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Figure 27: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 11.16.08 PM.png

- If $\alpha c < 8\mu$, equilibria: $(K, 0)$ stable.
- If $\alpha c > 8\mu$ and $K < n_1^* < n_2^*$, equilibria: $(K, 0)$ stable (Fig. 2(d)).
- If $\alpha c > 8\mu$ and $n_1^* < K < n_2^*$, equilibria: $\left(n_1^*, p_1^*\right)$ stable and $(K, 0)$ unstable (Fig. 2(c)).
- If $\alpha c > 8\mu$ and $n_1^* < n_2^* < K$, equilibria: $\left(n_1^*, p_1^*\right)$ stable, $\left(n_1^*, p_1^*\right)$ unstable and $(K, 0)$ stable.

Results from Model 2

Figure 28: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 11.16.54 PM.png

- If $K < n_3^*$, equilibria: $(K, 0)$ stable (Figs. 1(b) and 2(b)).
- If $K > n_3^*$, equilibria: $\left(n_3^*, p_{31}^*\right)$ stable and $(K, 0)$ unstable (Figs. 1(a) and 2(a)).

There is a relationship between prey density (carrying capacity) and the strategy adopted by predators. Hawk behavior is a strategy in high prey densities, whereas a polymorphism with the presence of both doves and hawks, is found in low densities.

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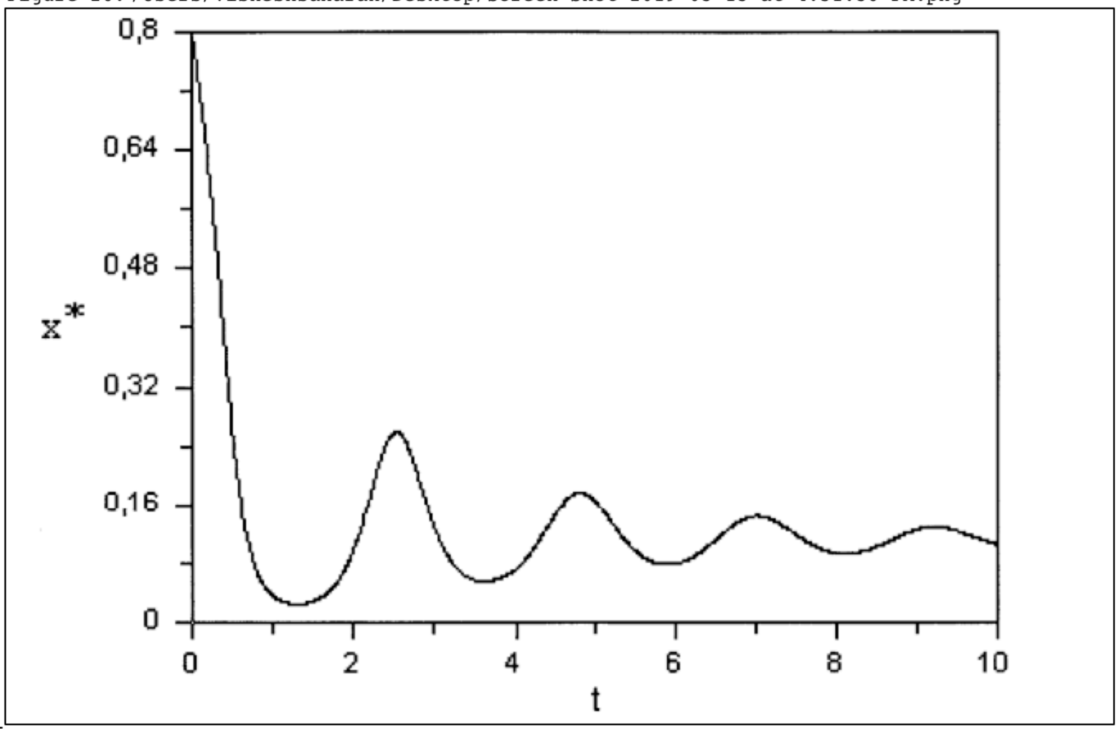
(%o377)

Figure 26: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.51.36 PM.png



Results from Model 1

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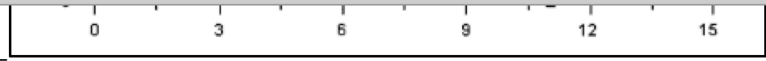


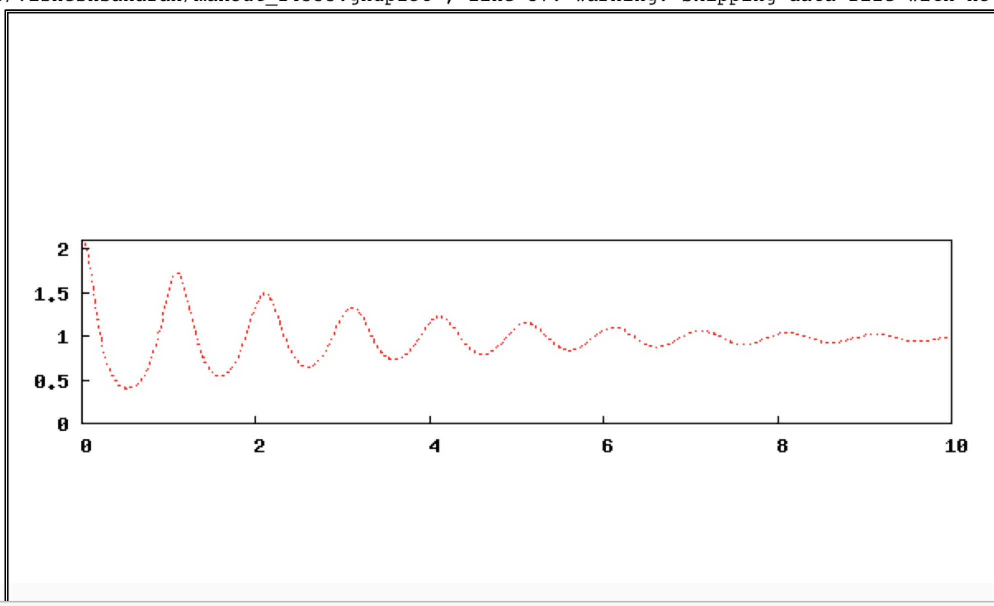
The time evolution of the proportion of hawks when the trajectory of the aggregated model is slowly spiraling to the polymorphic stable eq of (n1*,p1*) of Model II.

```
--> params5:[r=20,K=30,a=1,a1=4.2,C=0.5,u=1];
(%o353) [r=20,K=30,a=1,a1=4.2,C=0.5,u=1]

--> f:rkf45(subst(params5,aggmodel),[n,p],[2.1,18.4],[t,0,10])$

--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=red,
        implicit(subst(params5,zi1[1]),n,0,10,p,0,1),
        sol_points(f,first,second)
    );
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
"/Users/Visheshsaharan/maxout_24555.gnuplot", line 37: warning: Skipping data file with no valid points
```

(%t377)



