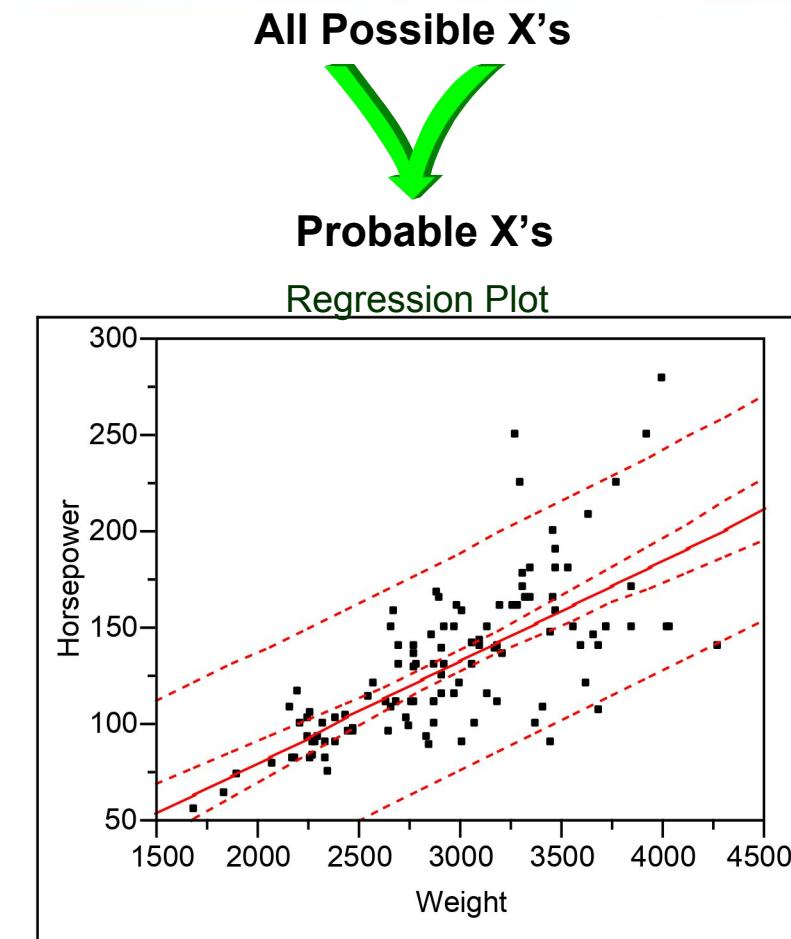


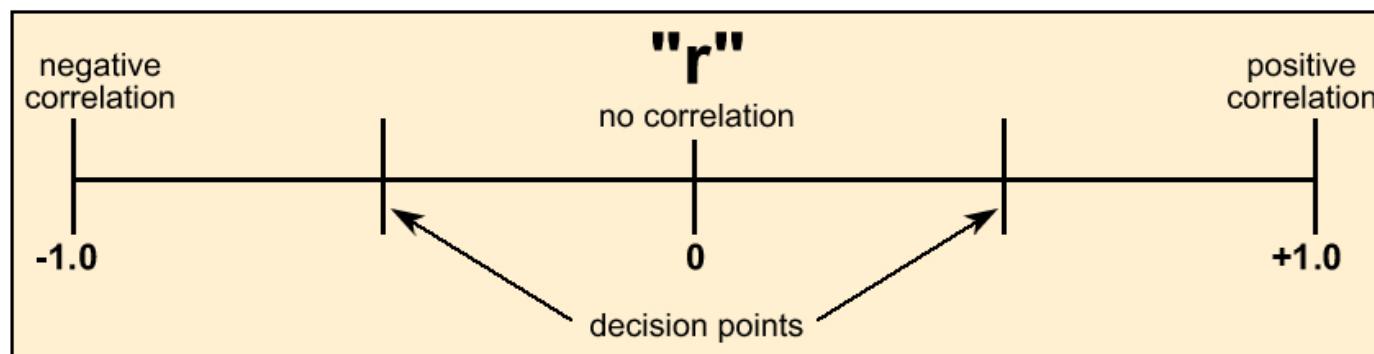
# Why Do We Use These Tools?

- Gather meaningful data about a process without interruption.
- This type of study leaves the process in a natural operating state (without DOE) versus artificially inducing variation (with DOE).
- Correlation provides a graphical analysis and quantifier of the strength between independent and dependent variables.
- Regression can be used as a means of obtaining prediction equations so independent variables can be controlled.



# Correlation

- Correlation is a measure of strength of association between two quantitative variables (e.g., pressure and yield) which aids in establishing  $Y = F(X)$ .
  - Correlation measures the strength of the relationship between two variables using the correlation coefficient,  $r$ .
  - The correlation coefficient,  $r$ , will always be between -1 and +1.
  - Guideline (usually based on sample size):
    - If  $|r| > 0.80$ , then relationship is significant
    - If  $|r| < 0.20$ , then relationship is not significant



\*NOTE: JMP uses the Pearson's formula

# Correlation

- Table for decision points in determining the correlation (positive or negative) for different sample sizes of n:

n	Decision point	n	Decision point
5	0.878	18	0.468
6	0.811	19	0.456
7	0.754	20	0.444
8	0.707	22	0.423
9	0.666	24	0.404
10	0.632	26	0.388
11	0.602	28	0.374
12	0.576	30	0.361
13	0.553	40	0.312
14	0.532	50	0.279
15	0.514	60	0.254
16	0.497	80	0.22
17	0.482	100	0.196

# Data Requirements

- To conduct a correlation study, one must have:
- **Bivariate Data:** data from two variables from the same object/person
- Bivariate data is made up of ordered pairs

(Factor)	(Response)
X (input)	Y (output)
x1	y1
x2	y2
x3	y3
x4	y4
x5	y5
x_nth	y_nth

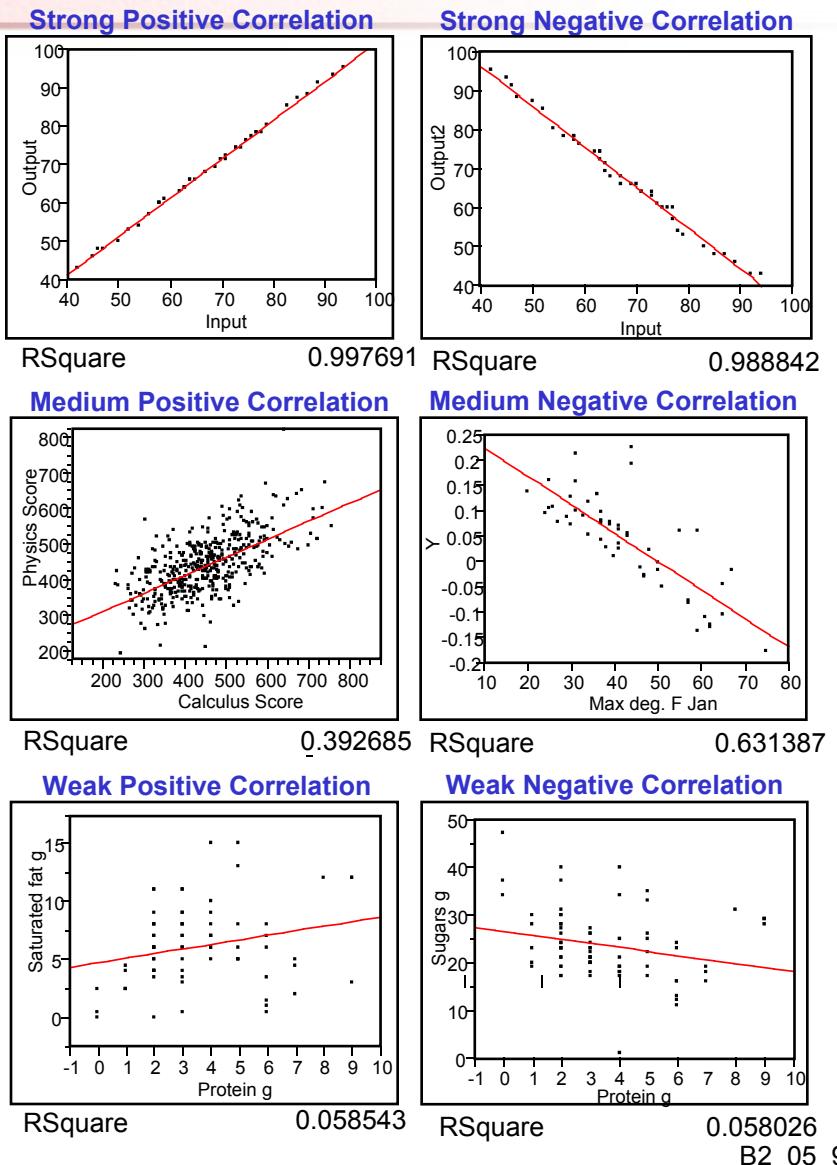
# Correlation

**The correlation coefficient ( $r$ ):**

- Always falls between -1 and +1
- Is a positive value – as the value of one variable increases, so does the other.
- Is a negative value – as the value of one variable increases, the other decreases.

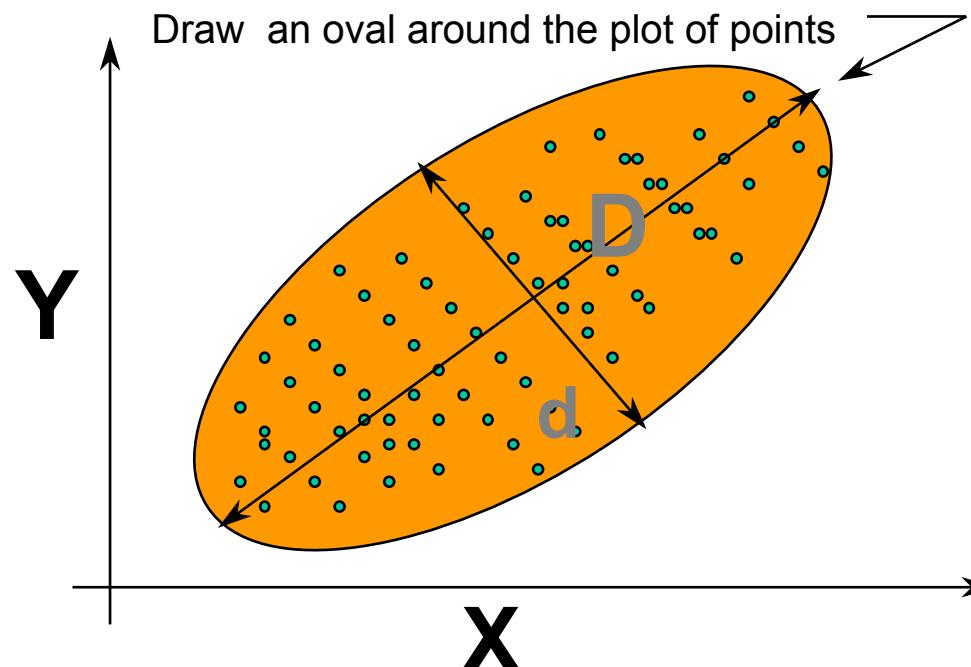
**The Correlation Formula:**

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$



# Estimating the Correlation Coefficient

1. Measure the length of the MAX diameter (D) of the oval with a scale
2. Measure the length of the MIN diameter (d) of the oval with a scale
3. Estimate the value of “ r ” by calculating:  $\pm (1 - [d/D])$
4. Attach the sign in the direction of the slope of D



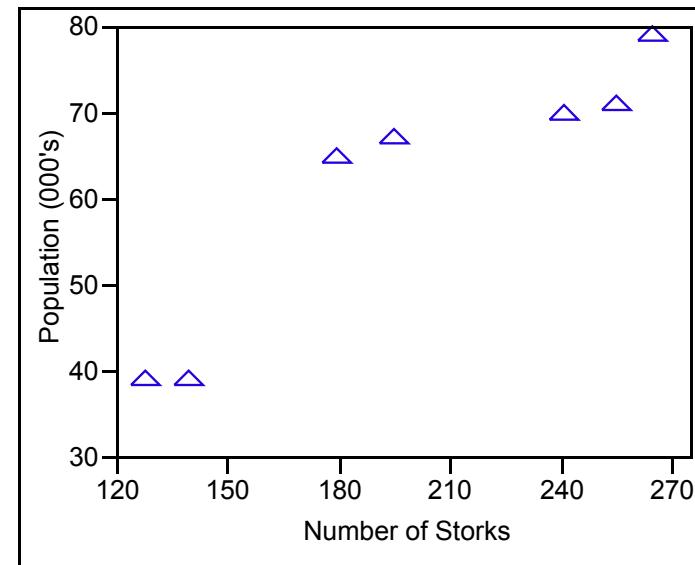
# Abuse and Misuse of Correlation

- If we establish a correlation between  $y$  and  $x_1$ , that does NOT necessarily mean variation in  $x_1$  caused variation in  $y$ .
- A third variable may be ‘lurking’ that causes both  $x_1$  and  $y$  to vary.
- In conclusion, an association between two variables does NOT mean there is a cause-and-effect relationship.

**Correlation does NOT determine causation!**

# Stork Example

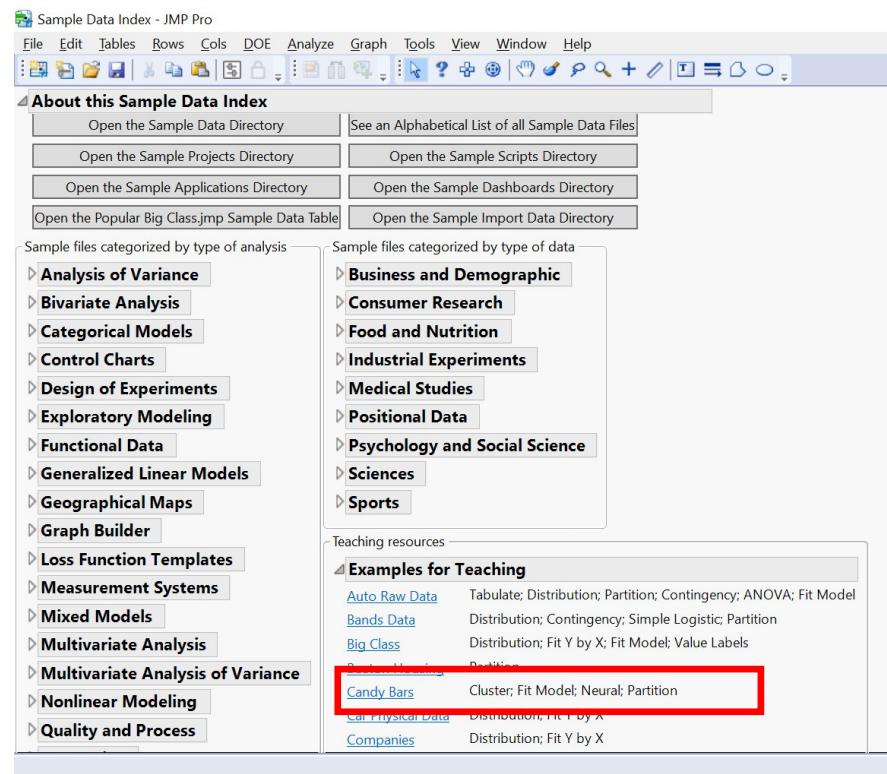
- **Correlation does not imply causation.**
- Look at the graph of population and storks below.
- Question: Would killing storks be an adequate method of birth control?
- We may identify a relationship by observing a process; two variables tend to increase together and decrease together. However, this does not necessarily mean that we can adjust one variable by manipulating the other variable.



