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Integrated Production Planning of Orders with Priorities and Due Dates: A Case Study in the Automotive Industry

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Abstract

This paper addresses the integrated production planning problem by developing a mathematical optimization model. A case study using real industrial data from a Renault automotive manufacturing site demonstrates the model's effectiveness in solving automotive production planning challenges. The study showcases the model's ability to integrate and coordinate planned inbound logistics and outbound distribution activities with the factory's production plan, reflecting the complex interdependencies inherent in an industrial environment. The model facilitates the planning of order assemblies with commercial priorities and due dates while also managing inbound and outbound logistics and the emergency shipment of critical components within a short-term planning horizon of three weeks. We implemented our model on a use case using data from a manufacturing plant. The results indicate that the production plan of our model adapts to inbound and outbound operational constraints, improving the adherence to release dates of 35.49% of the manufacturing orders compared to the current industrial tool used by the car manufacturer and reducing the holding cost of components by 80% on the studied horizon. Further, we assess the model's flexibility in accommodating disruptions in the supply chain. We demonstrated savings in money and time needed to find a feasible scenario in our use case. We introduce parameters that capture real-world operational conditions to enhance the model's practical applicability. These operational parameters ensure that the production plan scenario can be implemented.

Keywords:
Production Planning, Automotive Industry, Integrated Production Planning, End to End Supply Chain, Manufacturing systems, Logistics

1. Introduction

Automobile manufacturers are facing multiple challenges. Cars are assembled at manufacturing sites that follow a production plan. The supply chain department often builds the production plan to meet client due dates and respect production capacity constraints and component availability. One of the primary challenges is improving visibility on resource constraints to find the optimal global production plan with the most negligible impact on the supply chain and the minimum operational cost. The supply chain manages procurement from suppliers, inbound and outbound logistics, and distribution plans. Consequently, the subset of required cars for each model/market group within a specific period must not violate resource constraints. However, with constructors offering more personalization possibilities to the clients and recurrent supply chain disruptions, innovative solutions are essential to ensure sustainable production plans.

The supply chain cost per vehicle can reach up to 1000 Euros, and between 200 and 400 Euros are incurred for transport from suppliers to the factory alone (Baller, R. et al. , 2022). Supply chain costs can be much higher during disruptions, such as having a rupture in a component stock due to quality conditions or when the demand exceeds supply. Car manufacturer supply chains are constantly innovating to reduce these costs. Integrating operations within the supply chain by elaborating a production plan that adapts to the supply of components and distribution of product plans has effectively reduced global operational costs in assembly-based industries (Hrabec, D., Hvattum, L. M., & Hoff, A. , 2022).

This research article focuses on constructing a production plan based on commercial demand while respecting the company's upstream and downstream resource constraints through an integration approach. We are motivated by real industrial challenges that Renault is currently overcoming. Following recurrent disruptions since the COVID-19 pandemic and the electronic components crisis, we have collaborated closely with the industry to rethink how the production plan can be more reactive to changes resulting from internal or external disruptions, minimizing its impact on the supply chain and the overall operational costs.

The paper contributes to the existing literature by demonstrating, based on real industrial data, the coordination between the production plan and the inbound/outbound operations, the impact of supply chain disruptions on the production plan, and how to be reactive in response to such disruptions. As far as we know, the model proposed in our paper presents the most thorough integration of inbound and outbound operations with the production plan in the automotive industry. On the industrial level, our work is being tested and running on real use cases. This direct link with the industry allowed us to render our solutions feasible by introducing automotive industry-specific parameters related to supply planning and outbound distribution costs to our model.

The organization of the paper is as follows: First, we review existing production planning systems for supply chains (section 2). Second, we present the problem statement (section 3) and the mathematical model (section 4). The work presented in this article is based on the current challenges facing Renault, with a use case built from a real industrial context and a discussion of the results shown in (section 5) conclusion and the following future work shown in (section 6).

2. Literature Review

This section reviews integrated production planning approaches, identifies existing gaps, and highlights the contributions of our model. Production planning in the automotive industry has been challenging since the start of mass production (Dörmer, J., Günther, H.-O., & Gujjula, R. , 2015); factories do not have unlimited resources to cope with any conceivable mix of cars. Instead, manufacturing facilities are designed to accommodate certain levels of models and features, and production plans must adhere to these limitations (Hindi, K. S., & Ploszajski, G. , 1994).

The production planning problem involves selecting from the large number of car orders presented by the marketing and sales department (order bank) a subset to manufacture in the studied planning horizon, ensuring that overall capacities are maintained (Hindi, K. S., & Ploszajski, G. , 1994; Volling, T., & Spengler, T. S. , 2011; Volling, T. et al. , 2013).

The production planning process involves selecting orders for the assembly lines of production sites and integrating them into the master production plan, which is established during the Sales and Operations Planning (S&OP) process. These orders are typically generated by a separate entity responsible for managing client orders, which is part of the automobile manufacturer's commercial business unit (Volling, T., & Spengler, T. S. , 2011; Volling, T. et al. , 2013). The process is two-fold; first, orders are placed into production periods, where, according to the product being produced, the production period can represent a month, a week, a day or a shift in a day. In the automotive industry, a period usually represents a day. Orders selected must respect constraints such as component availability and platform-related production capacity. Then, for each period, orders are scheduled to form a sequence that respects the operational constraints such as human resources and machine capacities. This production sequence is typically defined over one to three months, where the first six to fourteen days of the horizon are frozen (i.e., it cannot be modified unless for emergency measures). Production dates within the frozen period cannot be changed, but the sequence can be modified based on short-term variations up until a week before production. Sometimes, the factory modifies the sequence on the same day due to disruptions such as a quality issue with a component or a machine breakdown (Jana, P. H. , 2018).

Integrated planning approaches can be more efficient because this enables to take into account multiple constraints directly; thus, fewer plans have to be generated in fewer iteration loops Quetschlich, M., Moetz, A., & Otto, B. (2021); integrated production planning in the automotive industry involves the simultaneous planning of supply, production, and distribution activities. This approach is essential due to supply chains' complexity and interconnected nature. Various studies have demonstrated the significant cost-saving potential of integrated production, inventory, and distribution decisions (Hrabec, D., Hvattum, L. M., & Hoff, A. , 2022).

The concept of an integrated supply chain dates to the 1950s Clark, A. J. (1958) focusing on inventory problems and economic order quantities in multi-echelon systems. Integration focuses on synchronizing different supply chain processes, such as production and distribution. Moons, S. et al. (2017) discusses the operational-level integration of production scheduling and vehicle routing, emphasizing the benefits of combined decision-making in reducing total costs. Similarly, Hrabec, D., Hvattum, L. M., & Hoff, A. (2022) quantifies the potential savings from integrated planning compared to sequential planning, demonstrating substantial cost reductions when production, inventory, and routing decisions are considered jointly.

The integration of inbound logistics with production planning has been explored in the literature (Comelli, M., Gourgand, M., & Lemoine, D. , 2008; Garcia-Sabater, J., Maheut, J., & Marin-Garcia, J. A. , 2013; Hrabec, D., Hvattum, L. M., & Hoff, A. , 2022), but the relationship between production planning and vehicle routing has received less attention. In many cases, these are solved sequentially, leading to sub-optimal solutions. In the context of our study, vehicle routing refers to the coordination of transportation resources to ensure the delivery of required components and finished products within the supply chain on the promised dates. Optimizing both production planning and vehicle routing offers the potential for operational cost savings and improved service levels.

Several studies focus on integrating production-distribution scheduling problems by incorporating vehicle routing decisions into operational planning. For instance, Garcia-Sabater, J., Maheut, J., & Marin-Garcia, J. A. (2013) introduces the *Generic Materials and Operations* (GMOP) model, which integrates inbound logistics with production planning. Their approach uses decision variables based on resource capabilities rather than just materials. Various researchers have highlighted the importance of coordination in optimizing inbound logistics. Baller, R. et al. (2022) discuss how inbound logistics management affects company performance, involving complex operational processes and systems. Additionally, better visibility into demand can reduce transportation distances and minimize backorders. Fabri, M., & Ramalhinho, H. (2024) underscore the impact of inbound logistics on operational efficiency. Production routing problem, which includes the supply of required components, has been further explored by Hein, F., & Almeder, C. (2016), who found that the value of coordination increases as the problem size grows, especially in larger, more complex systems. In the automotive sector, where build-to-order processes are standard, Lim, L. L., Alpan, G., & Penz, B. (2017) emphasize the role of Sales and Operations Planning (S&OP) in aligning production capacities with unpredictable market demand, mainly when sourcing components from distant locations.

Regarding the integration of outbound logistics, production planning is extended with distribution planning, which can be treated as vehicle routing planning. Boudia, M., Louly, M. A. O., & Prins, C. (2006) addresses the production distribution problem, determining the optimal production quantities, distribution schedules, and delivery routes to minimize total costs, including setup, holding, and distribution expenses. Zhang, X. et al. (2024) emphasizes the need for flexible and adaptive scheduling techniques in automotive production, particularly given the industry's susceptibility to disruptions in outbound distributions, which can significantly impact production and lead to the saturation of the manufacturing site's garages.

Despite progress in integrated production planning research, several gaps still need to be addressed. Quetschlich, M., Moetz, A., & Otto, B. (2021) notes the need for mathematical models that address routing products, sub-products, and raw materials through multi-echelon supply networks. Their work presents a generic optimization model to fill this gap, offering a reference for routing complex products in intricate supply chains. Our work is similar to the work of Quetschlich, M., Moetz, A., & Otto, B. (2021), especially in applying the model to a use case; in our model, we consider the outbound distribution operations in addition to the inbound supply and production planning. More models need to incorporate elements such as due dates and priorities in integrated production planning. Our model includes decision variables for emergency shipments and production order priorities, ensuring the feasibility of the production plan under real industrial conditions. We also introduce industry-specific supply and distribution rules and solution metrics to evaluate the quality of the solution and help the users better understand the solution's impact.