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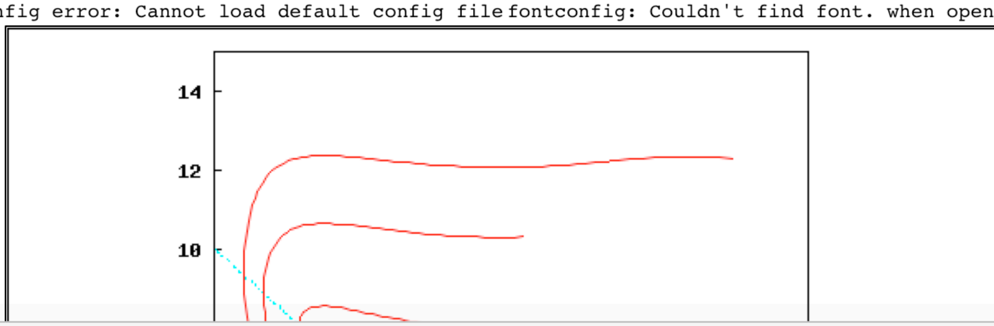
In Fig. 2B, there is a separatix. Based on inital conditions, the predator and prey coexist at a low desnity or the predator goes to exctinction and the prey tends towards it's carrying capacity

```
--> params4:[r=10,K=11,a=1,a1=1,C=10,u=1];
(%o76) [r=10,K=11,a=1,a1=1,C=10,u=1]

--> sd1:rkf45(subst(params4,aggmodel),[n,p],[9.3,8.1],[t,0,40])$

--> sd2:rkf45(subst(params4,aggmodel),[n,p],[3.3,2.1],[t,0,40])$

--> sd3:rkf45(subst(params4,aggmodel),[n,p],[7.8,10.3],[t,0,40])$

--> sd4:rkf45(subst(params4,aggmodel),[n,p],[12.6,3.1],[t,0,40])$

--> sd5:rkf45(subst(params4,aggmodel),[n,p],[13.1,12.3],[t,0,40])$

--> sd6:rkf45(subst(params4,aggmodel),[n,p],[7.1,4.3],[t,0,40])$

--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,
        implicit(subst(params4,zi1[1]),n,0,15,p,0,15),
        color=magenta,
       implicit(subst(params4,zi2[1]),n,0,15,p,0,15),
        line_type=solid,points_joined=true, point_type=0,color=red,
        sol_points(sd1,second,third),sol_points(sd2,second,third),
    sol_points(sd3,second,third),sol_points(sd4,second,third),
    sol_points(sd5,second,third),sol_points(sd6,second,third)
    );
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```



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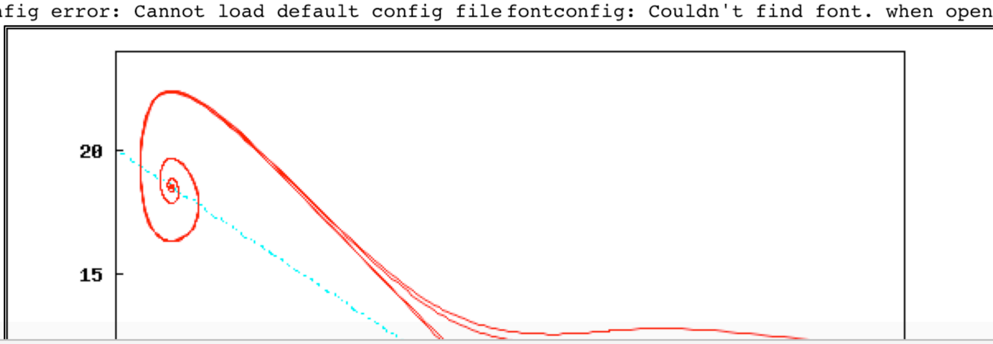
In Fig. 2A, there is a separatix. Two stable equilibria and according to inital conditions, the prey and predator coexist at one of these equilbira.

```
--> params3:[r=20,K=30,a=1,a1=1,C=20,u=1];
(%o63) [r=20,K=30,a=1,a1=1,C=20,u=1]

--> sc1:rkf45(subst(params3,aggmodel),[n,p],[9.3,8.1],[t,0,40])$

--> sc2:rkf45(subst(params3,aggmodel),[n,p],[11.1,4.8],[t,0,40])$

--> sc3:rkf45(subst(params3,aggmodel),[n,p],[16.8,5.3],[t,0,40])$

--> sc4:rkf45(subst(params3,aggmodel),[n,p],[16.2,10.1],[t,0,40])$

--> sc5:rkf45(subst(params3,aggmodel),[n,p],[16.1,12.3],[t,0,40])$

--> sc6:rkf45(subst(params3,aggmodel),[n,p],[28.1,12.3],[t,0,40])$

--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,
        implicit(subst(params3,zi1[1]),n,0,32,p,0,24),
        color=magenta,
       implicit(subst(params3,zi2[1]),n,0,32,p,0,24),
        line_type=solid,points_joined=true, point_type=0,color=red,
        sol_points(sc1,second,third),sol_points(sc2,second,third),
    sol_points(sc3,second,third),sol_points(sc4,second,third),
    sol_points(sc5,second,third),sol_points(sc6,second,third)
    );
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```



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```
--> sb3:rkf45(subst(params2,aggmodel),[n,p],[2.8,3.3],[t,0,40])$

--> sb4:rkf45(subst(params2,aggmodel),[n,p],[0.2,6.3],[t,0,40])$

--> sb5:rkf45(subst(params2,aggmodel),[n,p],[0.1,4.3],[t,0,40])$

--> sb6:sb1:rkf45(subst(params2,aggmodel),[n,p],[0.3,5.1],[t,0,40])$

--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,
       implicit(subst(params2,zi1[1]),n,0,8,p,0,10),
        color=magenta,
       implicit(subst(params2,zi2[1]),n,0,8,p,0,10),
        line_type=solid,points_joined=true, point_type=0,color=red,
        sol_points(sb1,second,third),sol_points(sb2,second,third),
    sol_points(sb3,second,third),sol_points(sb4,second,third),
    sol_points(sb5,second,third),sol_points(sb6,second,third)
    );
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```

(%t61)

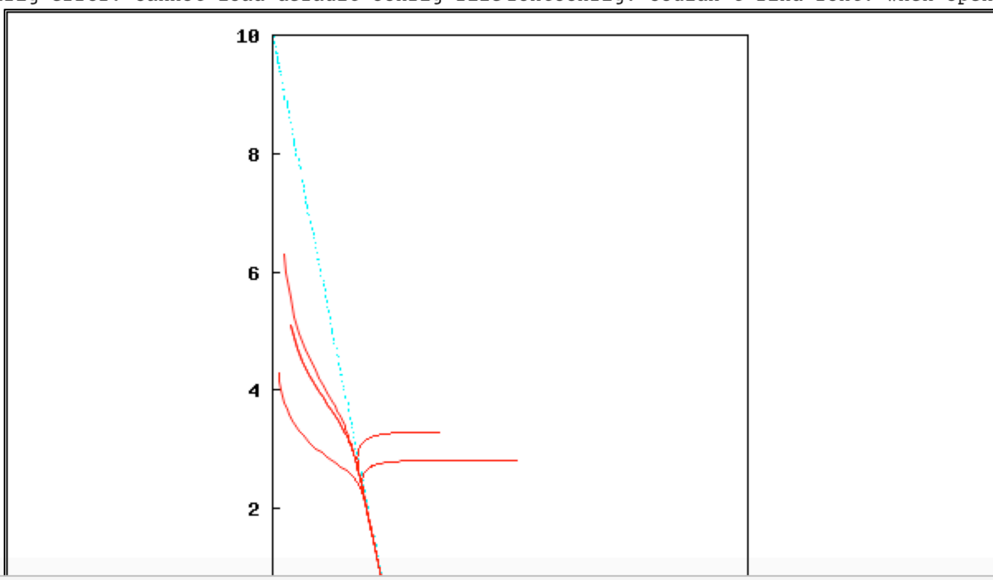

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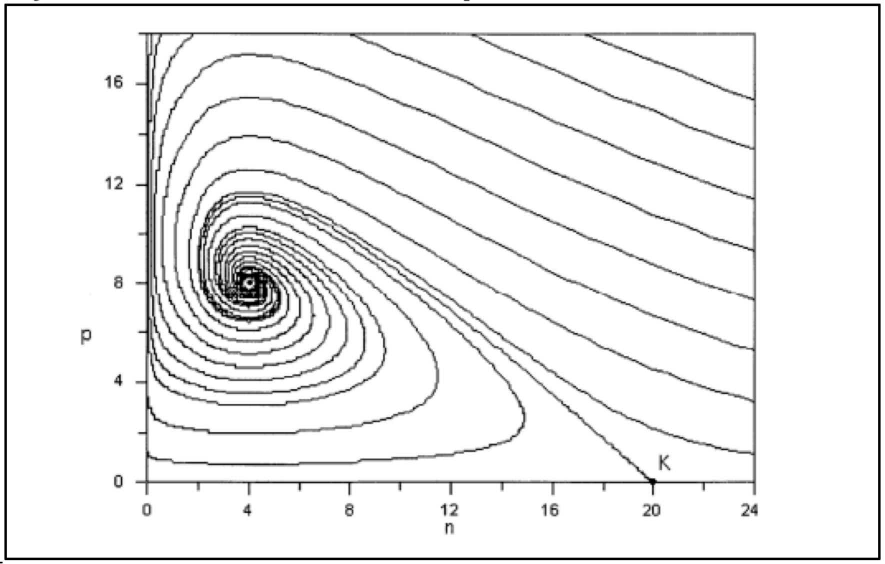
Figure 22: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 10.31.24 AM.png



Let us now look at predator exlcusion. Fig. 1B and Fig. 2D show the predator going to exctinction and the prey tends to its carrying capacity.

```
--> params2:[r=10,K=2,a=1,a1=1,C=10,u=1];
(%o110) [r=10,K=2,a=1,a1=1,C=10,u=1]

--> sb1:rkf45(subst(params2,aggmodel),[n,p],[9.17,1.1],[t,0,40])$

--> sb2:rkf45(subst(params2,aggmodel),[n,p],[4.1,2.8],[t,0,40])$

--> sb3:rkf45(subst(params2,aggmodel),[n,p],[2.8,3.3],[t,0,40])$

--> sb4:rkf45(subst(params2,aggmodel),[n,p],[0.2,6.3],[t,0,40])$

--> sb5:rkf45(subst(params2,aggmodel),[n,p],[0.1,4.3],[t,0,40])$
```

