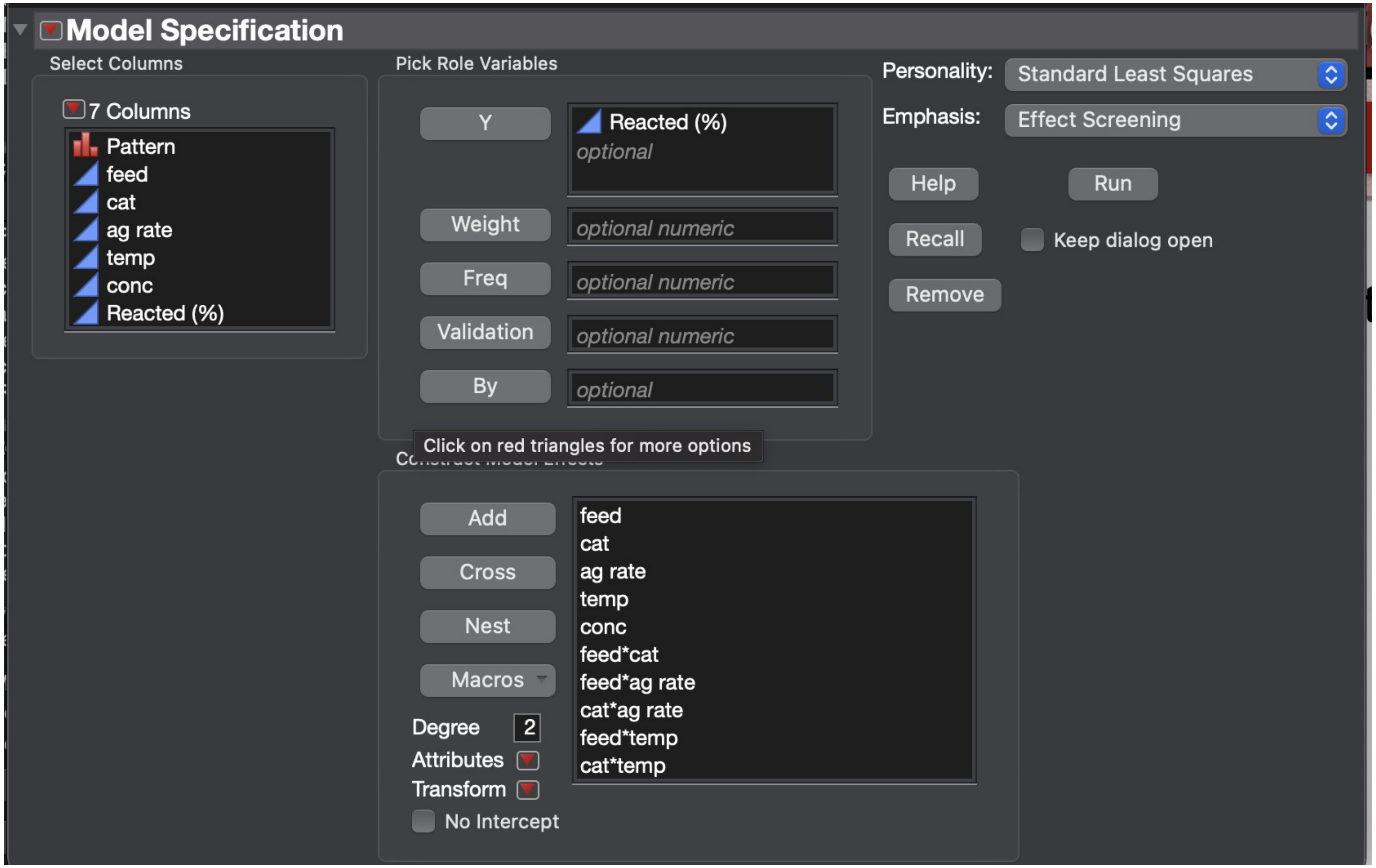
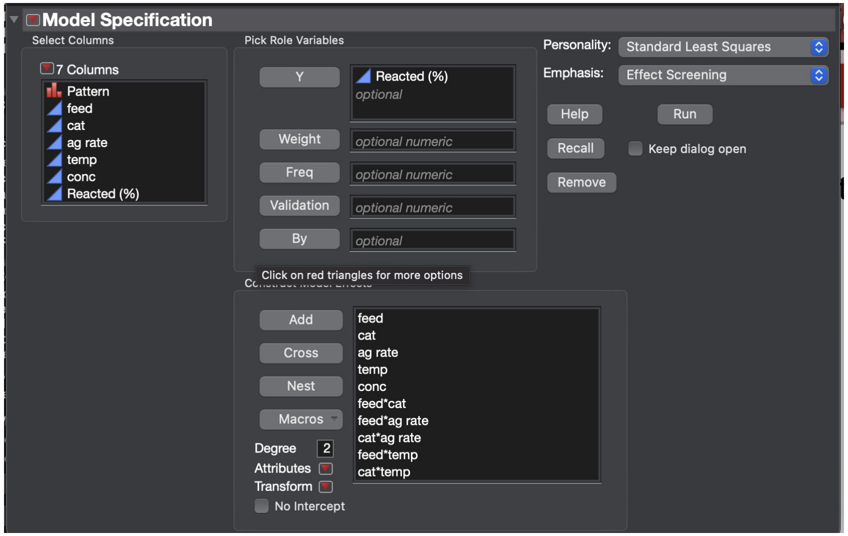
JMP DOE Design *(Cont.)*

- Select **Model** and **Run** when new screen appears.



JMP DOE Design (Cont.)

- Select **Run**
- Note: We will examine all of the main effects (ex. Feed Rate) and 2nd order interactions (Feed*Cat)
- Click Response Reacted %
- Under Regression Reports
- Select **Parameter Estimates and Summary of Fit**



JMP DOE Design *(Cont.)*

- JMP provides graphical and statistical analysis of the model as well as model predictions of the output, you must click on what you want.
- Since we do not have an estimate of the error in the current model, the statistical analysis cannot be completed.
- Let's continue to explore the results graphically!

Response Reacted (%)

Summary of Fit

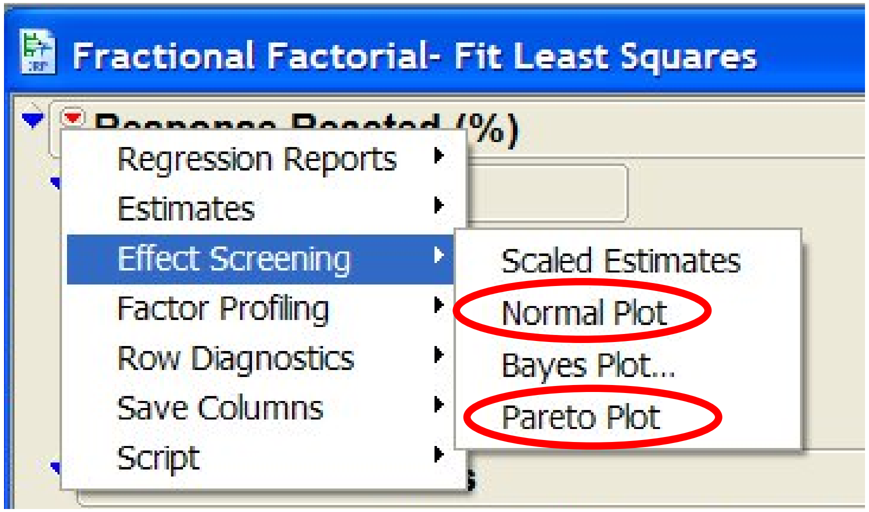
RSquare	1
RSquare Adj	.
Root Mean Square Error	.
Mean of Response	65.25
Observations (or Sum Wgts)	16

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.25	.	.	.
Feed(10,15)	-1	.	.	.
Cat(1,2)	10.25	.	.	.
Ag Rate(100,120)	0	.	.	.
Temp(140,180)	6.125	.	.	.
Conc(3,6)	-3.125	.	.	.
Feed*Cat	0.75	.	.	.
Feed*Ag Rate	0.25	.	.	.
Cat*Ag Rate	0.75	.	.	.
Feed*Temp	-0.375	.	.	.
Cat*Temp	5.375	.	.	.
Ag Rate*Temp	0.125	.	.	.
Feed*Conc	0.625	.	.	.
Cat*Conc	0.625	.	.	.
Ag Rate*Conc	1.125	.	.	.
Temp*Conc	-4.75	.	.	.

