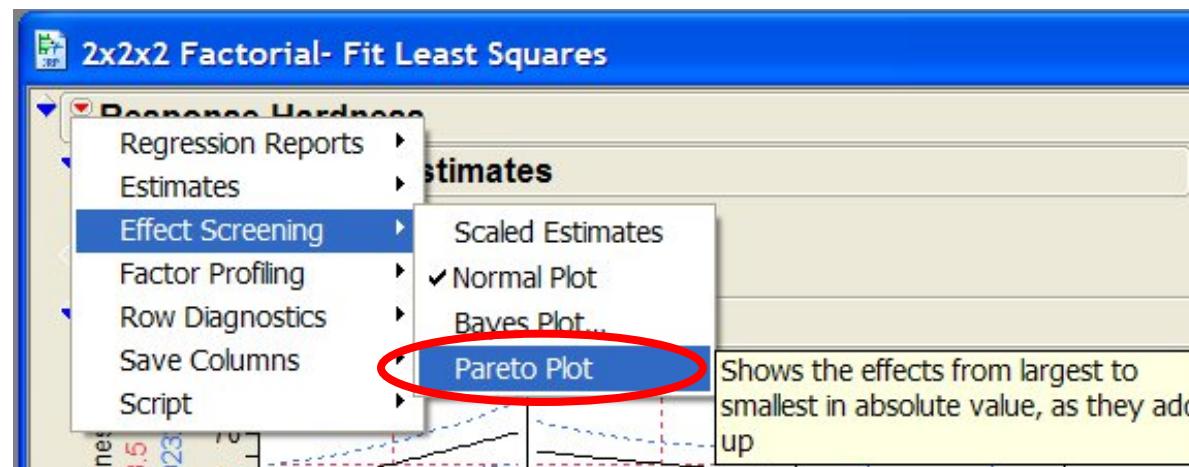


Pareto Plot

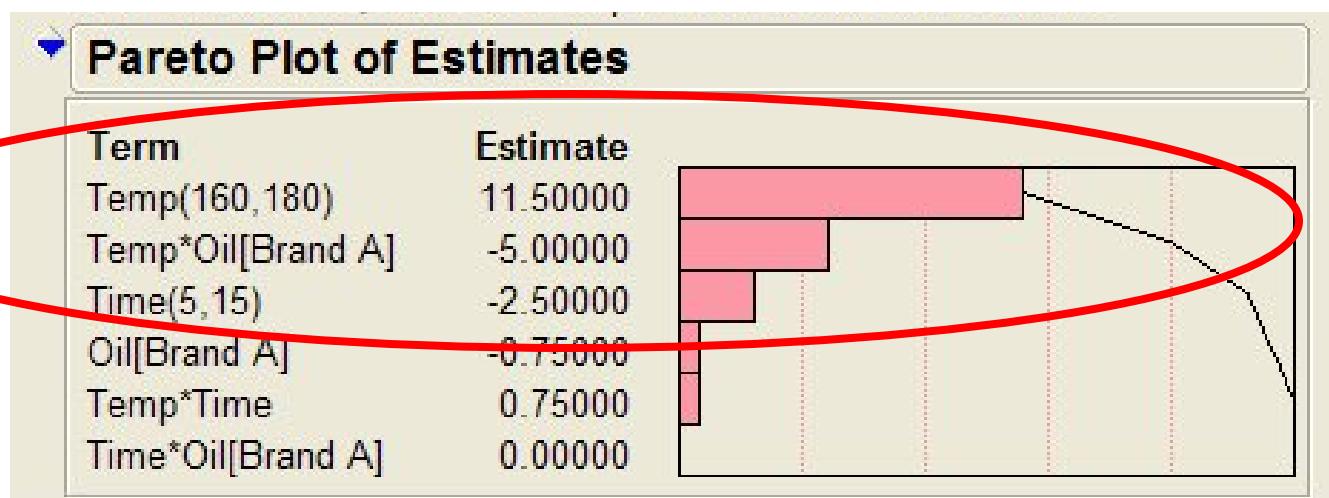
- Under the Y Response Red Triangle, select Effect Screening>Pareto Plot



Six Sigma – 2^k Factorial Experiments

Pareto Plot

- u The Pareto Plot shows that **Temperature, Time, and the interaction of Temperature and Oil** are the most significant model estimates.



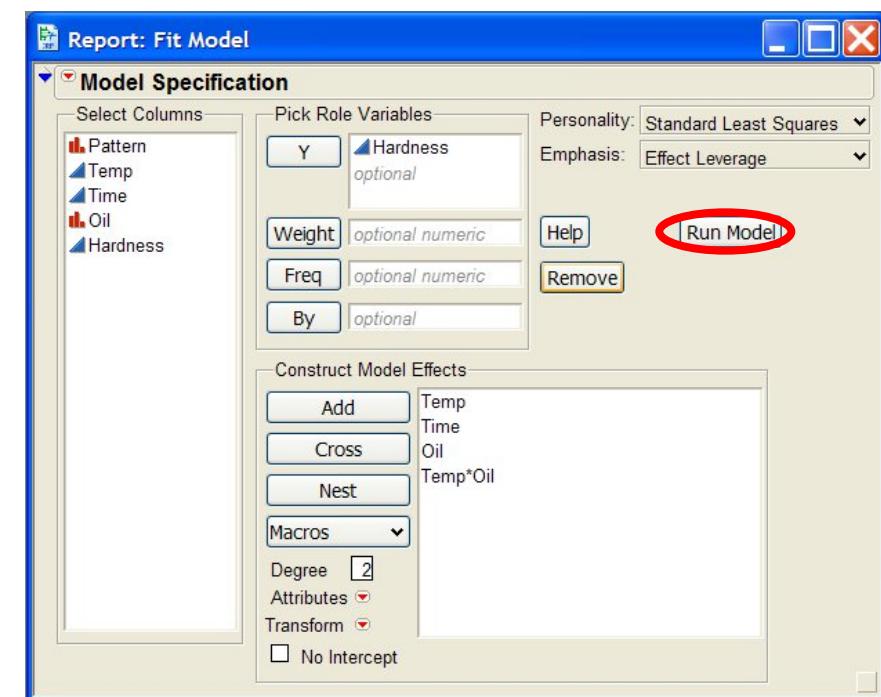
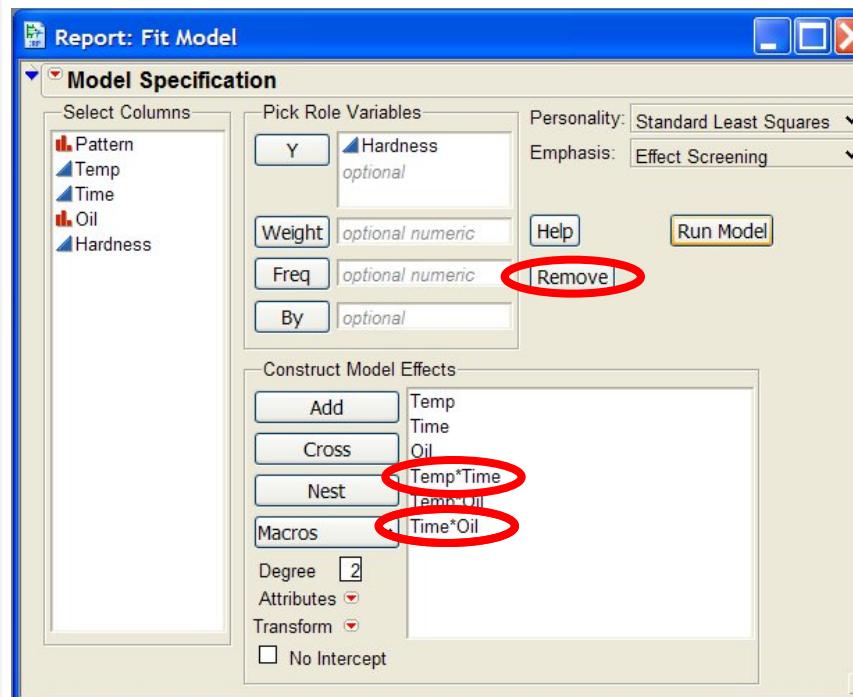
JMP Analysis (Cont.)

- u Step 6: Reduce the Model to include the largest estimates.
- u Note: If an interaction estimate is selected, then the interaction main factors must be included in the model.

Six Sigma – 2^K Factorial Experiments

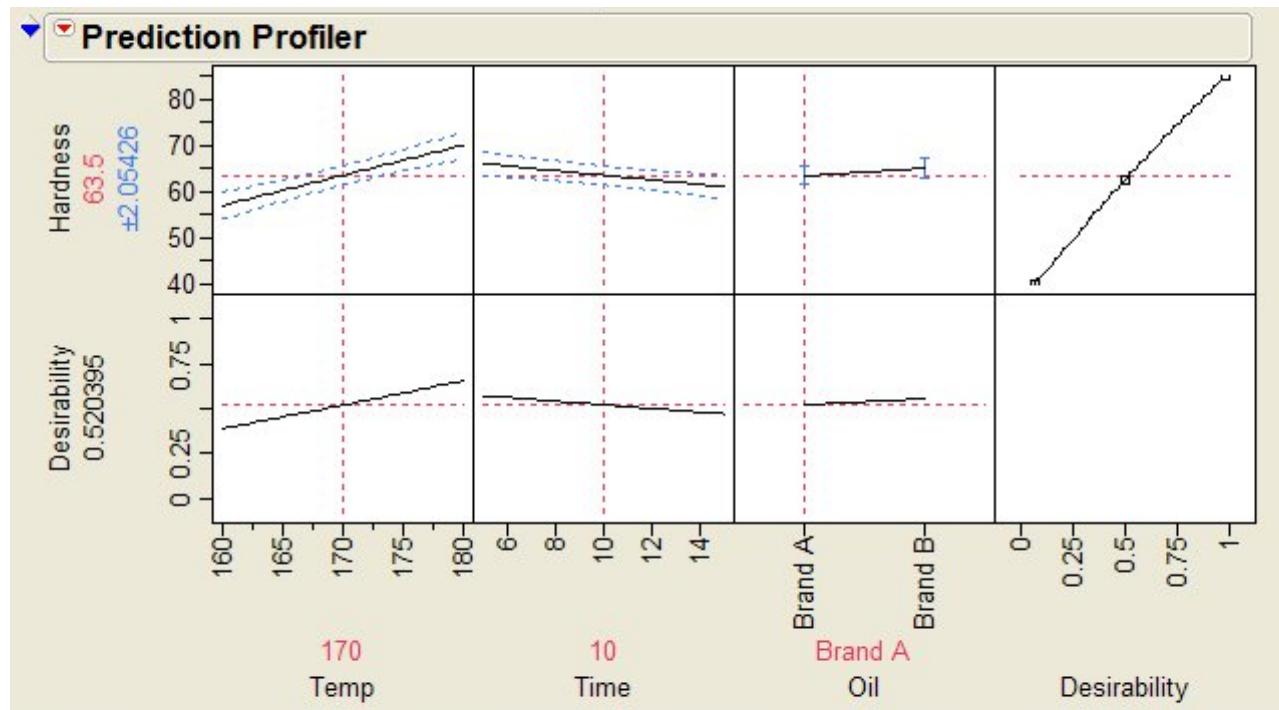
JMP Analysis (Cont.)

- u Click-on the Fit Model Window from Analyze
- u Click-on the non-significant estimates and then click-on **Remove**, and **Run Model**.



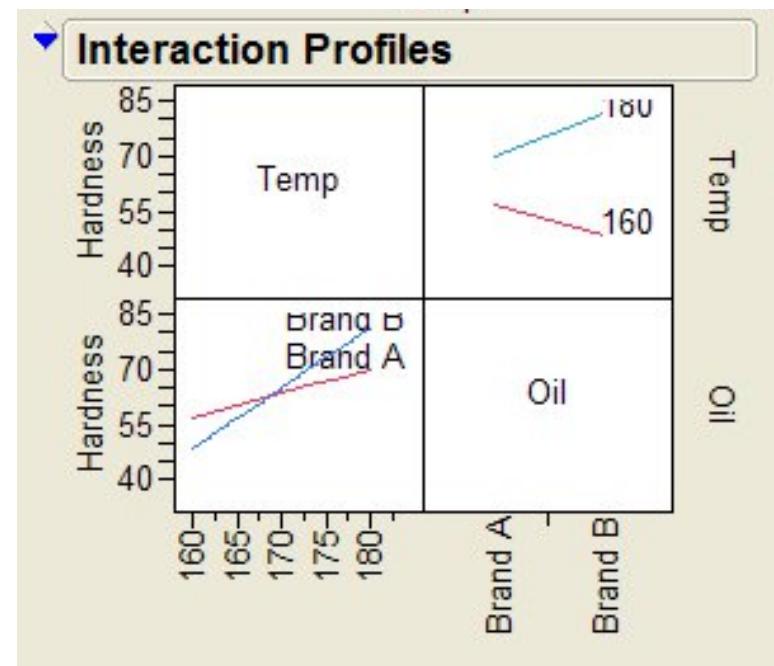
JMP Analysis (Cont.)

- From the **Response Y Red Triangle**, select **Factor Profiling>Profiler**



JMP Analysis (Cont.)

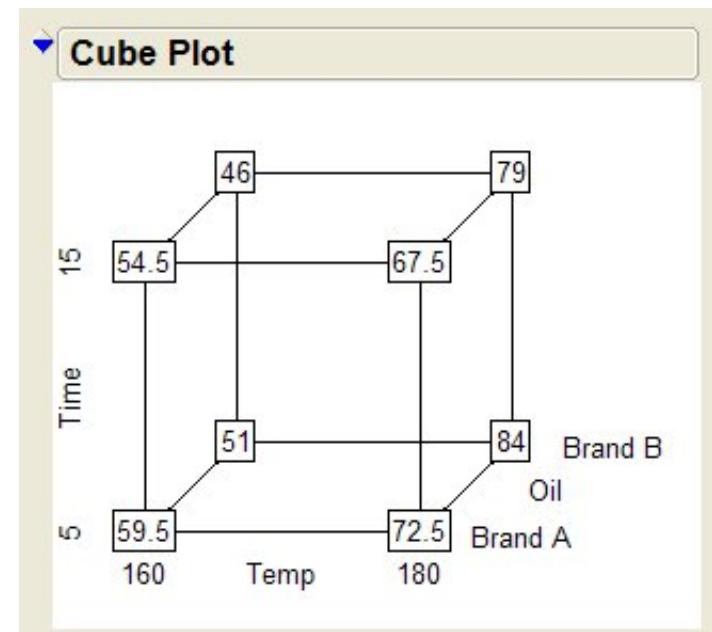
- From the **Response Y Red Triangle**, select **Factor Profiling>Interaction Plots**



Six Sigma – 2^k Factorial Experiments

JMP Analysis (Cont.)

- From the **Response Y Red Triangle**, select **Factor Profiling>Cube Plots**



JMP Analysis (Cont.)

u Step 7: Construct a mathematical model.

