11.9.4.7

EE23BTECH11063 - Vemula Siddhartha

Question:
Find the sum to n terms of the series:
$1^2 + \left(1^2 + 2^2\right) + \left(1^2 + 2^2 + 3^2\right) + ...$
Solution:

Variable	Description
$y(n)$	Sum of $n+1$ terms of the series
$x(n)$	General term

TABLE 0
VARIABLES USED

$$y(n) = 1^2 + \left(1^2 + 2^2\right) + \left(1^2 + 2^2 + 3^2\right) + ... \quad (1)$$

Let,

$$x(n) = (n+1)^2 u(n) \quad (2)$$
$$\implies y(n) = x(n) * u(n) * u(n) \quad (3)$$
$$Y(z) = X(z)\left(U(z)\right)^2 \quad (4)$$

From (**??**),

$$n^2 u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3} \quad \{|z| > 1\} \quad (5)$$

Using (**??**),

$$(n+1)^2 u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1+z^{-1}}{\left(1-z^{-1}\right)^3} \quad \{|z| > 1\} \quad (6)$$

From (6),

$$Y(z) = \left(\frac{1+z^{-1}}{\left(1-z^{-1}\right)^3}\right)\left(\frac{1}{1-z^{-1}}\right)^2 \quad (7)$$
$$= \frac{1+z^{-1}}{\left(1-z^{-1}\right)^5} \quad (8)$$
$$= \frac{1}{\left(1-z^{-1}\right)^5} + \frac{z^{-1}}{\left(1-z^{-1}\right)^5} \quad \{|z| > 1\} \quad (9)$$

From (**??**), using (**??**),

$$\frac{(n+1)(n+2)(n+3)(n+4)}{24} u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{\left(1-z^{-1}\right)^5} \{|z| > 1\} \quad (10)$$

$$\frac{(n)(n+1)(n+2)(n+3)}{24} u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{\left(1-z^{-1}\right)^5} \{|z| > 1\} \quad (11)$$

From (10) and (11), taking the Inverse Z Transform,

$$y(n) = \left(\frac{(n+1)(n+2)(n+3)(n+4)}{24} u(n)\right) + \left(\frac{(n)(n+1)(n+2)(n+3)}{24} u(n)\right) \quad (12)$$

$$\implies y(n) = \frac{(n+1)(n+2)^2(n+3)}{12} u(n) \quad (13)$$

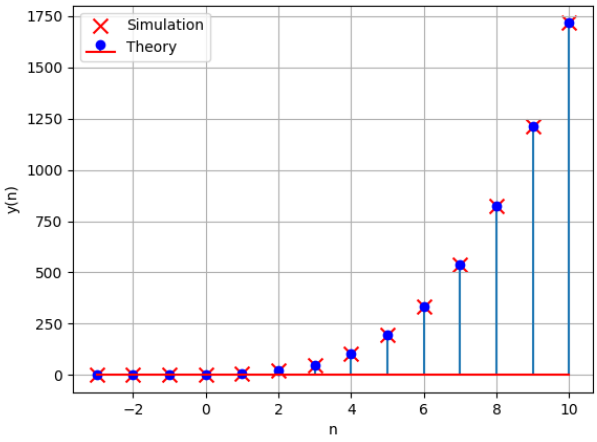

Fig. 0. Stem Plot of $y(n)$