Macro C - exam solutions (August 5, 2014)

General remarks

Please grade each question/item between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

In Problem 1, items a) and b), an answer that assumes $\gamma = 0$ should get half the points.

1 Question 1

False. It is true that when money is superneutral changes in its growth rate have no effect on real variables c and k. But there can be changes in real money holdings, m and money would be still considered to be superneutral. The second part of the statement (about irrelevance from policy perspective) is only there to give students a hint, since in the course we covered models where money was superneutral that had an optimal rate of growth of money (Friedman rule).

2 Question 2

False. By construction a currency peg with an escape clause is always preferable to a hard peg. In fact it includes the hard peg when it it found that optimally the escape clause is never invoked. The argument about volatility of supply shocks is used to justify the use of a peg instead of discretionary policy, but is irrelevant for answering this question.

3 Problem 1

a) The wage and interest rate are determined by imposing equilibrium in factor markets where firms competitively demand labor and capital from households. Thus the student needs to maximize profit function for firms

$$\max_{L_t, K_t} K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t^L K_t$$

From FOC of firms' problem of maximizing profits we get

$$\begin{aligned}(1-\alpha)K_t^{\alpha}L_t^{-\alpha} &= (1-\alpha)k_t^{\alpha} = w_t \\ \alpha K_t^{\alpha-1}L_t^{1-\alpha} &= \alpha k_t^{\alpha-1} = r_t^L\end{aligned}$$

where we impose that in equilibrium all firms work with the same capital labor ratio, k, which must be equal to the ratio of aggregate capital to labor.

Note that the following requires using the relations $d_t = a_t = \frac{k_t}{1-\gamma}$. The market wage is $(1-\alpha)k_t^{\alpha} = (1-\alpha)((1-\gamma)a_t)^{\alpha}$. Deposit rate r_t^D *times* saving per capita are payments to households (in per capita terms) for saving. Thus household income on saving is $a_t r_t^D = a_t\left(\alpha((1-\gamma)a_t)^{\alpha-1}(1-\gamma)\right) = \alpha((1-\gamma)a_t)^{\alpha}$. Partial credit if wrongly derived but intuition is correct.

b) Control and state variables respectively are c and a.

Hamiltonian (it is irrelevant if set up as current value or present value, what matters is that the FOC are correct for each setup) :

$$\begin{aligned}H_t &= \ln(c_t)e^{-(\rho-n)t} + \lambda_t\left(w_t + r_t^D a_t - c_t - na_t\right) \\ H_t^c &= \ln(c_t) + \mu_t\left(w_t + r_t^D a_t - c_t - na_t\right)\end{aligned}$$

with $\mu_t \equiv \lambda_t e^{(\rho-n)t}$.

Student gets full points if stating FOC assuming an interior solution (even if there is no explicit assumption of this, i.e. no penalty from failing to consider corner solution):

$$\begin{aligned}\frac{dH_t^c}{dc_t} &= \frac{1}{c_t} - \mu_t = 0 \\ \dot{\mu}_t &= -\frac{dH_t^c}{da_t} + (\rho-n)\mu_t = -\mu_t(r_t^D - \rho) \\ \lim_{t\to\infty} e^{-\rho t}\mu_t a_t &= 0\end{aligned}$$

Note that the law of motion of the state variable is also a FOC (derivative of Hamiltonian with respect to costate variable λ_t or μ_t). Not writing it has no penalty. If using H then FOC should be adjusted to that formulation.

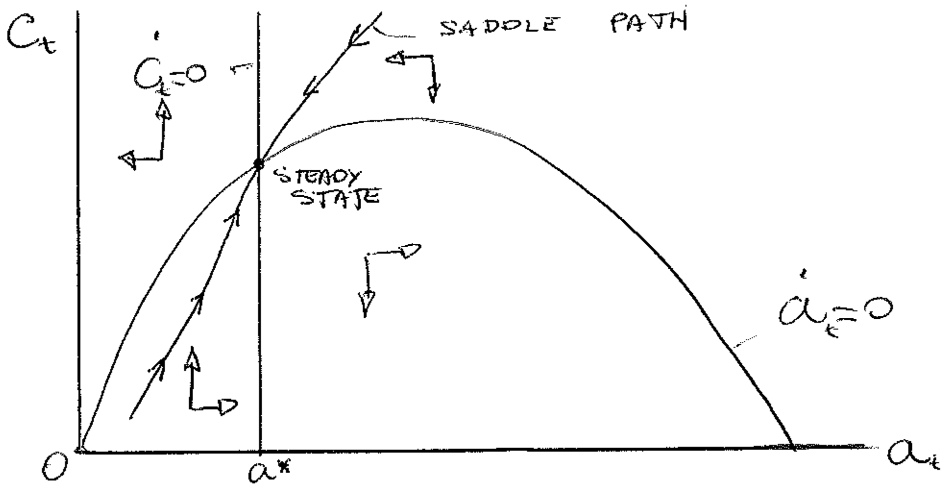
for the Euler equation, or Keynes-Ramsey condition, it is important that r^D and not r or r^L or the marginal productivity of capital, be in it:

$$\frac{\dot{c}_t}{c_t} = r_t^D - \rho = \alpha(1-\gamma)^{\alpha}a_t^{\alpha-1} - \rho.$$

