10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:
The sum of first r terms of an Arithmetic Progression (AP) S_r , whose first term is a and common difference is d is:

$$S_r = \frac{r}{2}(2a + (r-1)d) \tag{1}$$

Let the given AP have first term a and common difference d.
Given, the sum of first 7 terms of the AP is 49.

$$\begin{aligned} S_7 &= 49 \\ 49 &= \frac{7}{2}(2a + (7-1)d) \\ 49 &= \frac{7}{2}(2a + 6d) \\ a + 3d &= 7 \end{aligned} \tag{2}$$

Also given, the sum of first 17 terms of the AP is 289.

$$\begin{aligned} S_{17} &= 289 \\ 289 &= \frac{17}{2}(2a + (17-1)d) \\ 289 &= \frac{17}{2}(2a + 16d) \\ a + 8d &= 17 \end{aligned} \tag{3}$$

Subtracting equation 2 from equation 3 we get:

$$\begin{aligned} 5d &= 10 \\ d &= 2 \end{aligned} \tag{4}$$

Substituting the value of d in equation 1 we get:

$$\begin{aligned} a + 6 &= 7 \\ a &= 1 \end{aligned} \tag{5}$$

The sum of first n terms of the AP is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values of a and d:

$$\begin{aligned} S_n &= \frac{n}{2}(2(1) + (n-1)(2)) \\ S_n &= n(1 + n - 1) \\ S_n &= n^2 \end{aligned} \tag{6}$$

The signal corresponding to this will be:

$$x(n) = n^2 u(n)$$

Applying z-transform:

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} (n^2 u(n))z^{-n} \\ X(z) &= 0 + \sum_{n=1}^{\infty} (n^2) z^{-n} \\ X(z) &= (1^2) z^{-1} + (2^2) z^{-2} + (3^2) z^{-3} + ... \\ X(z) &= z^{-1} + 4z^{-2} + 9z^{-3} + 16z^{-4} + ... \end{aligned} \tag{7}$$

Multiplying the equation 7 with z^{-1}:

$$z^{-1}X(z) = z^{-2} + 4z^{-3} + 9z^{-4} + 16z^{-5} + ... \tag{8}$$

Subtracting equation 8 from equation 7:

$$X(z)(1 - z^{-1}) = z^{-1} + 3z^{-2} + 5z^{-3} + 7z^{-4} + ... \tag{9}$$

Multiplying the equation 9 with z^{-1}:

$$X(z)(z^{-1} - z^{-2}) = z^{-2} + 3z^{-3} + 5z^{-4} + 7z^{-5} + ... \tag{10}$$

Subtracting equation 10 from equation 9:

$$\begin{aligned} X(z)(1 - 2z^{-1} + z^{-2}) &= z^{-1} + 2(z^{-2} + z^{-3} + z^{-4} + ...) \\ X(z)(1 - z^{-1})^2 &= z^{-1} + 2\frac{(z^{-2})}{(1 - z^{-1})} \\ X(z) &= \frac{z^{-1}(1 - z^{-1}) + 2z^{-2}}{(1 - z^{-1})^3} \\ X(z) &= \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \end{aligned} \tag{11}$$