# Six Sigma – Correlation & Regression

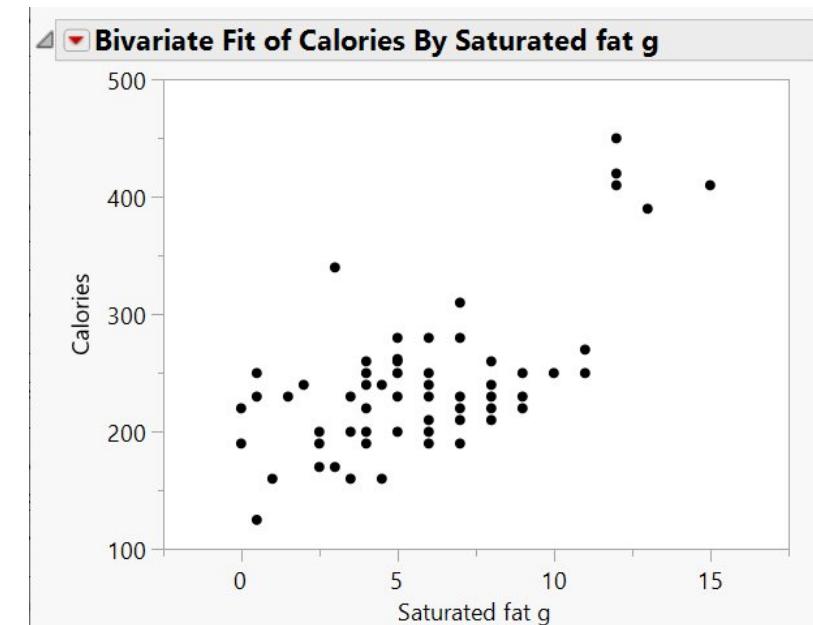
## Correlation Example 1

- Open the data table JMP>Help>Sample Index>Examples for Teaching> Candy Bars.jmp



# Correlation Example 1

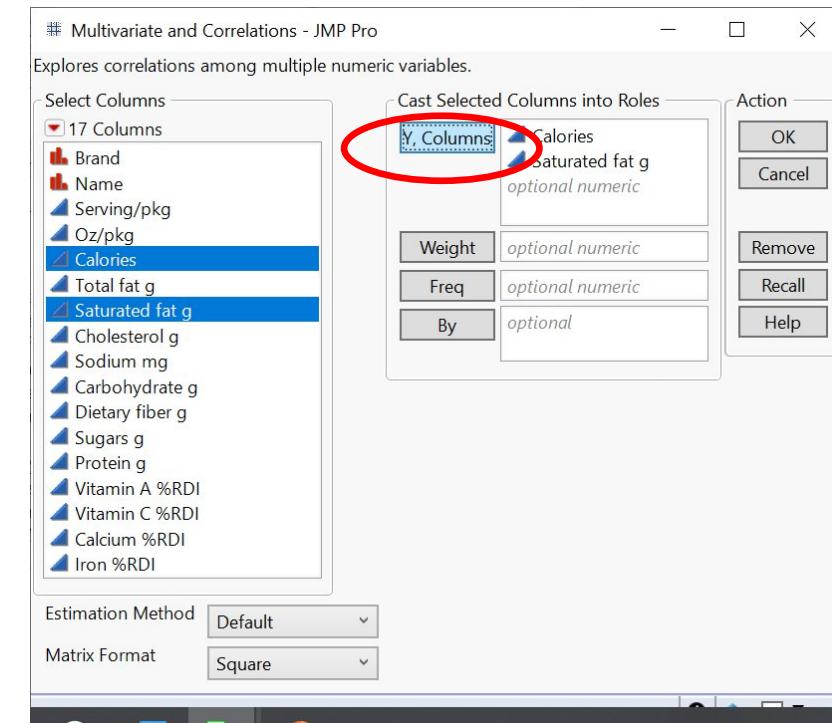
- Plot the data first
  - JMP>Analyze>Fit Y By X
  - For Y, Response select Calories
  - For X, Factor select Saturated fat g
  - Click OK



# Correlation Example 1

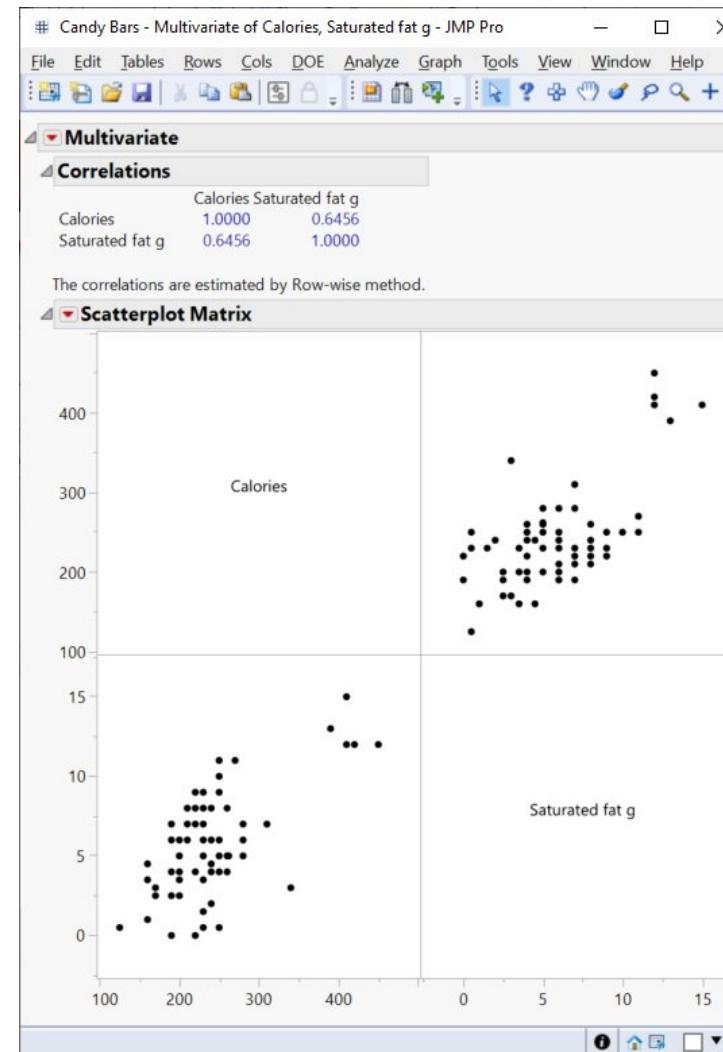
Run the Correlation Analysis

- **Analyze>Multivariate Methods>Multivariate**
- For **Y, Columns**, choose *Calories* and then *Saturated fat g*
- Click **OK**



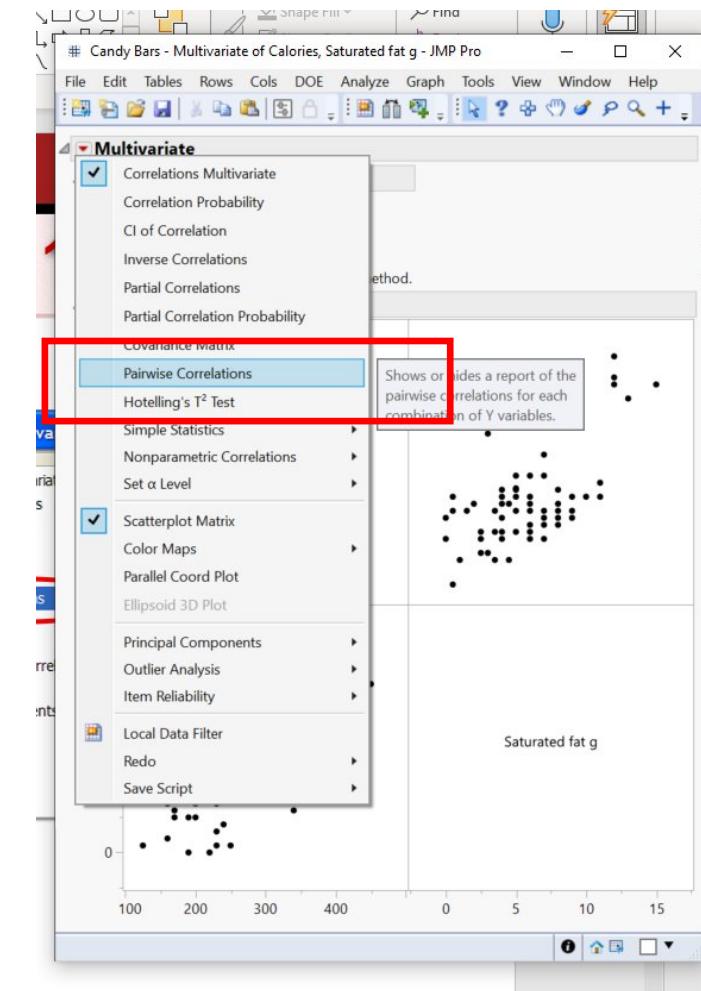
## Six Sigma – Correlation &amp; Regression

## Correlation Example 1



# Correlation Example 1

- Click-on the **Multivariate Red Triangle** and select **Pairwise Correlations** to see the p-values.



# Correlation Example 1

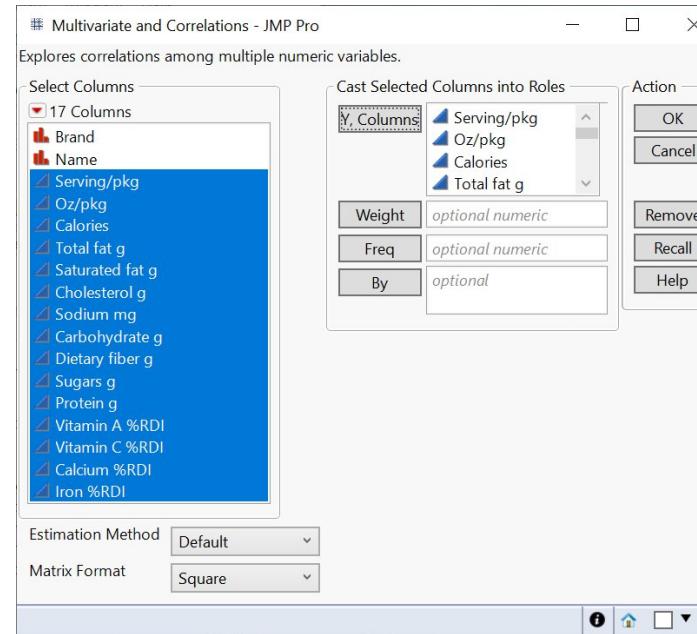
Pairwise Correlations															
Variable	by Variable	Correlation	Count	Lower 95%	Upper 95%	Signif Prob	-.8	-.6	-.4	-.2	0	.2	.4	.6	.8
Saturated fat g	Calories	0.6456	75	0.4906	0.7611	<.0001*									

Graphical picture of  
Correlation ( $r = 0.6456$ )

- Are the two variables related? What is  $r$ ?
- What are your conclusions?

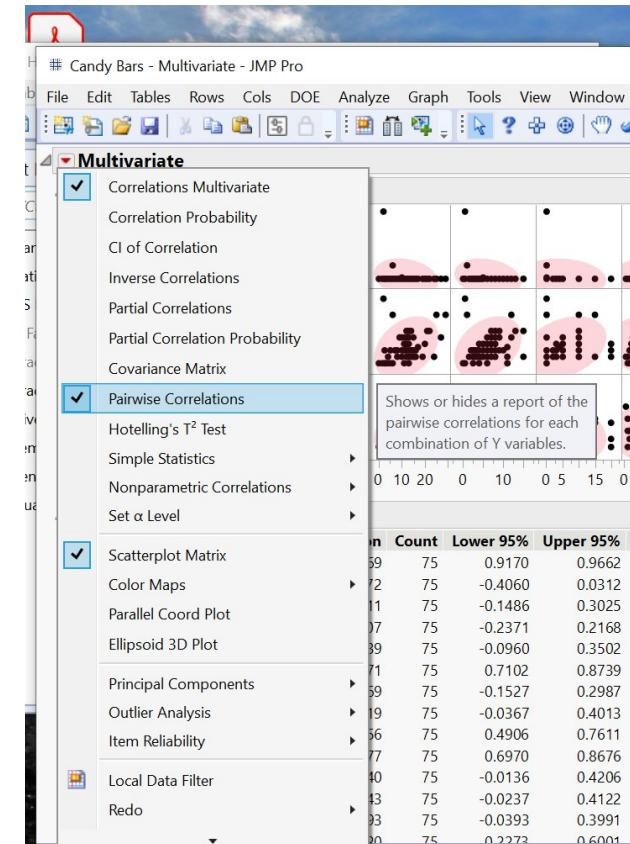
# Correlation Example 1

- **Analyze>Multivariate Methods>Multivariate**
- For **Y,Columns**, Choose *Servings/pkg* through *Iron %RDI*
- Click **OK**
- All the different combinations of the correlation graphs for the selected variables are generated in the output window.



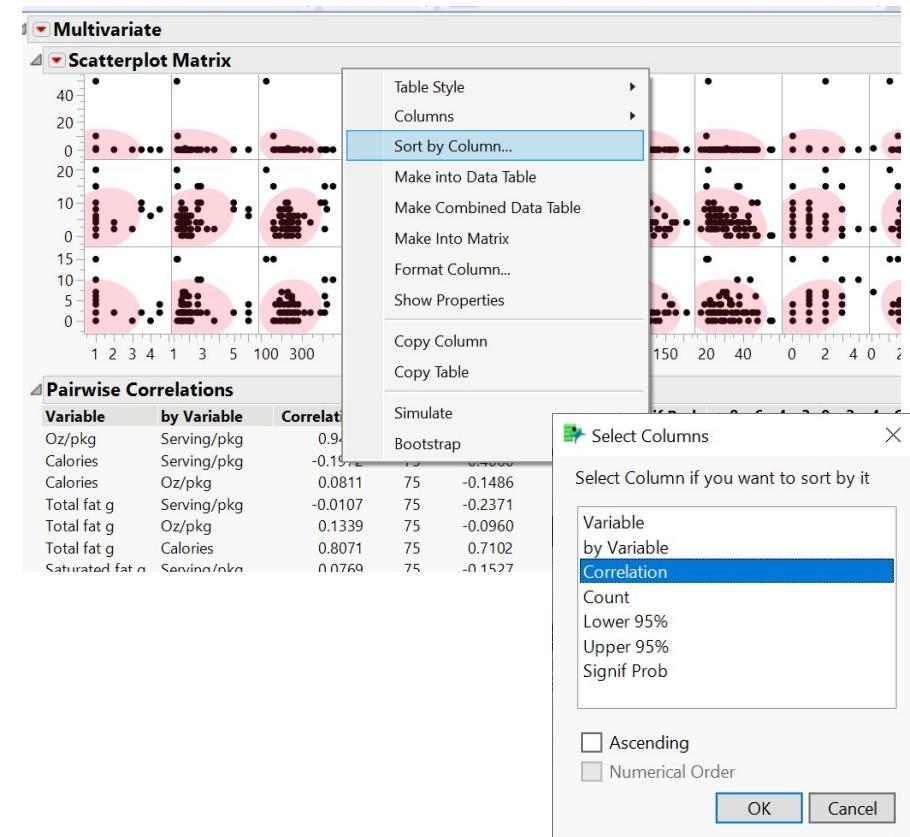
# Correlation Example 1

- u Click on red diamond in Multivariate, chose
- u Multivariate> Pairwise Correlations
- u Analyze all combinations of Scatterplots and the Correlation Coefficients



# Correlation Example 1

- u TIP: You can order the correlation values by right-clicking on the Pairwise Correlations table and selecting **Sort by Column**. Then select the “Correlation” column
- u The table is rank ordered by correlation coefficient from +1 to -1.