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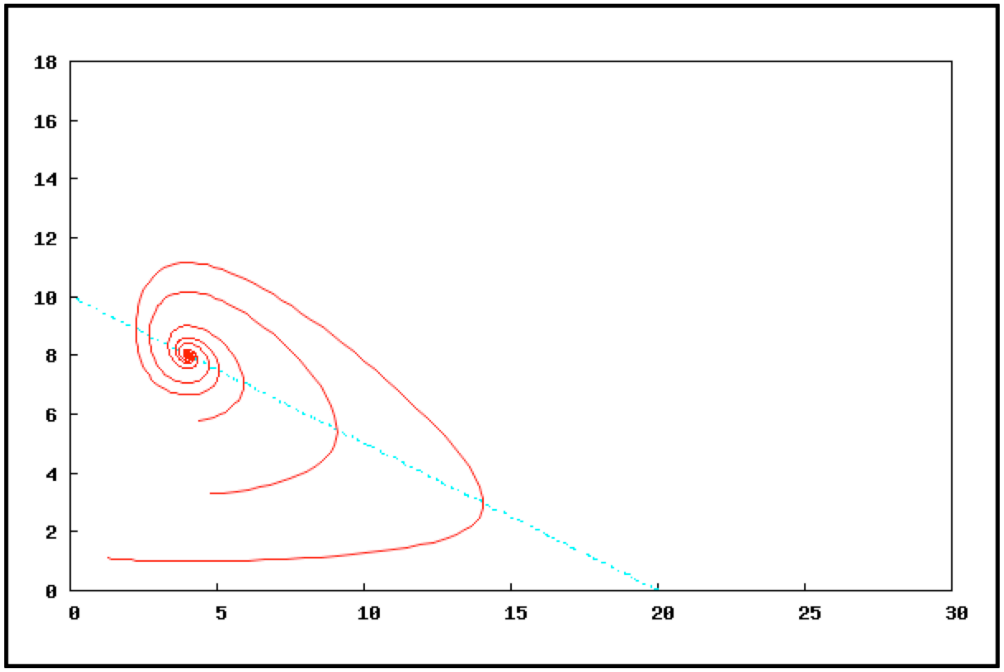
```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,
        implicit(subst(params1,zil[1]),n,0,30,p,0,18),
        line_type=solid,points_joined=true, point_type=0,color=red,
        sol_points(sa1,second,third),sol_points(sa2,second,third),
    sol_points(sa3,second,third)
    );
```

Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font

(%t79)



(%o79)

Figure 22: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 10.31.24 AM.png

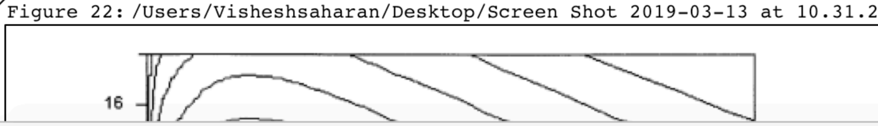


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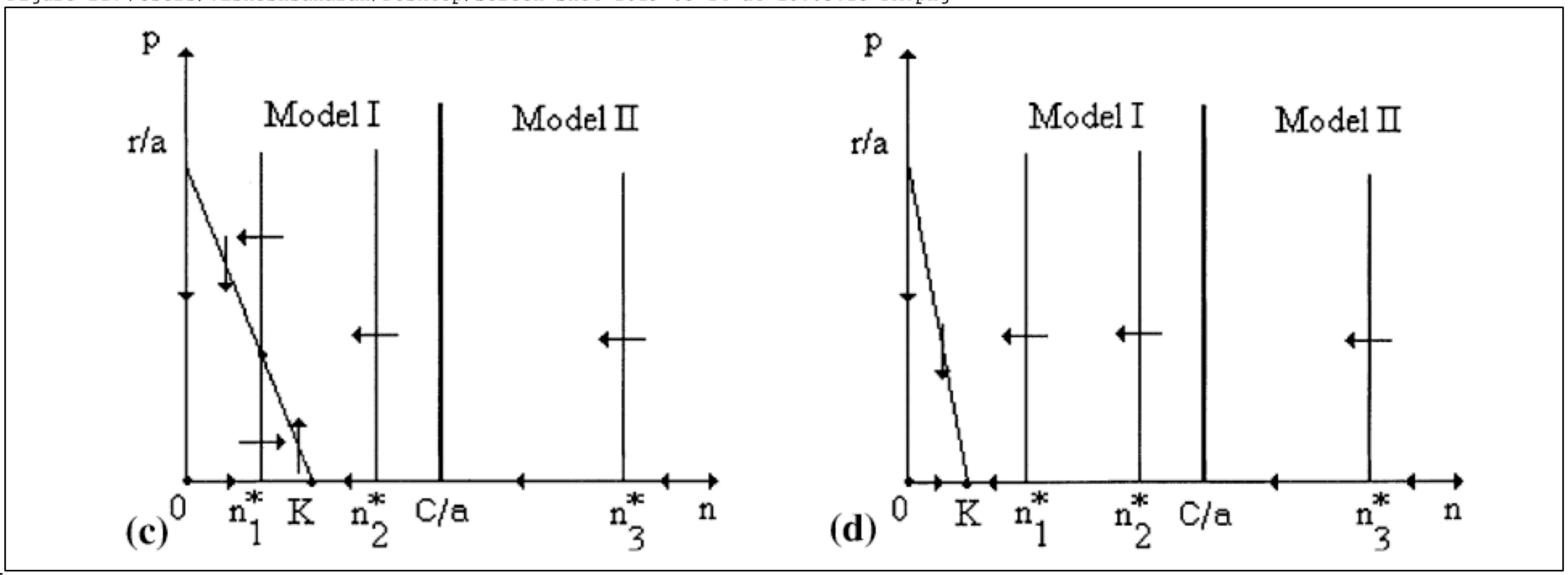
Figure 21: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.05.18 PM.png



We have 6 different phase portraits, so lets take into account four possibilites concerning the stability. Let's look at coexistence, predator exclusion, coexistence or predator exclusion based on inital conditions, and coexistance at two different density levels based on inital conditions.

First, let's look at coexistence. We can see that Fig. 1A and Fig. 2C demonstrate coexistence, so we can make a phase plot of these cases.

```
--> params1;
(%o69) [r=10,K=20,a=1,a1=1,C=2,u=1]

--> sa1:rkf45(subst(params1,aggmodel),[n,p],[1.3,1.1],[t,0,40])$

--> sa2:rkf45(subst(params1,aggmodel),[n,p],[4.4,5.8],[t,0,40])$

--> sa3:rkf45(subst(params1,aggmodel),[n,p],[4.8,3.3],[t,0,40])$
```