Schedule instability in production networks is a significant concern, with limited research from a network perspective, as noted by Moetz, A., Stylos-Duesmann, P., & Otto, B. (2019). In our work, we have identified relevant research that has been instructive to our work and also shows examples of integration in production planning; for integration approaches in the automotive industry, we refer the readers to (Quetschlich, M., Moetz, A., & Otto, B. , 2021; Müllerklein, D., Fontaine, P., & Ostermeier, F. , 2022; Moons, S. et al. , 2017; Zhang, X. et al. , 2024; Sarker, Bhaba R., and Ahmad Diponegoro., 2009). For references on automotive production planning, readers are referred to (Dörmer, J., Günther, H.-O., & Gujjula, R. , 2015; Bolat, A. , 2003; Hindi, K. S., & Ploszajski, G. , 1994). Our model is similar to the mathematical models referenced and presents new contributions:

(1) We present an optimization problem and the mathematical formulation that solves the integrated production planning problem using real industry data in a use case. (2) We present industry-specific supply planning flexibility rules to enhance the model's practical applicability of its solutions. (3) We introduce multiple supply scenarios where components can be shipped in an emergency setting in a short lead-time. However, the expected shipments scheduled can be modified if the supplier's capacities allow. (4) Manufacturing orders have commercial priorities that influence the degree of respect to the promised release dates. We highlight how coordinating decisions can impact overall planning and present opportunities to reduce operational costs by implementing our model. Moreover, our research emphasizes sustainability by integrating environmental and economic objectives. Our model contributes to more sustainable production practices by optimizing logistics, such as minimizing the number of trucks needed for distribution.

3. Problem Description

Multiple models of production planning and integration of planning tasks have been proposed in the literature. Some focus on specific problems or the integration of multiple planning tasks. Our main contribution is to present a mathematical optimization model that facilitates finding a solution to the main production planning problem while ensuring that it minimizes the impact on the supply of components, inventory levels and distribution plans. To contextualize, in this section, we highlight the challenges facing automobile manufacturers and the value of integration, and we introduce the problem in the context of the automotive industry.

Car manufacturers often construct production plans based on the Master Production Schedule (MPS), which is then communicated to supply chain divisions such as the manufacturing site and supply planning teams. The production planning team must respect communicated industrial constraints and work better to understand the complex interdependencies within the supply chain. Despite these efforts, the visibility into the broader impact of the production plan remains fragmented, leading to potential disruptions (due to quality problems or unexpected delays) that can affect production efficiency, inventory levels, operational costs, and client satisfaction. Current industrial tools primarily optimize manufacturing objectives, such as meeting release dates, without integrating other objectives, such as maximizing the utilization of the outbound distribution planned capacities to the destinations (markets) of the manufacturing orders.

Consequently, the production plan needs to be better coordinated with pre- and post-production stages, including inbound/outbound logistics, warehousing, and distribution, resulting in sub-optimal processes. This limited scope that we undertake in our research restricts the car manufacturer's ability to achieve a holistic, end-to-end supply chain perspective, which is crucial for better decision-making, optimized resource allocation, and improved client satisfaction, for example, a production planner can satisfy the objective of adherence to the promised release dates in the factory but generate high costs in the supply of components required for the orders. Unexpected disruptions, such as component damage, delivery delays due to accidents or weather, and truck unavailability, further complicate the situation. The disruptions generate instability in the production plan and all the supply chain components; taking into account disruptions can be done by introducing into the production planning model other components of the supply chain, such as the delays and costs of supply of components from the suppliers and the inventory levels.

Clients submit orders $o \in \mathcal{O}$, where $\forall\ o\ \exists\ d \in \mathcal{D}$ where $\mathcal{D}$ is the set of demands. Each demand d is a quantity of product $i \in \mathcal{P}$ where $\forall i \in \mathcal{P}\ \exists\ g \in \mathcal{G}$ where $\mathcal{G}$ is the set of model groups, such that $(i, g) \in Mg$ where $Mg\ \subseteq \mathcal{P} \times \mathcal{G}$ defines the relationship between products and their model groups. For example, a product i may be in one model group for leather seats and another for panoramic rooftops. Additionally, each product i is assigned a commercial priority w, where delays in release are penalized based on the product's priority. Each product also has a destination q, representing the final delivery city.

To produce each product i, a set of components k is required. Each component k has at least one supplier s with a standard lead time for delivery from the supplier lt and an emergency lead time lte for urgent deliveries. For instance, components that are needed in a shorter timeframe can be ordered with lte. The expected deliveries from the suppliers $Ds_t^{\prime k,s}$ can be adjusted while respecting the supplier's capacity.

The problem consists of selecting a specific number of manufacturing orders for each production period t, which is defined as one day in the planning horizon $\mathcal{T}$. The manufacturing facility has a daily production capacity of Cap orders, alongside resource constraints rc. Other constraints include the availability of components k in inventory and limits on production quantities for each product's model group g. For example, products $i \in$ model group g (e.g., vehicles with right-hand drive for the UK market) cannot exceed 10% of the daily production capacity Cap. Some resource constraints apply daily, while others apply weekly.

Orders are selected based on the availability of components and alignment with both the promised release date (the start of production based on facility capacity) and the expected delivery date (the forecasted date of product delivery to the client, factoring in lead times and processing durations). Orders that cannot be shipped (dispatched) at the same period are stored in the manufacturing site garage, which has a limited capacity Mfp, until transportation is available. If capacity Mfp is reached, orders are dispatched at a higher cost explained in details in section 5.1.

Selecting manufacturing quantities generates costs, including those associated with emergency component supply, holding costs for components, and penalties for producing orders after their promised release dates or delivering them past their expected delivery dates. Another critical factor is the stability of the production plan, as deviations from the initial plan should be avoided. The production plan is calculated daily, with quantities of manufacturing orders initially selected the day before. The notations used in our model are introduced in the following section.

4. Mathematical Formulation

We introduce our mixed-integer linear programming model, which optimizes production plans by coordinating logistics and outbound distribution while minimizing operational costs. These costs include holding costs for produced orders and components, emergency shipment costs, and lateness and setup costs.
To progressively present our model, we follow an iterative approach. Hence, we first introduce the mathematical formulation for the production planning problem PPP (4.1) without the integration of the inbound and outbound operations. We use similar formulations that were introduced previously in the literature. We then introduce the production distribution problem PDP (4.2), where we show how the outbound distribution operations can be integrated with the production planning problem. We then introduce the production routing problem PRP (4.3), where the inbound supply of components is integrated with the production plan. Finally, the mathematical formulation, including sets, indices, parameters, objective function, and constraints, is detailed in (4.4), where we use the same structure to differentiate between the mathematical model components that are linked to production planning, supply of components and inventory levels planning or distribution of productions planning. The main objective of this iterative presentation approach is to facilitate the understanding of our mathematical model.

4.1. The production planning problem

The production planning problem PPP or the order selection problem is the selection from the large number of car orders presented by the marketing and sales department (order bank) a subset to manufacture in the studied planning horizon, ensuring that overall capacities are not violated (Hindi, K. S., & Ploszajski, G. ,

1994) where over a studied horizon $\mathcal{T}$, let P be the number of products we need to select at periods $t \in \mathcal{T}$ where $t1$ denotes the first period in the studied horizon, the set of products $\mathcal{P}$ is indexed with i. $m_{i,t}$ denotes the decision variable of products selected such as,

$$m_{i,t} = \begin{cases} 1 & \text{if product } i \text{ is to be produced at period } t \\ 0 & \text{otherwise} \end{cases}$$

rc_t^k maximum acceptable level of attribute k at period t, attribute k can be, for example, a component, which by transitivity defines the maximum number of products having a quantity of attribute k that can be included in the schedule at period t. The quantity of k is defined with $G_{i,k}$ which describes products i that require attribute k (Gozinto factor, Comelli, M., Gourgand, M., & Lemoine, D. (2008)), with a variable c that defines the quantity needed of the attribute k where $c \in \mathbb{N}$, also called the Bill of Materials BOM, such as,

$$G_{i,k} = \begin{cases} c & \text{if order } i \text{ requires attribute } k \\ 0 & \text{otherwise} \end{cases}$$

Then, with the objective of maximizing the number of orders selected in the studied horizon, the problem can be written as:

$$\min \sum_{i=1}^{P} -m_{i,t}$$

subject to

$$\begin{aligned} \sum_{i=1}^{P} G_{i,k} \times m_{i,t} \leqslant rc_t^k \quad \forall k \\ m_{i,t} \in \{0,1\} \quad \forall I \in P \end{aligned} \tag{1}$$

The main constraint ensures that the capacity limits at the respective periods are not violated such as the consumption capacity of an attribute k per period t. Extending the model to include other operations, such as the distribution of the products or the supply of components, requires adding other decision variables to the production planning problem, as we show in section 4.4.

4.2. The production distribution problem

In addition to the model presented in 4.1. The production distribution problem PDP, is to find an optimal solution of both the production plan and distribution of the orders (Boudia, M., Louly, M. A. O., & Prins, C. , 2006). We introduce Q the number of destinations (markets) the manufacturing orders are are going to be delivered to, the set $\mathcal{Q}$ indexed with q, $c_{q,t}$ denotes the maximum number of products going to market q at period t and Q_i defines the destination q of product i.

Two decision variables are introduced, $gr_{i,t}, dp_{i,t}$, that describe if order i is stored in the manufacturing site garage after production or dispatched at period t respectively. The model then evolves and is additionally subject to:

$$\begin{aligned} m_{i,t1} = gr_{i,t1} + dp_{i,t1}, \quad \forall i \in P \\ m_{i,t} + gr_{i,t-1} = gr_{i,t} + dp_{i,t}, \quad \forall t > t1, i \in P \\ \sum_{i,\text{ where } Q_i=q}^{P} dp_{i,t} \leqslant Dps_{q,t}, \quad \forall q, t \\ m_{i,t}, gr_{i,t}, dp_{i,t} \in \{0,1\}, \quad \forall i \end{aligned} \tag{2}$$

The constraints define a continuity relationship between manufacturing, storing and dispatching an order i at period t and ensuring we do not exceed the available distribution scheduled capacity $Dps_{q,t}$ for each destination q at period t. The model can include other constraints, such as respecting the manufacturing site's garage's maximum capacity, and other objectives, such as minimizing unassigned capacities in distribution. We add these constraints and objectives in our model in (4.4).

