Suggestive solution for
Written Exam for the B.Sc. in Economics 2011-I
Macroeconomics C

Competence description: At the end of the course, the student should be able to demonstrate:

- Understanding of the main model frameworks for long-run macroeconomics. This includes the Diamond model with overlapping generations in discrete time and the Ramsey model in continuous time.
- Proficiency in the application of the concepts and methods from these frameworks, including competence in dynamic optimization and dynamic analysis in discrete and continuous time.
- Understanding of the role of expectations and basic knowledge of macroeconomic models with forwardlooking expectations under both perfect foresight and uncertainty and rational expectations.
- Proficiency in the application of the related concepts and methods.
- Competence in analyzing a macroeconomic problem, where the above-mentioned concepts and methodsare central, that is competence in solving such models and explaining in economic terms the results andimplications and how they derive from the assumptions of the model.

The particularly good performance, corresponding to the top mark, is characterized by a complete fulfilment of these learning objectives.

Problem A

1. **False.** The Ramsey model may be described by the following two differential equations

$$\dot{\hat{c}}_t = \hat{c}_t \frac{f'\left(\hat{k}_t\right) - \delta - \rho - \theta x}{\theta} \tag{1}$$

$$\dot{\hat{k}}_t = f\left(\hat{k}_t\right) - \hat{c}_t - (x + n + \delta)\,\hat{k}_t \tag{2}$$

where $\hat{c}_t$ and $\hat{k}_t$ denote consumption and physical capital, respectively, per unit of effective labour. From (1) and (2) one can construct the phase diagram, which can be shown to be saddle point stable, and further analysis shows that the economy will converge towards the steady state where (from eq. (1) with $\dot{\hat{c}}_t = 0$)

$$f'\left(\hat{k}^*\right) = \delta + \rho + \theta x \tag{3}$$

which defines $\hat{k}^*$, and $\hat{c}_t$ is then equal to (from eq. (2) with $\dot{\hat{k}}_t = 0$)

$$\hat{c}^* = f\left(\hat{k}^*\right) - (x + n + \delta)\,\hat{k}^* \tag{4}$$

We can thus conclude that the statement is false, since it follows from (3) that the steady state level of $\hat{k}^*$ will not be affected by the population growth rate, n, but according to eq. (4) a higher value of n will, given the value of $\hat{k}^*$, result in a lower value of $\hat{c}^*$. The economic intuition is that since employment equals population, a higher population growth rate will also cause employment to grow at a higher rate, and in a steady state where $\hat{k}^*$ is unaffected, investment has to be increased in order to keep $\hat{k}^*$ - capital *per unit* of effective *labour* - constant and this necessarily depresses consumption.

However, it is correct that aggregate variables will end up growing at a higher rate in steady state, since aggregate variables (e.g. consumption) is given by $C_t = A_t L_t \hat{c}_t$, where A_t is the labour efficiency index and L_t is employment. Now with $\hat{c}_t$ constant in steady state, C_t will be growing at rate $x + n$, where x is the growth rate of A_t and n is the growth rate of L_t, and thus a higher value of n will increase the growth rate of C_t.

It should be noted that these conclusions do not hinge upon the fact that labour efficieny grows or that physical capital depreciates, so an exposition which has $x = \delta = 0$ and which reaches the same conclusion should of course be accepted.

2. **True.** Assuming that the monopoly does not reset its price, the price chosen will be higher than the one that maximizes profits after the contraction in demand. This price will result in lower demand and thus consumer surplus will have decreased (compared to the situation where the monopoly resets its price). In addition, monopoly profits will have decreased, since the price is not profit maximizing. As a consequence, social surplus will have decreased. Resetting the price is socially optimal if doing so causes an increase in the social surplus which is greater than the cost of changing the cost (the 'menu cost'). However, since it is the decision of the monopoly whether to reset the price, the monopoly will do so only if that the gain in producer surplus is greater than the menu cost. But if this is the case, it will also be socially optimal to reset the price, since the increase in producer surplus is only a fraction of the increase in social surplus from resetting the price. It could be added that there may, however, be situations in which resetting the price is socially optimal but nontheless the firm does not do so because it is not privately optimal for the firm.

3. **False.** Ricardian equivalence (between taxes and debt) holds that the decision by government of how at any given point in time to finance public consumption by either taxes or debt does not affect the equilibrium (path) of the economy. The intuition for why Ricardian should hold is basically that consumers realize that since government has to obey its intertemporal budget constraint, taxes that are not levied today will have to be levied in the future at an equivalent present value, and given that the utility maximization on the part of the consumer is conducted subjext to the consumer's intertemporal

budget constraint, the consumer will not care when in time the taxes are actually levied. Now, it is correct that consumers in the OLG model maximize utility subject to an intertemporal budget constraint, however, since consumers in the OLG model have *finite time horizons* they will not think of present and future taxes as equivalent if the future taxes are levied at a point in time when said consumers are no longer alive. Thus, Ricardian equivalence will not hold in the OLG model.