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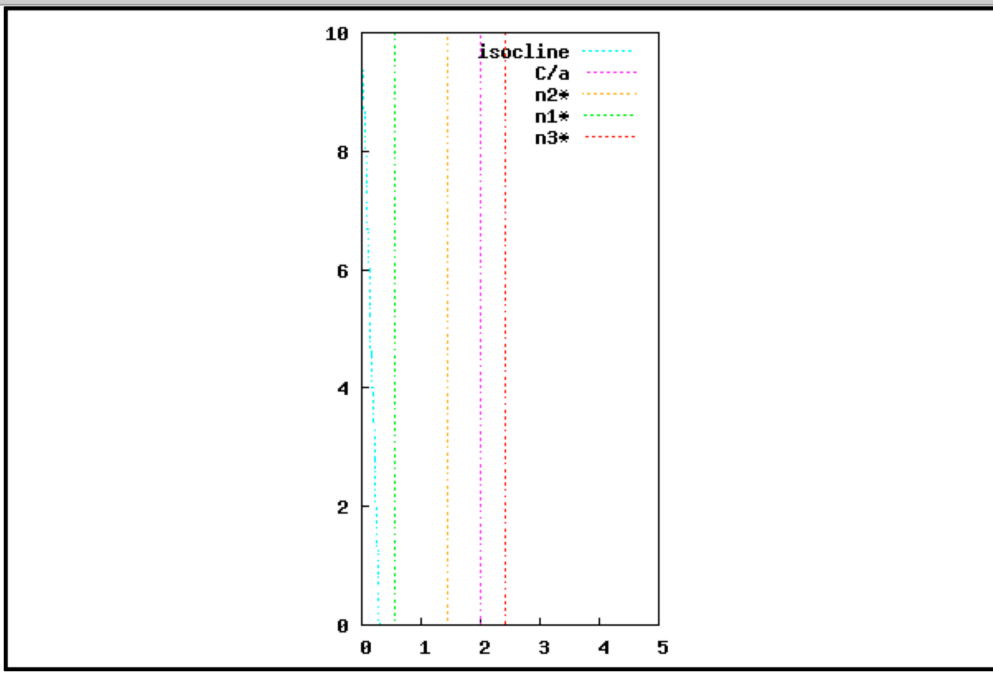
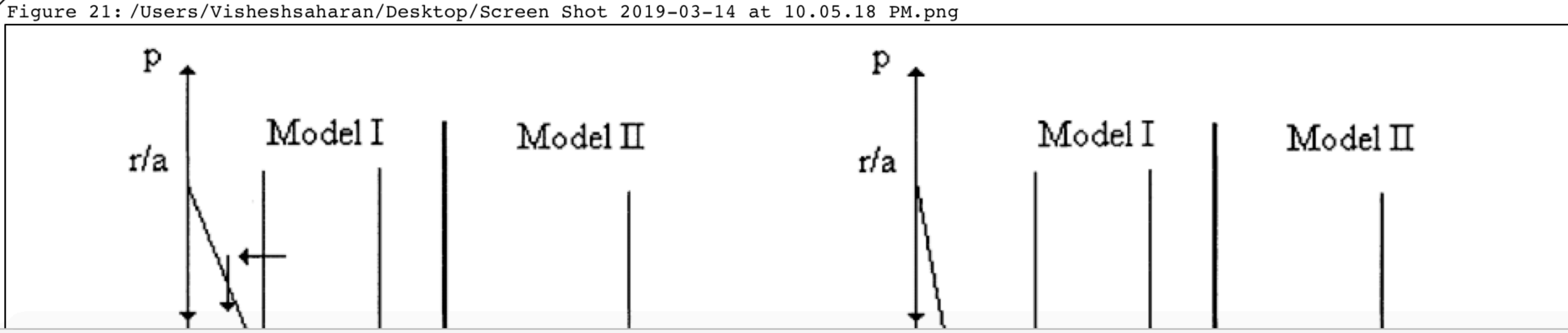
(%t55)



(%o55)

Figure 21: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.05.18 PM.png



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```
--> params1d:[r=10,K=0.3,a=1,a1=5,C=2,u=1];
```

(%o42) $[r=10, K=0.3, a=1, a1=5, C=2, u=1]$

```
--> eq1d:solve(subst(params1d,model1),[n,p]);
rat: replaced -3.333333333333333 by -10/3 = -3.333333333333333
```

(%o44) $[[n=0, p=0], [n=\frac{3}{10}, p=0], [n=-\frac{\sqrt{5}-5}{5}, p=\frac{4\,5^{3/2}-70}{3}], [n=\frac{\sqrt{5}+5}{5}, p=-\frac{4\,5^{3/2}+70}{3}]]$

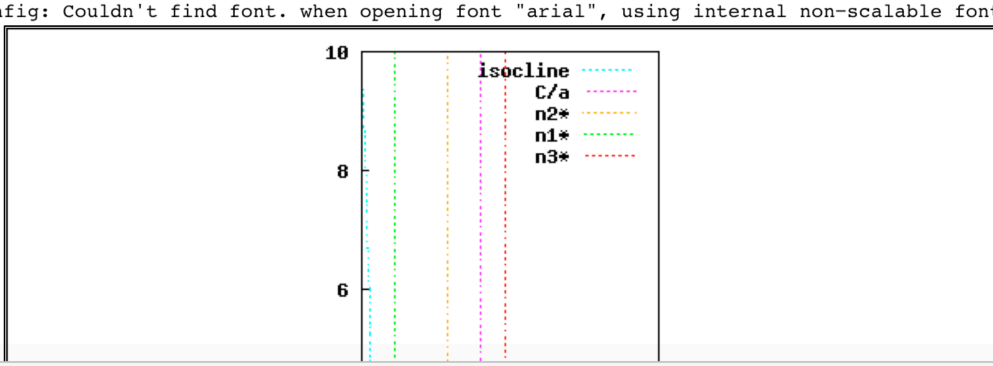
```
--> eq2d:solve(subst(params1d,model2),[n,p]);
rat: replaced -3.333333333333333 by -10/3 = -3.333333333333333
```

(%o45) $[[n=0, p=0], [n=\frac{3}{10}, p=0], [n=\frac{12}{5}, p=-70]]$

```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params1d,zil[1]),n,0,5,p,0,10),
        color=magenta,key="C/a",
       implicit(n=2,n,0,5,p,0,10),
    color=orange,key="n2*",implicit(n=sqrt(5)/5+1,n,0,5,p,0,10),
    color=green,key="n1*",implicit(n=-sqrt(5)/5+1,n,0,5,p,0,10),
    color=red,key="n3*",implicit(n=12/5,n,0,5,p,0,10),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
rat: replaced -3.333333333333333 by -10/3 = -3.333333333333333rat: replaced 3.0 by 3/1 = 3.0rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794Fontconfig error: Cannot load default config file
fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```

(%t55)



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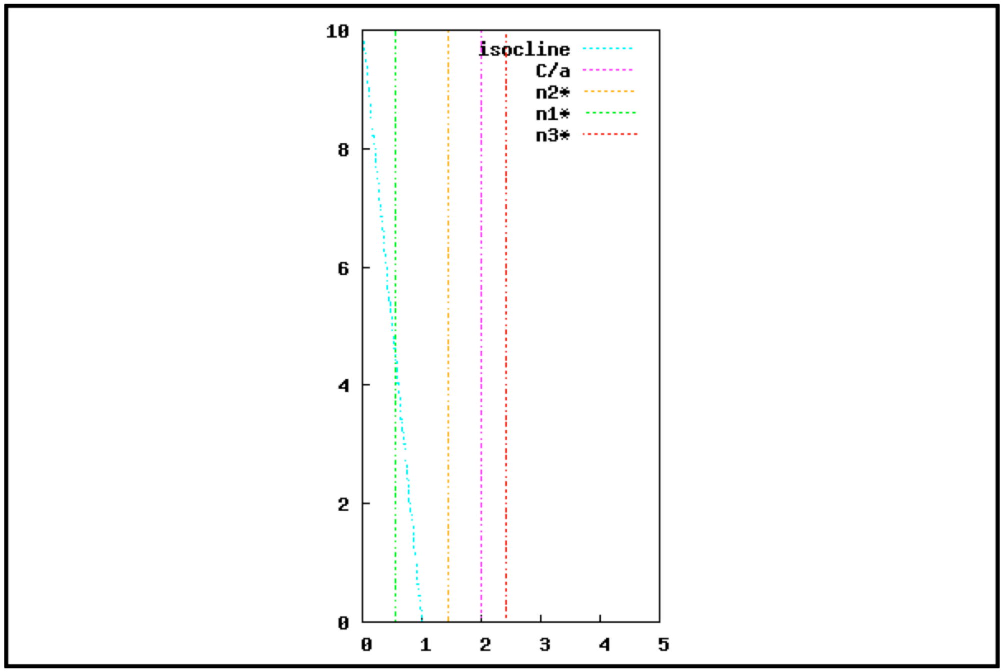
```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params1c,zi1[1]),n,0,5,p,0,10),
        color=magenta,key="C/a",
      implicit(n=2,n,0,5,p,0,10),
    color=orange, key="n2*",implicit(n=sqrt(5)/5+1,n,0,5,p,0,10),
    color=green, key="n1*",implicit(n=-sqrt(5)/5+1,n,0,5,p,0,10),
    color=red, key="n3*",implicit(n=12/5,n,0,5,p,0,10),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```

(%t56)



(%o56)