JMP DOE Design *(Cont.)*

- Under **Response Reacted (%)** select **Effect Screening>Pareto Plot**
- Next, select **Effect Screening>Normal Plot**



JMP DOE Design *(Cont.)*

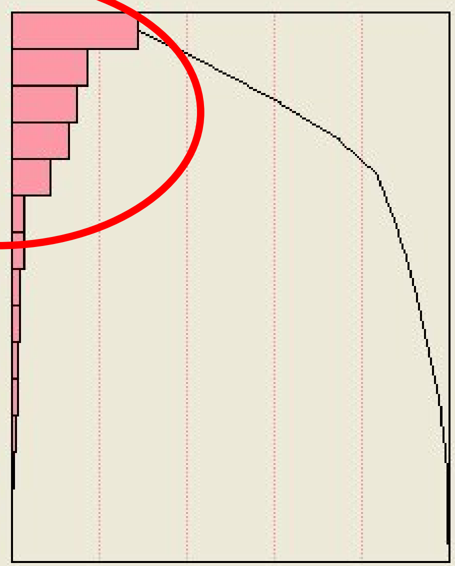
- The Pareto Plot shows us which factors most influence the response.
- Factors and interactions to keep in the model:

  Cat, Temp, Conc,

  Cat*Temp, Temp*Conc
- Note: The estimates are the coefficients of the prediction equation.

Pareto Plot of Estimates

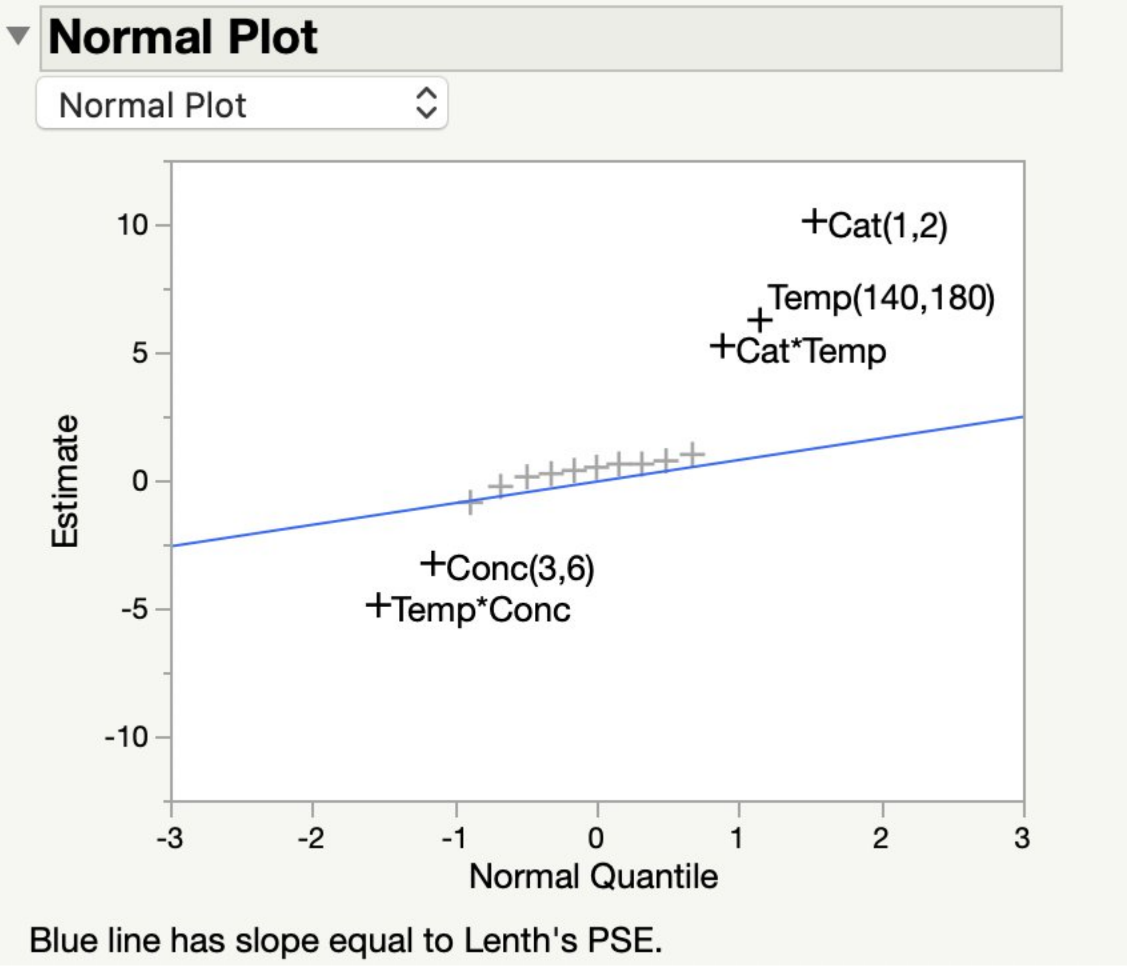
Term	Estimate
Cat(1,2)	10.25000
Temp(140,180)	6.12500
Cat*Temp	5.37500
Temp*Conc	-4.75000
Conc(3,6)	-3.12500
Ag Rate*Conc	1.12500
Feed(10,15)	-1.00000
Feed*Cat	0.75000
Cat*Ag Rate	0.75000
Cat*Conc	0.62500
Feed*Conc	0.62500
Feed*Temp	-0.37500
Feed*Ag Rate	0.25000
Ag Rate*Temp	0.12500
Ag Rate(100,120)	0.00000