Problem B

1. From eq. (B.6) we get

$$\dot{p} = 0 \Rightarrow \gamma\eta Q - \gamma\beta p + \gamma\left(z - \bar{y}\right) = 0 \Leftrightarrow$$

$$\dot{p} = 0 : Q = \frac{\beta}{\eta}p - \frac{1}{\eta}\left(z - \bar{y}\right) \tag{1}$$

where thus eq. (1) is the equation for the $\dot{p} = 0$ locus (Goods Market, GM curve). From eq. (B.7) we get

$$\dot{Q} = 0 \Rightarrow \left(r^f - (1-\tau)\,\alpha\eta\right)Q + (1-\tau)\,\alpha\beta p - (1-\tau)\,\alpha z = 0 \Leftrightarrow$$

$$\dot{Q} = 0 : Q = -\frac{(1-\tau)\,\alpha\beta}{r^f - (1-\tau)\,\alpha\eta}p + \frac{(1-\tau)\,\alpha}{r^f - (1-\tau)\,\alpha\eta}z \tag{2}$$

(Provided that $r^f \neq (1-\tau)\,\alpha\eta$, which holds since $r^f < (1-\tau)\,\alpha\eta$.) Eq. (2) is the equation for the $\dot{Q} = 0$ locus (the Asset Market, AM curve).

The phase diagram is constructed by first drawing the loci where $\dot{p} = 0$ and $\dot{Q} = 0$, i.e. the loci given by eqs. (1) and (2).

Eq. (1) shows the $\dot{p} = 0$ locus to be an upward-sloping line with a slope of $\frac{\beta}{\eta} > 0$ (in a (p, Q)-diagram) and an intercept with the vertical axis of $-\frac{1}{\eta}\left(z - \bar{y}\right)$. Eq. (2) shows the $\dot{Q} = 0$ to be an upward-sloping line with a slope of $-\frac{(1-\tau)\alpha\beta}{r^f-(1-\tau)\alpha\eta} > 0$ (when $r^f < (1-\tau)\,\alpha\eta$), and an intercept with the vertical axis equal to $\frac{(1-\tau)\alpha}{r^f-(1-\tau)\alpha\eta}z < 0$. With respect to the slope of the $\dot{Q} = 0$ locus we find that

$$\begin{aligned} -\frac{(1-\tau)\,\alpha\beta}{r^f - (1-\tau)\,\alpha\eta} &= \frac{(1-\tau)\,\alpha\beta}{(1-\tau)\,\alpha\eta - r^f} = \frac{(1-\tau)\,\alpha\beta}{(1-\tau)\,\alpha\eta - r^f}\frac{\eta}{\eta} \\ &= \frac{(1-\tau)\,\alpha\eta}{(1-\tau)\,\alpha\eta - r^f}\frac{\beta}{\eta} = \frac{1}{1 - \frac{r^f}{(1-\tau)\alpha\eta}}\frac{\beta}{\eta} > \frac{\beta}{\eta} \end{aligned} \tag{3}$$

Given that the two loci are stated to intersect in the positive orthant, they must look as in figure B.1.

The directions of motions indicated by the arrows in figure 1 are found from eqs. (B.6) and (B.7) in the following way: Beginning at any point on the $\dot{p} = 0$ locus moving either vertically up (increasing Q with p unchanged) or horizontally to the left (decreasing p with Q unchanged), it may be concluded from eq. (B.6) that at the new point it is the

case that $\dot{p} > 0$, i.e. that p will be increasing over time. This is indicated in figure 1 by the horizontal and rightward-pointing arrows to the left of (above) the $\dot{p} = 0$ locus. In a similar manner it may be concluded that $\dot{p} < 0$ to the right of (below) the $\dot{p} = 0$ locus explaining the horizontal leftward-pointing arrows there.
Beginning at any point on the $\dot{Q} = 0$ locus moving either vertically down or horizontally to the right, it follows from eq. (B.7) that at these points it is the case that $\dot{Q} > 0$ (moving vertically down, i.e. decreasing Q with p unchanged, one should use the assumption that $r^f < \alpha\eta$). Consequently Q is increasing over time below (to the right of) the $\dot{Q} = 0$ locus which explains the upward-pointing vertical arrows there. In a similar manner one can explain the downward-pointing vertical arrows to the left of (below) the $\dot{Q} = 0$ locus indicating that at these points Q will be decreasing over time.