▼ Parameter Estimates					
Term	Estimate	Std Error	t Ratio	Prob> t	
Intercept	64.25	0.456435	140.76	<.0001*	
Temp(160,180)	11.5	0.456435	25.20	0.0001*	
Time(5,15)	-2.5	0.456435	-5.48	0.0120*	
Oil[Brand A]	-0.75	0.456435	-1.64	0.1989	
Temp*Oil[Brand A]	-5	0.456435	-10.95	0.0016*	

Model coefficients (coded) for Oil Brand A
Coded: Temp= -1,1 Time = -1,1

JMP Analysis (Cont.)

- Step 7: Construct a mathematical model for Brand A.

Parameter Estimates					
Term	Estimate	Std Error	t Ratio	Prob> t	
Intercept	64.25	0.456435	140.76	<.0001*	
Temp(160,180)	11.5	0.456435	25.20	0.0001*	
Time(5,15)	-2.5	0.456435	-5.48	0.0120*	
Oil[Brand A]	-0.75	0.456435	-1.64	0.1989	
Temp*Oil[Brand A]	-5	0.456435	-10.95	0.0016*	

- Hardness = 64.25 + 11.50(Temp) – 2.5(Time)
- 0.75 – 5.00 (Temp*Oil Type)

Where Temp=-1,1 Time=-1,1 For Brand A

JMP Analysis (Cont.)

- u Construct a mathematical model for Brand B. Change the sign on the coefficients that contain Oil Type

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	64.25	0.456435	140.76	<.0001*
Temp(160,180)	11.5	0.456435	25.20	0.0001*
Time(5,15)	-2.5	0.456435	-5.48	0.0120*
Oil Type[Brand A] B	+0.75	0.456435	-1.64	0.1989
Temp*Oil Type[Brand A] B	+5	0.456435	-10.95	0.0016*

- u Hardness = $64.25 + 11.50(\text{Temp}) - 2.5(\text{Time})$
 $+ 0.75 + 5.00 (\text{Temp} * \text{Oil Type})$

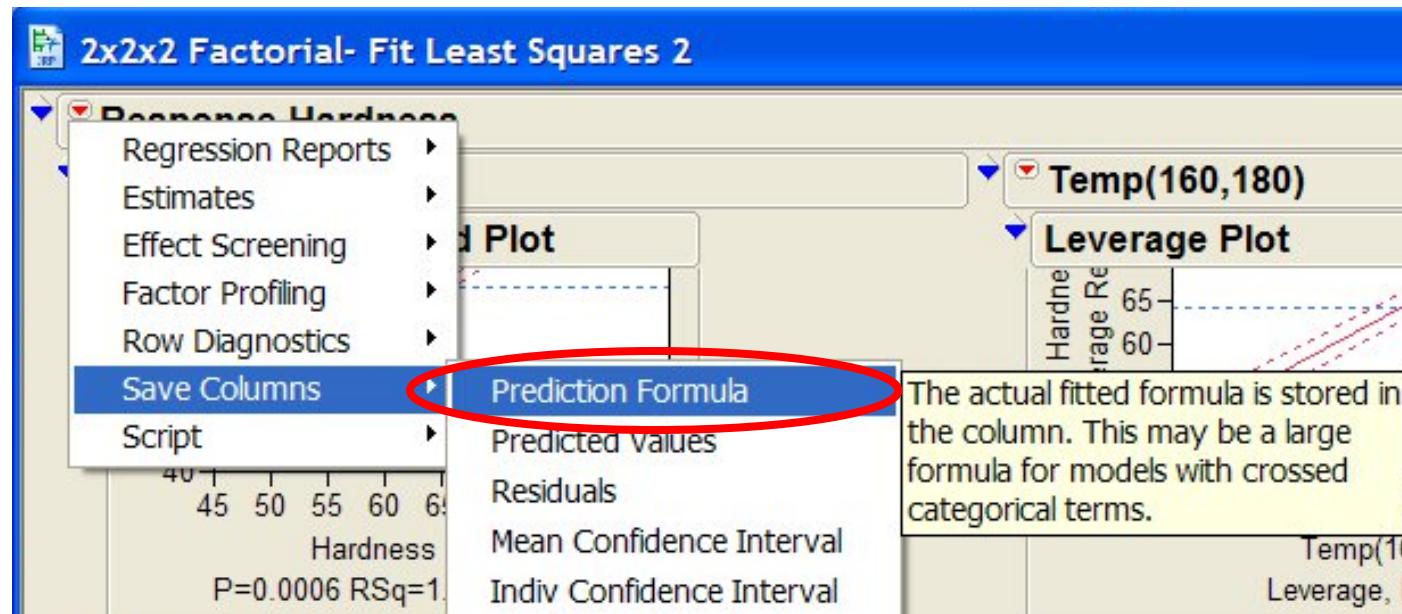
Where Temp=-1,1 Time=-1,1 For Brand B

Questions

- u **Question 1:** What is the hardness when Temp = 1, Time=1 and Oil=Brand A?
- u **Question 2:** What does that value represent?
- u **Question 3:** What is the hardness when Temp = 1, Time=1 and Oil=Brand B?
- u Check your answers using JMP.

Six Sigma – 2^k Factorial Experiments

JMP Analysis (Cont.)

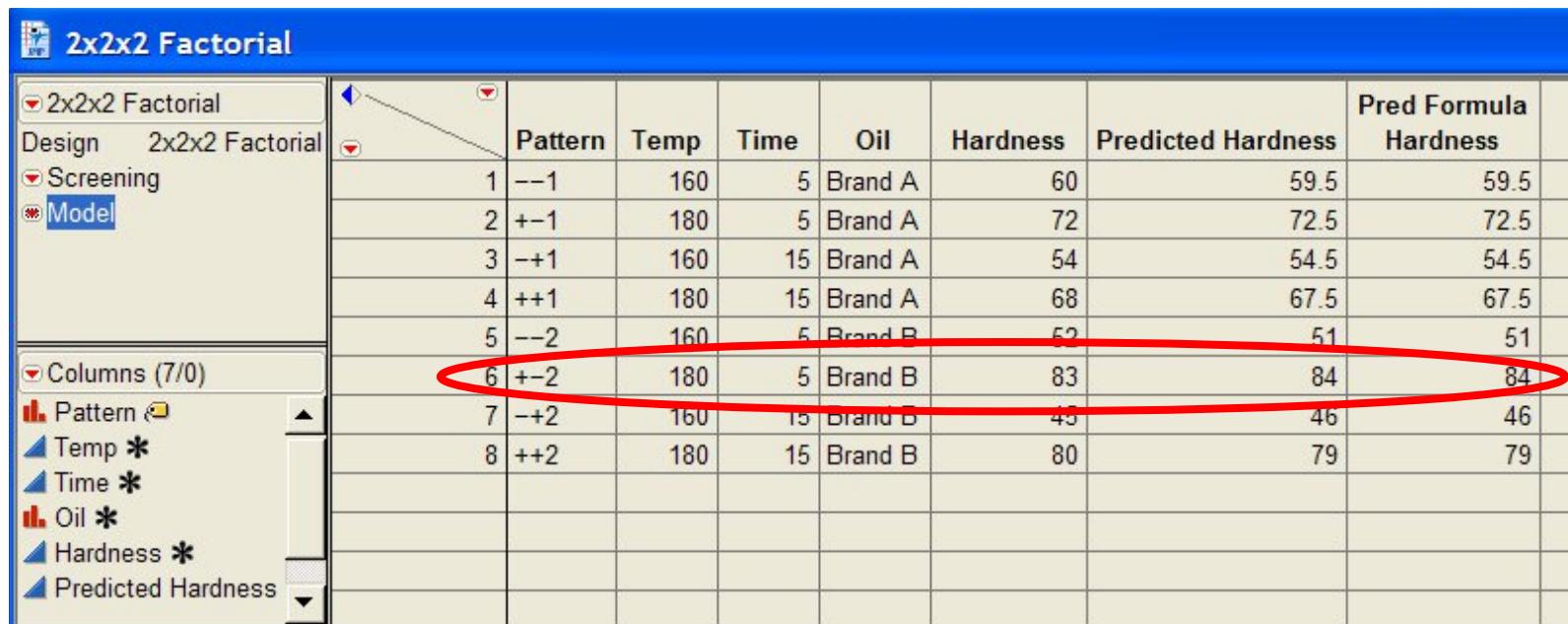


- u **Save Columns>Prediction Formula**

Six Sigma – 2^K Factorial Experiments

JMP Analysis (Cont.)

JMP 2x2x2 Factorial



	Pattern	Temp	Time	Oil	Hardness	Predicted Hardness	Pred Formula Hardness
1	--1	160	5	Brand A	60	59.5	59.5
2	+-1	180	5	Brand A	72	72.5	72.5
3	-+1	160	15	Brand A	54	54.5	54.5
4	++1	180	15	Brand A	68	67.5	67.5
5	--2	160	5	Brand B	52	51	51
6	+-2	180	5	Brand B	83	84	84
7	-+2	160	15	Brand B	45	46	46
8	++2	180	15	Brand B	80	79	79

u Maximum Hardness: Temp = 180, Time = 5,
Oil Brand B

Conclusions

- u **Step 8:** Translate the mathematical model into process terms. Formulate conclusions and recommendations.
 - Conclusions: Temperature has the biggest effect on Hardness and will be a controlling factor in determining the hardness levels. Hardness can be maximized using Oil Type Brand B at Temp = 180°C and Time = 5 sec.
- u **Step 9:** Replicate optimum conditions. Plan the next experiment or institutionalize the change.

Adding Center Points to 2^K Factorials

- u There is always a risk in 2-level designs of missing a curvilinear relationship by only including two levels of the input variable.
- u The addition of “Center Points” is an efficient way to test for curvature without adding a large number of experimental runs.

Example with Center Points

- u A process engineer wants to improve the yield for two different die-castings. There are two inputs of interest: pressure and temperature. The engineer decides to conduct the experiment using a 2×2 design augmented with five center points to estimate experimental error and curvature.

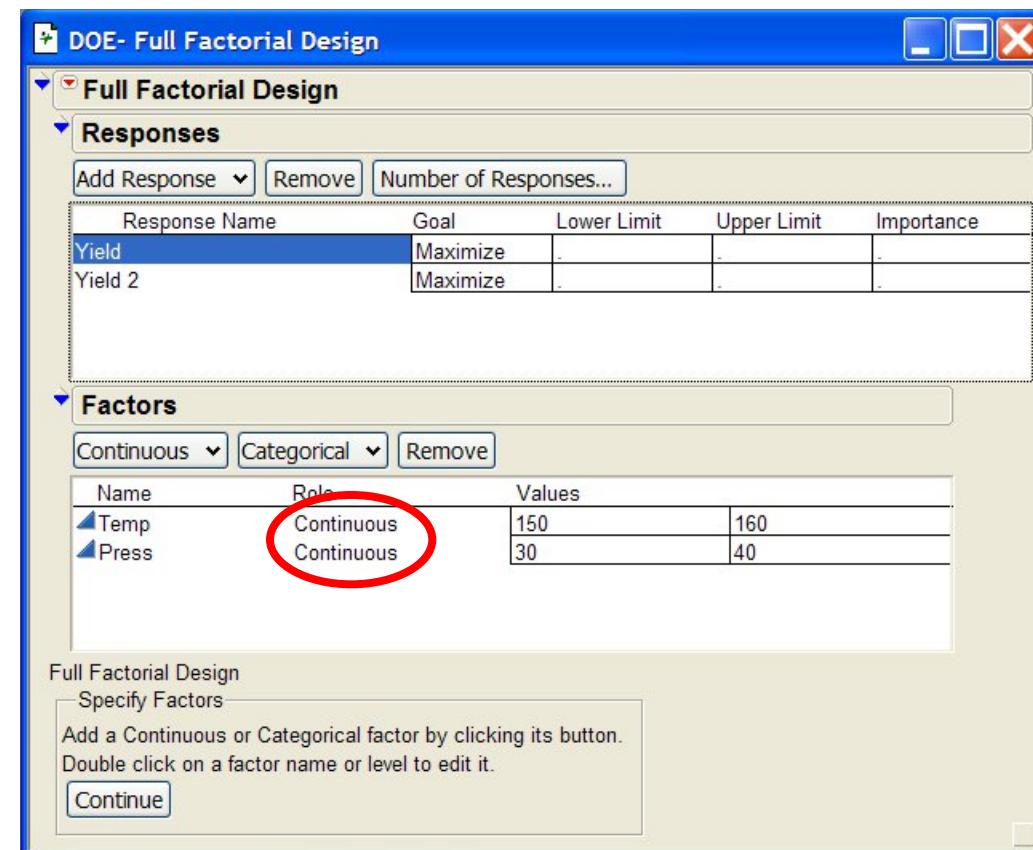
Example with Center Points

- u State the problem: A process engineer wants to improve yield for two different die-castings. There are two inputs of interest: pressure and temperature.
- u State the factors and levels of interest, and create a JMP experimental data sheet.
 - Temperature: 150, 160
 - Pressure: 30, 40

Design Matrix – Using JMP

- u JMP>DOE>Classical>Full Factorial Design

In order to insert center points into JMP, the variables are selected as continuous.

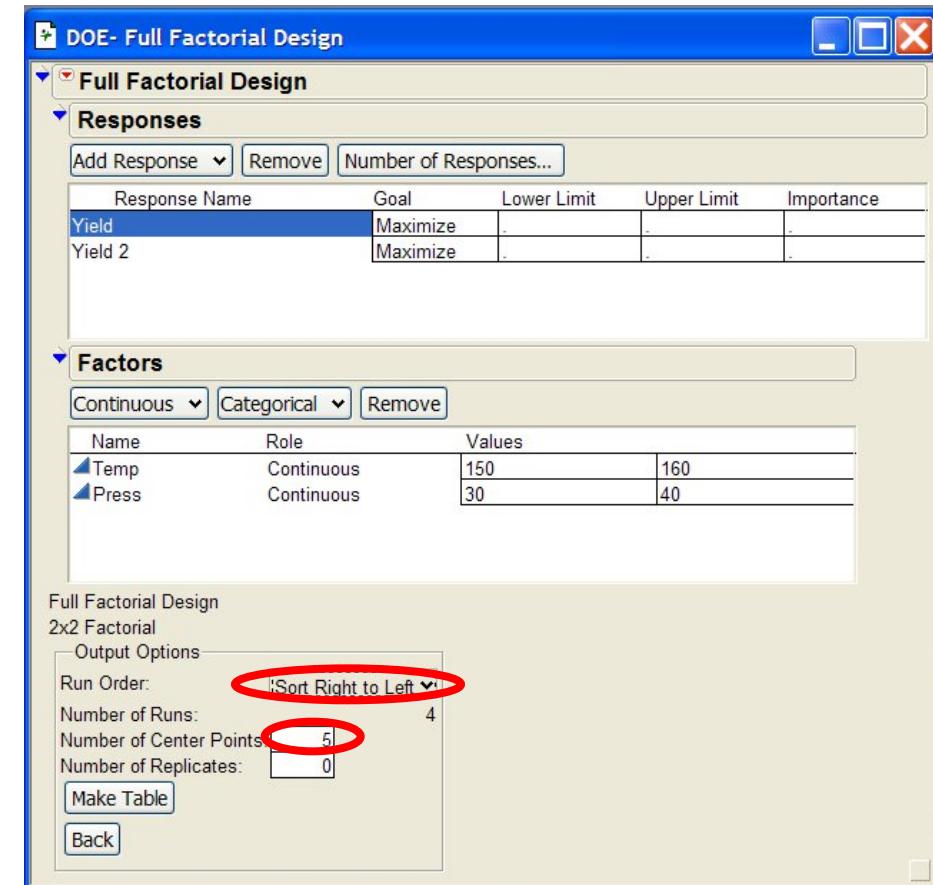


Response Name	Goal	Lower Limit	Upper Limit	Importance
Yield	Maximize			
Yield 2	Maximize			

Name	Role	Values
Temp	Continuous	150 160
Press	Continuous	30 40

JMP DOE Design (Cont.)

