Vortex Shedding and its dependence on Reynold's Number

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Abstract

Vortex shedding is a phenomenon commonly observed in fluid dynamics. It is an oscillating flow which forms when fluid (liquid or gas) passes past a body. The flow will be depended upon the size and shape of the body. The shedding of vortexes is also depended on Reynold's number. This write-up mainly focuses on Vortex Shedding and its dependency on Reynold's Number. We will majorly use a cylinder and vary its Reynold's number and discuss the findings.

1 Vortex Shedding

Vortex Shedding is the phenomenon which is observed in fluids where alternating vortexes are formed behind an object placed in a flowing fluid. The shedding of the vortexes are periodical and its interaction with the object leads to complex flow patterns, which in turn causes vibrations or drag forces.

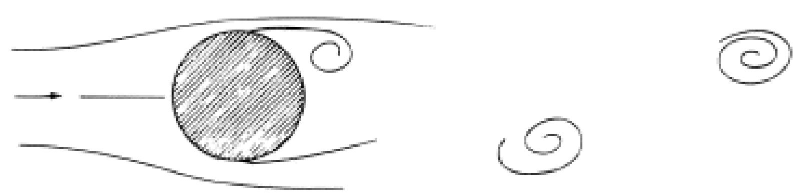

Figure 1: Vortex Shedding

1.1 Characteristics of Vortex Shedding

1.1.1 Alternating Vortexes

Vortex shedding involves alternate vortexes formation behind the object which is brought under fluid flow. These vortexes are shed periodically and also can vary in size and shape.

1.1.2 Shedding Frequency

The frequency in which vortexes shed are typically proportional to the velocity of the fluid flow. It also depends upon the shape and size of the body which is subjected to the fluid flow. Higher fluid velocities will result in higher frequency of shedding

1.1.3 Wake Formation

The Vortex Shedding creates a wake behind the object, which is characterised by certain turbulent flow patterns. This wake can extend downstream of the object and can affect the aerodynamic properties of the object.

1.1.4 Flow Instability

Vortex shedding also comprises of Flow instability, specifically in the wake region just behind the object. The vortex sheddings can cause variations in pressure and velocity, which in turn results in dynamic forces in the object.

1.1.5 Vortex Street Formation

In some of the cases, vortex shedding can also result in the formation of a unique pattern called von Kármán vortex street. It is named after the Hungarian-American mathematician and aerospace engineer, Theodore von Kármán. This pattern will have alternating vortexes which are shed symmetrically from the object and can be visualised in the wake region.

1.1.6 Frequency Relationship

The shedding frequency of vortexes are related to the Strouhal Number. It is a dimensionless parameter and it characterises the flow regime. It can be used to predict the shedding frequency based on velocity and characteristic length (diameter in the case of cylinder) of the object. Stourhal Number can be calculated by:

$$\boldsymbol{S = \frac{f}{U_\infty/D}} \tag{1}$$

where f = the frequency of vortex shedding, D = diameter of the cylinder and U_∞ = the free stream fluid velocity.

1.1.7 Drag Effect

Vortex shedding can have a significant impact on the resultant drag force which is experienced by an object when it travels through fluid. The alternating vortexes will contribute to drag, especially at certain shedding frequencies at which resonance effect occurs.

1.2 Causes of Vortex Shedding

Vortex Shedding is primarily caused by the interaction between the fluid flow and the body which is kept on flowing fluid. When some fluid flows around an object, it creates regions of low pressure on the leeward side (the side which is shielded from the wind.) and high pressure on the windward side (the side from which the wind is blowing). This pressure difference will cause vortexes to form and shed alternatively, which leads to a characteristic pattern of vortex shedding.

1.3 Applications of Vortex Shedding

1.3.1 Structural Engineering Applications

Vortex Shedding effect can cause vibrations in structures when they are exposed to fluid flows like bridges, tall buildings, chimneys etc. For maintaining structural integrity and safety of these kind of structures, it is important to understand how vortex shedding behaves in such structures.