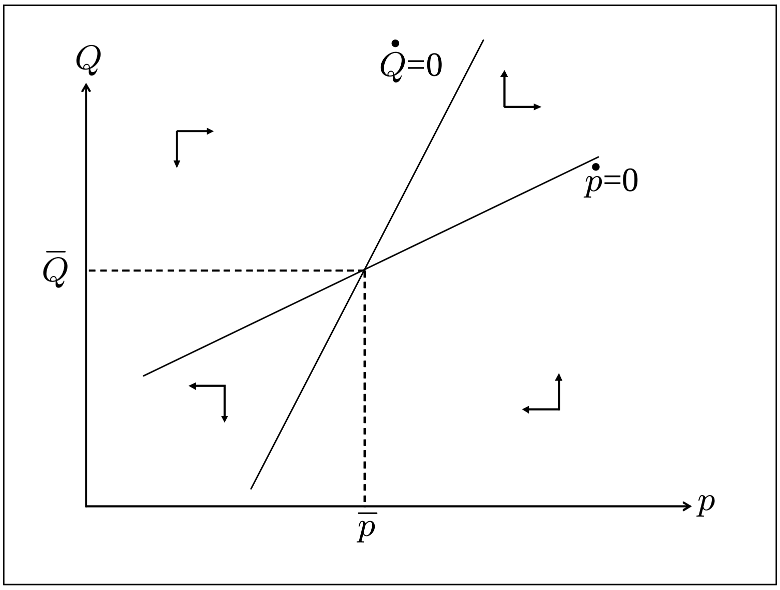

Figure B.1

The directions of motion shown in figure B.1 reveal that the economy is saddle point stable meaning that there is a unique path, the negatively sloped saddle path SS in figure B.2, approaching (from either side) the long run equlibrium at E, where p and Q are constant, while all other paths are diverging from the long run equilibrium at E. The diverging paths may be thought of as bubbles in the stock price where the stock price is eventually ever increasing or decresing due to self-fullfilling expectations. However, since there is rational expectations and no uncertainty which together imply perfect foresight, such evolutions may be ruled out on the argument that rational agents will not believe the stock price to be forever increasing or decreasing.[1] Since at any point in time p is

[1] It could be added that an evolution where the economy follows a diverging path for a while and then jumps to the saddle path is also not consistent with rational behaviour, since with perfect foresight, the time of the jump would be known to all agents. If therefore, e.g., the stock price were to increase discretely (jump) all agents would want to purchase shares the instant before the jump. This, however, would drive up the price the instant before, making agents want to purchase shares even earlier making the price increase even earlier and so forth. By continuing to drive this argument backwards in time, it may be concluded that such an evolution could never get started.

predetermined, while Q is free to jump/adjust because, as explained later and sa seen from eq. (B.8), it is based on the future, we can conclude that for any initial value of p, Q will be chosen such that the economy is on the saddle path and evolves along this to the long run equilibrium at point E.

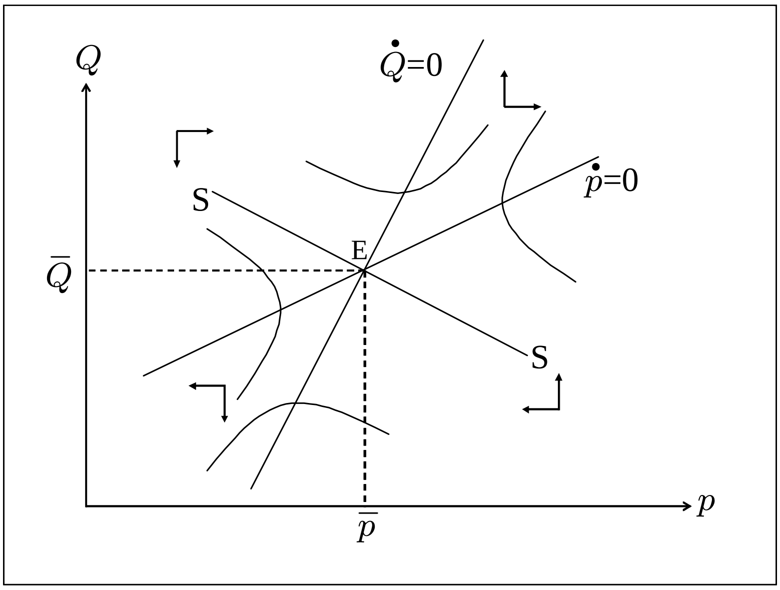

Figure B.2

2. The phase diagram is shown in figure B.3. It is seen from eqs. (1) that the $\dot{p} = 0$ locus is unaffected by the change in τ. The $\dot{Q} = 0$ locus however, becomes flatter, since the slope (see eq. (3)) is $-\frac{(1-\tau)\alpha\beta}{r^f-(1-\tau)\alpha\eta} = \frac{1}{1-\frac{r^f}{(1-\tau)\alpha\eta}}\frac{\beta}{\eta}$, which is seen to be decreasing in τ. At the same time, the intercept between the $\dot{Q} = 0$ locus and the horizontal axis is unafffected, (and equal to $\frac{1}{\beta}z$) since from eq. (2):

$$\dot{Q} = 0 : Q = -\frac{(1-\tau)\,\alpha\beta}{r^f - (1-\tau)\,\alpha\eta}p + \frac{(1-\tau)\,\alpha}{r^f - (1-\tau)\,\alpha\eta}z \Rightarrow$$

$$p = -\frac{r^f - (1-\tau)\,\alpha\eta}{(1-\tau)\,\alpha\beta}Q + \frac{1}{\beta}z$$

