Written Exam for the B.Sc. / M.Sc. in Economics 2010-I

Corporate Finance and Incentives

Elective Course/ Master's Course

December 21, 2009

(4-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub questions that do not necessarily have equal weight.

Please provide intermediate calculations.

Problem 1 (Factor Models, 20%)

You are considering the three stocks: Gizmo Corp., ACME Bricks and Widget Ltd. as well as their systematic risk sensitivities towards the following risk factors: Unemployment, Consumer Confidence and Producer Price Inflation (PPI).

These risk factor betas (β) are shown below:

	Unemployment	Consumer Confidence	Producer Price Inflation (PPI)
Gizmo Corp.	1.20	0.80	1.10
ACME Bricks	0.80	1.10	1.20
Widget Ltd.	0.90	0.80	0.70

The three risk factors are measured by their deviation from trend outcomes and are assumed to be completely uncorrelated. That is, their expected outcomes are 0 and all covariances between any two different risk factors are also 0. Therefore we may apply a factor model to predict asset and portfolio returns. Assume further that there are no capital restrictions (i.e. we may take short positions in any stock or combination of stocks).

1. Find the absolute portfolio weights to create each of the three possible pure factor portfolios.

Hint: Think of the above table as a 3x3 asset beta matrix (β_{Assets}). Solve the equation system: $\beta_{Assets}X^T = I(3)$, with respect to the 3x3 weights matrix X. I(3) is the 3x3 dimensional identity matrix consisting of pure factor portfolio betas for each of the three possible pure factor portfolios.

2. What are the three risk factor betas for the pure factor portfolio that only has risk exposure to Consumer Confidence?

Assume that Consumer Confidence is forecast to become 2% (above trend) and that the pure factor portfolio for Consumer Confidence has a risk premium: $\alpha_{PFP\ Consumer\ Confidence} = 4\%$.

3. What is the expected return on the pure factor portfolio that only has risk exposure to Consumer Confidence given the observed forecast?

We observe the following risk factor trend deviation outcomes:

Table 1: Risk factor trend deviation outcomes

Unemployment	Cons. Confidence	Producer Price Inflation		
2%	4%	-3%		

Assume that a fourth stock, Compost Esq., can be described by the following risk factor model:

 $r_{Compost\ Esq.} = 8\% + 0.90 \times Unemployment + 1.20 \times Consumer\ Confidence + 1.00 \times PPI$

- 4. What return would the factor model predict for Compost Esq.?
- 5. Create the tracking portfolio that mimics the risk behaviour of Compost Esq. by utilising the three different pure factor portfolios found in 1). What are the asset weights for Gizmo Corp., ACME Bricks, Widget Ltd. and the risk-free rate?

Assume that the tracking portfolio has an expected return of 10%

6. Is there an arbitrage opportunity? If so, how would you create arbitrage profits and would the arbitrage returns be?

Problem 2 (Real Investments, 25%)

Assume a firm producing tyres at an annual production cost of 38 million. The risk free rate is 1.5%. The firm may choose between investing two mutually exclusive types of projects that may serve to temporarily reduce production costs.

Project	Lifetime	Initial investment outlay	Subsequent annual production costs
A	4	75	12
В	8	50	26

- 1. What is the net present value (NPV) to the firm of investing in project A and B respectively?
- 2. What two methods may be used by an investor with long term horizon to compare the attractiveness of two mutually exclusive types of investment projects? Describe each method briefly.
- 3. Assume the firm has an 8-year investment horizon and that project A can be replicated immediately at the end of the 4th year. How should a profit maximizing firm invest if it has no budget restrictions? Calculate the corresponding net present value (NPV).

Assume an investor considering the following list of investment projects:

Project	Cost	Profitability Index (PI)
С	960	4,17
D	360	3,34
Е	800	4,80
F	120	2,68
G	640	3,76

- 4. Calculate the present value (PV) and the net present value (NPV) of each project.
- 5. Assume you have a budget constraint of 1700. What investments would maximize the investors profit (NPV) given his budget constraint? What is the NPV?
- 6. Why would you not necessarily choose the investments with the top Profitability Index scores?

Problem 3 (Options, 25%)

A 2-year European call option on a share of Hijacking Inc. has an exercise price of DKK 110. The current price of a share of Hijacking Inc. is DKK 100, the annual compounded risk-free interest rate is 10% per year, and the annual volatility of the share price is 22,314%. Each period lasts one year.

- 1. Set up the binomial tree.
- 2. Estimate the price of the European call option.
- 3. Briefly comment on what the price of a comparable American call option would be.

After you have just estimated the price of the European call option, Hijacking Inc. announces that it with certainty pays a dividend in of DKK 20 in 1 year and 6 months.

- 4. Estimate the price of the European and American call option when Hijacking Inc. it pays a dividend and briefly interpret the result. (hint: strip the share price of dividends)
- 5. Briefly interpret the difference in the price of the American call option when Hijacking Inc. pays and does not pay a dividend.
- 6. Briefly comment on whether it is possible to estimate the price of a comparable American put using the put-call parity.

