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Water Engineering

Sediment Threshold under Stream Flow: A State-of-the-Art Review

By Subhasish Dey* and Athanasios Papanicolaou**

Abstract

The doctoral research of Albert Frank Shields on sediment movement conducted in Technischen Hochshcule Berlin becomes a legend that is most often referred by many authors and has initiated a sizable number of researches over last seven decades. The Shields diagram is famous due to its application in ascertaining the threshold of sediment motion that is an essential prerequisite for the estimation of sediment transport in an alluvial stream. Since his pioneering work, numerous attempts have so far been made to study the sediment threshold both experimentally and theoretically. This paper presents a comprehensive state-of-the-art review of the important laboratory experimental and theoretical investigations on sediment threshold under steady stream flow highlighting the empirical concepts, hydrodynamic background and the mathematical treatment of the problem. The role of the turbulent bursting on sediment threshold is also discussed.

Keywords: *sediment transport, sediment threshold, fluvial hydraulics, open channel flow, stream flow*

1. Introduction

In alluvial streams, hydrodynamic forces are exerted on the sediment particles at the bed surface. An increase in flow velocity induces an increased magnitude of hydrodynamic forces. Consequently, sediment particles begin to move if a condition is eventually reached when the hydrodynamic forces go beyond a critical value. The initial motion of sediment particles is commonly called *incipient motion*. The condition that is just adequate to initiate sediment motion is termed *threshold* or *critical condition*. The threshold of sediment motion in fluvial geomorphology or mobile-bed hydraulics is an important module of the management of river systems and constitutes the fundamental mechanism of the sediment transport theory. Importantly, if the stream flow is increased beyond that for the threshold condition, then sediment transport takes place.

The doctoral dissertation of Albert Frank Shields (1936) on sediment movement submitted in Technischen Hochshcule Berlin was a phenomenal work [see Kennedy (1995)]. The Shields diagram is considered to be the benchmark of any sediment transport research. His pioneering work has inspired numerous researchers and is widely applied in the fields. Although a few attempts were made before Shields (1936), they were basically empirical formulation with limited applicability. While his diagram is widely used (Task Committee, 1966), Miller *et al.* (1977), Mantz (1977), Yalin and Karahan (1979) and Buffington (1999) expressed some dissatisfaction, because the Shields diagram departs from the experimental data in smooth and rough-flow regimes (Yalin and Karahan, 1979). That is why numerous attempts have so far been made to propose the modifications of Shields diagram, to study the sediment threshold experimentally and to develop theoretically models based on analytical and probabilistic approaches. Miller *et al.* (1977), Buffington and Montgomery (1997) and Paphitis (2001) put forward a survey of the empirical curves, that are commonly used for the predication of sediment threshold, using sets of threshold data of previous investigator; whereas Lavelle and Mofjeld (1987) compiled a bibliographical review.

The aim of this paper is to present a comprehensive state-of-the-art review of the important laboratory experimental and theoretical investigations on sediment threshold under steady stream flow. Attentions are primarily paid on the empirical concepts, hydrodynamic background and the mathematical treatment of the problem. The influence of the turbulent bursting on sediment threshold is also discussed.

2. Definition of Sediment Threshold

The first type of definition of sediment threshold is based on sediment flux. Shields (1936) proposed a concept of sediment threshold that the bed shear stress has a value for which the extrapolated sediment flux becomes zero. Alternatively, USWES (1936) put forward a concept of sediment threshold that the tractive force brings about *general motion* of bed particles. For

*Professor, Department of Civil Engineering, Indian Institute of Technology, Kharagpur 721302, West Bengal, India (Corresponding Author, E-mail: sdey@iitkgp.ac.in)
**Associate Professor, IIHR-Hydroscience and Engineering, The University of Iowa, Iowa City, IA 52242-1585, USA (E-mail: apapanic@engineering.uiowa.edu)

the median diameter of sediment particles less than 0.6 mm, this concept was found to be inadequate and general motion was redefined that sediment in motion should reasonably be represented by all sizes of bed particles and that sediment flux should go beyond 4.1×10^{-4} kg/sm.

The second type of definition is based on bed particle motion. Kramer (1935) designated four types of bed shear stress conditions for sediment bed for which:

1. No particles are in motion, termed *no transport*.
2. A few of the smallest particles are in motion at isolated zones, termed *weak transport*.
3. Many particles of mean size are in motion, termed *medium transport*.
4. Particles of all sizes are in motion at all points and at all times, termed *general transport*.

However, Kramer (1935) pointed out the difficulty of setting up clear limits between these regimes but defined threshold bed shear stress to be that stress initiating *general transport*. Vanoni (1964) proposed that the sediment threshold is the condition of particle motion in every two seconds at any location of a bed.

Sediment threshold is directly associated with the characteristics of the particle size distribution of the bed sediment sample. The result of the sieve analysis of an adequate number of representative sediment samples is presented by a *frequency curve* (also known as *probability density function curve*) [Fig. 1(a)] or as a *cumulative frequency curve* (also known as *particle size distribution curve*) [Fig. 1(b)]. Very often the distribution curve of sediments approaches the log-normal probability curve when plotted, as shown in Fig. 1(a), so that the distribution function is log-normal and is given by

$$f(d) = \frac{1}{\sqrt{2\pi}d\ln(\sigma_g)}\exp\left\{-0.5\left[\frac{\ln(d/d_{50})}{\ln(\sigma_g)}\right]^2\right\} \tag{1}$$

where d = sediment diameter; σ_g = geometric standard deviation, given by $(d_{84}/d_{16})^{0.5}$; and d_{50} = median particle diameter or 50 percent finer (by weight) particle diameter. Similarly, d_{84} and d_{16} are 84 and 16 percent finer diameters, respectively. For uniformly graded sediments, σ_g is less than 1.4. In the cumulative frequency curve [Fig. 1(b)], the ordinate indicates how much percent (by weight) of the total sample is finer than the diameter d of the abscissa.

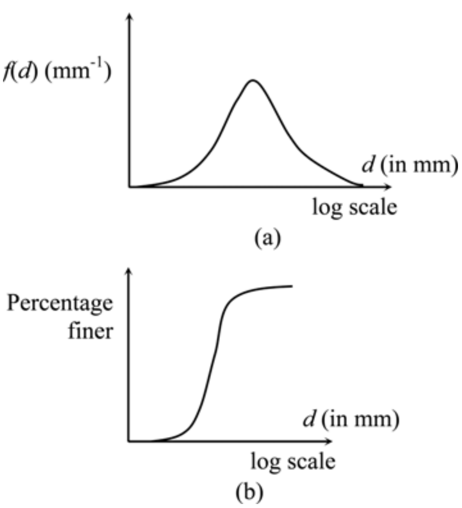

Fig. 1. (a) Frequency Curve and (b) Cumulative Frequency Curve

3. Competent Velocity Concept

A competent bed velocity or competent mean velocity is a velocity at particle level or mean velocity, which is just enough to move the particles of a given size. Though most of the earlier investigators provided valuable information regarding competency, many of them had not clearly reported the exact particle size and location of the bed velocity to be taken. Goncharov (1964) defined the threshold velocity as detachment velocity U_n, which was defined as the lowest average velocity at which individual particles continually detaches from the bed for which the mean value of the fluctuating lift force nearly equals the submerged weight of particle in fluid. He gave an equation as

$$U_n = \log(8.8h/d)\sqrt{0.57\Delta gd} \tag{2}$$

where h = flow depth; d = representative particle diameter, that is median particle diameter; g = acceleration due to gravity; $\Delta = s - 1$; s = relative density of sediment particles, that is ρ_s/ρ; ρ_s = mass density of sediment; and ρ = mass density of fluid.

Carstens (1966) reported an equation of critical or threshold velocity u_{cr} at the particle level having analyzed a large number of published data on threshold of sediment motion. It is

$$u_{cr}^2/\Delta gd \approx 3.61(\tan\varphi\cos\theta - \sin\theta) \tag{3}$$

where φ = angle of repose of sediment; and θ = angle made by the streamwise sloping bed with the horizontal.

Neill (1968) presented a conservative design curve for the movement of coarse uniform gravel in terms of average threshold velocity U_{cr} and represented it in an equation as

$$U_{cr}^2/\Delta gd = 2(h/d)^{1/3} \tag{4}$$

Zanke (1977) proposed the following equation:

$$U_{cr} = 2.8\sqrt{\Delta gd} + 14.7c_1\upsilon/d \tag{5}$$

where c_1 = a coefficient for cohesiveness varying from 1 for non-cohesive to 0.1 for cohesive sediments; and υ = kinematic viscosity of fluid. Many researchers have validly criticized the use of critical velocity equation as a criterion for threshold of sediment motion. The unanswered question is as to what is meant by competent velocity at particle level u_{cr} and average velocity for threshold condition U_{cr}. This confusion has led the hydraulicians to accept a more satisfactory quantity, the bed shear stress as a sediment threshold. Nevertheless, Yang (1973) developed a promising model for the estimation of average velocity for sediment threshold.

3.1 Yang's Competent Velocity Model

The forces acting on a spherical sediment particle at the bottom

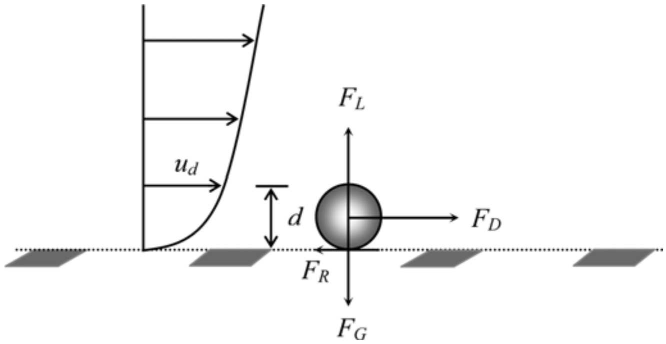

Fig. 2. Forces Acting on a Spherical Sediment Particle at the Bottom of an Open Channel

of an open channel, as considered by Yang (1973), are shown in Fig. 2. The drag force F_D is expressed as

$$F_D = C_D\frac{\pi}{8}d^2\rho u_d^2 \tag{6}$$

where C_D = drag coefficient; and u_d = velocity at a distance d above the bed.

The terminal fall velocity w_{ss} of a spherical particle is reached when there is a balance between the drag force F_D and submerged weight F_G of the particle. Thus, one can write

$$C_{D1}\frac{\pi}{8}d^2\rho w_{ss}^2 = \frac{\pi}{6}d^3(\rho_s-\rho)g(=F_G) \tag{7}$$

where C_{D1} = drag coefficient at w_{ss}, assumed as $\psi_1 C_D$. Eliminating C_D from Eqs. (6) and (7), the drag force becomes

$$F_D = \frac{\pi}{6\psi_1 w_{ss}^2}d^3(\rho_s-\rho)gu_d^2 \tag{8}$$

Considering the semi-logarithmic law for velocity distribution, velocity at particle level u_d and depth-averaged velocity U are obtained as:

$$u_d = B_r u_* \tag{9a}$$

$$U = u_*\left[5.75\left(\log\frac{h}{d}-1\right)+B_r\right] \tag{9b}$$

where B_r = roughness function; and u_* = shear velocity. Using Eqs. (9a) and (9b) into Eq. (8), yields

$$F_D = \frac{\pi}{6\psi_1}d^3(\rho_s-\rho)g\left(\frac{U}{w_{ss}}\right)^2\frac{B_r^2}{\left[5.75\left(\log\frac{h}{d}-1\right)+B_r\right]^2} \tag{10}$$

The lift force F_L acting on the particle is given by

$$F_L = C_L\frac{\pi}{8}d^2\rho u_d^2 \tag{11}$$

where C_L = lift coefficient, assumed as C_D/ψ_2.

Thus, using Eqs. (9a) and (9b) into Eq. (11), yields

$$F_L = \frac{\pi}{6\psi_1\psi_2}d^3(\rho_s-\rho)g\left(\frac{U}{w_{ss}}\right)^2\frac{B_r^2}{\left[5.75\left(\log\frac{h}{d}-1\right)+B_r\right]^2} \tag{12}$$

The drag force F_D is balanced by the resistance force F_R. Thus, one can write

$$F_D = F_R = \psi_3(F_G-F_L) \tag{13}$$

where ψ_3 = friction coefficient.