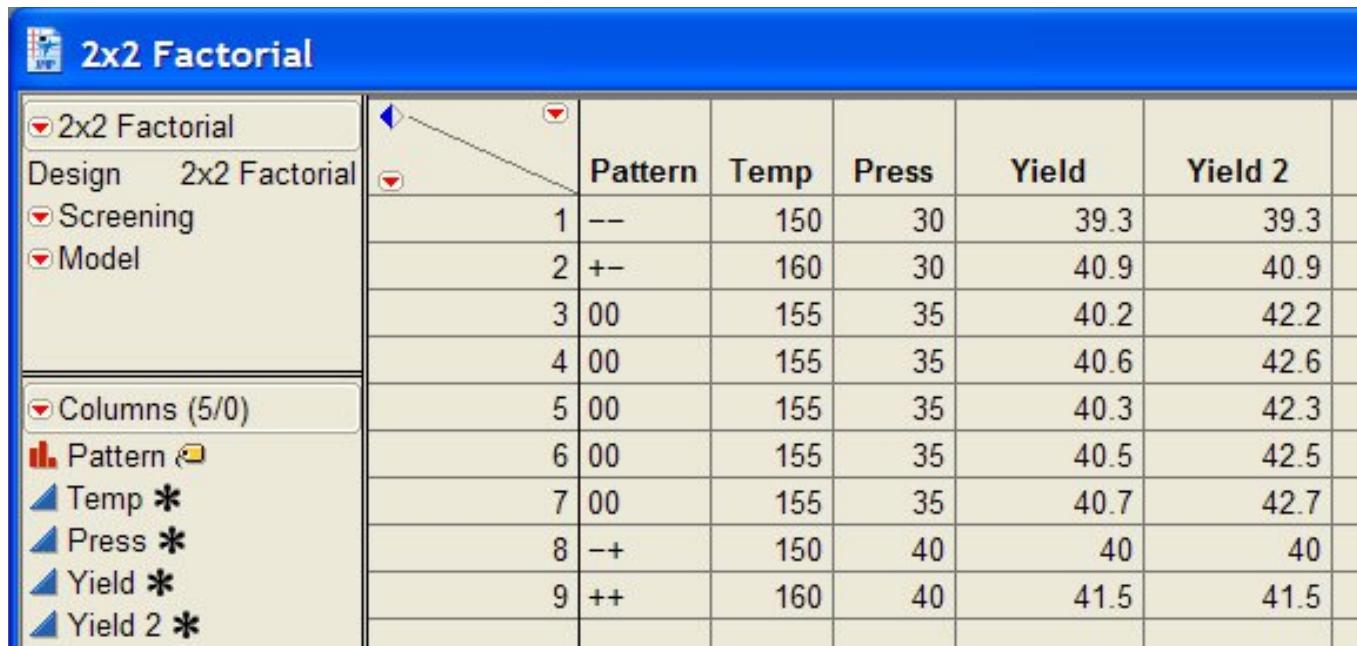
- u Change the **Number of Center Points** to 5.
- u Change **Randomize** to **Sort Right to Left** for teaching purposes.
- u Select **Make Table**.



Six Sigma – 2^k Factorial Experiments

JMP DOE Design (Cont.)

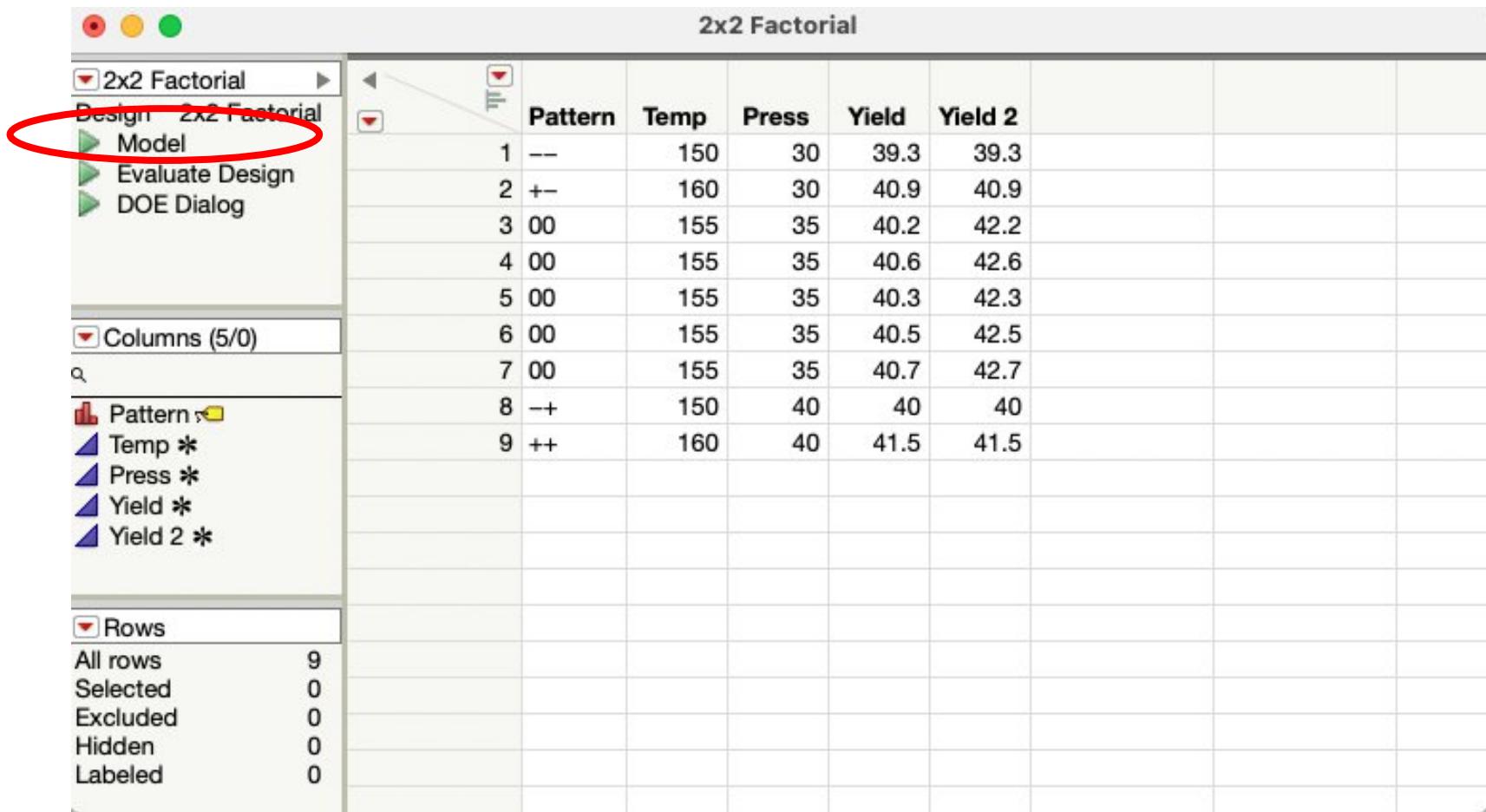
2x2 Factorial



	Pattern	Temp	Press	Yield	Yield 2
1	--	150	30	39.3	39.3
2	+-	160	30	40.9	40.9
3	00	155	35	40.2	42.2
4	00	155	35	40.6	42.6
5	00	155	35	40.3	42.3
6	00	155	35	40.5	42.5
7	00	155	35	40.7	42.7
8	-+	150	40	40	40
9	++	160	40	41.5	41.5

Analysis With JMP (Cont.)

- From the Model select the Green Triangle



The screenshot shows the JMP software interface with the title "2x2 Factorial". On the left, there is a navigation pane with the following items:

- 2x2 Factorial (selected)
- Model (circled in red)
- Evaluate Design
- DOE Dialog

Below the navigation pane, there is a search bar and a list of columns:

- Pattern
- Temp *
- Press *
- Yield *
- Yield 2 *

At the bottom, there is a summary of rows:

Rows	
All rows	9
Selected	0
Excluded	0
Hidden	0
Labeled	0

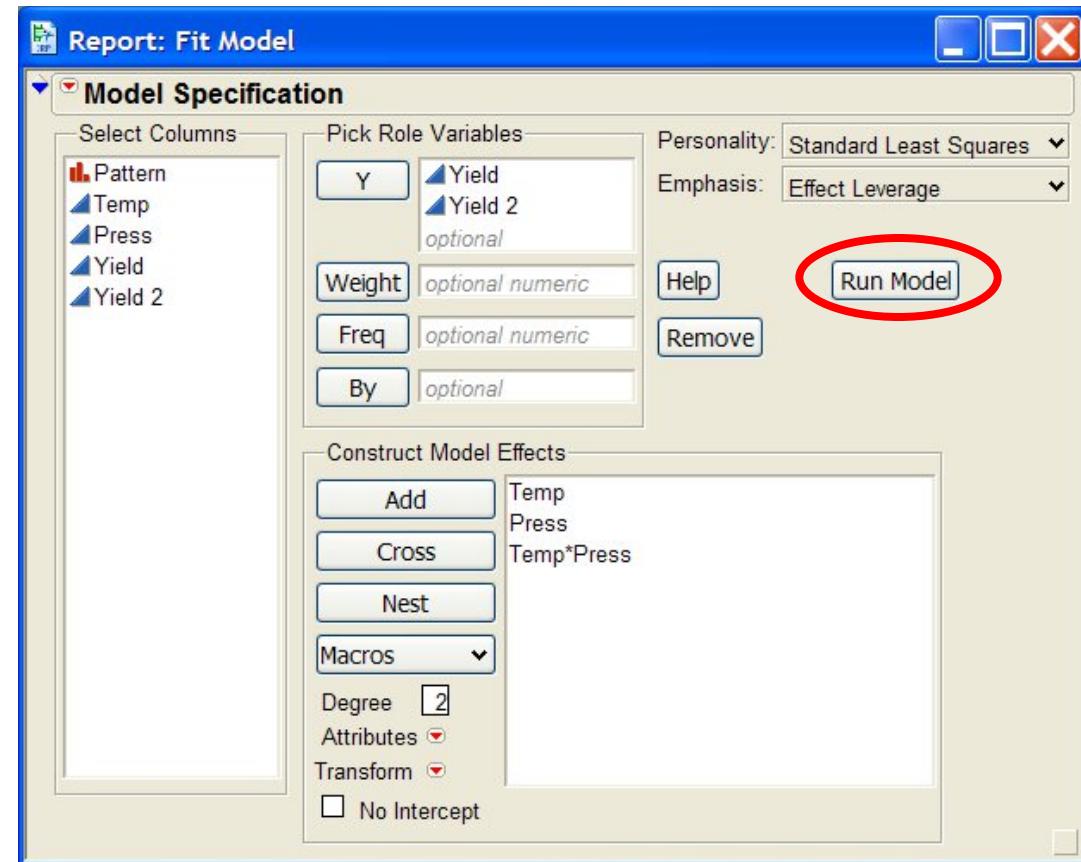
The main area displays the following data table:

	Pattern	Temp	Press	Yield	Yield 2
1	--	150	30	39.3	39.3
2	+-	160	30	40.9	40.9
3	00	155	35	40.2	42.2
4	00	155	35	40.6	42.6
5	00	155	35	40.3	42.3
6	00	155	35	40.5	42.5
7	00	155	35	40.7	42.7
8	-+	150	40	40	40
9	++	160	40	41.5	41.5

Six Sigma – 2^k Factorial Experiments

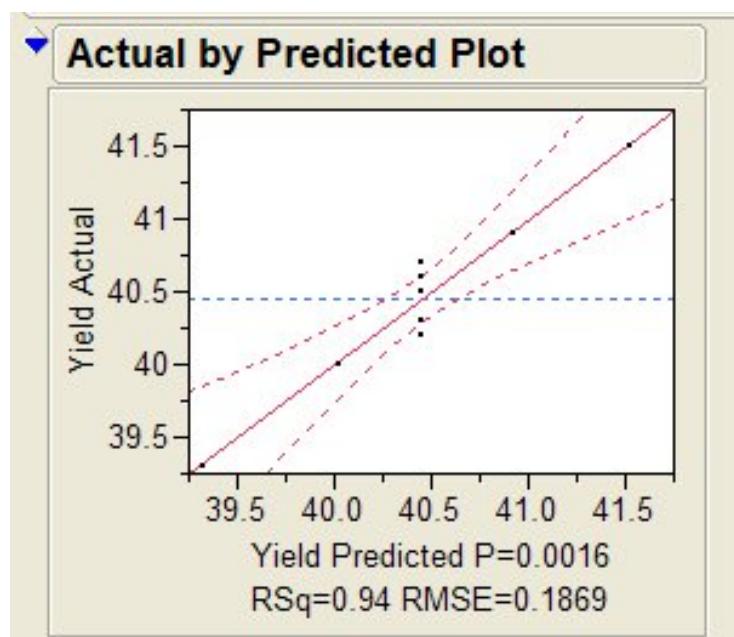
JMP Analysis (Cont.)

u Click Run

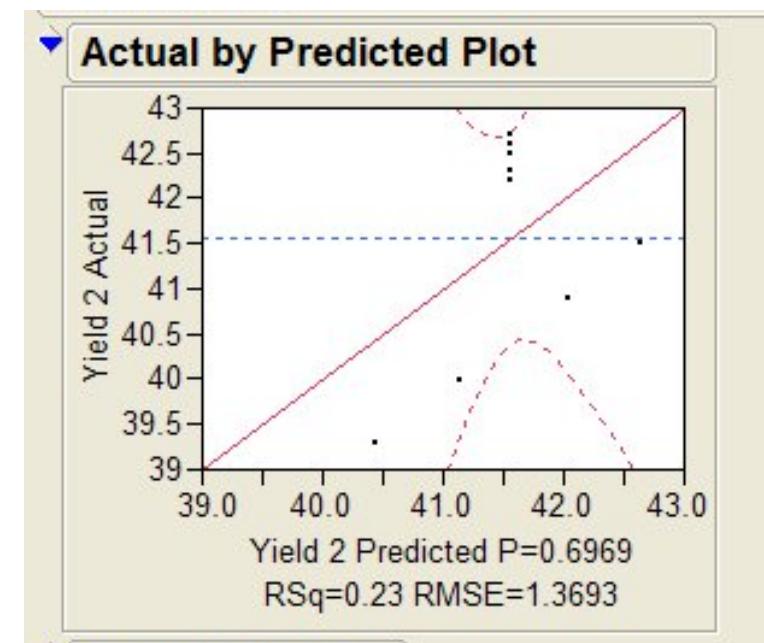


JMP Analysis (Cont.)

