JACOBIAN PSEUDO-INVERSE IK

DH TABLE $\longrightarrow$ TRANSFORMATION MATRICIES

(DENAVIT-HARTENBERG)

$$ {}^{0}_{EE}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdots {}^{n}_{EE}T = \left[\begin{array}{c|c} R & P \\ \hline 000 & 1 \end{array}\right] $$

P_x, P_y, P_z

$$ \begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} = {}^{0}J \begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \\ \dot{\Theta}_3 \end{bmatrix} $$

$$ \boxed{\text{SINGULARY @ } \det(J) = 0} $$

VELOCITIES

$$ \left.\begin{aligned} y_1 &= f_1(x_1, x_2, x_3, \ldots) \\ y_2 &= f_2(x_1, x_2, x_3, \ldots) \\ &\vdots \\ y_n & \end{aligned}\right\} \text{POSITION OF JOINTS} $$

$\Downarrow$ DIFFERENTIATE

$$ \partial y_1 = \frac{\partial f_1}{\partial x_1} \partial x_1 + \frac{\partial f_1}{\partial x_2} \partial x_2 + \cdots $$

$$ \partial y_2 = \frac{\partial f_2}{\partial x_1} \partial x + \frac{\partial f_2}{\partial x_2} \partial x_2 + \cdots $$

$$ \vdots $$

$$ \partial y_n $$

$$ \Longrightarrow \partial Y = \frac{\partial F}{\partial X} \partial x \longrightarrow \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \end{bmatrix} $$

$$\longrightarrow \boxed{\dot{Y} = J\dot{x}}$$

PARTIAL DIFFERENTIATION

① DH TABLE $\longrightarrow$ TRANSFORMATIONS MATRICIES

$${}^{0}_{1}T,\ {}^{1}_{2}T,\ {}^{2}_{3}T,\ \ldots$$

② $${}^{0}_{n}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot \ \ldots\ \cdot {}^{n-1}_{n}T$$

$$\longrightarrow \left[\begin{array}{c|c} {}^{0}_{n}R & {}^{0}_{n}P \\ \hline 000 & 1 \end{array}\right] \qquad \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

POSITION OF THE TOOL RELATIVE TO THE BASE.

③ $$\begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} = {}^{0}J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

EXAMPLE:

$$\left.\begin{aligned} P_x &= r_1 \cos\theta \\ P_y &= r_2 \sin\theta \\ P_z &= 0 \end{aligned}\right\} \longrightarrow \underbrace{\begin{aligned} \dot{P}_x &= -r_1 \sin\theta \times \dot{\theta} \\ \dot{P}_y &= r_2 \cos\theta \times \dot{\theta} \\ \dot{P}_z &= 0 \times \dot{\theta} \end{aligned}}$$

$$\begin{bmatrix} -r_1 \sin\theta \\ r_2 \cos\theta \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \end{bmatrix}$$

$${}^{0}_{n}J$$

JACOBIAN IN BASE FRAME

BASE $\longrightarrow$ FINAL

④ $${}^{2}\vec{J} = {}^{2}_{0}\vec{R} \cdot {}^{0}\vec{J}$$

$${}^{0}_{2}T = \left[\begin{array}{c|c} {}^{0}_{2}R & {}^{0}_{2}P \\ \hline 000 & 1 \end{array}\right]$$

$${}^{0}_{2}R \longrightarrow {}^{2}_{0}R = {}^{0}_{2}R^{-1}$$

CHECK $\det({}^{2}J) = 0$? FOR SINGULARITIES

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