CE-25

EE23BTECH11063 - Vemula Siddhartha

Question:
The following function is defined over the integral $[-L, L]$:

$$f(x) = px^4 + qx^5$$

It is expressed as a Fourier series,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \sin\left(\frac{\pi n x}{L}\right) + b_n \cos\left(\frac{\pi n x}{L}\right) \right\}$$

which options amongst the following are true?

(a) $a_n,\ n = 1, 2, .., \infty$ depend on p
(b) $a_n,\ n = 1, 2, .., \infty$ depend on q
(c) $b_n,\ n = 1, 2, .., \infty$ depend on p
(d) $b_n,\ n = 1, 2, .., \infty$ depend on q

Solution:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \sin\left(\frac{\pi n x}{L}\right) + b_n \cos\left(\frac{\pi n x}{L}\right) \right\} \quad (1)$$

Finding the Fourier Coefficient a_0,

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\ dx \quad (2)$$

$$= \frac{1}{2L} \int_{-L}^{L} \left(px^4 + qx^5\right)\ dx \quad (3)$$

$$= \frac{1}{2L} \left(2 \int_{0}^{L} px^4\ dx + 0\right) \quad (4)$$

$$= \frac{p}{L} \frac{x^5}{5} \bigg|_{-L}^{L} \quad (5)$$

$$\implies a_0 = \frac{2pL^4}{5} \quad (6)$$

Finding the Fourier Coefficients a_n,

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{\pi n x}{L}\right)\ dx \quad (7)$$

$$= \frac{1}{L} \int_{-L}^{L} \left(px^4 + qx^5\right) \sin\left(\frac{\pi n x}{L}\right)\ dx \quad (8)$$

$$= 0 + \frac{1}{L} \int_{-L}^{L} qx^5 \sin\left(\frac{\pi n x}{L}\right)\ dx \quad (9)$$

$$= -\frac{q}{\pi n}\left(x^5 \cos\left(\frac{\pi n x}{L}\right)\right) + \frac{5qL}{(\pi n)^2}\left(x^4 \sin\left(\frac{\pi n x}{L}\right)\right) + \frac{20qL^2}{(\pi n)^3}\left(x^3 \cos\left(\frac{\pi n x}{L}\right)\right) - \frac{60qL^3}{(\pi n)^4}\left(x^2 \sin\left(\frac{\pi n x}{L}\right)\right) - \frac{120qL^4}{(\pi n)^5}\left(x \cos\left(\frac{\pi n x}{L}\right)\right) + \frac{120qL^5}{(\pi n)^6} \sin\left(\frac{\pi n x}{L}\right)\bigg|_{-L}^{L} \quad (10)$$

$$= -\frac{2q}{\pi n}\left(L^5 \cos(\pi n)\right) + \frac{40qL^2}{(\pi n)^3}\left(L^3 \cos(\pi n)\right) - \frac{240qL^4}{(\pi n)^5}\left(L \cos(\pi n)\right) \quad (11)$$

$$\implies a_n = (-1)^{n+1}\left(2qL^5\right)\left(\frac{1}{\pi n} - \frac{2}{(\pi n)^3} + \frac{120}{(\pi n)^5}\right) \quad (12)$$

Finding the Fourier Coefficients b_n,

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{\pi n x}{L}\right)\ dx \quad (13)$$

$$= \frac{1}{L} \int_{-L}^{L} \left(px^4 + qx^5\right) \cos\left(\frac{\pi n x}{L}\right)\ dx \quad (14)$$

$$= \frac{1}{L} \int_{-L}^{L} px^4 \cos\left(\frac{\pi n x}{L}\right)\ dx + 0 \quad (15)$$

$$= \frac{p}{\pi n}\left(x^4 \sin\left(\frac{\pi n x}{L}\right)\right) + \frac{4pL}{(\pi n)^2}\left(x^3 \cos\left(\frac{\pi n x}{L}\right)\right) - \frac{12pL^2}{(\pi n)^3}\left(x^2 \sin\left(\frac{\pi n x}{L}\right)\right) - \frac{2pL^3}{(\pi n)^4}\left(x \cos\left(\frac{\pi n x}{L}\right)\right) + \frac{24pL^4}{(\pi n)^5} \sin\left(\frac{\pi n x}{L}\right)\bigg|_{-L}^{L} \quad (16)$$

$$= \frac{8pL}{(\pi n)^2}\left(L^3 \cos(\pi n)\right) - \frac{2pL^3}{(\pi n)^4}\left(L \cos(\pi n)\right) \quad (17)$$

$$\implies b_n = (-1)^n\left(2pL^4\right)\left(\frac{4}{(\pi n)^2} + \frac{1}{(\pi n)^4}\right) \quad (18)$$