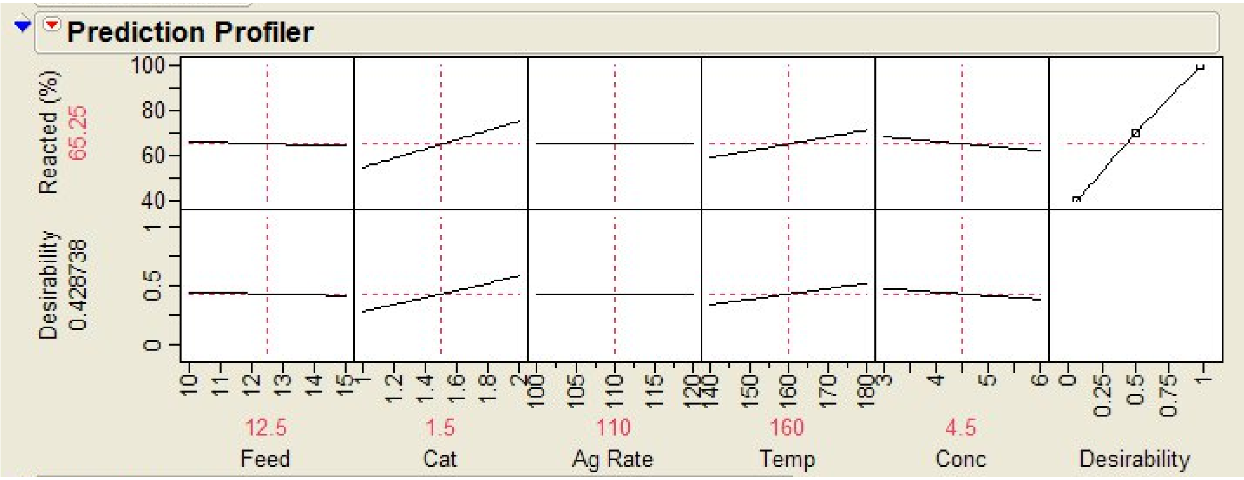
Note: If an interaction is used, then its main factors must be kept in the model [Ex. Temp*Conc, the Temp and Conc must be in the model].

JMP DOE Design *(Cont.)*

- The **Normal Plot** also shows us which factors most influence the response.
- The normal plot graphically shows the “outliers” (the factors that tend to be significantly different from the “random noise” of the experiment).



JMP DOE Design *(Cont.)*



- Before we reduce the model and keep only the factors that have the greatest effect on the response, look at the Prediction Profiler.
- Is there any useful information in the factors that were not considered significant (Feed, Ag rate)? Think about the optimizing the process. For the process to run, a level of each factor must be selected.

JMP DOE Design *(Cont.)*

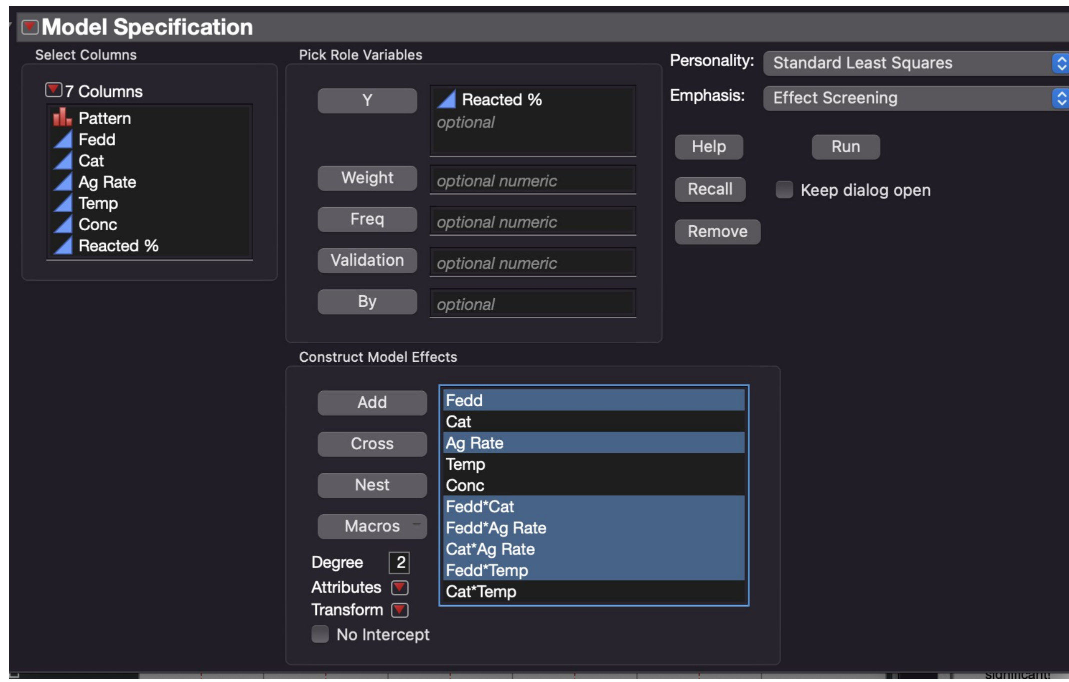
- Now, let’s reduce the model to include only those factors having the most influence on the response.
- Click on the Fit Model Window.

(Model Dialog) Or Under Analyze

- **Remove** all model factors <u>except</u>:

  Cat, Temp, Conc,

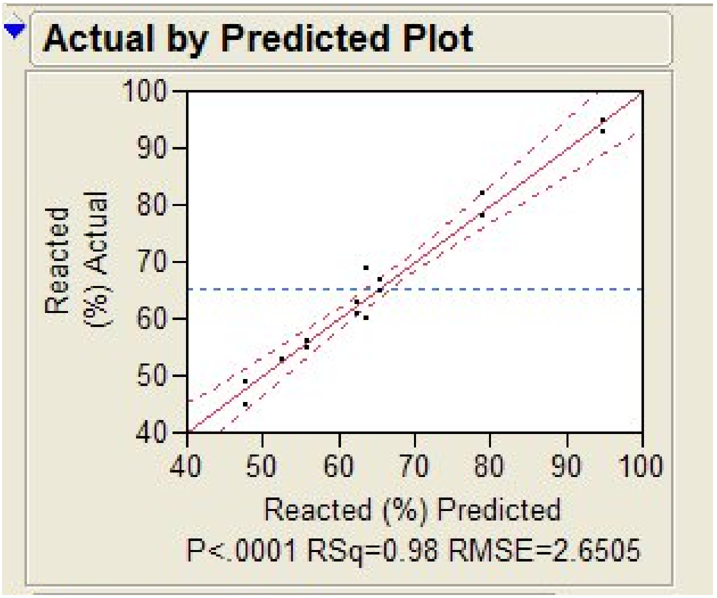
  Cat*Temp, Temp*Conc
- **Run Model.**



JMP DOE Design *(Cont.)*

Actual by Predicted shows:

1) p<.0001, the model is significant!
2) RSq = 0.98, 98% of the variation can be explained by the model. (This is a good model!)
3) Graphically, the confidence bands are tight around the diagonal solid line (The model is a close approximation of the data).



JMP DOE Design *(Cont.)*

- Expanded Estimates show the Prediction Formula Coefficients.
- Effect Tests show that all of the model factors are significant.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.25	0.662618	98.47	<.0001*
Cat(1,2)	10.25	0.662618	15.47	<.0001*
Temp(140,180)	6.125	0.662618	9.24	<.0001*
Conc(3,6)	-3.125	0.662618	-4.72	0.0008*
Cat*Temp	5.375	0.662618	8.11	<.0001*
Temp*Conc	-4.75	0.662618	-7.17	<.0001*

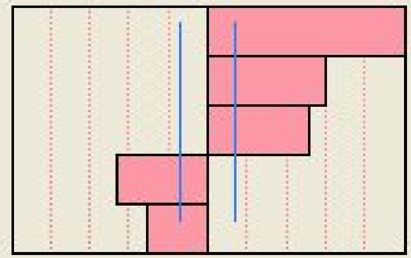
Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Cat(1,2)	1	1	1681.0000	239.2883	<.0001*
Temp(140,180)	1	1	600.2500	85.4448	<.0001*
Conc(3,6)	1	1	156.2500	22.2420	0.0008*
Cat*Temp	1	1	462.2500	65.8007	<.0001*
Temp*Conc	1	1	361.0000	51.3879	<.0001*

JMP DOE Design *(Cont.)*

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Cat(1,2)	10.25	0.662618	15.47	<.0001*
Temp(140,180)	6.125	0.662618	9.24	<.0001*
Cat*Temp	5.375	0.662618	8.11	<.0001*
Temp*Conc	-4.75	0.662618	-7.17	<.0001*
Conc(3,6)	-3.125	0.662618	-4.72	0.0008*



- **Sorted Estimates** summarizes the model and its effect on the response.
- You can go directly to this output and see the relative effect of the model terms with the model coefficients listed and the corresponding p-values.
- Question: For Conc (3,6), what does -3.125 mean practically? (See the Prediction Profile Graph).