Yield



Yield 2



JMP Analysis (Cont.)

Yield

Yield 2

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	40.44444	0.062311	649.07	<.0001*
Temp(150,160)	0.775	0.093467	8.29	0.0004*
Press(30,40)	0.325	0.093467	3.48	0.0177*
Temp*Press	-0.025	0.093467	-0.27	0.7998

Effect Tests

Source	Nparm	DF	Sum of Squares		
			F Ratio	Prob > F	
Temp(150,160)	1	1	2.4025000	68.7520	0.0004*
Press(30,40)	1	1	0.4225000	12.0906	0.0177*
Temp*Press	1	1	0.0025000	0.0715	0.7998

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	41.555556	0.456429	91.05	<.0001*
Temp(150,160)	0.775	0.684643	1.13	0.3090
Press(30,40)	0.325	0.684643	0.47	0.6550
Temp*Press	-0.025	0.684643	-0.04	0.9723

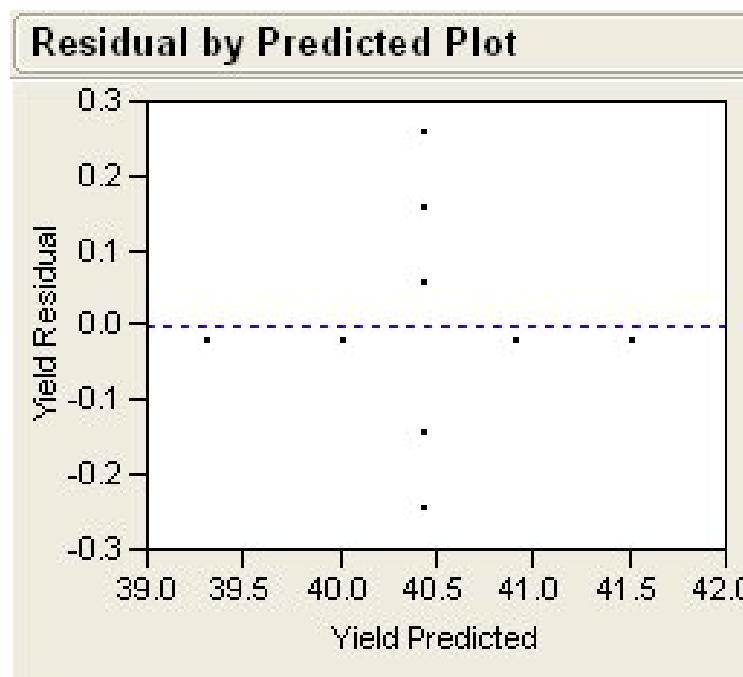
Effect Tests

Source	Nparm	DF	Sum of Squares		
			F Ratio	Prob > F	
Temp(150,160)	1	1	2.4025000	1.2814	0.3090
Press(30,40)	1	1	0.4225000	0.2253	0.6550
Temp*Press	1	1	0.0025000	0.0013	0.9723

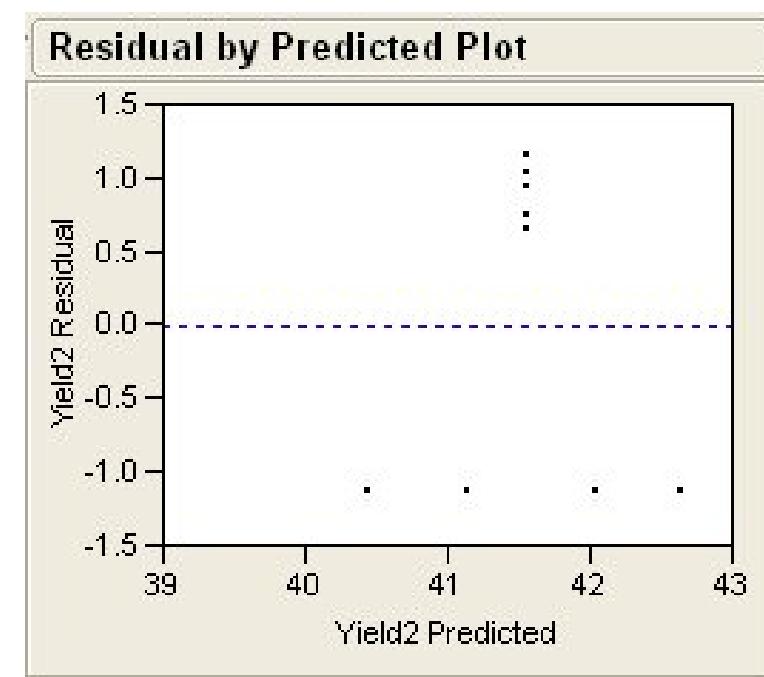
Six Sigma – 2^K Factorial Experiments

JMP Analysis (Cont.)

Yield

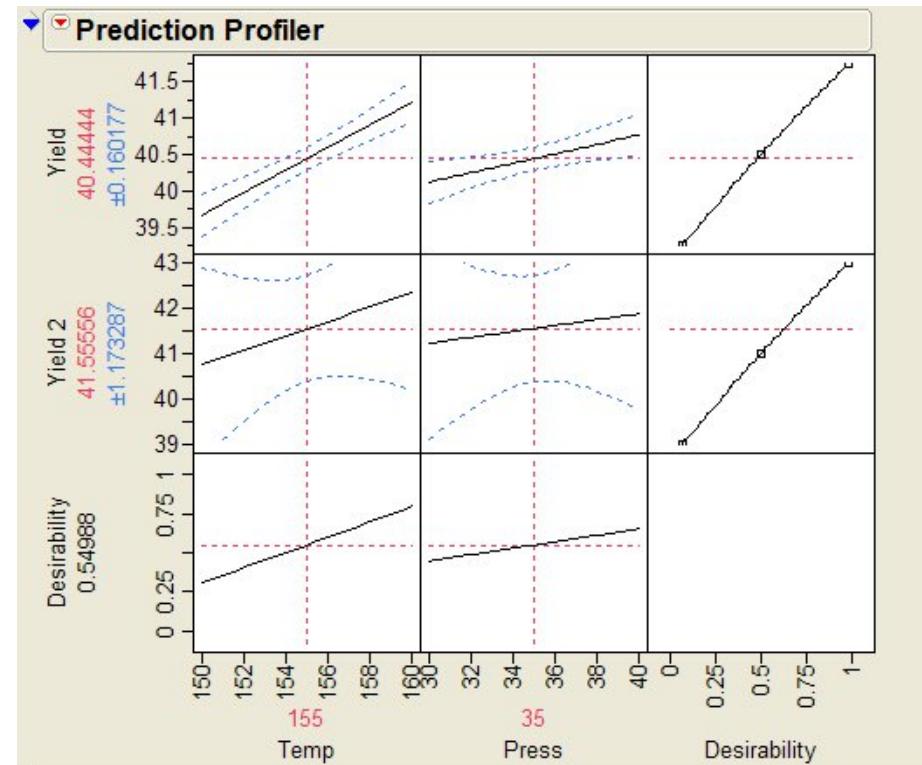


Yield 2



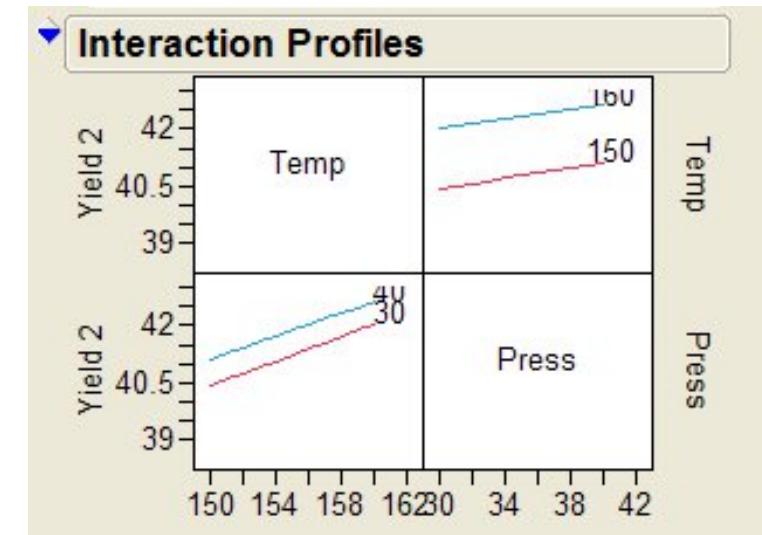
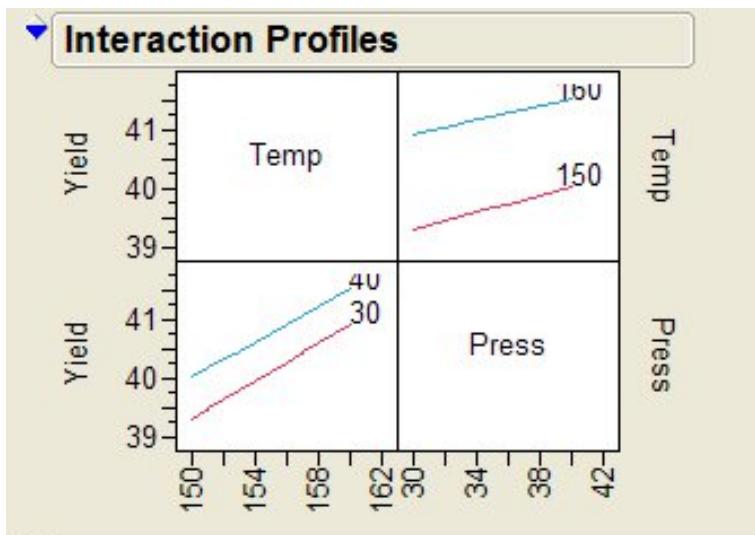
JMP Analysis (Cont.)

- From the Response Yield Red Triangle, select Factor Profiling>Profiler



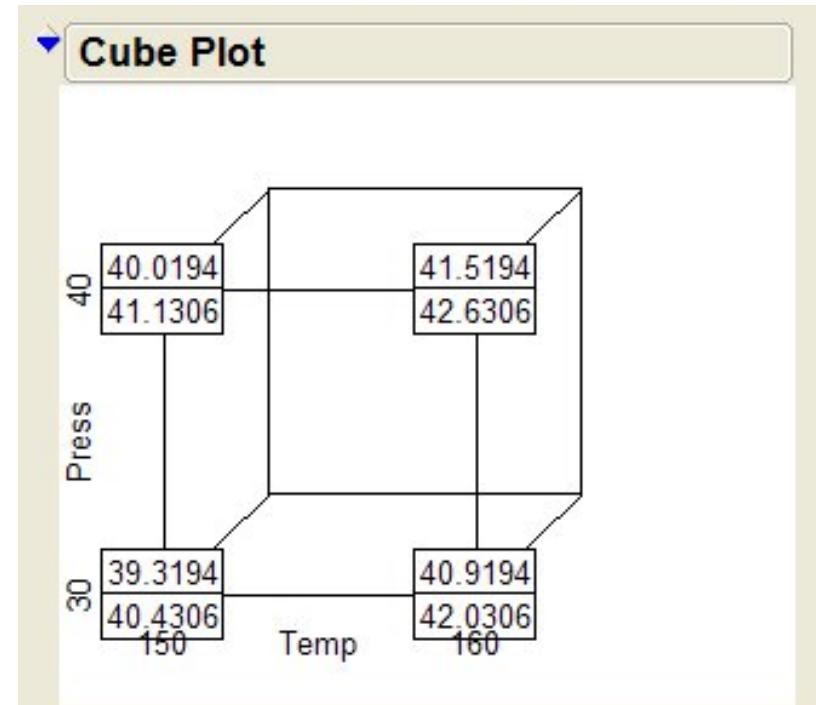
JMP Analysis (Cont.)

- From the **Response Yield Red Triangle**, select **Factor Profiling>Interaction Plots**



JMP Analysis (Cont.)

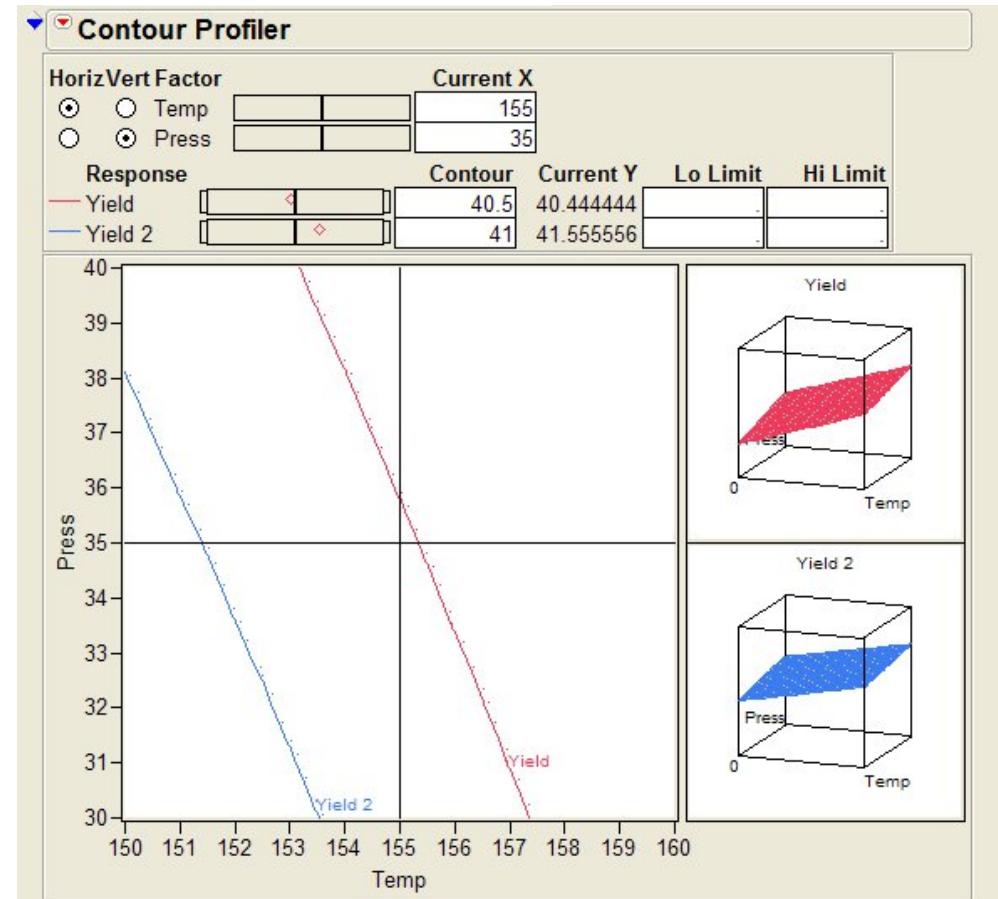
- From the Response Yield Red Triangle, select Factor Profiling>Cube Plots



Six Sigma – 2^K Factorial Experiments

JMP Analysis (Cont.)