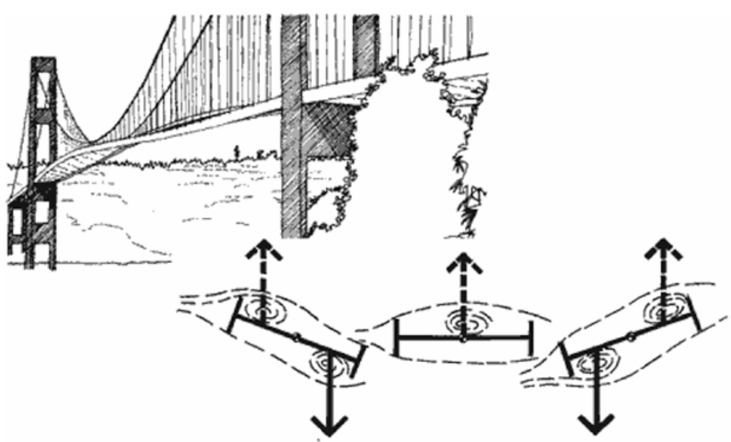

Figure 2: Tacoma bridge which collapsed due to van Kármán vortex street

1.3.2 Flow Measurement

Vortex Shedding is used in flow meters to measure the flow of liquids. As the vortex shedding is directly proportional to the flow velocity, flow measurement can be taken accurately.

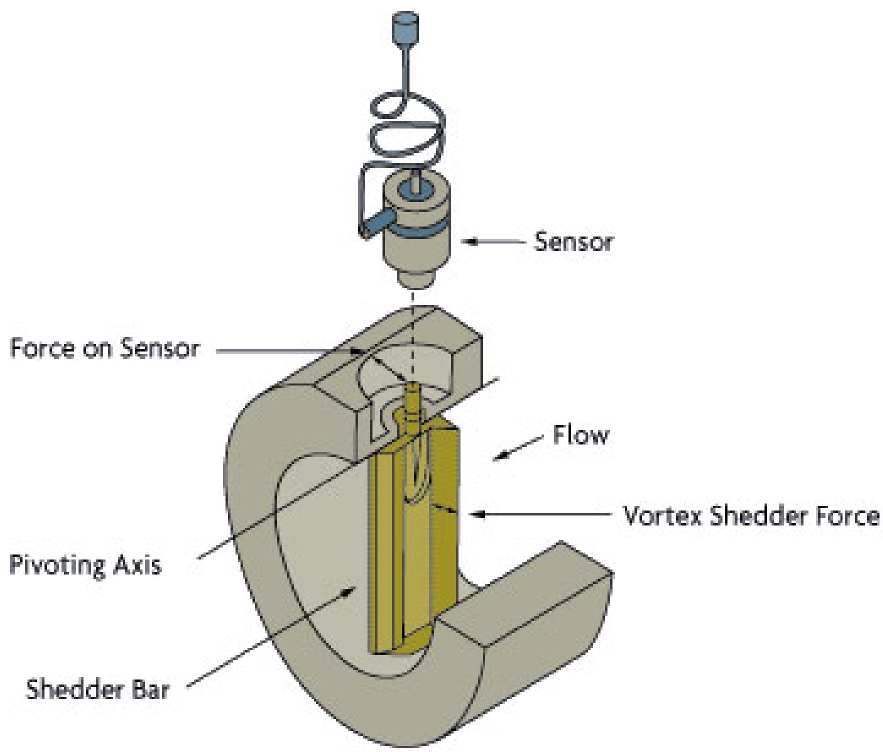

Figure 3: Vortex Flow Meter

1.3.3 Aerospace Applications

Vortex Shedding can be used in understanding the behaviour of airfoils, aircraft wings and other aerodynamic structures. As vortex shedding can significantly affect the lift and drag forces, the effect should be taken into consideration while designing aircrafts.

2 Reynold's Number

Reynold's Number is crucial in determining the occurence of vortex shedding. It is the ratio of inertial forces to viscous forces within a fluid that experiences due to relative internal movement as a result of varying fluid velocities. The formula for Reynold's number (Re) is given by

$$\boldsymbol{Re = \frac{\rho \cdot U_\infty \cdot D}{\mu}} \tag{2}$$

Where:

- ρ is the fluid density,
- U_∞ is the free stream fluid velocity,
- D is the diameter of the cylinder,
- μ is the dynamic visocity of the fluid

2.1 Derivation of Reynold's Number

When we take incompressible Navier-Stokes Equation (convective form):

$$\boldsymbol{\frac{\partial u}{\partial t} + (u \cdot \nabla)u - \nu \nabla^2 u = -\frac{1}{\rho}\nabla p + g} \tag{3}$$

