11.9.4.7

EE23BTECH11063 - Vemula Siddhartha

Question:
Find the sum to n terms of the series:
$1^2 + \left(1^2 + 2^2\right) + \left(1^2 + 2^2 + 3^2\right) + \ldots$
Solution:

Variable	Description
$y(n)$	Sum of $n+1$ terms of the series
$x(n)$	General term

TABLE 0
VARIABLES USED

$$y(n) = 1^2 + \left(1^2 + 2^2\right) + \left(1^2 + 2^2 + 3^2\right) + \ldots \tag{1}$$

Let,

$$y(n) = \sum_{k=0}^{n} x(k) \tag{2}$$
$$y(n) = x(n) * u(n) \tag{3}$$

Then,

$$x(n) = \sum_{k=0}^{n} (k+1)^2 u(k) \tag{4}$$
$$x(n) = \left((n+1)^2 u(n)\right) * u(n) \tag{5}$$
$$X(z) = Z\left\{(n+1)^2 u(n)\right\} U(z) \tag{6}$$

From (**??**),

$$n^2 u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}\left(1 + z^{-1}\right)}{(1 - z^{-1})^3} \quad \{|z| > 1\} \tag{7}$$

Using (**??**),

$$(n+1)^2 u(n) \overset{Z}{\longleftrightarrow} \frac{1 + z^{-1}}{(1 - z^{-1})^3} \tag{8}$$

From (8),

$$X(z) = \left(\frac{1 + z^{-1}}{(1 - z^{-1})^3}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{9}$$
$$X(z) = \frac{1 + z^{-1}}{(1 - z^{-1})^4} \quad \{|z| > 1\} \tag{10}$$

$$\implies Y(z) = X(z) U(z) \tag{11}$$
$$= \left(\frac{1 + z^{-1}}{(1 - z^{-1})^4}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{12}$$
$$= \frac{1 + z^{-1}}{(1 - z^{-1})^5} \tag{13}$$
$$= \frac{1}{(1 - z^{-1})^5} + \frac{z^{-1}}{(1 - z^{-1})^5} \quad \{|z| > 1\} \tag{14}$$

From (**??**), using (**??**),

$$\frac{(n+1)(n+2)(n+3)(n+4)}{24} u(n) \overset{Z}{\longleftrightarrow} \frac{1}{(1 - z^{-1})^5} \{|z| > 1\} \tag{15}$$

$$\frac{(n)(n+1)(n+2)(n+3)}{24} u(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^5} \{|z| > 1\} \tag{16}$$

From (15) and (16), taking the Inverse Z Transform,

$$y(n) = \left(\frac{(n+1)(n+2)(n+3)(n+4)}{24} u(n)\right) + \left(\frac{(n)(n+1)(n+2)(n+3)}{24} u(n)\right) \tag{17}$$

$$\implies y(n) = \frac{(n+1)(n+2)^2(n+3)}{12} u(n) \tag{18}$$

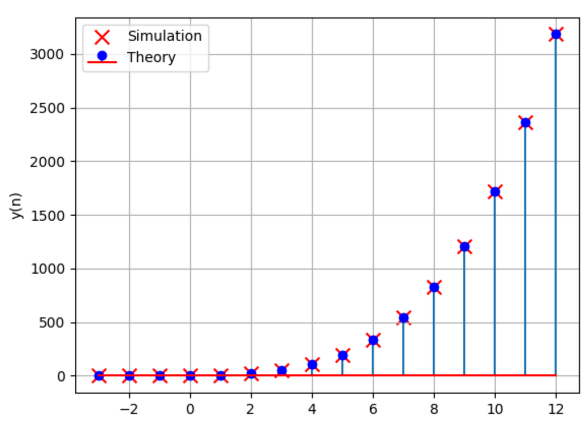

Fig. 0. Stem Plot of $y(n)$