JMP DOE Design *(Cont.)*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.25	0.662618	98.47	<.0001*
Cat(1,2)	10.25	0.662618	15.47	<.0001*
Temp(140,180)	6.125	0.662618	9.24	<.0001*
Conc(3,6)	-3.125	0.662618	-4.72	0.0008*
Cat*Temp	5.375	0.662618	8.11	<.0001*
Temp*Conc	-4.75	0.662618	-7.17	<.0001*

Prediction Formula:

Reacted (%) = 65.25 + 10.25(Cat) + 6.125(Temp) – 3.125(Conc) + 5.375(Cat)(Temp) – 4.75(Temp)(Conc)

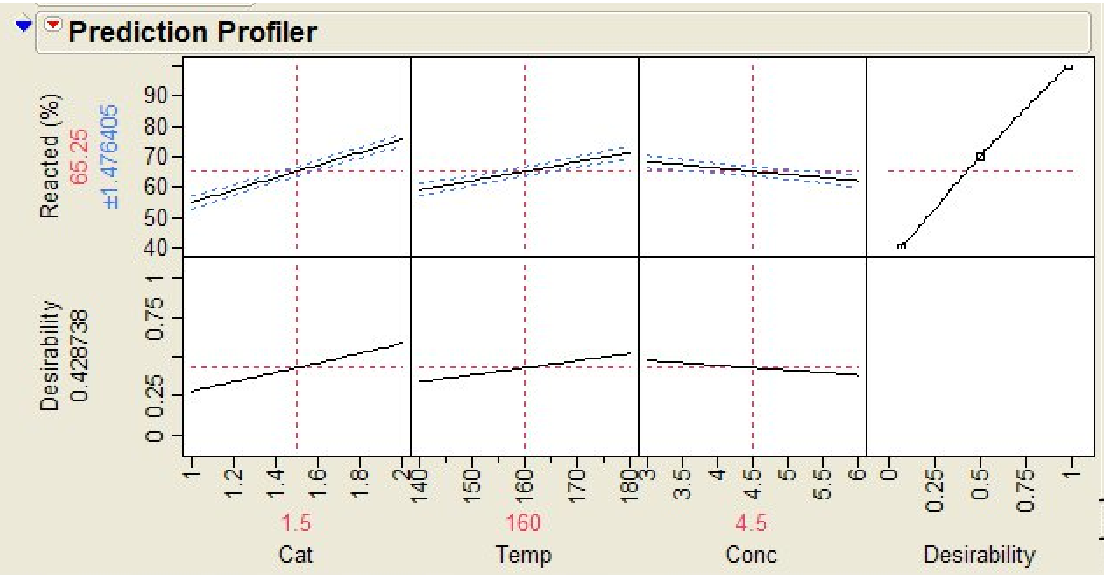
Note: Use “coded” values: Cat(-1,1), Temp(-1,1), Conc(-1,1)

Select the levels of Temp, Conc, & Cat to maximize the Reacted (%)?

Calculate the maximum predicted Reacted (%). [94.875 %]

Note: You cannot have two different levels of the same factor!!!

JMP DOE Design *(Cont.)*

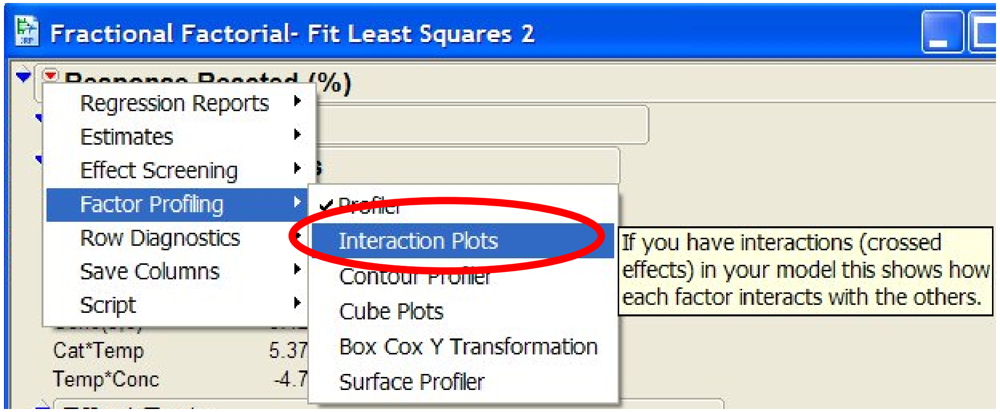


- Notice that the reduced model does not have Feed and Ag rate, in the model. We have to recall from our first analysis what levels to set these factors when we conduct additional trials.
- Concentration level 6 has an average 3.125 Reacted (%) reduction compared to Conc. Level 3 (“-3.125” means that the response goes down from Level 1 to Level 2).

JMP DOE Design *(Cont.)*

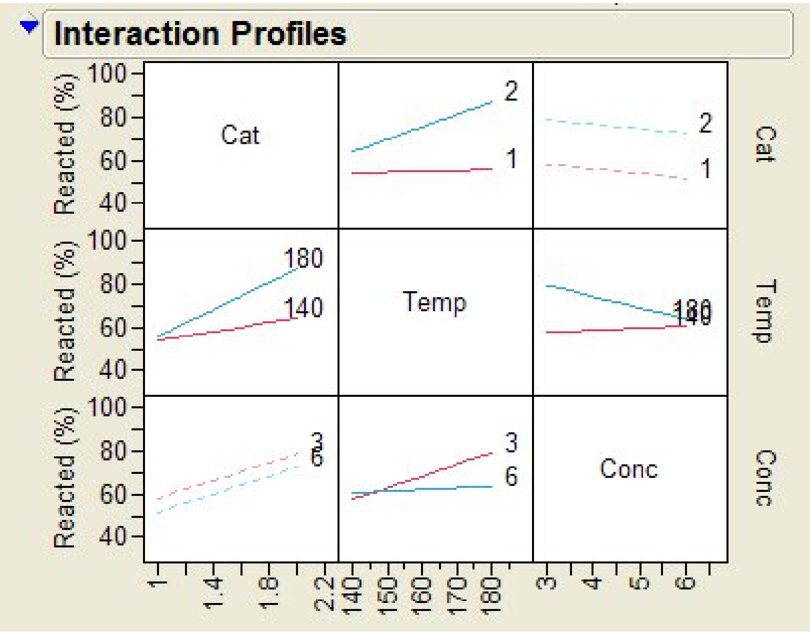
- Next, let's examine the **Interaction Plots**

