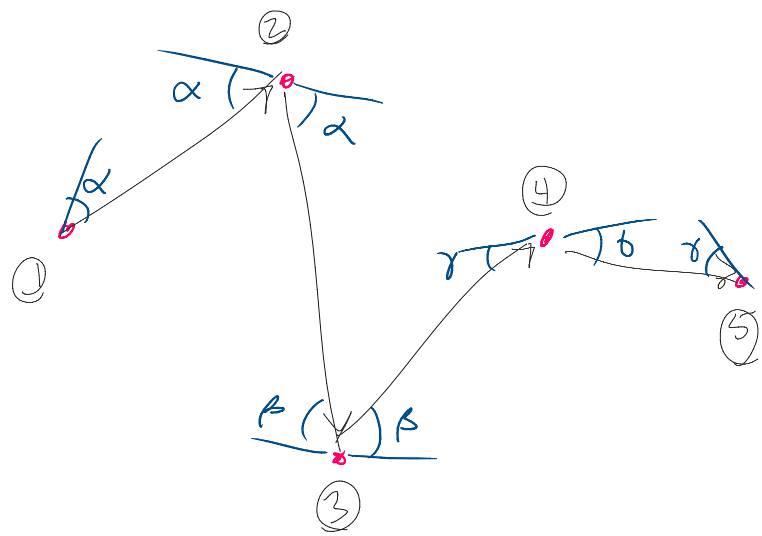
Curvature Method

Wednesday, January 19, 2022 3:16 PM



$$\vec{C}_0(t) = (1-t)^3\vec{P}_0 + 3(1-t)^2 t\vec{P}_1 + 3(1-t)t^2\vec{P}_2 + t^3\vec{P}_3$$

$$\vec{C}_1(t) = (1-t)^3\vec{P}_4 + 3(1-t)^2 t\vec{P}_5 + 3(1-t)t^2\vec{P}_6 + t^3\vec{P}_7$$

$$C_0 = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3$$

$$C_0' = 3(1-t)^2(-1)P_0 + \left[6(1-t)(-1)t + 3(1-t)^2\right]P_1 + \left[3(1-t)2t + 3(-1)t^2\right]P_2 + 3t^2 P_3$$

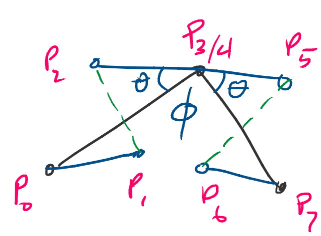
$$= -3(1-t)^2 P_0 + \left[-6(1-t)t + 3(1-t)^2\right]P_1 + \left[6(1-t)t - 3t^2\right]P_2 + 3t^2 P_3$$

$$= -3(1-t)^2 P_0 - 6(1-t)tP_1 + 3(1-t)^2 P_1 + 6(1-t)tP_2 - 3t^2 P_2 + 3t^2 P_3$$

$$= (1-t)^2(-3P_0 + 3P_1) + (1-t)(t)(-6P_1 + 6P_2) + 3t^2(-P_2 + P_3)$$

$$= 3(1-t)^2(P_1 - P_0) + 6t(1-t)(P_2 - P_1) + 3t^2(P_3 - P_2)$$

$$\boxed{\begin{aligned} C_0' + C_1' = \; & 3(1-t)^2(P_1 - P_0 + P_5 - P_4) \\ & + 6t(1-t)(P_2 - P_1 + P_6 - P_5) \\ & + 3t^2(P_3 - P_2 + P_7 - P_6) \end{aligned}}$$



$\rightarrow P_3 = P_4$

$$\theta = \frac{\pi - \phi}{2}$$

$$C_0'' + C_1'' = -6(1-t)$$

$\theta = \frac{\pi - \psi}{2}$

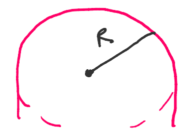
$$C_0'' + C_1'' = -6(1-t) + 6(1-t) - 6t + 6t$$

$$6(1-t) - 6t \rightarrow 6(1-t-t) \rightarrow 6(1-2t)$$

$$\boxed{\begin{aligned} C_0'' + C_1'' = &-6(1-t)(P_1 - P_0 + P_5 - P_4) \\ &+ 6t(1-t)(P_2 - P_1 + P_6 - P_5) \\ &+ 6t(P_3 - P_2 + P_7 - P_6) \end{aligned}}$$

$k = 0 \Rightarrow$

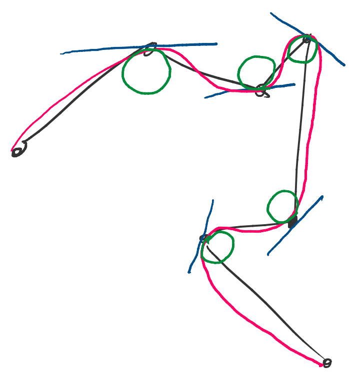
$k = 1 \Rightarrow$

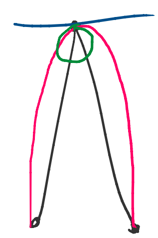


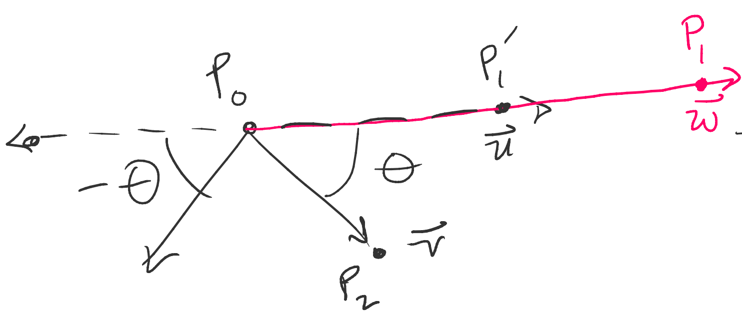
$R = \frac{1}{k}$

$$\boxed{\text{SET} \quad k = \frac{\pi - \phi}{\pi}}$$

$$k = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3} \quad \text{WHERE } t = 0.5$$







$$\boxed{\begin{aligned} \vec{w} &= \frac{\vec{u}}{|\vec{u}|}\ell \\ \vec{u} &= \vec{v}\cos\theta \\ \vec{v} &= P_f - P_i \end{aligned}}$$