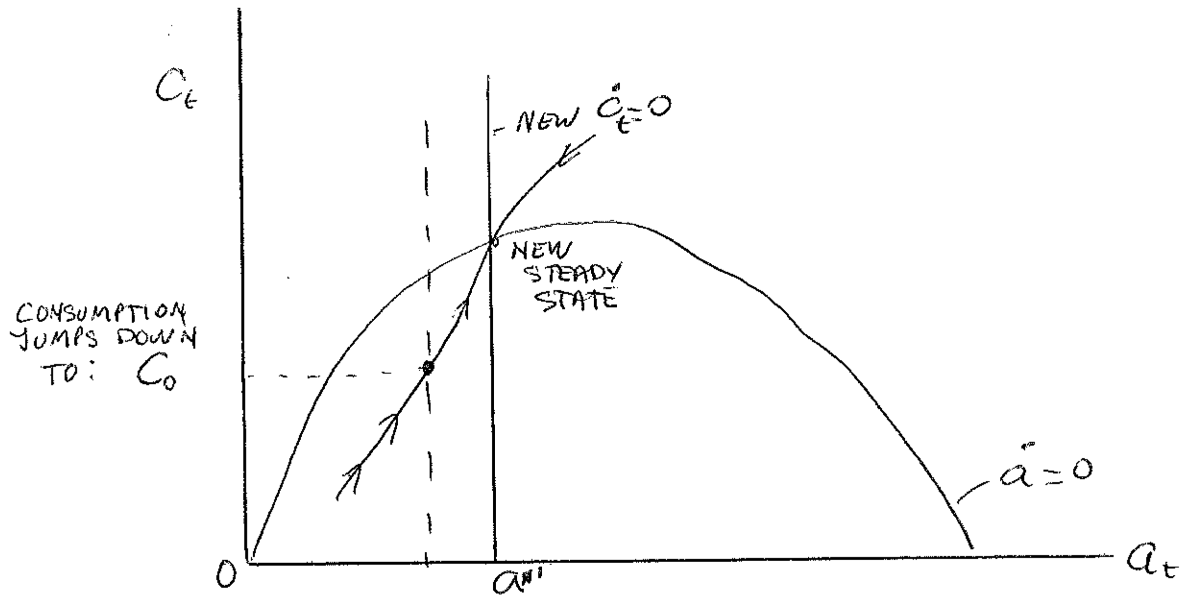
Interpretation is that consumption (in per capita terms) is increasing/falling over time as long as interest rate on savings is above/below rate at which future consumption is discounted, and that for the logarithmic utility function, the instantaneous elasticity of substitution (inverse of coefficient of relative risk aversion), measuring the response of consumption growth rate to a given difference between r_t^D and ρ, is one.

Steady state is characterized by $\dot{a}_t = \dot{c}_t = 0$ (if using $\dot{k}_t = 0$ pay attention whether the student makes a coherent analysis, and not just answering from memory). Thus $\dot{a}_t = 0$ implies that $c_t = w_t + r_t^D a_t - n a_t = (a_t(1-\gamma))^\alpha - n a_t$. $\dot{c}_t = 0$ implies that $r_t^D = (1-\gamma)\alpha(a_t(1-\gamma))^{\alpha-1} = \rho$. This pins down the steady state saving per capita $(a^* = \frac{1}{1-\gamma}\left(\frac{\alpha(1-\gamma)}{\rho}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{\rho}\right)^{\frac{1}{1-\alpha}}(1-\gamma)^{\frac{\alpha}{1-\alpha}})$.

