

10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

Variable	Description
$x(0)$	First term of the AP
d	Common difference of the AP
$y(n)$	Sum of $n + 1$ terms of the AP
$x(n)$	General term

TABLE 0: Variables Used

$$y(n) = \frac{n+1}{2} (2x(0) + nd) \quad u(n) \quad (0.1)$$

$$y(6) = 49 \quad (0.2)$$

$$y(16) = 289 \quad (0.3)$$

Then,

$$x(0) + 3d = 7 \quad (0.4)$$

$$x(0) + 8d = 17 \quad (0.5)$$

From equations 0.4 and 0.5, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \quad (0.6)$$

$$\xrightarrow{R_1 \leftarrow R_1 - \frac{3}{5}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 10 \end{pmatrix} \quad (0.7)$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad (0.8)$$

$$\Rightarrow \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.9)$$

$$x(n) = (1 + 2n) u(n) \quad (0.10)$$

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{(1 - z^{-1})^2} \quad \{z \in \mathbb{C} : |z| > 1\} \quad (0.11)$$

$$y(n) = x(n) * u(n) \quad (0.12)$$

$$Y(z) = X(z) U(z) \quad (0.13)$$

$$\Rightarrow Y(z) = \left(\frac{1}{1-z^{-1}} + \frac{2z^{-1}}{(1-z^{-1})^2} \right) \left(\frac{1}{1-z^{-1}} \right) \quad (0.14)$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3} \quad (0.15)$$

$$(n+1)u(n) \xleftrightarrow{Z} \frac{1}{(1-z^{-1})^2} \{z \in \mathbb{C} : |z| > 1\} \quad (0.16)$$

$$n((n+1)u(n)) \xleftrightarrow{Z} \frac{2z^{-1}}{(1-z^{-1})^3} \{z \in \mathbb{C} : |z| > 1\} \quad (0.17)$$

From equations (??) and (??), taking the inverse Z Transform,

$$y(n) = (n+1)u(n) + n((n+1)u(n)) \quad (0.18)$$

$$\Rightarrow y(n) = (n+1)^2 u(n) \quad (0.19)$$

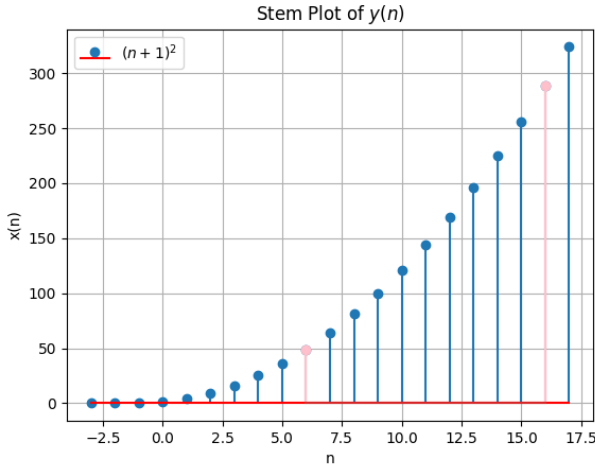


Fig. 0.1: Stem Plot of y(n)