Macro C - exam solutions (Feb 16, 2015)

General remarks

Please grade each item of each question between 0 and 20 points. Thus the maximum possible grade of the exam is 180.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that capital holdings are negative).

1. False. In these models it is hyperinflations that can be ruled out by having money to be essential. A sufficient condition to rule out hyperdeflations is to print money at a constant positive rate.

2. True. The existence of the escape clause implies that inflation expectations exceed the expected inflation in the country whose currency we are pegged to (in the model, target inflation is always zero). Thus in normal times, when the escape clause is not invoked, since actual inflation is the same as in the target country, our country experiences unexpected deflation. And unexpected deflation translates mechanically through the Phillips curve into unexpected output loses.

3 a) This requires solving first for the firms maximization of profits problem (taking factor prices as given) and then imposing equilibrium to get expressions for r and w as function of aggregate factor use. (to simplify I don't have time indexes)

$$\max_{K^i,N^i} (K^i)^\alpha (KN^i)^{1-\alpha} - rK^i - wN^i$$

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$$\begin{aligned}
\alpha(K^i)^{\alpha-1}(KN^i)^{1-\alpha} - r &= 0, \\
(1-\alpha)(K^i)^\alpha K^{1-\alpha}(N^i)^{-\alpha} - w &= 0.
\end{aligned}$$

Using the fact that $N = 1$ these can be rewritten as

$$\begin{aligned}
\alpha(\frac{K^i}{N^i})^{\alpha-1}(\frac{K}{N})^{1-\alpha} &= r, \\
(1-\alpha)(\frac{K^i}{N^i})^\alpha(\frac{K}{N})^{1-\alpha} &= w.
\end{aligned}$$

Now we impose equilibrium such that every firm uses factors with the same intensity as the average in the economy, i.e. $\frac{K^i}{N^i} = \frac{K}{N} = K$. This gives desired result:

$$\begin{aligned} r &= \alpha, \\ w &= (1-\alpha)K. \end{aligned}$$

Characterizing individual saving behavior requires setting up the problem of workers.

$$\begin{aligned} \max_{c_{1t}, c_{2t+1}} \quad & \ln(c_{1t}) + \frac{1}{1+\rho}\ln(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t} = w_t(1-\tau) - s_t \\ & c_{2t+1} = s_t(1+r_{t+1}) + \tau w_{t+1} \end{aligned}$$

Solving this problem and finding the Euler equation, from which

$$c_{2t+1} = \frac{1+r_{t+1}}{1+\rho} c_{1t}$$

Replacing from period constraints we get individual savings

$$s_t = \frac{1}{2+\rho} w_t(1-\tau) - \left(\frac{1+\rho}{2+\rho}\right) \frac{\tau w_{t+1}}{(1+r_{t+1})}$$

b) To get capital accumulation we replace individual savings with next period capital K_{t+1} (note there is no $\frac{1}{1+n}$ term since there is no population growth, and because $N = 1$ we have that $k = K$), and we use equilibrium expressions for wage and interest rate from a)

$$K_{t+1} = \frac{1}{2+\rho}(1-\alpha)K_t(1-\tau) - \left(\frac{1+\rho}{2+\rho}\right) \frac{1-\alpha}{1+\alpha} \tau K_{t+1}$$

Combining terms with K_{t+1}

$$K_{t+1} = \frac{1}{\left[1 + \frac{1+\rho}{2+\rho}\frac{1-\alpha}{1+\alpha}\tau\right]} \frac{1}{2+\rho}(1-\alpha)K_t(1-\tau)$$

IMPORTANT NOTE: From the above equation it is clear that there is no steady state with positive capital level in this model. The reason for this is a mistake in having made social security proportional to wages and not a transfer whose size was independent of output (or capital). Thus students would be unable to solve the question on the steady state K^* and thus point b) should only be graded on the correct derivation of the above capital dynamics relation and the dynamic efficiency as described below. Also for remaining points c) and d) as the student is assumed to start on a steady state, the answers should be based on the correct derivation of the relative effect of the policy change in question.

The economy would be dynamically inefficient if $1+r < 1+n$, since the rate of return of social security is $1+n = 1$, and $r = \alpha > 0$ the economy is always dynamically efficient (note that for this question it is not an issue that there is no steady state).

c, d) We now have to consider the effects of social security privatization in the problem of households. First we look at generic γ

$$\begin{aligned} \max_{c_{1t}, c_{2t+1}} \quad & \ln(c_{1t}) + \frac{1}{1+\rho}\ln(c_{2t+1}) \\ \text{s.t.} \quad & c_{1t_0} = w_{t_0}(1-\gamma\tau) - s_{t_0} \\ & c_{2t_0+1} = s_{t_0}(1+r_{t_0+1}) \end{aligned}$$

