Rettevejledning til
Eksamen på Økonomistudiet 2011-I
Reeksamen
Makro A og Macro A, 2. årsprøve
Efterårssemestret 2010
(Tre-timers prøve uden hjælpemidler)

Målbeskrivelse:

Faget videreudvikler langsigtsdelen af Økonomiske Principper 2, Makro.

I Makro A opstilles og analyseres alternative formelle modeller til forståelse af de langsigtede, trendmæssige tendenser i de vigtigste makroøkonomiske variable, såsom aggregeret indkomst og forbrug (per capita), indkomstfordeling, realløn og realrente, nettofordringsposition overfor udlandet, teknologisk niveau og produktivitet samt ledighed. I sammenhæng hermed præsenteres empirisk materiale under anvendelse af simple statistiske metoder.

Faget bygger op til Makro B ved at beskrive det forankringspunkt, økonomiens fluktuationer foregår omkring. Det bygger også op til Makro C ved at omfatte de mest fundamentale versioner af de langsigtsmodeller, som også indgår i Makro C.

De studerende skal lære de vigtigste såkaldte stiliserede empiriske fakta om økonomisk vækst og strukturel ledighed at kende, og de skal kende til og forstå den række af økonomisk teoretiske modeller, som i kurset inddrages til forklaring af disse fakta og til forståelse af økonomiens trendmæssige udvikling i det hele taget.

En vigtig kundskab, der begyndende skal erhverves i dette kursus, er selvstændig opstilling og analyse af formelle, makroøkonomiske modeller, som af type er som kendt fra faget, men som kan være variationer heraf. Der vil typisk være tale om modeller, som er formulerede som, eller er tæt på at være formulerede som, egentlige generelle ligevægtsmodeller. En del af denne kundskab består i en verbal formidling af en forståelse af modellernes egenskaber.

En anden vigtig kundskab er at kunne koble teori og empiri, så empirisk materiale kan tilvejebringes og analyseres på en måde, der er afklarende i forhold til teorien. Igen er verbal formidling af de konklusioner, der kan drages ud af samspillet mellem teori og empiri, en vigtig del af den beskrevne færdighed.

De typer af modeller, der skal kunne analyseres, omfatter modeller for lukkede såvel som for åbne økonomier, statiske såvel som dynamiske modeller, dynamiske modeller med såvel diskret tid som kontinuert tid. Modellerne skal både kunne analyseres generelt og ved numerisk simulation (sidstnævnte dog kun af ikke-stokastiske dynamiske modeller i diskret tid).

De studerende skal opnå færdigheder i at foretage økonomiske analyser i de typer af modeller, faget beskæftiger sig med, herunder analyser af strukturelle, økonomisk politiske indgreb og formidle analysens indsigter.

Topkarakteren 12 opnås, når de beskrevne færdigheder mestres til en sådan fuldkommenhed, at den studerende er blevet i stand til selvstændigt at analysere nye (fx økonomisk politiske) problemstillinger ved egen opstilling og analyse af varianter af de fra kurset kendte modeller under inddragelse og analyse af relevant empiri og afgive absolut fyldestgørende verbal forklaring af de opnåede analyseresultater.

Problem 1.

1.1 Stylized fact 5: Over long periods of 130-200 years, many countries in Western Europe and North America have had relatively constant annual rates of growth in GDP per capita in the range 1.5–2 per cent.

Stylized fact 6: During the long periods of relatively constant growth rates in GDP per worker in the typical Western economy, labour's share of GDP has stayed relatively constant, and the average real wage of a worker has grown by approximately the same rate as GDP per worker.

Stylized fact 7: During the long periods of relatively constant growth in GDP per worker in the typical Western economy, capital's share and the rate of return on capital have shown no trend upwards or downwards, the capital–output ratio has been relatively constant, and the capital intensity has grown by approximately the same rate as GDP per worker.