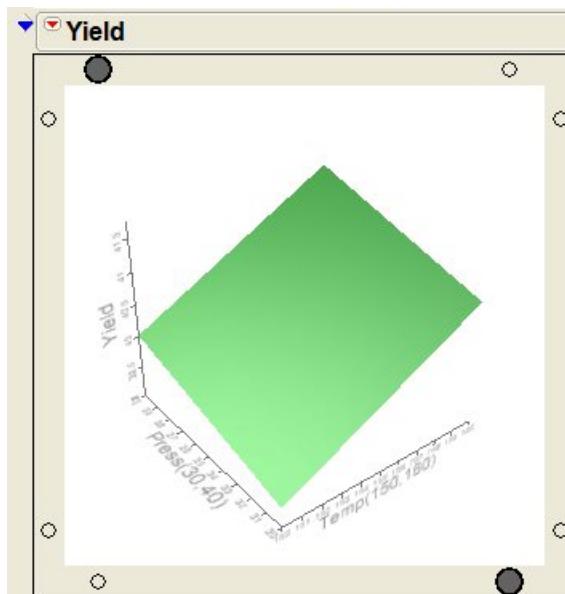
- From the Response Yield Red Triangle, select Factor Profiling>Contour Profiler



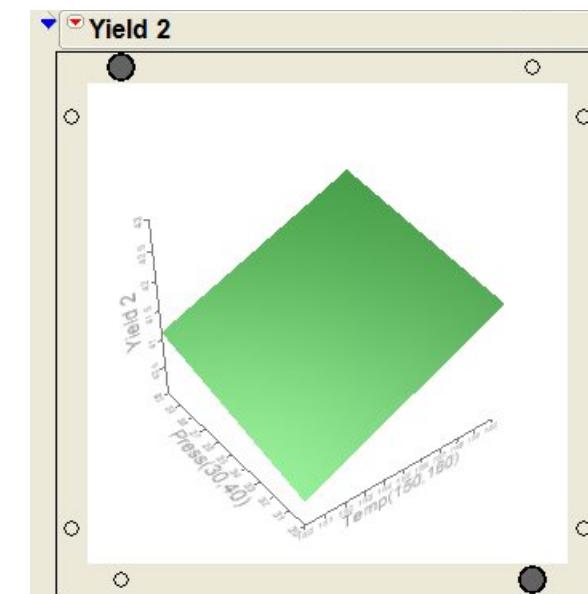
JMP Analysis (Cont.)

- From the Response Yield Red Triangle, select Factor Profiling>Surface Profiler

Yield



Yield 2



Mathematical Model

- u State the mathematical model obtained. A simpler model is always preferable (Why?)
 - For the yield of product 1 we can reduce the model because the interaction is not significant.
 - For the yield of product 2 (yield 2), we haven't yet obtained a working model (no main effects or interactions are significant). At this point, we simply know that curvature exists.

u Product 1:

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	40.444444	0.062311	649.07	<.0001
Temp	0.775	0.093467	8.29	0.0004
Pressure	0.325	0.093467	3.48	0.0177
Temp *Pressure	-0.025	0.093467	-0.27	0.7998

$$\text{Yield} = 40.44 + .775 * \text{Temp} + .325 * \text{Press}$$

- Note: The above equation is for coded variables only!

u Product 2:

- Step 4 shows that Curvature is significant. Stop there!
- Re-do the DOE. Change your level settings.

Conclusions

- u Translate the mathematical model into process terms. Formulate conclusions and recommendations.
 - For product 1: To increase yield, set the current process to run at 160 degrees and 40 psi.
 - We also recommend a follow-up investigation to see if yield can be further improved at factor levels not tested in this experiment.
 - For product 2: Conduct additional experiments to model curvature.
- u Replicate optimum conditions. Plan the next experiment or institutionalize the change.

Adding A Block To 2^k Factorials

- **Blocking Variable:** A factor in an experiment that has undesired influence as a source of variability is called a “block.” A block can be a batch of material or a set of conditions likely to produce experimental runs that are more homogenous within the block (or batch) than between blocks.

Run	A	B	C	Block
1	-1	-1	-1	I
2	+1	-1	-1	I
3	-1	+1	-1	I
4	+1	+1	-1	I
5	-1	-1	+1	II
6	+1	-1	+1	II
7	-1	+1	+1	II
8	+1	+1	+1	II

Adding A Block To 2^K Factorials

- Suppose we wanted to run a $2 \times 2 \times 2$ factorial. We would like to run the experiment under as homogeneous conditions as possible. But, we find that two batches of raw material are needed to run the entire experiment.

Run	A	B	C	Block
1	-1	-1	-1	I
2	+1	-1	-1	I
3	-1	+1	-1	I
4	+1	+1	-1	I
5	-1	-1	+1	II
6	+1	-1	+1	II
7	-1	+1	+1	II
8	+1	+1	+1	II

Adding A Block To 2^k Factorials

- It's easy to see that if we ran the first four runs with Batch 1 of raw material and the second four runs with Batch 2, we would completely "**confound**" Factor C. We could not distinguish between the effect due to Factor C and the effect of variation in the raw material.

Run	A	B	C	Block
1	-1	-1	-1	I
2	+1	-1	-1	I
3	-1	+1	-1	I
4	+1	+1	-1	I
5	-1	-1	+1	II
6	+1	-1	+1	II
7	-1	+1	+1	II
8	+1	+1	+1	II

Adding A Block To 2^K Factorials

- u We must figure out a way to “spread” the raw material effect across the experiment so the differences in material batches are “seen” by all the main effects. Recall the expanded design matrix for a 2^3 factorial experiment showing all contrasts.

Adding A Block To 2^k Factorials

- In general, we make the assumption that “higher order interactions” are not significant ($p\text{-value} > 0.05$). Here we can use the contrast for the 3-way interactions to define our blocking variable. The new design would look like:

Run	A	B	C	$A*B$	$A*C$	$B*C$	$A*B*C$	Block
1	-1	-1	-1	1	1	1	-1	I
2	1	-1	-1	-1	-1	1	1	II
3	-1	1	-1	-1	1	-1	1	II
4	1	1	-1	1	-1	-1	-1	I
5	-1	-1	1	1	-1	-1	1	II
6	1	-1	1	-1	1	-1	-1	I
7	-1	1	1	-1	-1	1	-1	I
8	1	1	1	1	1	1	1	II

Adding A Block To 2^k

Factorials

- We would run runs 1, 4, 6 and 7 with Batch 1 and runs 2, 3, 5 and 8 with Batch 2. This experiment would not allow us to test for the significance of the 3-way interaction, but it would allow us to investigate the main effects and 2-way interactions without worrying about confounding these effects with the Raw Material batch.

Note: In an actual experiment, you would randomize the runs within each Block. JMP will do that for you.

Run	A	B	C	Block
1	-1	-1	-1	I
2	1	-1	-1	II
3	-1	1	-1	II
4	1	1	-1	I
5	-1	-1	1	II
6	1	-1	1	I
7	-1	1	1	I
8	1	1	1	II

Run	A	B	C	Block
1	-1	-1	-1	I
4	1	1	-1	I
6	1	-1	1	I
7	-1	1	1	I
2	1	-1	-1	II
3	-1	1	-1	II
5	-1	-1	1	II
8	1	1	1	II



Lean Six Sigma 2^K Factorial Experiments

2^K Factorial Experiments

- Objective:** To develop a mathematical relationship between the significant factors and the response variable.
- Deliverables:** $Y=f(x)$, Mathematical Model, DOE Report, Updated FMEA

2^K Factorial Experiments

Objective	Project Deliverables
DOE Planning	<ul style="list-style-type: none">u Experiment PlanningExperiments
Screening Experiments (What are the significant factors?)	<ul style="list-style-type: none">u Fractional Factorial Experimentsu Reduced List of Input Variables
$Y=f(x)$ (What is the mathematical relationship between the significant factors and the response variable?)	<ul style="list-style-type: none">u Mathematical Modelu 2^K Factorial Experimentsu Full Factorial Experimentsu Established Input Levels
Process Documentation	<ul style="list-style-type: none">u Updated Process FMEAu DOE Report(s)

2^K Factorial Experiments

Agenda

- u Why do we need 2^K factorial experimentation?
- u 2^K Standard Order Designs
 - Exercise:
 - Calculating main effects
 - Calculating interactions
 - 2^K Example (using JMP)
 - 2^K Exercises (using JMP)

2^K Factorial Experiments

Agenda

- u Adding Center Points
 - 2^K Example with center points (using JMP)
- u Adding Blocking
 - 2^K Example with Blocking
- u 2^K Vocabulary
- u Steps for DOE Analysis

Why Use 2^K Factorial Experiments?

- u The **GOAL** is to obtain a mathematical relationship which characterizes:
 - $Y = f(x_1, x_2, x_3, \dots)$
- u The mathematical relationship allows us to identify not only the *critical factors* but also the *best levels* for those factors.
- u 2^K Factorial Experiments investigate multiple factors where each factor is studied at only two levels.

Why Use 2^K Factorial Experiments?

- u 2^K Factorial Experiments allow us to investigate a large number of factors simultaneously in relatively few runs compared to full factorial designs.
- u Finally, 2^K designs are used most frequently in industrial DOE applications because they are very easy to analyze and lend themselves well to sequential studies.

Types of 2^K Factorials

- u **One observation per treatment combination**
 - Usually low statistical power
 - Use Normal Quantile Plots or Pareto charts instead of F-tests
 - Can create full factorials by leaving unimportant factors out

Types of 2^K Factorials

- u **More than one observation per treatment combination (known as repeats or replicates)**
 - Better estimates of error
 - Better statistical power
 - Can still run reduced models
 - F-tests, normal plots and pareto charts can be used

Standard Order Of 2^k Designs

- u **2^k factorials refer to k factors, each with 2 levels.** A 2^2 factorial is a 2×2 factorial. This design has two factors with two levels and can be executed in only 2×2 or 4 runs.
Likewise a 2^3 factorial has 3 factors, each with two levels. This experiment can be done $2 \times 2 \times 2$ or 8 runs.

Standard Order Of 2^K Designs

(Cont.)

- The design matrix for 2^K factorials is usually shown in standard order. The low level of a factor is designated with “-” or -1 and the high level is designated with “+” or 1. An example of a design matrix follows.

2^2 Factorial

Temp	Conc
-1	-1
1	-1
-1	1
1	1

2^3 Factorial

Temp	Conc	Catalyst
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1



Standard Order Of 2^k Designs

Exercise

- u Create a 2^4 Factorial Design Matrix**
- u What are the minimum number of runs needed?**
- u Verify your results using JMP:
JMP>DOE>Classical>Full Factorial Design**
- u Add two 2-level continuous factors and two 2-level categorical factors**

Standard Order Of 2^k Designs

Exercise

Open JMP – new data table

Click on DOE

Select Classical > Full Factorial Design

Y is response, Maximize

**Select 2 continuous, 2 categorical variables,
2 levels each, click continue**

Six Sigma – 2^K Factorial ExperimentsStandard Order Of 2^K Designs

Exercise

DOE- Full Factorial Design

Full Factorial Design

Responses

Add Response Remove Number of Responses...

Response Name	Goal	Lower Limit	Upper Limit	Importance
Y optional item	Maximize	.	.	.

Factors

Continuous Categorical Remove

Name	Role	Values
X1	Continuous	-1 1
X2	Continuous	-1 1
X3	Categorical	L1 L2
X4	Categorical	L1 L2

Full Factorial Design
2x2x2x2 Factorial

Output Options

Run Order:

Number of Runs:

Number of Center Points:

Number of Replicates:

Keep the Same
Sort Left to Right
Randomize
Sort Right to Left

Make Table
Back



Six Sigma – 2^k Factorial Experiments

Standard Order Of 2^k Designs

Exercise

2x2x2x2 Factorial

	Pattern	X1	X2	X3	X4	Y
1	--11	-1	-1	L1	L1	.
2	+−11	1	-1	L1	L1	.
3	−+11	-1	1	L1	L1	.
4	++11	1	1	L1	L1	.
5	--21	-1	-1	L2	L1	.
6	+−21	1	-1	L2	L1	.
7	−+21	-1	1	L2	L1	.
8	++21	1	1	L2	L1	.
9	--12	-1	-1	L1	L2	.
10	+−12	1	-1	L1	L2	.
11	−+12	-1	1	L1	L2	.
12	++12	1	1	L1	L2	.
13	--22	-1	-1	L2	L2	.
14	+−22	1	-1	L2	L2	.
15	−+22	-1	1	L2	L2	.
16	++22	1	1	L2	L2	.

Columns (6/0)

- Pattern
- X1 *
- X2 *
- X3 *
- X4 *
- Y *

Rows

All rows	16
Selected	0
Excluded	0
Hidden	0
Labelled	0

Exercise: Calculating Main Effects

Step 1: Problem Statement: A process engineer would like to determine the effect of Quench Temperature, Quench Time, and Quench Oil types on the hardness of a steel shaft. Goal: Maximize Hardness.

Step 2: The factors and levels:

- Quench Temp: 160° C (-1), 180° C (1)
- Quench Time (sec): 5 (-1), 15 (1)
- Quench Oil Type: Brand A (-1), Brand B (1)

Exercise: Calculating Main Effects

Step 3: The design matrix with the results column is as follows:

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

This is an example of 2^K Factorial Experiment with only one observation per treatment combination (experimental run).

Exercise: Calculating Main Effects

- u Step 4: We will now calculate the effects of the experiment by hand. First, look at Temperature.
- u We simply add the yields associated with (-1) and the yields associated with (1) and calculate the average (Sum/4). The “1’s” and “-1’s” in the temperature column are called the “contrast” for the main effect of temperature.



Exercise: Calculating Main Effects

$$\begin{aligned} \text{TempMainEffect} &= \frac{\overline{x}_{-1} - \overline{x}_1}{\overline{e}} = \frac{72 + 68 + 83 + 80}{4} - \frac{60 + 54 + 52 + 45}{4} \\ &= 75.75 - 52.75 = 23 \end{aligned}$$

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

Total -	-211
Total +	303
Sum	92
Avg Effect	23

The Hardness increases, on average, by 23 points as temperature moves from Low to High (160 to 180).

Exercise: Calculating Main Effects

- Now use the contrast for Time to calculate the effect Time has on Hardness.

$$\begin{aligned} \text{TimeMainEffect} &= \frac{\overline{x}_5 + 68 + 45 + 80}{4} - \frac{\overline{x}_{\emptyset} + 60 + 72 + 52 + 83}{4} \\ &= 61.75 - 66.75 = -5 \end{aligned}$$

As the Time moves from 5 sec to 15 sec, the Hardness drops by 5 points on average.

