1. Consider a 1-D diffusion equation. **(10)**

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

(a) Using $0 \le x \le 1$ and initial square wave ($u = 1$ for $0.4 \le x \le 0.6$ and 0 elsewhere), write a code to solve the problem using a finite difference FTCS approach using 64 grid points. Take $\nu = 0.501$ and $nt = nx$ with $t_{max} = 0.05$. Plot the solution at t = 0.00,0.01 and 0.05.

(b) Now, double the time-size size (Δt). What do you observe? Can you comment on the stability of the scheme?

2. Consider a 1-D linear convection equation. **(10)**

$$\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x}$$

(a) For an initial sinusoidal wave of wavenumber $k = 4\pi$ (2 periods in 1 m), and convection speed of 1 m/s, Solve this problem using an upwind approach, LaxWendroff Approach, and the Lax-Fredrichs approach and compare with the theoretical solution at t = 1.5 s. You can choose the time-step of your choice for numerical stability but justify your choice.

(b) Comment on relative features of numerical approaches employed in terms ofnumerical accuracy and stability.

3. Write a code for solving the 1-D Non-linear Convection-Diffusion (Burgers') equation for the initial input of a Heaviside function using the (i) Richtmyer and (ii) MacCormack schemes. Choose a domain of x in [0 4] m with 401 grid points with the step at x = 2 m. Plot the solutions at t = 0.5 s and compare two schemes. **(10)**

4. Consider the following PDE: **(20)**

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial x^2}$$

Initial conditions: $\phi(x,0) = \sin(\pi x), \quad \frac{\partial \phi}{\partial x}(x,0) = \frac{1}{4}\sin(2\pi x)$

Boundary conditions: $\phi(0,t) = 0 \qquad \phi(1,t) = 0$

(a) Solve the equation analytically using the separation of variables.

(b) Solve the equation numerically using the explicit Euler approach. Take no. ofgrid points as $nx = 51$. Take CFL number $\lambda = 0.5$ such that $\Delta t = \lambda \Delta x$. Plot $\phi(x,t)$ vs x at times t = 0.4,0.8 and 1.2 on single graph. Also plot the error $\phi(x,t) - \phi_a(x,t)$ at these times. Now change the CFL number to $\lambda = 2.0$ and repeat the above exercise.

(c) Use the implicit Euler scheme to solve the above equation with both CFLnumbers. Plot the solutions and errors at the same time instants as earlier. Comment on the accuracy and stability of the solutions between explicit and implicit schemes.

All the best !