```
--> params1d:[r=10,K=0.3,a=1,a1=5,C=2,u=1];
```

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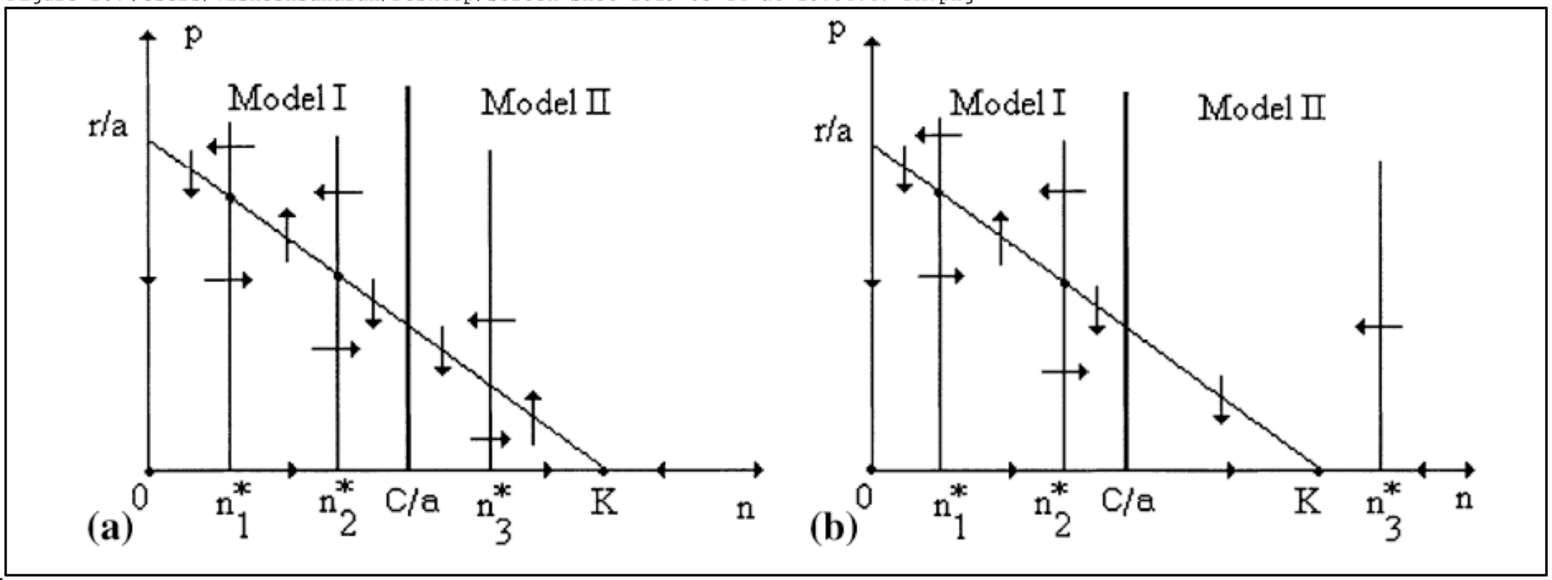
Figure 20: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.04.47 PM.png



Now let us look at when K<C/a

```
--> params1c:[r=10,K=1,a=1,a1=5,C=2,u=1];
```

(%o37) $[r=10, K=1, a=1, a1=5, C=2, u=1]$

```
--> eq1c:solve(subst(params1c,model1),[n,p]);
```

(%o38) $[[n=0,p=0],[n=1,p=0],[n=-\frac{\sqrt{5}-5}{5},p=2\sqrt{5}],[n=\frac{\sqrt{5}+5}{5},p=-2\sqrt{5}]]$

```
--> eq2c:solve(subst(params1c,model2),[n,p]);
```

(%o39) $[[n=0,p=0],[n=1,p=0],[n=\frac{12}{5},p=-14]]$

```
--> wxdraw2d(proportional_axes=xy,
     line_type=dots,line_width=0.5,
```

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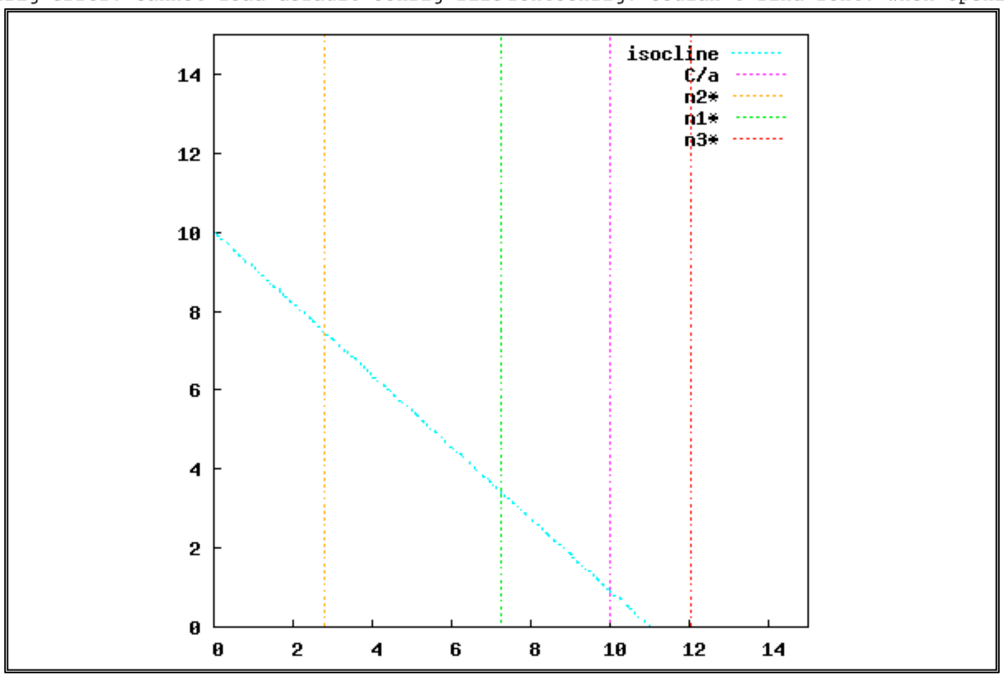
```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params4,zil[1]),n,0,15,p,0,15),
        color=magenta,key="C/a",
       implicit(n=10,n,0,15,p,0,15),
    color=orange,key="n2*",implicit(n=5-sqrt(5),n,0,15,p,0,15),
    color=green,key="n1*",implicit(n=5+sqrt(5),n,0,15,p,0,15),
    color=red,key="n3*",implicit(n=12,n,0,15,p,0,15),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794 rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```

(%t64)



(%o64)

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```
--> params4:[r=10,K=11,a=1,a1=1,C=10,u=1];
```

(%o60) $[r=10, K=11, a=1, a1=1, C=10, u=1]$

```
--> eq14:solve(subst(params4,model1),[n,p]);
```

(%o61) $[[n=0, p=0], [n=11, p=0], [n=5-\sqrt{5}, p=\frac{2\,5^{3/2}+60}{11}], [n=\sqrt{5}+5, p=-\frac{2\,5^{3/2}-60}{11}]]$

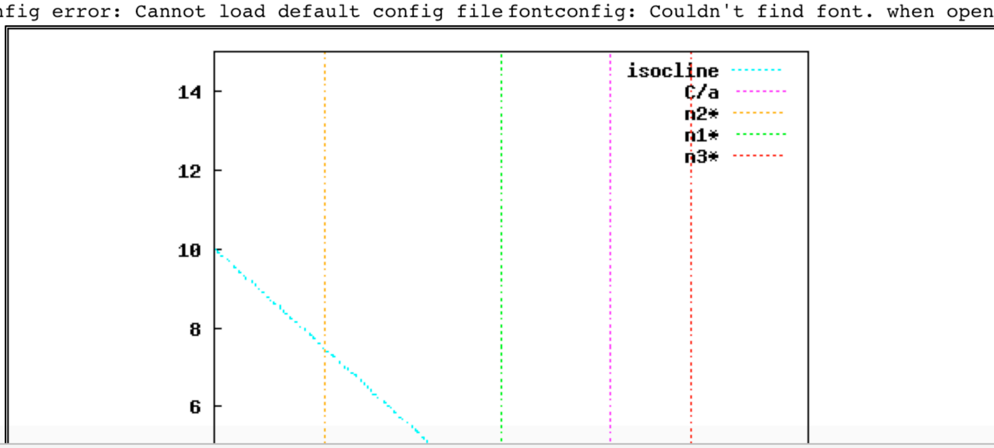
```
--> eq24:solve(subst(params4,model2),[n,p]);
```

(%o62) $[[n=0, p=0], [n=11, p=0], [n=12, p=-\frac{10}{11}]]$

```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params4,zi1[1]),n,0,15,p,0,15),
        color=magenta,key="C/a",
       implicit(n=10,n,0,15,p,0,15),
    color=orange,key="n2*",implicit(n=5-sqrt(5),n,0,15,p,0,15),
    color=green,key="n1*",implicit(n=5+sqrt(5),n,0,15,p,0,15),
    color=red,key="n3*",implicit(n=12,n,0,15,p,0,15),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794rat: replaced 2.23606797749979 by 16692641/7465176 = 2.236067977499794
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```

(%t64)



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```
--> zi2;
```

(%o68) $[p=\frac{r\,K-n\,r}{a\,K}]$