Solving this problem and finding the Euler equation, from which (same as before)

$$c_{2t+1} = \frac{1+r_{t+1}}{1+\rho}c_{1t}$$

Replacing from period constraints we get individual savings

$$s_{t_0} = \frac{1}{2+\rho}w_{t_0}(1-\gamma\tau)$$

To get capital accumulation we subtract from individual savings the debt that the government issues and is bought by the young

$$K_{t_0+1} = \frac{1}{2+\rho}w_{t_0}(1-\gamma\tau) - (1-\gamma)\tau w_{t_0}$$

Now all that it rests to do is to consider $\gamma = 0$ and $\gamma = 1$. And for comparison, saving a capital accumulation without a policy change would have been (under the assumption that t_0 was a steady state, i.e. $w_{t_0+1} = w_{t_0}$), with superscript ss for steady state or status quo:

$$\begin{aligned} s_{t_0}^{ss} &= \frac{1}{2+\rho}w_{t_0}(1-\tau) - \left(\frac{1+\rho}{2+\rho}\right)\frac{\tau w_{t_0}}{(1+\alpha)} \\ K_{t_0+1}^{ss} &= s_{t_0}^{ss} = \frac{1}{2+\rho}w_{t_0}(1-\tau) - \left(\frac{1+\rho}{2+\rho}\right)\frac{\tau w_{t_0}}{1+\alpha} \end{aligned}$$

Case $\gamma = 0$

$$\begin{aligned} s_{t_0} &= \frac{1}{2+\rho} w_{t_0} \\ K_{t_0+1} &= \frac{1}{2+\rho} w_{t_0} - \tau w_{t_0} \end{aligned}$$

We see that consumption for the young increases, as does savings (trivially from equations).

$$c_{1t_0} - c_{1t_0}^{ss} = \tau w_{t_0} \frac{1+\rho}{2+\rho}\left(1 - \frac{1}{1+\alpha}\right)$$

Capital accumulation is decreased (the debt burden overweights the increase in savings)

$$K_{t_0} - K_{t_0}^{ss} = -\tau w_{t_0} \frac{1+\rho}{2+\rho}\left(1 - \frac{1}{1+\alpha}\right)$$

Consumption for the old is unchanged as they are paid their benefits. Since the interest rate is unchanged, it must be the case that consumption when old of the current young also goes up (from Euler equation), thus they are better off with this policy, the old are indifferent.

Case $\gamma = 1$

$$\begin{aligned} s_{t_0} &= \frac{1}{2+\rho} w_{t_0}(1-\tau) \\ K_{t_0+1} &= \frac{1}{2+\rho} w_{t_0}(1-\tau) \end{aligned}$$

Consumption for the old is unchanged as they are paid their benefits, and savings and capital accumulation go up in this case (trivially from equations). And it is also straightforward to see that the young's consumption is reduced (by the present value of the benefits they will no longer receive). Again, since the interest rate is unchanged, it must be the case that consumption when old of the current young also goes down (from Euler equation), thus they are worse off with this policy, the old are indifferent.

4 a) The prices are set according to the following equations (1) and (2)

$$\begin{aligned} p^f &= E[p_i^*|m] = (1-\phi)p + \phi m && (1) \\ p^r &= E[p_i^*] = (1-\phi)E[p] + \phi E[m] && (2) \end{aligned}$$

Since when some rigid price firms are allowed to set prices they have the same information as flexible price firms it is immediate that $p^u = p^f$. The average price is given, after the realization of m, by

$$\begin{aligned} p &= qp^r + (1-q)p^f & (3) \\ p &= q(1-\alpha)p^r + (1-q(1-\alpha))p^f & (4) \end{aligned}$$

where (3) corresponds to shocks that make adjusting prices not beneficial (this happens with probability $1-\theta$, we denote these states as L), and (4) corresponds to shocks that make adjusting prices beneficial if being able to do so (this happens with probability θ, we denote these states as H)

For notational simplicity we call $\hat{q}$ the ex post fraction of firms with rigid prices (i.e. $\hat{q} = q$ or $= q(1-\alpha)$), and $E[q] = (1-\theta)q + \theta q(1-\alpha)$ (i.e. the expected fraction of rigid price firms in the economy)

