higher the value of t. It could be noticed that installations costs makes firms smooth investments across time periods.

Though not asked for it could be noted, that specifying the investment function in terms of  $q_t$  has the advantage that this variable can be measured empirically since both the stock market value and the replacement costs (or changes therein) can be observed. This is in opposition to the possibility of observing expected dividends (which are underlying V and q). Hence by introducing q the model for investments may be taken to the data. In relation to this it could be noted, that the investment decision by the firm depends on the marginal value of q, whereas the estimated value of q is an average value between the total market value and the total replacement cost.

As  $q_t = \frac{V_t}{K_t}$  we have that anything that can induce fluctuations in  $V_t$  may also cause fluctuations in investments. Thus, the three explanations for market volatility stated above may also explain volatility in investments. Also, as

stock markets are quite volatile it may come as no surprise that investments are highly volatile as well.

6. From the textbook we have the uncovered interest parity (UIP) which is a financial arbitrage condition whereby the return on domestically denominated financial assets are tied to the returns on foreign denominated assets. Assuming investors are risk neutral and capital mobility is perfect so that domestic and foreign assets are perfect substitutes. In this situation investors may reallocate their portfolios instantaneously and without any costs so that the arbitrage condition stated below holds

$$1 + i = (1 + i^f) \times \frac{E_{+1}^e}{E},$$

where E is the current exchange rate and  $E_{+1}^e$  is the expected exchange rate and i and  $i^f$  are the nominal interest rate in the domestic and foreign economy respectively. Using the approximation  $\ln(1+x) \approx x$  when x is small the uncovered interest parity (UIP)

$$i = i^f + e^e_{+1} - e,$$

follows directly. If the fixing of the exchange rate is perceived perfectly credible we have that  $e_{+1}^e = e$  and as an implication we have that  $i = i^f$ .

7. Any change in foreign interest rates spills directly into the domestic interest rate. This follows from the UIP and the reasoning behind the UIP. In the foreign interest rate decrease then the domestic interest rate also has to be lower in order to sustain the fixing of the exchange rate. If  $i > i^f$  capital will flow into of the domestic economy as investors buy domestic denominated assets (and thereby domestic currency) and sell foreign denominated assets (and thereby foreign currency). This creates a pressure for an appreciation of the domestic currency and eventually the domestic interest rate has to adapt to the level of foreign interest rate in order for the fixed exchange rate regime to be sustained.

Assuming inflation expectations are unaffected this will decrease the real interest rate (defined by  $1+r = \frac{1+i}{1+\pi_{+1}^e}$ . Using the approximation  $\ln(1+x) \approx x$ 

when x is small we have  $r \approx i - \pi_{+1}^e$  which is the ex ante real interest rate used in the text book). With a lower real interest rate discounting is milder meaning that the present value of future expected dividends and hence the market value of the company is higher. According to the q-theory this stimulates investments.

8. If investors are risk averse the answer in question 6 is modified as a risk premium is added to the domestic interest rate i = i<sup>f</sup> + ρ where ρ is the risk premium demanded for holding domestic denominated financial assets (bonds). Investors worry about both the average return of their investment and the degree of certainty (or variance) of that return. To be induced to invest in a risky asset, they require a higher average return to compensate for increased uncertainty. According to the UIP modified to include a risk premium a change in the risk premium is reflected one-to-one in the domestic interest rate. Say, investors for some reason find it less risky to invest in the domestic economy. Then the risk premium is lower as investors demand a lower average return on their investment. This corresponds to a lower nominal interest rate and ceteris paribus a lower real interest rate. Hence, investments are stimulated.

## Problem B

1. It is socially desirable to stabilize inflation around a constant target value because it makes it easier for wage setters, consumers and firms to forecast inflation. A fluctuating inflation rate typically leads to unanticipated inflation, causing the ex post real interest rate and the ex post real wage to deviate from their ex ante expected levels. Because of these expectation errors, economic agents will make suboptimal decisions and hence experience lower welfare relative to a situation where they had anticipated inflation correctly. Unanticipated inflation leads to an arbitrary redistribution of real incomes between lenders and borrowers and real wages deviate from target.

