

10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

$$S(n) = \frac{n}{2} (2x(0) + (n-1)d) \quad (1)$$

$$S(7) = 49 \quad (2)$$

$$49 = \frac{7}{2} (2x(0) + (7-1)d) \quad (3)$$

$$49 = \frac{7}{2} (2x(0) + 6d) \quad (4)$$

$$x(0) + 3d = 7 \quad (5)$$

$$S(17) = 289 \quad (6)$$

$$289 = \frac{17}{2} (2x(0) + (17-1)d) \quad (7)$$

$$289 = \frac{17}{2} (2x(0) + 16d) \quad (8)$$

$$x(0) + 8d = 17 \quad (9)$$

From equations 5 and 9, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \quad (10)$$

$$R_2 \rightarrow R_2 - R_1 \quad (11)$$

$$\begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix}$$

$$R_1 \rightarrow 5R_1 - 3R_2 \quad (12)$$

$$\begin{pmatrix} 5 & 0 & 5 \\ 0 & 5 & 10 \end{pmatrix}$$

$$R_1 \rightarrow \frac{1}{5}R_1 \quad (13)$$

$$R_2 \rightarrow \frac{1}{5}R_2 \quad (14)$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (15)$$

$$\Rightarrow S(n) = \frac{n}{2} (2x(0) + (n-1)d) \quad (16)$$

$$S(n) = n^2 \quad (17)$$

$$x(n) = (x(0) + nd)u(n) \quad (18)$$

From Table 0:

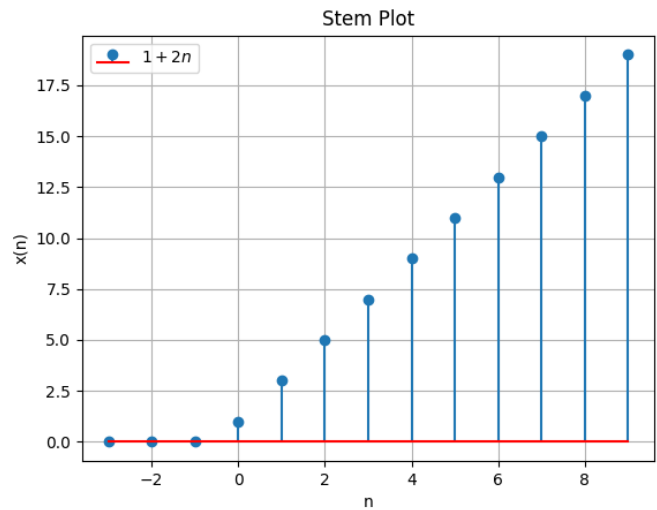
$$\Rightarrow x(n) = (1 + 2n)u(n) \quad (19)$$

$$x(n) \xleftrightarrow{Z} X(z)$$

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (20)$$

From Table 0:

$$\Rightarrow X(z) = \frac{1+z^{-1}}{(1-z^{-1})^2} \quad \{z \in \mathbb{C} : |z| > 1\} \quad (21)$$



(12) Fig. 0. Stem Plot of x(n)

Variable	Description	Value
$x(0)$	First term of the AP	1
d	Common difference of the AP	2
$S(n)$	Sum of n terms of the AP	n^2
$S(7)$	Sum of 7 terms of the AP	49
$S(17)$	Sum of 17 terms of the AP	289
$x(n)$	General term	$(1 + 2n)u(n)$
$X(z)$	Z- transform of $x(n)$	$\frac{1+z^{-1}}{(1-z^{-1})^2} \{z \in \mathbb{C} : z > 1\}$

TABLE 0
VARIABLES USED