# Six Sigma – Correlation & Regression

## Correlation Example 2

- Open the menu **JMP>Help>Sample Index**
- Click on See an Alphabetical List of all Sample Data Files
- Scroll & Select **Scores.jmp**

### About this Sample Data Index

This is an Index to some of the sample data tables provided with JMP. Choose a subject heading below and open it to see a list of data tables you can use to explore that topic. Click the blue underlined file name to open a data table. Then, within each data table, study the table notes and column notes for more information.

Open the Sample Data Directory      See an Alphabetical List of all Sample Data Files  
 Open the Sample Projects Directory      Open the Sample Scripts Directory  
 Open the Sample Applications Directory      Open the Sample Dashboards Directory  
 Open the Popular Big Class.jmp Sample Data Table      Open the Sample Import Data Directory

Sample files categorized by type of analysis      Sample files categorized by type of data

**Analysis of Variance**      **Business and Demographic**

File Edit Tables Rows Cols DOE Analyze Graph Tools View Window Help

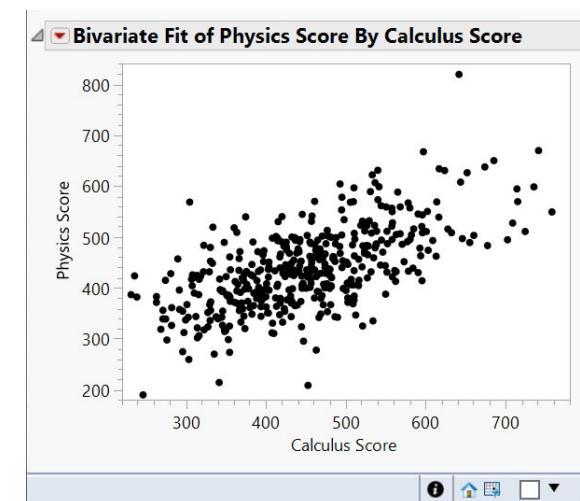
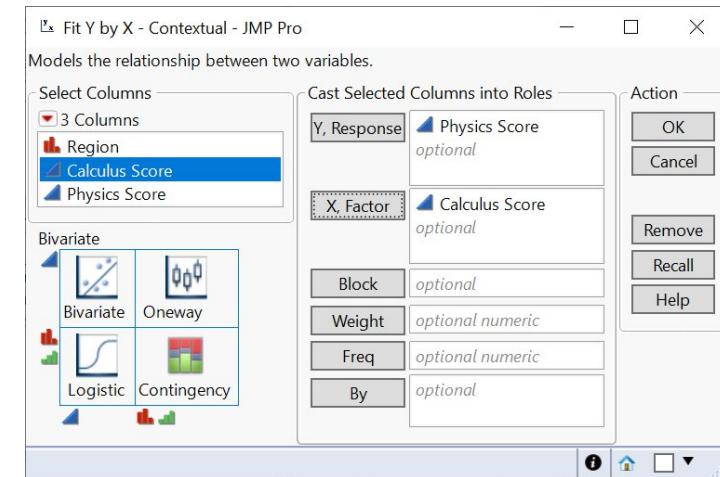
Big Class      Golf Balls  
 Big Class Families      Gossel's Corn  
 Billion Dollar Events      Grandfather Clocks  
Binomial Experiment (Design Experiment)      Gravel (Quality Control)  
Binomial Optimal Start (Design Experiment)      Grocery Purchases  
Bioassay (Nonlinear Examples)      Growth  
 Birth Death      Growth Measurements  
 Birth Death Subset      Hair Care Product  
 BirthDeathYear      Half Reactor  
 Bladder Cancer (Reliability)      Health Risk Survey  
 Blenders (Reliability)      Hearing Loss  
 Blood Pressure      Hollywood Movies  
 Blood Pressure by Time      Hot Dogs  
blsPriceData      Hot Dogs2  
 Body Fat      Holthand  
 Body Measurements      Hwtv12  
Borehole Factors (Design Experiment)      Hwtv15  
Borehole Latin Hypercube (Design Experiment)      Hurricanes  
Borehole Sphere Packing (Design Experiment)      Hybrid Fuel Economy  
Borehole Uniform (Design Experiment)      ICDevice02 (Reliability)  
 Boston Housing      Ingots  
 Bottle Tops (Quality Control)      Ingots2  
Bounce Data (Design Experiment)      InjectionMolding  
 Bounce Factors (Design Experiment)      Investment Castings  
 Bounce Response (Design Experiment)      Iris  
Box Corrosion Split-Plot (Design Experiment)      IRS Example (Nonlinear Examples)  
 BoxCov      Ichikawa

Reading Study  
 Readings  
 Resistor (Reliability)  
 Restaurant Tips  
 Ro  
 Runners Covariates (Design Experiment)  
 Runners Factors (Design Experiment)  
 S4 Temps  
 S4-Name  
 S4-XY  
 Salt in Popcorn  
 San Francisco Crime  
 San Francisco Crime Distances  
 SAS Offices  
 SAT  
 SATByYear  
**Scores**      Semiconductor Capability  
 Seriesa (Time Series)  
 Seriesa1 (Time Series)  
 Seriesa2 (Time Series)  
 Seriesa3 (Time Series)  
 Seriesb (Time Series)  
 Seriesc (Time Series)  
 Seriesd (Time Series)  
 Seriese (Time Series)

# Six Sigma – Correlation & Regression

## Correlation Example 2

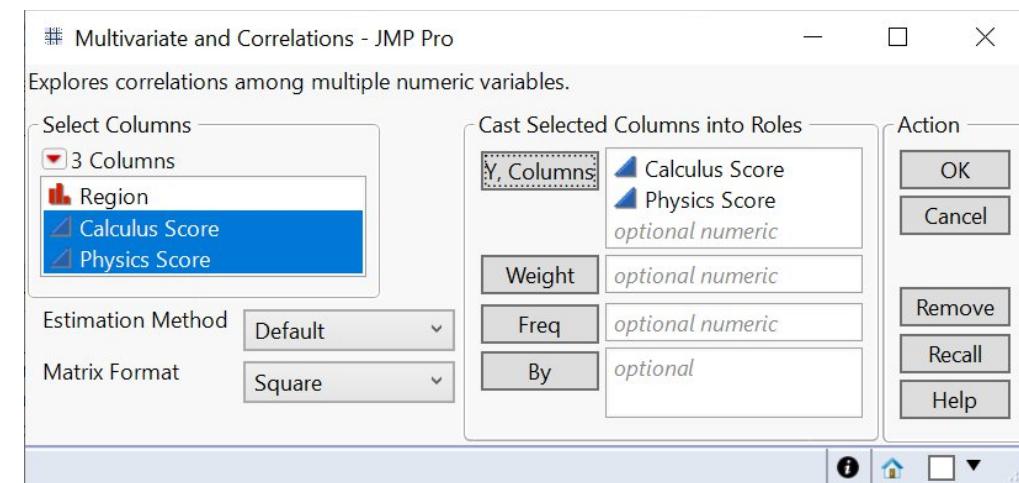
- Always plot the data first
  - JMP>Analyze>Fit Y by X**
  - For **Y, Response** select *Physics Score*
  - For **X, Factor** select *Calculus Score*
  - Click **OK**



# Correlation Example 2

Run the Correlation Analysis

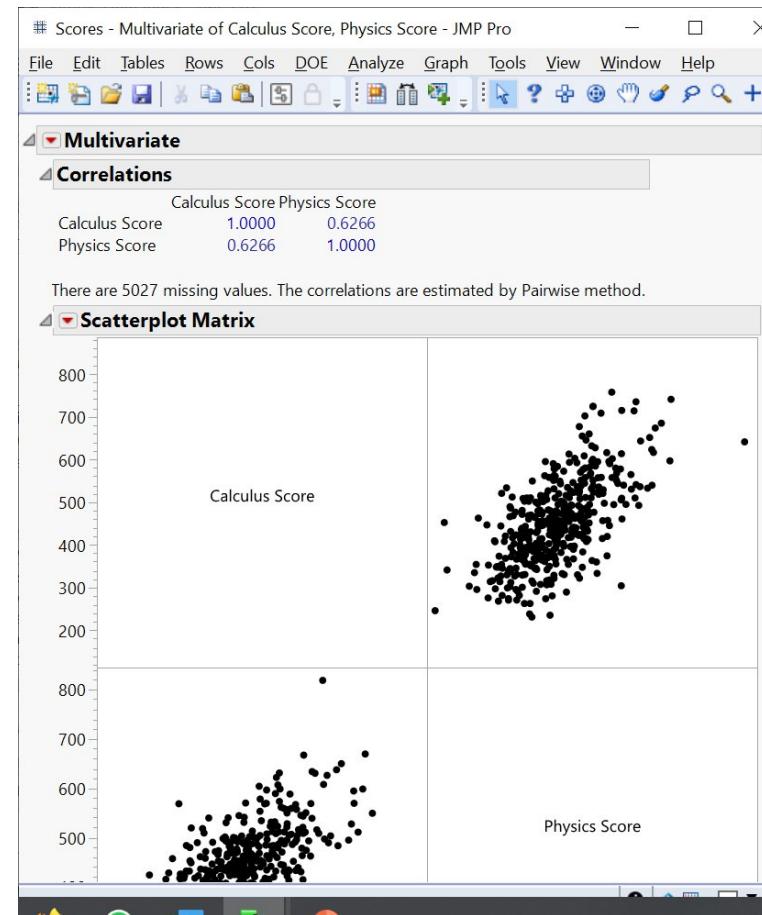
- **Analyze>Multivariate Methods>Multivariate**
- For **Y, Columns**, choose *Calculus Score* and then *Physics Score*
- Click **OK**





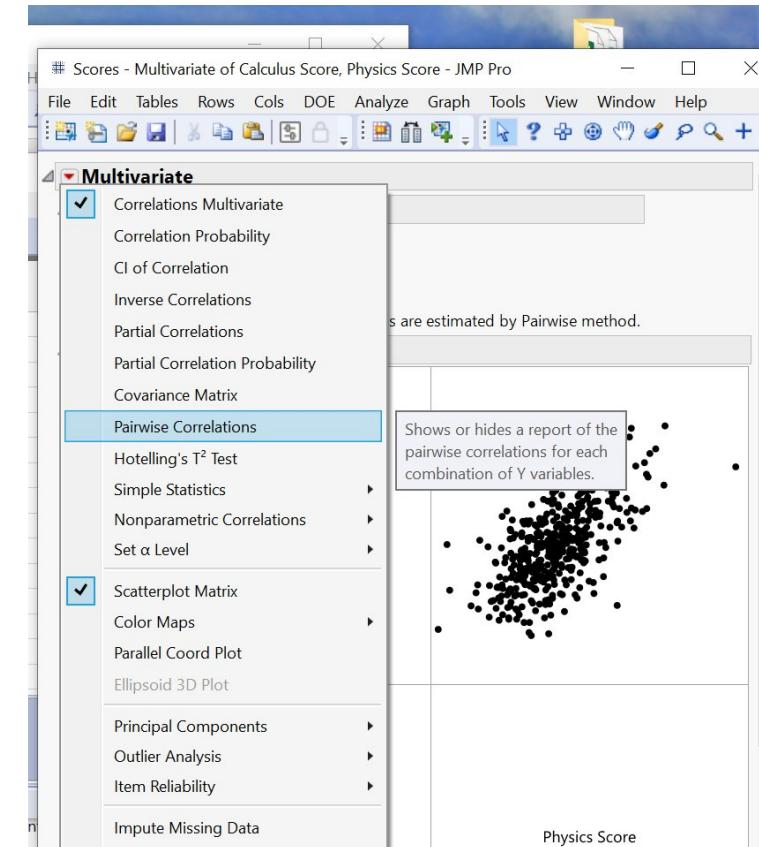
# Six Sigma – Correlation & Regression

## Correlation Example 2



# Correlation Example 2

- Click-on the **Multivariate Red Triangle** and select **Pairwise Correlations** to see the p-values.



# Correlation Example 2

Pairwise Correlations																
Variable	by Variable	Correlation	Count	Lower 95%	Upper 95%	Signif	Prob	-.8	-.6	-.4	-.2	0	.2	.4	.6	.8
Physics Score	Calculus Score	0.6266	436	0.5660	0.6805	<.0001*										

- Are the two variables related? What is r?
- What are your conclusions?

# Regression Analysis

- Correlation tells us the strength of a relationship, not the exact numerical relationship.
- The next step for analyzing continuous data is the determination of the regression equation.
- Regression analysis calculates a “prediction equation” which can mathematically predict Y for any given X.
- The primary objective of regression analysis is to make **PREDICTIONS**.
- The regression equation is simply the one that **BEST FITS** the plotted data.
- Examples of prediction equations:

$$Y = a + b x$$

(linear model)

$$Y = a + b x + c x^2$$

(with quadratic term)

$$Y = a + b x + c x^2 + d x^3$$

(with cubic term)

$$Y = a ( b ^ x )$$

(exponential model)

# Coefficient of Determination, R-

## Squared

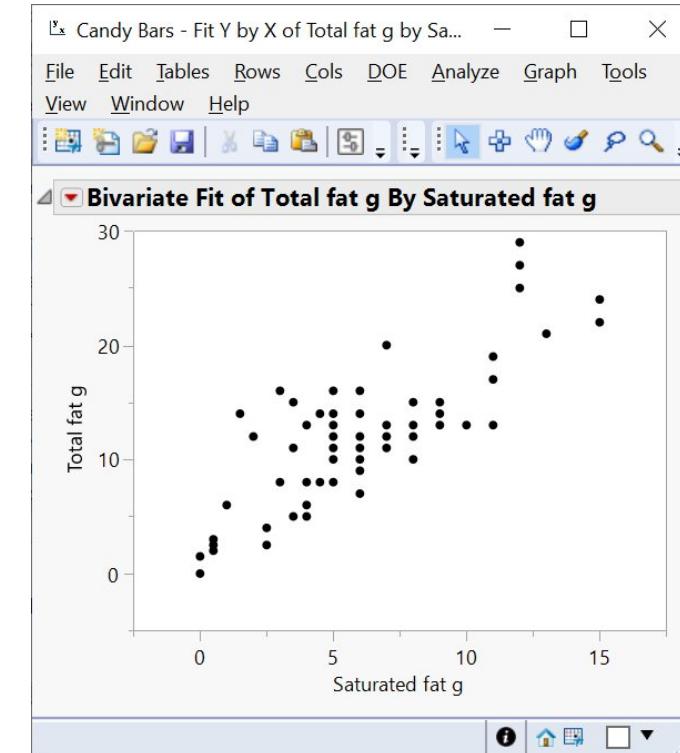
- The output from the fitted line plot contains an equation which relates the predictor (input variable) to the response (output variable).
- The **R-sq.** value is the square of the correlation coefficient. It is also the **fraction of the variation in the output (response) variable that is explained by the equation.**
- What is a good value? It depends on the process and the industry. For example, a chemist may require an R-sq of 0.99. However, the fact that one input variable may account for 65% of the variation in your final product may be phenomenal too!

