

11.9.4.7

EE23BTECH11063 - Vemula Siddhartha

Question:

Find the sum to n terms of the series:

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$$

Solution:

Variable	Description
$y(n)$	Sum of $n+1$ terms of the series
$x(n)$	General term

TABLE 0
VARIABLES USED

$$y(n) = 1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots \quad (1)$$

(2)

Let,

$$y(n) = \sum_{k=0}^n x(k) \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

Then,

$$x(n) = \sum_{k=0}^n (k+1)^2 u(k) \quad (5)$$

$$x(n) = ((n+1)^2 u(n)) * u(n) \quad (6)$$

From (??) and (??),

$$X(z) = Z\{(n+1)^2 u(n)\} U(z) \quad (7)$$

$$= \left[z \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \right] \left(\frac{1}{1-z^{-1}} \right) \quad (8)$$

$$X(z) = \frac{1+z^{-1}}{(1-z^{-1})^4} \quad \{|z| > 1\} \quad (9)$$

From (4),

$$Y(z) = X(z) U(z) \quad (10)$$

$$= \left(\frac{1+z^{-1}}{(1-z^{-1})^4} \right) \left(\frac{1}{1-z^{-1}} \right) \quad (11)$$

$$= \frac{1+z^{-1}}{(1-z^{-1})^5} \quad \{|z| > 1\} \quad (12)$$

$$= z^5 \frac{z^{-5}}{(1-z^{-1})^5} + z^4 \frac{z^{-5}}{(1-z^{-1})^5} \quad (13)$$

From (??) and (??), taking the Inverse Z Transform,

$$y(n) = \left(\frac{(n+1)(n+2)(n+3)(n+4)}{24} u(n) \right) + \left(\frac{(n)(n+1)(n+2)(n+3)}{24} u(n) \right) \quad (14)$$

$$\Rightarrow y(n) = \frac{(n+1)(n+2)^2(n+3)}{12} u(n) \quad (15)$$

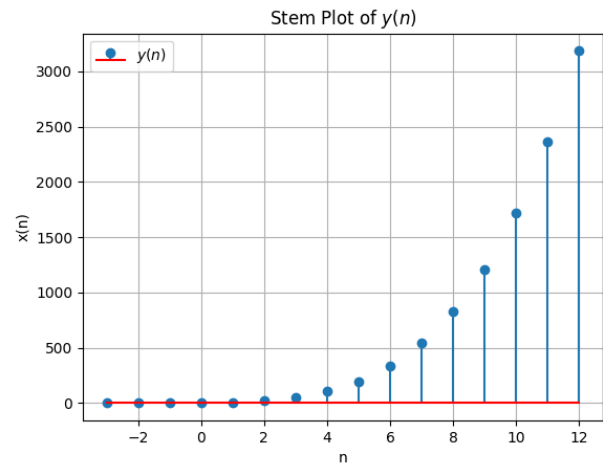


Fig. 0. Stem Plot of $y(n)$