4.3. The production routing problem

PRPs involve routing decisions that manage inventory distribution to meet demand while minimizing transportation and holding costs. Common formulations include the Multi-Level Capacitated Lot-Sizing Problem (MLCLSP) or the Capacitated Lot-Sizing Problem (CLSP), where the primary objective is to link the product demand with internal component requirements (Comelli, M., Gourgand, M., & Lemoine, D. , 2008).

In addition to the model discussed in 4.1, we introduce $I_{k,t0}$, representing the initial inventory level of component k. The decision variable $pc_{k,t}$ determines the quantities of component k ordered from the supplier in period t, while lt_k represents the lead time required for the component to be delivered to the manufacturing facility. $pc_{k,t}, lt_k$ can be extended by adding a supplier subscript s, to differentiate between various suppliers as we show later in the main model (section (4.4)). The inventory level of component k at period t is defined by $I_{k,t}$. This extended model incorporates the Gozinto factor to map the consumption of components, ensuring a more accurate representation of the supply-production interface.

$$
\begin{aligned}
I_{k,t1} = I_{k,t0} - \sum_{i=1}^{P}(G_{i,k} \times m_{i,t1}) + pc_{k,t1} \quad \forall k, lt_k = 0 \\
I_{k,t} = I_{k,t-1} - \sum_{i=1}^{P}(G_{i,k} \times x_{i,t}) + pc_{k,(t-lt_k)} \quad \forall k, t - lt_k > t1 \\
pc_{k,t}, I_{k,t} \in \mathbb{N} \quad \forall i
\end{aligned}
\tag{3}
$$

Where the constraints ensure the inventory balance, non-negativity conditions, the model can include other constraints, such as respecting maximum inventory levels, and objectives, such as minimizing the supply of components. We add these constraints in our general model in (4.4).

4.4. Intergated Production Planning Problem

In previous sections, we introduced the formulation of the production planning problem. We discussed extending the model to incorporate supply and distribution planning. These preliminary formulations are a foundation for our main model, the *integrated production planning* model. This model employs mixed-integer linear programming to combine order planning, component supply, and distribution operations. In section (5), we apply our model to an industrial use case using data from the automotive industry. The formulation of our model is described below.

		Sets related to production planning:
$\mathcal{T}$	:	set of planning periods indexed by t. (Cardinality: T), first period : $t1$, index τ represents the weeks, first week : $\tau 1$.
$\mathcal{O}$	:	set of client orders indexed by o. (Cardinality: O) order o is a set of demands where each demand is a quantity of product i with priority w.
$\mathcal{D}$	:	set of demands indexed by d. (Cardinality: D), $\mathcal{D} \subset \mathcal{O}$
$\mathcal{P}$	:	set of products indexed by i. (Cardinality: P)
$\mathcal{G}$	:	set of model groups indexed by g. (Cardinality: G) items of $\mathcal{P}$ are linked to at least one item of $\mathcal{G}$
$\mathcal{W}$	:	set of priorities indexed by w. (Cardinality: W) $w \in [0, 3]$ where "0" is the most prioritised.
$\mathcal{L}$	:	set of assembly lines indexed by l. (Cardinality: L)
		Sets related to supply and components:
$\mathcal{K}$	:	set of components indexed by k. (Cardinality: K)
$\mathcal{S}$	:	set of suppliers indexed by s. (Cardinality: S)
		Sets related to outbound distribution:
$\mathcal{Q}$	:	set of destinations indexed by q. (Cardinality: Q)

Parameters

The parameters introduced in this model can be categorized into input data such as demand, lead-time and constraint-related parameters such as maximum production capacity. Our parameters can be linked to the studied horizon $\mathcal{T}$ by introducing either a daily subscript or a weekly subscript. For example, resource capacities may constrain the production of a model group either on a daily basis (using subscript t) or at an aggregated level over a week of production (using subscript τ).

		Production Order parameters:
$mi_{i,t}$	:	Initial production plan over the same horizon $\mathcal{T}$.
$mi_{i}^{'}$	:	A binary indicator for demands that are cancelled.
$DD_{o,d}$	:	Promised release date t of demand d of order o.
$DD_{o,d}^{'}$	:	Expected delivery date t of demand d of order o to the destination.
$Pr_{o,d}$	:	Priority w of demand d of order o.
		Product parameters:
O_i	:	Order o linked to the product i.
D_i	:	Demand d linked to the product i.
Q_i	:	Destination q of the product i.
$\mathcal{G}_k^i$	:	Components needed to build product i, Bill of Material (BOM).
$Mg_{i,g}$	:	A binary indicator if product i is in model group g.
		Components (parts) parameters:
Inv^k	:	Initial Inventory of components at $t = t1$.
$Sq_{k,s}$	:	Supply quantity of a shipment of component k from supplier s.
Sc_k^s	:	Supply cost of component k from supplier s.
Sec_k^s	:	Emergency supply cost of component k from supplier s.
H_k	:	Holding cost of component k per period t.
		Manufacturing Site Parameters:
Cap_t	:	Production capacity of the factory in period t.
$Cap_t^{'g}$	:	Production capacity of the factory in period t for group g.
$KL_{g,t}$	:	Production flag, 1 means we could produce products from the model group g at period t.
rc_t^k	:	Maximum consumption capacity of component k at period t.
rcw_τ^g	:	Weekly maximum production capacity for group g in week τ.
		Logistics Parameters:
$Ds_t^{k,s}$	:	Quantities of component k **confirmed** shipments from supplier s to be delivered in period t.
$Ds_t^{'k,s}$	:	Quantities of component k **expected** shipments from supplier s to be delivered in period t.
M_k	:	Maximum capacity of inventory for each part k
m_k	:	Minimum inventory to keep at the end of each period t.
$lt_{k,s}$	:	Delivery lead time of component k from supplier s in classic shipment.
$lte_{k,s}$	:	Delivery lead time of component k from supplier s in urgent shipment.
HG	:	Holding product cost in the manufacturing site garage.
Mfp	:	Maximum inventory of Finished Products
Dps_t^q	:	Dispatch schedule for destination q in period t
ltd^q	:	Delivery lead-time of order going to destination q.

		Due date and production cost parameters:
$\alpha_{o,d}(t, DD_{o,d})$	:	Earliness cost of demand d of order o.
$\beta_{o,d}(t, DD_{o,d})$	:	Tardiness cost of demand d of order o.
$\gamma_{o,d}(t)$	:	Additional production costs of demand d of order o.
$\mathcal{L}(t)^{o,d}$	:	Cost of producing the demand d of order o in the period t.

Decision variables

The decision variables in this model can be categorized into manufacturing-related variables, such as deciding which manufacturing order to produce at which period, and logistics-related variables, such as how many completed (produced) orders to hold in stock (to keep in the manufacturing site garages) and when to dispatch them. Finally, supply-related variables include which part to purchase as an emergency, a classic shipment, or modifying an existing expected delivery at which period while respecting delivery lead-time. Emergency shipments have shorter lead times and are more expensive. Then, we could choose between modifying an expected shipment or ordering a new classic shipment while respecting the constraints, which is changing the quantities expected to be delivered from suppliers.

		Production Planning:
ls_i	:	Manufacturing orders not positioned in the studied horizon $\mathcal{T}$.
$m_{i,t}$	:	Binary variable if product i is produced at period t.
$gr_{i,t}$	:	Binary variable if product i is stored in the manufacturing site's garages at period t.
		Supply Planning:
$cp_{k,t}$	:	Quantity of component k consumed over a period t.
$I_{k,t}$	:	Inventory level of each component k at the end of period t.
$pc^s_{k,t}$	:	Quantity of component k to purchase from supplier s at period t.
$pm^s_{k,t}$	:	Quantity of component k to add to a current delivery from supplier s at period t.
$pe^s_{k,t}$	:	Quantity of component k to purchase from supplier s at period t in emergency mode.
		Distribution Planning:
$dp_{i,t}$	:	Binary variable if product i is dispatched at period t.
$pd^q_{i,t}$	:	Binary variable if product i is dispatched at period t but the capacity planned Dps^q_t is exceeded.

Objective Function

The objective function (F) expressed in (4) is designed to minimize the total cost, which is the sum of four specific cost components. These costs, detailed in Table (4.4), are categorized as follows:
(1) Inventory costs such as the holding cost of components and finished products (HC); (2) The cost of purchasing components over the planning horizon (emergency shipments) and emergency dispatch (ESC) (3) the total cost of manufacturing a product, including tardiness: reduce the cost of tardiness and earliness in the studied horizon (MC) (4) negative (unassigned) capacities costs (UC). The minimization is done over a defined planning horizon.

HC	Holding cost of finished products in the manufacturing site's garages and components in the manufacturing site's inventories.
$\boxed{\phi}$	$\sum_{i\in\mathcal{P}}\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}(Mg_{i,g}\times \mathrm{HG}\times \mathrm{gr}_{i,t})+\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}(H_k\times \mathrm{I}_{k,t})$
MC	Manufacturing cost includes the lateness, earliness, setup, and instability costs.
$\boxed{\epsilon}$	$\sum_{i\in\mathcal{P}}(\mathcal{L}(t)^i\times m_{i,t}+\Theta_{o,d}\times dp_{i,t})+\sum_{i\in\mathcal{P}} stability(t^{'}\times mi_{i,t'},t\times m_{i,t})\ \forall\ t\in\mathcal{T},\ t^{'}\in\mathcal{T}$, where $\mathcal{L}(t)$ is expressed in equation (5) and $stability(t^{'},t)$ is expressed in equation (8).
UC	Unassigned capacity cost over the studied horizon.
$\boxed{\nu,\mu}$	$\nu\times(Cap_t-\sum_{i\in\mathcal{P}}m_{i,t})+\mu\times\sum_{i\in\mathcal{P}}Dps_t^{Q_i}-dp_{i,t},\ \forall t\ in\ \mathcal{T}$
ESC	Emergency supply cost of component k purchased from supplier s and emergency dispatch cost of product i at period t during the studied horizon.
$\boxed{\psi}$	$\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}pe^s_{k,t}\times Sec^s_k+\sum_{i\in\mathcal{P}}pd_{i,t}$

Table 1: Costs considered, coefficients and the expressions are in the row following each cost.

$$Minimize\ F=\phi\times HC+\epsilon\times\sum_{t\in\mathcal{T}}\sum_{t'\in\mathcal{T}}MC+UC+\psi\times ESC \tag{4}$$

Total cost of production:

$$\mathcal{L}(t)^i = \begin{cases} \alpha_{o,d}(t, DD_{o,d}) - \\ \alpha_{o,d}(t+1, DD_{o,d}) + \gamma_{o,d}(t) & \text{if } DD_{o,d} > t, \\ Pr_{o,d}^{-1} \times [\beta_{o,d}(t+1, DD_{o,d}) - \\ \beta_{o,d}(t, DD_{o,d})] + \gamma_{o,d}(t) & \text{if } DD_{o,d} < t, \\ \gamma_{o,d}(t) & \text{if } DD_{o,d} = t \end{cases} \tag{5}$$

We pre-set the coefficients: δ as the penalty factor for delay (1), ω as the weight factor for deviation from the ideal date (0.2), and θ as the weight factor for favouring smaller desired dates (0.01). The tardiness cost function $\beta_{o,d}(t, DD_{o,d})$ is monotonically increasing with the number of late days, represented by $t - DD_{o,d}$. This tardiness function is quadratic, with the coefficients δ, ω, and θ. Similarly, the earliness cost function $\alpha_{o,d}(t, DD_{o,d})$ is analogous to the tardiness cost function but with smaller coefficients, signifying a higher penalty for lateness compared to earliness. Moreover, the order priority coefficient is multiplied by the lateness cost function, indicating that higher priority orders incur higher penalties for lateness. o, d are defined using O_i and D_i, the choice of a quadratic penalty cost function to avoid single products having extreme delays.