```
--> params3:[r=20,K=30,a=1,a1=1,C=20,u=1];
```

(%o48) $[r=20,K=30,a=1,a1=1,C=20,u=1]$

```
--> eq13:solve(subst(params3,model1),[n,p]);
```

(%o50) $[[n=0,p=0],[n=30,p=0],[n=10-2\sqrt{15},p=\frac{4\sqrt{3}\sqrt{5}+40}{3}],[n=2\sqrt{15}+10,p=-\frac{4\sqrt{3}\sqrt{5}-40}{3}]]$

```
--> eq23:solve(subst(params3,model1),[n,p]);
```

(%o51) $[[n=0,p=0],[n=30,p=0],[n=10-2\sqrt{15},p=\frac{4\sqrt{3}\sqrt{5}+40}{3}],[n=2\sqrt{15}+10,p=-\frac{4\sqrt{3}\sqrt{5}-40}{3}]]$

```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params3,zi1[1]),n,0,30,p,0,24),
        color=magenta, key="C/a",
      implicit(n=20,n,0,30,p,0,24),
    color=orange, key="n2*",implicit(n=2*sqrt(15)+10,n,0,30,p,0,24),
    color=green,key="n1*",implicit(n=10-2*sqrt(15),n,0,30,p,0,24),
    color=red,key="n3*",implicit(n=22,n,0,30,p,0,24),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
```

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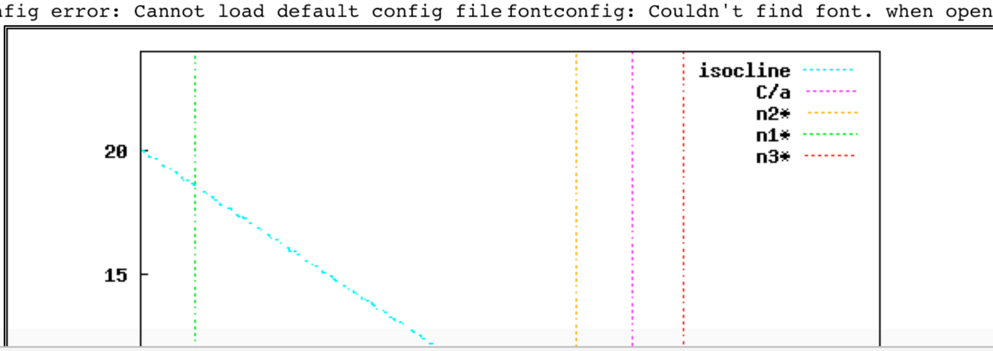

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(%o66)

Figure 19: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.49.58 PM.png

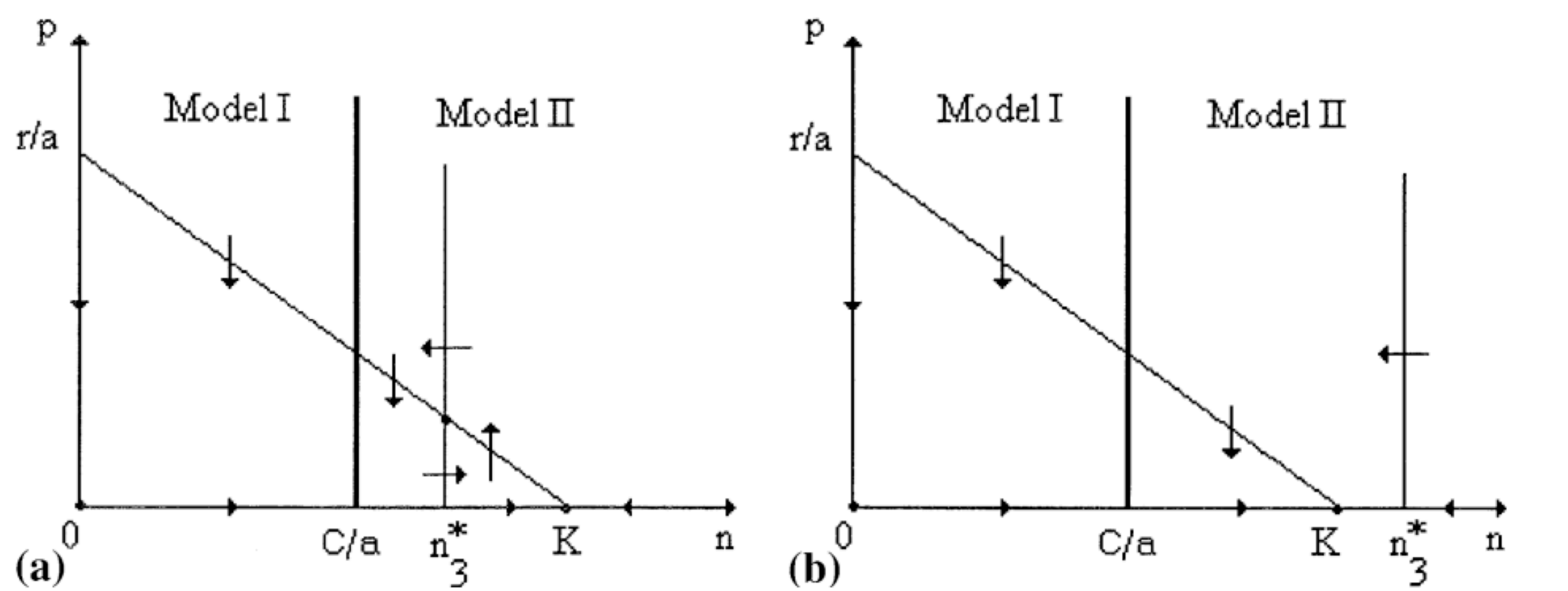

Fig. 1. Phase portrait of the aggregated system when $\alpha C < 8\mu$: (a) $K > n_3^*$ and (b) $K < n_3^*$.

Now let's consdier the case of αC>8μ. Because we can use Model I and Model II for this, we will have four cases according to the position of K with respect to the equilibrium solutions n1*, n2*, and n3*. Let's observe what happens when we put K>C/a