- **Response Reacted (%)>Factor Profiling>Interaction Plots**



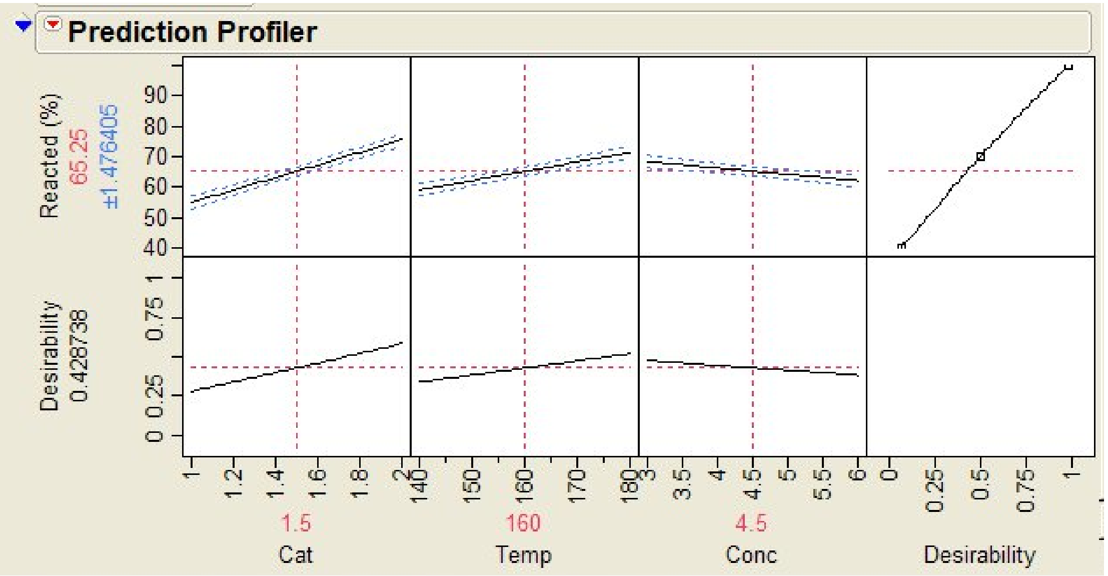
JMP DOE Design *(Cont.)*

- What does the interaction plots tell us?
- We determined from our analysis that Cat*Temp and Temp*Conc were statistically significant.
- We can see that these interactions are not parallel, indicating an interactive effect on the model



- Key Question: What levels of the Temperature, Concentration, and Catalyst would you select to **maximize** the response, Reacted (%)? [Look at the interaction plot and select the levels that maximize the Reacted (%).]

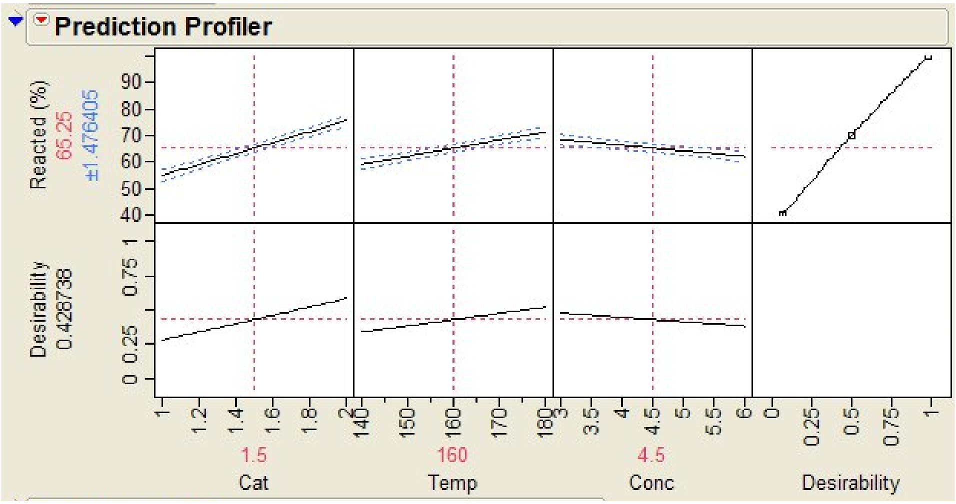
JMP DOE Design *(Cont.)*

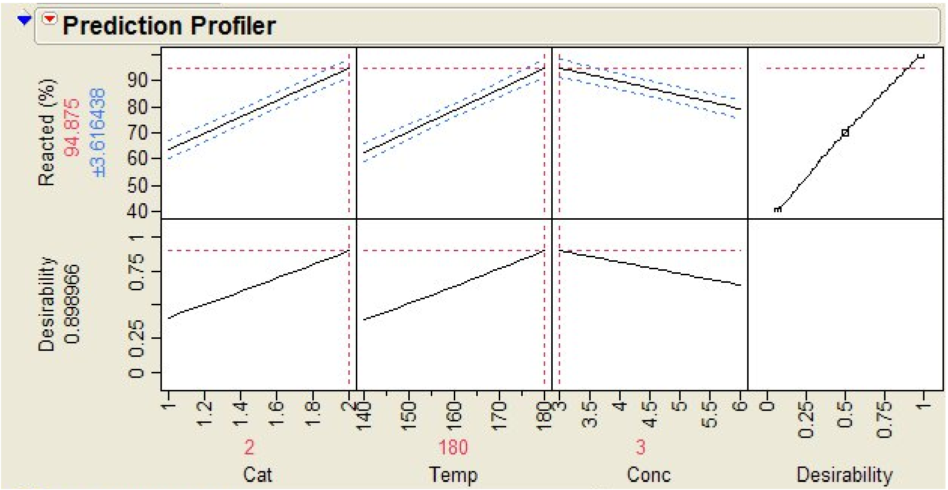


- Now drag Conc over to 3
- Drag Temp over to 180
- Drag Cat over to 2
- You should now be at 94.88% reacted and desirability around 0.89

