10.5.3.9

EE23BTECH11063 - Vemula Siddhartha

Question:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

$$s(n) = \frac{n+1}{2}\left(2x(0) + nd\right) \tag{1}$$

From Table 0:

$$49 = \frac{(6+1)}{2}\left(2x(0) + 6d\right) \tag{2}$$

$$49 = \frac{7}{2}\left(2x(0) + 6d\right) \tag{3}$$

$$x(0) + 3d = 7 \tag{4}$$

From Table 0:

$$289 = \frac{(16+1)}{2}\left(2x(0) + 16d\right) \tag{5}$$

$$289 = \frac{17}{2}\left(2x(0) + 16d\right) \tag{6}$$

$$x(0) + 8d = 17 \tag{7}$$

From equations 4 and 7, the augmented matrix is:

$$\begin{pmatrix} 1 & 3 & 7 \\ 1 & 8 & 17 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 3 & 7 \\ 0 & 5 & 10 \end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{3}{5}R_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 10 \end{pmatrix} \tag{9}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{5}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \tag{10}$$

$$\implies \begin{pmatrix} x(0) \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{11}$$

$$x(n) = \left(x(0) + nd\right) u(n) \tag{12}$$

From Table 0:

$$\implies x(n) = (1 + 2n)\, u(n) \tag{13}$$

$$x(n) \overset{z}{\longleftrightarrow} X(z)$$

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{\left(1 - z^{-1}\right)^2} \tag{14}$$

From Table 0:

$$\implies X(z) = \frac{1 + z^{-1}}{\left(1 - z^{-1}\right)^2} \quad \{z \in \mathbb{C} : |z| > 1\} \tag{15}$$

$$s(n) = x(n) * u(n) \tag{16}$$

$$s(n) \overset{z}{\longleftrightarrow} S(n)$$

$$\implies S(z) = X(z)\, U(z) \tag{17}$$

$$\implies S(z) = \left(\frac{1 + z^{-1}}{\left(1 - z^{-1}\right)^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{18}$$

$$S(z) = \frac{1 + z^{-1}}{\left(1 - z^{-1}\right)^3} \tag{19}$$

$$S(z) = \frac{1}{\left(1 - z^{-1}\right)^2} + \frac{2z^{-1}}{\left(1 - z^{-1}\right)^3} \tag{20}$$

$$S(z) = \frac{1}{\left(1 - z^{-1}\right)} + \left(-z\frac{d}{dz}\left(\frac{1}{\left(1 - z^{-1}\right)^2}\right)\right) \tag{21}$$

$$S(z) = \frac{1}{1 - z^{-1}} + \frac{z^{-1}}{\left(1 - z^{-1}\right)^2} + \left(z\frac{d}{dz}\left(\frac{1}{\left(1 - z^{-1}\right)^2}\right)\right) \tag{22}$$

$$\implies s(n) = \left(u(n) + nu(n)\right) + n\left(u(n) + nu(n)\right) \tag{23}$$

$$\implies s(n) = (n+1)^2\, u(n) \tag{24}$$

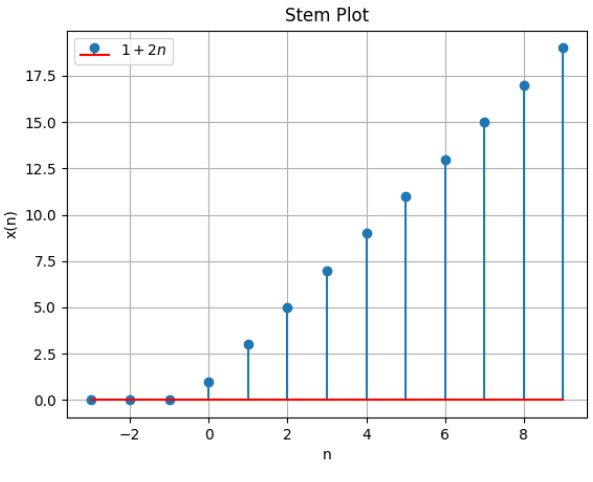

Fig. 0. Stem Plot of x(n)

Variable	Description	Value
$x(0)$	First term of the AP	?
d	Common difference of the AP	?
$s(n)$	Sum of n terms of the AP	?
$s(6)$	Sum of 7 terms of the AP	49
$s(16)$	Sum of 17 terms of the AP	289
$x(n)$	General term	?
$X(z)$	Z- transform of $x(n)$	?

TABLE 0
VARIABLES USED