Exercise: Calculating Main Effects

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

Total -	-211	-267	
Total +	303	247	
Sum	92	-20	
Avg Effect	23	-5	

Exercise: Calculating Main Effects

- Next use the contrast for Oil to calculate the effect Oil type has on Hardness.

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

-211	-267
303	247
92	-20
23	-5



Exercise: Calculating Main Effects

$$\text{OilMainEffect} = \frac{\overline{y}_{++} + \overline{y}_{+-}}{\overline{e}} - \frac{\overline{y}_{-+} + \overline{y}_{--}}{\overline{e}}$$
$$= \frac{\overline{y}_{++} + \overline{y}_{+-}}{\overline{e}} - \frac{\overline{y}_{-+} + \overline{y}_{--}}{\overline{e}} =$$

Interpretation: _____

Exercise: Calculating Interactions

- u We have just finished calculating the Main Effects for this experiment. We've only investigated the independent effects of Temperature, Time and Oil Type. This is similar to conducting three 2-Sample T tests.
- u The benefit of factorial experiments is that they provide the ability to assess “interactions” between factors. “Is there a particular combination of input settings that improve Hardness over and above the singular (main) effects?”



Exercise: Calculating Interactions

- The interaction contrast is derived by multiplying the columns to be represented.

Calculate the Interaction Contrasts below.

Temp(T)	Time (S)	Oil (O)	T*S	T*O	S*O	T*S*O	Hardness
-1	-1	-1					60
1	-1	-1					72
-1	1	-1					54
1	1	-1					68
-1	-1	1					52
1	-1	1					83
-1	1	1					45
1	1	1					80

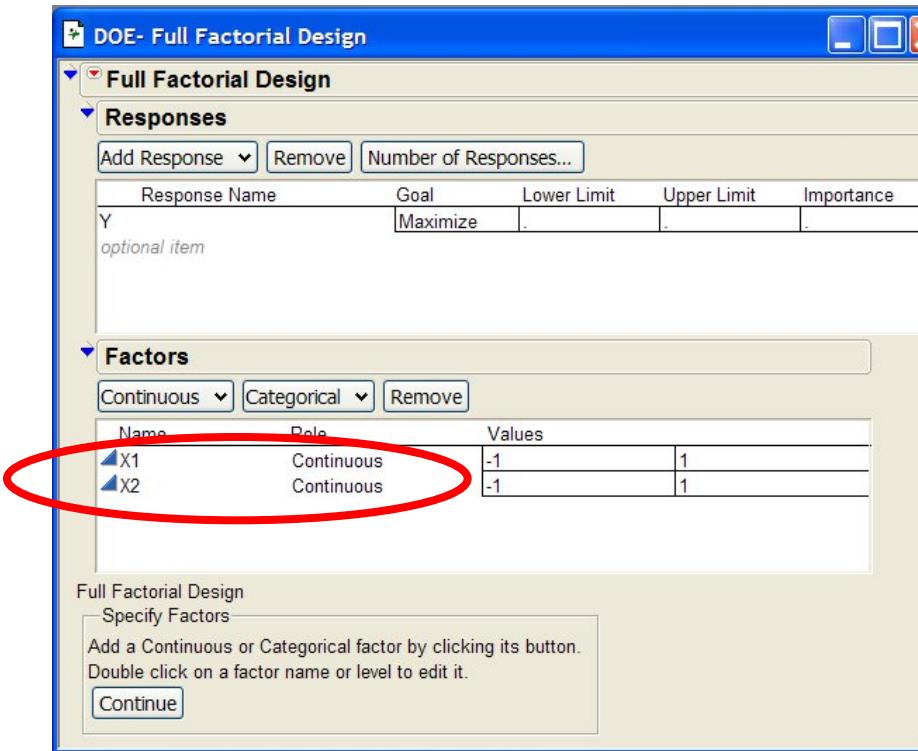
Exercise: Calculating Interactions

-1	-1	-1	1	1	1	-1	60
1	-1	-1	-1	-1	1	1	72
-1	1	-1	-1	1	-1	1	54
1	1	-1	1	-1	-1	-1	68
-1	-1	1	1	-1	-1	1	52
1	-1	1	-1	1	-1	-1	83
-1	1	1	-1	-1	1	-1	45
1	1	1	1	1	1	1	80
Total -	-211	-267					
Total +	303	247					
Sum	92	-20					
Effect	23	-5	1.5	1.5	10	0	0.5
Estimate	11.5	-2.5	0.75	0.75	5	0	0.25

- Verify the “Contrast” for each interaction using JMP.

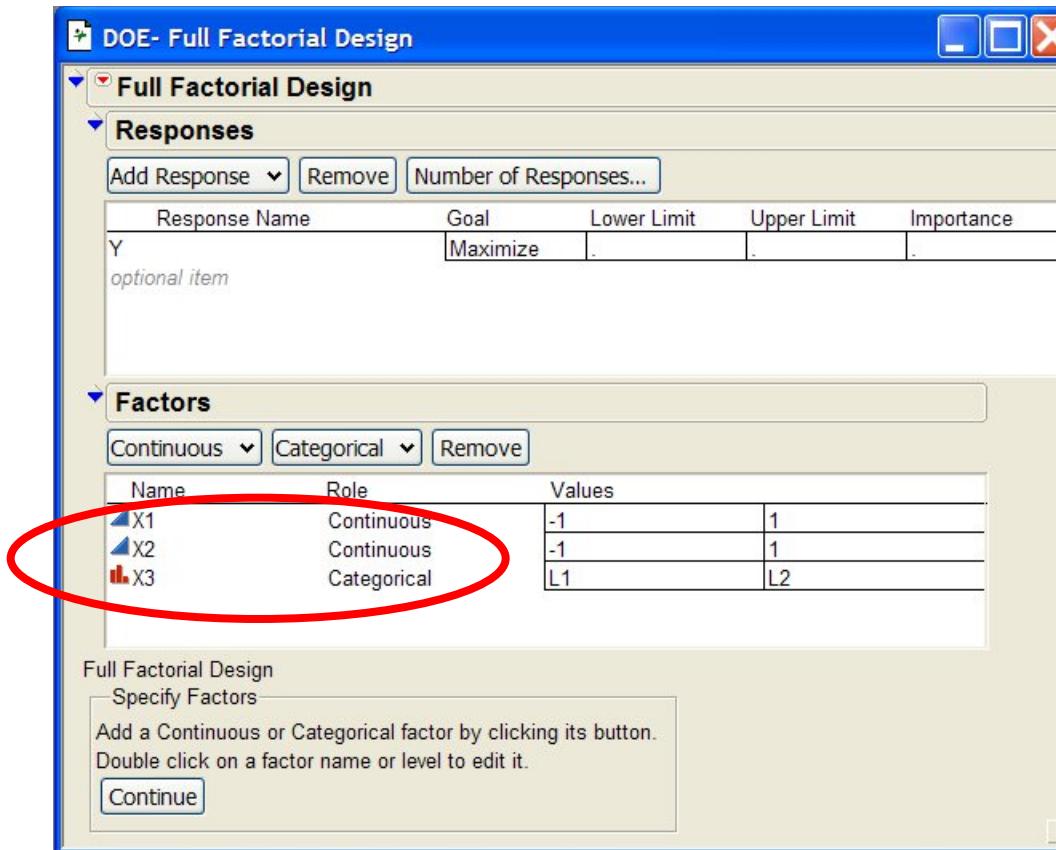
Design Matrix – Using JMP

- u JMP> DOE > Classical > Full Factorial Design
 - **Continuous>2 Level**
 - Repeat the previous step (**Continuous>2 Level**)



Design Matrix – Using JMP

- u JMP > DOE > Classical > Full Factorial Design
 - Categorical>2 Level



The screenshot shows the 'DOE- Full Factorial Design' dialog box. In the 'Factors' section, there are three entries:

Name	Role	Values
X1	Continuous	-1 1
X2	Continuous	-1 1
X3	Categorical	L1 L2

A red oval highlights the X1, X2, and X3 entries in the 'Factors' table.

JMP DOE Design (Cont.)

- u Double-click in the Name Column to change X1, X2 and X3 to Temp, Time and Oil.
 - Quench Temp: 160° C (-1), 180° C (1)
 - Quench Time (sec): 5 (-1), 15 (1)
 - Quench Oil: Brand A (L1), Brand B (L2)
- u Double-click in the Values Column to change the Temp row to 160 and 180, the Time row to 5 and 15, and the Oil row to Brand A and Brand B.

Six Sigma – 2^K Factorial Experiments

JMP DOE Design (Cont.)

DOE- Full Factorial Design

Full Factorial Design

Responses

Add Response ▾ Remove Number of Responses...

Response Name	Goal	Lower Limit	Upper Limit	Importance
Y optional item	Maximize	.	.	.

Factors

Continuous ▾ Categorical ▾ Remove

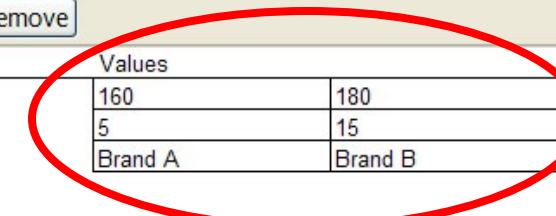
Name	Role	Values	
Temp	Continuous	160	180
Time	Continuous	5	15
Oil	Categorical	Brand A	Brand B

Full Factorial Design

Specify Factors

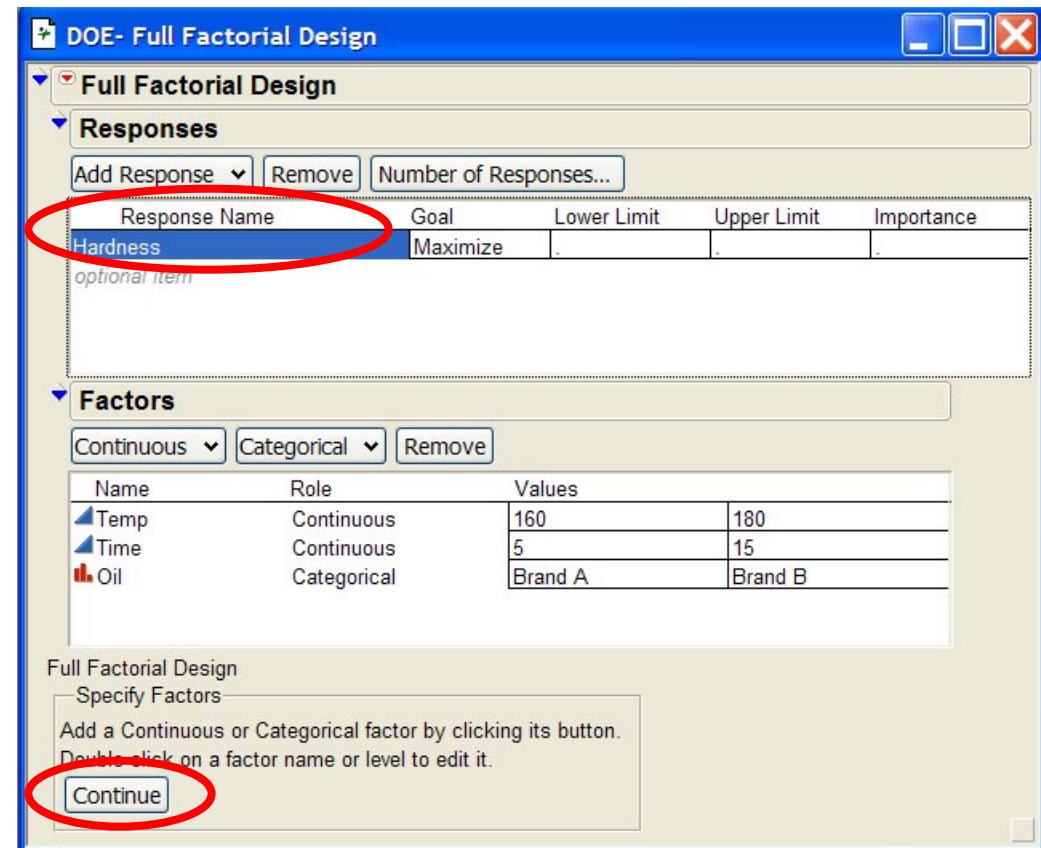
Add a Continuous or Categorical factor by clicking its button.
Double click on a factor name or level to edit it.

Continue



Design Matrix – Using JMP

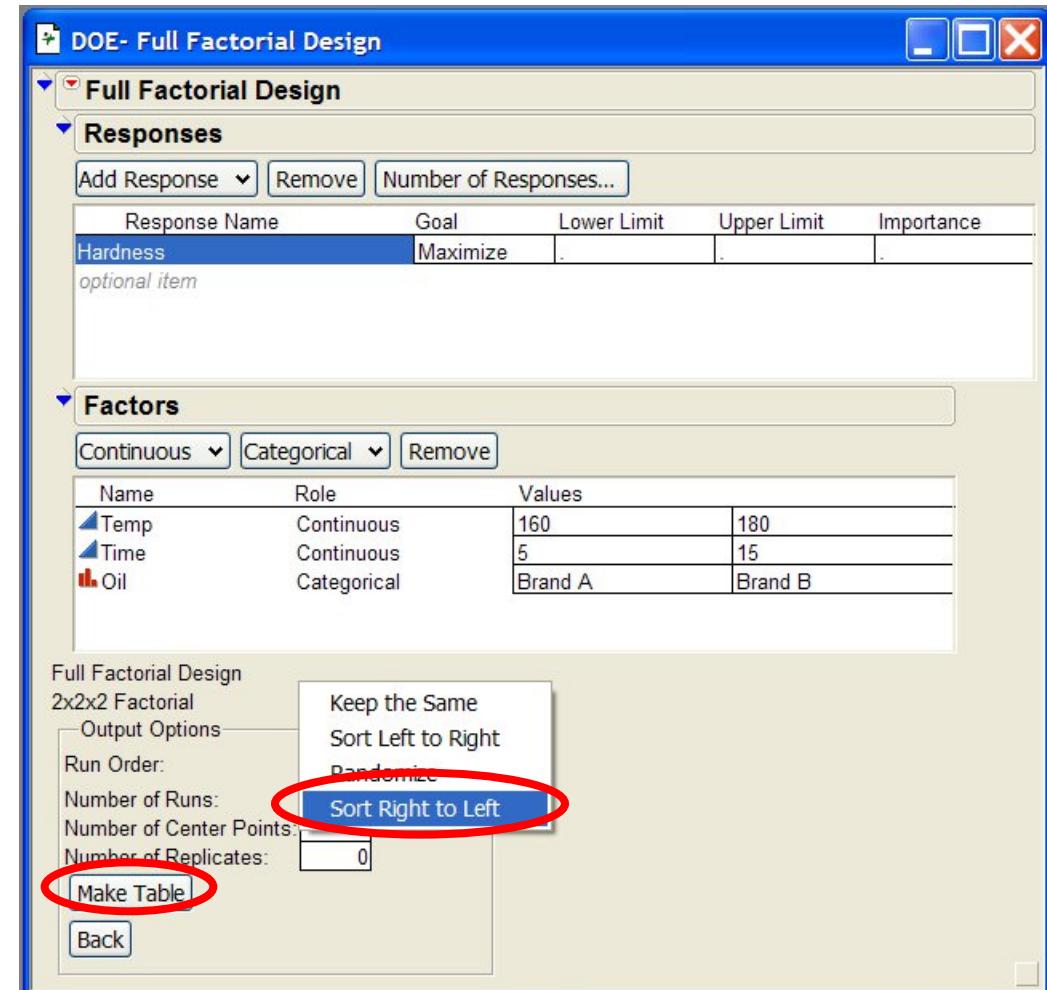
- u Enter the Response column heading.
- u Enter *Hardness* as the Response Name.
- u Select **Continue**.