JMP DOE Design *(Cont.)*

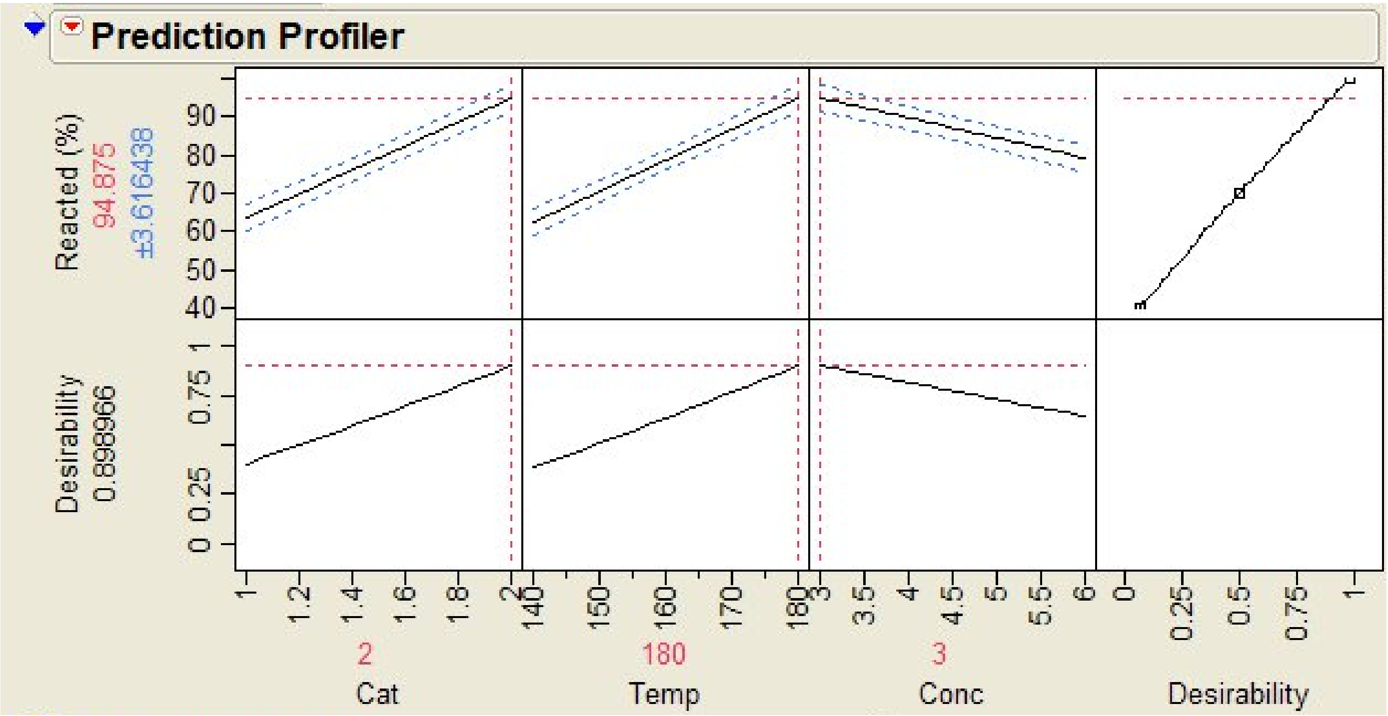
- Return to the **Prediction Profiler** and drag the vertical dashed lines to the levels that will maximize the reacted (%). (Or double click on the red x-axis value and enter the level value.)
- The Vertical Axis shows the predicted response value by the horizontal red dashed line and the predicted value in red(94.875).



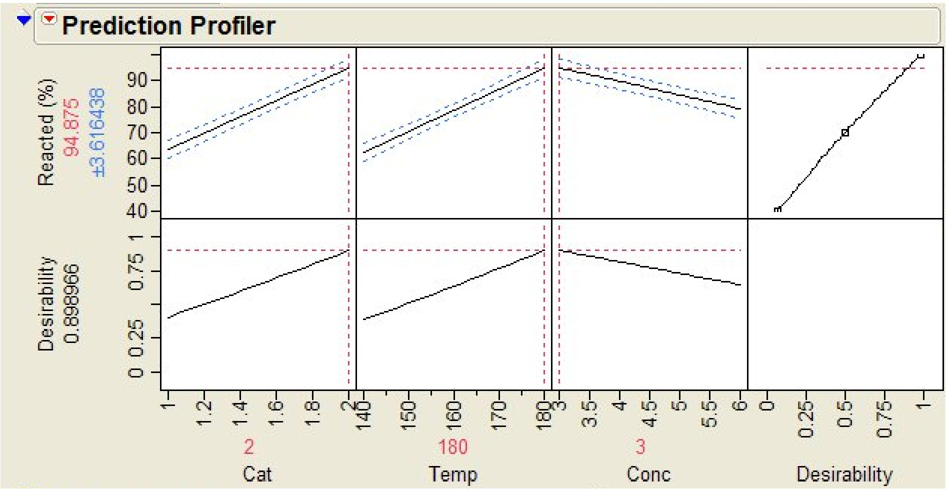


JMP DOE Design *(Cont.)*

- The **Desirability Function** automatically provides the best combinations.



JMP DOE Design *(Cont.)*



- The Prediction Profiler is using the prediction formula derived from the estimates for each factor in the reduced model.

<u>Prediction Formula:</u>

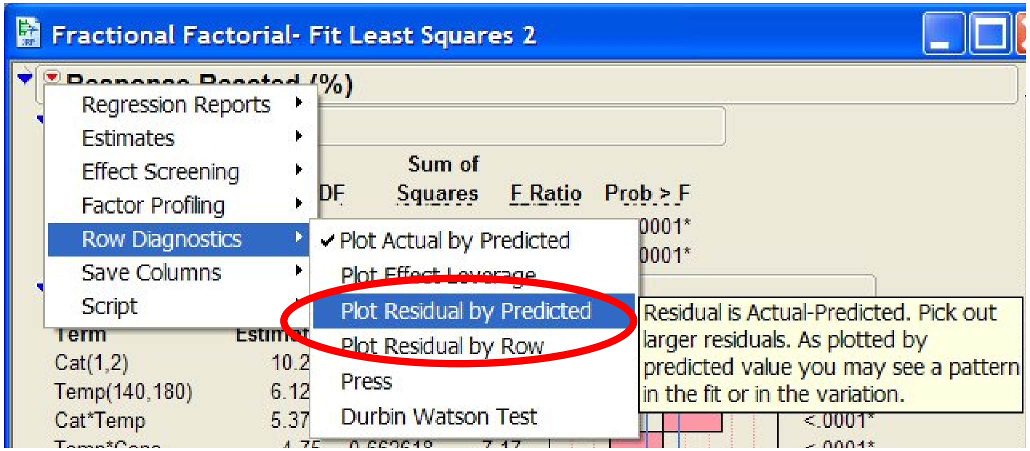
Reacted (%) = 65.25 + 10.25(Cat) + 6.125(Temp) – 3.125(Conc) + 5.375(Cat)(Temp) – 4.75(Temp)(Conc)

Factor Settings: Cat = 1(2%), Temp = 1(180C), Conc = -1(3%)

Reacted (%) = 65.25 + 10.25(1) + 6.125(1) – 3.125(-1) + 5.375(1)(1) – 4.75(1)(-1) = 94.875 %

JMP DOE Design *(Cont.)*

- We are almost finished with the analysis. We always want to check the residuals to see if we have missed something that may not be considered random noise!
- **Response Reacted (%)>Row Diagnostics>Plot Residual by Row**, and **Plot Residual by Predicted** (JMP output default)



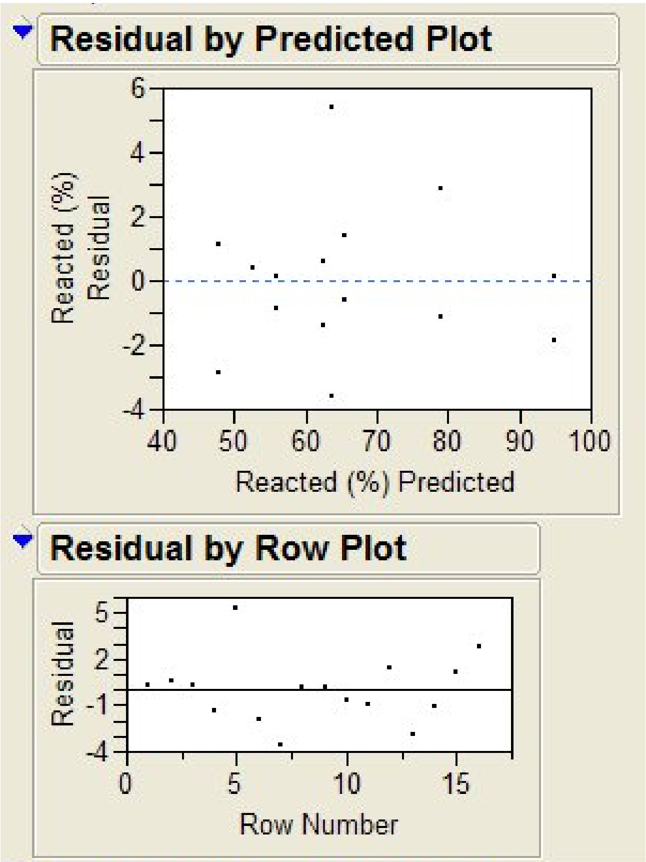
JMP DOE Design *(Cont.)*

Residual by Predicted Values

- We are expecting to see the residuals randomly distributed (STD = constant) around a mean = 0. (i.e., no patterns)

Residual by Row

- If the order of the rows represent the sequential (time) order when the data was collected, then any patterns in the plot would indicate possible variation that occurred over the duration of the experiment.



