## 10.5.3.9

## EE23BTECH11063 - Vemula Siddhartha

## **Question:**

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

## **Solution:**

Variable	Description
x (0)	First term of the AP
d	Common difference of the AP
y (n)	Sum of $n + 1$ terms of the AP
x(n)	General term

TABLE 0 Variables Used

$$y(n) = \frac{n+1}{2} (2x(0) + nd) u(n)$$
 (1)

$$y(6) = 49$$
 (2)

$$y(16) = 289 (3)$$

Then,

$$x(0) + 3d = 7 (4)$$

$$x(0) + 8d = 17 \tag{5}$$

From equations 4 and 5, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \tag{6}$$

$$\stackrel{R_1 \leftarrow R_1 - \frac{3}{5}R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 1\\ 0 & 5 & 10 \end{pmatrix} \tag{7}$$

$$\stackrel{R_2 \leftarrow \frac{R_2}{5}}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{8}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{9}$$

$$y(n) = x(n) * u(n)$$
(12)

$$Y(z) = X(z) U(z)$$
(13)

$$\implies Y(z) = \left(\frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{(1 - z^{-1})^2}\right) \left(\frac{1}{1 - z^{-1}}\right) \tag{14}$$

$$= \frac{1}{(1-z^{-1})^2} + \frac{2z^{-1}}{(1-z^{-1})^3}$$

$$(n+1)u(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})^2} \{ z \in \mathbb{C} : |z| > 1 \}$$

$$(16)$$

$$n((n+1)u(n)) \stackrel{\mathcal{Z}}{\longleftrightarrow} \frac{2z^{-1}}{(1-z^{-1})^3} \{z \in \mathbb{C} : |z| > 1\}$$
(17)

Taking the inverse Z Transform,

$$y(n) = (n+1)u(n) + n((n+1)u(n))$$
 (18)

$$\implies y(n) = (n+1)^2 u(n) \tag{19}$$

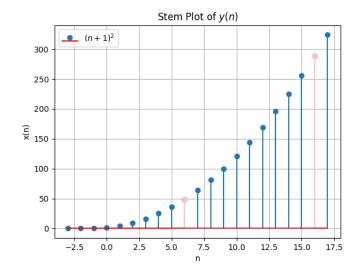


Fig. 0. Stem Plot of y(n)

$$x(n) = (1 + 2n) u(n)$$
 (10)

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{(1 - z^{-1})^2} \quad \{z \in \mathbb{C} : |z| > 1\} \quad (11)$$