Even a stable positive inflation rate is socially costly due to so-called 'shoe-leather costs', 'menu costs', relative price distortions due to staggered price setting. Also the tax system may fail to distinguish between nominal and real income from capital. As long as the rate of inflation is kept low these cost are likely to be small.

The target rate should be above zero as policy makers should take into account that a very low inflation rate and hence a low level of nominal interest rates reduces the scope for a cut in the real interest rate when the economy is hit by a negative shock, since the nominal interest rate cannot fall below zero. A very low inflation rate may also reduce the scope for a downward adjustment of real wages if nominal wages are downward sticky.

To sum up there is a trade of between on the one hand the cost of inflation and on the other hand the risk of ending up in a liquidity strap and a limited scope for reducing real wages through (unexpected) high inflation.

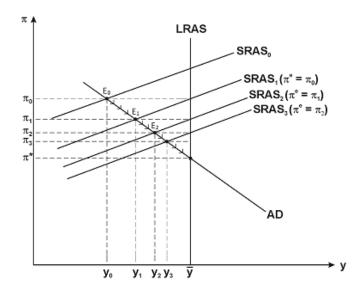
2. The "overshooting" behavior refer to situations where economic variables overreact to for instance changes in monetary or fiscal policy. This overreactions involves that the instant change in the economic variable is so large that it surpasses the new long run equilibrium value so that it has to adjust gradually to the new long run equilibrium value.

Since explaining "overshooting" requires forward-looking expectations (rational expectations) it is sufficient that the "overshooting" mechanism is explained in words. An explanation may be as follows: Financial investors

may correctly anticipate that e.g. a positive demand shock calls for a long run appreciation of the exchange rate. Due to the positive demand shock domestic inflation increase. As a result the domestic central bank increase interest rates so much that the real interest rate is increased. This means domestic interest rates are higher than than interest rates abroad. Therefore assets denominated in domestic currency are other things equal relative attractive. The currency appreciates, and it does so instantly in order to make room for the expected future depreciations needed to fulfill the uncovered interest parity (UIP) (hence domestic assets are not more attractive than foreign assets). Therefore the currency overshoots its new long run equilibrium value and therefore this "overshooting" is perfectly rational.

3. For the closed economy the central bank play a crucial role as the "mechanism" that ensures convergence to equilibrium. For instance in case of a recession inflation is falling. The central bank responds to this by lowering the policy rate According to the Taylor principle the nominal rate is lowered so much that the real interest rate is lower too. This stimulates private demand. It could be noted that according to theory the effects on total private demand from a change in the real interest rate is ambiguous. A lower real interest rate stimulates investments but the effect on private consumption is indeterminate due to the possibility of counteracting income and substitution effects Hence the effect on total private demand can not be determined with certainty. However empirical evidence tells that there is a negative relationship so that a lower real interest rate is accompanied by higher total private demand. The convergence process for the closed economy may be illustrated as in the following figure where downward revisions in economic agents inflation expectations translates into lower wage demands which dampens inflation (through lower marginal costs and thereby lower product prices inflation). This leads the central bank to cut nominal interest rates so much that the expected (ex ante) real interest rate is lowered. This stimulates total private demand.

For the open economy with fixed exchange rates the real exchange rate plays a crucial role. Assume we start out in an equilibrium, where both the domestics and the foreign economy is in a long run equilibrium. In this situation



inflation in the domestic economy is equal to inflation abroad. Then the domestic economy is hit by a shock so that the economy end up in a recession. Domestic inflation is now below inflation abroad (as wage formation is dampened) This leads to an improvement in domestic competitiveness and accordingly demand for domestically produced goods increase (both in the domestic economy and abroad). The convergence process may be illustrated in a figure like the following.