```
--> zi1:solve(model1[1],p);
```

(%o67) $[p=\frac{r K - n r}{a K}]$

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```
);
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```
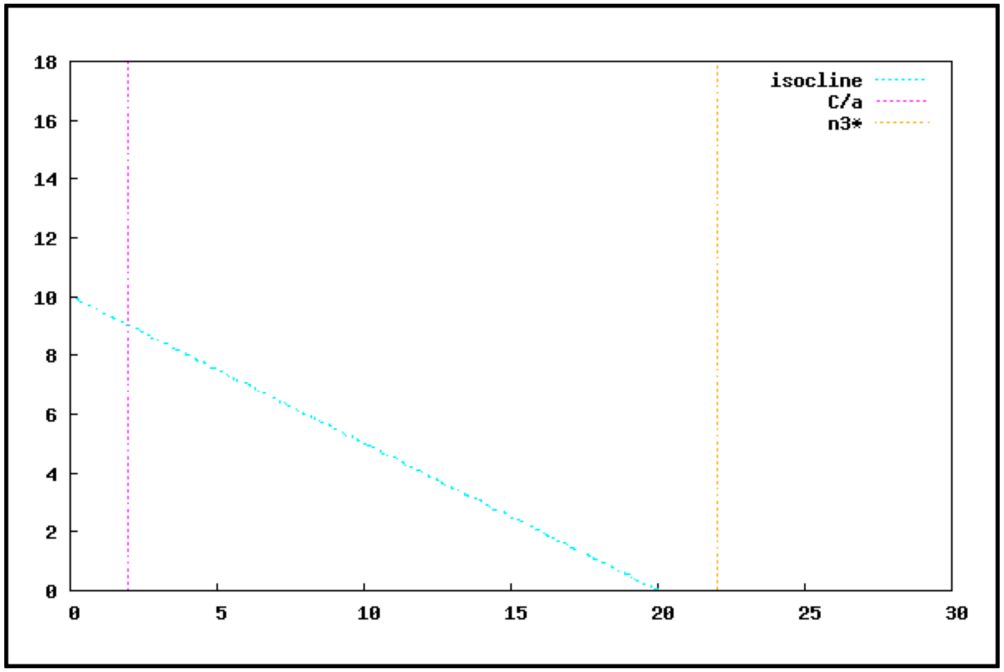
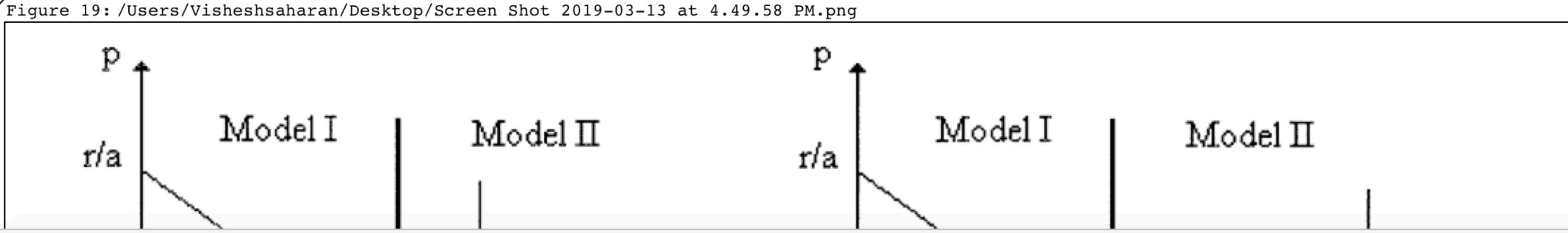

(%t66)



(%o66)

Figure 19: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.49.58 PM.png



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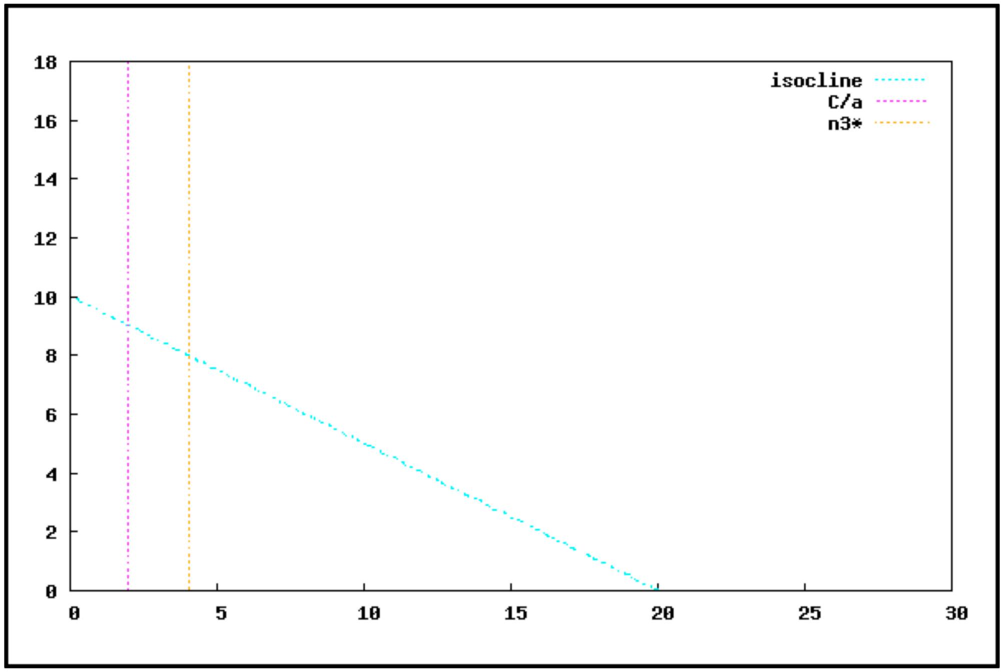
(%t65)



(%o65)

```
--> params1b:[r=10,K=20,a=1,a1=.111,C=2,u=1];
(%o111) [r=10,K=20,a=1,a1=0.111,C=2,u=1]

--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params1b,zi2[1]),n,0,30,p,0,18),
        color=magenta,key="C/a",
       implicit(n=2,n,0,30,p,0,18),
    color=orange, key="n3*",implicit(n=22,n,0,30,p,0,18),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font
```

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```
--> aggmodel:[r*n*(1-(n)/(K))-a*n*p,
    if n<C/a then -u*p+((a1*a)/(2))*n*p-((a1*a^2)/(2*C)*n^2*p)
    else -u*p+((a1*a)/(2))*n*p-((a1*C)/(2)*p) ];
```

(%o11) $\left[n\, r \left(1-\frac{n}{K}\right) - a\, n\, p,\ \text{if } n<\frac{C}{a} \text{ then } -\frac{a^2\, a1\, n^2\, p}{2\, C} - p\, u + \frac{a\, a1\, n\, p}{2} \text{ else } -\frac{a1\, p\, C}{2} - p\, u + \frac{a\, a1\, n\, p}{2} \right]$

```
--> eq1:solve(model1,[n,p]);
```

(%o17) $\left[[n=0, p=0], [n=K, p=0], \left[n=-\frac{\sqrt{a1^2\, C^2 - 8\, a1\, u\, C} - a1\, C}{2\, a\, a1}, p=\frac{2\, a\sqrt{a1}\, r\, K + r\sqrt{C}\sqrt{a1\, C - 8\, u} - \sqrt{a1}\, r\, C}{2\, a^2\sqrt{a1}\, K} \right], \left[n=\frac{\sqrt{a1^2\, C^2 - 8\, a1\, u\, C} + a1\, C}{2\, a\, a1}, p=\frac{2\, a\sqrt{a1}\, r\, K - r\sqrt{C}\sqrt{a1\, C - 8\, u} - \sqrt{a1}\, r\, C}{2\, a^2\sqrt{a1}\, K} \right] \right]$

```
--> eq2:solve(model2,[n,p]);
```

(%o16) $\left[[n=0, p=0], [n=K, p=0], \left[n=\frac{a1\, C + 2\, u}{a\, a1}, p=\frac{a\, a1\, r\, K - a1\, r\, C - 2\, r\, u}{a^2\, a1\, K} \right] \right]$

```
--> params1:[r=10,K=20,a=1,a1=1,C=2,u=1];
```

(%o18) $[r=10, K=20, a=1, a1=1, C=2, u=1]$

2<8, so we're good. Because of our condition, we will use Model II

```
--> zi2:solve(model2[1],p);
```

(%o20) $\left[p=\frac{r\, K - n\, r}{a\, K} \right]$

```
--> wxdraw2d(proportional_axes=xy,
        line_type=dots,line_width=0.5,
        color=cyan,key="isocline",
        implicit(subst(params1,zi2[1]),n,0,30,p,0,18),
        color=magenta, key="C/a",
       implicit(n=2,n,0,30,p,0,18),
    color=orange, key="n3*",implicit(n=4,n,0,30,p,0,18),
        line_type=solid,points_joined=true, point_type=0,color=red

    );
```

Fontconfig error: Cannot load default config file fontconfig: Couldn't find font. when opening font "arial", using internal non-scalable font

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Figure 17: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.49.12 PM.png

$$n = n_1^* = \frac{C}{2a} - \frac{\sqrt{\alpha a^2 C(\alpha C - 8\mu)}}{2\alpha a^2}$$

and

$$n = n_2^* = \frac{C}{2a} + \frac{\sqrt{\alpha a^2 C(\alpha C - 8\mu)}}{2\alpha a^2}.$$

For Model II, we have two nullclines, p=0 and the vertical line

Figure 18: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.49.32 PM.png

$$n = n_3^* = \frac{2\mu + \alpha C}{\alpha a}.$$

Here's where I'll begin some actual Maxima work. Let's consider the case αC<8μ where K>n3* and K<n3*

```
--> model1:[r*n*(1-(n)/(K))-a*n*p,-u*p+((a1*a)/(2))*n*p-((a1*a^2)/(2*C)*n^2*p)];
```

(%o8) $\left[n\,r\left(1-\frac{n}{K}\right)-a\,n\,p,\ -\frac{a^2\,a1\,n^2\,p}{2\,C}-p\,u+\frac{a\,a1\,n\,p}{2}\right]$

```
--> model2:[r*n*(1-(n)/(K))-a*n*p,-u*p+((a1*a)/(2))*n*p-((a1*C)/(2)*p)];
```

(%o9) $\left[n\,r\left(1-\frac{n}{K}\right)-a\,n\,p,\ -\frac{a1\,p\,C}{2}-p\,u+\frac{a\,a1\,n\,p}{2}\right]$

```
--> aggmodel:[r*n*(1-(n)/(K))-a*n*p,
```

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Figure 16: /Users/Vishesshsaharan/Desktop/Screen Shot 2019-03-13 at 4.48.36 PM.png

Model I: $n < \frac{C}{a}$, $x^* = \frac{an}{C}$ is asymptotically stable in Eq. (12),

$$\begin{cases} \frac{\mathrm{d}n}{\mathrm{d}t} = rn\left(1 - \frac{n}{K}\right) - anp, \\ \frac{\mathrm{d}p}{\mathrm{d}t} = -\mu p + \frac{\alpha a}{2} np - \frac{\alpha a^2}{2C} n^2 p. \end{cases}$$

Model II: $n > \frac{C}{a}$, $x^* = 1$ is asymptotically stable in Eq. (12),

$$\begin{cases} \frac{\mathrm{d}n}{\mathrm{d}t} = rn\left(1 - \frac{n}{K}\right) - anp, \\ \frac{\mathrm{d}p}{\mathrm{d}t} = -\mu p + \frac{\alpha a}{2} np - \frac{\alpha C}{2} p. \end{cases}$$

These two models connect at the vertical line $n = C/a$ of the phase space (n, p).