We can remove the gravity term g. Then the left side of the equation will consists the internal forces $\frac{\partial u}{\partial t} + (u \cdot \nabla)u$ and the viscous force, $\nu\nabla^2 u$. Their ratio will be have the order of :

$$\boldsymbol{\frac{(u \cdot \nabla)u}{\nu \nabla^2 u} \sim \frac{u^2/L}{\nu u/L^2} = \frac{uL}{\nu}} \tag{4}$$

In case of cylinder the characteristic length, L = Diameter of the cylinder (D) and the ν is kinematic viscocity where $\nu = \frac{\mu}{\rho}$, where μ is the dynamic viscocity of the fluid and ρ is the density of the fluid. Therefore the Reynold's Number can be rewritten as:

$$\boldsymbol{Re = \frac{\rho \cdot U_\infty \cdot D}{\mu}} \tag{5}$$

2.2 Relation of Strouhal number with Reynold's number

For flows around cylinders, the Strouhal number can be found varying with Reynold's number according to a certain empirical relationship.

$$\boldsymbol{S = A \times Re^{-B}} \tag{6}$$

A and B are constants which depends on specific flow configuration and the range of Reynolds number. In general, Strouhal Numbers increases with the increase in Reynolds Number. Given the Graph by MIT OCW. Data is taken from Lienhard (1966) and Achenbach and Heinecke (1981).

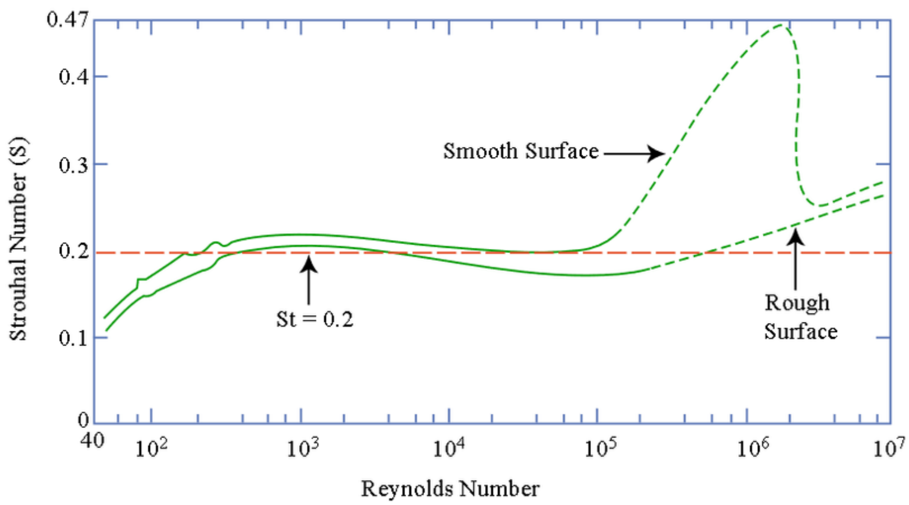

Figure 4: Relationship between Stouhal Number and Reynold's Number for Vortex Shedding

3 Dependence of Vortex shedding with Reynold's Number

3.1 When Re is low (<5)

For flow around a cylinder at low Re(<5), the vorticity generated at the surface is diffused (not advected) and there is a symmetry of flow. There will be a regime of unseparated flow.

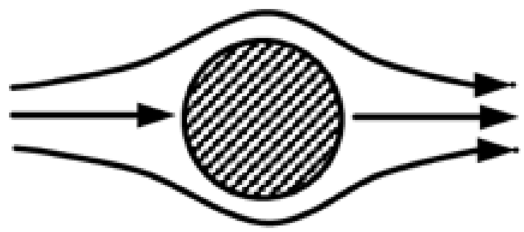

Figure 5: Re < 5

3.2 When $5 \leq \text{Re} \leq 40$

When Reynold's Number is in the range of $5 \leq \text{Re} \leq 40$, the flow regime can be typically characterised as laminar. Here there will be a fixed pair of vortexes in the wake. The vortexes shed are organised and predictable.