# Regression Example (Fitted Line Plot)

- Open the data table

**JMP>Help>Sample Index>Examples for Teaching>Candy Bars.jmp**

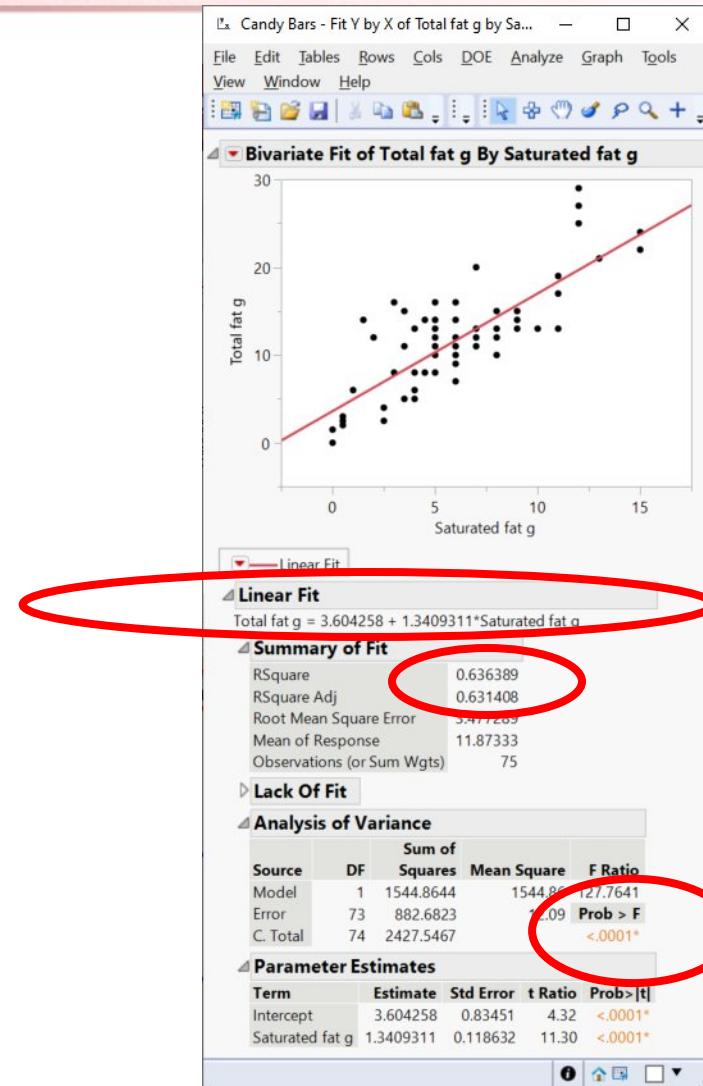
- Plot the data first
  - JMP>Analyze>Fit Y by X**
  - For Y, Response** select *Total Fat g*
  - For X, Factor** select *Saturated Fat g*
  - Click OK**



# Six Sigma – Correlation & Regression

## Regression Example (Fitted Line Plot)

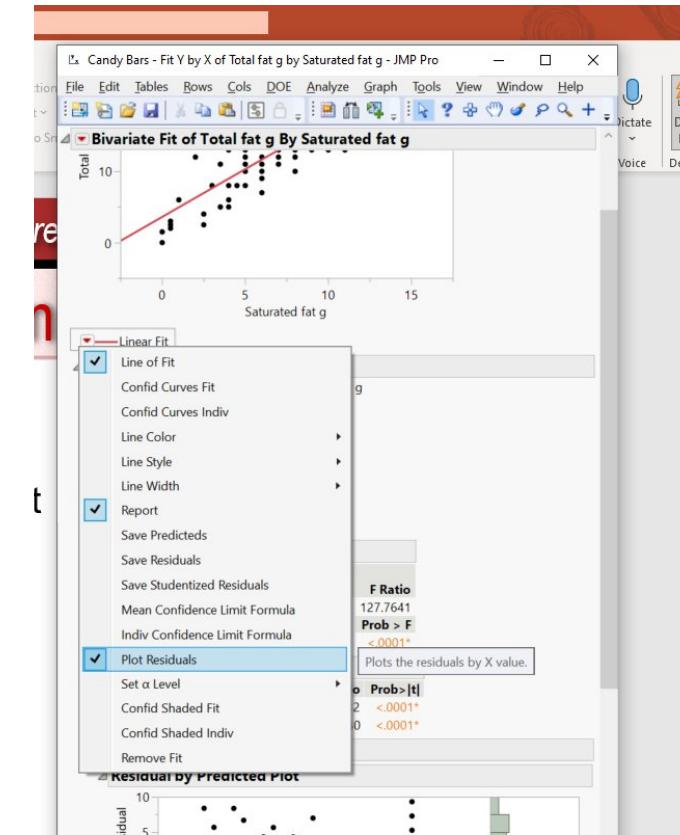
- Click-on the **Bivariate Fit of Total Fat by Saturated Fat** and select **Fit Line**.
- Note the Prediction Equation and the RSquare value.
- 63.6% of the variation in the data can be explained by the model.



# Six Sigma – Correlation & Regression

## Regression Example (Fitted Line Plot)

- u Examine the Residuals
- u Click-on the **Linear Fit Red Triangle** and select **Plot Residuals**

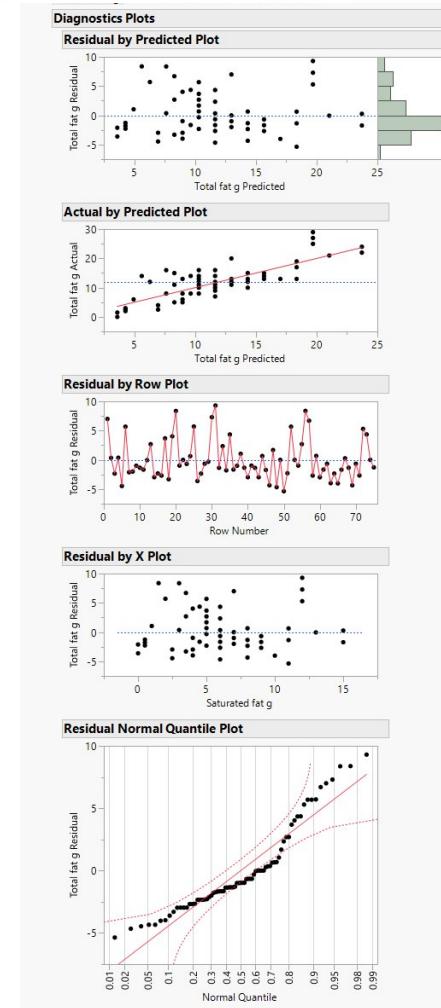




# Six Sigma – Correlation & Regression

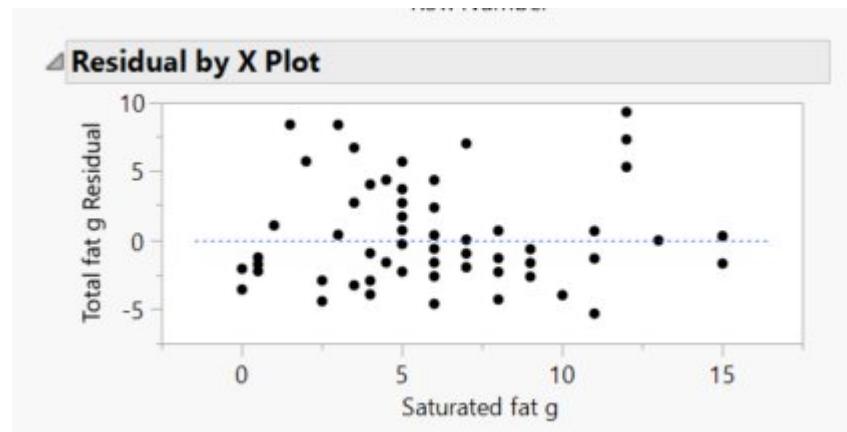
## Regression Example (Residuals)

- u What do residuals show you?
- u What does residual by predicted plot mean?
- u What does the Normal Quantile Plot tell you?



## Six Sigma – Correlation &amp; Regression

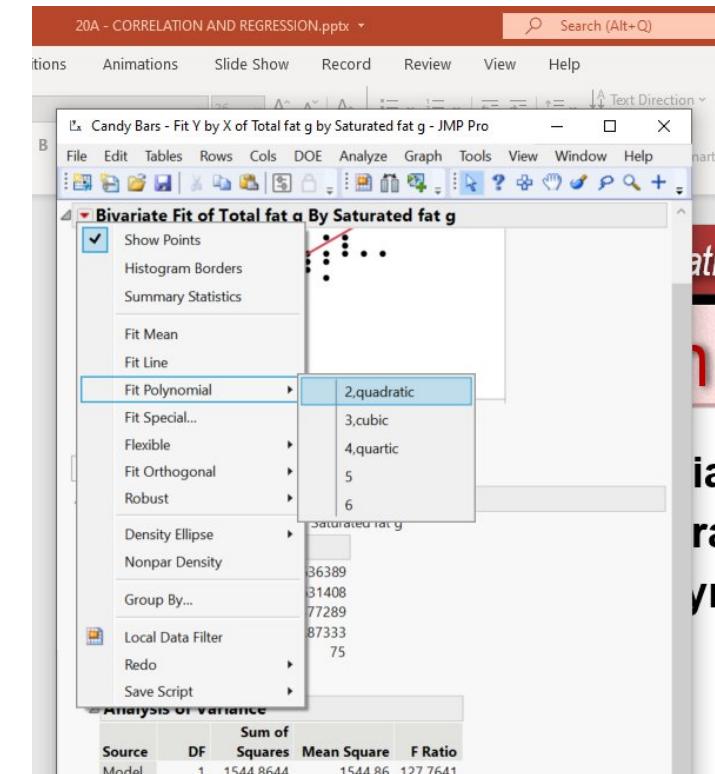
# Regression Example (Fitted Line Plot)



- u Obtain a prediction equation:
  - Do you need a higher order model?
  - Investigate the residuals plot.

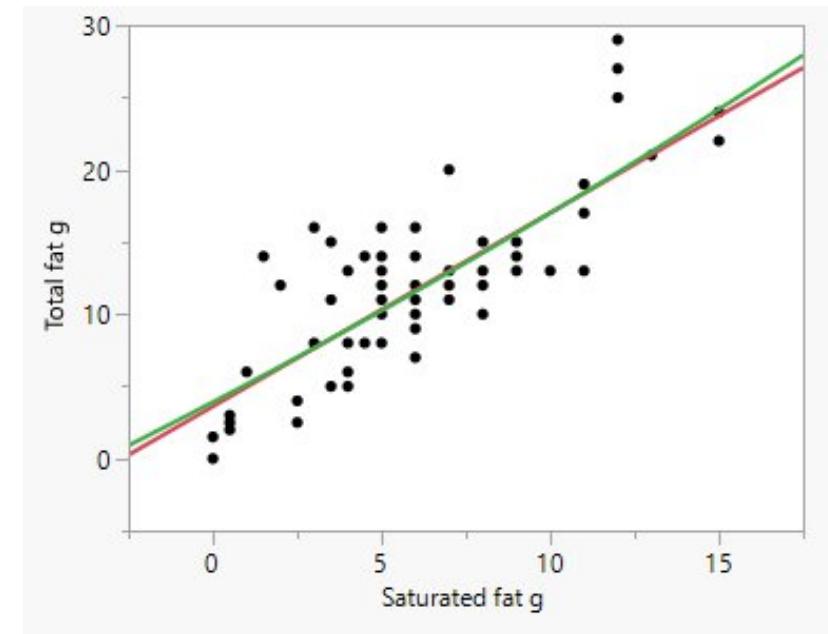
# Regression Example (Fitted Line Plot)

- Click-on the **Bivariate Fit of Total Fat by Saturated Fat** and select **Fit Polynomial>2,quadratic**.



# Regression Example (Fitted Line Plot)

u Note that the Quadratic Model is not an improvement over the linear model.



# Regression Example (Fitted Line Plot)

u The RSquare value for the quadratic model is 0.636941.

u The squared term in the model does not improve the model, and it is not significant as shown by the p>value of .7417

Polynomial Fit Degree=2														
Total fat g = 3.5810782 + 1.3285343*Saturated fat g + 0.0086968*(Saturated fat g-6.16667)^2														
Summary of Fit														
<table><tr><td>RSquare</td><td>0.636941</td></tr><tr><td>RSquare Adj</td><td>0.626856</td></tr><tr><td>Root Mean Square Error</td><td>3.498695</td></tr><tr><td>Mean of Response</td><td>11.87333</td></tr><tr><td>Observations (or Sum Wgts)</td><td>75</td></tr></table>					RSquare	0.636941	RSquare Adj	0.626856	Root Mean Square Error	3.498695	Mean of Response	11.87333	Observations (or Sum Wgts)	75
RSquare	0.636941													
RSquare Adj	0.626856													
Root Mean Square Error	3.498695													
Mean of Response	11.87333													
Observations (or Sum Wgts)	75													
Lack Of Fit														
Analysis of Variance														
Source	DF	Sum of Squares	Mean Square	F Ratio										
Model	2	1546.2044	773.102	63.1575										
Error	72	881.3422	12.241	Prob > F										
C. Total	74	2427.5467		<.0001*										
Parameter Estimates														
Term		Estimate	Std Error	t Ratio	Prob> t									
Intercept		3.5810782	0.842565	4.25	<.0001*									
Saturated fat g		1.3285343	0.125105	10.62	<.0001*									
(Saturated fat g-6.16667)^2		0.0086968	0.026285	0.33	0.7417									

# Class Examples

Use correlation/regression to analyze the Candy Bars.jmp file. Does the Total Fat, X, predict the Calories, Y, in the candy bars?

