10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

Variable	Description
$x(0)$	First term of the AP
d	Common difference of the AP
$s(n)$	Sum of $n+1$ terms of the AP
$x(n)$	General term

TABLE 0
VARIABLES USED

$$s(n) = \frac{n+1}{2}\left(2x(0) + nd\right) u(n) \tag{1}$$

From Table 0:

$$x(0) + 3d = 7 \tag{2}$$
$$x(0) + 8d = 17 \tag{3}$$

From equations 2 and 3, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \tag{4}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{3}{5}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 10 \end{pmatrix} \tag{5}$$
$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{6}$$
$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{7}$$

$$x(n) = (1 + 2n)\, u(n) \tag{8}$$
$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z)$$
$$X(z) = \frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{\left(1 - z^{-1}\right)^2} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{9}$$

$$s(n) = x(n) * u(n) \tag{10}$$
$$s(n) \overset{\mathcal{Z}}{\longleftrightarrow} S(z)$$
$$S(z) = X(z)\, U(z) \tag{11}$$
$$\implies S(z) = \left(\frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{\left(1 - z^{-1}\right)^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{12}$$
$$= \frac{1}{\left(1 - z^{-1}\right)^2} + \frac{2z^{-1}}{\left(1 - z^{-1}\right)^3} \tag{13}$$
$$= z\frac{z^{-1}}{\left(1 - z^{-1}\right)^2} + \left(-z\frac{d}{dz}\left(z\frac{z^{-1}}{\left(1 - z^{-1}\right)^2}\right)\right) \tag{14}$$

From (**??**) and (**??**), taking the inverse Z Transform,

$$s(n) = (n+1)\, u(n) + n\left((n+1)\, u(n)\right) \tag{15}$$
$$\implies s(n) = (n+1)^2\, u(n) \tag{16}$$

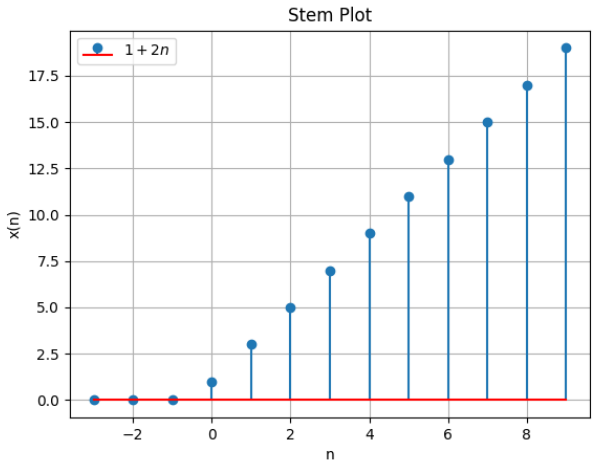

Fig. 0. Stem Plot of x(n)