10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

$$S_n = \frac{n}{2}\left(2x(0) + (n-1)\,d\right) \tag{1}$$

$$S_7 = 49 \tag{2}$$

$$49 = \frac{7}{2}\left(2x(0) + (7-1)\,d\right) \tag{3}$$

$$49 = \frac{7}{2}\left(2x(0) + 6d\right) \tag{4}$$

$$x(0) + 3d = 7 \tag{5}$$

$$S_{17} = 289 \tag{6}$$

$$289 = \frac{17}{2}\left(2x(0) + (17-1)\,d\right) \tag{7}$$

$$289 = \frac{17}{2}\left(2x(0) + 16d\right) \tag{8}$$

$$x(0) + 8d = 17 \tag{9}$$

From equations 5 and 9, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \tag{10}$$

$$R_2 \to R_2 - R_1$$

$$\begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \tag{11}$$

$$R_1 \to 5R_1 - 3R_2$$

$$\begin{pmatrix} 5 & 0 & 5 \\ 0 & 5 & 10 \end{pmatrix} \tag{12}$$

$$R_1 \to \frac{1}{5}R_1$$

$$R_2 \to \frac{1}{5}R_2$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{13}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{14}$$

$$\implies S_n = \frac{n}{2}\left(2x(0) + (n-1)\,d\right) \tag{15}$$

$$S_n = n^2 \tag{16}$$

$$x(n) = \left(x(0) + nd\right)u(n) \tag{17}$$

$$\implies x(n) = (1+2n)\,u(n) \tag{18}$$

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{19}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (1+2n)\,u(n)\,z^{-n} \tag{20}$$

$$X(z) = 0 + \sum_{n=0}^{\infty} (1+2n)\,z^{-n} \tag{21}$$

$$\text{If: } x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z)$$

$$\implies n^k x(n) \overset{\mathcal{Z}}{\longleftrightarrow} (-1)^k z^k \frac{d^k}{dz^k}\left(X(z)\right)$$

$$\implies X(z) = \frac{1+z^{-1}}{\left(1-z^{-1}\right)^2} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{22}$$

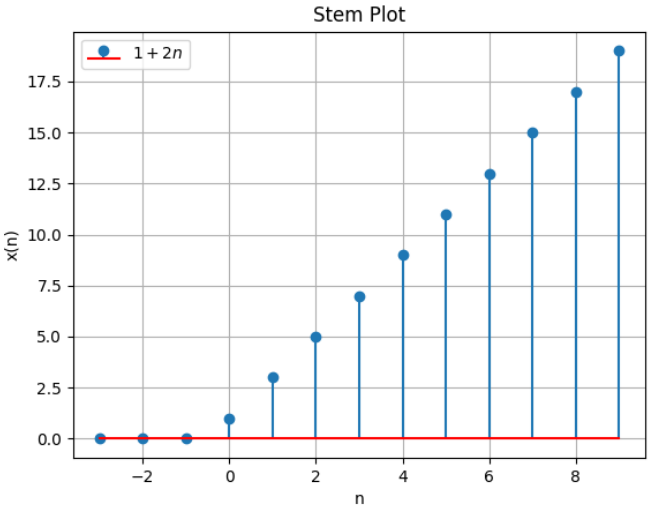

Fig. 0. Stem Plot of x(n)

Generalizing the problem:
Let the sum of first n_1 terms of the AP be 49, and

the sum of first n_2 terms of the AP be 289.
From equation 1:

$$\implies \frac{n_1}{2}\left(2x\left(0\right)+\left(n_1-1\right)d\right)=49 \tag{23}$$

$$\implies n_1x\left(0\right)+d\frac{\left(n_1-1\right)\left(n_1\right)}{2}=49 \tag{24}$$

Also,

$$\frac{n_2}{2}\left(2x\left(0\right)+\left(n_1-1\right)d\right)=289 \tag{25}$$

$$\implies x\left(0\right)\left(n_2\right)+d\frac{\left(n_2-1\right)\left(n_2\right)}{2}=289 \tag{26}$$