Inserting Eqs. (7), (10) and (12) in Eq. (13), one gets the equation of average critical or threshold velocity U_c as

$$\frac{U_c}{w_{ss}} = \sqrt{\frac{\psi_1\psi_2\psi_3}{\psi_2+\psi_3}}\left[\frac{5.75}{B_r}\left(\log\frac{h}{d}-1\right)+1\right] \tag{14}$$

Yang (1973) gave the equations for both smooth and rough boundaries as follows:

$$\frac{U_c}{w_{ss}} = \frac{2.5}{\log R_*-0.06}+0.66 \quad \text{for } 0<R_*<70 \tag{15}$$

$$\frac{U_c}{w_{ss}} = 2.05 \quad \text{for } R_*\geq 70 \tag{16}$$

where R_* = particle Reynolds number, that is u_*d/υ.

4. Lift Force Concept

Einstein (1950), Velikanov (1955), Yalin (1963), Gessler (1966) and Ling (1995) assumed the sediment entrainment is solely due to lift. Principally the lift force may be induced for three reasons:

- The sediment particle laying on the bottom of a channel where the velocity gradient is the maximum; and thus a pressure difference is created resulting in a lift acting on the particle.
- The sediment particle might experience lift due to the upward velocity component in the vicinity of the bed as a result of turbulence fluctuations.
- The slip spinning motion of sediment particle results in lift due to Magnus effect [see Dey (1999)].

Importantly, if the magnitude of the lift tends to equal the submerged weight of the particle, the nominal drag force would be enough to cause a threshold motion of the particle.

Jeffreys (1929) showed that the conventional hydrodynamics provides a simple justification of lifting and carrying solid particles by the fluid. Assuming a potential flow over a circular cylinder having its axis perpendicular to the flow, lift prevails if the following condition is maintained:

$$(3+\pi^2)U^2 > 9\Delta g r_1 \tag{17}$$

where r_1 = radius of the cylinder. On experimental verification of the above model, it was felt that modified factors should be taken into account, as the two-dimensional model behaves in dissimilar way for flow past a spherical particle in three-dimension. The basic deficiency of Jeffreys' model is that the drag forces are totally ignored.

Reitz (1936) discussed a similar idea and suggested to express the threshold of sediment motion with a lift model. Circulation and viscosity are imperative parameters of the analysis.

Lane and Kalinske (1939) stressed on turbulence for the determination of lift and assumed that: (a) particles that have a settling velocity smaller than the instantaneous vertical velocity fluctuations in the vicinity of the bed experience lift; (b) the velocity fluctuations vary following the normal-error law; and (c) the velocity fluctuations and shear velocities are correlated.

White (1940) carried out a single experiment and found that the lift on an individual particle is very small compared to its weight. However, Einstein and El-Samni (1949) measured the lift force directly as a pressure difference. They carried out experiments using plastic spherical balls (d = 68.6 mm) and gravel (d_{50} = 68.6 mm) having the considerable spread of particle size. They proposed

$$f_L = 0.5 C_L \rho u_{0.35d}^2 \tag{18}$$

where f_L = lift force per unit area of the particle; C_L = lift coefficient assumed as 0.178; and $u_{0.35d}$ = measured velocity of flow at a distance of 0.35 diameter from the theoretical wall. They also studied the turbulent fluctuations on the lift. These experiments revealed a constant average lift force with random fluctuations superimposed, following the normal-error law.

Iwagaki (1956) worked on the problem of sediment threshold using the shear stress concept. His analysis with and without considering lift does not change the critical tractive force significantly, thereby concluding that the lift force is of secondary significance.

The results of the study of Einstein and El-Samni (1949) were used by the Task Committee (1966), who calculated f_L/τ_c; where τ_c = threshold bed shear stress. This ratio was found to be about 2.5, suggesting that the lift forces are of considerable importance in the mechanism of the threshold of sediment motion. Nevertheless, once the particle moves, lift and drag forces tend to diminish and increase, respectively, as pointed out by Chepil (1961). In a wind stream on surface roughness elements as hemispheres, Chepil (1961) observed that the lift to drag ratio is about 0.85 for $47 < UD/\upsilon < 5\times10^3$. Aksoy (1972) measured forces on a 20 mm sphere at $R_* = 300$ and found a ratio of lift to drag of about 0.1. Bagnold (1974) investigated lift and drag on a 16 mm sphere for R_* values of about 800 and obtained lift to drag ratio about 0.5. Brayshaw *et al.* (1983) measured lift and drag on a 115 mm hemisphere at $R_* = 5.2\times10^4$ and found lift to drag ratio about 1.8.

Saffman (1965) studied the lift on a sphere for $R_* < 5$ to present an expression for lift that Yalin (1977) found to be compatible with the Shields criterion for that range. Apperley (1968) measured hydrodynamic forces on a 6.4 mm sphere laid on 6.4 mm gravels for $R_* = 70$ and found lift to drag ratio as 0.5 that increases to 0.78 if the sphere was raised by 0.25 times its diameter and further rise reduces this value. Coleman (1967) studied the lift forces acting on a sphere placed on a hypothetical streambed. Plastic and steel spheres were tested and variation of lift coefficient with Reynolds number was obtained. For Reynolds number less than 100, the negative (downwards) values of lift force could not be explained. Watters and Rao (1977) measured lift and drag forces on 95.3 mm sphere in different bed configurations and observed negative lift for $20 < R_* < 100$. Davies and Samad (1978) reported that the lift force on a sphere adjacent to a boundary becomes negative if both significant underflows occur beneath the sphere and the flow has a condition $R_* < 5$. However, for $R_* \geq 5$, lift is positive.

Although the lift forces obviously contribute to the sediment threshold problem, the phenomenon of lift on sediment particles is still incompletely understood and inadequate experimental results are available to determine reliable quantitative relationships, as such no critical lift criterion has been presented as yet which could be a ready reference for the determination of the sediment threshold condition. The occurrence of negative lift at low R_* has been well established, but its variation with R_* and magnitude remain uncertain. It was seen that besides the lift force, the drag force always exists to contribute towards the threshold movement of the bed sediment. For higher R_*, the ratio of lift to drag (or the correlation between lift and drag) is another uncertain aspect, although the lift is indisputably positive.

5. Threshold Shear Stress Concept

5.1 Empirical Equations of Threshold Shear Stress

Several attempts have been made in laboratory and field studies to correlate the threshold bed shear stress and sediment properties. Kramer (1935) carried out experiments in a flume using quartz particles of relative density 2.7. On the basis of these experiments and data available from other sources, he proposed

$$\tau_c = 29\sqrt{(\rho_s - \rho)gd/M} \tag{19}$$

where τ_c = threshold or critical bed shear stress (in g/m^2); M = uniformity coefficient of Kramer; and d is in mm. Eq. (19) is based on d ranging from 0.24 mm to 6.52 mm and the uniformity coefficient varying from 0.265 to 1.

USWES (1936) recommended the formula as

$$\tau_c = 0.285\sqrt{\Delta d/M} \tag{20}$$

where τ_c is in Pa; and d is in mm. This relationship is valid for d ranging from 0.205 mm to 4.077 mm and M ranging from 0.280 to 0.643.

Leliavsky (1966) gave a simple threshold bed shear stress equation as

$$\tau_c = 166d \tag{21}$$

where τ_c is in g/m^2; and d is in mm. It can be seen that none of the formulae account for the effect of fluid viscosity, and further each of these formulae gives results that differ from each other considerably. This discrepancy might be a result of variation in the definition of sediment threshold. However, the empirical formulae can estimate the approximate threshold bed shear stress but their use is not recommended since more reliable methods are available.

5.2 Theoretical and Semi-Theoretical Analyses

5.2.1 Shields Diagram

Fig. 3 shows the steady state flow over a bed composed of noncohesive sediment particles. These particles do not move at very low velocity. As the flow velocity increases to a certain value, the driving forces on the sediment particles exceed the stabilizing forces, and the sediment starts to move. Shields (1936) was the pioneer to present a semi-theoretical solution to the sediment threshold problem. The threshold of sediment particle motion is governed by the ratio of the driving (as drag force) to the stabilizing forces.

The driving force is the drag force F_D due to the flow exerted on the sediment particle and is given by

$$F_D = C_D\frac{1}{2}\rho u^2 A = f_1\left(a_1, \frac{ud}{\upsilon}\right)\rho d^2 u^2 \tag{22}$$

where u = velocity at elevation $z = a_2 d$; A = frontal area of the particle; and a_1 = particle shape factor. The semi-logarithmic velocity distribution for the flow over rough and smooth beds is given by

$$\frac{u}{u_*} = 5.75\log\frac{z}{k_s} + \frac{zu_*}{\upsilon} = 5.85\log a_2 + f_2(R_*) \tag{23}$$

where k_s = roughness height being proportional to d. Thus, the drag force is

$$F_D = \tau_0 d^2 f_3(a_1, a_2, R_*) \tag{24}$$

The resistance to motion F_R was assumed to be dependent only upon the bed roughness and the submerged weight F_G of the particle. That is

$$F_R = a_3 \Delta\rho g d^3 \tag{25}$$

where a_3 = roughness factor.

At the threshold condition, when the sediment particle is about to move, $u_* \rightarrow u_{*c}$ (that is the critical shear velocity), and then the drag force is balanced by the resistance. Therefore, one can write

$$F_D = F_R \tag{26}$$

Rearranging the terms

$$\frac{u_{*c}^2}{\Delta g d} = \frac{\tau_c}{\Delta\rho g d} = f(R_*) \tag{27}$$

The Shields parameter Θ is defined as

$$\Theta = \frac{u_*^2}{\Delta g d} \tag{28}$$

Therefore, Eq. (27) is expressed as a critical Shields parameter Θ_c that is

$$\Theta_c = f(R_*) \tag{29}$$

Fig. 4 that shows the Shields' experimental results, which relate critical Shields parameter Θ_c and R_*, is known as *Shields diagram*. The threshold of sediment motion occurs when $\Theta \rightarrow \Theta_c$ or $\tau_0 \rightarrow \tau_c$ or $u_* \rightarrow u_{*c}$. The Fig. 4 depicts three distinct zones:

1. Hydraulically smooth flow for $R_* \leq 2$: In this case, d is much smaller than the thickness of viscous sub-layer; and experimentally it was found that $\Theta_c = 0.1/R_*$.
2. Hydraulically rough flow for $R_* \geq 500$: The viscous sub-layer does not exist. The critical Shields parameter Θ_c is invariant of the fluid viscosity and has a constant value of 0.056.
3. Hydraulically transitional flow for $2 \leq R_* \leq 500$: Sediment particles are of the order of the thickness of viscous sub-layer. There is a minimum value of Θ_c = 0.032 corresponding to $R_* = 10$.