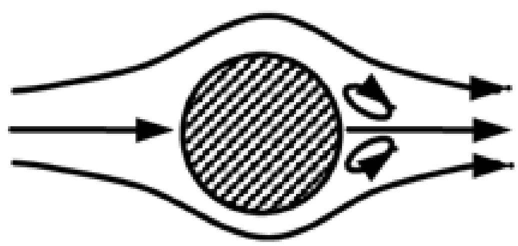

Figure 6: $5 \leq \text{Re} \leq 40$

3.3 When $40 \leq \text{Re} \leq 150$

In this range of Reynold's number, Laminar vortex street will be seen. Here the flow regime is typically transitional, that it exhibits both laminar and turbulent flow. The shedding pattern might be intermediate and irregular, and vortexes are less stable compared to laminar flow but not as chaotic as that in turbulent flow.

3.3.1 When $40 \leq \text{Re} \leq 90$

Periodicity of oscillations are governed by wake instability.

3.3.2 When 90 $\leq$ Re $\leq$ 150

Periodicity of oscillations are governed by Vortex Shedding.

Figure 7: $40 \leq \mathrm{Re} \leq 150$

3.4 When 150 $\leq$ Re $\leq$ 3x10^5

This region of the varying Reynold's number is actually a transition of turbulence in the wake.The vortex shedding is more stable and regular when compared to transitional and laminar regimes. Vortexes are more predictable with a consistent shedding frequency. The wake behind the object will be complex in structure.

3.4.1 When 150 $\leq$ Re ≤ 300

This is the transition range to turbulence in vortex.

3.4.2 When 300 $\leq$ Re $\leq$ 3x10^5

Now the vortex street is fully turublent.

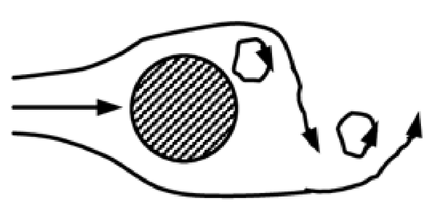

Figure 8: $150 \leq \mathrm{Re} \leq 3\mathrm{x}10^5$

3.5 When $3\text{x}10^5 \leq$ Re $\leq 3.5\text{x}10^6$

In this region, the laminar boundary layer will undergo turbulent transition and wake will get narrower and disorganised. Here no vortex street will be apparent.

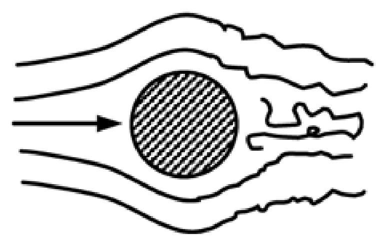

Figure 9: $3\text{x}10^5 \leq \text{Re} \leq 3.5\text{x}10^6$

3.6 When $3.5\text{x}10^6 \leq$ Re $\leq \infty$

On this region, the re-establishment of the turbulent vortex street which was evident when $300 \leq \text{Re} \leq 3\text{x}10^5$. This time the boundary layer will be turbulent while the wake is thinner.