The setup cost function $\gamma_{o,d}(t)$ is introduced for specific product types, such as *"bi-tone"* or *"opening rooftop"*, requiring additional manoeuvres. The notation in equation (5) is previously presented in the work of Bolat, A. (2003), and the priority is introduced into the cost:

$$\beta_{o,d}(t, DD_{o,d}) = [\delta \times (\omega \times (t - DD_{o,d}) + \theta \times (DD_{o,d} - t1))]^2 \quad \text{if } DD_{o,d} < t, \tag{6}$$

Delivery Lateness:

$DD_{o,d}$ describes the promised release date, $DD'_{o,d}$ describes the promised delivery date to the destination, which is a prognostic date calculated using the average delays of manufacturing and transportation. In equation (7), we introduce the delivery lateness cost function; the function is used to calculate the cost of lateness that will be later minimized in the objective function (4). The function is used later to calculate a satisfaction indicator that measures the percentage of orders delivered within the grace period of the expected delivery, which is up to one week of a delay from the promised delivery date, as shown in the results in a table (3), $\Theta_{o,d}$ is only calculated if the dispatch period t is beyond the promised delivery date $DD^{'}_{o,d}$ otherwise the cost is null. In the industry, however, it is not recommended to have early deliveries because clients with big orders usually do not have enough space to store the products; it is recommended to not exceed two weeks before the promised delivery date, according to our interviews with the experts.

$$\Theta_{o,d}\left(t, DD^{'}_{o,d}\right) = [\Gamma \times (t + ltd^{Q_i} - DD^{'}_{o,d})] \quad \text{if } DD^{'}_{o,d} < t, \tag{7}$$

Film stability:

The objective is to generate a production plan that aligns with the initial production plan. In general, the initial production plan is the production plan generated by the supply planners last time, if supply planner calculate the production plan on a daily basis, then the initial production plan is the production plan of the day before. This is critical in the industry as manufacturing sites adjust human resources and machinery based on specific model mixes for each production period. Maintaining stability ensures that these adjustments are not impacted. Stability is measured by comparing the initial production plan with the updated solution, accounting for cancelled demands. The stability indicator calculates the total difference between the initial date of assembly product i in $mi_{i,t}$ and its new date in $m_{i,t}$, as shown in equation (8) calculated for each product i.

$$stability(t^{'}, t) = a \times [(b \times |t^{'} - t| + c \times |t^{'} - t1|)]^2 \tag{8}$$

Where a, b, c are coefficients of predefined values currently used by the automobile manufacturer, t is the period where the demand d of order o is produced, and $t^{'}$ is the period where the demand has been previously positioned and $t1$ is the first period in the studied horizon. The indicator does not consider a new demand added or a previous demand cancelled. We multiply the indicator with the coefficient a, which represents the percentage of demands changed or cancelled using a simple formulation $(\sum_{i\in\mathcal{P}} mi^{'}_{i})/P$; it allows the production planner to understand better the impact of order changes on production film stability.

Constraints

In an industrial context, the manufacturing site is subject to specific constraints, such as the manufacturing site's maximum production capacity, the inventory of components and the garage where produced orders are stored at maximum capacity. The model is also subject to continuity (balance) constraints to balance between the components consumed and the orders produced, the orders stored in the garage, and the orders dispatched to the clients. The objective function is subject to the following constraints:

Continuity constraints ensure the correct balance between inventory levels, consumption and supply of components, production of orders, storage in the garages, and dispatch (distribution). These are the main constraints ensuring the correct balance between multiple operations in the supply chain. Equations (9) and (10) initialise the balance at $'t1'$ and equations (11) and (12) ensure balance from $'t2'$ until end of the horizon.

– *Produced orders initial balance:*

$$\mathrm{m}_{i,t1} = \mathrm{dp}_{i,t1} + \mathrm{pd}_{i,t1} + \mathrm{gr}_{i,t1}, \forall i \in \mathcal{P} \tag{9}$$

– *Components supply initial inventory balance:*

$$\begin{aligned}\mathrm{I}_{k,t1} = \mathrm{ds}_{t1}^{k,s} \times Sq_{k,s} + \mathrm{Inv}^{k} - \mathrm{cp}_{k,t1} + \mathrm{pm}_{k,t1} \times Sq_{k,s}& \\ (\mathrm{lt}_{k,s} == 0 : + \,\mathrm{pc}_{k,t1} \times Sq_{k,s}, +0)& \\ \forall k \in \mathcal{K}, s \in \mathcal{S}&\end{aligned} \tag{10}$$

– *Produced orders balance:*

$$\begin{aligned}\mathrm{dp}_{i,t} + \mathrm{gr}_{i,t} + \mathrm{pd}_{i,t} = \mathrm{gr}_{i,t-1} + \mathrm{m}_{i,t},& \\ \forall i \in \mathcal{P}, t \in \mathcal{T} \;\&\; t > t1&\end{aligned} \tag{11}$$

– *Components supply inventory balance*:

$$\begin{aligned}\mathrm{I}_{k,t} = \mathrm{I}_{k,t-1} - \mathrm{cp}_{k,t} + (t - \mathrm{lt}_{k,s} >= 0 : \mathrm{pc}_{k,t-lt_{k,s}} \times Sq_{k,s}, +0)& \\ + (\mathrm{ds}_{t}^{k} \times Sq_{k,s}) + \mathrm{pm}_{k,t} \times \mathcal{Q}_{k}& \\ \forall k \in \mathcal{K}, s \in \mathcal{S}, t \in \mathcal{T} \;\&\; t > t1&\end{aligned} \tag{12}$$

Production Planning Related Constraints:
– *Production capacity constraint:* the maximum production capacity in the manufacturing site. Production should also not exceed the resource constraint, whereas

the daily and weekly resource constraints are translated in Cap'$_{g,t}$, rc_t^k and rcw_τ^g.

$$\sum_{i \in \mathcal{P}} m_{i,t} \leq \text{Cap}_t,\ \forall t \in \mathcal{T}, \tag{13}$$

$$\sum_{i \in \mathcal{P}} (m_{i,t} \times Mg_{i,g}) \leq \text{Cap}^{'}_{g,t} \times KL_{g,t}, \forall t \in \mathcal{T}, i \in \mathcal{P}, g \in \mathcal{G} \tag{14}$$

$$\sum_{i \in \mathcal{P}} G_{i,k} \times m_{i,t} \leqslant rc_t^k \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \tag{15}$$

$$\sum_{i \in \mathcal{P}} (m_{i,\tau} \times Mg_{i,g}) \leq rcw_\tau^g, \forall \tau \in \mathcal{T} \tag{16}$$

– *Demand constraint:* orders produced should not exceed the demand.

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} m_{i,t} \leq P \tag{17}$$

– *Lost Sales constraint:* balance products between produced or a lost sale; the sum of both should equal the total demand.

$$\text{ls}_i + \sum_{t \in \mathcal{T}} \text{m}_{i,t} = \mathcal{P} \tag{18}$$

– *Produced orders maximum capacity constraint:* to respect the maximum number of produced orders we could keep in the garage of the manufacturing site.

$$\sum_{i \in \mathcal{P}} gr_{i,t} \leq \text{Mfp},\ \forall t \in \mathcal{T} \tag{19}$$

Supply Related Constraints
– *Inventory capacity constraint:*

$$I_{k,t} \leq M_k,\ \forall k \in \mathcal{K}, t \in \mathcal{T} \tag{20}$$

$$I_{k,t} \geq m_k,\ \forall k \in \mathcal{K}, t \in \mathcal{T} \tag{21}$$

– *Consume constraint:* updating components consumed at each period.

$$\text{cp}_{k,t} = \sum_{i \in \mathcal{P}} \text{m}_{i,t} \times \mathcal{G}_k^i\ \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{22}$$

– *Modify purchase:* The purchase can only be modified if there is an expected delivery scheduled for the same period t. Let R be a sufficiently large number.

$$pm^{s}_{k,t} \leq R \times Ds^{'k,s}_{t} \ \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{23}$$
$$pm^{s}_{k,t} \geq 0 \ \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{24}$$

Outbound Distribution constraint:
– *Scheduled dispatch capacity:* Respect the available capacities to distribute produced orders.

$$\sum_{i,\text{ where } Q_i=q}^{P} dp_{i,t} \leqslant Dps^{q}_{t}, \quad \forall q \ \in \mathcal{Q},\ t \in \mathcal{T} \tag{25}$$

Purchase dispatch:

Only purchase dispatch if the planned distribution capacity exceeds the orders we need to dispatch at period t. This is done by respecting the dispatch capacity constraint and adding cost to $pd_{i,t}$ while including it in the *Produced orders initial balance* constraint.