1.2 The figures indicate constant labour's shares and no trend upwards or downwards in real interest rates on long-term bonds in the long run. The income share of factors other than labour must also have been relatively constant, and we may crudely summarize these other factors as 'capital'. Hence Figure 1 indicates relatively constant capital's shares in the long run. Over long periods the rate of return on (physical) capital cannot diverge from the real interest rate on long-term bonds, so Figure 2 indicates no long run trend in the real rate of return on capital. In usual notation we have that capital's share $r_t K_t / Y_t$ is relatively constant and the rate of return r_t also shows no long run trend. In that case the capital-output ratio, K_t / Y_t, must have no trend and hence can be viewed as relatively constant for long run issues.

1.3 The three facts of Question 1.1 can be stated in terms of three constancies: the growth rate of GDP per worker is relatively constant, the functional income distribution between labour and 'capital' is relatively constant, and the rate of return on 'capital' is relatively constant. These constancies plus underlying assumptions of a labour force growing at constant rate and a constant savings ratio can be collected in the concept of 'balanced growth':

Along a balanced growth path GDP per worker, consumption per worker, the real wage rate, and the capital intensity all grow at one and the same constant rate, g, the labour force (population) grows at constant rate, n, GDP, consumption, and capital grow at the common rate, $g + n$, the capital–output ratio is constant, and the rate of return on capital is constant.

In growth theory balanced growth is used as an empirical 'consistency check' on growth models: Under the assumption of a constant growth rate of labour supply, the model should exhibit balanced growth in the long run (in steady state). Otherwise it is considered empirically implausible.

Problem 2.

The model repeated from the problem set:

$$\begin{aligned}
Y_t &= BK_t^{\alpha}L_t^{1-\alpha}, \quad 0<\alpha<1, \quad B>0 && (1)\\
r_t &= \alpha B\left(\frac{K_t}{L_t}\right)^{\alpha-1} && (2)\\
w_t &= (1-\alpha)\,B\left(\frac{K_t}{L_t}\right)^{\alpha} && (3)\\
S_t &= sY_t, \quad 0<s<1 && (4)\\
K_{t+1} &= S_t+K_t && (5)\\
L_{t+1} &= (1+n)\,L_t, \quad n>0 && (6)
\end{aligned}$$

2.1 Dividing on both sides of (1) by L_t gives:

$$\frac{Y_t}{L_t}=\frac{BK_t^{\alpha}L_t^{1-\alpha}}{L_t^{\alpha}L_t^{1-\alpha}} \Leftrightarrow$$

$$\begin{aligned}
y_t &= B\left(\frac{K_t}{L_t}\right)^{\alpha}\\
&= Bk_t^{\alpha} && (7)
\end{aligned}$$

Dividing on both sides of (5) by L_{t+1} ($=(1+n)L_t$) etc. gives:

$$\frac{K_{t+1}}{L_{t+1}}=\frac{S_t+K_t}{(1+n)\,L_t} \Leftrightarrow$$

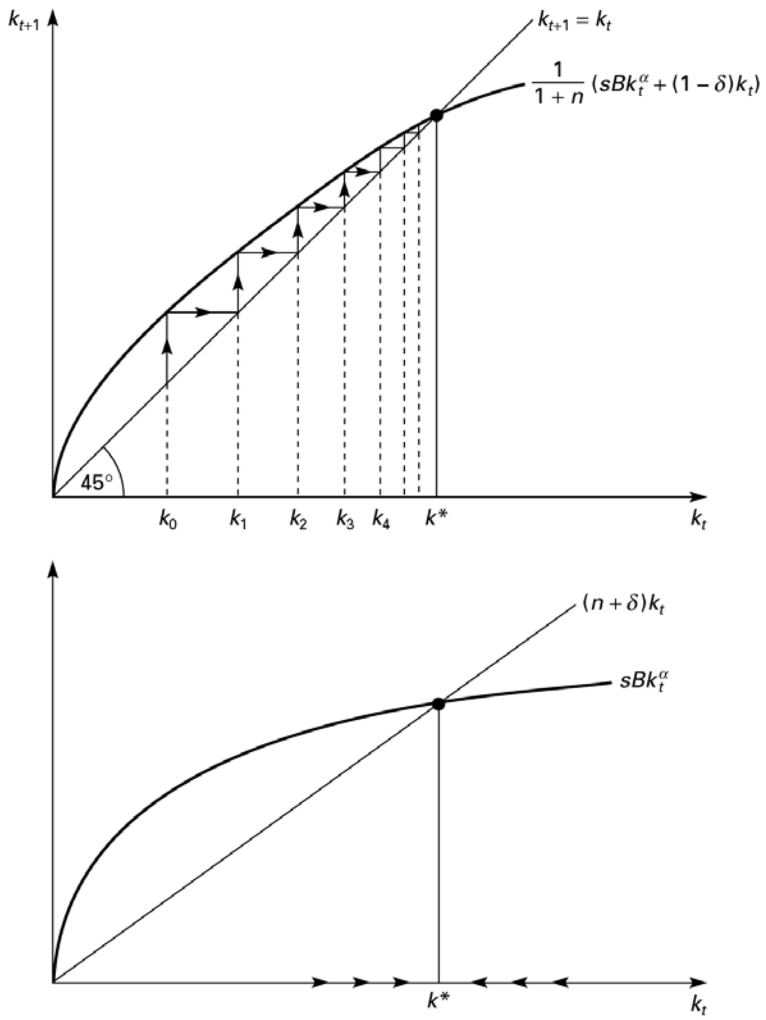
$$\begin{aligned}
k_{t+1} &= \frac{1}{1+n}\frac{sY_t+K_t}{L_t}\\
&= \frac{1}{1+n}\left(sy_t+k_t\right)\\
&= \frac{1}{1+n}\left(sBk_t^{\alpha}+k_t\right) && (8)
\end{aligned}$$

