

PH-26

EE23BTECH11063 - Vemula Siddhartha

Question:

If $G(f)$ is the Fourier Transform of $f(x)$, then which of the following are true?

- (a) $G(-f) = +G^*(f)$ implies $f(x)$ is real.
- (b) $G(-f) = -G^*(f)$ implies $f(x)$ is purely imaginary.
- (c) $G(-f) = +G^*(f)$ implies $f(x)$ is purely imaginary.
- (d) $G(-f) = -G^*(f)$ implies $f(x)$ is real.

(GATE 2022 PH Question 26)

Solution:

Symbol	Description
$f(x)$	Function
$G(f)$	Fourier Transform of the function $f(x)$
$f^*(x)$	Complex Conjugate of $f(x)$
$G^*(f)$	Complex Conjugate of $G(f)$
$\text{Im}(G(f))$	Imaginary Part of $G(f)$

TABLE 4
GIVEN INFORMATION

$$f(x) \xleftrightarrow{\mathcal{F}} G(f) \quad (1)$$

$$G(f) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi f x} dx \quad (2)$$

$$\Rightarrow G(-f) = \int_{-\infty}^{\infty} f(x) e^{j2\pi f x} dx \quad (3)$$

$$\Rightarrow G^*(f) = \int_{-\infty}^{\infty} f^*(x) e^{j2\pi f x} dx \quad (4)$$

If $G(-f) = +G^*(f)$, from (3) and (4),

$$f(x) = f^*(x) \quad (5)$$

Hence, $f(x)$ is real.

Consider, $G(f) = \frac{j}{2} (\delta(f + f_0) - \delta(f - f_0))$,

$$G(-f) = -\frac{j}{2} (\delta(f + f_0) - \delta(f - f_0)) \quad (6)$$

$$G^*(f) = -\frac{j}{2} (\delta(f + f_0) - \delta(f - f_0)) \quad (7)$$

$$\Rightarrow G(-f) = +G^*(f) \quad (8)$$

Here, $f(x) = \sin(2\pi f_0 x)$, is real.

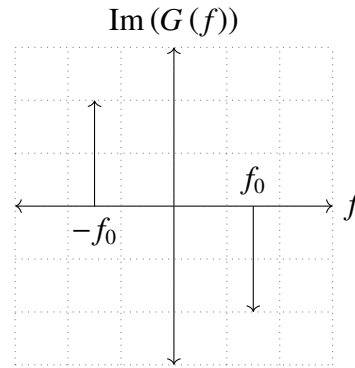


Fig. 4. Plot of $\text{Im}(G(f))$ vs f

If $G(-f) = -G^*(f)$, from (3) and (4),

$$f(x) = -f^*(x) \quad (9)$$

Hence, $f(x)$ is purely imaginary.

Consider, $G(f) = \frac{j}{2} (\delta(f - f_0) + \delta(f + f_0))$,

$$G(-f) = \frac{j}{2} (\delta(f + f_0) + \delta(f - f_0)) \quad (10)$$

$$G^*(f) = -\frac{j}{2} (\delta(f - f_0) + \delta(f + f_0)) \quad (11)$$

$$\Rightarrow G(-f) = -G^*(f) \quad (12)$$

Here, $f(x) = j \cos(2\pi f_0 x)$, is purely imaginary.

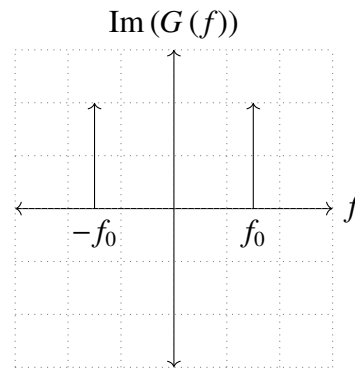


Fig. 4. Plot of $\text{Im}(G(f))$ vs f

Therefore, (a) and (b) are true.