The mathematical model's objective function aims to find an optimal solution while respecting global industrial constraints. However, applying the model to a complex system makes interpreting the solution challenging. To address this, we incorporated aggregated indicators to help users of the model better understand the impact of the solution on orders and introduced industry-specific rules to meet the use case's requirements. In section 5, we present indicators related to the service level. We introduce supply and distribution rules, which provide a more accurate representation of the costs associated with these operations.

5. Application: a use case in a Renault manufacturing plant

In this section, we will apply our mathematical optimization model to an actual use case in the automotive industry in a Renault manufacturing plant that is facing disruption in the outbound distribution plans, limiting their capacity to deliver the orders as previously planned. Coordination between manufacturing and distribution teams is unsystematic, relying on experience, and is both personnel intensive. A case such as the one we will present can take multiple days until the planners can find a feasible solution that cannot be thoroughly evaluated and compared across different scenarios. This results in sub-optimal and slow responses to disruptions in the supply chain. The objective of this case study is to show a real example of the use of the model using real industrial data and show the value of integration in production planning. This use case is introduced in two parts. In the first part, we will compare the production plan resulting from our model vs. the production plan of the industrial tool. In the second part, we will introduce a scenario with a disruption in the supply chain impacting the outbound distribution operations, and we will present the production plan our model created and compare it with the production plan of the industrial tool currently used by the production planning teams at Renault.

Classically, the planning is sequential, and each team in the supply chain is the client of another team. In the conventional organizational structure, the main client of the manufacturing site is the commercial team. The commercial team manages the order bank and communicates with the client on a prognostic delivery date based on the expected delays in assembling and delivering the product. The prognostic date evolves to be a promised delivery date. The commercial team participates in the S&OP meetings to better grasp the manufacturing capacity. Following that, conventionally, the manufacturing site is the primary client of the supply and distribution divisions, where they are designed to adapt to feed the assembly line with the necessary components to build vehicles and deliver assembled orders to their respective markets. We can translate this by showing that the information flow between the factory (the production planning team) and the supply and distribution teams is uni-directional, see figure (1) which represents the information and physical flows in the supply chain. We mean by the "1-way information flow", the situation in which the information about the factory is visible to the supply and distribution planners but not otherwise.

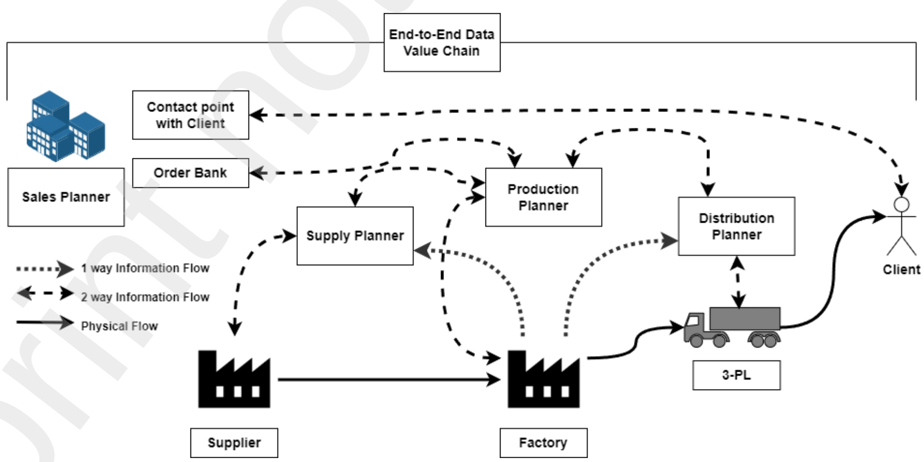

Figure 1: Supply chain network information and physical flows

The primary objective of the production planning team is to respect the planned release dates of the manufacturing orders and adhere to the industrial constraints within the manufacturing site. Additionally, any changes to the production plan must not cause a deficit in the inventory levels of components or impact the promised release dates of other manufacturing orders. However, the objectives are manufacturing-facility specific, which increases the risk of constructing a production plan that has a high impact on the different supply chain components, such as the inventory levels (the holding of components) and the utilization of the outbound distribution capacities. The distribution planning team aim to optimize the available capacity by filling the spaces efficiently, avoiding extra costs when dispatch volumes exceed capacity, minimizing the inventory in the distribution centre (the garages of the finished products), and ensuring the expected delivery dates to clients are met. Our model aligns these objectives, where both teams can work cohesively to improve overall efficiency and client satisfaction. A production plan that is optimal for the factory and adheres perfectly to the release dates is only sometimes the optimal one if we consider the overall supply chain objectives.

5.1. Data collection, industry-specific cost structures and solution indicators

This section introduces how we collected the data for the use case and industry-specific rules and policies. In our work, we introduce two rules specific to the automotive industry: one related to the outbound distribution operations costs and one related to the supply decisions.

Data Collection

The data is collected from different industrial tools where the tables are uploaded into the company's data cloud; depending on the tool, the data is exported on a defined frequency (i.e. daily, weekly). Figure (A.7) displays in detail the four main blocs of data sources we collected data from a high-level perspective. We categorized data into four main blocs; (1) Manufacturing orders data, such as the release dates and the commercial priority, (2) the industrial capacities data, such as the manufacturing capacity and the human resources, (3) the inbound supply and supplier's data, such as the expected shipments and the suppliers' capacity, (4) the outbound distribution data, such as the lead times of deliver, costs and capacities.

The data is collected using a framework we developed during our work using *SQL* queries; the data is then filtered and validated using *Python* to create instances with which we feed our optimisation model to complete data for the parameters such as the initial inventories and data for the constraints such as the manufacturing capacity in the studied horizon.

Distribution cost over capacity

The contracts with third-party logistics (3-PL) companies are usually over a long term (from 1 to 3 years) where the client (the car manufacturer) defines a volume of vehicles to be transported from one factory to a certain market or a destination; the volumes are based on the long term master production scheduling (from 1 to 2 years), contractually, certain flexibility can be implemented depending on the agreement between both parties (e.g. for a route factory/destination (OEM, q) the volume of cars that can be transported is 1000 +/- 10%) the flexibility percentage can range from 0 to 15%, usually in maritime transport flexibility is lower than road transport. Exceeding this flexibility, the distribution team negotiates additional spots with their suppliers at an additional cost, and in some cases, they subcontract with another 3-PL supplier if the first supplier cannot offer more spaces; in the latter case, the additional cost is more consequential.

In this context, we introduce the following distribution cost structure: for each week τ of the studied horizon with length T, the volumes (number of products) we should distribute v, we define a general transportation cost function $DC_{\tau,q}(v)$ a nondecreasing and piecewise linear in the total volume transported $v \geq 0$. Let V_q denote the volumes set for each destination q representing a different cost level; for simplification, we denote the intervals with numbers $\{V_q 1, ..., V_q n\}$, knowing that the first volume level is within the contractual flexibility. The cost function $DC_{\tau,q}(v)$ is formulated as follows:

$$DC_{o,d}(v) = \begin{cases} 0 & \text{if } v < V_q 1, \\ b1 & \text{if } v \in [V_q 1; V_q 2), \\ ... & \\ bn & \text{if } v \in [V_q n - 1; V_q n], \\ \infty & \text{if } v > V_q n \end{cases} \tag{26}$$

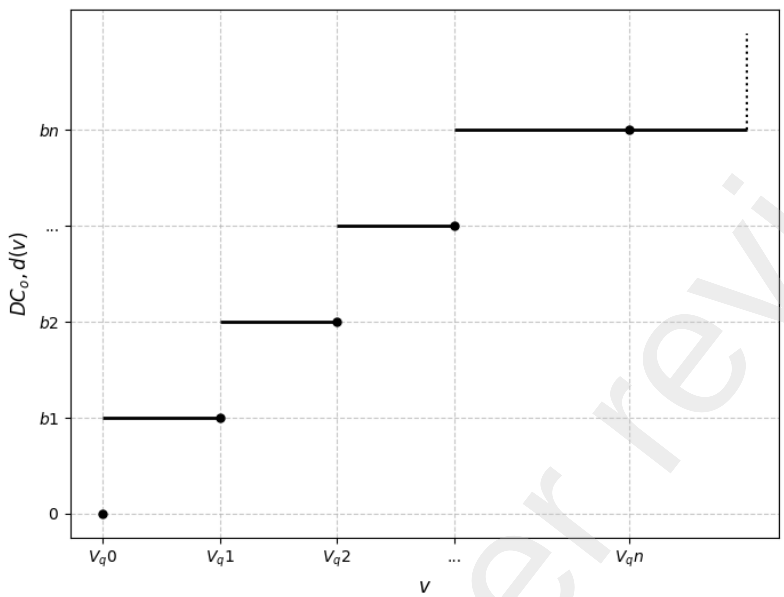

Figure 2: Step-wise cost structure

Supply rules in the horizon

We compare the ordered quantities against the supplier's available capacity to manage supplier capacity constraints in our production planning model. For instance, if a supplier can provide 2000 parts in an industrial week Sx, this quantity is initially assumed to be available. However, if an order is already confirmed, the supplier may allocate this capacity to another client, limiting the supply planner from increasing the order quantity from 400 to 2000 parts on short notice.

Based on interviews with supply planners, we developed a rule to handle this scenario more accurately than relying solely on supplier capacity, which is often erroneous for short-term planning. The constraints and parameters we established, while rough estimates, provide a more realistic approach. These parameters are indicative and open to discussion.

The rules developed are as follows (compared with the date of the order confirmation): *Horizon 1* (0 to 3 weeks): No flexibility versus the programmed quantity (frozen). *Horizon 2* (4 to 6 weeks): Maximum flexibility of 4% versus the programmed quantity within capacity limits. *Horizon 3* (7 to 9 weeks): Maximum flexibility of 6% versus the programmed quantity within capacity limits. *Horizon 4* (3 months and beyond): The full capacity can be considered. These rules enable more realistic planning by accounting for the supplier's potential capacity reallocation and providing a structured framework for flexibility based on the planning horizon, see figure (3).

We can notice that there is a difference between distribution and component suppliers. The first has allocated space that we can consume and usually paid upfront by contracts that endure from 1 to 3 years; for the latter, the manufacturing facility is more dynamic and has a production line that produces for many clients (car manufacturers), so maybe the distribution supply should be more flexible and modify the politics of contracting. Still, vehicle distribution is limited in capacity in markets such as Europe. This is why car manufacturers prefer maintaining stable long-term contracts instead of short-term ones and ensure client satisfaction.