Subtracting k_t on both sides in (8) gives:

$$\begin{aligned}
k_{t+1}-k_t &= \frac{1}{1+n}\left(sBk_t^{\alpha}+k_t\right)-\frac{1+n}{1+n}k_t\\
&= \frac{1}{1+n}\left(sBk_t^{\alpha}+k_t-(1+n)\,k_t\right) \Leftrightarrow
\end{aligned}$$

$$k_{t+1}-k_t=\frac{1}{1+n}\left(sBk_t^{\alpha}-nk_t\right) \qquad (9)$$

2.2 The figures should look like ...



... but with the δ in these equal to zero (the figure is copied from the text book).

The following properties of the transition curve in (8) should be established:

1. It passes through (0,0) - can be seen directly from (8).

2. It is everywhere increasing - can be seen directly from (8).

Its slope

$$\frac{dk_{t+1}}{dk_t} = \frac{s\alpha k_t^{\alpha-1} + 1}{1+n}$$

3. goes to infinity as k_t goes to zero and

4. goes to $\frac{1}{1+n} < 1$ as k_t goes to infinity.

It follows that there is a unique, strictly positive intersection between the transition curve and the 45°-line in some k^*, and it follows by "stair-case iteration" in the upper figure that k_t converges to this steady state value from any initial $k_0 > 0$. The lower figure illustrates the adjustment of k_t in the Solow diagram.

2.3 To find k^*, one can set the left hand side, and hence also the right hand side, of (9) equal to zero:

$$sBk_t^{\alpha} = nk_t \Leftrightarrow$$
$$k_t^{1-\alpha} = B\frac{s}{n} \Leftrightarrow$$
$$k_t = B^{\frac{1}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{1}{1-\alpha}} \equiv k^*. \qquad (*)$$

Inserting into (7), (2) and (3) gives:

$$\begin{aligned} y_t &= B\left(B^{\frac{1}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha} \\ &= BB^{\frac{\alpha}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{\alpha}{1-\alpha}} \\ &= B^{\frac{1}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{\alpha}{1-\alpha}} \equiv y^* \qquad (10) \end{aligned}$$

and

$$\begin{aligned} r_t &= \alpha B\left(B^{\frac{1}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha-1} \\ &= \alpha BB^{-1}\left(\frac{s}{n}\right)^{-1} \\ &= \frac{\alpha n}{s} \equiv r^* \qquad (11) \end{aligned}$$

and

$$\begin{aligned} w_t &= (1-\alpha)\,B\left(B^{\frac{1}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha} \\ &= (1-\alpha)\,BB^{\frac{\alpha}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{\alpha}{1-\alpha}} \\ &= (1-\alpha)\,B^{\frac{1}{1-\alpha}}\left(\frac{s}{n}\right)^{\frac{\alpha}{1-\alpha}} \equiv w^* \qquad (12) \end{aligned}$$

