

10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

The sum of first r terms of an Arithmetic Progression (AP) $x(r)$, whose first term is a and common difference is d is:

$$x(r) = \frac{r}{2} (2a + (r - 1)d) \quad (1)$$

Let the given AP have first term a and common difference d .

Given, the sum of first 7 terms of the AP is 49.

$$\begin{aligned} x(7) &= 49 \\ 49 &= \frac{7}{2} (2a + (7 - 1)d) \\ 49 &= \frac{7}{2} (2a + 6d) \\ a + 3d &= 7 \end{aligned} \quad (2)$$

Also given, the sum of first 17 terms of the AP is 289.

$$\begin{aligned} x(17) &= 289 \\ 289 &= \frac{17}{2} (2a + (17 - 1)d) \\ 289 &= \frac{17}{2} (2a + 16d) \\ a + 8d &= 17 \end{aligned} \quad (3)$$

Subtracting equation 2 from equation 3 we get:

$$\begin{aligned} 5d &= 10 \\ d &= 2 \end{aligned} \quad (4)$$

Substituting the value of d in equation 1 we get:

$$\begin{aligned} a + 6 &= 7 \\ a &= 1 \end{aligned} \quad (5)$$

The sum of first n terms of the AP is:

$$S_n = \frac{n}{2} (2a + (n - 1)d)$$

Substituting the values of a and d :

$$\begin{aligned} x(n) &= \frac{n}{2} (2(1) + (n - 1)(2)) \\ x(n) &= n(1 + n - 1) \\ x(n) &= n^2 \end{aligned} \quad (6)$$

Applying z-transform:

$$\begin{aligned} X(z) &= \sum_{n=1}^{\infty} (n^2) z^{-n} \\ X(z) &= (1^2) z^{-1} + (2^2) z^{-2} + (3^2) z^{-3} + \dots \\ X(z) &= z^{-1} + 4z^{-2} + 9z^{-3} + 16z^{-4} + \dots \end{aligned} \quad (7)$$

Multiplying the equation 7 with z^{-1} :

$$z^{-1}X(z) = z^{-2} + 4z^{-3} + 9z^{-4} + 16z^{-5} + \dots \quad (8)$$

Subtracting equation 8 from equation 7:

$$X(z)(1 - z^{-1}) = z^{-1} + 3z^{-2} + 5z^{-3} + 7z^{-4} + \dots \quad (9)$$

Multiplying the equation 9 with z^{-1} :

$$X(z)(z^{-1} - z^{-2}) = z^{-2} + 3z^{-3} + 5z^{-4} + 7z^{-5} + \dots \quad (10)$$

Subtracting equation 10 from equation 9:

$$\begin{aligned} X(z)(1 - 2z^{-1} + z^{-2}) &= z^{-1} + 2(z^{-2} + z^{-3} + z^{-4} + \dots) \\ X(z)(1 - z^{-1})^2 &= z^{-1} + 2 \frac{(z^{-2})}{(1 - z^{-1})} \\ X(z) &= \frac{z^{-1}(1 - z^{-1}) + 2z^{-2}}{(1 - z^{-1})^3} \\ X(z) &= \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3} \end{aligned} \quad (11)$$