- Are the two variables related?
- What is “r”? What is R-Sq?
- What are your conclusions for each analysis?
- Try a polynomial of degree = 2.
- Try a polynomial of degree = 3.
- Do either improve the R-Sq?

# Summary

- Correlation is a very useful tool in the process industry.
- Correlation is the measure of the **relationship** between two quantitative variables.
- Correlation does NOT determine causation!
- Regression analysis seeks to find a relationship between the variables in the form of a prediction equation which may or may not be linear.
- In regression, the equation may be the desired answer, or it may be the means to the desired prediction.

## TRUE or FALSE Question:

When the calculated value of  $r$  is between the table value of  $r$  and 1 (positive or negative), have we established a cause- and-effect relationship between the two variables?

# INTRODUCTION TO IMPROVE

# Objectives

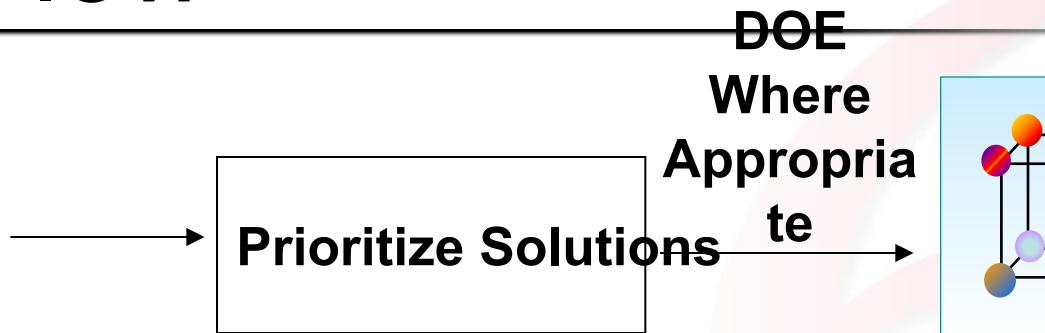
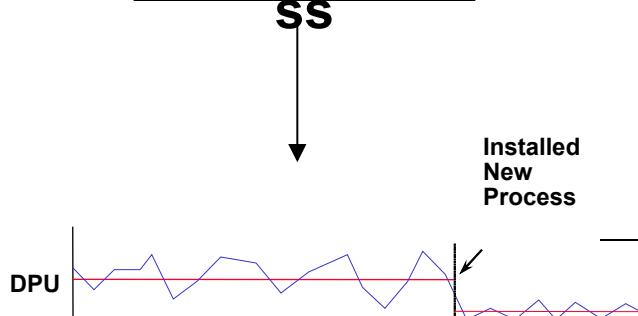
- Discuss the typical flow through the Improve phase of a project
- Overview of the types of improvement strategies
- Use a Pick Chart to evaluate solutions

# Improve Flow

Based on root

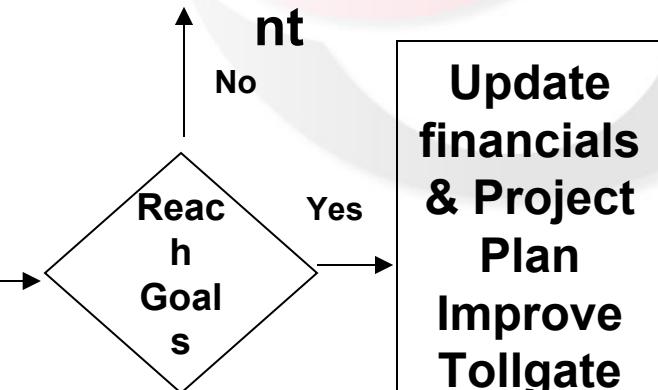
cause analysis,  
brainstorm  
potential  
solutions, risks,  
costs and  
benefits.

Pilot  
solutions  
and  
measure  
effectivene



Review /  
Approve  
solutions  
with  
manageme

Solution  
Implementati  
on Plan



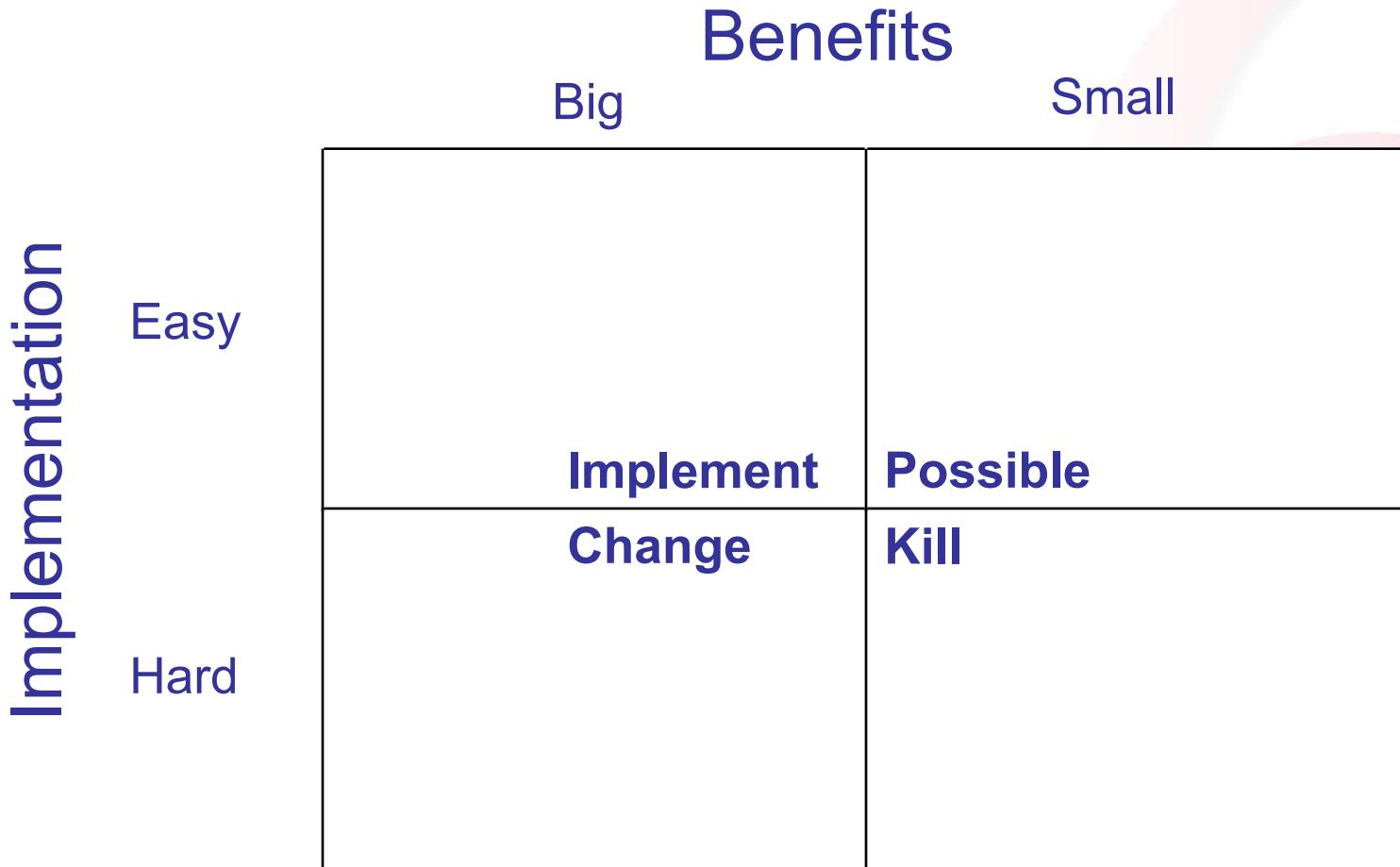
# Root Causes

- Identified in Analyze.
- Rank in relative importance.
- Brainstorm potential solutions for each root cause.
- Prioritize root causes with solutions.

# Prioritizing Root Causes

Root Cause	Solutions	Cost	Benefits
1.	1. 2. 3.		
2.			
3.			

# Pick Chart



# Types of Solutions

- Lean Strategies
- DOE
- Pilot Study
- Error Proofing

# LEAN IMPROVEMENT TOOLS

# Objectives

- Review the eight types of wastes found in processes
- Understand how to apply the 5 S + 1 principles to improving a process

# Lean Six Sigma

- Reduce complexity in processes
- Create efficient, continuous flow processes
- Identify and eliminate waste

# Waste

Non – Value Added activities are considered to be WASTE in a process.

- Defects
- Overproduction
- Transportation
- Waiting
- Inventory
- Motion
- Processing
- Intellect



# 5 S + 1

## Sort

- Keep only what is needed in your area

## Set in Order

- A place for everything and everything in its place

## Shine

- Clean the work space

## Standardize

- Develop systems to maintain the improvements

## Sustain

- Self discipline to maintain established procedures

## Safety

- Removing hazards and dangers

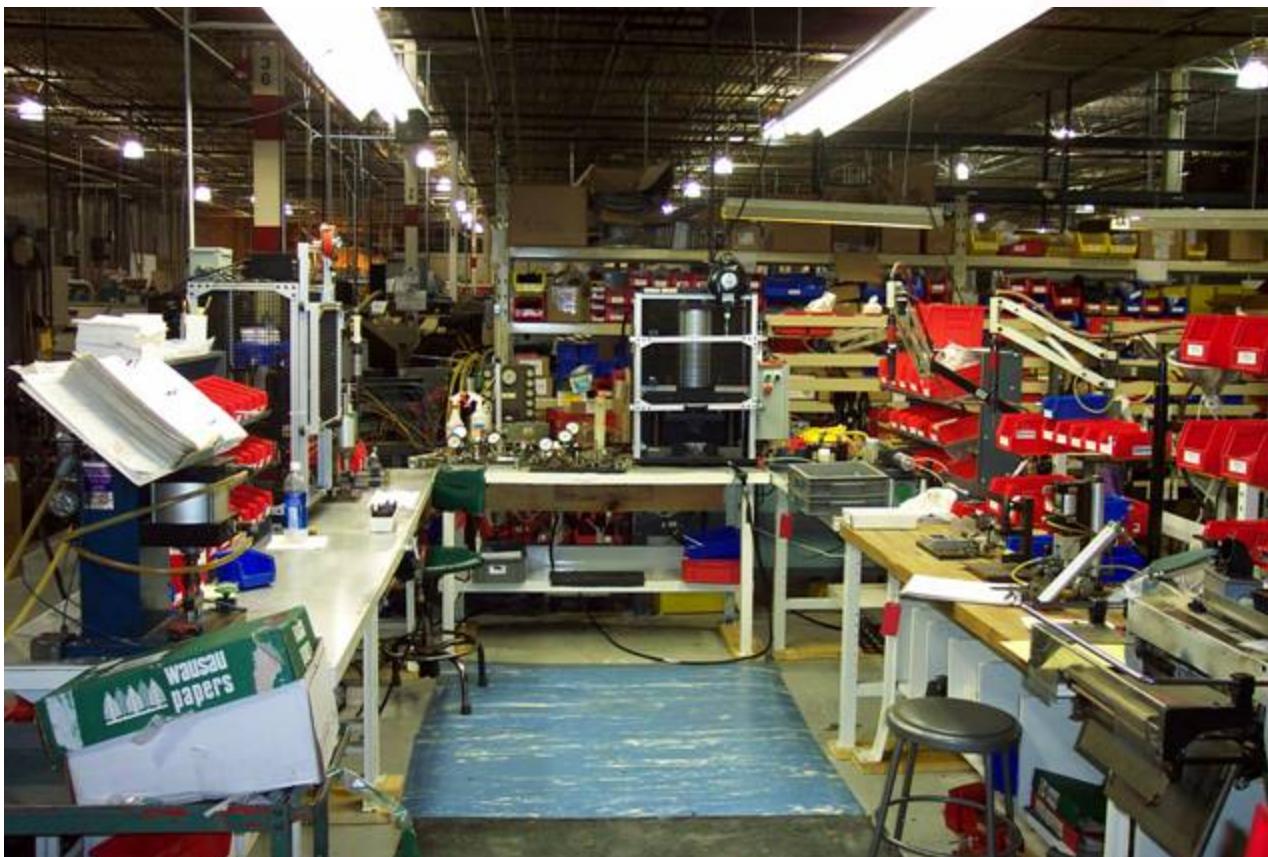
# 5 S + 1

## Before



# 5 S + 1

After



# 5 S + 1



# KAIZEN WORKSHOPS

# Objectives

- Define Kaizen.
- Understand how Kaizen events may be used for improvement within the DMAIC framework.
- Discuss the steps included in a Kaizen workshop.

# Kaizen

A core lean concept, Kaizen is the philosophy of continuous improvement through the removal of wastes in a process.

# Kaizen Workshops

- Implement rapid process improvements
- Short, focused projects (3 – 5 days)
- Cross-functional teams
- Solutions implemented immediately



# Kaizen Workshops

Kaizen Workshops may be used effectively to address:

- Customer complaints
- Processes that are not clearly defined
- Processes that generate a large number of defects
- Processes that have not been standardized
- Processes that are not error proofed
- Processes that have very lengthy cycle times

# Roles and Responsibilities

- Management
- Kaizen lead or facilitator
- Team members
- Subject matter experts

# Management

- Executives, managers and process owners
- Assist in identifying projects
- Provide resources and remove barriers to success
- Attend kickoff meeting
- Celebrate success

# Kaizen Lead or Facilitator

- Responsible for completing pre-event planning
  - Objective setting
  - Selecting and notifying participants
  - Preparing training materials
  - Assemble background information
  - Logistics
  - Coordinate with participants' departments and management to make resources available for the workshop
- Facilitates the workshop
- Manages implementation of solutions

# Team Members

- Participate in the workshop
- Process expertise
- Carry out project tasks as assigned
- Provide updates on the status of solution implementations or action plan

# Subject Matter Experts

- Participate in events as needed
- Provide expertise
- Assist in managing the implementation of solutions

# Tools

- Brainstorming
- Cause & Effect Diagrams
- 5 S + 1
- Error Proofing
- Value Matrix or Value Stream Maps

# Typical Schedule

- Pre-event Activities
  - Objectives
  - Resources
  - Logistics
  - Background
  - Collect data
  - Validate value stream

# Typical Schedule

- Day 1
  - Management kickoff
  - Brief team on project objectives and pre-work
  - Training
  - Review and validate value matrix/value stream
- Days 2 – 3
  - Complete any data collection
  - Identify and verify root causes
  - Identify sources of waste
  - Brainstorm improvements to remove wastes and reduce variability

# Typical Schedule

- Day 4
  - Create a solution action list
  - Implement improvements
  - Train employees
  - Test and refine solution
- Day 5
  - Document new procedures
  - Present results to management team
  - Create a control plan

# LEAN IMPROVEMENT TOOLS

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- Waiting
- Inventory
- Motion
- Processing
- Intellect



