

11.9.4.7

EE23BTECH11063 - Vemula Siddhartha

Question:

Find the sum to n terms of the series:

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$$

Solution:

Variable	Description	Value
$y(n)$	Sum of $n + 1$ terms of the series	?
$x(n)$	General term	$(n + 1)^2 u(n)$

TABLE 0
VARIABLES USED

$$y(n) = 1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots \quad (1)$$

Let,

$$x(n) = (n + 1)^2 u(n) \quad (2)$$

$$\Rightarrow y(n) = x(n) * u(n) * u(n) \quad (3)$$

$$Y(z) = X(z)(U(z))^2 \quad (4)$$

From (??),

$$n^2 u(n) \xleftrightarrow{z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \quad \{|z| > 1\} \quad (5)$$

Using (??),

$$(n + 1)^2 u(n) \xleftrightarrow{z} \frac{1 + z^{-1}}{(1 - z^{-1})^3} \quad \{|z| > 1\} \quad (6)$$

From (6),

$$Y(z) = \left(\frac{1 + z^{-1}}{(1 - z^{-1})^3} \right) \left(\frac{1}{1 - z^{-1}} \right)^2 \quad (7)$$

$$= \frac{1 + z^{-1}}{(1 - z^{-1})^5} \quad (8)$$

$$= \frac{1}{(1 - z^{-1})^5} + \frac{z^{-1}}{(1 - z^{-1})^5} \quad \{|z| > 1\} \quad (9)$$

From (??), using (??),

$$\frac{(n + 1)(n + 2)(n + 3)(n + 4)}{24} u(n) \xleftrightarrow{z} \frac{1}{(1 - z^{-1})^5} \quad \{|z| > 1\} \quad (10)$$

$$\frac{(n)(n + 1)(n + 2)(n + 3)}{24} u(n) \xleftrightarrow{z} \frac{z^{-1}}{(1 - z^{-1})^5} \quad \{|z| > 1\} \quad (11)$$

From (10) and (11), taking the Inverse Z Transform,

$$y(n) = \left(\frac{(n + 1)(n + 2)(n + 3)(n + 4)}{24} u(n) \right) + \left(\frac{(n)(n + 1)(n + 2)(n + 3)}{24} u(n) \right) \quad (12)$$

$$\Rightarrow y(n) = \frac{(n + 1)(n + 2)^2(n + 3)}{12} u(n) \quad (13)$$

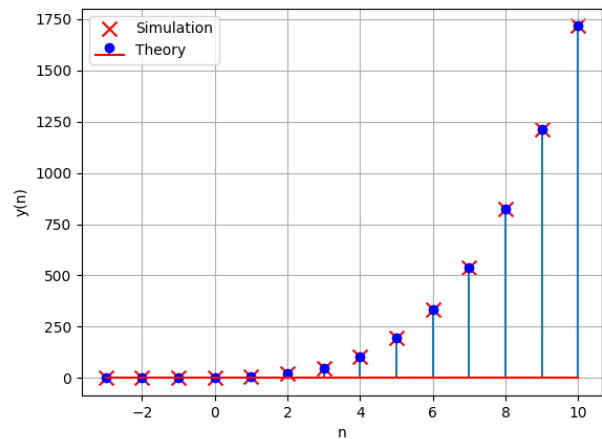


Fig. 0. Stem Plot of $y(n)$