The drawback of the Shields theory is that the viscous sub-layer does not have any effect on the velocity distribution when $R_* \geq 70$, but his diagram shows that Θ_c still varies with R_* when the latter is greater than seventy. Furthermore, Shields used the bed shear stress and the shear velocity in his diagram as dependent and independent variables, which is not appropriate as they are interchangeable. Consequently the threshold bed shear

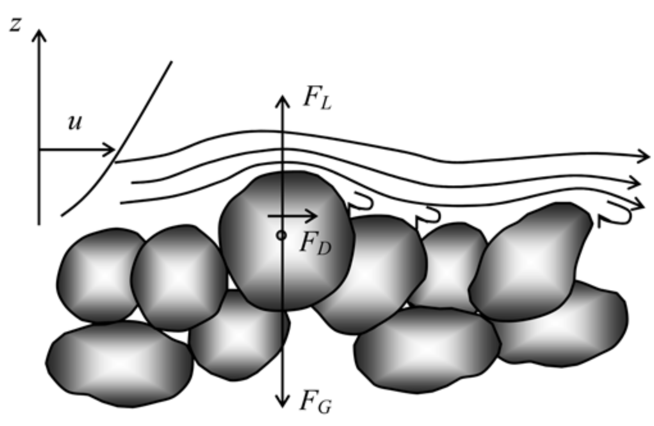

Fig. 3. Forces Acting on a Sediment Particle Resting on Bed

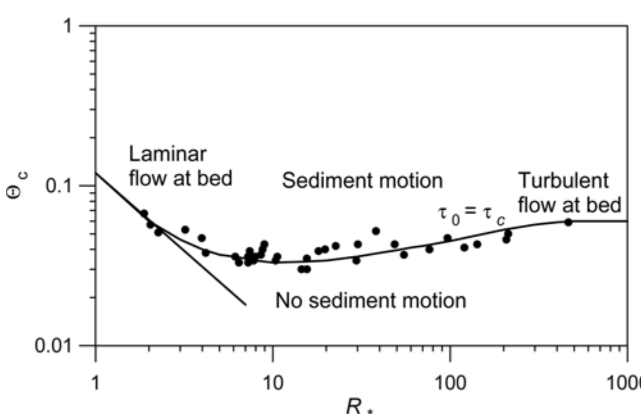

Fig. 4. Shields Parameter Θ_c as a Function of Particle Reynolds number R_*

stress must be determined through trial and error method. However, van Rijn (1984) gave the empirical equations for the Shields curve as

$$\Theta_c(D_* \le 4) = 0.24/D_* \tag{30a}$$

$$\Theta_c(4 \le D_* \le 10) = 0.14/D_*^{0.64} \tag{30b}$$

$$\Theta_c(10 \le D_* \le 20) = 0.04/D_*^{0.1} \tag{30c}$$

$$\Theta_c(20 < D_* \le 150) = 0.013 D_*^{0.29} \tag{30d}$$

$$\Theta_c(D_* > 150) = 0.055 \tag{30e}$$

where D_* = particle parameter, that is $d(\Delta g/\upsilon^2)^{1/3}$. Also, Julien (1998) proposed the empirical equations for the Shields curve as a function of particle parameter and angle of repose φ as

$$\Theta_c(D_* \le 0.3) = 0.5\tan\varphi \tag{31a}$$

$$\Theta_c(0.3 < D_* \le 19) = 0.25\tan\varphi/D_*^{0.6} \tag{31b}$$

$$\Theta_c(19 < D_* \le 150) = 0.013 D_*^{0.4}\tan\varphi \tag{31c}$$

$$\Theta_c(D_* > 150) = 0.06\tan\varphi \tag{31d}$$

Fenton and Abbott (1977) included the particle protrusion in Shields' work and found that the threshold shear stress for sediment particles resting on the top of an otherwise flat bed in turbulent-rough regime is one-sixth of that for beds where all particles are at same level.

5.2.3 White's Analysis

If one neglects the lift force, at limiting equilibrium the drag force (shear drag) is balanced by the frictional resistance. White (1940) classified a high-speed case ($R_* \ge 3.5$) and a low-speed case ($R_* < 3.5$).

High-speed case ($R_* \ge 3.5$): High flow velocity is capable of moving larger sediment particles. In this case, the drag due to skin friction is negligible as compared to the drag due to pressure difference. If p_f is the packing coefficient defined by Nd^2, where N is the number of particles per unit area, the shear drag per particle (that is τ_0/N) is given by $\tau_0 d^2/p_f$. At the threshold of motion of a particle resting on a horizontal bed, the shear drag equals the product of the submerged weight of the particle and the frictional coefficient $\tan\varphi$. Therefore, one gets

$$\Theta_c = \frac{\pi}{6} p_f \tan\varphi \tag{32}$$

In Eq. (32), White introduced a factor termed *turbulence factor* T_f, which is the ratio of the instantaneous bed shear stress to the mean bed shear stress. Hence, Eq. (32) becomes

$$\Theta_c = \frac{\pi}{6} p_f T_f \tan\varphi \tag{33}$$

He experimentally obtained $p_f = 0.4$ and $T_f = 4$ for fully developed turbulent flow.

Low-speed case ($R_* < 3.5$): Low flow velocity is capable of moving smaller sediment particles. In this case, the drag due to pressure difference acting on the particle is very insignificant as compared to the viscous force. However, the upper portion of the particle is exposed to the shear drag that acts above the center of gravity of the particle. This effect is taken into account introducing a coefficient α_f. Therefore, the equation of sediment threshold is

$$\Theta_c = \frac{\pi}{6} p_f \alpha_f \tan\varphi \tag{34}$$

He experimentally obtained $p_f\alpha_f = 0.34$ as an average value.

5.2.4 Wilberg and Smith's Approach

On a horizontal bed, the expression for the force balance given by Wiberg and Smith (1987) becomes

$$(F_G - F_L)\tan\varphi = F_D \tag{35}$$

They expressed the submerged weight of the particle F_G, drag force F_D and lift force F_L as follows:

$$F_G = \Delta\rho g V \tag{36}$$

$$F_D = C_D \frac{1}{2}\rho u^2 A_x = C_D \frac{1}{2}\tau_0 [f^2(z/z_0)] A_x \tag{37}$$

$$F_L = C_D \frac{1}{2}\rho(u_T^2 - u_B^2) A_x = C_D \frac{1}{2}\tau_0 [f^2(z_T/z_0) - f^2(z_B/z_0)] A_x \tag{38}$$

where V = volume of the particle; A_x = frontal area of the particle; u = velocity at z above the bed; z_0 = zero-velocity level; u_T = velocity at the top of the particle; u_B = velocity at the bottom of the particle; z_T = height of the top point of the particle from the bed; and z_B = height of the bottom point of the particle from the bed. They assumed the bed level passing through the mid points (those are the contact points) of the bed particles.

Using Eqs. (36) – (38), the following expression for Θ_c is obtained:

$$\Theta_c = \frac{2}{C_D\alpha_0} \cdot \frac{1}{f^2(z/z_0)} \cdot \frac{\tan\varphi}{1 + (F_L/F_D)_c \tan\varphi} \tag{39}$$

where $\alpha_0 = A_x d/V$. They used C_D as a function of particle Reynolds number (Schlichting 1960), $C_L = 0.2$ and $\cos\varphi = [(d/k_s) + z_*]/[(d/k_s) + 1]$. For natural sands, $z_* = -0.02$. For smooth regime ($R_* < 3$) and transitional regime ($3 \le R_* < 100$), Reichardt's (1951) equation of velocity distribution was used (see Eq. (72)), while for rough regime ($R_* \ge 100$), universal semi-logarithmic velocity distribution (see Eq. (75)) was taken into consideration. Fig. 5 shows Θ_c as a function of R_* for different d/k_s.

5.2.5 Equations of Other Investigators

Kurihara (1948) made an extension of the work of White (1940). He considered that the bed shear stress is a combination of the time-averaged bed shear stress due to main flow and the bed shear stress resulting from turbulent fluctuations. He obtained an expression for the turbulence factor T_f in terms of R_*, turbulence intensity and the probability of bed shear stress increment. Since his theoretical equations were quite complicated, he proposed the following empirical equations of threshold bed shear stress:

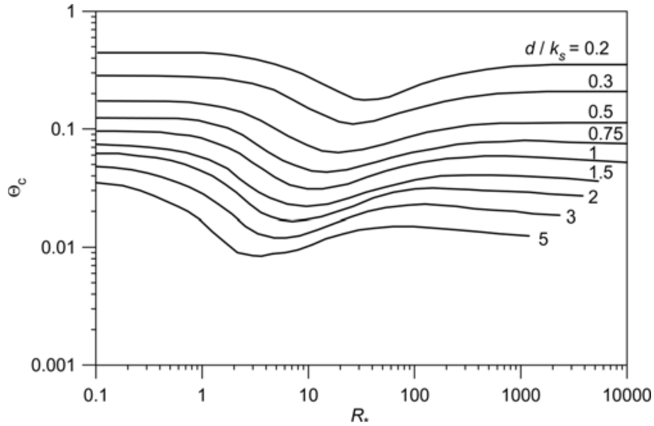

Fig. 5. Θ_c as a Function of R_* for Different d/k_s

$$\Theta_c(X_2 \le 0.1) = (0.047\log X_2 - 0.023)\beta_2 \quad (40a)$$

$$\Theta_c(0.1 < X_2 \le 0.25) = (0.01\log X_2 + 0.034)\beta_2 \quad (40b)$$

$$\Theta_c(X_2 > 0.25) = (0.0517\log X_2 + 0.057)\beta_2 \quad (40c)$$

where $X_2 \approx 4.67\times10^{-3}[\Delta g/(\upsilon^2\beta_2)]^{1/3}d$; $\beta_2 = (M+2)/(1+2M)$; and M = uniformity coefficient of Kramer (1935) varying from 0.265 to 1.

Iwagaki (1956) considered the equilibrium of a single spherical particle, placed on a rough surface and found the conditions necessary for the equilibrium of a particle. However, in practice, this condition seldom occurs because of the existence of other particles. The theoretical equation given by Iwagaki (1956) is of the form

$$\Theta_c = \frac{\tan\varphi}{\varepsilon_x \Psi_s R_*} \quad (41)$$

where ε_s = empirical coefficient to take care of the sheltering effect; and Ψ_s = function of R_*.

Egiazaroff (1965) presented yet another derivation for Θ_c as a function of R_*. The essential feature of his analysis is the assumption that at threshold condition, the velocity at an elevation of $0.63d$ (above the bottom of particle) equals the fall velocity w_{ss} of particle. He gave the equation as

$$\Theta_c = \frac{1.33}{C_D[a_r + 5.75\log(0.63)]} \quad (42)$$

where a_r = 8.5; and C_D = drag coefficient = 0.4 for large R_*, and both a_r and C_D increase for low R_*. His results do not agree quantitatively with the Shields curve.

Mantz (1977) proposed the extended Shields diagram for a flat sedimentary bed to obtain the condition of maximum stability (Fig. 6). Yalin and Karahan (1979) presented a graphical presentation of Θ_c versus R_*, using a large volume of data collected by various investigators (Fig. 6). Their curve is regarded as a superior curve to the commonly used Shields curve.

Soulsby and Whitehouse (1997) presented the critical Shields parameter Θ_c in terms of the nondimensionl particle size D_* to avoid the trial and error estimation of τ_c. It is

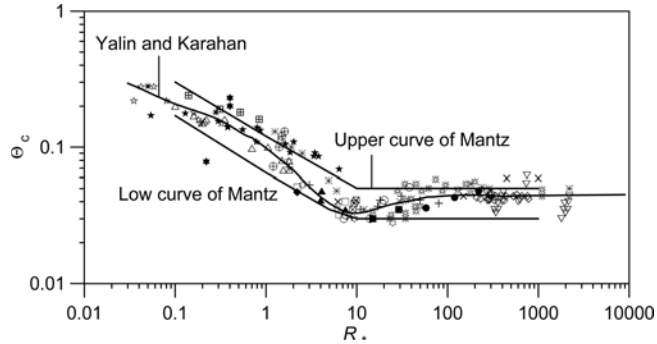

Fig. 6. Curves (Θ_c Versus R_*) of Mantz (1977) and Yalin and Karahan (1979)

$$\Theta_c = \frac{0.24}{D_*} + 0.055[1 - \exp(-0.02D_*)] \quad (43)$$

6. Probabilistic Concept

The threshold of sediment motion is probabilistic in nature. It depends primarily on the turbulence characteristics of flow in association with the location of a specific particle relative to the surrounding particles of various sizes and their orientations. The concept gives the mean condition that there is a fifty percent chance for a given particle to move under specific flow and sediment conditions.

Gessler (1970) estimated the probability that particles of a specific size remain stationary. It was revealed that the probability of a given particle to remain stationary depends strongly on the Shields parameter and feebly on particle Reynolds number. The ratio τ_c/τ_0 is directly related to the probability that a sediment particle remains stationary. This concept is useful to determine the particle size distribution in the armor layer. Therefore, the probability of particles of a specific size to remain stationary is

$$P_0(d) = \int_{d_{\min}}^{d} p_0(d)dd \quad (44)$$

where p_0 = frequency function of the original distribution. The armor layer particle size frequency is

$$p_a(d) = k_1 q p_0(d) \quad (45)$$

where q = probability for a particle size d; and k_1 = constant. The quantity q varies with the particle size d that can be determined by

$$\int_{d_{\min}}^{d_{\max}} p_a(d)dd = 1 \quad (46)$$

The expression for particle size distribution of the armor layer is

$$P(d) = \frac{\int_{d_{\min}}^{d} q p_0(d)dd}{\int_{d_{\min}}^{d_{\max}} q p_0(d)dd} \quad (47)$$

The expression for particle size distribution of the moving particles is

$$P(d) = \frac{\int_{d_{\min}}^{d} (1-q)p_0(d)dd}{\int_{d_{\min}}^{d_{\max}} (1-q)p_0(d)dd} \tag{48}$$

The sediment particles of finer size than that obtained from Eq. (48) cease to move with the formation of an armor layer.