Problem 4 (Essay questions, 30%)

- 1. Define the concepts duration and convexity and explain what they can be used for.
- 2. Explain why comparable firms preferably should be used when trying to estimate the price of a firm based on multiples.
- 3. Discuss which tax issues influences the decision to distribute earnings to shareholders as either dividends or share repurchases.

Fixed Income

$$\pi = Cd$$

$$y(0,t) = \left(\frac{1}{d_t}\right)^{\frac{1}{t}} - 1 = r_t$$

yield to maturity, y solves: $\pi = \sum_{t=1}^{T} \frac{c_t}{(1+y)^t}$

 c_t i_t δ_t

	payment	interest	deduction of principal
Annuity	$F\alpha_{\tau \mid R}^{-1}$	$R_{\alpha_{\tau} \mid R}^F \alpha_{\tau-t+1} \mid R$	$\frac{F}{\alpha_{\tau \mid R}} (1 - R\alpha_{\tau - t + 1 \mid R})$
Bullet	$RF \text{ for } t < \tau$ $(1+R)F \text{ for } t = \tau$	RF	$ 0 \text{ for } t < \tau \\ F \text{ for } t = \tau $
Serial	$\frac{F}{\tau} + R\left(F - \frac{t-1}{\tau}F\right)$	$R\left(F - \frac{t-1}{\tau}F\right)$	$\frac{F}{\tau}$

$$D(c;r) = \sum_{t=1}^{T} t w_t$$

$$K(c;r) = \sum_{t=1}^{T} t^2 w_t$$

$$w_t = \frac{c_t}{(1+r)^t} \frac{1}{PV(c;r)}$$

Mean-Variance Optimization, CAPM, APT and Factor Models

$$E(\widetilde{r_i}) = \sum_{s=1}^{S} q_s \times \widetilde{r}_{i,s} = \bar{r}_i$$

$$Var(\widetilde{r_i}) = E[(\widetilde{r_i} - E(\widetilde{r_i}))^2] = \sum_{s=1}^{S} q_s \times (\widetilde{r}_{i,s} - \bar{r}_i)^2 = \sigma_i^2$$

$$Cov(\widetilde{r_1}, \widetilde{r_2}) = E[(\widetilde{r_1} - E(\widetilde{r_1}))(\widetilde{r_2} - E(\widetilde{r_2}))] = \sigma_{12}$$

$$\rho_{12} = \frac{Cov(\widetilde{r_1}, \widetilde{r_2})}{\sigma_1 \sigma_2}$$

$$Var(\widetilde{R_P}) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \ \sigma_{ij}$$

$$\beta_i = \frac{Cov(\widetilde{r_i}, \widetilde{R_T})}{Var(\widetilde{R_T})}$$

$$\widetilde{r}_i = \alpha_i + \beta_{i1}\widetilde{F}_1 + \beta_{i2}\widetilde{F}_2 + \dots + \beta_{iK}\widetilde{F}_K + \widetilde{\varepsilon}_i$$
, Factor Model

$$\widetilde{r}_i = r_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \dots + \beta_{iK}\lambda_K + \widetilde{\varepsilon}_i$$
, APT Model

Derivatives

$$F_0 = S_0 \big(1 + r_f \big)^T$$

$$f = (F_0 - K)(1 + r_f)^{-T}$$

$$c_0 - p_0 = S_0 - PV(K)$$

$$S_0 - K \le C_0 - P_0 \le S_0 - PV(K)$$

$$u = e^{\sigma\sqrt{(T/N)}}$$

$$d = \frac{1}{u}$$

$$\pi = \frac{1 + r_f - d}{u - d}$$

$$c_0 = S_0 N(d_1) - PV(K) N(d_2)$$

$$d_1 = \frac{\ln \left(\frac{S_0}{K}\right) + \left(r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

Real Investments:

