# PH-26

EE23BTECH11063 - Vemula Siddhartha

**Question**:
If $G(f)$ is the Fourier Transform of $f(x)$, then which of the following are true?

(a) $G(-f) = +G^*(f)$ implies $f(x)$ is real.
(b) $G(-f) = -G^*(f)$ implies $f(x)$ is purely imaginary.
(c) $G(-f) = +G^*(f)$ implies $f(x)$ is purely imaginary.
(d) $G(-f) = -G^*(f)$ implies $f(x)$ is real.

(GATE 2022 PH Question 26)

**Solution:**

| Symbol | Description |
|---|---|
| $f(x)$ | Function |
| $G(f)$ | Fourier Transform of the function $f(x)$ |
| $f^*(x)$ | Complex Conjugate of $f(x)$ |
| $G^*(f)$ | Complex Conjugate of $G(f)$ |
| Im($G(f)$) | Imaginary Part of $G(f)$ |

TABLE 4
GIVEN INFORMATION

$$f(x) \xleftrightarrow{\mathcal{F}} G(f) \tag{1}$$

$$G(f) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi f x}\, dx \tag{2}$$

$$\implies G(-f) = \int_{-\infty}^{\infty} f(x) e^{j2\pi f x}\, dx \tag{3}$$

$$\implies G^*(f) = \int_{-\infty}^{\infty} f^*(x) e^{j2\pi f x}\, dx \tag{4}$$

If $G(-f) = +G^*(f)$, from (3) and (4),

$$f(x) = f^*(x) \tag{5}$$

Hence, $f(x)$ is real.
Consider, $G(f) = \frac{j}{2}(\delta(f+f_0) - \delta(f-f_0))$,

$$G(-f) = -\frac{j}{2}(\delta(f+f_0) - \delta(f-f_0)) \tag{6}$$

$$G^*(f) = -\frac{j}{2}(\delta(f+f_0) - \delta(f-f_0)) \tag{7}$$

$$\implies G(-f) = +G^*(f) \tag{8}$$

Here, $f(x) = \sin(2\pi f_0 x)$, is real.

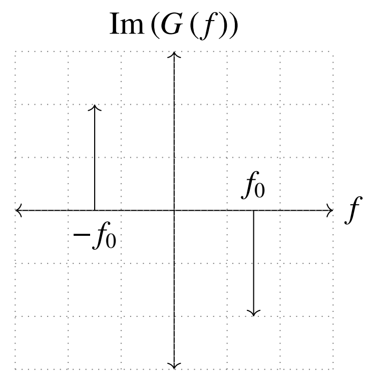


Fig. 4. Plot of Im($G(f)$) vs $f$

If $G(-f) = -G^*(f)$, from (3) and (4),

$$f(x) = -f^*(x) \tag{9}$$

Hence, $f(x)$ is purely imaginary.
Consider, $G(f) = \frac{j}{2}(\delta(f-f_0) + \delta(f+f_0))$,

$$G(-f) = \frac{j}{2}(\delta(f+f_0) + \delta(f-f_0)) \tag{10}$$

$$G^*(f) = -\frac{j}{2}(\delta(f-f_0) + \delta(f+f_0)) \tag{11}$$

$$\implies G(-f) = -G^*(f) \tag{12}$$

Here, $f(x) = j\cos(2\pi f_0 x)$, is purely imaginary.

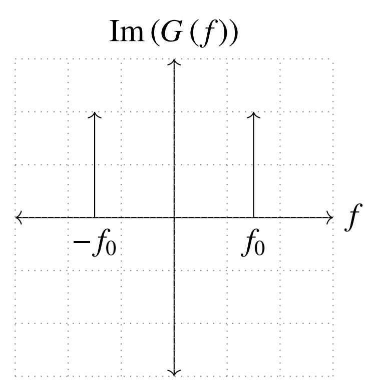


Fig. 4. Plot of Im($G(f)$) vs $f$

Therefore, (a) and (b) are true.