Phase diagram should show the $\dot{a}_t = 0$ and $\dot{c}_t = 0$ curves and the local dynamics of the variables in the four quadrants they define. The phase diagram should also have the saddle path of convergent dynamics to the steady state (and this correctly identified as the intersection of the $\dot{a}_t = 0$ and $\dot{c}_t = 0$ curves).



c) This shocks moves the $\dot{c}_t = 0$ curve to the right and leaves the $\dot{a}_t = 0$ curve unaffected. New steady state saving per capita comes from previous formula $(a^{*'} = \frac{1}{1-\gamma}\left(\frac{\alpha(1-\gamma)}{\rho'}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{\rho'}\right)^{\frac{1}{1-\alpha}}(1-\gamma)^{\frac{\alpha}{1-\alpha}})$ now with new $\rho' < \rho$ which immediately implies $a^{*'} > a^*$. Steady state capital per capita is simply $k^{*'} = (1-\gamma)a^{*'}$. Graphical analysis follows. Intuition is that the shock makes households more patient and thus at the prevailing interest rate decide to reduce consumption and increase saving. This translated into increase capital accumulation up to the point that the marginal productivity of capital decreases such that the perceived interest on saving, r^D, equals the new discount factor ρ'.



d) Yes, if income on saving is taxed (and receipts are returned lump sum) the household will be unwilling to save more, and thus steady state a will remain at initial level a^*. With tax the Euler equation is now (tax rate is τ)

$$\frac{\dot{c}_t}{c_t} = (1-\tau)r_t^D - \rho' = (1-\tau)\alpha(1-\gamma)^\alpha a_t^{\alpha-1} - \rho'.$$

This policy prevents the $\dot{c}_t = 0$ curve from shifting to the right after the shock, and given that the $\dot{a}_t = 0$ curve does not shift, this policy leaves the economy at its initial steady state. Thus there is no change in initial or steady state consumption. The reason for this is that the policy makes saving undesirable in the same extent as the shock made them more desirable. Thus marginally the incentive to save does not change. Since the tax receipts are returned to the household there is no effect on income and thus allocation of income to saving and consumption remains the same as in the old steady state.

4 Problem 2

a) The profit function for firms is given

$$AK_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t$$

The FOC of firms' problem of maximizing profits (taking factor prices as given) are

$$\begin{aligned} (1-\alpha)A(K_t)^\alpha(L_t)^{-\alpha} - w_t &= 0 \\ \alpha A(K_t^j)^{\alpha-1}(L_t^j)^{1-\alpha} - r_t &= 0 \end{aligned}$$

In what follows, since population is constant we can assume that $L_t \equiv 1$, such that the capital labor ratio, $k \equiv \frac{K}{L}$ satisfies $k_t = K_t$ (not necessary for any result).

The wage and interest rates are determined by imposing equilibrium in factor markets where firms competitively demand labor from households and capital from intermediaries. Thus

$$\begin{aligned} w_t &= (1-\alpha)Ak_t^{\alpha} \\ r_t &= \alpha A k_t^{\alpha-1} \end{aligned}$$

Characterizing individual saving behavior requires setting up the problem of workers.

$$\begin{aligned} \max_{c_{1t},c_{2t+1}} \quad & \ln(c_{1t}) + \frac{1}{1+\rho}\ln(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t} = w_t(1-\tau) - s_t \\ & c_{2t+1} = s_t r_{t+1} + \tau w_{t+1} \end{aligned}$$

Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \frac{r_{t+1}}{1+\rho}c_{1t}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{1}{2+\rho}w_t(1-\tau) - \left(\frac{1+\rho}{2+\rho}\right)\frac{1}{(r_{t+1})}\tau w_{t+1}$$

b) To get capital accumulation we replace individual savings with next period capital per worker $k_{t+1} = s_t$ (note there is no $\frac{1}{1+n}$ term since there is no population growth), and we use equilibrium expressions for wage and interest rates from a)

$$k_{t+1} = \left[\frac{1}{2+\rho}(1-\alpha)Ak_t^{\alpha}(1-\tau) - \left(\frac{1+\rho}{2+\rho}\right)\frac{(1-\alpha)k_{t+1}}{\alpha}\tau\right]$$