The most detailed experimental observations on the bed shear stress fluctuation carried out so far are due to Grass (1970). He proposed to use a probabilistic description of the stresses acting on a single particle to achieve motion. He proposed two probability distributions: one for the bed shear stress τ_w induced by the fluid and other for the bed shear stress τ_{wc} required to put the particle in motion. When these two distributions start overlapping, the particles that have the lowest threshold bed shear stress start to move. The concept of Grass is insightful, as it is evident that sediment threshold takes place where two probability distributions overlap, as shown in Fig. 7. The representative magnitudes of the probability distributions are their standard deviations being used to describe the distance of the two mean bed shear stresses as $\bar{\tau}_{wc} - \bar{\tau}_w = n(\sigma_c - \sigma_\tau)$. From the observations, he obtained the relationships as $s_t = 0.4\bar{\tau}_w$ and $\sigma_c = 0.3\bar{\tau}_{wc}$, which leads to $\bar{\tau}_w = \bar{\tau}_{wc}(1 - 0.3n)/(1 + 0.4n)$. It is found that for $n = 0.625$, the result collapses with that of Shields.

Mingmin and Qiwei (1982) developed a stochastic model for the incipient motion of sediment particles. They expressed the statistical parameters using the velocity of bottom flow and particle size. The probability of incipient motion, the life distribution of stationary particles, the number of distributions of particles in incipient motion, and the intensity of incipient motion were derived. Wu and Chou (2003) studied the rolling and lifting probabilities for sediment entrainment by incorporating the probabilistic features of the turbulent fluctuation and bed particle geometry. These probabilities were linked to the two separate criteria for incipient motion to explore the critical entrainment probabilities. The critical entrainment probabilities are highly variable functions of the particle Reynolds number. Thus, an inconsistent probability corresponding to the critical particle entrainment was found.

The discovery of the so-called turbulent bursting phenomenon in turbulent flows by Kline *et al.* (1967) generated a new interest in further studying the structures of boundary turbulence and then applying this new knowledge to the incipient motion or sediment threshold problem. In an attempt to link the characteristics of turbulent episodes with the initial entrainment of sediment, several researchers (e.g., Nelson *et al.*, 1995; Clifford *et al.*, 1991) suggested that the Reynolds stress component is not the most relevant component to sediment entrainment. Recently, the studies of Krogstad *et al.* (1992), and Papanicolaou *et al.* (2001) provided further evidence that the presence of bed packing conditions in gravel bed streams affects the turbulence characteristics of the flow and as a result the entrainment of sediment. Flows over lower packing density bed configurations (i.e., beds that are loosely packed) behave differently than flows over higher density bed configurations (Church, 1978). Along the same lines, the quadrant analysis of Papanicolaou *et al.* (2001) shows that the ratio of the Reynolds stress to the standard deviation of the downstream velocity is smaller in the low-density cases than the densely packed cases. Hence, incipient motion criteria based solely on time-averaged bed shear stress may under-predict transport, especially in low-density packing cases.

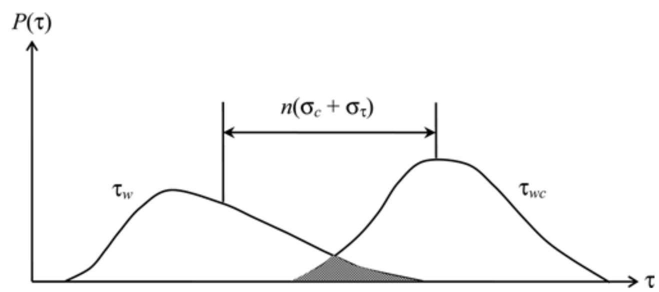

Fig. 7. Probability Distributions of Bed Shear Stress τ_w Due to Flow and Critical Bed Shear Stress τ_{wc} Associated with the Movement of Individual Particles

Based on the above considerations, Papanicolaou *et al*. (2002) developed a stochastic sediment threshold model that considers the role of near-bed turbulent structures and bed microtopography upon the initiation of unisized particle motion. The model was primarily based on the hypothesis that the probability of exceedance of the minimum moment required to initiate rolling motion equals the probability of first displacement of a particle. The theoretical derivation was complemented by the experimental detections of the probability and near-bed turbulence for three different packing regimes; namely, isolated, wake interference and skimming. They found it reasonable to consider that on average (temporal and spatial) and for a sufficient large number of data the probability of the occurrence of intermittent turbulent events equals the sediment entrainment probability. The co-relationship of flow and sediment was accounted for by deriving a general non-central chi-squared distribution for the resultant moment that acts on a particle for the isolated and wake interference regimes. The shape of this distribution revealed the changes in bed microtopography. Isolated flow regimes were represented by skewed bell-shaped distributions, whereas wake interference regimes by an exponential distribution. Skimming flow regimes were described by a standard chi-squared type distribution. Higher packing conditions resulted in less frequent small magnitude events.

Dancey *et al*. (2002) introduced a new criterion for the experimental characterization of the threshold of sediment motion as bed load. It was implemented in an experimental study of the role of sediment packing density on bed behavior near the threshold of motion. The criterion that might be interpreted as the probability of individual particle motion considers the statistical nature of sediment motion in turbulent flow and the time scale of the flow. The sediment threshold was specified by a fixed value of the probability. However, it was found that a threshold criterion based upon the probability of particle motion could yield relatively active sediment beds where the mechanism is strongly dependent upon sediment packing density.

7. Sediment Threshold Model for Horizontal Bed (Dey, 1999)

In a unidirectional steady-uniform flow over a sedimentary bed, the most stable three-dimensional configuration of a spherical solitary sediment particle of diameter D resting over three closely packed spherical particles of identical diameter d forming the sediment bed is shown in Fig. 8. Depending on the orientation of the bed particles, the solitary particle has a tendency either to roll over the valley formed by the two particles or to roll over the summit of a single particle due to the hydrodynamic forces.

When the solitary particle is about to dislodge downstream from its original position, the equation of moment about the point of contact M of the solitary particle downstream is

$$(F_L - F_G)X + F_D Z = 0 \tag{49}$$

where X and Z = horizontal and vertical lever arms, respectively (Fig. 8). The expressions of X and Z given by Dey *et al.* (1999) [also see Dey (1999)] are

$$X = \frac{\sqrt{3}}{4} \cdot \frac{Dd}{D+d} \tag{50}$$

$$Z = \frac{1}{2\sqrt{3}} \cdot \frac{D}{D+d}(3D^2 + 6Dd - d^2)^{0.5} \tag{51}$$

The submerged weight of the solitary particle is

$$F_G = \frac{\pi}{6}D^3(\rho_s - \rho)g \tag{52}$$

The drag force developed due to pressure and viscous skin frictional forces is given by

$$F_D = C_D \frac{\pi}{8} D^2 \rho u_m^2 \tag{53}$$

where C_D = drag coefficient; and u_m = mean flow velocity received by the frontal area (the projected area of the particle being right angles to the direction of flow) of the solitary particle. The empirical equation of the drag coefficient C_D given by Morsi and Alexander (1972) is used. It is

$$C_D = a + bR^{-1} + cR^{-2} \tag{54}$$

where R = flow Reynolds number at particle level (= $u_m D/\upsilon$); and a, b and c = coefficients dependent on R.

The lift force, caused by the velocity gradient, in a shear flow is termed *lift due to shear effect* (F_{Ls}). For a sphere in a viscous flow, Saffman (1968) proposed the following equation:

$$F_{Ls} = C_L \rho D^2 u_m \left(\upsilon \frac{\partial u}{\partial z}\right)^{0.5} \tag{55}$$

where $\partial u/\partial z$ = velocity gradient; and u = time-averaged flow velocity at z.

For low particle Reynolds number R_*, Eq. (55) is applicable. However, for large Reynolds number ($R_* > 3$), the solitary particle spins into the groove, formed by the three closely packed bed particles, just before dislodging downstream from its original position due to large velocity gradient at the particle level (Dey *et al*., 1999). To be more explicit, the hydrodynamic force acting on the upper portion of particle is significantly greater than that acting on the lower portion of particle, resulting in a turning moment to the particle. Therefore, the inclusion of slip-spinning mode is significant in the analysis of the threshold of sediment motion. The lift force, caused by the spinning mode of particle, is termed *lift due to Magnus effect* (F_{Lm}). Rubinow and Keller (1961) formulated it as

$$F_{Lm} = C_L \rho D^3 u_m \omega \tag{56}$$

where ω = angular velocity of spinning particle. According to Saffman (1965), the maximum angular velocity achieved by a solitary particle equals $0.5\partial u/\partial z$. Thus, Eq. (56) is rewritten as

$$F_{Lm} = 0.5 C_L \rho D^3 u_m \frac{\partial u}{\partial z} \tag{57}$$

The total lift force F_L, a combination of F_{Ls} and F_{Lm}, is expressed as

$$F_L = C_L \rho D^2 u_m \left(\frac{\partial u}{\partial z}\right)^{0.5} \left[\upsilon^{0.5} + 0.5 f(R_*) D \left(\frac{\partial u}{\partial z}\right)^{0.5}\right] \tag{58}$$

where $f(R_*) = 1$ for $R_* \geq 3$; $f(R_*) = 0$ for $R_* < 3$; and R_* = particle Reynolds number (= $u_* d/\upsilon$). For low values of R_* ($R_* < 3$), particles do not spin.

Using Eqs. (50) - (53) and (58) into Eq. (49), the equation of the threshold of sediment motion is obtained as

$$\Theta_c = \frac{2\pi\hat{d}}{\pi C_D \hat{u}_m^2 (3 + 6\hat{d} - \hat{d}^2)^{0.5} + 6 C_L \hat{d} \hat{u}_m (\partial\hat{u}/\partial\hat{z})\{2[(R_*/\hat{d})\partial\hat{u}/\partial\hat{z}]^{-0.5} + f(R_*)\}} \tag{59}$$

where $\hat{u}_m = u_m/u_*$; $\hat{d} = d/D$; $\hat{u} = u/u_*$; and $\hat{z} = z/D$.

The particle diameter ratio $\hat{d}$ is determined from the information on angle of repose of bed sediments, using the expression

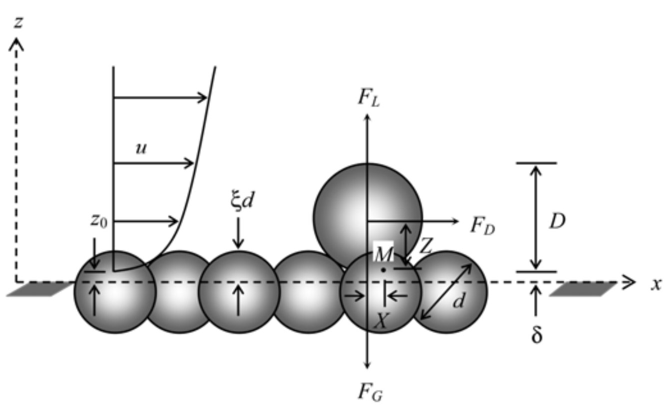

Fig. 8. Diagrammatic Presentation of Forces Acting on a Solitary Particle

given by Ippen and Eagleson (1955) for the spherical sediments as

$$\hat{d} = \frac{2\tan\varphi[6\tan\varphi + (48\tan^2\varphi + 27)^{0.5}]}{4\tan^2\varphi + 9} \tag{60}$$

The threshold of sediment motion over sedimentary bed is controlled by the applied instantaneous shear stress at the bed due to the turbulent fluctuations. The most important event for the threshold of sediment motion is the sweep event, which has a dominant role in entraining the sediment particles at the bed. The sweep event applies shear in the direction of the flow and provides additional forces to the viscous shear stress. Keshavarzy and Ball (1996) reported that the magnitude of instantaneous bed shear stress in a sweep event is much larger than the time-averaged bed shear stress. Thus, they proposed the following equation of total bed shear velocity for rough-turbulent regime:

$$u_{*t} = (1 + p\sqrt{\alpha - 1}\cos\psi)u_* = \eta_t u_* \tag{61}$$

where u_{*t} = total shear velocity (= $u_* + u_t$); u_t = instantaneous shear velocity [= $u_* p(\alpha - 1)^{0.5}\cos\psi$ or $(\tau_t/\rho)^{0.5}$]; τ_t = instantaneous bed shear stress; p = probability of occurring sweep event; $\alpha = \tau_t/\tau_0$; and ψ = sweep angle. Therefore, Θ_c calculated from Eq. (59) is modified as

$$\Theta_c = \Theta_c(\text{Eq. } 59)/\eta_t^2 \tag{62}$$

Keshavarzy and Ball (1996) observed that the frequency of sweep event p and the sweep angle ψ are 30 percent and 22°, respectively, in the vicinity of the bed. In smooth regime, η_t is considered as unity. To solve Eq. (62), one needs additional information as given below.