JMP DOE Design *(Cont.)*

Summary

1) We started with 5 factors at 2 levels, 2^5 = 32 runs. We decided to design a ½ **fractional factorial, 2^{5-1} = 16 runs, Resolution V Design**. We did not replicate the design to minimize costs.

2) We analyzed the results graphically to select the factors/interactions that had the most influence on the response. Then we **reduced the model** and kept the significant interactions and main factors. *We selected the optimum levels of the main factors not considered significant (Feed Rate, Ag rate).*

3) We ran the reduced model (3 factors: Cat, Temp, Conc, Cat*temp, Temp*Conc) - a 2^3 design. Therefore, with 16 runs, we now have replication and can estimate the error!

JMP DOE Design *(Cont.)*

Summary (Continued)

4) We analyzed the reduced model by:

- Looking at the **Scaled Estimates** to check to see if the factors were statistically significant. We could also see the magnitude of the effect of each factor.
- Plotting the **Prediction Profiles** and the **Interaction Plots** to graphically see the effect of the factors on the response.
- Optimize the factor levels using the **Prediction Profiler**. In this example, we selected factor levels to maximize the response. (Cat(2%), Temp(180°C), and Conc(3%)

5) Next, we analyzed the residuals using **Residual by Predicted Values** and **Residual by Row** to look for any trends that may suggest other factors influenced the experiment that were not in the model.

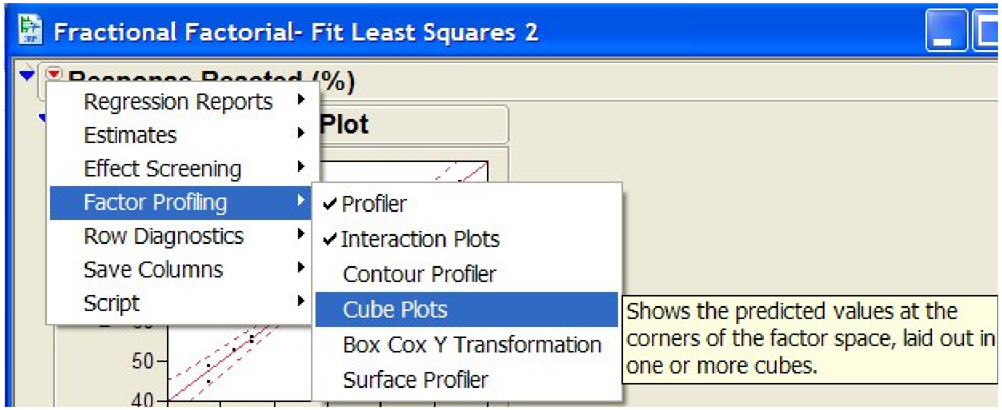
JMP DOE Design *(Cont.)*

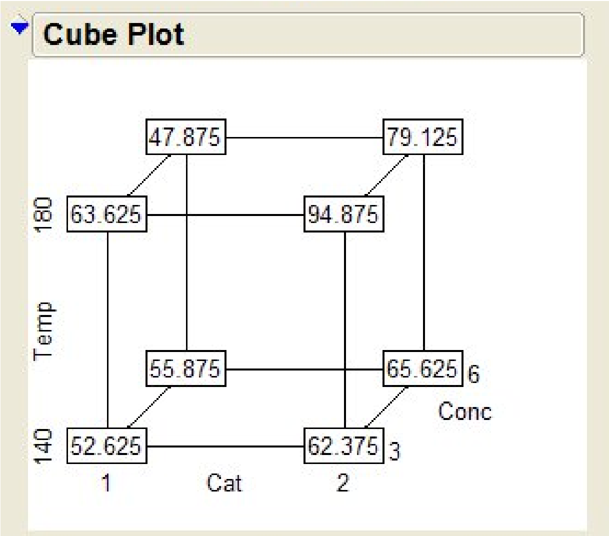
Additional Graphical Output

JMP also provides additional graphical output that helps to optimize the response using the predictive model.

- Cube Plots
- Contour Plots
- Surface Plots

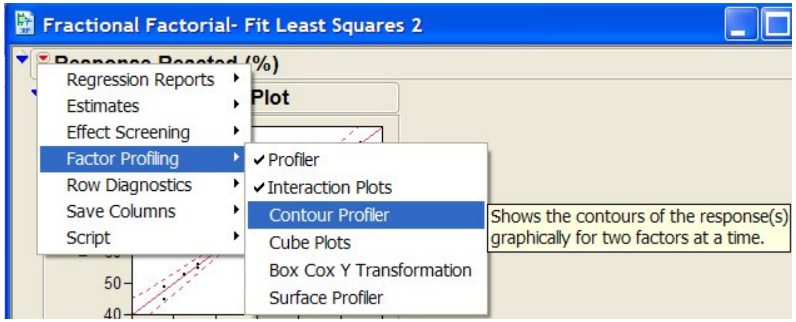
JMP DOE Design *(Cont.)*

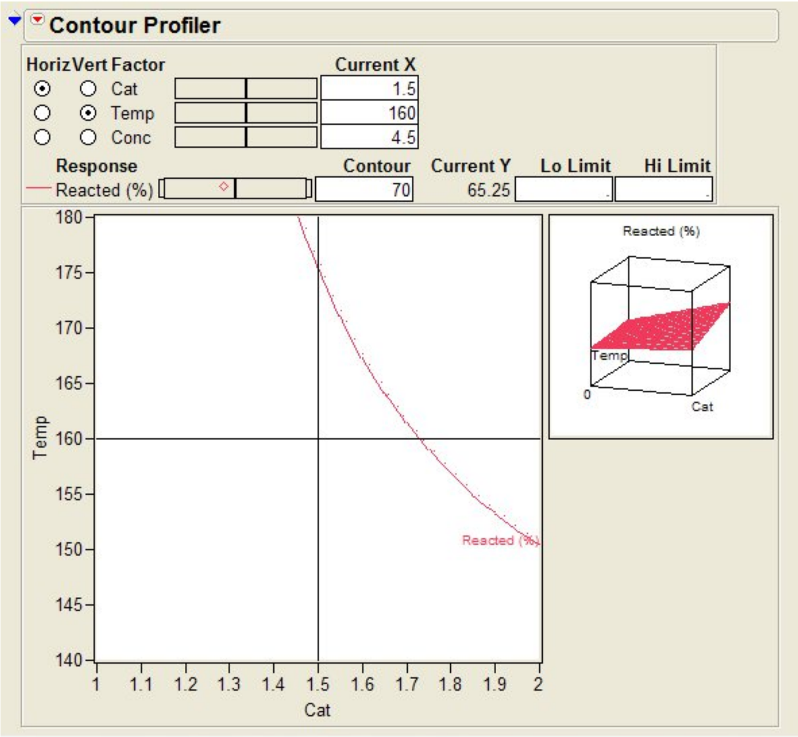




- **Cube Plots** enable us to look at the "response space" as a function of three factors. If we want to maximize the response, we look for the corner with the maximum value.