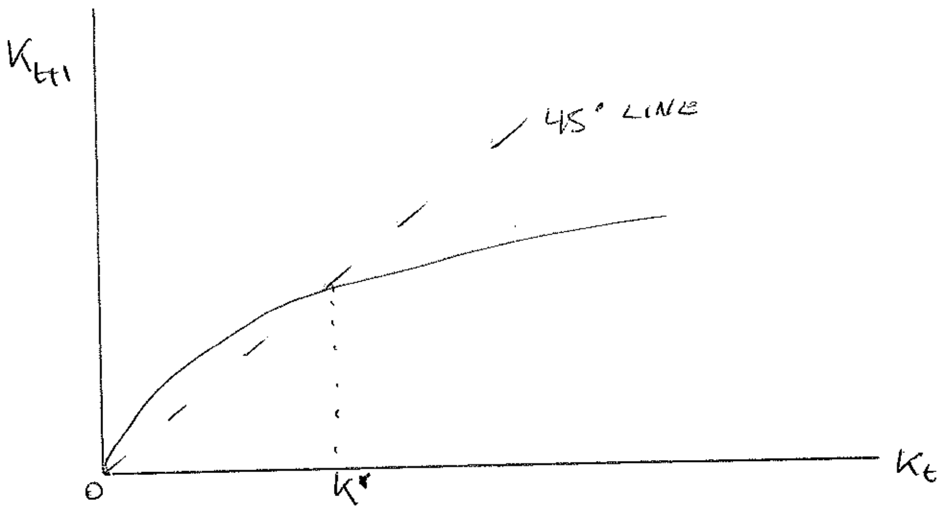
Combining terms with k_{t+1}

$$k_{t+1} = \frac{1}{\left[1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau\right]}\frac{1}{2+\rho}(1-\alpha)Ak_t^{\alpha}(1-\tau)$$

From here imposing steady state we get the following

$$k^* = \left[\frac{1}{\left[1+\frac{1+\rho}{2+\rho}\frac{1-\alpha}{\alpha}\tau\right]}\frac{1}{2+\rho}(1-\alpha)A(1-\tau)\right]^{\frac{1}{1-\alpha}}.$$

Graphically (OPTIONAL)

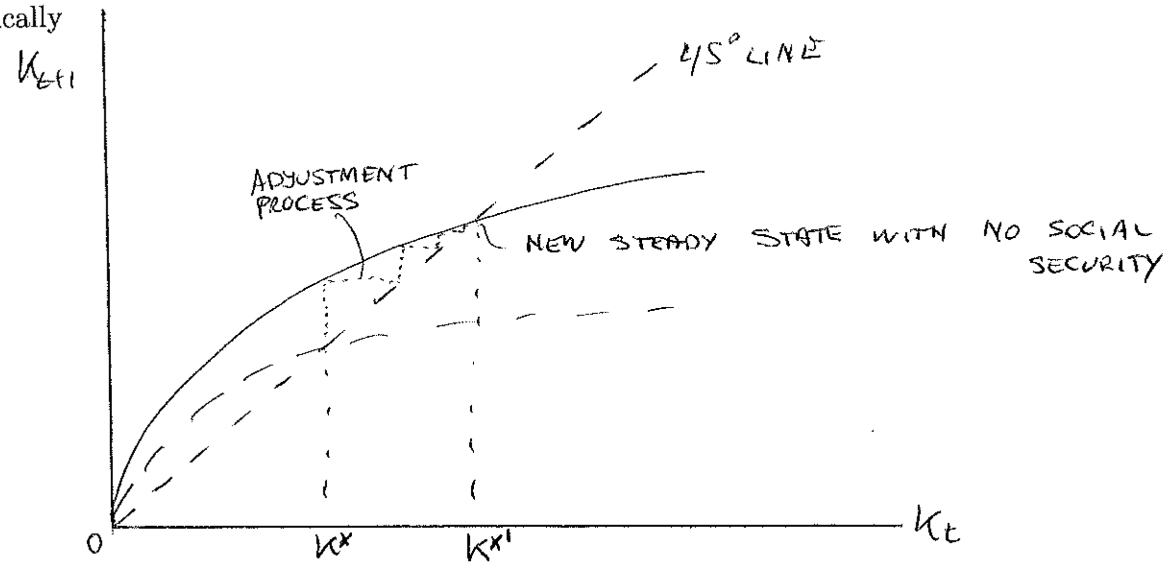


The economy can be dynamically inefficient in this steady state if $r^* < 1$, since the rate of return of social security is $1 + n = 1$. It is up to parameters whether the economy will be dynamically efficient or not.

c) From b) above we know that the new steady state satisfies

$$k^{*'} = \left[\frac{1}{2+\rho}(1-\alpha)A\right]^{\frac{1}{1-\alpha}} > k^*,$$

Graphically



In the first period we have that wage is equal to $w^* = (1-\alpha)A(k^*)^\alpha$ and capital accumulation satisfies

$$k_1 = \frac{1}{2+\rho}w^*$$

Thus, $k_1 > k^*$. The effect on first period consumption on impact is ambiguous. This is given by

$$c_{1t} - c_1^* = \tau w^* \frac{1+\rho}{2+\rho} \left(1 - \frac{1}{r^*}\right).$$

For example, if the economy was dynamically inefficient such that $r^* < 1$ then the shock reduces first period consumption relative to the previous steady state. Since the economy was dynamically inefficient, the shock makes the young worse off by reducing the size of the social security which had a higher rate of return. Thus the income effect leads them to allocate less resources to first and second period consumption. This explains the reduction in first period consumption.

From the above reasoning the young are better off only when the economy initially was dynamically efficient, i.e. when $r^* > 1$. The old are clearly always worse off since they do not receive contributions.