The particle parameter $\tilde{d}$ is given by $(d/\upsilon)[gd(\rho_s - \rho)/\rho]^{0.5}$. The following equation is used to compute $\tilde{d}$:

$$\tilde{d} = R_*(\hat{d}/\Theta_c)^{0.5} \tag{63}$$

The virtual bed level is considered to be at a depth of ξd below the top of the bed particles (Fig. 8). Thus, the normal distance δ between the virtual bed level and the bottom level of the solitary sediment particle given by Dey *et al*. (1999) is

$$\delta = \frac{1}{2\sqrt{3}}(3D^2 + 6Dd - d^2)^{0.5} - \frac{1}{2}(D + d) + \xi d \tag{64}$$

According to van Rijn (1984), $\xi = 0.25$.

The mean velocity of flow received by the frontal area of the solitary particle is given by

$$u_m = \frac{2\zeta}{A}\int_{\varepsilon}^{D+\delta} u[(z-\delta)(D+\delta-z)]^{0.5}dz \tag{65}$$

where A = frontal area of the solitary particle exposed to the flow, that is $(\pi D^2/4)\{1 - \arccos(1 - 2\hat{h}) + 2(1 - 2\hat{h})[\hat{h}(1 - \hat{h})]^{0.5}\}$; $\hat{h} = h/D$; $h = \varepsilon - \delta$; ζ = coefficient being less than unity; and ε = normal distance between the bottom level of the solitary particle or zero-velocity level and the virtual bed level. The introduction of ζ is pertinent here, because the summits of the bed particles upstream of the solitary particle obstruct the flow velocity to some extent. It was found that the value of ζ being 0.5 produced satisfactory results. The normalized mean velocity $\hat{u}_m$ is obtained as

$$\hat{u}_m = \frac{2\zeta}{\hat{A}}\int_{\hat{\varepsilon}}^{1+\hat{\delta}} \hat{u}[(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5}d\hat{z} \tag{66}$$

where $\hat{A} = A/D^2$; $\hat{\delta} = \delta/D$; and $\hat{\varepsilon} = \varepsilon/D$.

The velocity gradient $\partial u/\partial z$ can be obtained as follows:

$$\frac{\partial u}{\partial z} = \frac{1}{D+\delta+\varepsilon}\int_{\varepsilon}^{D+\delta}\frac{\partial u}{\partial z}dz = \frac{u_{D+\delta} - u_{\varepsilon}}{D+\delta-\varepsilon} \tag{67}$$

Thus, the normalized velocity gradient $\partial\hat{u}/\partial\hat{z}$ is given by

$$\frac{\partial\hat{u}}{\partial\hat{z}} = \frac{\hat{u}_{1+\hat{\delta}} - \hat{u}_{\hat{\varepsilon}}}{1+\hat{\delta}-\hat{\varepsilon}} \tag{68}$$

Case 1 ($R_* < 3$): The flow is hydraulically smooth when R_* is less than three because the bed roughness lies within the viscous sub-layer (Schlichting, 1960).

It is assumed that the velocity distribution of the flow is solely linear for $R_* < 3$. Hence, the expression for the velocity distribution is

$$\hat{u} = \frac{zu_*}{\upsilon} \tag{69}$$

Thus, the mean flow velocity $\hat{u}_m$ obtained using Eq. (69) is

$$\hat{u}_m = \frac{2\zeta R_*}{\hat{A}\hat{d}}\int_{\hat{\varepsilon}}^{1+\hat{\delta}}[(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5}\hat{z}d\hat{z} \tag{70}$$

where $\hat{\varepsilon} = 0$ if $\hat{\delta} \le 0$ and $\hat{\varepsilon} = \hat{\delta}$ if $\hat{\delta} > 0$.

The velocity gradient determined using Eq. (70) is

$$\frac{\partial\hat{u}}{\partial\hat{z}} = \frac{R_*}{\hat{d}} \tag{71}$$

Case 2 ($3 \le R_* \le 70$): The range of particle Reynolds number $3 \le R_* \le 70$ can be considered as transitional regime (Schlichting, 1960).

The equation of the velocity distribution for transitional regime proposed by Reichardt (1951) is used. It is

$$\hat{u} = \frac{1}{\kappa}\left\{\ln\left(1 + \frac{\kappa\hat{z}R_*}{\hat{d}}\right) - \left[1 - \exp\left(-\frac{\hat{z}R_*}{11.6\hat{d}}\right) - \frac{\hat{z}R_*}{11.6\hat{d}}\exp\left(-\frac{\hat{z}R_*}{3\hat{d}}\right)\right]\ln\left(\frac{\kappa\hat{z}_0R_*}{\hat{d}}\right)\right\} \tag{72}$$

where κ = von Karman constant (= 0.4); z_0 = zero-velocity level above the virtual bed level (= $0.033k_s$); and k_s = equivalent roughness height of Nikuradse, assumed as d (Wiberg and Smith, 1987).

The mean flow velocity $\hat{u}_m$ determined using Eq. (72) is

$$\hat{u}_m = \frac{2\zeta}{\kappa\hat{A}}\int_{\hat{\varepsilon}}^{1+\delta}[(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5}\left\{\ln\left(1+\frac{\kappa\hat{z}R_*}{\hat{d}}\right) - \left[1-\exp\left(-\frac{\hat{z}R_*}{11.6\hat{d}}\right) - \frac{\hat{z}R_*}{11.6\hat{d}}\exp\left(-\frac{\hat{z}R_*}{3\hat{d}}\right)\right]\ln\left(\frac{\kappa\hat{z}_0R_*}{\hat{d}}\right)\right\}d\hat{z} \tag{73}$$

where $\varepsilon = \hat{z}_0$ if $(\hat{z}_0 - \hat{\delta}) \geq 0$ and $\varepsilon = \hat{\delta}$ if $(\hat{z}_0 - \hat{\delta}) < 0$.

The velocity gradient obtained using Eq. (72) is

$$\frac{\partial \hat{u}}{\partial \hat{z}} = \frac{1}{\kappa(1+\hat{\delta}-\hat{\varepsilon})}\left\{\ln\left[1+\frac{\kappa(1+\hat{\delta})R_*}{\hat{d}}\right]-\ln\left(1+\frac{\kappa\hat{\varepsilon}R_*}{\hat{d}}\right)\right\}+\frac{1}{\kappa(1+\hat{\delta}-\hat{\varepsilon})}$$
$$\left\{\exp\left[-\frac{(1+\hat{\delta})R_*}{11.6\hat{d}}\right]-\exp\left(-\frac{\hat{\varepsilon}R_*}{11.6\hat{d}}\right)+\frac{(1+\hat{\delta})R_*}{11.6\hat{d}}\exp\left[-\frac{(1+\hat{\delta})R_*}{3\hat{d}}\right]\right.$$
$$\left.-\left(\frac{\hat{\varepsilon}R_*}{11.6\hat{d}}\right)\exp\left(-\frac{\hat{\varepsilon}R_*}{3\hat{d}}\right)\right\}\ln\left(\frac{\kappa\hat{z}_0R_*}{\hat{d}}\right) \tag{74}$$

Case 3 ($R_* > 70$): The flow over a sedimentary bed is completely rough when R_* exceeds a value of seventy (Schlichting, 1960).

The universal semi-logarithmic velocity distribution in rough regime is given below:

$$\hat{u} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \tag{75}$$

The mean flow velocity $\hat{u}_m$ derived using Eq. (75) is

$$\hat{u}_m = \frac{2\zeta}{\kappa\hat{A}}\int_{\hat{\varepsilon}}^{1+\delta}[(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5}\ln\left(\frac{\hat{z}}{z_0}\right)d\hat{z} \tag{76}$$

The velocity gradient can be determined using Eq. (75) as

$$\frac{\partial \hat{u}}{\partial \hat{z}} = \frac{1}{\kappa(1+\hat{\delta}-\hat{\varepsilon})}\ln\left(\frac{1+\hat{\delta}}{\hat{\varepsilon}}\right) \tag{77}$$

Simpson's rule could be applied to solve Eqs. (70), (73) and (76).

As the exact expression for the lift coefficient C_L as a function of R_* was not available, Eq. (62) was calibrated extensively. The experimental data (Θ_c and R_*) on sediment threshold reported by Gilbert (1914), Casey (1935), Kramer (1935), Shields (1936), USWES (1936), White (1940), Vanoni (1946), Meyer-Peter and Müller (1948), Iwagaki (1956), Neill (1967), Grass (1970), White (1970), Karahan (1975), Mantz (1977) and Yalin and Karahan (1979) were used to calibrate Eq. (62), using C_L as a free parameter. Fig. 9 shows the dependency of C_L on R_*. The negative values of C_L for low range of R_* ($R_* < 3$) were also reported by Watters and Rao (1971) and Davies and Samad (1978).

The dependency of Θ_c on particle parameter $\tilde{d}$ for different φ is presented in Fig. 10, that enables direct estimation of Θ_c.

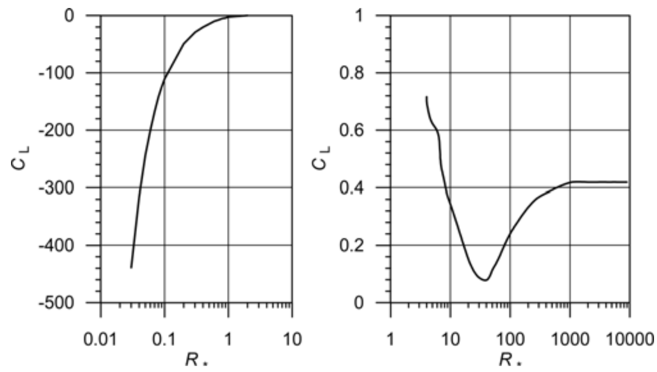

Fig. 9. Dependency of C_L on R_*

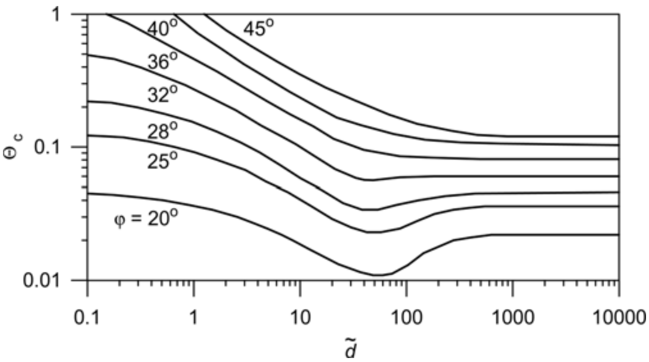

Fig. 10. Dependency of Θ_c on Particle Parameter $\tilde{d}$ for Different φ

7.1 Some Other Models on Sediment Threshold on Horizontal Beds

James (1990) presented a generalized model of sediment threshold for nonuniform sediments based on an analysis of moments of forces acting on a sediment particle. Particle geometry and packing arrangements were accounted for in addition to variations of near-bed flow velocity, drag and lift coefficients for a wide range of flow conditions. Ling (1995) studied the equilibrium of a solitary particle on a sediment bed, considering spinning motion of particle. The model proposed by him has two limits – rolling and lifting. The value of lift coefficient was assumed as 1.615, although it is not a constant in reality. He considered a depth-averaged method for the estimation of mean flow velocity received by a solitary particle. However, an area-averaged method is the most accurate one as the averaging is done over the whole projected area of the particle, being right angles to the direction of flow. The limitation of this model is that it is not applicable to non-uniform sediments. McEwan and Heald (2001) analyzed the stability of randomly deposited sediment beds using a discrete particle model where the particles are considered as spheres. The results indicate that the threshold shear stress could not be adequately described by a single-valued parameter and was best represented by a distribution of values. A Shields parameter of 0.06 being commonly used to define threshold of gravels found to correspond to a point on the distribution where 1.4 percent (by weight) of surface particles is on motion. An analysis on sheltering of particles reveals that remote sheltering induced by the prominent upstream particles has a significant effect to enhance the apparent threshold shear stress of exposed surface particles. Dey and Zanke (2004) modified the threshold model of Dey (1999) applicable to the sediment threshold by the stream flow subject to upward seepage.

