# 10.5.3.9

### EE23BTECH11063 - Vemula Siddhartha

**Question**:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

**Solution:**
The sum of first $r$ terms of an Arithmetic Progression (AP) $x(r)$, whose first term is $a$ and common difference is $d$ is:

$$x(r) = \frac{r}{2}(2a + (r-1)d) \tag{1}$$

Let the given AP have first term $a$ and common difference $d$.
Given, the sum of first 7 terms of the AP is 49.

$$x(7) = 49$$
$$49 = \frac{7}{2}(2a + (7-1)d)$$
$$49 = \frac{7}{2}(2a + 6d)$$
$$a + 3d = 7 \tag{2}$$

Also given, the sum of first 17 terms of the AP is 289.

$$x(17) = 289$$
$$289 = \frac{17}{2}(2a + (17-1)d)$$
$$289 = \frac{17}{2}(2a + 16d)$$
$$a + 8d = 17 \tag{3}$$

Subtracting equation 2 from equation 3 we get:

$$5d = 10$$
$$d = 2 \tag{4}$$

Substituting the value of $d$ in equation 1 we get:

$$a + 6 = 7$$
$$a = 1 \tag{5}$$

The sum of first n terms of the AP is:

$$x(n) = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values of $a$ and $d$:

$$x(n) = \frac{n}{2}(2(1) + (n-1)(2))$$
$$x(n) = n(1 + n - 1)$$
$$x(n) = n^2 \tag{6}$$

Applying z-transform:

$$X(z) = \sum_{n=1}^{\infty}(n^2)z^{-n}$$
$$X(z) = (1^2)z^{-1} + (2^2)z^{-2} + (3^2)z^{-3} + ...$$
$$X(z) = z^{-1} + 4z^{-2} + 9z^{-3} + 16z^{-4} + ... \tag{7}$$

Multiplying the equation 7 with $z^{-1}$:

$$z^{-1}X(z) = z^{-2} + 4z^{-3} + 9z^{-4} + 16z^{-5} + ... \tag{8}$$

Subtracting equation 8 from equation 7:

$$X(z)(1 - z^{-1}) = z^{-1} + 3z^{-2} + 5z^{-3} + 7z^{-4} + ... \tag{9}$$

Multiplying the equation 9 with $z^{-1}$:

$$X(z)(z^{-1} - z^{-2}) = z^{-2} + 3z^{-3} + 5z^{-4} + 7z^{-5} + ... \tag{10}$$

Subtracting equation 10 from equation 9:

$$X(z)(1 - 2z^{-1} + z^{-2}) = z^{-1} + 2(z^{-2} + z^{-3} + z^{-4} + ...)$$
$$X(z)(1 - z^{-1})^2 = z^{-1} + 2\frac{(z^{-2})}{(1 - z^{-1})}$$
$$X(z) = \frac{z^{-1}(1 - z^{-1}) + 2z^{-2}}{(1 - z^{-1})^3}$$
$$X(z) = \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \tag{11}$$