10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
Solution:

Variable	Description
$x(0)$	First term of the AP
d	Common difference of the AP
$y(n)$	Sum of $n+1$ terms of the AP
$x(n)$	General term

TABLE 0
VARIABLES USED

$$y(n) = \frac{n+1}{2}(2x(0)+nd)\,u(n) \tag{1}$$

$$y(6) = 49 \tag{2}$$

$$y(16) = 289 \tag{3}$$

Then,

$$x(0)+3d = 7 \tag{4}$$

$$x(0)+8d = 17 \tag{5}$$

From equations 4 and 5, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xleftarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \tag{6}$$

$$\xleftarrow{R_1 \leftarrow R_1 - \frac{3}{5}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 10 \end{pmatrix} \tag{7}$$

$$\xleftarrow{R_2 \leftarrow \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{8}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{9}$$

$$x(n) = (1+2n)\,u(n) \tag{10}$$

$$X(z) = \frac{1}{1-z^{-1}} + \frac{2z^{-1}}{(1-z^{-1})^2} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{11}$$

$$y(n) = x(n) * u(n) \tag{12}$$

$$Y(z) = X(z)\,U(z) \tag{13}$$

$$\implies Y(z) = \left(\frac{1}{1-z^{-1}} + \frac{2z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{14}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3} \tag{15}$$

$$(n+1)\,u(n) \xleftrightarrow{z} \frac{1}{(1-z^{-1})^2} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{16}$$

$$n((n+1)\,u(n)) \xleftrightarrow{z} \frac{2z^{-1}}{(1-z^{-1})^3} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{17}$$

From equations (**??**) and (**??**), taking the inverse Z Transform,
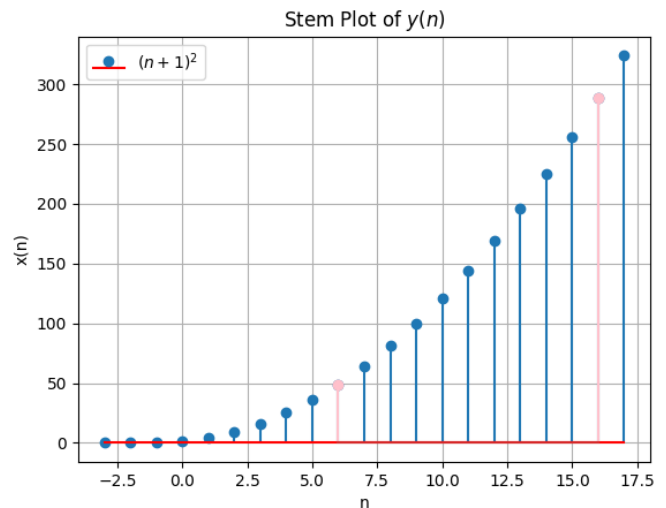
$$y(n) = (n+1)\,u(n) + n((n+1)\,u(n)) \tag{18}$$

$$\implies y(n) = (n+1)^2\,u(n) \tag{19}$$

Fig. 0. Stem Plot of y(n)