From equations 24 and 26, the augmented matrix is:

$$\begin{pmatrix} n_1 & \frac{\left(n_1-1\right)\left(n_1\right)}{2} & 49 \\ n_2 & \frac{\left(n_2-1\right)\left(n_2\right)}{2} & 289 \end{pmatrix} \tag{27}$$

$$R_1 \to \frac{1}{n_1}R_1$$

$$\begin{pmatrix} 1 & \frac{\left(n_1-1\right)}{2} & \frac{49}{n_1} \\ n_2 & \frac{n_2\left(n_2-1\right)}{2} & 289 \end{pmatrix} \tag{28}$$

$$R_2 \to R_2-n_2R_1$$

$$\begin{pmatrix} 1 & \frac{\left(n_1-1\right)}{2} & \frac{49}{n_1} \\ 0 & \frac{n_2\left(n_2-n_1\right)}{2} & 289-\frac{49}{n_1} \end{pmatrix} \tag{29}$$

$$R_2 \to \frac{2}{n_2\left(n_2-n_1\right)}R_2$$

$$\begin{pmatrix} 1 & \frac{\left(n_1-1\right)}{2} & \frac{49}{n_1} \\ 0 & 1 & \frac{2(49n_2-289n_1)}{n_1n_2(n_1-n_2)} \end{pmatrix} \tag{30}$$

$$R_1 \to R_1-\left(\frac{n_1-1}{2}\right)R_2$$

$$\begin{pmatrix} 1 & 0 & \frac{289n_1^2-289n_1-49n_2^2+49n_2}{n_1n_2(n_1-n_2)} \\ 0 & 1 & \frac{2(-289n_1+49n_2)}{n_1n_2(n_1-n_2)} \end{pmatrix} \tag{31}$$

$$\implies x\left(0\right)=49\left(\frac{-n_2+1}{n_1^2-n_1n_2}\right)+289\left(\frac{-n_1+1}{n_2^2-n_1n_2}\right) \tag{32}$$

$$\implies d=49\left(\frac{2}{n_1^2-n_1n_2}\right)+289\left(\frac{2}{n_2^2-n_1n_2}\right) \tag{33}$$

From the equations 15, 32 and 33:

$$\implies S_n=\frac{n}{2}\left(2\left(49\left(\frac{-n_2+1}{n_1^2-n_1n_2}\right)\right)+289\left(\frac{-n_1+1}{n_2^2-n_1n_2}\right)\right)+\left(n-1\right)\left(49\left(\frac{2}{n_1^2-n_1n_2}\right)+289\left(\frac{2}{n_2^2-n_1n_2}\right)\right) \tag{34}$$

The general term of the AP is:

$$x\left(n\right)=49\left(\frac{-n_2+1}{n_1^2-n_1n_2}\right)+289\left(\frac{-n_1+1}{n_2^2-n_1n_2}\right)+n\left(49\left(\frac{2}{n_1^2-n_1n_2}\right)+289\left(\frac{2}{n_2^2-n_1n_2}\right)\right) \tag{35}$$

$$\forall\ n\geq 0$$

$$\implies x\left(n\right)=\left(49\left(\frac{-n_2+1}{n_1^2-n_1n_2}\right)+289\left(\frac{-n_1+1}{n_2^2-n_1n_2}\right)+n\left(49\left(\frac{2}{n_1^2-n_1n_2}\right)+289\left(\frac{2}{n_2^2-n_1n_2}\right)\right)\right)u\left(n\right) \tag{36}$$

Variable	Description	Value
$x\left(0\right)$	First term of the AP	1
d	Common difference of the AP	2
S_n	Sum of n terms of the AP	n^2
S_7	Sum of 7 terms of the AP	49
S_{17}	Sum of 17 terms of the AP	289
$x(n)$	General term	$\left(1+2n\right)u\left(n\right)$
$X(z)$	Z- transform of $x(n)$	$\frac{1+z^{-1}}{\left(1-z^{-1}\right)^2}\ \{z\in\mathbb{C}:\left\|z\right\|>1\}$

TABLE 0
VARIABLES USED