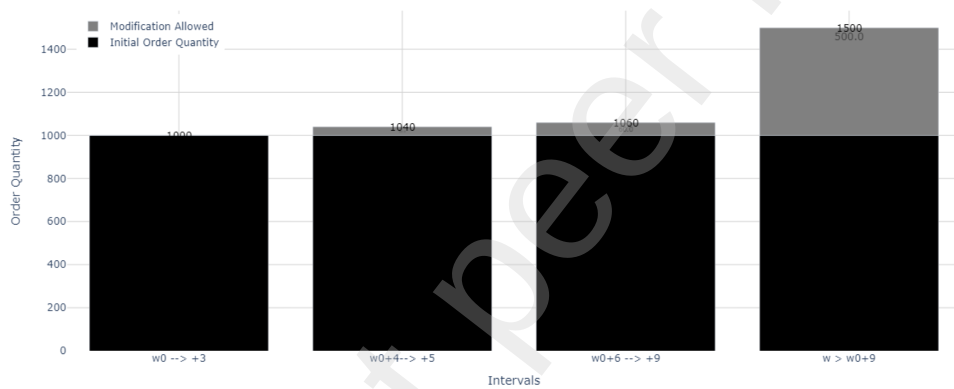

Figure 3: An example of supply quantity modification rule

Solution Quality Indicators

The model aims to minimize the holding cost of finished products, purchased materials, and lateness-associated costs. However, not all production plans (scenarios) are equal. To evaluate and compare different production plans, we introduce the following indicators. These indicators allow users to better understand the solution provided by the optimization model and its impact on the orders, such as the impact of the new production plan concerning the promised release dates of each order.

Service Level

We calculate the percentage of stockouts compared to the total demand for the service level indicators. This indicator translates the percentage of demand not satisfied. By designating the quantity out of stock on the reference at the end of the day, the calculation of this indicator is illustrated by the following equations:

$$SL1 = \frac{\sum_{i \in \mathcal{P}} \sum_{t \in \mathcal{T}} dp_{i,t}}{P} \tag{27}$$

Equation (27) shows the global service level, which is the ratio of demand fulfilled over the total number of products to build; a demand is fulfilled when the order is dispatched.

$$SL2(o) = \frac{\sum_{i \in \mathcal{P}} \sum_{t \in \mathcal{T}} dp_{i,t}}{\sum_{i \in \mathcal{P}, O_i = o} i} \qquad \forall o \in \mathcal{O} \tag{28}$$

Equation (28) shows the service level of each order. This allows the production planner to evaluate if the service level impact affects a specific group of orders, and the model allows the user to evaluate the service level over each demand simply by replacing O_i by D_i.

The indicators provide production planners with insights into the solution. For instance, they can identify how significantly a particular disruption affects specific orders, allowing them to respond effectively. In our application, these indicators are secondary and not included in the objective function for (1) to avoid overcomplicating the objective function and (2) due to the complexity of quantifying these indicators. The lack of visibility on the volume of each order makes it challenging to generalize the objective function, as the impact varies greatly depending on whether an order consists of 1000 manufacturing units or just 10. Client satisfaction is also calculated and the delivery date should be within a certain time window that does not exceed two weeks before the promised delivery date and one week after that date.

5.2. Experiment Setting and Results

In this study, we presented the industrial context, and the data collected spans over the period from the industrial week ($S8$) to the industrial week ($S10$) of 2024; the data was collected during the industrial week ($S5$) the reason is twofold: (1) to be able to study the horizon before entering into the frozen period (between 6 and 14 days) which is the period in the industry where we cannot change the production plan except for disruptions within the manufacturing site such as a disruption in the human resources due to social movements or a technical incident, (2) this allows us to have flexibility over the supply quantities ordered (see section 5.1).

In our experiment, we first compare the *end-to-end* production plan of our model with the industrial tool's production plan (Sc1). Secondly, we show how the production plan can adapt to a distribution constraint while minimizing the impact on the supply chain components (Sc2) and compare the outcome with the industrial tool's production plan. In both tests, we compare plans over the following criteria: (a) supply of components cost, (b) production plan stability, (c) the holding cost of components and, the holding cost of assembled vehicles, (d) the lateness impact on vehicles that were rescheduled.

The program is coded in Python and runs on a laptop with an AMD Ryzen 3 PRO 5450U with Radeon Graphics (2.60 GHz) in 50 seconds based on an average of 10 runs. We use Gurobi ® to solve integer and linear programs. In total, 8251 manufacturing orders were planned over a horizon of three weeks; the problem is finding an integrated supply, production, and transportation plan that fulfils all manufacturing orders to minimize inventory holding and transportation costs while minimizing additional operational costs related to emergency shipments and distributions and respecting client satisfaction by respecting the promised delivery and release dates.

In the 8251 manufacturing orders planned for assembly in a manufacturing site that has a daily production capacity of Cap_t of 650 manufacturing orders, 902 manufacturing orders are delivered to a certain market (destination) and have planned release dates in the industrial week $S10$, henceforward called *Target Population*, in our first study, we run our model using our end to end data framework (fig A.7) where we compare the production plan of the model with the production plan generated by the current industrial tool (see figure 4 and 6) where the target population is shown in black and other manufacturing orders are shown in grey.

In the data we collected, the industrial tool's production plan is visible, however, the tool doesn't show the costs of the plan so in order to compare the industrial tool's production plan with our model's we introduced the same dates of production that were planned in the industrial tool as a constraint to the model. We did so by implementing a hard constraint where $m_{i,t} == mi_{i,t}, \forall i \in \mathcal{P}, t \in \mathcal{T}$. This constraint allowed us to evaluate the performance of the industrial tool's production plan on the same objective costs that our current model is minimizing. The results of this experiment are presented in the sections below.

5.2.1. Comparing our model's production plan with the industrial tool production plan

In the first part, we test our model on an instance at a Renault manufacturing site. To highlight the differences between both plans, we will focus on the target population that is going to a specific destination. Later on, the same population will be analysed in the disruptive event.

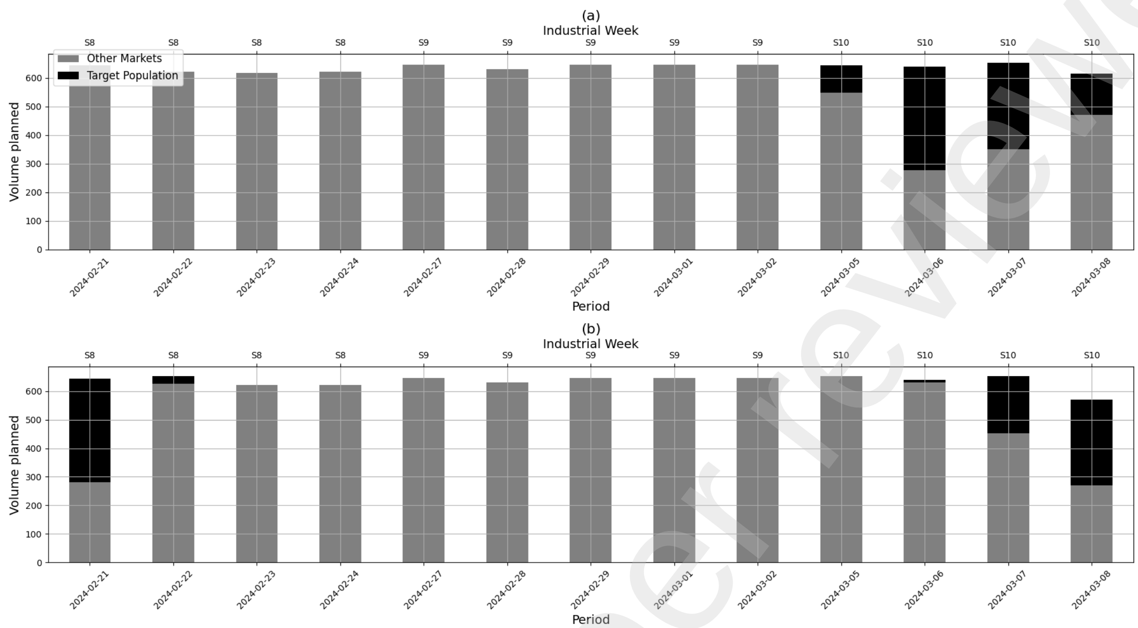

Figure 4: (a) the production plan using the current industrial tool at Renault, (b) the production plan using our model

We first analyse the production plans from the target population's perspective. We can see that there is a significant shift in the dates planned for the target population where 510 manufacturing orders are planned to be released on the industrial week $S10$ and the remaining 392 manufacturing orders are planned to be released on the industrial week $S8$, our analysis shows that there were no additional costs linked to buying more spaces for the distribution capacities, however, we notice a significant reduction in the holding of components with a reduction of 80.1%, this change of dates increased the instability. In total, 5088 manufacturing orders changed the planned release dates compared with the industrial tool's production plan, displaying a larger spectrum of lateness that varies between 0 to 5 days with 35.44% of the manufacturing orders have better adherence to the promised release dates. We also noticed in the production plan that the volume of orders in the last period $(08 - 03 - 24)$ is slightly lower in our model because the industrial tool has an additional objective to smoothen the volumes over the horizon, which we decided to eliminate. After all, it contradicted the objective of minimizing the unassigned production capacities we introduced in our model.

This change in the production plan better adapts to the objective of reduction of the inventory levels which has a direct impact on the holding cost. In terms of supply of components, only an increase of orders from suppliers of 2% is generated. No emergency supply of components was required for both production plans, the average lateness was increased by 0.7 days which impacted the adherence to the release dates, and the client satisfaction dropped from 97.84% to 95.64% which is not a significant drop but shows how small compromises in other objectives (in our case in the adherence to the release dates and the client satisfaction) can serve the overall supply chain operational costs, so by reducing the holding of components.

5.2.2. Introducing a disruptive event impacting the planned outbound distribution capacities

In October 2023, the outbound distribution team informed the production planning team of limited space on a maritime transport scheduled for the 10^{th} industrial week of 2024 (S10, first week of March). They requested that only 200 vehicles destined for a specific market be released during this week, as any additional orders would incur high holding costs in either the manufacturing site's or port's garages. The production planning team had initially scheduled the release of 902 vehicles in $S10$, but with the transport disruption, only 200 could be shipped. Proceeding with the original plan would leave 702 vehicles waiting in the port garage if they leave the garage of the manufacturing site, generating holding costs.