# 5 S + 1

## Sort

- Keep only what is needed in your area

## Set in Order

- A place for everything and everything in its place

## Shine

- Clean the work space

## Standardize

- Develop systems to maintain the improvements

## Sustain

- Self discipline to maintain established procedures

## Safety

- Removing hazards and dangers

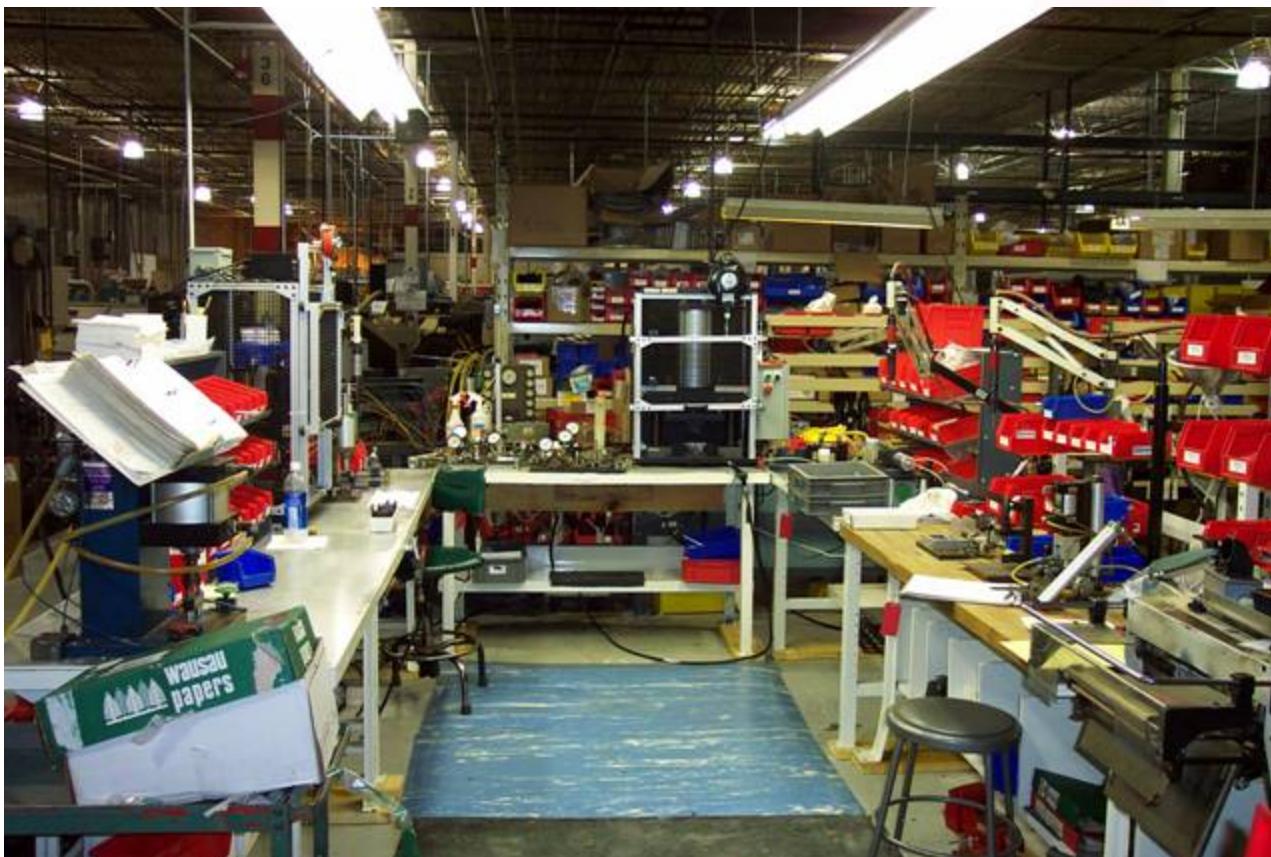
# 5 S + 1

## Before



# 5 S + 1

After



# 5 S + 1



# KAIZEN WORKSHOPS

# Objectives

- Define Kaizen.
- Understand how Kaizen events may be used for improvement within the DMAIC framework.
- Discuss the steps included in a Kaizen workshop.

# Kaizen

A core lean concept, Kaizen is the philosophy of continuous improvement through the removal of wastes in a process.

# Kaizen Workshops

- Implement rapid process improvements
- Short, focused projects (3 – 5 days)
- Cross-functional teams
- Solutions implemented immediately



# Kaizen Workshops

Kaizen Workshops may be used effectively to address:

- Customer complaints
- Processes that are not clearly defined
- Processes that generate a large number of defects
- Processes that have not been standardized
- Processes that are not error proofed
- Processes that have very lengthy cycle times

# Roles and Responsibilities

- Management
- Kaizen lead or facilitator
- Team members
- Subject matter experts

# Management

- Executives, managers and process owners
- Assist in identifying projects
- Provide resources and remove barriers to success
- Attend kickoff meeting
- Celebrate success

# Kaizen Lead or Facilitator

- Responsible for completing pre-event planning
  - Objective setting
  - Selecting and notifying participants
  - Preparing training materials
  - Assemble background information
  - Logistics
  - Coordinate with participants' departments and management to make resources available for the workshop
- Facilitates the workshop
- Manages implementation of solutions

# Team Members

- Participate in the workshop
- Process expertise
- Carry out project tasks as assigned
- Provide updates on the status of solution implementations or action plan

# Subject Matter Experts

- Participate in events as needed
- Provide expertise
- Assist in managing the implementation of solutions

# Tools

- Brainstorming
- Cause & Effect Diagrams
- 5 S + 1
- Error Proofing
- Value Matrix or Value Stream Maps

# Typical Schedule

- Pre-event Activities
  - Objectives
  - Resources
  - Logistics
  - Background
  - Collect data
  - Validate value stream

# Typical Schedule

- Day 1
  - Management kickoff
  - Brief team on project objectives and pre-work
  - Training
  - Review and validate value matrix/value stream
- Days 2 – 3
  - Complete any data collection
  - Identify and verify root causes
  - Identify sources of waste
  - Brainstorm improvements to remove wastes and reduce variability

# Typical Schedule

- Day 4
  - Create a solution action list
  - Implement improvements
  - Train employees
  - Test and refine solution
- Day 5
  - Document new procedures
  - Present results to management team
  - Create a control plan



# Lean Six Sigma $2^K$ Factorial Experiments

# $2^K$ Factorial Experiments

- Objective:** To develop a mathematical relationship between the significant factors and the response variable.
- Deliverables:**  $Y=f(x)$ , Mathematical Model, DOE Report, Updated FMEA

# $2^K$ Factorial Experiments

Objective	Project Deliverables
DOE Planning	<ul style="list-style-type: none"><li>u Experiment Planning</li><li>Experiments</li></ul>
Screening Experiments (What are the significant factors?)	<ul style="list-style-type: none"><li>u Fractional Factorial Experiments</li><li>u Reduced List of Input Variables</li></ul>
$Y=f(x)$ (What is the mathematical relationship between the significant factors and the response variable?)	<ul style="list-style-type: none"><li>u Mathematical Model</li><li>u <math>2^K</math> Factorial Experiments</li><li>u Full Factorial Experiments</li><li>u Established Input Levels</li></ul>
Process Documentation	<ul style="list-style-type: none"><li>u Updated Process FMEA</li><li>u DOE Report(s)</li></ul>

# $2^K$ Factorial Experiments

## Agenda

- u Why do we need  $2^K$  factorial experimentation?
- u  $2^K$  Standard Order Designs
  - Exercise:
    - Calculating main effects
    - Calculating interactions
  - $2^K$  Example (using JMP)
  - $2^K$  Exercises (using JMP)

# $2^K$ Factorial Experiments

## Agenda

- u Adding Center Points
  - $2^K$  Example with center points (using JMP)
- u Adding Blocking
  - $2^K$  Example with Blocking
- u  $2^K$  Vocabulary
- u Steps for DOE Analysis

# Why Use $2^K$ Factorial Experiments?

- u The **GOAL** is to obtain a mathematical relationship which characterizes:
  - $Y = f(x_1, x_2, x_3, \dots)$
- u The mathematical relationship allows us to identify not only the *critical factors* but also the *best levels* for those factors.
- u  $2^K$  Factorial Experiments investigate multiple factors where each factor is studied at only two levels.

# Why Use $2^K$ Factorial Experiments?

- u  $2^K$  Factorial Experiments allow us to investigate a large number of factors simultaneously in relatively few runs compared to full factorial designs.
- u Finally,  $2^K$  designs are used most frequently in industrial DOE applications because they are very easy to analyze and lend themselves well to sequential studies.

# Types of $2^K$ Factorials

- u **One observation per treatment combination**
  - Usually low statistical power
  - Use Normal Quantile Plots or Pareto charts instead of F-tests
  - Can create full factorials by leaving unimportant factors out

# Types of $2^K$ Factorials

- u **More than one observation per treatment combination (known as repeats or replicates)**
  - Better estimates of error
  - Better statistical power
  - Can still run reduced models
  - F-tests, normal plots and pareto charts can be used

# Standard Order Of $2^k$ Designs

- u  **$2^k$  factorials refer to k factors, each with 2 levels.** A  $2^2$  factorial is a 2x2 factorial. This design has two factors with two levels and can be executed in only 2x2 or 4 runs.  
**Likewise a  $2^3$  factorial has 3 factors, each with two levels. This experiment can be done 2x2x2 or 8 runs.**

# Standard Order Of $2^K$ Designs

(Cont.)

- u The design matrix for  $2^K$  factorials is usually shown in standard order. The low level of a factor is designated with “-” or -1 and the high level is designated with “+” or 1. An example of a design matrix follows.

**$2^2$  Factorial**

Temp	Conc
-1	-1
1	-1
-1	1
1	1

**$2^3$  Factorial**

Temp	Conc	Catalyst
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1



# Standard Order Of $2^k$ Designs

## Exercise

- u Create a  $2^4$  Factorial Design Matrix**
- u What are the minimum number of runs needed?**
- u Verify your results using JMP:  
JMP>DOE>Classical>Full Factorial Design**
- u Add two 2-level continuous factors and two 2-level categorical factors**

# Standard Order Of $2^k$ Designs

## Exercise

**Open JMP – new data table**

**Click on DOE**

**Select Classical > Full Factorial Design**

**Y is response, Maximize**

**Select 2 continuous, 2 categorical variables,  
2 levels each, click continue**

Six Sigma –  $2^K$  Factorial ExperimentsStandard Order Of  $2^K$  Designs

## Exercise

**DOE- Full Factorial Design**

**Full Factorial Design**

**Responses**

Add Response Remove Number of Responses...

Response Name	Goal	Lower Limit	Upper Limit	Importance
Y optional item	Maximize	.	.	.

**Factors**

Continuous Categorical Remove

Name	Role	Values
X1	Continuous	-1 1
X2	Continuous	-1 1
X3	Categorical	L1 L2
X4	Categorical	L1 L2

Full Factorial Design  
2x2x2x2 Factorial

Output Options

Run Order:

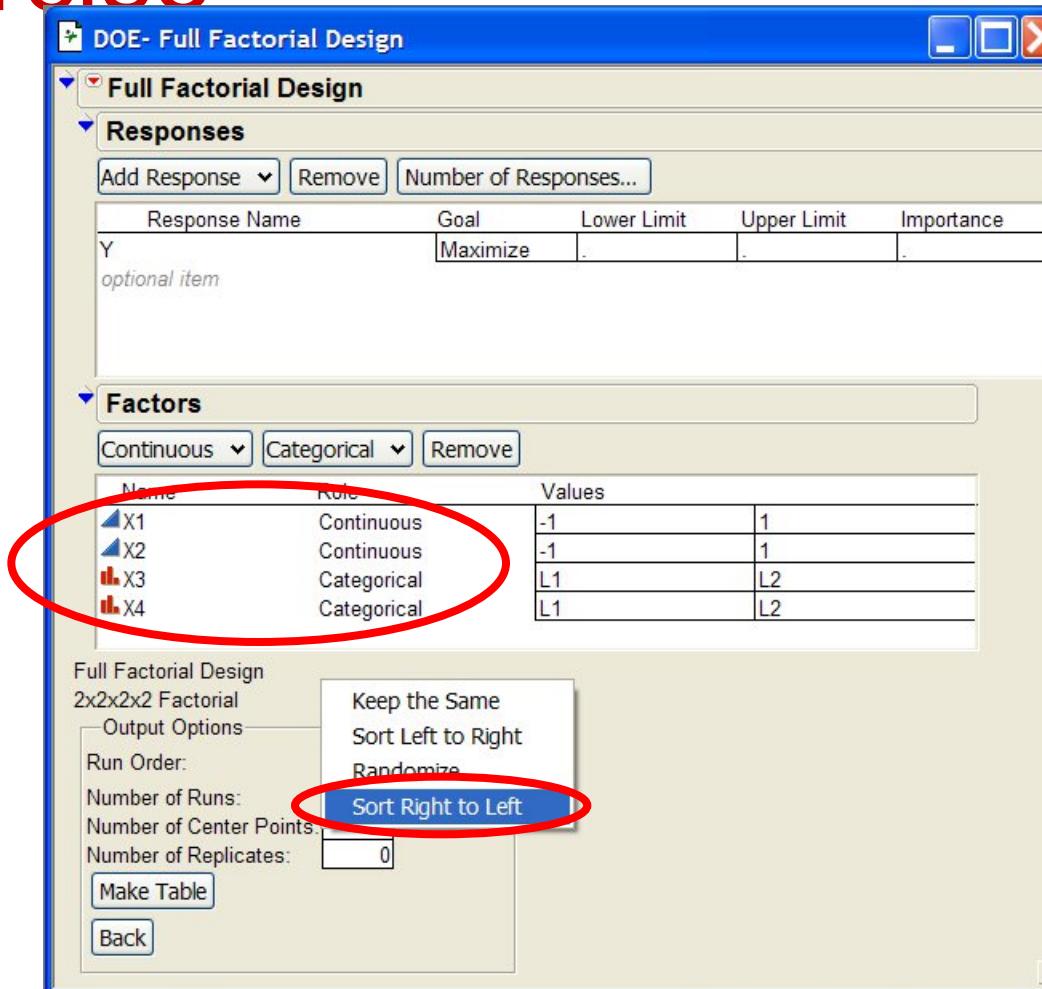
Number of Runs:

Number of Center Points:

Number of Replicates:

Make Table Back

Keep the Same  
Sort Left to Right  
Randomize  
**Sort Right to Left**





# Six Sigma – $2^k$ Factorial Experiments

## Standard Order Of $2^k$ Designs

### Exercise

**2x2x2x2 Factorial**

	Pattern	X1	X2	X3	X4	Y
1	--11	-1	-1	L1	L1	.
2	+−11	1	-1	L1	L1	.
3	−+11	-1	1	L1	L1	.
4	++11	1	1	L1	L1	.
5	--21	-1	-1	L2	L1	.
6	+−21	1	-1	L2	L1	.
7	−+21	-1	1	L2	L1	.
8	++21	1	1	L2	L1	.
9	--12	-1	-1	L1	L2	.
10	+−12	1	-1	L1	L2	.
11	−+12	-1	1	L1	L2	.
12	++12	1	1	L1	L2	.
13	--22	-1	-1	L2	L2	.
14	+−22	1	-1	L2	L2	.
15	−+22	-1	1	L2	L2	.
16	++22	1	1	L2	L2	.