As a consequence, the long run equilibrium changes from E_1 to E_2 where both Q and p have increased. (All solutions which conclude that *only* the $\dot{Q} = 0$ locus shifts, and that both Q and p will increase in the new long run equilibrium, should be accepted.)
Thus, the long run equlibrium changes from E_1 to E_2 as shown in figure B.3. (Figure B.3 only shows the directions of motion associated with the *new* loci and the original loci are dotted.)
Before continuing we note that eq. (B.8) in the text states that the equlibrium stock price is the sum (integral) of all present and future discounted after-tax dividends between time t and the future time T plus the discounted value of the stock price at the future time T (which is known since the model assumes perfect foresight). This is easily interpretable:

When buying shares one basically buys the right to a future stream of dividend income and the payment that one is willing to make (the stock price) is the total discounted value of this stream of dividends plus whatever the share is worth in the future and can thus be sold for.

In question 1 it was argued that the economy will always be moving along a path that eventually reaches the saddle path and thus ends up in long run equilibrium where $Q = \bar{Q}$, i.e. is constant. We thus have

$$\lim_{T\to\infty} e^{r(T-t)} Q\left(T\right) = \lim_{T\to\infty} e^{r(T-t)} \bar{Q} = \bar{Q} \lim_{T\to\infty} e^{r(T-t)} = 0 \tag{4}$$

and using this when letting $T = \infty$ in eq. (B.8) we find

$$Q\left(t\right) = \int_t^\infty \left(1-\tau\right) D\left(s\right) e^{-r(s-t)} ds \tag{5}$$

which is just saying that the stock price is the sum of all present and future discounted after-tax dividends. This is also known as the fundamental value of the shares.

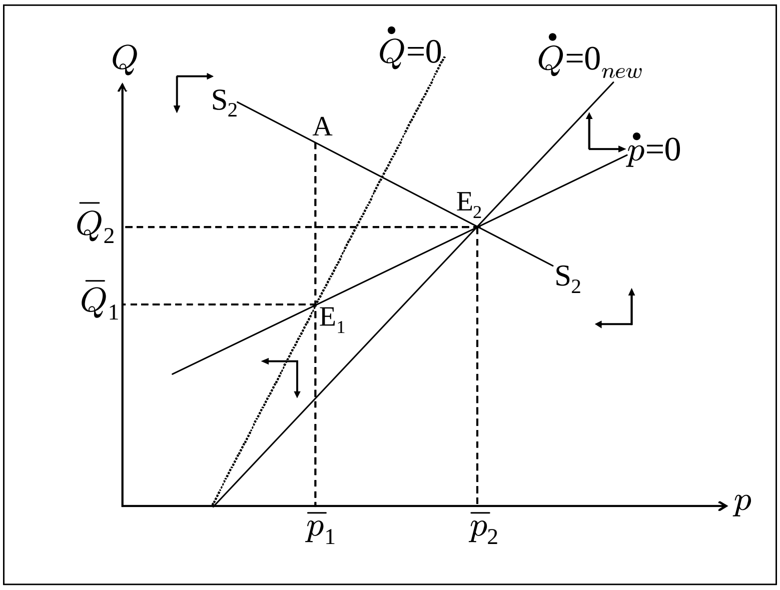

Figure B.3

We now make the following considerations:

a) Before time t_0 the economy is at point E_1 and thus at time t_0 p is predetermined at $\bar{p}_1$, while Q is free to jump, since, as just argued, it depends on the future.

b) Sooner or later the economy must reach the new saddle path, S_2S_2, since otherwise it will forever follow one of the diverging paths which we have ruled out.

c) If Q is to jump at any point in time it must be at time t_0, since otherwise the jump would be expected/known meaning that agents would sit around waiting for capital gains or losses and this is not compatible with rational behaviour.