If $\vec{v}_0 \rightarrow -\theta$

$\vec{v}_1 \rightarrow \theta$

$$P_R = \frac{(P_f - P_i)\cos\theta}{|(P_f - P_i)\cos\theta|}\ell$$

$P_L \ldots -\theta$

GIVEN:

GIVEN!

SETPOINTS — P_i, $i_3 = 0$, for $0 \le i \le n$

ENDPOINTS — $P_1 = P_0$, $P_{n-1} = P_n$

$P_L \cdots -\theta$

r'

$$\boxed{\begin{aligned} &3(1-t)^2(P_1 - P_0 + P_5 - P_4) \\ &+ 6t(1-t)(P_2 - P_1 + P_6 - P_5) \\ &+ 3t^2(P_3 - P_2 + P_7 - P_6) \end{aligned}}$$

$P_4 \to P_3$

$P_5 \to P_4$

$P_6 \to P_5$

$P_7 \to P_6$

SINCE $P_4 = P_3$

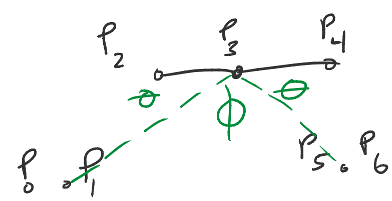
$$\begin{aligned} &3(1-t)^2(P_0 - P_0 + P_5 + P_3) \\ &+ 6t(1-t)(P_2 - P_0 + P_7 - P_5) \\ &+ 3t^2(P_3 - P_2 + P_7 - P_7) \end{aligned}$$

$$\boxed{\begin{aligned} r'(t) = {} &3(1-t)^2(P_4 + P_3) \\ &+ 6t(1-t)(P_2 - P_0 + P_6 - P_4) \\ &+ 3t^2(P_3 - P_2) \end{aligned}}$$

r''

$$\boxed{\begin{aligned} &-6(1-t)(P_1 - P_0 + P_5 - P_4) \\ &+ 6t(1-t)(P_2 - P_1 + P_6 - P_5) \\ &+ 6t(P_3 - P_2 + P_7 - P_6) \end{aligned}}$$

$$\boxed{\begin{aligned} r''(t) = {} &-6(1-t)(P_4 + P_3) \\ &+ 6t(1-t)(P_2 - P_0 + P_6 - P_4) \\ &+ 6t(P_3 - P_2) \end{aligned}}$$



$$P_2 = P_L = \frac{Vec_{30} \cos -\theta}{|Vec_{30} \cos -\theta|} l$$

$$P_4 = P_R = \frac{Vec_{36} \cos \theta}{|Vec_{36} \cos \theta|} l$$

$$\phi = \angle P_{31}, P_{35}$$

$$\theta = \frac{\pi - \phi}{2}$$

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curvature =

(abs(((3*p0_2)/4 + (3*p1_2)/4 - (3*p5_2)/4 - (3*p6_2)/4 - (3*l*vec30_2*cos(theta))/(4*(abs(vec30_1*cos(theta))^2 + abs(vec30_2

>> solution_l = solve(curvature == k, l)
Warning: Unable to find explicit solution. For options, see help.
> In sym/solve (line 317)

solution_l =

Empty sym: 0-by-1

>>
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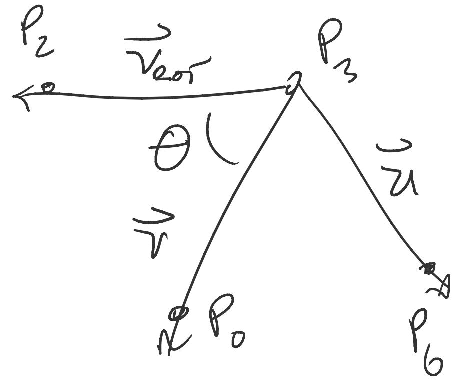
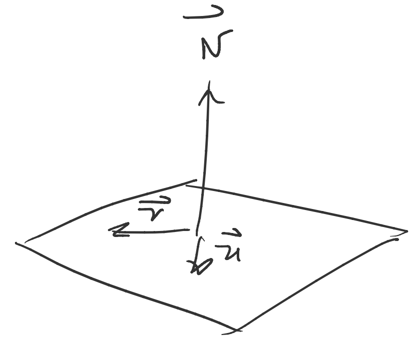
P_2 $\vec{V}_{0}$ P_1 $\vec{N}$





$$\vec{N} = \vec{v} \times \vec{u}$$

$$\vec{k} = \frac{\vec{N}}{|\vec{N}|}$$

RODRIGUES' ROTATION FORMULA

$$\vec{v}_{ROT} = \vec{v}\cos\theta + (\vec{k} \times \vec{v})\sin\theta + \vec{k}(\vec{k} \cdot \vec{v})(1 - \cos\theta)$$