PH-26

EE23BTECH11063 - Vemula Siddhartha

Question:
If $G(f)$ is the Fourier Transform of $f(x)$, then which of the following are true?

(a) $G(-f) = +G^*(f)$ implies $f(x)$ is real.
(b) $G(-f) = -G^*(f)$ implies $f(x)$ is purely imaginary.
(c) $G(-f) = +G^*(f)$ implies $f(x)$ is purely imaginary.
(d) $G(-f) = -G^*(f)$ implies $f(x)$ is real.

(GATE 2022 PH Question 26)

Solution:

Symbol	Description
$f(x)$	Function
$G(f)$	Fourier Transform of the function $f(x)$
$f^*(x)$	Complex Conjugate of $f(x)$
$G^*(f)$	Complex Conjugate of $G(f)$

TABLE 4
Given Information

$$f(x) \overset{\mathcal{F}}{\longleftrightarrow} G(f) \tag{1}$$

$$G(f) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi f x}\, dx \tag{2}$$

$$\implies G(-f) = \int_{-\infty}^{\infty} f(x) e^{j2\pi f x}\, dx \tag{3}$$

$$\implies G^*(f) = \int_{-\infty}^{\infty} f^*(x) e^{j2\pi f x}\, dx \tag{4}$$

If $G(-f) = +G^*(f)$, from (3) and (4),

$$f(x) = f^*(x) \tag{5}$$

Hence, $f(x)$ is real.
Consider, $f(x) = \sin(x)$,

$$G(f) = -\frac{j}{2}\left(\delta(f - f_0) - \delta(f + f_0)\right) \tag{6}$$

$$\implies G(-f) = \frac{j}{2}\left(\delta(f - f_0) - \delta(f + f_0)\right) \tag{7}$$

$$G^*(f) = \frac{j}{2}\left(\delta(f - f_0) - \delta(f + f_0)\right) \tag{8}$$

Here, $f(x)$ is real and $G(-f) = +G^*(f)$.
If $G(-f) = -G^*(s)$, from (3) and (4),

$$f(x) = -f^*(x) \tag{9}$$

Hence, $f(x)$ is purely imaginary.
Consider, $f(x) = j\cos(x)$,

$$G(f) = \frac{j}{2}\left(\delta(f - f_0) + \delta(f + f_0)\right) \tag{10}$$

$$\implies G(-f) = \frac{j}{2}\left(\delta(f + f_0) + \delta(f - f_0)\right) \tag{11}$$

$$G^*(f) = -\frac{j}{2}\left(\delta(f - f_0) + \delta(f + f_0)\right) \tag{12}$$

Here, $f(x)$ is purely imaginary and $G(-f) = -G^*(f)$.
Therefore, (a) and (b) are true.