Applying these considerations to figure B.3 it is seen that is must be the case that exactly at time t_0 when τ is decreased, the economy jumps from E_1 to A and then moves continously along the new saddle path to the new long run equlibrium at E_2 where both Q and p are higher than in the original long-run equilibrium.
The economic intuition is the following: When it becomes known that the dividend tax rate has been decreased, the present discounted value of future after-tax dividends will immediately increase and this increases the stock price, since it equals the present discounted value of present and future after-tax dividends dividends. If before-tax dividends were *not* affected, this would immediatly increase Q from $\bar{Q}_1$ to $\bar{Q}_2$. However, according to the goods market eqilibrium in eq. (B.2) an increase in Q will increase demand for goods and will thus increasereal output. This in turn increases dividends according to eq. (B.5) and will cause a further increase in Q. This explains why initially the stock price overshoots its long run level when the economy moves to point A.
At point A it is now the case that $y > \bar{y}$ (since at E_1 we have $\dot{p} = 0 \Rightarrow y = \bar{y}$ and y has increased from E_1 to A because of the increase in Q). According to the SRAS curve in eq. (B.1) this will cause the price level to be increasing over time. This will then worsen the competitiveness of domestic goods and over time real output and thus dividends and consequently the stock price will be decreasing. This explains the economic mechanisms along the new saddle path from A to E_2. It could be added that the intution for the long-run effect increase in Q is that in a long-run equilibrium we have $\dot{p} = 0$ and thus $y = \bar{y}$, according to (B.1) and from (B.5) it thus follows that before-tax dividends are given as $D = \alpha\bar{y}$ in the long run, i.e. they are unaffected, but with the lower taxation, after-tax dividends have increased and this increases the long-run value of the stock price.

3. The phase diagram is shown in figure C.4 where now only the directions of motion (dotted) associated with the *original* $\dot{Q} = 0$ locus are shown.
   In addition to considerations a)-c) i question 2 above we now also have:

   d) Between time t_0 and time t_1 the economy is governed by the original directions of motion and from time t_1 by the new directions of motion.

   Applying these considerations to figure B.4 we conclude that the economy must at time t_0 jump from E_1 to A which is below the new saddle path. Between time t_0 and time t_1 the economy then moves according to the original directions of motion from A to B which is reached exactly at time t_1 after which point in time the economy moves along the new saddle path to the new long run equlibrium at E_2. The path followed from A to B crosses the (original) $\dot{Q} = 0$ locus as required.

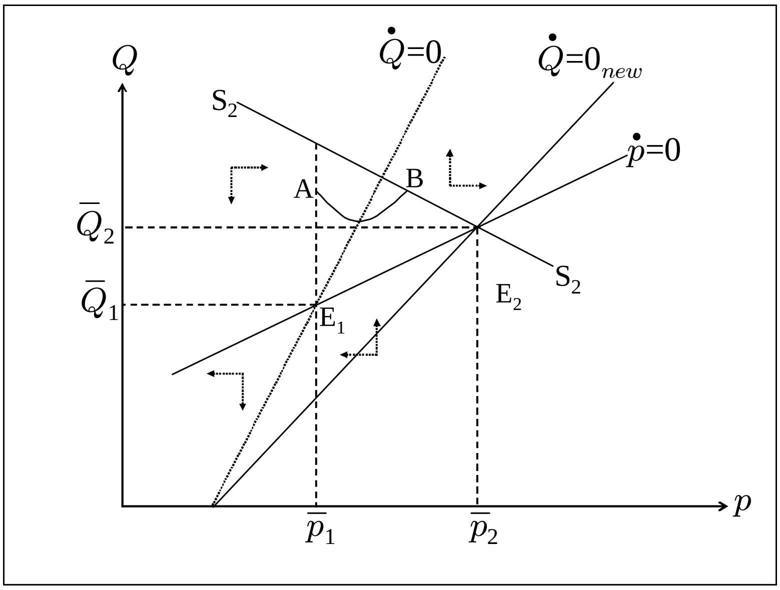

Figure B.4

The economic intuition is the following: At time t_0 it is learned that at the future time t_1 the dividend tax rate will be decreased. Ceteris Paribus (for given values of real dividends) this immediately increases (discretely) the stock price due to the fact that it is the present discounted value of present and future after-tax dividends. When the stock price increases this will increase demand and thus equlibrium production (according to eq. (B.2)) and this will in turn increase dividends (eq. (B.5)) and thus further increase the stock price. This explains the initial upward jump in the stock price from E_1 to A. These mechanisms are exactly the same as in question 2, except that it now takes a while before the dividend tax rate is actutally increased. As a result, dividends between time t_0 and t_1 should still be taxed using the original higher tax rate, and this dampens the initial increase in Q compared with the situation in question 2. This also explains why in figure B.4 the stock price does not immmediately jump to the value on the new saddle path. It could be added, that the initial jump from E_1 to A will be larger the shorter the time span between t_0 and t_1. Whether initially the stock price undershoots or overshoots the new long run value cannot be determined without further information. At point A it is now the case that $y > \bar{y}$ and consequently there will be upward pressure on prices (according to eq. (B.1)). This explains why from A to B the price level is increasing. With respect to the stock price there are two counteracting effects: one is that when the price level increases, net exports and thus real output and dividends will decrease and this leads to a decrease in the stock price. The other effect is that as time evolves, the future time t_1 where the decrease in τ will cause after-tax dividends to increase moves closer in time. This effects acts to increase the stock price. As is seen from Figure B.4 the first effects dominates initially, while the second effect dominates when time has come close enough to t_1 (specifically, after the orignial $\dot{Q} = 0$ locus has been crossed). It could be added that with different parameter values, the second effect could be dominating for the entire period until t_1, i.e. Q would be increasing until t_1. At time t_1 when the economy reaches the new saddle path, the decrease in τ takes place but this does not cuase the stock price to jump, since the effects are fully anticipated. From that point in time the economy moves continously along the new saddle path to the new long-run equilibrium due to the same economic mechanisms as in question 2.