Both teams should ensure that this change will not impact the supply planning; The manufacturing facility and its suppliers are linked directly. In turn, component suppliers are indirectly connected by unique client orders and the associated Bill of Materials (BOM). Consequently, the manufacturing site's weekly production schedule changes impact the material requirements planning and other suppliers' associated quoted call orders. This interconnectedness means initiating countermeasures can lead to cascading disturbances in the supply chain, see figure (1).

If any operational costs are generated (e.g. emergency shipments of some components), they should be much less if we compare it with storing the assembled products in the port. In normal settings, the objective is to minimize inventory, but according to industry experts, having excess volumes due to unconsumed components is tolerated during supply chain disruptions, as it is preferable to not having sufficient quantities of the necessary components. The advantage of our study is that we have the real industry costs of holding, the supply, and the emergency supply of components. The only cost we couldn't calculate is client satisfaction. Still, we know that the promised delivery date can be exceeded by a defined number of days, so we implemented that into our formulation (see 5.1). Figure (5) displays two configurations: on the left, we present how the production plan adapts to the distribution plan and holding costs while presenting the orders to the studied destination with the flag with zigzag lines and other markets with the flag with straight lines. On the right side, the condition is to keep 200 vehicles in this industrial week (Sb), and we show how the production plan reacts accordingly. The figure reflects this change by the number of vehicles with straight lines and zigzag lines.

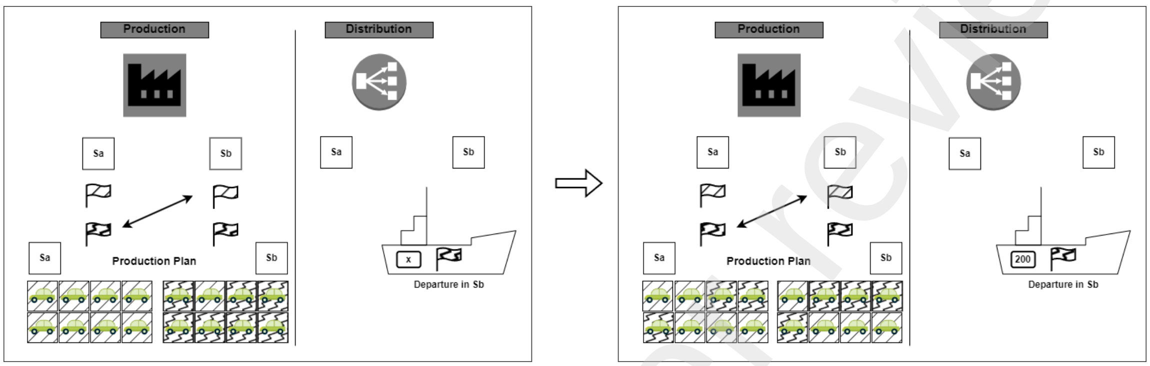

Figure 5: Use Case Description

Our model found an optimal solution by selecting 200 orders out of the 902 orders in the industrial week $S10$ instead of the 902 initially selected. The only metric we could compare was the average lateness compared with the promised release dates, where the average lateness in our model for the 8251 manufacturing orders is 1.817 days, which is a decrease of the adherence to the release dates compared with the current industrial tool by 44.96%.

Our optimization model succeeded in only selecting 200 vehicles from the target population, which resulted in positioning the rest (702 orders) in earlier weeks, as we can see in figure (6). This is done by changing the data in $Dps^q_\tau = 200$, and $pd^q_{i,t} = 0$ at $t \in \tau = S10$ and $q =$ destination of Target Population. The other 702 were in the industrial week $S9$ which is logical since the objective is to maintain stability and respect the initial promised release dates. This change satisfies the objectives of the distribution team but comes at a cost. We also notice the choice of producing the orders of the target population in the last production day of the industrial week $S10$, to reduce the holding cost of the assembled orders.

The most significant impact of introducing this disruption is the increase of the cost of supply of components of 87.41% and a decrease of the cost of the modification of programmed shipments of components of 30.24%, this explains how a change in the production plan can impact supply decisions. In further analysis, the model found the optimal solution by suggesting new commands from suppliers since the expected shipment dates do not align with the change in the production plan, these new command had an impact on the expected deliveries later where we reduced the quantities and resulted in few modifications in the expected deliveries. Table (2) displays the data of supply and consumption of a component that was chosen randomly to represent the impact of the production film on the inbound operations. Client satisfaction is also impacted with a decrease of 19.8%, which is logical since 702 orders for other markets (destinations) are pushed to the industrial week $S10$, which increased the average lateness of 44.96%. Finally, the stability of the production plan is impacted where 5477 manufacturing orders changed the planned release dates compared with the industrial tool's production plan.

We could compare the case with only 200 vehicles from the target population in industrial week $S10$ to the case where the current production planning tool used by the team placed 902 vehicles if we introduce some assumptions on the operations. The distribution team's request was not fulfilled. Two potential costs arise: (a) in the best-case setting, we assume there are 702 available spaces in the garage, and the additional cost would be holding these vehicles in the manufacturing site's garage since they cannot be transported to the port; (b) in the worst-case setting, there is no capacity to hold these vehicles in the garage, and they must be dispatched to the port to wait for shipment. The additional cost of not coordinating with the distribution team while using the current industrial production planning tool lies between these extremes that we calculated : $[30,221;37,231]$ € and the details of our cost estimation are in Appendix B. In addition to the objectives linked to the inbound supply, we further analyse the following indicators that can be categorized into aggregated indicators and indicators on the component level. For the first category, we look into the inventory levels at each period in the studied horizon, the total cost of holding, supply, emergency operations. We also analyse inventory levels that are close to the security stock level. On the component level, we look into the consumption, the modifications made on expected deliveries and the supplier information. Table (2) shows an excerpt of the data for a component. The values have been modified to preserve industrial confidentiality.

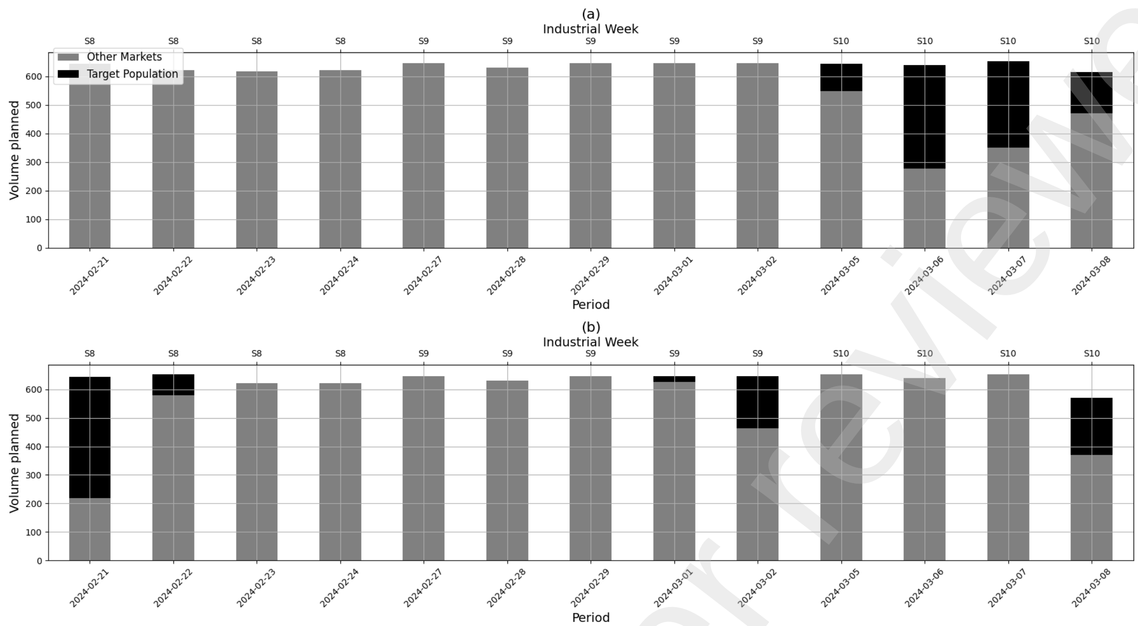

Figure 6: (a) the production plan using the current industrial tool at Renault, (b) the production plan using our model

Reference code		MOT-6954R													
Description		Engine													
Initial Inventory		357													
Supplier Code		SE-297													
Supplier Name		Engine Supplier													
Lead Time		1													
Supplier Cap. ST		50													
Supplier Cap. LT		150													
Cost		900													
LT Emergency		1													
Cost Emergency		500+900													
Expected Confirmed Deliveries															
Dates		20/02	-	-	-	-	-	-	-	-	-	-	-	-	-
Quantity		143	-	-	-	-	-	-	-	-	-	-	-	-	-
Expected Provisional Deliveries															
Dates		-	-	-	-	-	-	-	-	-	-	05/03	06/03	07/03	08/03
Quantity		-	-	-	-	-	-	-	-	-	-	40	40	40	40
Dates		20/02	21/02	22/02	23/02	24/02	27/02	28/02	29/02	01/03	02/03	05/03	06/03	07/03	08/03
Cons.	Sc1	0	427	73	0	0	0	0	0	0	0	42	237	411	70
	Sc2	0	427	73	0	0	0	0	6	7	234	17	176	250	70
Supply	Sc1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Sc2	0	0	0	0	0	0	6	0	234	0	0	0	0	0
Supply M.	Sc1	0	0	0	0	0	0	0	0	0	0	42	237	411	70
	Sc2	0	0	0	0	0	0	0	0	7	0	17	176	250	70

Table 2: An excerpt of the supply data on a component level

Table (2) displays multiple information about the components consumed in the assembly of the orders. In this example, we present the engine *MOT-6954R*, where we have visibility on the supplier's long-term and short-term capacity, where the long term supply capacity is for orders that are for dates in the horizon 6 months from the date the order was confirmed, and the short-term is for other orders below 6 months. This is done while respecting the industry-specific rules presented in (5.1), lead time of the engine is one day since the supplier of the engine is an OEM, the lead time is small, the cost of one unit is calculated in euros and the emergency cost is added to the normal cost if the component is ordered in the emergency mode. The emergency lead time is much shorter than the normal lead time for other components since the components in normal scenarios are delivered with maritime transport. The component is delivered in an emergency in an air transport. For each component, we have visibility of expected deliveries, where some are confirmed where we cannot modify the quantity, and other deliveries are provisional, where we can modify the quantity until it is confirmed from the supplier side.

