Written Exam for M.Sc. in Economics Summer School 2013

Investment Theory

Master Course

19th August 2013

3 hours closed books exam

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

Question 1:

Consider an investment project. The investment cost is $I > 0$. The dividend of the project switches randomly between two states both before and after the project is started. Indeed, the dividend of the project is either $XP - C_X$ or $YP - C_Y$, where

$$dP = \alpha P dt + \sigma P dz$$

with $\sigma > 0$, $Y > X > 0$ and $C_Y > C_X > 0$. The dividend of the project changes from $XP - C_X$ to $YP - C_Y$ with probability $\lambda_{XY} dt$ in the small time interval dt and from $YP - C_Y$ to $XP - C_X$ with probability $\lambda_{YX} dt$ in the small time interval dt, where $\lambda_{XY}, \lambda_{YX} > 0$.

The discount rate is $\rho > 0$ with $\rho > \alpha$.

Let $F_X(P)$, resp. $F_Y(P)$, be the value of the option to invest when the dividend is $XP - C_X$, resp. $YP - C_Y$. Let $V_X(P)$, resp. $V_Y(P)$, be the value of the project when the dividend is $XP - C_X$, resp. $YP - C_Y$.

(a) Explain the project. Give an example of an investment project that fits the above project.

(b) State possible strategies for starting the project. Use the strategies to relate $F_X(P)$, $F_Y(P)$, $V_X(P)$ and $V_Y(P)$ and discuss their properties.

(c) State the Bellman equations for $V_X(P)$ and $V_Y(P)$. Use the Bellman equations to find differential equations in $V_X(P)$ and $V_Y(P)$.

(d) Find $V_X(P)$ and $V_Y(P)$. (Hint: Guess that both functions are linear and find the coefficients.) Compare and interpret your expressions for $V_X(P)$ and $V_Y(P)$.

(e) State the Bellman equations for $F_X(P)$ and $F_Y(P)$. Use the Bellman equations to find differential equations in $F_X(P)$ and $F_Y(P)$ as well as $F(P) = F_X(P) - F_Y(P)$.

(f) Find $F(P)$ up to undetermined constants. Find $F_X(P)$ and $F_Y(P)$ up to undetermined constants. Interpret your expressions for $F_X(P)$ and $F_Y(P)$.

(g) Show how the optimal strategies and the undetermined constants for $F_X(P)$ and $F_Y(P)$ can be found.

(h) Consider your example of an investment project in (a). Give an example of a real option for the active project that fits with your example. Discuss how incorporation of the real option would change your analysis in (b)-(g).