Problem C

1. When the policy rule is

$$\pi_t = \bar{\pi} \tag{1}$$

and thisn is believed to be followed by government, and expectations are thus formed based on the rule, rational expectations imply that (using eq. (C.4))

$$\pi^e_{t,t-1} = E\left(\pi_t \left| I_{t-1}\right.\right) = E\left(\bar{\pi} \left| I_{t-1}\right.\right) = \bar{\pi} \tag{2}$$

From (C.1) it then follows (given that the rule actually *is* followed, i.e. $\pi_t = \bar{\pi}$) that

$$\bar{\pi} = \bar{\pi} + y_t - \bar{y} \Rightarrow$$

$$y_t = \bar{y} \tag{3}$$

Inserting (1) and (3) into the social loss function (C.2) we find that, using (C.3),

$$SL = (\bar{y} - (\bar{y} + \theta))^2 + \eta\left(\bar{\pi} - \pi^*\right)^2 = \theta^2 + \eta\left(\bar{\pi} - \pi^*\right)^2$$

which is clearly minimized by choosing $\bar{\pi} = \pi^*$ in which case the social loss becomes

$$SL_R = \theta^2 \tag{4}$$

All this may also be derived less formally (and in more intuitive economic terms): It follows from the SRAS curve in (C.1) that real output can only deviate from its natural level when the policy makers are able to create surprise inflation, i.e. to create an actual inflation that deviates from the expected level, $\pi_t \neq \pi^e_{t,t-1}$. Given rational expectations, it is obviously not possible for government to create surprise inflation by following a preannounced rule. As a consequence, real output will equal its natural level in which case the term in the social loss function capturing output deviations from the target level will equal θ^2, due to output being at its inefficiently low level. All government can thus do with respect to minimizing the loss is to choose inflation in order to avoid a social loss caused by inflation deviating from its target level, i.e. to choose $\pi_t = \bar{\pi} = \pi^*$.

2. When government is expected to follow the and expectastions are based on this, we found in question 1 that

$$\pi^e_{t,t-1} = \bar{\pi} = \pi^*$$

Since the objective of government is to minimize the loss function, the problem of government, when it decides to choose π_t freely may then be formulated as

$$\begin{aligned}
\min_{\pi_t} SL_t &= (y_t - y^*)^2 + \eta\,(\pi_t - \pi^*)^2 && (5)\\
\text{s.t. } \pi^e_{t,t-1} &= \pi^* && (6)\\
y_t &= \bar{y} + \pi_t - \pi^e_{t,t-1} && (7)\\
y^* &= \bar{y} + \theta && (8)
\end{aligned}$$

where eq. (7) is a rewritten version of the SRAS curve from eq. (C.1), which the government has to take into account. One way of solving the problem in (5) – (8) is to insert eq. (6) and (8) into eq. (7) and the result into eq. (5). This produces

$$\min_{\pi_t} SL_t = (\pi_t - \pi^* - \theta)^2 + \eta\,(\pi_t - \pi^*)^2$$

The first order condition is[2]

$$\frac{\partial SL_t}{\partial \pi_t} = 2\,(\pi_t - \pi^* - \theta) + 2\eta\,(\pi_t - \pi^*) = 0 \Leftrightarrow$$

$$\begin{aligned}
\pi_t - \pi^* - \theta + \eta\pi_t - \eta\pi^* &= 0 \Leftrightarrow\\
(1+\eta)\,\pi_t &= (1+\eta)\,\pi^* + \theta \Leftrightarrow
\end{aligned}$$

$$\pi_t = \pi^* + \frac{1}{1+\eta}\theta \qquad (9)$$

Inserting eqs. (6) and (9) into the SRAS curve from eq. (7) we find

$$y_t = \bar{y} + \pi^* + \frac{1}{1+\eta}\theta - \pi^* \Leftrightarrow$$

$$y_t = \bar{y} + \frac{1}{1+\eta}\theta \qquad (10)$$

Inserting eqs. (9), (10) and (8) into eq. (5) we then find

$$\begin{aligned}
SL^{\text{Cheat}} &= \left(\bar{y} + \frac{1}{1+\eta}\theta - (\bar{y}+\theta)\right)^2 + \eta\left(\pi^* + \frac{1}{1+\eta}\theta - \pi^*\right)^2\\
&= \left(\frac{1}{1+\eta}\theta - \theta\right)^2 + \eta\left(\frac{1}{1+\eta}\theta\right)^2\\
&= \left(\frac{-\eta}{1+\eta}\theta\right)^2 + \eta\left(\frac{1}{1+\eta}\theta\right)^2\\
&= \frac{\eta^2}{(1+\eta)^2}\theta^2 + \frac{\eta}{(1+\eta)^2}\theta^2\\
&= \frac{\eta\,(\eta+1)}{(1+\eta)^2}\theta^2 \Leftrightarrow
\end{aligned}$$