When looking for nullclines, there are several different cases depending on parameter values. For Model 1, if αC<8μ, the only nullcline is p=0. If αC>8μ, we have besides p=0, two more nullclines which are vertical lines

Figure 17: /Users/Vishesshsaharan/Desktop/Screen Shot 2019-03-13 at 4.49.12 PM.png

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- $G < C$, $x^* = G/C$ is asymptotically stable for any initial condition $0 < x(0) < 1$. In this case, equilibrium, the population is polymorphic with a proportion G/C of hawks and $1 - G/C$ doves.
- $G > C$, G/C does not belong to the interval $[0,1]$. The equilibrium $x^* = 1$ is asymptotically st ble. The population is monomorphic and totally hawk at equilibrium.

We have to make the assumption that the fast process is at the fast equilibrium. We substitute the previous fast equilibrium and add the two predator equations to the compl model above to get.

Figure 14: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.46.36 PM.png

$$p_{\mathrm{H}} = x^* p, \qquad p_{\mathrm{D}} = (1 - x^*)p.$$

The two equations govern the total prey and predator densitites at the slow time scale. We can call this the aggregated model, which takes into account the behaviors of the

Figure 15: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.48.03 PM.png

$$\begin{cases} \dfrac{\mathrm{d}n}{\mathrm{d}t} = rn\left(1 - \dfrac{n}{K}\right) - anp, \\ \dfrac{\mathrm{d}p}{\mathrm{d}t} = -\mu p + \dfrac{\alpha G}{2} p - \dfrac{\alpha C}{2} (x^*)^2 p. \end{cases} \tag{14}$$

We can have two possibilites for the fast equilibrum and that the gain depends on the prey density. We can plug this into the equation to get two different aggregated model are valid on two domains of the phase plane.

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Deviation of aggregated model

The first step is to neglect the small terms of ε and to look for existence of a stable equillibrium for the fast part of the system.

Figure 12: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.45.32 PM.png

$$\begin{cases} \dfrac{\mathrm{d}p_{\mathrm{H}}}{\mathrm{d}\tau} = px(\Delta_{\mathrm{H}} - \Delta), \\ \dfrac{\mathrm{d}p_{\mathrm{D}}}{\mathrm{d}\tau} = py(\Delta_{\mathrm{D}} - \Delta). \end{cases} \quad (11)$$

Using the fact that x+y=1, the system can be reduced to

Figure 13: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.45.53 PM.png

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{x}{2}(1-x)(G-Cx). \quad (1$$

This equation has three equilibria, 0, 1 and G/C. 0 is always unstable. Let us denote x^* the stal non-trivial equilibrium. According to parameters values, two cases can occur:

- $G < C$, $x^* = G/C$ is asymptotically stable for any initial condition $0 < x(0) < 1$. In this case, equilibrium, the population is polymorphic with a proportion G/C of hawks and $1 - G/C$ doves.
- $G > C$, G/C does not belong to the interval $[0, 1]$. The equilibrium $x^* = 1$ is asymptotically st

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Figure 9: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.41.17 PM.png

$$\frac{\mathrm{d}p_{\mathrm{H}}}{\mathrm{d}t} = -\mu p_{\mathrm{H}} + \left(\alpha\left(\frac{G-C}{2}\right)\frac{p_{\mathrm{H}}}{p} + \alpha G\frac{p_{\mathrm{D}}}{p}\right)p_{\mathrm{H}}, \tag{7}$$

Figure 10: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.41.46 PM.png

$$\frac{\mathrm{d}p_{\mathrm{D}}}{\mathrm{d}t} = -\mu p_{\mathrm{D}} + \alpha\frac{G}{2}\frac{p_{\mathrm{D}}}{p}p_{\mathrm{D}}. \tag{8}$$

The complete model combines both fast and slow processes

Figure 11: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.44.26 PM.png

$$\begin{cases} \dfrac{\mathrm{d}n}{\mathrm{d}\tau} = \varepsilon\left(rn\left(1-\dfrac{n}{K}\right) - anp\right), \\ \dfrac{\mathrm{d}p_{\mathrm{H}}}{\mathrm{d}\tau} = px(\Delta_{\mathrm{H}} - \Delta) + \varepsilon\left(-\mu p_{\mathrm{H}} + \left(\alpha\left(\dfrac{G-C}{2}\right)\dfrac{p_{\mathrm{H}}}{p} + \alpha G\dfrac{p_{\mathrm{D}}}{p}\right)p_{\mathrm{H}}\right), \\ \dfrac{\mathrm{d}p_{\mathrm{D}}}{\mathrm{d}\tau} = py(\Delta_{\mathrm{D}} - \Delta) + \varepsilon\left(-\mu p_{\mathrm{D}} + \alpha\dfrac{G}{2}\dfrac{p_{\mathrm{D}}}{p}p_{\mathrm{D}}\right). \end{cases} \tag{10}$$

Deviation of aggregated model

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Figure 6: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.46.19 PM.png

$$\Delta = (x \quad y)\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}.$$

We now try to calculate the difference between the gain of each of them and the average gain of the population. We assume that the hawk-dove game is fast.

Figure 7: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.48.45 PM.png

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}\tau} = x(\Delta_{\mathrm{H}} - \Delta), \\ \dfrac{\mathrm{d}y}{\mathrm{d}\tau} = y(\Delta_{\mathrm{D}} - \Delta). \end{cases}$$

Prey equation can be represented by a classica Lotka-Volterra model. r is the growth rate, K is the carrying capacity of the population, a is a positive predation force paramter, ad p is the

Figure 8: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.39.51 PM.png

$$\frac{\mathrm{d}n}{\mathrm{d}t} = rn\left(1 - \frac{n}{K}\right) - anp, \tag{6}$$

For predators, we assume a constant mortality rate μ>0 for hawks and doves. G represents the gain that the predator gets. For hawks, they get the gain (G-C)/2 while doves get gain G. α represents a positive conversion coefficient of gain and cost into biomass of predators.

Figure 9: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-13 at 4.41.17 PM.png

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Figure 2: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.42.43 PM.png

$$p(t) = p_H(t) + p_D(t).$$

Let x(t) and y(t) be repectively the hawks and doves proportions in the population of predators at time t.

Figure 3: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.43.42 PM.png

$$x(t) = \frac{p_H(t)}{p(t)}, \qquad y(t) = 1 - x(t) = \frac{p_D(t)}{p(t)}.$$

Using replicator equations that describe that change of tactics of predators that have been described, we can get an average gain of an individual playing the two tactics

Figure 4: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.47.18 PM.png

$$\Delta_H = (1 \quad 0)\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Figure 5: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.47.23 PM.png

$$\Delta_D = (0 \quad 1)\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Figure 6: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.46.19 PM.png

$$\Delta = (x \quad y)\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}.$$

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```
--> load("/Users/Visheshsaharan/Downloads/MATH280.mac")$
MATH280.mac:  help(MATH280) for contents
```

Figure 1: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 8.52.48 PM.png

A predator–prey model with predators using hawk and dove tactics

Pierre Auger [a,*], Rafael Bravo de la Parra [b], Serge Morand [c], Eva Sánchez [d]

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[b] *Departamento de Matemáticas, Universidad de Alcalá, 28871 Alcalá de Henares (Madrid), Spain*

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[d] *Departamento de Matemáticas, E.T.S.I. Industriales, Universidad Politécnica de Madrid, c/José Gutiérrez Abascal, 2, 28006 Madrid, Spain*

The authors aimes to study the effects of individual predator behavior on the dynamics of the predator-prey system. They refer to the two behavioral tactics used by predators as classical hawk and dove tactics. Authors assume two different time scales, fast and slow. Fast time scale corresponds to the disputes and fights between the predators. Slow time scale corresponds to the growth of the prey population, the mortality of the predator and the predator-prey populations. Let n(t) be the size of the prey population and p(t) be the size of the predator population.

The total density of predators is given by the following equation, with ph(t) represetning the hawk predator desnity and pd(t) representing the dove pedator density

Figure 2: /Users/Visheshsaharan/Desktop/Screen Shot 2019-03-14 at 10.42.43 PM.png

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