Six Sigma – 2^K Factorial Experiments

JMP DOE Design (Cont.)

- u Number of Replicates
= Number of Additional Replicates
- u For this example, there are no additional replicates.
- u Change **Randomize** to **Sort Right to Left** for teaching purposes.
- u Click **Make Table**.



JMP DOE Design (Cont.)

- Use the table to enter in the Y-values.

2x2x2 Factorial

	Pattern	Temp	Time	Oil	Hardness
1	--1	160	5	Brand A	60
2	+ -1	180	5	Brand A	72
3	- +1	160	15	Brand A	54
4	++1	180	15	Brand A	68
5	--2	160	5	Brand B	52
6	+ -2	180	5	Brand B	83
7	- +2	160	15	Brand B	45
8	++2	180	15	Brand B	80

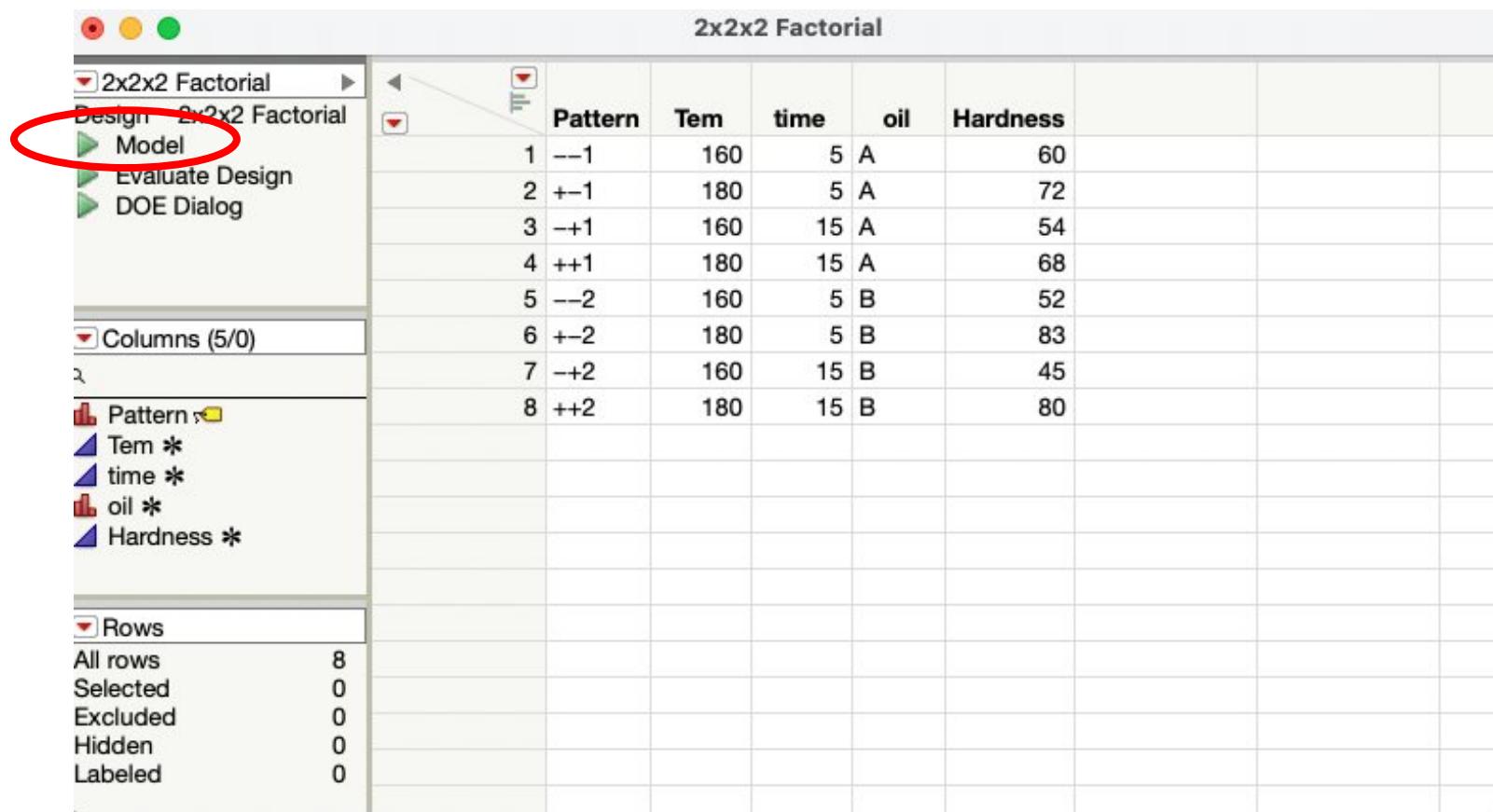
Columns (5/0)
Pattern
Temp *
Time *
Oil *
Hardness *

Analysis With JMP

- u This experiment only has one observation per treatment combination. Therefore, we can't analyze the "full factorial" using the ANOVA procedure until we learn a few analysis tricks.
- u When there is only one observation per treatment combination, we use the Normal Probability Plot or Pareto Chart to interpret which effects are likely to be significant.
- u If a factor or its interaction is insignificant (the null hypothesis, effect = 0, is true), then we expect to see the effects normally distributed around a mean of zero. Any outlying effect is considered significant.

Analysis With JMP (Cont.)

Step 4: From the **Model**, select the **Green Triangle**



The screenshot shows the JMP software interface with the title "2x2x2 Factorial". On the left, there is a navigation pane with the following sections:

- Design:** 2x2x2 Factorial (with a red circle around the "Model" option)
- Evaluate Design**
- DOE Dialog**
- Columns (5/0)**
- Pattern** (selected)
- Tem ***
- time ***
- oil ***
- Hardness ***
- Rows**
- All rows: 8
- Selected: 0
- Excluded: 0
- Hidden: 0
- Labeled: 0

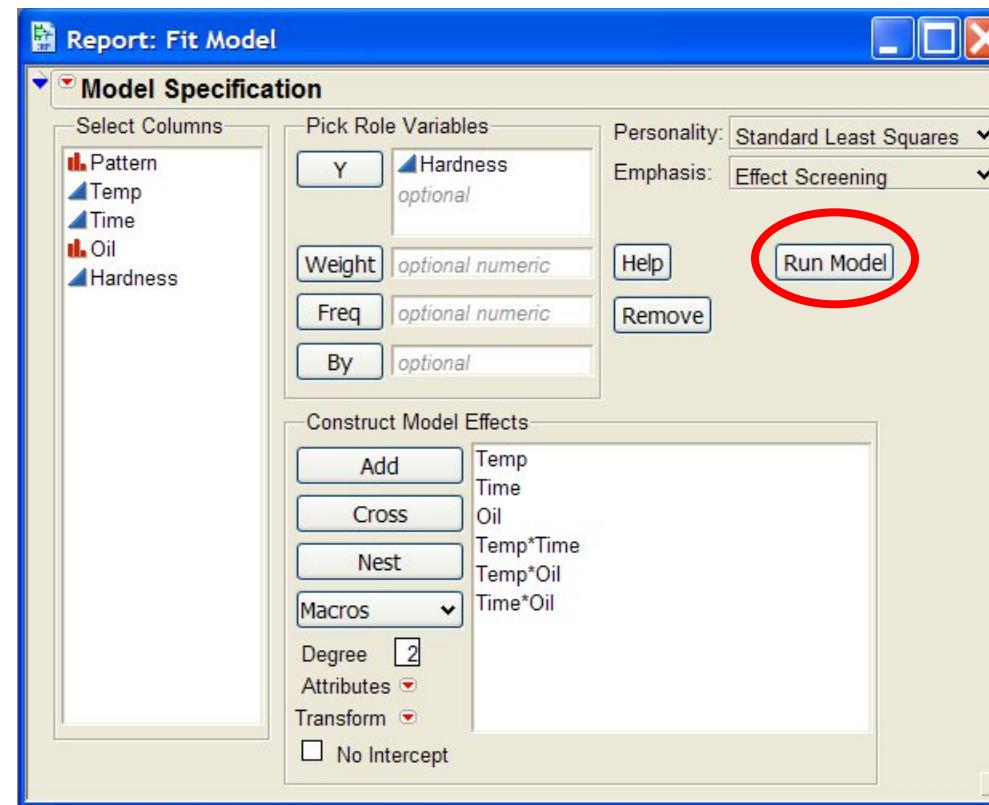
The main table area displays the following data:

	Pattern	Tem	time	oil	Hardness
1	--1	160	5 A		60
2	+--1	180	5 A		72
3	--+1	160	15 A		54
4	++1	180	15 A		68
5	--2	160	5 B		52
6	+--2	180	5 B		83
7	--+2	160	15 B		45
8	++2	180	15 B		80

Six Sigma – 2^K Factorial Experiments

JMP Analysis (Cont.)

u Click Run





Six Sigma – 2^k Factorial Experiments

JMP Analysis (Cont.)

JMP Output (Use Red Triangle under Response)>
Regression Report>summary of Fit
Regression Report> Analysis of Variance

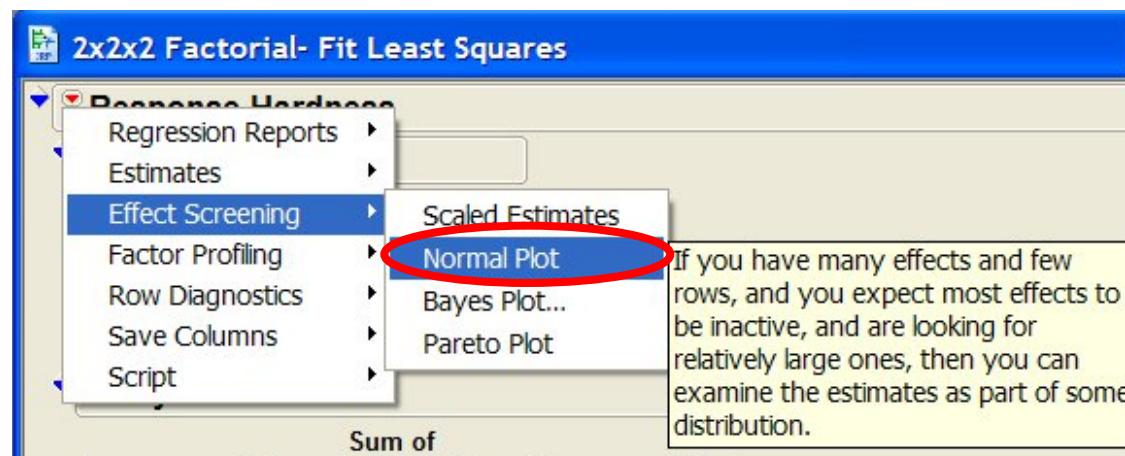
Summary of Fit				
RSquare		0.99962		
RSquare Adj		0.997343		
Root Mean Square Error		0.707107		
Mean of Response		64.25		
Observations (or Sum Wgts)		8		

Analysis of Variance				
Source	DF	Sum of Squares		
		Mean Square	F Ratio	Prob > F
Model	6	1317.0000	219.500	439.0000
Error	1	0.5000	0.500	
C. Total	7	1317.5000		0.0365*

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	64.25	0.25	257.00	0.0025*
Temp(160,180)	11.5	0.25	46.00	0.0138*
Time(5,15)	-2.5	0.25	-10.00	0.0635
Oil[Brand A]	-0.75	0.25	-3.00	0.2048
Temp*Time	0.75	0.25	3.00	0.2048
Temp*Oil[Brand A]	-5	0.25	-20.00	0.0318*
Time*Oil[Brand A]	0	0.25	0.00	1.0000

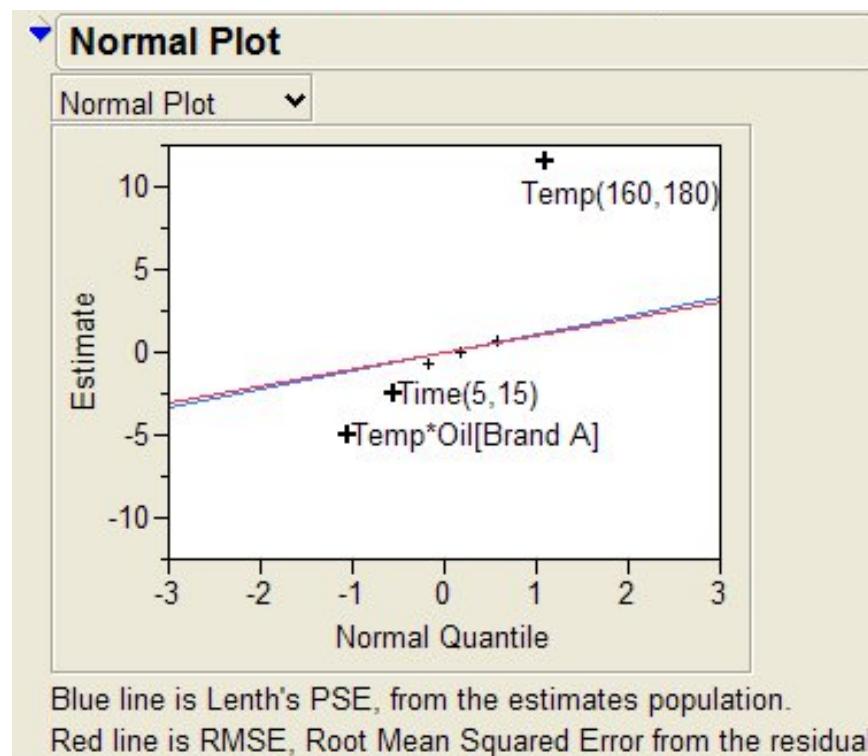
JMP Analysis (Cont.)

- u Step 5: Identify the most significant Model Effects or **Estimates** by looking at the Normal Plot and Pareto Plot of the Estimates.