[2] The second order condition is

$$\frac{\partial SL_t}{\partial \pi_t} = 2 + 2\eta > 0$$

as required for the FOC to be sufficient for yielding a minimum.

$$SL_{\text{Cheat}} = \frac{\eta}{1+\eta}\theta^2 \tag{C.5}$$

If it is believed that govenment will set inflation equal to π^*, government will therefore actually choose $\pi_t = \pi^* + \frac{1}{1+\eta}\omega > \pi^* = \pi^e_{t,t-1}$ thereby creating surprice inflation, which enables y_t to increase above $\bar{y}$, according to the SRAS curve, eq. (C.1). The reason is that the target value, y^*, is greater than natural output, $\bar{y}$, and the social loss is thus decreased by increasing y_t above $\bar{y}$.

On a more technical level we have that when

$$\pi_t = \pi^e_{t,t-1} = \pi^*$$

and consequently

$$y_t = \bar{y} < y^*$$

the marginal social cost of a slight rise in inflation (due to the second term in the social loss function, $\eta\left(\pi_t - \pi^*\right)^2$) is zero, whereas the marginal social benfit from a slight rise in y_t (due to the first term in the social loss function, $\left(y_t - y^*\right)^2$) is positive.

3. Since government does not want to actually follow the policy once it is announced and believed, the policy is said to be *dynamically inconsistent* or *time inconsistent* and government has a *credibility problem*, since rational private agents will realize that the government will not want to follow the rule in the first place, and consequently the equilibrium of question 2 it not a true rational expectations equilirium when government is free to act in a discretionary manner.

   The true rational expectations equilibrium with discretionary policy is the one where policy is dynamically consistent. *A policy is said to be dynamically consistent (or time consistent) if, after the policy has been announced and expectations have been formed based on the policy, it is still optimal to implement the policy.*

   In order to find the time consistent equilibrium, we first must first find out what government does when it mimizes the social loss *given* π^e_t. In this case the problem is

$$\begin{aligned} \min_{\pi_t} SL_t &= \left(y_t - y^*\right)^2 + \eta\left(\pi_t - \pi^*\right)^2 & (11) \\ \text{s.t. } y_t &= \bar{y} + \pi_t - \pi^e_t & (12) \\ y^* &= \bar{y} + \theta & (13) \end{aligned}$$

   Inserting eqs. (13) and (12) into eq. (11) we get

$$\min_{\pi_t} SL_t = \left(\pi_t - \pi^e_t - \theta\right)^2 + \eta\left(\pi_t - \pi^*\right)^2$$

   The first order condition is[3]

$$\frac{\partial SL_t}{\partial \pi_t} = 2\left(\pi_t - \pi^e_t - \theta\right) + 2\eta\left(\pi_t - \pi^*\right) = 0 \Leftrightarrow$$

[3]The SOC is

$$\frac{\partial SL_t}{\partial \pi_t} = 2 + 2\eta > 0$$

as required.

$$
\begin{aligned}
(\pi_t - \pi_t^e - \theta) + \eta (\pi_t - \pi^*) &= 0 \Leftrightarrow \\
(1+\eta)\pi_t &= \pi_t^e + \theta + \eta\pi^* \Leftrightarrow
\end{aligned}
$$

$$
\pi_t = \frac{1}{1+\eta}\pi_t^e + \frac{1}{1+\eta}\theta + \frac{\eta}{1+\eta}\pi^* \tag{14}
$$