**Columns (6/0)**

- Pattern
- X1 \*
- X2 \*
- X3 \*
- X4 \*
- Y \*

**Rows**

All rows	16
Selected	0
Excluded	0
Hidden	0
Labelled	0

# Exercise: Calculating Main Effects

Step 1: Problem Statement: A process engineer would like to determine the effect of Quench Temperature, Quench Time, and Quench Oil types on the hardness of a steel shaft. Goal: Maximize Hardness.

Step 2: The factors and levels:

- Quench Temp:  $160^{\circ}\text{ C}$  (-1),  $180^{\circ}\text{ C}$  (1)
- Quench Time (sec): 5 (-1), 15 (1)
- Quench Oil Type: Brand A (-1), Brand B (1)

# Exercise: Calculating Main Effects

Step 3: The design matrix with the results column is as follows:

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

This is an example of  $2^K$  Factorial Experiment with only one observation per treatment combination (experimental run).

# Exercise: Calculating Main Effects

- u Step 4: We will now calculate the effects of the experiment by hand. First, look at Temperature.
- u We simply add the yields associated with (-1) and the yields associated with (1) and calculate the average (Sum/4). The “1’s” and “-1’s” in the temperature column are called the “contrast” for the main effect of temperature.



# Exercise: Calculating Main Effects

$$\text{TempMainEffect} = \frac{\overline{x}_{-1} - \overline{x}_{+1}}{\overline{x}_{\emptyset} - \overline{x}_{\emptyset}}$$

$$= \frac{72 + 68 + 83 + 80}{4} - \frac{60 + 54 + 52 + 45}{4}$$

$$= 75.75 - 52.75 = 23$$

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

Total -	-211
Total +	303
Sum	92
Avg Effect	23

The Hardness increases, on average, by 23 points as temperature moves from Low to High (160 to 180).

# Exercise: Calculating Main Effects

- Now use the contrast for Time to calculate the effect Time has on Hardness.

$$\begin{aligned} TimeMainEffect &= \frac{\overline{x}_5 + 68 + 45 + 80}{4} - \frac{\overline{x}_{\emptyset} + 60 + 72 + 52 + 83}{4} \\ &= 61.75 - 66.75 = -5 \end{aligned}$$

As the Time moves from 5 sec to 15 sec, the Hardness drops by 5 points on average.

# Exercise: Calculating Main Effects

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

Total -	-211	-267	
Total +	303	247	
Sum	92	-20	
Avg Effect	23	-5	

# Exercise: Calculating Main Effects

- Next use the contrast for Oil to calculate the effect Oil type has on Hardness.

Temp	Time	Oil	Hardness
-1	-1	-1	60
1	-1	-1	72
-1	1	-1	54
1	1	-1	68
-1	-1	1	52
1	-1	1	83
-1	1	1	45
1	1	1	80

-211	-267
303	247
92	-20
23	-5



# Exercise: Calculating Main Effects

$$\text{OilMainEffect} = \frac{\overline{y}_{++} + \overline{y}_{+-}}{\overline{e}} - \frac{\overline{y}_{-+} + \overline{y}_{--}}{\overline{e}}$$
$$= \frac{\overline{y}_{++} + \overline{y}_{+-}}{\overline{e}} - \frac{\overline{y}_{-+} + \overline{y}_{--}}{\overline{e}} =$$

Interpretation: \_\_\_\_\_

# Exercise: Calculating Interactions

- u We have just finished calculating the Main Effects for this experiment. We've only investigated the independent effects of Temperature, Time and Oil Type. This is similar to conducting three 2-Sample T tests.
- u The benefit of factorial experiments is that they provide the ability to assess “interactions” between factors. “Is there a particular combination of input settings that improve Hardness over and above the singular (main) effects?”



# Exercise: Calculating Interactions

- The interaction contrast is derived by multiplying the columns to be represented.

**Calculate the Interaction Contrasts below.**

Temp(T)	Time (S)	Oil (O)	T*S	T*O	S*O	T*S*O	Hardness
-1	-1	-1					60
1	-1	-1					72
-1	1	-1					54
1	1	-1					68
-1	-1	1					52
1	-1	1					83
-1	1	1					45
1	1	1					80

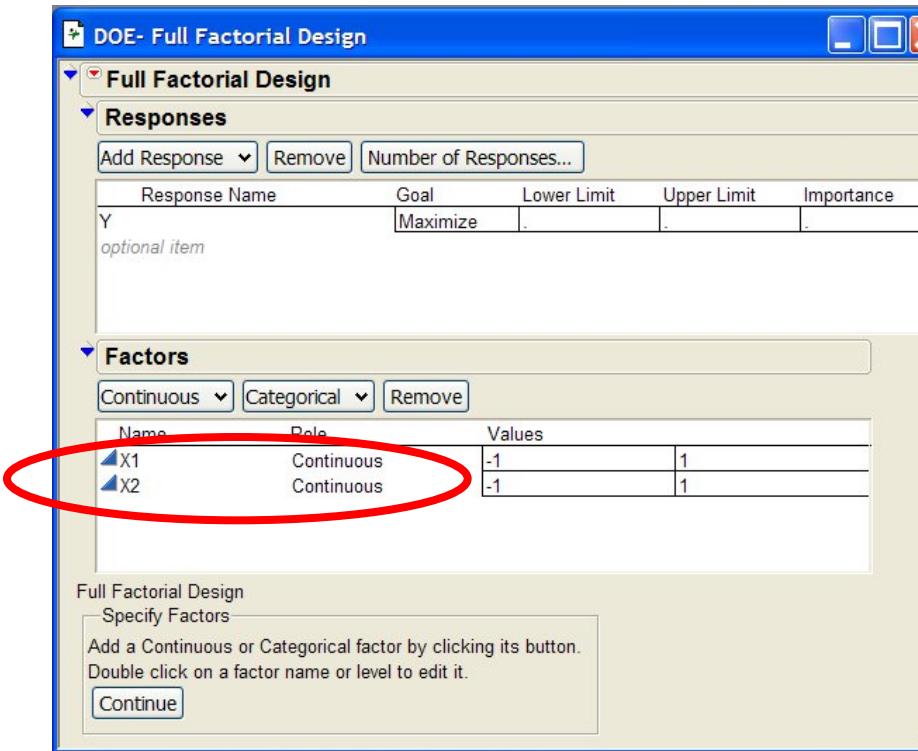
# Exercise: Calculating Interactions

-1	-1	-1	1	1	1	-1	60
1	-1	-1	-1	-1	1	1	72
-1	1	-1	-1	1	-1	1	54
1	1	-1	1	-1	-1	-1	68
-1	-1	1	1	-1	-1	1	52
1	-1	1	-1	1	-1	-1	83
-1	1	1	-1	-1	1	-1	45
1	1	1	1	1	1	1	80
Total -	-211	-267					
Total +	303	247					
Sum	92	-20					
Effect	23	-5	1.5	1.5	10	0	0.5
Estimate	11.5	-2.5	0.75	0.75	5	0	0.25

- Verify the “Contrast” for each interaction using JMP.

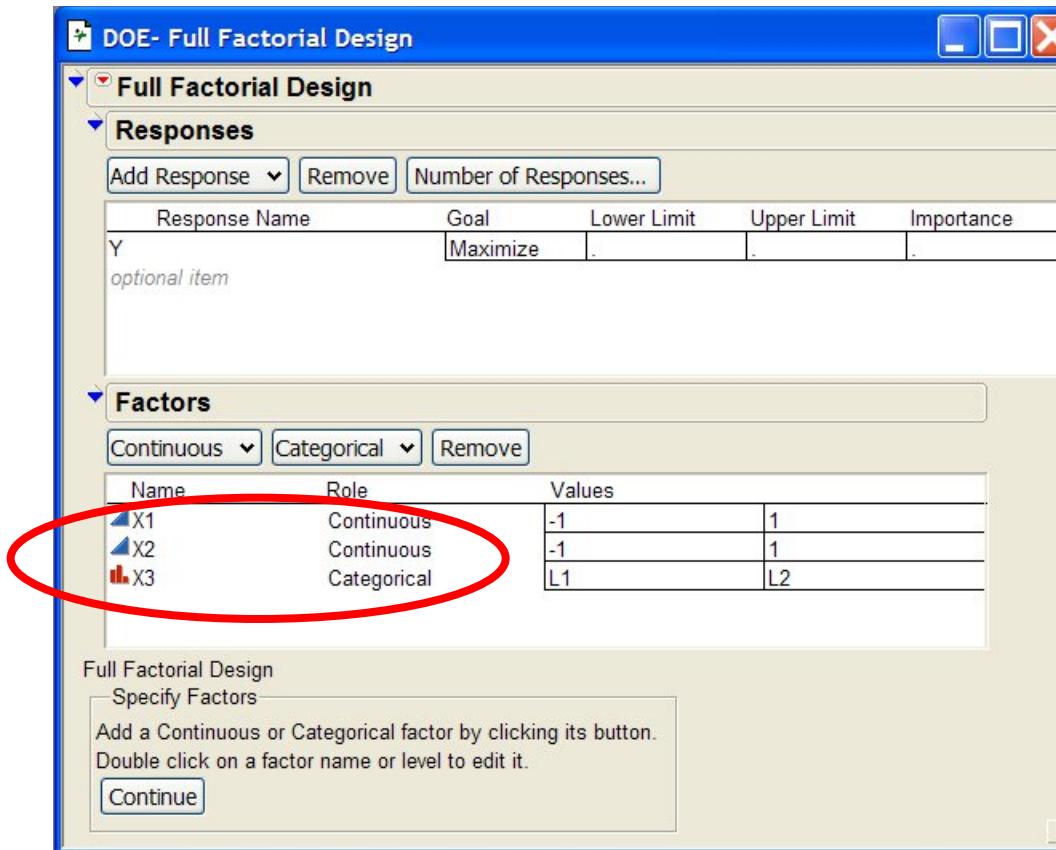
# Design Matrix – Using JMP

- u JMP> DOE > Classical > Full Factorial Design
  - **Continuous>2 Level**
    - Repeat the previous step (**Continuous>2 Level**)



# Design Matrix – Using JMP

- u JMP > DOE > Classical > Full Factorial Design
  - Categorical>2 Level



The screenshot shows the 'DOE- Full Factorial Design' dialog box. In the 'Factors' section, there are three entries:

Name	Role	Values
X1	Continuous	-1 1
X2	Continuous	-1 1
X3	Categorical	L1 L2

A red oval highlights the X1, X2, and X3 entries in the 'Factors' table.

# JMP DOE Design (Cont.)

- u Double-click in the Name Column to change X1, X2 and X3 to Temp, Time and Oil.
  - Quench Temp: 160° C (-1), 180° C (1)
  - Quench Time (sec): 5 (-1), 15 (1)
  - Quench Oil: Brand A (L1), Brand B (L2)
- u Double-click in the Values Column to change the Temp row to 160 and 180, the Time row to 5 and 15, and the Oil row to Brand A and Brand B.

Six Sigma –  $2^K$  Factorial Experiments

## JMP DOE Design (Cont.)

DOE- Full Factorial Design

Full Factorial Design

Responses

Add Response ▾ Remove Number of Responses...

Response Name	Goal	Lower Limit	Upper Limit	Importance
Y optional item	Maximize	.	.	.

Factors

Continuous ▾ Categorical ▾ Remove

Name	Role	Values	
Temp	Continuous	160	180
Time	Continuous	5	15
Oil	Categorical	Brand A	Brand B

Full Factorial Design

Specify Factors

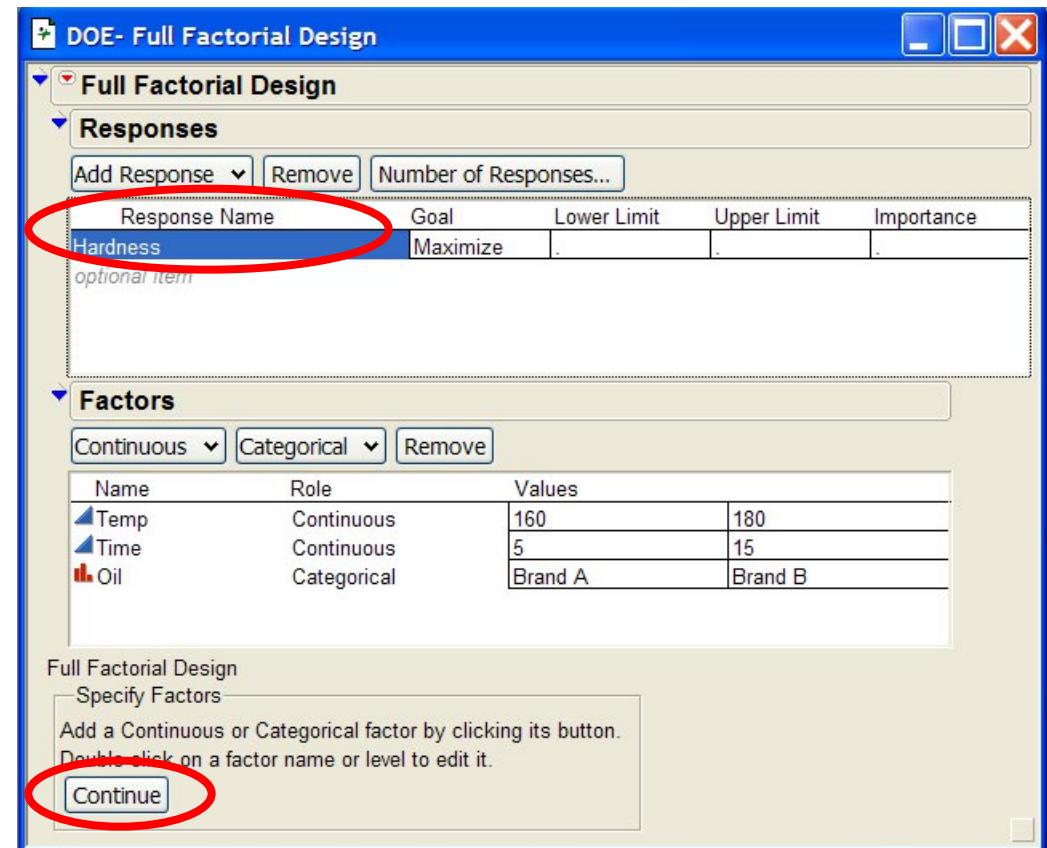
Add a Continuous or Categorical factor by clicking its button.  
Double click on a factor name or level to edit it.

