Guide to answers to Written Exam for the B.Sc. or M.Sc. in Economics
Winter 2010/2011

Mikroøkonomi A

Final Exam

January 2011

Problem 1

Consider a Koopmans economy in which the consumer's initial endowment consists of 24 hours of time (and no food). Time can either be consumed as leisure or used as labor input in a firm whose output is food.
The consumer's preferences can be represented by the utility function $u(x_1, x_2) = x_1 \cdot x_2$, with x_1 being the units of leisure consumed, x_2 being the quantity of food consumed, both non-negative.
The firm has a production function given by $y = \max \{ q - 2, 0\}$, with q being the non-negative quantity of labor input and y being the quantity of food output.

- Illustrate this economy in a diagram
- Find the Pareto-Optimal (Pareto Efficient) allocation (i.e. compute values for production plan and consumption plan):
- Is it possible to implement this allocation in a market equilibrium (i.e. finding a suitable price system)?

The plan is q = 13, y = 11, x = (11,11); it is not possible to implement this as a market equilibrium allocation, as the price system supporting the consumer's choice, (1,1), will entail negative profits (-2) for the producer; the underlying problem, of course, is the non-convexity of the production possibility set, breaking a crucial assumption in the second welfare theorem.

Problem 2

Define, discuss, and compare these two concepts:
- Compensating variations (CV)
- Equivalent variations (EV)

See Varian, section 14.8.

Problem 3

Consider a consumer living in a private-ownership and pure-exchange economy. There are two goods, food and housing. Consider a situation in which the consumer is a net seller of housing. Imagine that the price of housing increases (relative to the price of food):
- Will the consumer ever wish to become a net buyer of housing?
- Will the consumer necessarily want to sell more housing than before?
- Will the consumer be worse or better off after the price change?

Varian chapter 9: By WARP, the consumer will stay a net seller and never become a net buyer; the consumer will be better off (or at least as well as before); but will not necessarily wish to sell more, as the endowment income effect may lead to an increase in the gross demand for housing.

Problem 4

A consumer has the utility function $u(x_1,x_2) = 2 \cdot x_1^{\frac{1}{2}} + x_2$ (with both quantities being non-negative), good 1 being housing and good 2 being food.

- Solve the utility maximization problem when prices are (p_1,p_2) and the exogenous income is m, all taking on strictly positive values.

The consumer has quasi-linear preferences. The interior solution is $x_1^* = (p_2/p_1)^2$. So the solution is:
$x_1 = min \{ (p_2/p_1)^2, m/p_1 \}, x_2 = max \{ m/p_2 - p_2/p_1, 0 \}$

Problem 5

Consider an Edgeworth economy with two consumers Agnes and Burt, who have the utility functions $u_A(x_{1A},x_{2A}) = x_{1A}{}^a \cdot x_{2A}{}^{(1-a)}$ and $u_B(x_{1B},x_{2B}) = x_{1B}{}^b \cdot x_{2B}{}^{(1-b)}$, with $0 < a < 1$ og $0 < b < 1$, good 1 being food, good 2 being drinks. They have private ownership to bundles (w_{1A},w_{2A}) and (w_{1B},w_{2B}), respectively, with all these quantities being strictly positive.

- Using good 2 as numeraire, find an expression for the Walrasian equilibrium value of the price of good 1 (food)
- Discuss the way in which parameters a and b affect this equilibrium value of the food price

The demand functions are easily found, the consumers both having Cobb-Douglas preferences, and the equilibrium price hence turns out to be $p_1^* = [aw_{2A}+bw_{2B}]/[(1-a)w_{1A}+(1-b)w_{1B}]$. A higher value of a or b will both entail a higher equilbrium price, as the agents' preferences for commodity 1 are strengthened..

Problem 6

Consider a firm existing in a world of perfect competition.

- Describe how the firm's demand for an input (a factor of production) is determined
- Is it possible that the firm's demand for some input increases when the price of that input increases?

See Varian 20.1 and 20.2: Because of "the law of supply", demand for an input can never increase when the price f the input increases.