For the outbound stream, we are capable of calculating the following aggregated indicators: the total cost of outbound distribution and distance travelled, the consumption of available trucks and the percentage of unused capacity, the total number of vehicles to dispatch to their respective markets, and client satisfaction. On the vehicle level, we have the following information: the distribution cost, the truck identifier, the expected arrival date, and the lateness compared to the promised arrival date. Table (3) shows an excerpt of the data where the values have been modified for confidentiality.

Order ID	MO-5004T
Planned Prod.	2024/02/21
Holding in garage	0
Dispatch date	2024-02-21
Earliest arrival date	2024-02-27
Latest arrival date	2024-03-03
Expected arrival date	2024-03-08
Satisfaction flag	1
Travel Distance (km)	730
Transport mode	Truck
Transport supplier (3-PL)	Transport Company
Transport Identifier	SU-50017
Destination center	CE-0017
Arrival city	Cachoeirinha
Country	Brazil
Commercial priority	3

Table 3: An excerpt of the outbound distribution data for each manufacturing order

Table (3) displays information linked to the outbound distribution operations; the dispatch plan is based on availability constraints of transport modes headed to a certain destination that the (3-PL) supplier provides. Our model currently considers that outbound distribution occurs in a single step. Client satisfaction is measured by comparing the expected arrival date to the client and the calculated arrival date.

This model, however, has some limitations. For instance, even though we can access a rich amount of data, some operations remain partially visible to the model, especially in the outbound distribution part. The respective planners should first verify the constructed supply and distribution plans. The costs we have in the data are nominal costs, and in situations of emergency, the planners usually negotiate with the suppliers, which may result in lower costs than the ones calculated. The model suggests component orders based on the supply plan's expected delivery date, which is subject to change. Finally, the suppliers must confirm their commitment to the production plan, and the 3-PL sometimes change their schedule, which may impact the initially suggested distribution plan.

The production planners then assess the new scenario by comparing the additional operational costs with the holding costs of vehicles at the port. If the increase in operational costs is significantly lower than the holding costs, it is advantageous to implement the change. If a scenario is implemented, they have to ensure its feasibility with the stakeholders, such as the component suppliers and the 3-PL providers, so a minor modification to the scenario can be expected. For the first time, this scenario provides the supply chain team with a quantitative and prescriptive tool that facilitates informed decision-making, optimizing the performance of various components within the supply chain in Renault.

6. Conclusion

In this article, we developed a model for optimizing the automotive industry's production planning and supply chain management. The model demonstrates a capacity to adapt to changes in the whole supply chain, aiming to minimize operational costs. Through our case study, we addressed critical questions such as whether the distribution capacity could handle the full volume of deliveries to the port and the associated additional costs if it could not. We also explored whether the supply chain could meet all requirements and, if not, what the additional costs would be. Additionally, we assessed the potential impact of delays on other vehicles and their impact on client satisfaction.

The work presented here stands out due to its realistic approach, providing insights that align closely with industry scenarios. By comparing our production plans with outcomes from other data instances, we ensured that our solutions reflected feasible and practical situations. Industry experts further validated this scenario, reviewing our findings and confirming that the proposed model offers valuable, prescriptive guidance. The integration should be seen from a process lens, a key contribution of integrated planning would be a faster response to disruptions in the supply chain, process bottlenecks should be identified in research.

For future research, one potential direction could be to extend the studied horizon to a longer one to match the current output of industrial tools in the market. To achieve this, researchers could explore alternative mathematical modelling approaches to maintain acceptable execution times, such as aggregating the production plan by weeks instead of days or narrowing the focus to critical components. The industry moves at a fast pace to find the best business solutions to better satisfy their clients; models should be able to adapt to these changes and offer a level of parametrization, for example, during our work with the commercial priority, the process engineering department updated the priority levels to better match the current markets and opportunities, these changes in policies should be seamlessly integrated into the optimization frameworks. Additionally, future studies may benefit from incorporating a broader range of use cases from different industrial settings.

Appendix A. Data Diagram of all data sources presented in the use case

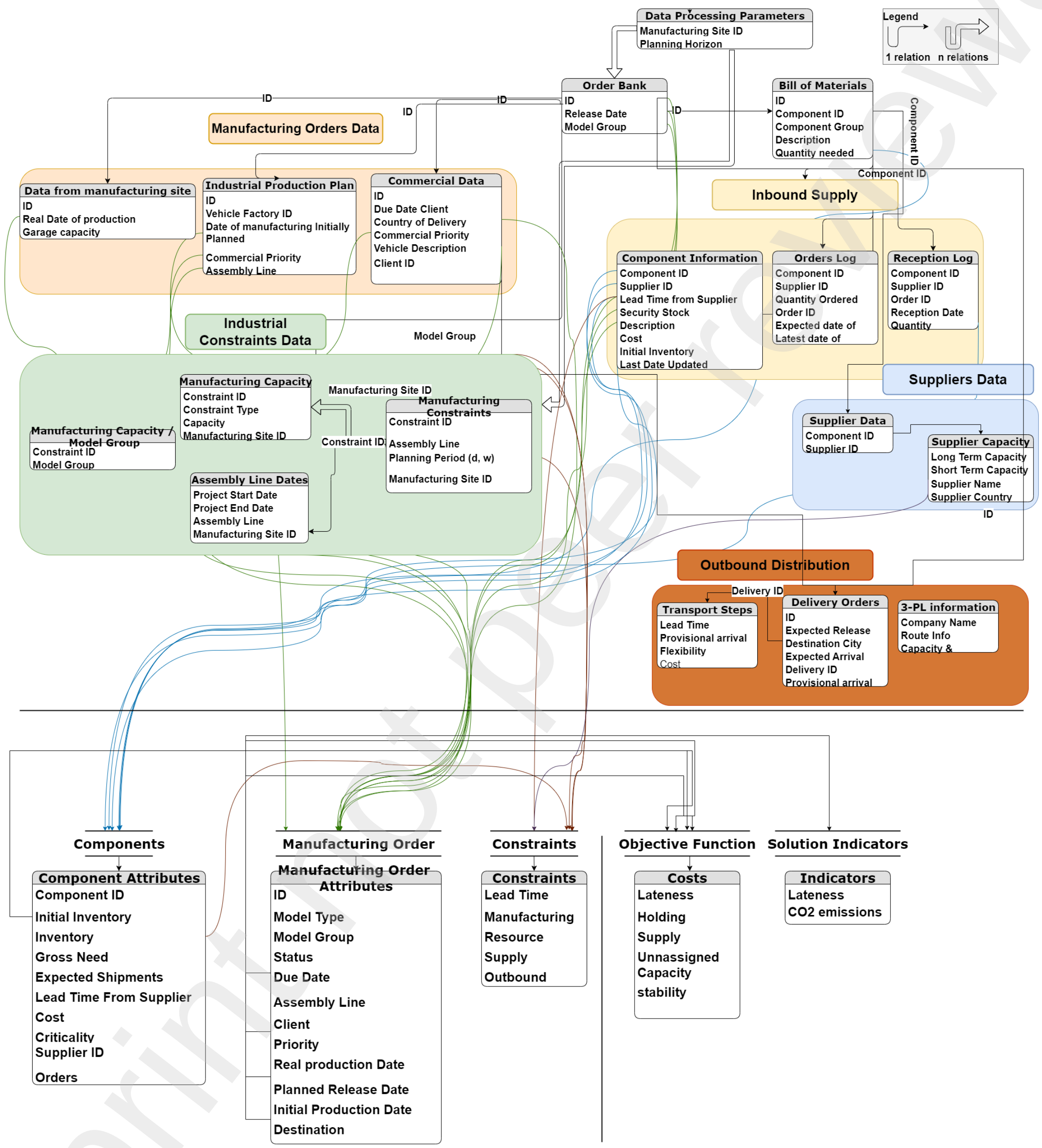

Figure A.7: Data Diagram

Appendix B. Calculating the cost of holding vehicles post assembly

Calculating the gains is challenging since not all information is available. In this section, we explain how the holding costs were calculated. As mentioned in our results, the manufacturing orders, if not dispatched, have to be stored. Ideally, they would be stored in the manufacturing site's garages. The garages, however, have limited capacities and come with a cost.

The cost generated is because of the immobilization value of the product, which has yet to be billed. The immobilization value refers to the value tied up in inventory that has yet to be sold or billed. This immobilization impacts Renault's free cash flow (FCF). This cost is known. Certain assumptions are made; we consider that for our *Target Population*, only **one** shipment is scheduled every week, so if we miss the first shipment, then the cars have to stay in the garage for at least one week, so we assume that for this one week, no other events will oblige the manufacturing site to dispatch the 702 vehicles. The holding cost in our model is HG, which is calculated in Renault due to the immobilization value and the insurance and is representative of the global average, which yields a holding cost $HG = 6.15$ € per day spent in the garage, so for 702 vehicles it will cost $4,317$ € of holding cost per day so $30,221$ € for a week of holding to distribution. This is the best scenario in our cost estimation since the cost of holding vehicles at the port is more significant. If there are no places left in the garages, the supply chain team will have to send the vehicles to wait for shipment in the port; after discussing with distribution planners, each port allows a number of *Free Days* where the car manufacturer can store the vehicles at the port, this is a common industrial practice to allow better flexibility in the logistics and time windows. In our calculation, we assume that the number of *Free Days* allowed is 2, the *Free Days* in the industry can vary however between 0 to 5 days, which means that the cost will be for storing the vehicles for 5 days, with a holding cost at the port of 2€ per vehicle per day including insurance, so that it will cost the distribution team in addition to the immobilization cost 7020€, in total this yields an additional cost of 37231€. With this calculation, we defined the *max & min* operational cost that the distribution team would pay if they did not react to the change in the distribution plan. Since the team can decide to keep some in the garages and some at the port, we estimate the actual cost between these two extremes $[30,221; 37,231]$€.

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