Continue



# Design Matrix – Using JMP

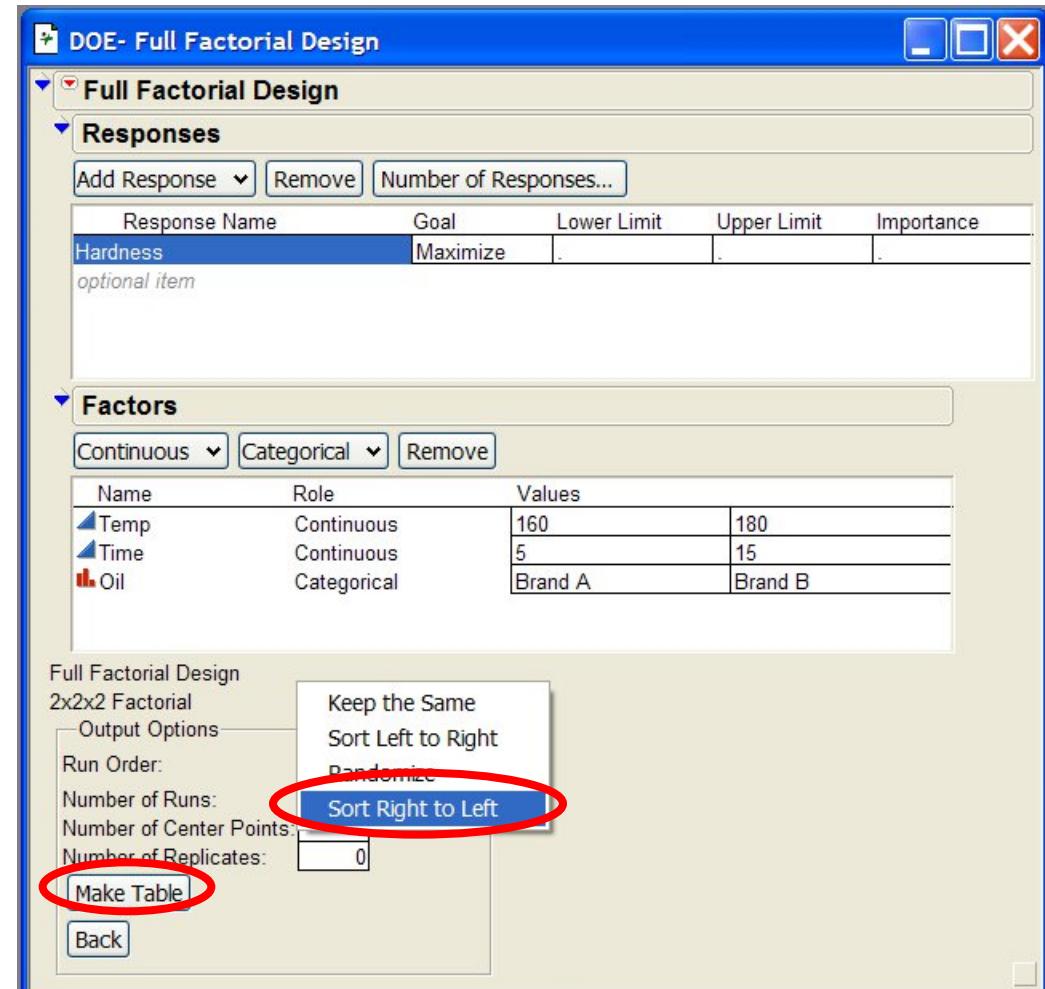
- u Enter the Response column heading.
- u Enter *Hardness* as the Response Name.
- u Select **Continue**.



# Six Sigma – $2^K$ Factorial Experiments

## JMP DOE Design (Cont.)

- u Number of Replicates  
= Number of Additional Replicates
- u For this example, there are no additional replicates.
- u Change **Randomize** to **Sort Right to Left** for teaching purposes.
- u Click **Make Table**.



# JMP DOE Design (Cont.)

- Use the table to enter in the Y-values.

2x2x2 Factorial

	Pattern	Temp	Time	Oil	Hardness
1	--1	160	5	Brand A	60
2	+ -1	180	5	Brand A	72
3	- +1	160	15	Brand A	54
4	++1	180	15	Brand A	68
5	--2	160	5	Brand B	52
6	+ -2	180	5	Brand B	83
7	- +2	160	15	Brand B	45
8	++2	180	15	Brand B	80

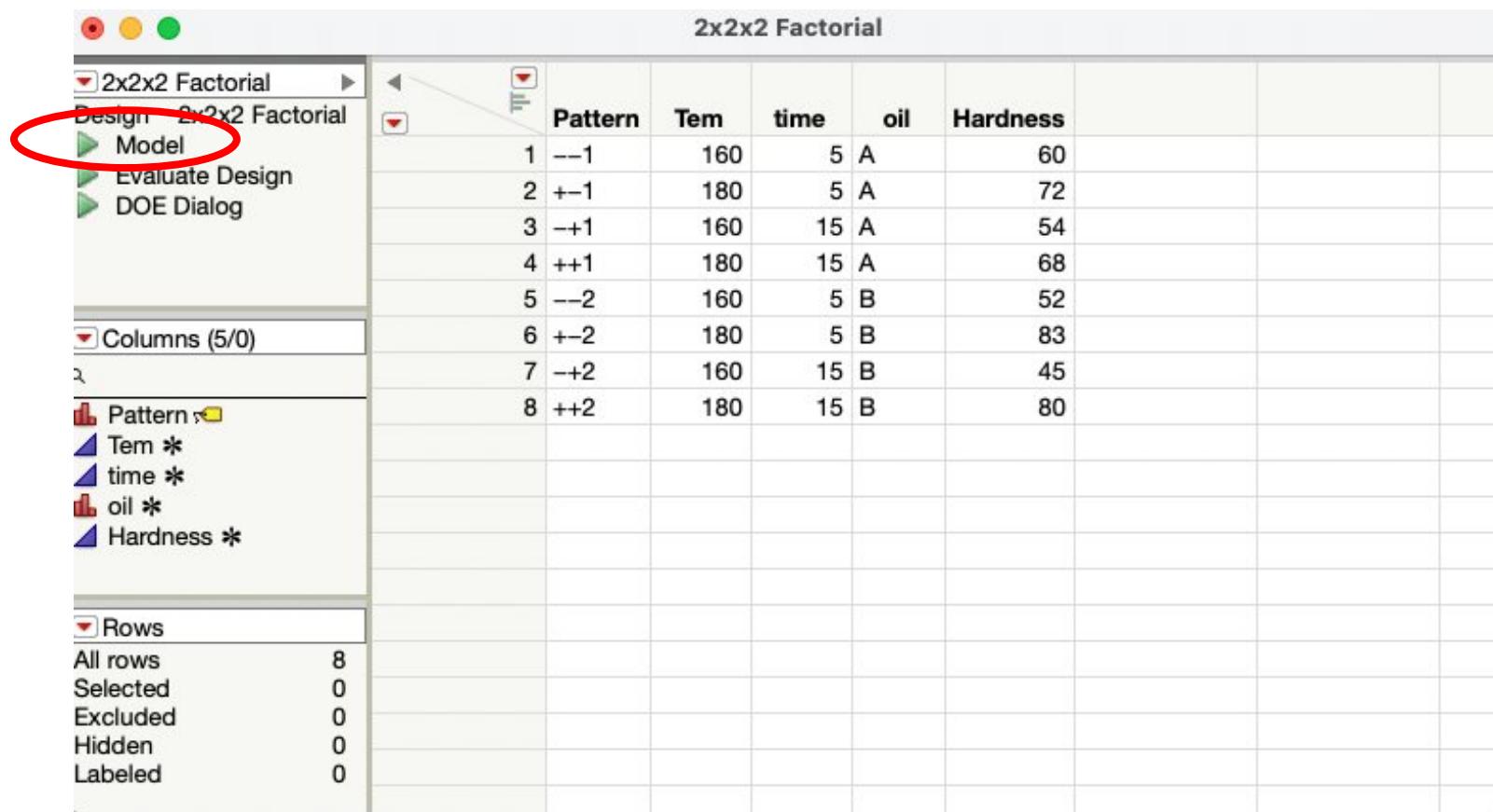
Columns (5/0)  
Pattern  
Temp \*  
Time \*  
Oil \*  
Hardness \*

# Analysis With JMP

- u This experiment only has one observation per treatment combination. Therefore, we can't analyze the "full factorial" using the ANOVA procedure until we learn a few analysis tricks.
- u When there is only one observation per treatment combination, we use the Normal Probability Plot or Pareto Chart to interpret which effects are likely to be significant.
- u If a factor or its interaction is insignificant (the null hypothesis, effect = 0, is true), then we expect to see the effects normally distributed around a mean of zero. Any outlying effect is considered significant.

# Analysis With JMP (Cont.)

Step 4: From the **Model**, select the **Green Triangle**



The screenshot shows the JMP software interface with the title "2x2x2 Factorial". On the left, there is a navigation pane with the following sections:

- Design**: 2x2x2 Factorial (selected)
- Model** (circled in red)
- Evaluate Design**
- DOE Dialog**

Below these are sections for **Columns (5/0)**, **Pattern**, **Tem \***, **time \***, **oil \***, and **Hardness \***. At the bottom, there is a section for **Rows** with counts for All rows (8), Selected (0), Excluded (0), Hidden (0), and Labeled (0).

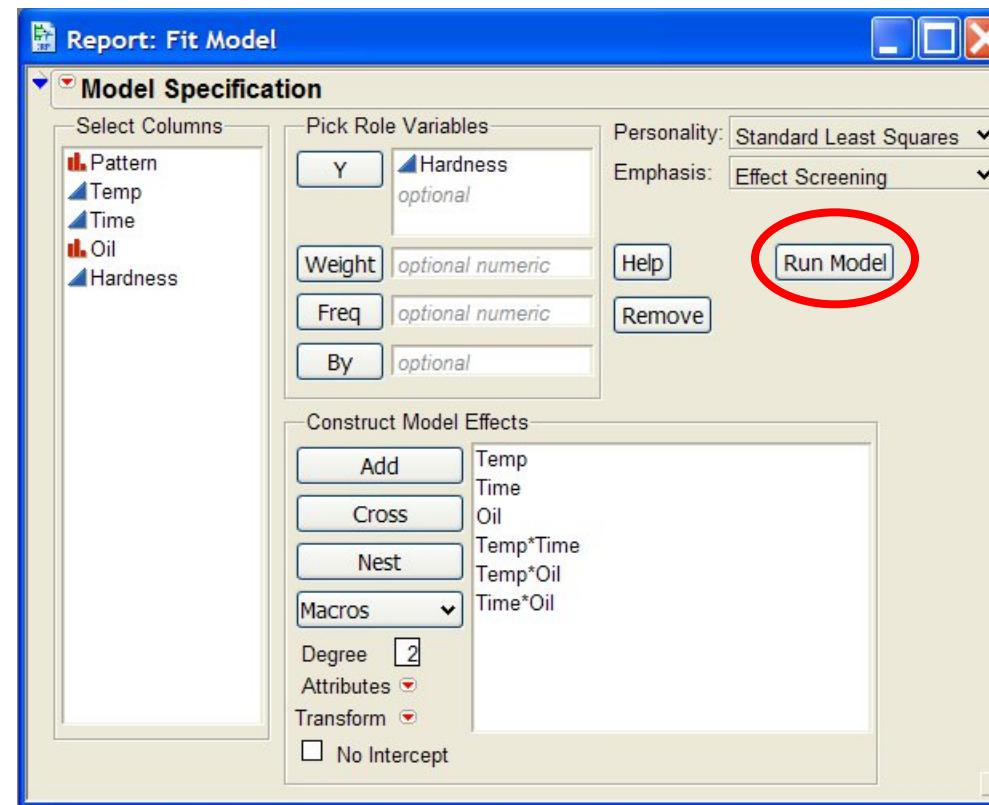
The main table area displays the following data:

Pattern	Tem	time	oil	Hardness
--1	160	5 A		60
+--1	180	5 A		72
--+1	160	15 A		54
++1	180	15 A		68
--2	160	5 B		52
+--2	180	5 B		83
--+2	160	15 B		45
++2	180	15 B		80

Six Sigma –  $2^K$  Factorial Experiments

## JMP Analysis (Cont.)

u Click Run





# Six Sigma – $2^k$ Factorial Experiments

## JMP Analysis (Cont.)

JMP Output (Use Red Triangle under Response)>  
Regression Report>summary of Fit  
Regression Report> Analysis of Variance

Summary of Fit				
RSquare		0.99962		
RSquare Adj		0.997343		
Root Mean Square Error		0.707107		
Mean of Response		64.25		
Observations (or Sum Wgts)		8		

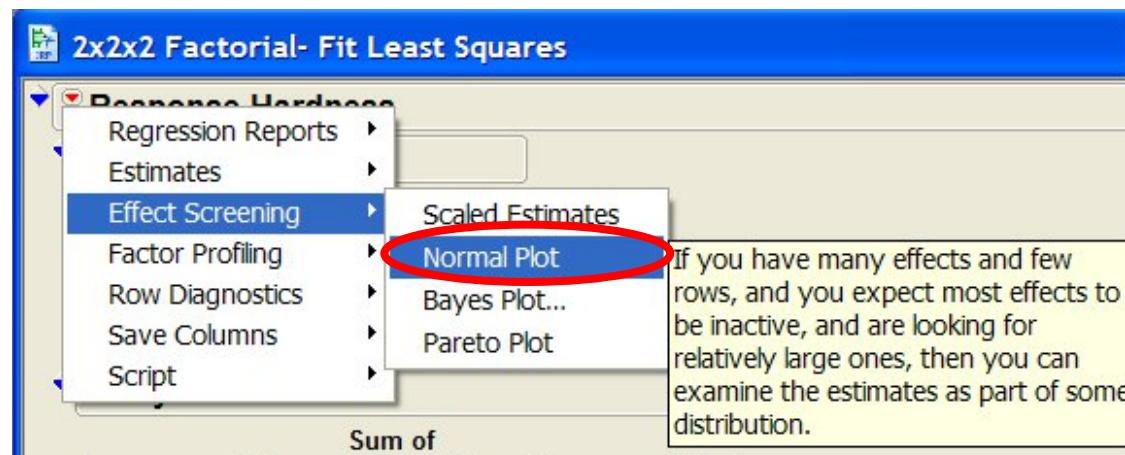
Analysis of Variance				
Source	DF	Sum of Squares		
		Mean Square	F Ratio	Prob > F
Model	6	1317.0000	219.500	439.0000
Error	1	0.5000	0.500	
C. Total	7	1317.5000		0.0365*

Parameter Estimates				
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	64.25	0.25	257.00	0.0025*
Temp(160,180)	11.5	0.25	46.00	0.0138*
Time(5,15)	-2.5	0.25	-10.00	0.0635
Oil[Brand A]	-0.75	0.25	-3.00	0.2048
Temp*Time	0.75	0.25	3.00	0.2048
Temp*Oil[Brand A]	-5	0.25	-20.00	0.0318*
Time*Oil[Brand A]	0	0.25	0.00	1.0000

# JMP Analysis (Cont.)

- u Step 5: Identify the most significant Model Effects or **Estimates** by looking at the Normal Plot and Pareto Plot of the Estimates.



# Normal Plot

- The Normal Plot shows that **Temperature**, **Time**, and the interaction of **Temperature and Oil** are the most significant model estimates.