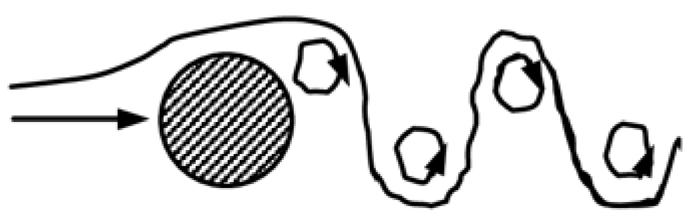

Figure 10: $3.5\text{x}10^6 \leq \text{Re} \leq \infty$

4 Experiment to prove dependence of Reynold's number in Vortex Shedding

4.1 Expiremental Setup

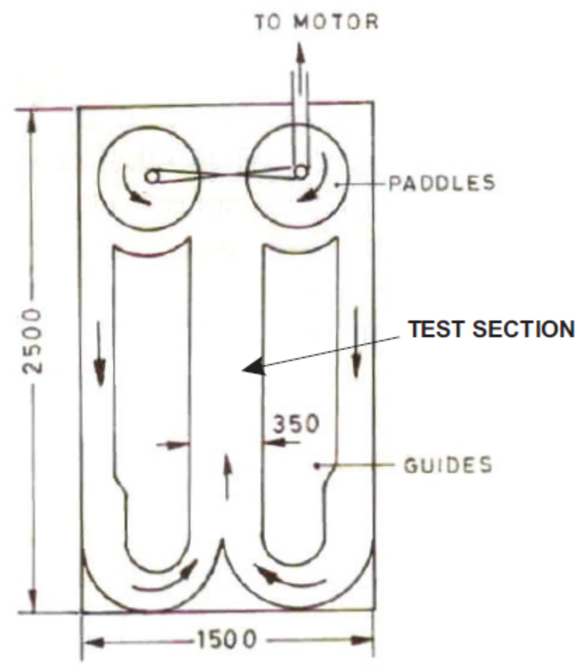

Figure 11: Top view of the experimental setup

The given figure is the schematic of the experimental setup. The setup consists of a tank 2.5m×1.5m and a depth of 150mm. On each end there are 2 sets of aluminium disks which will rotate and create a flow. After this, the fluid flow is guided to the test section where two cylinders are placed. The flow rate can be adjusted by varying rate of rotation of disks and velocities of a range 0.01 m/s to 0.2 m/s can be achieved. For fluid, water mixed with black dye can be taken and as the tracer, the aluminum powder can be used. Then the free stream velocity can be measured by noting down the time taken for a floating particle to go a fixed distance in the test section. Photographs of the flow phenomena can be taken by capturing the reflected light from the aluminum powder with any kind of head over camera.

4.2 Procedure

1. Fill the tank full of water with black dye dissolved in it. Now add sufficient amount of aluminum powder and switch on the power to the motor with the variac at lowest position.

2. When the variac is turned very slowly in an anti-clockwise manner, the rotation of vanes starts. Steady rotation of the disks can be obatined by adjusting the variac.

3. By noting the time taken for a floating particle to traverse a fixed distance (L) in the test region, velocity of flow can be measured. The floating particles should be away from walls. Mean time (t_m) and average free stream velocity ($U_\infty = L/T_m$)can be calculated by repeating the measurement

4. Flow pattern can be observed after placing the cylinder with diameter d=2cm in the test section.

4.3 Results

4.3.1 Velocity Calculation 1

Distance (L)(cm)	time(t)(s)
50	12.06
50	12.21
50	11.5
50	10.6

Mean Time (t_m) = 11.5925 s
U_∞ = L/t_m (m/s) = 0.043 m/s

4.3.2 Calculation of S and Re of fluid of velocity v1 when D1 = 3.5cm

D (cm)	No of vortexes(n)	time(t_v) (s)	f=n/t_v
3.5	4	10	0.4
3.5	6	15	0.4
3.5	8	20	0.4
3.5	10	25	0.4

$f_m = 0.4\ s^{-1}$
S = $f_m/(U_\infty/\mathrm{D}) = 0.3256$
Re = 1.69

4.3.3 Calculation of S and Re of fluid of velocity v2 when D2 = 5cm

D (cm)	No of vortexes(n)	time(t_v) (s)	f=n/t_v
5	5	10	0.5
5	4	15	0.27
5	7	20	0.35
5	10	25	0.4

$f_m = 0.38\ s^{-1}$
S = $f_m/(U_\infty/\mathrm{D}) = 0.442$
Re = 2.416

4.3.4 Velocity Calculation 2

Distance (L)(cm)	time(t)(s)
50	7.48
50	7.08
50	7.19
50	6.18

Mean Time (t_m) = 6.98 s
U_∞ = L/t_m (m/s) = 0.72 m/s

4.3.5 Calculation of S and Re of fluid of velocity v1 when D = 3.5cm

D (cm)	No of vortexes(n)	time(t_v) (s)	f=n/t_v
3.5	6	10	0.6
3.5	7	15	0.46
3.5	10	20	0.5
3.5	14	25	0.56

f_m = 0.53
S = $f_m/(U_\infty/\text{D})$ = 0.258
Re = 2.83

4.3.6 Calculation of S and Re of fluid of velocity v2 when D = 5cm

D (cm)	No of vortexes(n)	time(t_v) (s)	f=n/t_v
5	5	10	0.5
5	8	15	0.533
5	10	20	0.5
5	12	25	0.48

f_m = 0.5 s^{-1}
S = $f_m/(U_\infty/\text{D})$ = 0.349
Re = 4.04

4.4 Conclusion

The experiment confirms the occurence of the vortex shedding behind the cylinder. This shedding is characterised by a vortex formation and detachment of vortexes in the wake region of the cylinder. The shedding frequency was found related to the flow velocity. We saw that when we increase the flow velocity the shedding frequency will increase. If we increase the diameter of the cylinder, then the shedding frequency (f_m) will decrease.

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