## Written Exam at the Department of Economics summer 2019

## Macroeconomics III Final Exam

August 9, 2019

(3-hour closed book exam)

Answers only in English.

## This exam question consists of 4 pages in total

Falling ill during the exam If you fall ill during an examination at Peter Bangs Vej, you must:

- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams! You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

1 (20 points) Answer true, false, or uncertain. Justify your answer.

OECD countries faces a secular process of population aging. This will force those countries that have a pay-as-you go social security system to reduce benefits, and thus current workers should be given incentives to save more for their retirement privately.

2 (20 points) Answer true, false, or uncertain. Justify your answer.

In an open economy, a reduction in labor taxes financed with debt will have no effect on the current account. This is an example of Ricardian equivalence.

**3** (20 points) Answer true, false, or uncertain. Justify your answer.

If the government announces that next year a distortionary capital income tax will be replaced by a lump sum tax (keeping government spending constant), consumers will respond immediately by increasing consumption.

4 (60 points) Consider a Ramsey economy with a continuum of households and firms operating under perfect competition. There is no population growth, and the representative household is infinitely lived, has a unitary endowment of time each period, and maximizes the following objective function under perfect foresight:

$$\max_{c_t, x_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[ \log c_t + \frac{x_t^{1-\epsilon}}{1-\epsilon} \right]$$

subject to the budget constraint:

$$c_t + k_{t+1} = w_t(1 - x_t) + R_t k_t$$

where  $c_t$  is household consumption,  $x_t$  is the amount of leisure consumed (such that  $1 - x_t$  is labor supply),  $w_t$  is the wage rate,  $k_{t+1}$  is saving assumed to be in capital, and  $R_{t+1} = 1 + r_{t+1}$  is the gross return on that saving (we assume no depreciation of capital).  $0 < \beta < 1$  is the time discount factor, and  $0 < \epsilon < 1$  measures the concavity of leisure in preferences, and is thus related to the elasticity of labor supply.

Production technology is Cobb-Douglas such that the representative firm i takes factor prices as given and maximizes

$$\Pi_t^i = K_t^{i\alpha} L_t^{i1-\alpha} - r_t K_t^i - w_t L_t^i$$

where  $K_t^i$  is the demand for capital and  $L_t^i$  the demand for labor, and  $0 < \alpha < 1$ .

a) Write the Lagrangian for households' problem and derive its first order conditions with respect to  $c_t$ ,  $x_t$ , and  $k_{t+1}$ . Derive the Euler equation and interpret it. Characterize the steady state for this economy.

- b) Assume that the economy is in steady state and there is an unexpected permanent increase in parameter  $\epsilon$ . Explain how this affects the steady state levels of capital, consumption and leisure.
- c) Assume that the economy is in steady state, and the government introduces a capital income tax whose proceeds are rebated as a lump sum to households. How does this affects the steady state levels of capital, consumption and leisure? Explain.

**5** (60 points) Consider an economy where individuals live for two periods, and population is initially constant. Identical competitive firms maximize the following profit function:

$$\pi^F(K_t^i, L_t^i) = AK_t^{i\alpha}L_t^{i1-\alpha} - w_tL_t^i - r_tK_t^i,$$

where  $r_t$  is the interest rate at which firms can borrow capital,  $w_t$  is the wage rate,  $K_t^i$  and  $L_t^i$  denote the quantities of capital and labor employed by the firm, and A > 0 is total productivity. Assume  $0 < \alpha < 1$ . Capital fully depreciates  $(\delta = 1)$ . Utility for young individuals born in period t is

$$U_t = \ln(c_{1t}) + \beta \ln(c_{2t+1}), \quad \beta < 1$$

where  $c_{1t}$  is consumption when young, and  $c_{2t+1}$  consumption when old. Young agents work a unit of time (i.e. their labor income is equal to the wage they receive). Old agents do not work and provide consumption through saving and social security benefits. The old get gross return  $r_{t+1}$  for their savings.

Suppose that the government runs a balanced pay-as-you-go social security system in which the young contribute a fraction  $0 < \tau < 1$  of their wages that is received by the old  $(\tau w_t)$  are then the benefits received by the old in period t).

a) Characterize individual saving behavior by solving the individual's problem of optimal intertemporal allocation of resources. Find the capital accumulation equation that gives  $k_{t+1}$  as a function of  $k_t$ . Find the level of capital in steady state.

Assume that the economy is initially in the steady state. Now unexpectedly there is a permanent flow of immigrants at rate n per period (i.e. the economy moves to a regime of constant population growth at rate n driven by immigration). Immigrants are young, have same preferences as residents, and are assumed to get employment. They stay in the country when they get old and thus receive social security benefits. Both immigrants and residents receive the same wage, make the same contributions, and receive same benefits from social security.

The government decides to reduce the size of social security such that the initial old generation receives the same benefits that they would have received in the absence of immigration. Denote by  $\tau'$  the new contribution rate. Assume that parameters are such that the economy is always dynamically efficient.

Note that to solve what follows you have to consider the general equilibrium effects that the flow of immigrants has on wages and interest rate.

- b) Characterize  $\tau'$  as a function of  $\tau$ , n, and  $\alpha$ . How does this shock affect the economy? What are the effects of the shock on consumption and capital accumulation in the first period (compared to consumption and capital accumulation in the previous steady state)? And on the new steady state? Explain.
- c) Show that the initial old are strictly better off with immigration, even though they receive the same benefits. Show that for some parameters, the disposable income of the first young generation is lower, despite the reduction in contributions. Explain.