The elasticity of y^* with respect to B is $\frac{1}{1-\alpha} > 1$. Equation (7) indicates a direct elasticity (for constant k_t) of one. The steady state elasticity is larger because of the capital accumulation effect: If B increases by one per cent the effect on impact (in the same period) is an increase in y_t of one per cent because k_t is unaffected in that period as given by past capital accumulation. The increased y_t will, however, give rise to larger saving and hence in the next period more capital and even higher income and so on ...

2.4 In the coupled figures above, the two curves will move upwards with the increase in B. This will, as the revised figure will show, start an adjustment upwards in k_t from the old steady state value k^* to a new and higher one $k^{*\prime}$, where the new curves intersect the 45°-line and the line nk_t, respectively.

In period one (on impact), however, k_t will be unchanged, that is, k_1 will equal its old steady state value k^*. The reason is that $k_1 = K_1/L_1 = (K_0 + sY_0)/\left[(1+n)L_0\right] = (k_0 + sy_0/(1+n)$ is predetermined from period zero (determined by variable values in period zero) and therefore unaffected by events in period one. Of course, $y_1 = B'k_1^{\alpha} = B'\left(k^*\right)^{\alpha}$ increases above y^* because of the increase in B, and $r_1 = \alpha B' (k^*)^{\alpha-1}$ (equation (2)) and $w_1 = (1-\alpha) B' (k^*)^{\alpha}$ (equation (3)) both increase above their old values, r^* and w^*, respectively.

The increase in y_1 means that saving per worker, sy_1, will increase, which implies that k_2 will increase above $k_1 = k^*$. The higher level of capital per worker will increase y_2 further above y_1, thereby lifting k_3 above k_2 etc. through a process of capital accumulation. Due to diminishing returns in $y_t = Bk_t^{\alpha}$, the steps will be smaller and smaller leading k_t towards $k^{*\prime} = (B')^{\frac{1}{1-\alpha}} (s/n)^{\frac{1}{1-\alpha}}$ (equation (*)).

As k_t increases from period 2, y_t will increase further up from y_1 according to $y_t = B' k_t^{\alpha}$ and finally reach the level $y^{*\prime} = B (k^{*\prime})^{\alpha} = (B')^{\frac{1}{1-\alpha}} (s/n)^{\frac{\alpha}{1-\alpha}}$. The interest rate increased up to r_1 in period one, but will from period 2 decrease from that level according to $r_t = \alpha B' (k_t)^{\alpha-1}$ and reach the level $r^{*\prime} = \alpha B' (k^{*\prime})^{\alpha-1} = \alpha n/s$ (equation (11)), that is, it will return down to its old level. Finally, w_t will increase further up from w_1 according to $w_t = (1-\alpha) B' (k_t)^{\alpha}$ and reach $(1-\alpha)(B')^{\frac{1}{1-\alpha}} (s/n)^{\frac{\alpha}{1-\alpha}}$.

2.5 In the figures the curves move up for one period, period one, and then moves back to their old positions.

In period one (on impact) the same (as described above) happens: k_1 stays unchanged at k^*, while y_1, r_1, and w_1 increase with B.

The fact that y_1 is larger than $y_0 = y^*$ means that in period 2, $k_2 = (sy_1 + k_1)/(1+n)$ will be larger than $k_1 = k_0 = k^*$. This implies that y_2 will be larger than $y_0 = y^*$, but, of course, smaller than y_1 because the productivity shock has now vanished. Through the periods 3, 4, ... both k_t and y_t will decrease back towards their old values.

The effect that k_2 and y_2 are larger than their original values (k_0 and y_0) even though B is back at its old value in period 2 is entirely due to capital accumulation. The propagation is thus due to capital accumulation: The technology shock pulls y_1 upwards, this builds into capital and k_2, which means larger y_t than originally also in the periods 2, 3, ...