$$E = P \times n$$

$$EV = D + E$$

$$NI = EBT(1 - T_C)$$

$$CE(\tilde{C}) = E(\tilde{C}) - b(\bar{R}_T - r_f)$$

Cumulative Normal Distribution											
d	N(d)	d	N(d)	d	N(d)	d	N(d)	d	N(d)	d	N(d)
-3,00	0,0013	-1,58	0,0571	-0,76	0,2236	0,06	0,5239	0,86	0,8051	1,66	0,9515
-2,95	0,0016	-1,56	0,0594	-0,74	0,2296	0,08	0,5319	0,88	0,8106	1,68	0,9535
-2,90	0,0019	-1,54	0,0618	-0,72	0,2358	0,10	0,5398	0,90	0,8159	1,70	0,9554
-2,85	0,0022	-1,52	0,0643	-0,70	0,2420	0,12	0,5478	0,92	0,8212	1,72	0,9573
-2,80	0,0026	-1,50	0,0668	-0,68	0,2483	0,14	0,5557	0,94	0,8264	1,74	0,9591
-2,75	0,0030	-1,48	0,0694	-0,66	0,2546	0,16	0,5636	0,96	0,8315	1,76	0,9608
-2,70	0,0035	-1,46	0,0721	-0,64	0,2611	0,18	0,5714	0,98	0,8365	1,78	0,9625
-2,65	0,0040	-1,44	0,0749	-0,62	0,2676	0,20	0,5793	1,00	0,8413	1,80	0,9641
-2,60	0,0047	-1,42	0,0778	-0,60	0,2743	0,22	0,5871	1,02	0,8461	1,82	0,9656
-2,55	0,0054	-1,40	0,0808	-0,58	0,2810	0,24	0,5948	1,04	0,8508	1,84	0,9671
-2,50	0,0062	-1,38	0,0838	-0,56	0,2877	0,26	0,6026	1,06	0,8554	1,86	0,9686
-2,45	0,0071	-1,36	0,0869	-0,54	0,2946	0,28	0,6103	1,08	0,8599	1,88	0,9699
-2,40	0,0082	-1,34	0,0901	-0,52	0,3015	0,30	0,6179	1,10	0,8643	1,90	0,9713
-2,35	0,0094	-1,32	0,0934	-0,50	0,3085	0,32	0,6255	1,12	0,8686	1,92	0,9726
-2,30	0,0107	-1,30	0,0968	-0,48	0,3156	0,34	0,6331	1,14	0,8729	1,94	0,9738
-2,25	0,0122	-1,28	0,1003	-0,46	0,3228	0,36	0,6406	1,16	0,8770	1,96	0,9750
-2,20	0,0139	-1,26	0,1038	-0,44	0,3300	0,38	0,6480	1,18	0,8810	1,98	0,9761
-2,15	0,0158	-1,24	0,1075	-0,42	0,3372	0,40	0,6554	1,20	0,8849	2,00	0,9772
-2,10	0,0179	-1,22	0,1112	-0,40	0,3446	0,42	0,6628	1,22	0,8888	2,05	0,9798
-2,05	0,0202	-1,20	0,1151	-0,38	0,3520	0,44	0,6700	1,24	0,8925	2,10	0,9821
-2,00	0,0228	-1,18	0,1190	-0,36	0,3594	0,46	0,6772	1,26	0,8962	2,15	0,9842
-1,98	0,0239	-1,16	0,1230	-0,34	0,3669	0,48	0,6844	1,28	0,8997	2,20	0,9861
-1,96	0,0250	-1,14	0,1271	-0,32	0,3745	0,50	0,6915	1,30	0,9032	2,25	0,9878
-1,94	0,0262	-1,12	0,1314	-0,30	0,3821	0,52	0,6985	1,32	0,9066	2,30	0,9893
-1,92	0,0274	-1,10	0,1357	-0,28	0,3897	0,54	0,7054	1,34	0,9099	2,35	0,9906
-1,90	0,0287	-1,08	0,1401	-0,26	0,3974	0,56	0,7123	1,36	0,9131	2,40	0,9918
-1,88	0,0301	-1,06	0,1446	-0,24	0,4052	0,58	0,7190	1,38	0,9162	2,45	0,9929
-1,86	0,0314	-1,04	0,1492	-0,22	0,4129	0,60	0,7257	1,40	0,9192	2,50	0,9938
-1,84	0,0329	-1,02	0,1539	-0,20	0,4207	0,62	0,7324	1,42	0,9222	2,55	0,9946
-1,82	0,0344	-1,00	0,1587	-0,18	0,4286	0,64	0,7389	1,44	0,9251	2,60	0,9953
-1,80	0,0359	-0,98	0,1635	-0,16	0,4364	0,66	0,7454	1,46	0,9279	2,65	0,9960
-1,78	0,0375	-0,96	0,1685	-0,14	0,4443	0,68	0,7517	1,48	0,9306	2,70	0,9965
-1,76	0,0392	-0,94	0,1736	-0,12	0,4522	0,70	0,7580	1,50	0,9332	2,75	0,9970
-1,74	0,0409	-0,92	0,1788	-0,10	0,4602	0,72	0,7642	1,52	0,9357	2,80	0,9974
-1,72	0,0427	-0,90	0,1841	-0,08	0,4681	0,74	0,7704	1,54	0,9382	2,85	0,9978
-1,70	0,0446	-0,88	0,1894	-0,06	0,4761	0,76	0,7764	1,56	0,9406	2,90	0,9981
-1,68	0,0465	-0,86	0,1949	-0,04	0,4840	0,78	0,7823	1,58	0,9429	2,95	0,9984
-1,66	0,0485	-0,84	0,2005	-0,02	0,4920	0,80	0,7881	1,60	0,9452	3,00	0,9987
-1,64	0,0505	-0,82	0,2061	0,00	0,5000	0,82	0,7939	1,62	0,9474	3,05	0,9989
-1,62	0,0526	-0,80	0,2119	0,02	0,5080	0,84	0,7995	1,64	0,9495		
-1,60	0,0548	-0,78	0,2177	0,04	0,5160						