Forming rational expectations based on eq. (14) yields

$$
\begin{aligned}
\pi_t^e &= E(\pi_t | I_{t-1}) \\
&= E\left(\frac{1}{1+\eta}\pi_t^e + \frac{1}{1+\eta}\theta + \frac{\eta}{1+\eta}\pi^* | I_{t-1}\right) \\
&= \frac{1}{1+\eta}\pi_t^e + \frac{1}{1+\eta}\theta + \frac{\eta}{1+\eta}\pi^* \Leftrightarrow \\
(1+\eta)\pi_t^e &= \pi_t^e + \theta + \eta\pi^* \Leftrightarrow \\
\eta\pi_t^e &= \theta + \eta\pi^* \Leftrightarrow
\end{aligned}
$$

$$
\pi_t^e = \pi^* + \frac{\theta}{\eta} \tag{15}
$$

Inserting eq. (15) into eq. (14) produces

$$
\begin{aligned}
\pi_t &= \frac{1}{1+\eta}\left(\pi^* + \frac{1}{\eta}\theta\right) + \frac{1}{1+\eta}\theta + \frac{\eta}{1+\eta}\pi^* \\
&= \frac{1}{1+\eta}\frac{1}{\eta}\theta + \frac{1}{1+\eta}\theta + \frac{1}{1+\eta}\pi^* + \frac{\eta}{1+\eta}\pi^* \\
&= \frac{1}{1+\eta}\frac{1+\eta}{\eta}\theta + \frac{1+\eta}{1+\eta}\pi^* \Leftrightarrow
\end{aligned}
$$

$$
\pi_t = \pi^* + \frac{\theta}{\eta} \tag{C.6}
$$

and inserting eq. (15) and (C.6) into eq. (12) yields

$$
y_t = \bar{y} \tag{C.7}
$$

Equations (15), (C.6) and (C.7) is by construction the time consistent rational expectations equilibrium, since government has no incentive to deviate from the inflation in eq. (C.6) once expectations are given by (15). When private agents have rational expectations it is also the equilibrium that will result when government cannot make a binding commitment to stick to the policy rule of question 1. The unfortunate consequence is that the social loss is actually higher in the dynamically consistent equlibrium than in the rule-based equlibrium. By inserting eqs. (13), (C.6) and (C.7) into the social loss function we obtain

$$
\begin{aligned}
SL_{\mathrm{DC}} &= (\bar{y} - (\bar{y} + \theta))^2 + \eta\left(\pi^* + \frac{\theta}{\eta} - \pi^*\right)^2 \\
&= \theta^2 + \eta\frac{\theta^2}{\eta^2} \\
&= \left(1 + \frac{1}{\eta}\right)\theta^2 \Leftrightarrow
\end{aligned}
$$

$$SL_{\text{DC}} = \frac{1+\eta}{\eta}\theta^2 \tag{17}$$

Comparing with the social loss from eq. (4) when the rule-based policy is followed - and believed to be followed - we find that

$$SL_{\text{DC}} > SL_{\text{Rule}} \text{ since } \frac{1+\eta}{\eta}\omega^2 > \omega^2 \tag{18}$$

According to (18) government is thus hurt (the social loss is greater) when it cannot make a binding commitment and consequently the time consistent equilibrium results. The reason is that since the inflation is fully anticipated in both equilibria, output is given by its natural level. However, in the time consistent equlibrium inflation is driven above its target level in a futile attempt to increase oputput. According to eq. (C.6) the time consistent equilibrium thus creates an *inflation bias* (difference between actual inflation and the target level) of $\frac{\theta}{\eta}$, which increases the social loss. The inflation bias is rising with θ and decreasing with η, the relative weight on inflation in the social loss function.

4. Note that actual calculations are not required in this question.
With respect to government benefitting from delegating policy to a independent central bank there are various options of explaining this and any valid one should of course be accepted. Here we focus on the case where policy is delegated to a central bank which has a social loss function

$$SL_{CB} = (y_t - y^*)^2 + \eta_{CB}(\pi_t - \pi^*)^2$$

where it puts a higher weight, η_{CB}, on inflation deviations than government, i.e. $\eta_{CB} > \eta$. If the central bank is independent in its choice of policy, the resulting dynamically consistent equilibrium will be given by eqs. (C.6) and (C.7) with η_{CB} replaced by η, i.e.

$$\pi_t = \pi^* + \frac{\theta}{\eta_{CB}} \text{ and } y_t = \bar{y} \tag{19}$$

Since once again the resulting equilibrium real output is equal to its natural level but since inflation is now closer to the (government) target rate, we can conclude that when the equilibrium values of π_t and y_t are evaluated - using the government social loss function - the value of the social loss will have decreased due to the loss from the inflation bias having decreased.
When the economy is hit by supply shocks, s_t, the SRAS curve in (C.1) is replaced by

$$\pi_t = \pi^e_{t,t-1} + y_t - \bar{y} + s_t \tag{20}$$

The important thing to note is that given expected inflation, $\pi^e_{t,t-1}$, an (e.g.) negative supply shock, s_t, will have to either cause π_t to increase, cause y_t to decrease or a combination of the two, and the smaller the effect on π_t, the larger will be the effect on

y_t and vice versa. Now, assuming that the central bank is very inflation averse (has a large value of η_{CB}) the central bank will be reluctant to let supply shocks affect inflation and will instead allow them to affect real output, but this is not necessarily in the interest of government which cares about variation in both π_t and y_t - and who attaches different relative weights to deviations in the two variables than does the central bank.