8. Sediment Threshold on Arbitrary Sloping Beds (Dey, 2003)

The forces acting on a spherical sediment particle placed on a

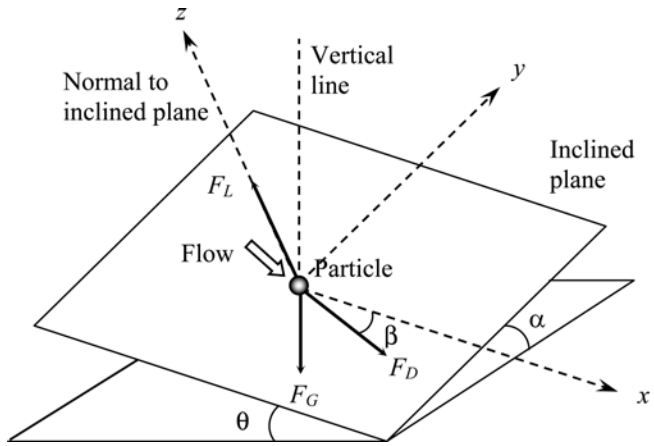

Fig. 11. Forces Acting on a Sediment Particle Lying on an Arbitrary Sloping Bed

bed having an arbitrary bed slope are shown in Fig. 11. When the solitary particle is about to move downstream from its original position, the equation of force balance is

$$F_s^2 = (F_D\cos\beta + F_G\sin\theta)^2 + (F_D\sin\beta + F_G\sin\alpha)^2 \tag{78}$$

where F_s = static Coulomb friction force between the particle and the bed; θ = longitudinal bed angle with the horizontal; α = transverse bed angle with the horizontal; and β = angle of inclination of flow with respect to the longitudinal axis of the channel (positive downward). The submerged weight of the particle is obtained from Eq. (52).

The static Coulomb friction force is equated to

$$F_s = (F_G\sqrt{\cos^2\theta - \sin^2\alpha} - F_L)\mu_c \tag{79}$$

where μ_c = static Coulomb friction factor at threshold condition, such that arctan $(\mu_c) = \varphi$. Equating Eqs. (78) and (79), one gets

$$\begin{aligned} &F_D^2 + 2F_G(\cos\beta\sin\theta + \sin\beta\sin\alpha) + F_G^2(\sin^2\theta + \sin^2\alpha) \\ &-(F_G\sqrt{\cos^2\theta - \sin^2\alpha} - F_L)^2\tan^2\varphi = 0 \end{aligned} \tag{80}$$

Normalizing the above equation, one can write

$$\begin{aligned} &(1-\eta^2\tan^2\varphi)\Theta_{cs}^2 + \frac{2}{\hat{F}_D}(\cos\beta\sin\theta + \sin\beta\sin\alpha + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha}\,\hat{\Theta}_{cs} \\ &-\frac{1}{\hat{F}_D}[(\cos^2\theta - \sin^2\alpha)\tan^2\varphi - \sin^2\theta - \sin^2\alpha] = 0 \end{aligned} \tag{81}$$

where $\eta = F_L/F_D$; Θ_{cs} = Shields parameter on an arbitrarily sloping bed, that is $\rho u_{*s}^2/[(\rho_s - \rho)gd]$ or $\tau_{0bs}/[(\rho_s - \rho)gd]$; u_{*s} = critical shear velocity on a sloping bed, that is $(\tau_{0s}/\rho)^{0.5}$; τ_{0s} = critical bed shear stress on a sloping bed; and $\hat{F}_D = 6F_D/(\pi\rho d^2 u_{*s}^2)$. The value of η proposed by Chepil (1958) is as 0.85. The positive solution of Eq. (81) is

$$\begin{aligned} \Theta_{cs} = \frac{1}{(1-\eta^2\tan^2\varphi)\hat{F}_D}\{&-(\cos\beta\sin\theta + \sin\beta\sin\alpha + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha}) \\ &+[(\cos\beta\sin\theta + \sin\beta\sin\alpha + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha})^2 \\ &+(1-\eta^2\tan^2\varphi)(\cos^2\theta\tan^2\varphi - \sin^2\alpha\tan^2\varphi - \sin^2\theta - \sin^2\alpha)]^{0.5}\} \end{aligned} \tag{82}$$

For a horizontal bed, θ and α become zero and Eq. (82) reduces to

$$\hat{\Theta}_c = \frac{\tan\varphi}{(1+\eta\tan\varphi)\hat{F}_D} \tag{83}$$

Dividing Eq. (82) by Eq. (83), yields

$$\begin{aligned} \tilde{\Theta}_{cs} = \frac{1}{(1-\eta\tan\varphi)\tan\varphi}\{&-(\cos\beta\sin\theta + \sin\beta\sin\alpha + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha}) \\ &+[(\cos\beta\sin\theta + \sin\beta\sin\alpha + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha})^2 \\ &+(1-\eta^2\tan^2\varphi)(\cos^2\theta\tan^2\varphi - \sin^2\alpha\tan^2\varphi - \sin^2\theta - \sin^2\alpha)]^{0.5}\} \end{aligned} \tag{84}$$

where Θ_{cs} = critical bed shear stress ratio, that is τ_{0s}/τ_0. However, in general, the flow through a river or a channel is in the longitudinal direction. Therefore, the equation of Θ_{cs} for this type of flow can be obtained using $\beta = 0$ in Eq. (84) as

$$\begin{aligned} \tilde{\Theta}_{cs} = \frac{1}{(1-\eta\tan\varphi)\tan\varphi}\{&-(\sin\theta + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha}) \\ &+[(\sin\theta + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha})^2 \\ &+(1-\eta^2\tan^2\varphi)(\cos^2\theta\tan^2\varphi - \sin^2\alpha\tan^2\varphi - \sin^2\theta - \sin^2\alpha)]^{0.5}\} \end{aligned} \tag{85}$$

For transverse bed slopes, using $\theta = 0$ and $\eta = 0$, Eq. (85) becomes

$$\tilde{\Theta}_{c\alpha} = \cos\alpha\sqrt{1 - \frac{\tan^2\alpha}{\tan^2\varphi}} \tag{86}$$

where $\Theta_{c\alpha} = \tau_{0\alpha}/\tau_0$; and $\tau_{0\alpha}$ = bed shear stress on a transverse sloping bed.

On the other hand, for longitudinal bed slopes, using $\alpha = 0$, Eq. (85) becomes

$$\tilde{\Theta}_{c\theta} = \cos\theta\left(1 - \frac{\tan\theta}{\tan\varphi}\right) \tag{87}$$

where $\Theta_{c\theta} = \tau_{0\theta}/\tau_0$; and $\tau_{0\theta}$ = bed shear stress on a longitudinal sloping bed.

Also, van Rijn (1993) and Dey (2004) proposed that critical bed shear stress on an arbitrary sloping bed is given by $\tau_{0b} = \tau_0\Theta_{c\alpha}\,\Theta_{c\theta}$.

8.1 Other Investigations on Sediment Threshold on Transverse Sloping Beds

The concept of the three-dimensional analysis of the gravity and tractive forces on a particle resting on a transverse or side slope of channels at the state of threshold motion was first given by Forchheimer (1924). A comprehensive analysis of a channel section using the concept of Forchheimer was first developed by Fan (1947). The same analysis was also developed independently by the US Bureau of Reclamation under the direction of Lane (1955) to suggest an equation of critical tractive force ratio as a function of side slope and angle of repose of the bed sediments. Glover and Florey (1951) proposed the profile of a minimum stable cross-section of a channel considering the particles on the side slopes of the channel being at threshold condition. Li *et al.*

(1976) extended the work of Lane for a channel in coarse alluvium. Parker (1978) used singular perturbation technique to determine bed shear stress distribution, which allows a mobile bed but immobile side slopes (particles at threshold condition on side slopes). Ikeda (1982) theoretically studied the threshold of sediment particle motion on side slopes considering the equilibrium of a sediment particle and conducted the experiments to verify the theoretical model. Ikeda's work was applied by Dey (2001) to propose a mathematical model for a threshold channel, considering sediment particles are at threshold condition on side slopes of a channel. There is a consensus of the equations used by the above investigators with Eq. (86).

8.2 Other Investigations on Sediment Threshold on Longitudinal Sloping Beds

Limited experiments by Luque and van Beek (1976) showed that the critical shear stress required for the initial movement of sediment on a longitudinal sloping bed decreases with increase in slope. Whitehouse and Hardisty (1988) conducted experiments on both adverse and streamwise longitudinal bed slopes to examine the effect of bed slopes and angle of repose on sediment threshold. Chiew and Parker (1994a) conduced experiments on beginning of sediment motion in closed-conduits having longitudinal (adverse and streamwise) slopes. Using dynamical considerations, an expression for the critical shear stress ratio of a longitudinal slope to a horizontal slope was derived by Chiew and Parker (1994b). Dey *et al.* (1999) put forward a theoretical model for the threshold of sediment motion on longitudinal sloping beds considering the equilibrium of a solitary sediment particle. They expressed the critical shear stress as a function of particle Reynolds number and angle of repose of sediment particles. Dey and Debnath (2000) carried out the experiments to validate the theoretical model developed by Dey *et al.* (1999). More of such investigations have been reported elsewhere (Stevens *et al*., 1976; Howard, 1977; Allen, 1982; Dyer, 1986; Sarre, 1987; Iversen and Rasmussen, 1994; Damgaard *et al.*, 1996). All the investigators found that Eq. (87) is valid for the threshold of sediment motion on longitudinal bed slopes.

9. Role of Turbulence on Sediment Threshold

Zanke (2003) developed a model for the sediment threshold considering the influence of turbulence. He recognized two additional effects as (a) the effective bed shear stress acting on a particle increases above the time-averaged bed shear stress owing to turbulent stress peaks, and (b) the particles exposed to the flow become effectively lighter due to lift forces. Both the turbulence induced effects are randomly distributed.

Dey and Raikar (2007) measured and analyzed the vertical distributions of time-averaged velocity and turbulence intensities in the flow on the near-threshold gravel beds. Importantly, these aspects were different from those of the immobile beds. The variation of the mixing-length is considerably linear with the depth within the inner-layer, whose thickness is 0.23 times the boundary-layer thickness; and the von Karman constant was obtained as 0.35. In the inner-layer, the semi-logarithmic-law of wall for the time-averaged streamwise velocity holds with the von Karman constant 0.35 and a constant of integration 7.8; while in the outer-layer, the law of the wake defines the velocity profiles with average value of the Coles' wake parameter 0.11. The turbulence intensities being non-isotropic were defined by an exponential-law; while the Reynolds stress varies almost linearly with the depth. The values of the Shields parameter for the threshold of gravel motion obtained experimentally corresponded closely with the curve obtained from the modified model of Dey (1999) considering the semi-logarithmic-law of velocity with von Karman constant 0.35 and a constant of integration 7.8.

9.1 Turbulent Burst

Investigations on the laminar sub-layer by Kline *et al.* (1967), Corino and Brodkey (1969) and Grass (1971) revealed a flow structure dominated by viscosity consists of large three-dimensional high- and low-speed velocity steaks. The near-bed region of flow has an extremely complex structure and most of the turbulence is produced there (Nezu and Nakagawa, 1993). The emission of fluid from the low-speed streaks initiates the concept of turbulent burst. The sequence is described by two significant features as ejection and sweep which have an important role on entrainment of bed sediments. During the ejection, the upward flow expands the shear layer and the associated small-scale flow structures to a broad region. It occurs as a low-speed fluid streak that oscillates in three dimensions lifts up from the bed and then collapses to entrain into the main body of flow. The ejected fluid which remains as a result of retardation is brushed away by high-speed fluid that approaches the bed in a process called the sweep. During sweep, the downward flow generates a narrow, highly turbulent shear layer containing multiple small-scale vortices. The bursting process can be described by a quadrant analysis.