Substituting (3) or (4) into (1)

$$p^f = (1-\phi)E[p|m] + \phi m = (1-\phi)\big(\hat{q}p^r + (1-\hat{q})p^f\big) + \phi m \quad \Leftrightarrow$$

$$p^f\big(1-(1-\phi)(1-\hat{q})\big) = (1-\phi)\hat{q}p^r + \phi m \quad \Leftrightarrow$$

$$p^f = \frac{(1-\phi)\hat{q}p^r + \phi m}{1-(1-\phi)(1-\hat{q})} = \frac{(1-\phi)\hat{q}p^r + \phi m}{\phi + (1-\phi)\hat{q}} = \frac{\big[(1-\phi)\hat{q} + \phi\big]p^r - \phi p^r + \phi m}{\phi + (1-\phi)\hat{q}}$$

$$p^f = p^r + (m - p^r)\frac{\phi}{\phi + (1-\phi)\hat{q}} \qquad (5)$$

Substituting (3) and (4) into (2) and using above expression (5) for p^f

$$p^r = (1-\phi)E[p] + \phi E[m] = (1-\phi)E[\hat{q}p^r + (1-\hat{q})p^f] + \phi E[m]$$

Thus, and using $E[m|L] = E[m|H] = E[m]$,

$$\begin{aligned} p^r &= (1-\phi)p^r E[q] + (1-\phi)\left((1-\theta)(1-q)\left[p^r + (E[m|L] - p^r)\frac{\phi}{\phi + (1-\phi)q}\right]\right. \\ &\left. + \theta(1-q(1-\alpha))\left[p^r + (E[m|H] - p^r)\frac{\phi}{\phi + (1-\phi)q(1-\alpha)}\right]\right) + \phi E[m] \\ &= (1-\phi)p^r E[q] + (1-\phi)\left((1-\theta)(1-q)\left[p^r + (E[m] - p^r)\frac{\phi}{\phi + (1-\phi)q}\right]\right. \\ &\left. + \theta(1-q(1-\alpha))\left[p^r + (E[m] - p^r)\frac{\phi}{\phi + (1-\phi)q(1-\alpha)}\right]\right) + \phi E[m] \\ &= (1-\phi)p^r + (1-\phi)(E[m] - p^r)\left(\frac{(1-\theta)(1-q)\phi}{\phi + (1-\phi)q} + \frac{\theta(1-q(1-\alpha))\phi}{\phi + (1-\phi)q(1-\alpha)}\right) + \phi E[m] \quad \Leftrightarrow \\ & - \phi(E[m] - p^r) = (1-\phi)(E[m] - p^r)\left(\frac{(1-\theta)(1-q)\phi}{\phi + (1-\phi)q} + \frac{\theta(1-q(1-\alpha))\phi}{\phi + (1-\phi)q(1-\alpha)}\right) \end{aligned}$$

This can only be satisfied if

$$p^r = E[m]. \tag{6}$$

Note that an alternative, more straightforward solution strategy starts by assuming $p^r = E[m]$, replacing this in (5), and then verifying the assumption (which in this case boils down to proving that $E[p^f] = E[m]$.

b) Substituting (5) and (6) into (3) and (4)

$$p = \hat{q}p^r + (1-\hat{q})p^f = \hat{q}p^r + (1-\hat{q})p^r + (m-p^r)\frac{\phi(1-\hat{q})}{\phi+(1-\phi)\hat{q}} = p^r + (m-p^r)\frac{\phi(1-\hat{q})}{\phi+(1-\phi)\hat{q}}$$
$$= E[m] + (m - E[m])\frac{\phi(1-\hat{q})}{\phi+(1-\phi)\hat{q}} \tag{7}$$

Since $y = m - p$, substituting (7) into this relation

$$y = m - E[m] - (m-E[m])\frac{\phi(1-\hat{q})}{\phi+(1-\phi)\hat{q}} = (m-E[m])\left(1 - \frac{\phi(1-\hat{q})}{\phi+(1-\phi)\hat{q}}\right)$$
$$= (m-E[m])\left(\frac{\phi+(1-\phi)\hat{q} - \phi(1-\hat{q})}{\phi+(1-\phi)\hat{q}}\right) = (m-E[m])\left(\frac{\hat{q}}{\phi+(1-\phi)\hat{q}}\right) \tag{8}$$

From this equation we see that only unanticipated changes in m have an effect on output. The reason for this is that output is given by the difference between m and the price level, and the latter fully reflects anticipated changes in m (changes in $E[m]$). This is expected in a static model where money is neutral.

c) This requires looking at (8), in particular

$$\frac{dy}{dm} = \frac{\hat{q}}{\phi+(1-\phi)\hat{q}}$$

This is higher the higher $\hat{q}$ which happens when no rigid price firms finds optimal to change prices ($\hat{q} = q$). To see the effect of an increase in α note that this will only impact $\frac{dy}{dm}$ when rigid firms find optimal to change prices. In that case $\hat{q} = q(1-\alpha)$, thus an increase in α makes output less responsive to unanticipated changes in m.

Since the response of output to demand shocks depends on the degree of nominal rigidities, then a higher flexibility (higher α), or situations when more prices change (when shocks are large making it optimal for those rigid price firms that have the option to do so), lead to lower response of output to unanticipated shocks.