2.6 The point is that for plausible parameters the pure propagation effect on income is very small even for considerable shocks. The key to this insight is that the capital-output ratio is (plausibly) quite large. In steady state, $k^*/y^* = s/n$. A somewhat small value will result from $s = 0.2$ and $n = 0.01$, where $k^*/y^* = 20$.

The temporary shock increases y_1 above y^*, in fact proportionally to the size of the shock. If B increases by 10 per cent, y_1 will be 10 per cent above y^*. However, since capital is so much larger than income, the change in y_t relative to k^* is small. For instance, if the increase in y_1 is 10 per cent, then the increase relative to k^* is 1/20'th of 10 per cent, that is, 0.5 per cent.

This is important since the only reason for an increased income level in period 2 is the more capital in period 2 coming from the increased income in period one: If the savings rate is 0.2, then the 10 per cent increase in income would give an additional 2 percentage points of saving or an increase in savings of 2 per cent relative to the old (before the shock) level of income. Relating to capital at the outset the increase in capital is down to 0,1 per cent, 1/20'th of 2 per cent. (Here we have even left out the effect of population growth, but this is small). Hence the increase in capital relative to its old level is plausibly at most 0.1 per cent in period 2 even after a large productivity shock of 10 per cent. If the relative increase in capital (per worker) is small, so must the relative increase in income (per worker) be from period 2 and onwards: From $y_t = Bk_t^{\alpha}$ it is seen that the elasticity from k_t to y_t is plausibly around 1/3, so the 0.1 per cent increase in capital gives an increase in income of around 0.03 per cent.

2.7 Without stating the full model, the key feature of the small open economy is capital arbitrage: Capital will run in or out of the open economy until the marginal product of capital, $\alpha B (K_t/L_t)^{\alpha-1} = \alpha B k_t^{\alpha-1}$, equals the international real interest rate, $\bar{r}$. The arbitrage condition, $\alpha B k_t^{\alpha-1} = \bar{r}$, determines the capital intensity, k^*, immediately $\alpha B (k^*)^{\alpha-1} = \bar{r}$. The wage rate and GDP per worker only depend on k_t and are hence determined accordingly, e.g., $w_t = w^* = (1-\alpha) B k_t^{\alpha}$. A steady state also involves a level of wealth per worker, v^*, and a level of national income per worker, y^{n*}.

If B increases to B' in period one, and only in that period, the left hand side of the arbitrage condition increases accordingly given k_t. This means that capital runs into the country until the condition is reestablished. Hence the capital intensity increases in period one to k_1: $\alpha B' (k_1)^{\alpha-1} = \bar{r}$. The wage rate increases according to $w_1 = (1-\alpha) B' (k_1)^{\alpha}$, that is, both from the higher B and from the higher k_1. Wealth per worker, however, is unchanged in period one as predetermined from former periods, $v_1 = v^*$. This means that national income per worker increases by the amount that the wage rate increases, $y_1^n = w_1 + \bar{r} v^*$, and saving increases accordingly, $s_1 = s y_1^n$.

In period 2, B falls back to its old level. Capital arbitrage sends the capital intensity back to its old level, k^*, immediately, and hence also brings the wage rate and GDP per worker to their old levels immediately. The larger national income and savings in period one means that in period 2 national wealth is higher than its old level, $v_2 > v_1 = v^*$. Therefore national income, $y_2^n = w^* + \bar{r} v_2$, and savings will be above their old steady state levels and so forth. Again, there is a propagation effect in period 2 and subsequent periods, this time coming from wealth accumulation rather than capital accumulation. However, the effect on y_t^n in periods from 2 and onwards comes entirely from the increase in wealth, which in turn comes entirely from the increase in national income in period one caused by the temporary productivity shock. The effects in periods 2, 3, ... is therefore small (again), because the ratio between wealth and national income is very large at the outset, $v^*/y^{n*} = s/n$.

2.8 Since the propagation effects through capital or wealth accumulation found in the closed and the open economy are very small, they do not in themselves give a convincing explanation of the persistence (of income and other aggregate variables) observed in the real world. If the shocks affecting production and income are completely uncorrelated over time, these effects will not in themselves be able to create a considerable amount of correlation in income over time. Something has to be added to achieve this. Correlation in the shocks themselves is one possibility studied in the literature.