9.2 Quadrant Analysis

To quantify the total Reynolds shear stress $\overline{u'v'}$ at a specific point as a sum of contributions from different bursting events, it is traditional to arrange the fluctuations of velocity components (u' and v') according to quadrant in $u'v'$-plane (Lu and Willmarth, 1973), as shown in Fig. 12. The hyperbolic shaded zone bounded by the curve $|u'v'|$ = constant is called a *hole*. Introducing a parameter H called *hole-size* that represents threshold level as explained by Nezu and Nakagawa (1993), the size of the hole is decided by the curve $|\overline{u'v'}| = H(\overline{u'u'})^{0.5}(\overline{v'v'})^{0.5}$. With this method, large contributions to $\overline{u'v'}$ from each quadrant can be extracted leaving the smaller fluctuations of velocity components (u' and v') that belong to the hole corresponding to more quiescent periods. Therefore, the hole-size H allows to differentiate between strong and weak events for small values of the hole size and only strong events for large values of the hole size. Let the four quadrants i (= 1, 2, 3 and 4) refer to the bursting events, which are outward interactions ($i = 1$; $u' > 0$, $v' > 0$), ejections ($i = 2$; $u' < 0$, $v' > 0$), inward interactions ($i = 3$; $u' < 0$, $v' < 0$) and sweeps ($i =$

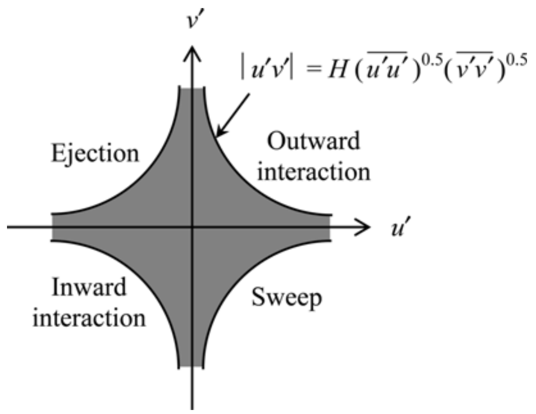

Fig. 12. Quadrant Analysis

4; $u' > 0$, $v' < 0$). The conditional stochastic analysis can be performed introducing a detection function $\lambda_{i,H}(t)$ defined as

$$\lambda_{i,H}(y,t) = \begin{cases} 1, \text{ if } (u',v') \text{ is in quadrant } i \text{ and if } |u'v'| = H(\overline{u'u'})^{0.5}(\overline{v'v'})^{0.5} \\ 0, \text{ otherwise} \end{cases} \tag{88}$$

At any point, contributions to the total Reynolds shear stress $\overline{u'v'}$ from the quadrant i outside the hyperbolic hole region of size H is given by

$$\langle u'v'\rangle_{i,H} = \lim_{T\to 0}\frac{1}{T}\int_0^T u'(t)v'(t)\lambda_{i,H}(y,t)dt \tag{89}$$

The Reynolds shear stress fractional contribution to each event is given by

$$S_{i,H} = \frac{\langle u'v'\rangle_{i,H}}{\overline{u'v'}} \tag{90}$$

The time fraction during which this contribution is made as

$$T_{i,H} = \frac{1}{T}\int_0^T \lambda_{i,H}(y,t)dt \tag{91}$$

It implies that $S_{i,H} > 0$ when $i = 2$ and 4 (sweeps and ejections), and $S_{i,H} < 0$ when $i = 1$ and 3 (inward and outward interactions). Therefore, $\sum_{i=0}^{i=4}[S_{i,H}]_{H=0} = 0$. In coherent motion of the bursting process, all the events directly influence the energy budget through turbulent diffusion. The Reynolds shear stress fractions are related to the higher-order moments and events in the bursting process by using probability distribution of u' and v'. If $p(\hat{u}', \hat{v}')$ is the joint probability density function of the rescaled velocity components, it can be specified by the infinite set of moments as

$$M_{i,H} = \overline{\hat{u}_i'\hat{v}_k'} \tag{92}$$

where $\hat{u}' = u'/(\overline{u'u'})^{0.5}$; and $\hat{v}' = v'/(\overline{v'v'})^{0.5}$. In Eq. (92), $i + k = 3$, so that M_{30} and M_{03} are skewness of u' and v'. Raupach (1981) expressed the difference of Reynolds shear stress between sweep and ejection events as

$$\Delta S_{i,H} = S_{4,H} - S_{2,H} \tag{93}$$

10. Closure

Though a large number of researches have been carried out on sediment threshold of different sizes of sediments and all the experimental and theoretical investigations bring us a step nearer to a better understanding of the problem, there remains an inadequate attention on many cases. The definite role of coherent structure on sediment threshold is yet to be known. Also, not many researchers tried to explore the sediment threshold of water worked beds. It is believed that the sediment threshold of water worked beds is quite different from that of laboratory prepared beds. Moreover, sediment threshold under sheet flow or shallow flow depth seems to remain unexplored. Therefore, further studies are encouraged on aforementioned cases.

References

Aksoy, S. (1973). "Fluid forces acting on a sphere near a solid boundary." *Proc. 15th IAHR Congr.*, Istanbul, Turkey, Vol. 1, pp. 217-224.

Allen, J.R.L. (1982). "Simple models for the shape and symmetry of tidal sand wave: (1) statically stable equilibrium forms." *Mar. Geol.*, Vol. 48, pp. 31-49.

Apperley, L.W. (1968). "Effect of turbulence on sediment entrainment." *PhD thesis*, University of Auckland, New Zealand.

Bagnold, R.A. (1974). "Fluid forces on a body in shear flow; experimental use of stationary flow." *Proc. R. Soc. London A*, Vol. 340, pp. 147-171.

Brayshaw, A.C., Frostick, L.E., and Reid, I. (1983). "The hydrodynamics of particle clusters and sediment entrainment in course alluvial channels." *Sedimentology*, Vol. 30, pp. 137-143.

Buffington, J.M. (1999). "The legend of A. F. Shields." *J. Hydraul. Eng.*, Vol. 125, No. 4, pp. 376-387.

Buffington, J.M. and Montgomery, D.R. (1997). "A systematic analysis of eight decades of incipient motion studies, with special reference to gravel-bedded rivers." *Wat. Resour. Res.*, Vol. 33, No. 8, pp. 1993-2029.

Carstens, M.R. (1966). "An analytical and experimental study of bed ripples under water waves." *Quart. Rep. 8 and 9*, Georgia Institute of Technology, School of Civil Engineering, Atlanta, USA.

Casey, H.J. (1935). "Über die geschiebebewegung." *PhD thesis*, Teknikal Hochschule-Scharlottenburg, Berlin, Germany.

Chepil, W.S. (1958). "The use of evenly spaced hemispheres to evaluate aerodynamic force on a soil surface." *Trans. Am. Geophy. Union*, Vol. 39, No. 3, pp. 397-404.

Chepil, W.S. (1961). "The use of spheres to measure lift and drag on wind-eroded soil grains." *Proc. Soil Sci. Soc. Am.*, Vol. 25, No. 5, pp. 343-345.

Chiew, Y.M. and Parker, G. (1994a). "Basic data on incipient sediment motion on non-horizontal slopes in closed-conduit flow." *Tech. Memo. No. 234*, St. Anthony Falls Hydraulics Laboratory, University of Minnesota.

Chiew, Y.M. and Parker, G. (1994b). "Incipient sediment motion on non-horizontal slopes." *J. Hydraul. Res.*, Vol. 32, No. 5, pp. 649-660.

Church, M. (1978). "Palaeohydraulic reconstructions from a Holocene valley fill in." Miall, A. D. (ed.) *Fluvial Sedimentology*, Canadian Society of Petroleum Geologists, Alberta, Canada, pp. 743-772.

Clifford, N.J., McClatchey, J., and French, J.R. (1991). "Measurements of turbulence in the benthic boundary layer over a gravel bed and comparison between acoustic measurements and predictions of the bedload transport of marine gravels." *Sedimentology*, Vol. 38, pp. 161-171.

Coleman, N.L. (1967). "A theoretical and experimental study of drag and lift forces acting on a sphere resting on a hypothetical stream bed." *Proc. 12th IAHR Congr.*, Fort Collins, Colorado, Vol. 3, pp. 185-192.

Corino, E.R. and Brodkey, R.S. (1969). "A visual investigation of the wall region in turbulent flow." *J. Fluid Mech.*, Vol. 37, pp. 1-30.

Damgaard, J.S., Whitehouse, R.J.S., and Wallingford, H.R. (1966). "Gravity effects in bed-load sediment transport." *Progress Rep. HR Wallingford MAFF Contract CSA 3051*, Ministry of Agriculture, Fisheries and Food, London, UK.

Dancy, C.L., Diplas, P., Papanicolaou, A., and Bala, M. (2002). "Probability of individual grain movement and threshold condition." *J. Hydraul. Eng.*, Vol. 128, No. 12, pp. 1069-1075.

Davies, T.R.H. and Samad, M.F.A. (1978). "Fluid dynamic lift on a bed particle." *J. Hydraul. Div.*, Vol. 104, No. 8, pp. 1171-1182.

Dey, S. (1999). "Sediment threshold." *Appl. Math. Modelling*, Vol. 23, No. 5, pp. 399-417.

Dey, S. (2001). "Bank profile of threshold channels: a simplified approach." *J. Irrig. and Drain. Eng.,* Vol. 127, No. 3, pp. 184-187.

Dey, S. (2003). "Threshold of sediment motion on combined transverse and longitudinal sloping beds." *J. Hydraul. Res.*, Vol. 41, No. 4, pp. 405-415.

Dey, S. (2004). "Critical bed shear for initial movement of sediments on a combined lateral and longitudinal slope." *Nordic Hydrol.*, Vol. 35, No. 2, pp. 153-164.

Dey, S. and Debnath, K. (2000). "Influence of stream-wise bed slope on sediment threshold under stream flow." *J. Irrig. and Drain. Eng.,* Vol. 126, No. 4, pp. 255-263.

Dey, S., Dey Sarker, H.K., and Debnath, K. (1999). "Sediment threshold under stream flow on horizontal and sloping beds." *J. Eng. Mech.*, Vol. 125, No. 5, pp. 545-553.

Dey, S. and Raikar, R.V. (2007). "Characteristics of loose rough boundary streams at near-threshold." *J. Hydraul. Eng.*, Vol. 133, No. 3, pp. 288-304.

Dey, S. and Zanke, U.C.E. (2004). "Sediment threshold with upward seepage." *J. Eng. Mech.*, Vol. 130, No. 9, pp. 1118-1123.

Dyer, K.R. (1986). *Coastal and estuarine sediment dynamics*. John Wiley and Sons Limited, London, UK.

Egiazaroff, J.V. (1965). "Calculation of non-uniform sediment concentrations." *J. Hydraul. Div.*, Vol. 91, No. 4, pp. 225-247.

Einstein, H.A. (1950). "The bed-load function for sediment transportation in open channel flows." *Tech. Bulletin No. 1026*, US Department of Agriculture.

Einstein, H.A. and El-Samni, E.A. (1949). "Hydrodynamic forces on rough wall." *Rev. Modern Phys.*, Vol. 21, No. 3, pp. 520-524.

Fan, C.H. (1947). "A study of stable channel cross section." *J. Hydraul. Eng.*, Chinese Society of Hydraulic Engineers, Vol. 15, No. 1, pp. 71-79 (in Chinese).

Fenton, J.D. and Abbott, J.E. (1977). "Initial movement of grains on a stream bed: the effect of relative protrusion." *Proc. R. Soc. London* A, Vol. 352, pp. 523-537.

Forchheimer, P. (1924). *Hydraulik.* Teubner Verlagsgesellschaft, Berlin, Germany (in German).

Gessler, J. (1966). "Geschiebetrieb bei mischungen untersucht an naturlichen, abpflasterungserscheinungen in kanalen." *Nr. 69*, Mitteilungen der Versuchsanstalt für Wasserbau und Erdbau, ETH Zurich, Germany.

Gessler, J. (1970). "Self-stabilizing tendencies of alluvial channels." *J. Waterways Harbors Div.*, Vol. 96, No. 2, pp. 235-249.

Glover, R.E. and Florey, Q.L. (1951). "Stable channel profiles." *Hydraul. Lab. Rep. Hyd-325*, US Bureau of Reclamation.