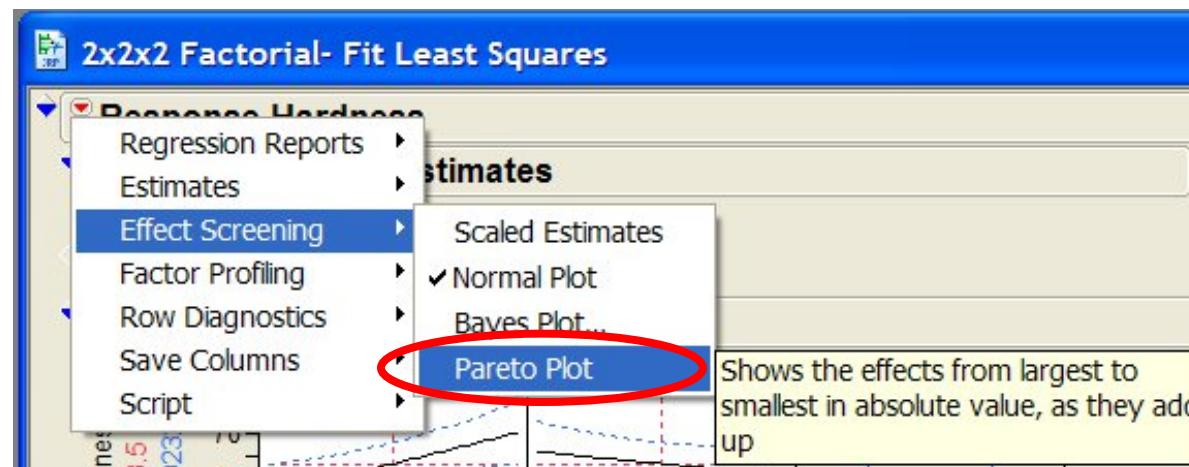
Normal Plot

- The Normal Plot shows that **Temperature**, **Time**, and the interaction of **Temperature and Oil** are the most significant model estimates.



Pareto Plot

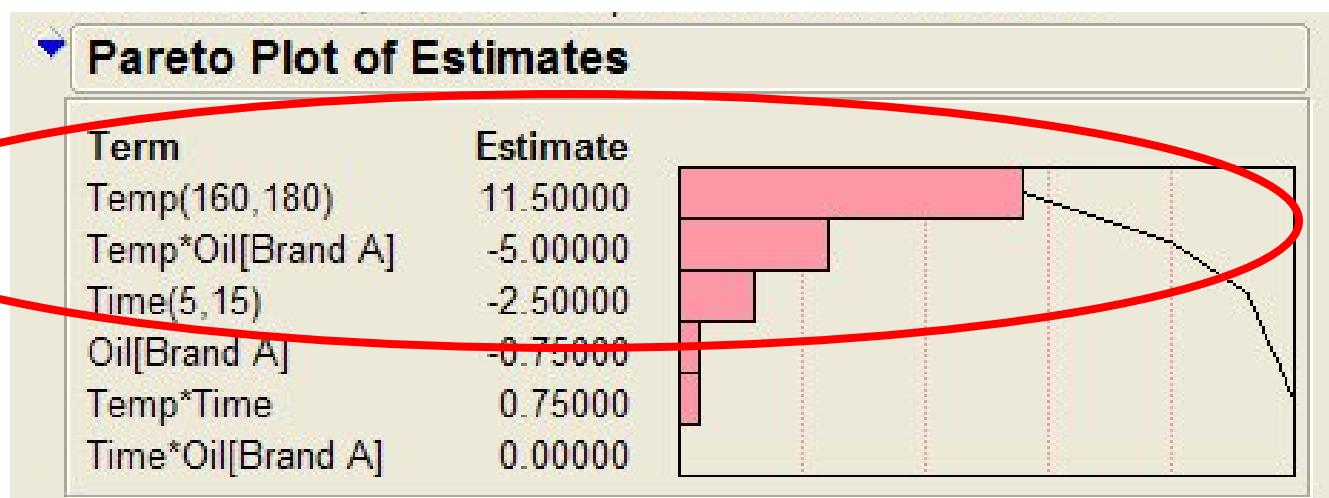
- Under the Y Response Red Triangle, select Effect Screening>Pareto Plot



Six Sigma – 2^k Factorial Experiments

Pareto Plot

- u The Pareto Plot shows that **Temperature, Time, and the interaction of Temperature and Oil** are the most significant model estimates.



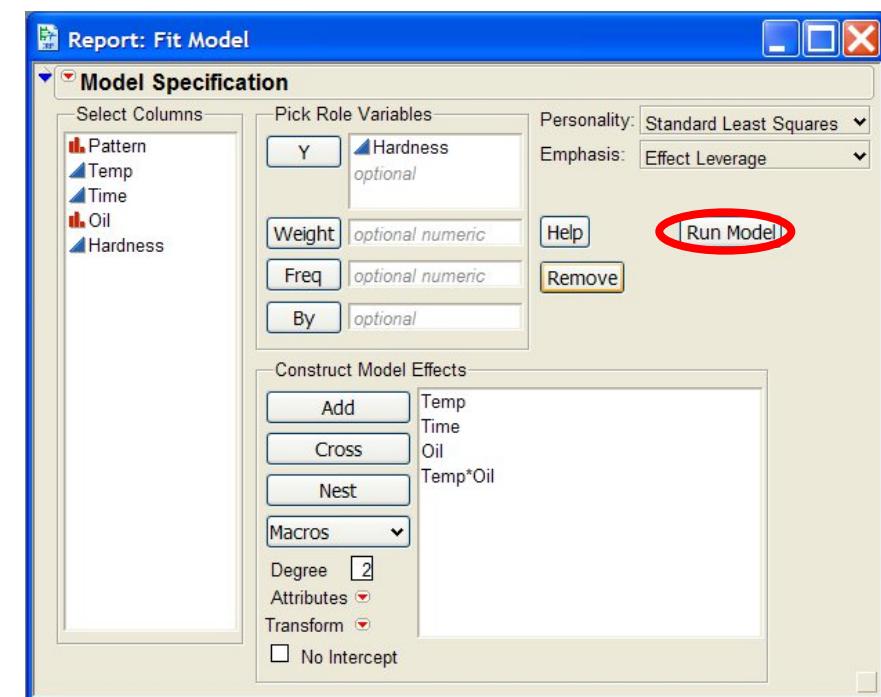
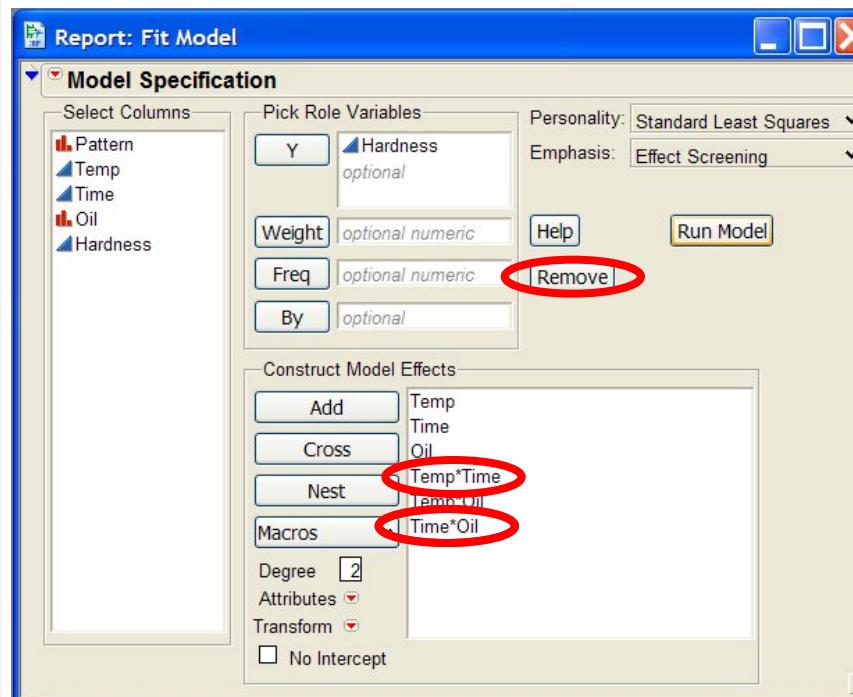
JMP Analysis (Cont.)

- u Step 6: Reduce the Model to include the largest estimates.
- u Note: If an interaction estimate is selected, then the interaction main factors must be included in the model.

Six Sigma – 2^K Factorial Experiments

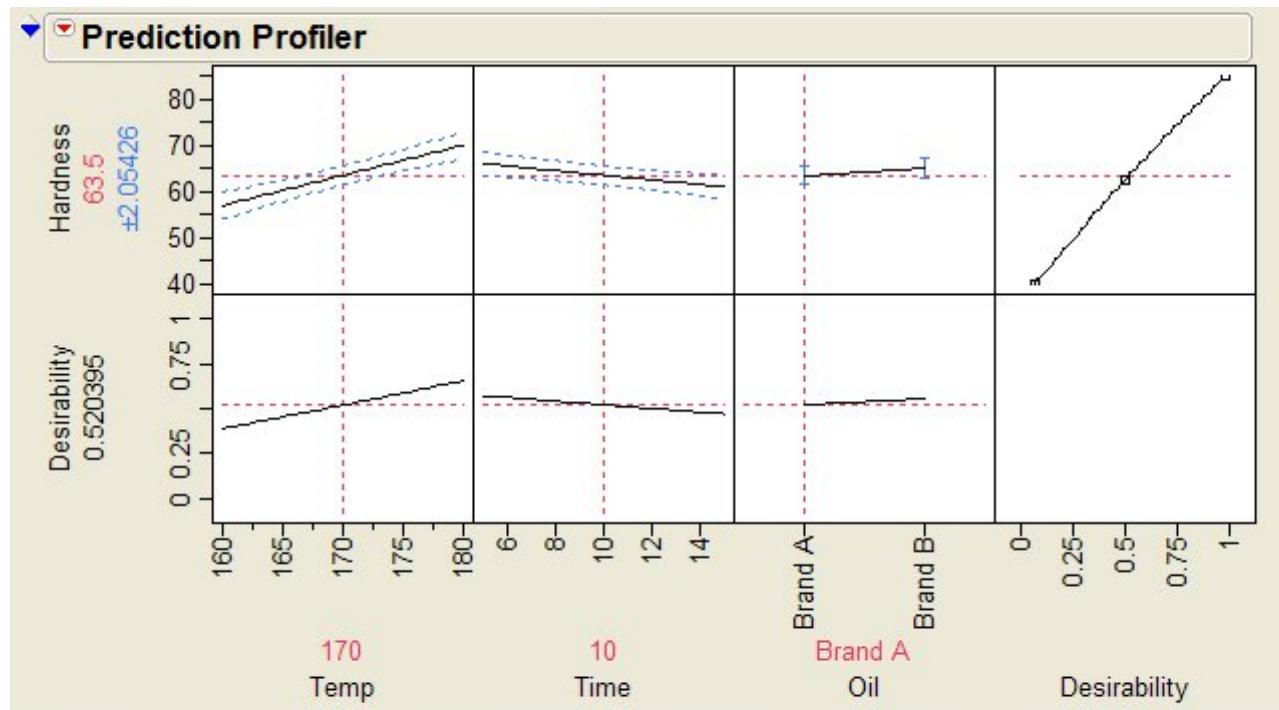
JMP Analysis (Cont.)

- u Click-on the Fit Model Window from Analyze
- u Click-on the non-significant estimates and then click-on **Remove**, and **Run Model**.



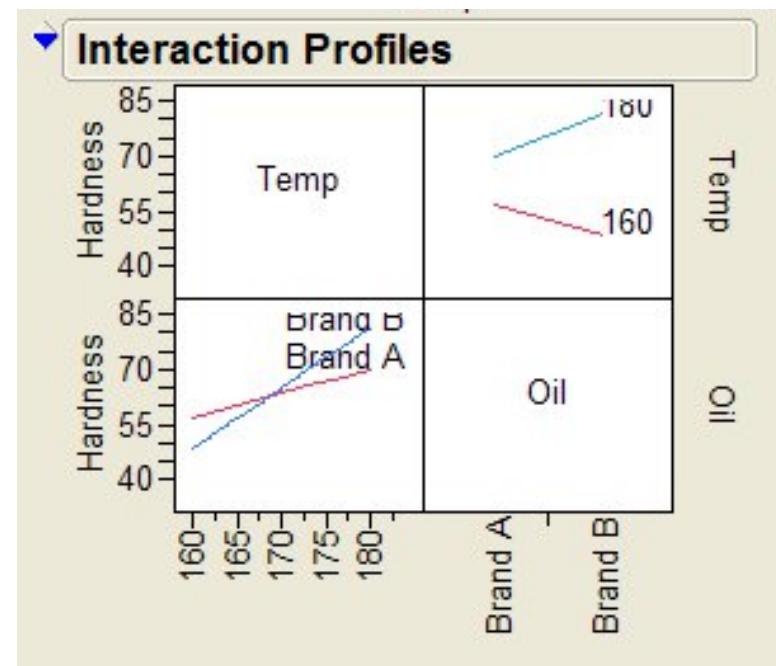
JMP Analysis (Cont.)

- From the **Response Y Red Triangle**, select **Factor Profiling>Profiler**



JMP Analysis (Cont.)

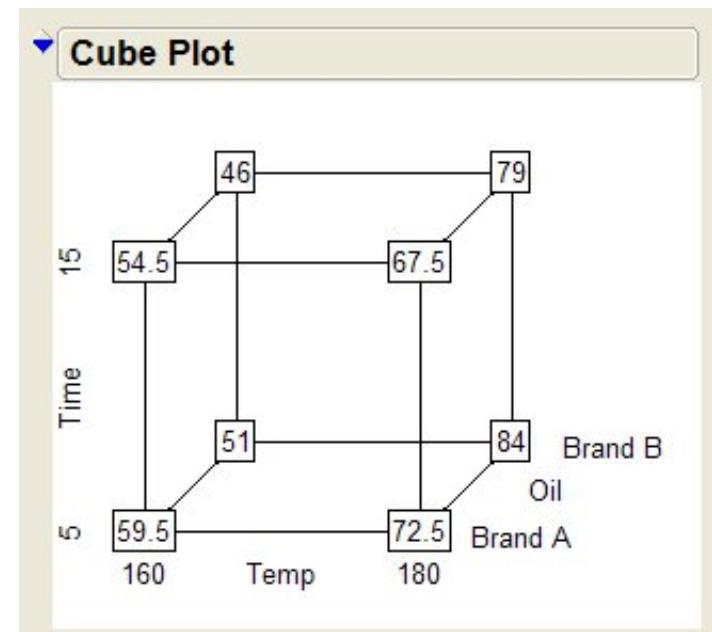
- From the **Response Y Red Triangle**, select **Factor Profiling>Interaction Plots**



Six Sigma – 2^k Factorial Experiments

JMP Analysis (Cont.)

- From the **Response Y Red Triangle**, select **Factor Profiling>Cube Plots**



JMP Analysis (Cont.)

u Step 7: Construct a mathematical model.

▼ Parameter Estimates					
Term	Estimate	Std Error	t Ratio	Prob> t	
Intercept	64.25	0.456435	140.76	<.0001*	
Temp(160,180)	11.5	0.456435	25.20	0.0001*	
Time(5,15)	-2.5	0.456435	-5.48	0.0120*	
Oil[Brand A]	-0.75	0.456435	-1.64	0.1989	
Temp*Oil[Brand A]	-5	0.456435	-10.95	0.0016*	

Model coefficients (coded) for Oil Brand A
Coded: Temp= -1,1 Time = -1,1