

10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

Variable	Description
$x(0)$	First term of the AP
d	Common difference of the AP
$s(n)$	Sum of $n + 1$ terms of the AP
$x(n)$	General term

TABLE 0
VARIABLES USED

$$s(n) = \frac{n+1}{2} (2x(0) + nd) u(n) \quad (1)$$

From (1):

$$x(0) + 3d = 7 \quad (2)$$

$$x(0) + 8d = 17 \quad (3)$$

From equations 2 and 3, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \quad (4)$$

$$\xrightarrow{R_1 \leftarrow R_1 - \frac{3}{5}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 10 \end{pmatrix} \quad (5)$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad (6)$$

$$\Rightarrow \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (7)$$

$$x(n) = (1 + 2n) u(n) \quad (8)$$

$$x(n) \xrightarrow{Z} X(z)$$

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{(1 - z^{-1})^2} \quad \{z \in \mathbb{C} : |z| > 1\} \quad (9)$$

$$s(n) = x(n) * u(n) \quad (10)$$

$$s(n) \xleftrightarrow{Z} S(z)$$

$$S(z) = X(z) U(z) \quad (11)$$

$$\Rightarrow S(z) = \left(\frac{1}{1 - z^{-1}} + \frac{2z^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (12)$$

$$= \frac{1}{(1 - z^{-1})^2} + \frac{2z^{-1}}{(1 - z^{-1})^3} \quad (13)$$

$$= z \frac{z^{-1}}{(1 - z^{-1})^2} + \left(-z \frac{d}{dz} \left(z \frac{z^{-1}}{(1 - z^{-1})^2} \right) \right) \quad (14)$$

From (??) and (??), taking the inverse Z Transform,

$$s(n) = (n+1) u(n) + n((n+1) u(n)) \quad (15)$$

$$\Rightarrow s(n) = (n+1)^2 u(n) \quad (16)$$

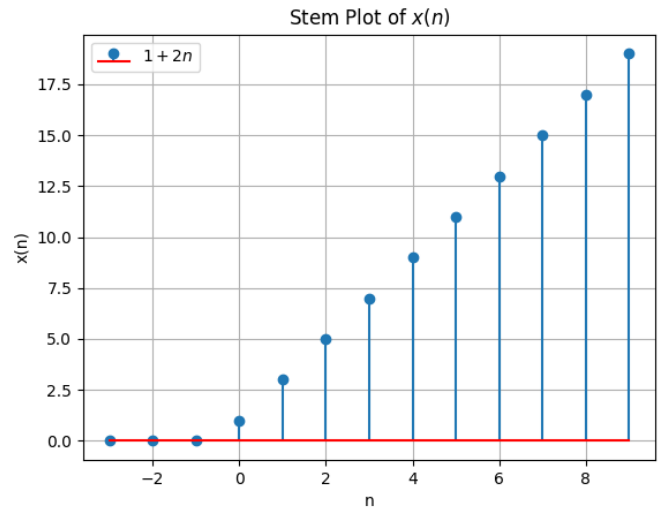


Fig. 0. Stem Plot of x(n)