Gilbert, G.K. (1914). "Transportation of debris by running water." *Prof. Paper No. 86*, United States Geological Survey, Washington DC, USA.

Goncharov, V.N. (1964). *Dynamics of channel flow*. Israel Programme for Scientific Translation, Moscow, Russia.

Grass, A.J. (1970). "Initial instability of fine bed sand." *J. Hydraul. Div.*, Vol. 96, No. 3, pp. 619-632.

Grass, A.J. (1971). "Structural features of turbulent flow over smooth and rough boundaries." *J. Fluid Mech.*, Vol. 50, 233-255.

Howard, A.D. (1977). "Effect of slope on the threshold of motion and its application to orientation of wind ripples." *Bull. Geol. Soc. Am.*, Vol. 88, pp. 853-856.

Ikeda, S. (1982). "Incipient motion of sand particles on side slopes." *J. Hydraul. Div.*, Vol. 108, No. 1, pp. 95-114.

Ippen, A.T. and Eagleson, P.S. (1955). "A study of sediment sorting by waves shoaling on a plane beach." *Tech. Memo. No. 63*, US Army Corps Engineers, Beach Erosion Board.

Iversen, J.D. and Rasmussen, K.R. (1994). "The effect of surface slope on saltation threshold." *Sedimentation*, Vol. 41, pp. 721-728.

Iwagaki, Y. (1956). "Fundamental study on critical tractive force." *Trans. Jap. Soc. Civ. Eng.*, Vol. 41, pp. 1-21.

James, C. (1990). "Prediction of entrainment conditions for nonuniform, noncohesive sediments." *J. Hydraul. Res.*, Vol. 28, No. 1, pp. 25-41.

Jeffreys, H. (1929). "On the transport of sediments in stream." *Proc. Camb. Phil. Soc.*, Vol. 25, pp. 272 .

Julien, P.Y. (1998). *Erosion and sedimentation*. Cambridge University Press, Cambridge, UK.

Karahan, E. (1975). "Initiation of motion for uniform and non-uniform materials." *PhD thesis*, Technical University, Istanbul, Turkey.

Kennedy, J.F. (1995). "The Albert Shields story." *J. Hydraul. Eng.*, Vol. 121, No. 11, pp. 766-772.

Keshavarzy, A. and Ball, J.E. (1996). "Characteristics of turbulent shear stress applied to bed particles in an open channel flow." *Proc. 7th IAHR Int. Symp. Stochastic Hydr. 96*, pp. 451-458.

Kline, S.J., Reynolds, W.C., Straub, F.A., and Runstadler, P.W. (1967). "The structure of turbulent boundary layers." *J. Fluid Mech.*, Vol. 30, pp. 741-773.

Krogstad, P.A., Antonia, R.A., and Browne, L.W.B. (1992). "Comparison between rough and smooth wall turbulent boundary layers." *J. Fluid Mech.*, Vol. 245, pp. 599-617.

Kramer, H. (1935). "Sand mixtures and sand movement in fluvial levels." *Trans. Am. Soc. Civ. Eng.*, Vol. 100, pp. 798-838.

Kurihara, M. (1948). "On the critical tractive force." *Rep. No.* 3, Vol. 4, Research Institute for Hydraulic Engineering.

Lane, E.W. (1955). "Design of stable channels." *Trans. ASCE*, Vol. 120, pp. 1234-1260.

Lane, E.W. and Kalinske, A.A. (1939). "The relation of suspended to bed materials in river." *Trans. Am. Geophys. Union*, Vol. 20, pp. 637.

Lavelle, J.W. and Mofjeld, H.O. (1987). "Bibliography on sediment threshold velocity." *J. Hydraul. Eng.*, Vol. 113, No. 3, pp. 389-393.

Leliavsky, S. (1966). *An introduction to fluvial hydraulics*. Dover, New York.

Li, R., Simons, D.B., and Stevens, M.A. (1976). "Morphology of cobble streams in small watersheds." *J. Hydraul. Div.*, Vol. 102, No. 8, pp. 1101-1117.

Ling, C.H. (1995). "Criteria for incipient motion of spherical sediment particles." *J. Hydraul. Eng.*, Vol. 121, No. 6, pp. 472-478.

Lu, S.S. and Willmarth, W.W. (1973). "Measurements of the structures of the Reynolds stress in a turbulent boundary layer." *J. Fluid Mech.*, Vol. 60, pp. 481-571.

Luque, R.F. and van Beek, R. (1976). "Erosion and transport of bed-load sediment." *J. Hydraul. Res.*, Vol. 14, No. 2, pp. 127-144.

Mantz, P.A. (1977). "Incipient transport of fine grains and flanks by fluids-extended Shields diagram." *J. Hydraul. Div.*, Vol. 103, No. 6, pp. 601-615.

McEwan, I. and Heald, J. (2001). "Discrete particle modeling of entrainment from flat uniformly sized sediment beds." *J. Hydraul. Eng.*, Vol. 127, No. 7, pp. 588-597.

Meyer-Peter, E. and Müller, R. (1948). "Formulas for bed-load transport." *Proc. 2nd IAHR Congr.*, Stockholm, Sweden, pp. 39-64.

Miller, M.C., McCave, I.N., and Komar, P.D. (1977). "Threshold of sediment motion under unidirectional currents." *Sedimentology*, Vol. 24, pp. 507-527.

Mingmin, H. and Qiwei, H. (1982). "Stochastic model of incipient sediment motion." *J. Hydraul. Div.*, Vol. 108, No. 2, pp. 211-224.

Morsi, S.A. and Alexander, A.J. (1972). "An investigation of particle trajectories in two-phase flow systems." *J. Fluid Mech.*, Vol. 55, pp. 193-208.

Neill, C.R. (1967). "Mean velocity criterion for scour of course uniform bed material." *Proc. 12th IAHR Congr.*, Fort Collins, Colorado, USA, Vol. 3, pp. 46-54.

Neill, C.R. (1968). "Note on initial movement of coarse uniform bed-material." *J. Hydraul. Res.*, Vol. 6, No. 2, pp. 173-176.

Nelson, J., Shreve, R.L., McLean, S.R, and Drake, T.G. (1995). "Role of near-bed turbulence structure in bed load transport and bed form mechanics." *Wat. Resour. Res.*, Vol. 31, No 8, pp. 2071-2086.

Nezu, I. and Nakagawa, H. (1993). *Turbulence in open-channel flows*. Balkema, Rotterdam, Netherlands.

Papanicolaou, A., Diplas, P., Dancey, C., and Balakrishnan, M. (2001). "Surface roughness effects in near-bed turbulence: implications to sediment entrainment." *J. Eng. Mech.*, Vol. 127, No. 3, pp. 211-218.

Papanicolaou, A.N., Diplas, P., Evaggelopoulos, N., and Fotopoulos, S. (2002). "Stochastic incipient motion criterion for spheres under various bed packing conditions." *J. Hydraul. Eng.*, Vol. 128, No. 4, pp. 369-380.

Paphitis, D. (2001). "Sediment movement under unidirectional flows: an assessment of empirical threshold curves." *Coastal Eng.*, Vol. 43, pp. 227-245.

Parker, G. (1978). "Self-formed straight rivers with equilibrium banks and mobile bed. Part 2. The gravel river." *J. Fluid Mech.*, Vol. 89, pp. 127-146.

Raupach, M.R. (1981). "Conditional statistics of Reynolds stress in rough-wall and smooth-wall turbulent boundary layers." *J. Fluid Mech.*, Vol. 108, pp. 363-382.

Reichardt, H. (1951). "Vollstandige darstellung der turbulenten geschwindig-keitsverteilung in glatten leitungen." *Z. Angew. Math. Mech.*, Vol. 31, No. 7, pp. 208-219.

Reitz, W. (1936). "Uber geschiebebewegung." *Wasserwirtsch. und Tech.*, pp. 28-30.

Rubinow, S. I. and Keller, J.B. (1961). "The transverse force on a spinning sphere moving in a viscous fluid." *J. Fluid Mech.*, Vol. 11, pp. 447-459.

Saffman, P.G. (1965). "The lift on a small sphere in a slow shear flow." *J. Fluid Mech.*, Vol. 22, pp. 385-400.

Saffman, P.G. (1968). "Corrigendum, the lift on a small sphere in a slow shear flow." *J. Fluid Mech.*, Vol. 31, pp. 624.

Sarre, R.D. (1987). "Aeolian sand transport." *Progr. Phys. Geogr.*, Vol. 11, pp. 157-182.

Schlichting, H. (1960). *Boundary layer theory*. McGraw-Hill Book Companies, New York, USA.

Shields, A.F. (1936). "Application of similarity principles and turbulence research to bed-load movement." *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau,* Berlin, Germany, Vol. 26, pp. 5-24.

Soulsby, R.L. and Whitehouse, R.J.S. (1997). "Threshold of sediment motion in coastal Environments." *Proc. Combined Australasian Coastal Eng. and Port Conf.*, Christchurch, New Zealand, pp. 149-154.

Stevens, M.A., Simons, D.B., and Lewis, G.L. (1976). "Safety factor for riprap protection." *J. Hydraul. Engrg.*, Vol. 102, No. 5, pp. 637-655.

Task Committee. (1966). "Sediment transportation mechanics: Initiation of motion." *J. Hydraul. Div.*, Vol. 92, No. 2, pp. 291-314.

USWES. (1936). "Flume tests made to develop a synthetic sand which will not form ripples when used in movable bed models." *Tech. Memo. 99-1*, United States Waterways Experiment Station, Vieksburg, Mississippi, USA.

Vanoni, V.A. (1946). "Transport of suspended sediment by water." *Trans. Am. Soc. Civ. Eng.*, Vol. 111, pp. 67-102.

Vanoni, V.A. (1964). "Measurements of critical shear stress." *Rep. No. KH-R*-7, California Institute of Technology, USA.

van Rijn, L.C. (1984). "Sediment transport, part I: bed-load transport." *J. Hydraul. Eng.*, Vol. 110, No. 10, pp. 1431-1456.

van Rijn, L.C. (1993). *Principles of sediment transport in rivers, estuaries and coastal seas*. Aqua Publications, The Netherlands.

Velikanov, M.A. (1955). *Dynamics of alluvial stream*. Vol. 2, State Publishing House of Theoretical and Technical Literature, Russia (in Russian).

Watters, G.Z. and Rao, M.V.P. (1971). "Hydrodynamic effects of seepage on bed particles." *J. Hydraul. Div.*, Vol. 97, No. 3, pp. 421-439.

White, C. M. (1940). "The equilibrium of grains on the bed of a stream." *Phil. Trans. Royal Soc.*, Vol. 174A, pp. 322-338.

White, S.J. (1970). "Plane bed thresholds of fine grained sediments." *Nature*, Vol. 228, No. October, pp. 152-153.

Whitehouse, R.J.S. and Hardisty, J. (1988). "Experimental assessment of two theories for the effect of bed slope on the threshold of bed-load transport." *Mar. Geol.*, Vol. 79, pp. 135-139.

Wiberg, P.L. and Smith, J.D. (1987). "Calculations of the critical shear stress for motion of uniform and heterogeneous sediments." *Wat. Resour. Res.*, Vol. 23, No. 8, 1471-1480.

Wu, F.C. and Chou, Y.J. (2003). "Rolling and lifting probabilities for sediment entrainment." *J. Hydraul. Eng.*, Vol. 129, No. 2, pp. 110-119.

Yalin, M.S. (1963). "An expression of bed-load transportation." *J. Hydraul. Div.*, Vol. 89, No. 3, pp. 221-250.

Yalin, M.S. (1977). *Mechanics of sediment transport*. Pergamon Press, Braunschweig, Germany.

Yalin, M.S. and Karahan, E. (1979). "Inception of sediment transport." *J. Hydraul. Div.*, Vol. 105, No. 11, pp. 1433-1443.

Yang, C.T. (1973). "Incipient motion and sediment transport." *J. Hydraul. Div.*, Vol. 99, No. 10, pp. 1679-1704.

Zanke, U.C.E. (1977). "Neuer Ansatz zur Berechnung des Transportbeginns von Sedimenten unter Stromungseinfluss." *Mitt. Des Franzius-Institut,* Technical University Hannover, Germany, Heft 46.

Zanke, U.C.E. (2003). "On the influence of turbulence on the initiation of sediment motion." *Int. J. Sediment Res.*, Vol. 18, No. 1, pp. 17-31.