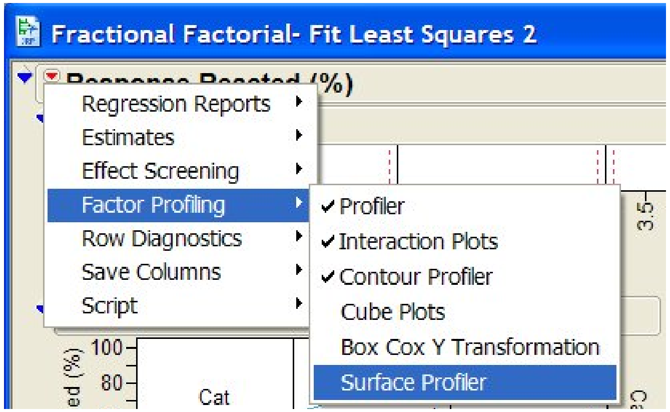
JMP DOE Design *(Cont.)*

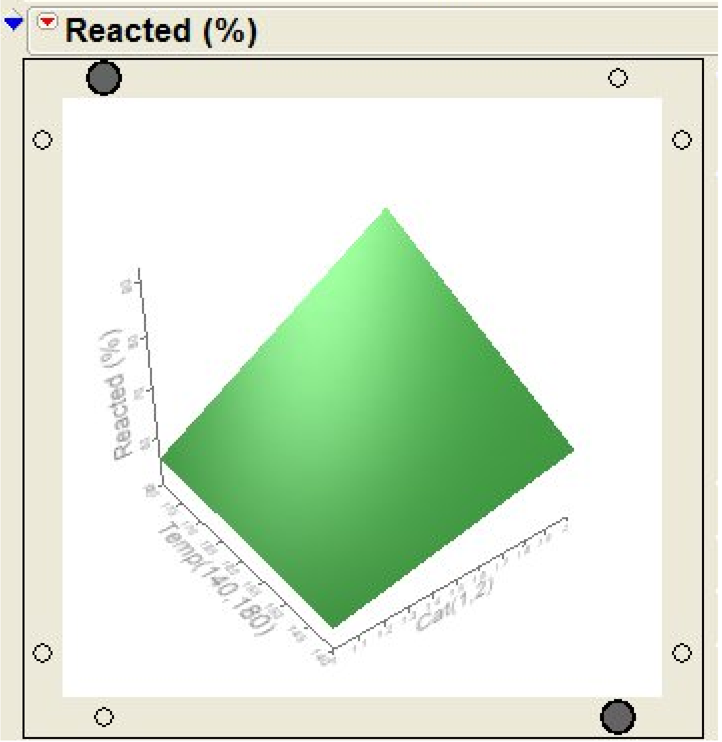




- **Contour Plots** enable us to look at a response contour as a function of two factors. Additional factors must be held at a selected constant level. (Ex. Given a conc=4.5, we can follow the contour line to select any combination of Temp and Cat to give us 70%).

JMP DOE Design *(Cont.)*



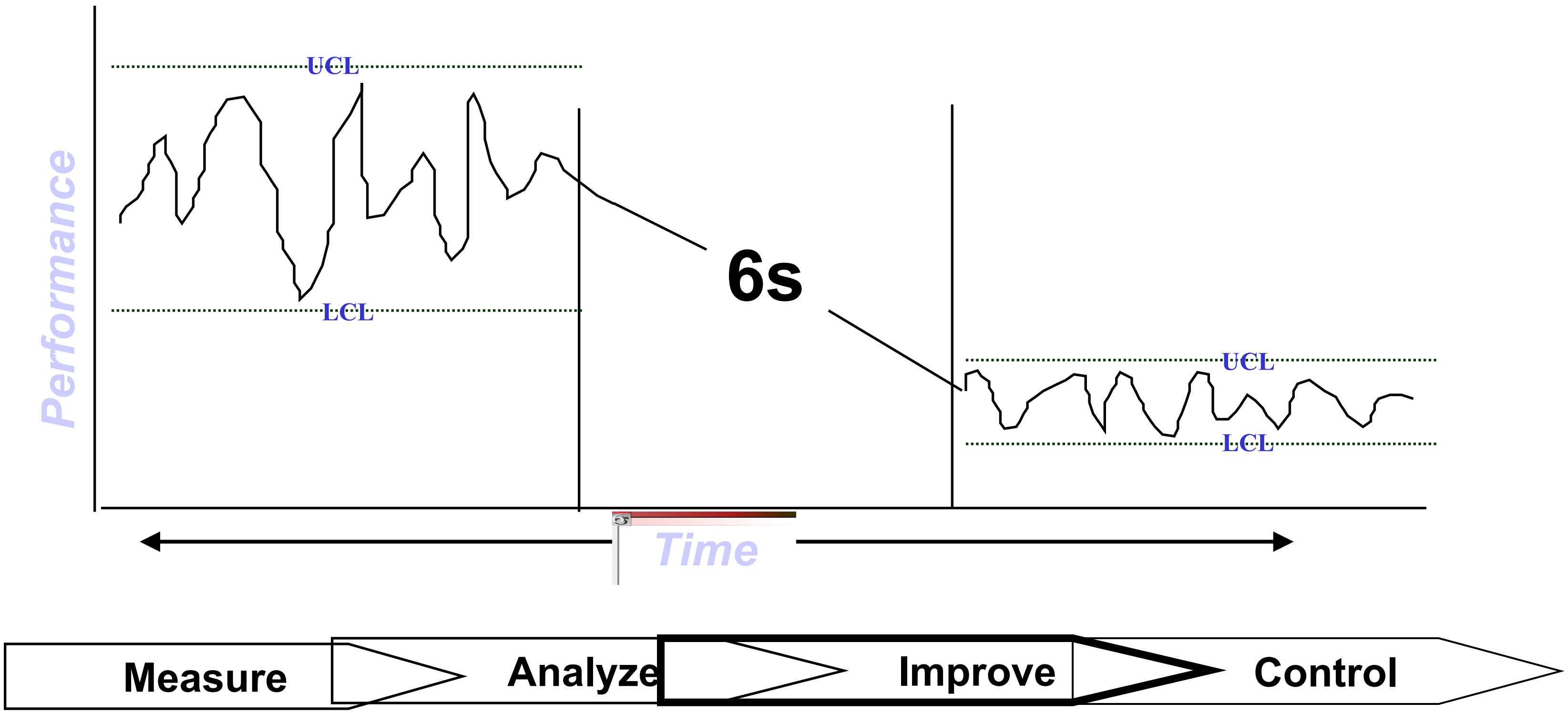


- **Surface Plots** enable us to look at a response surface as a function of three factors. You can dynamically rotate and move the plot to see the response space generated by the prediction model.

JMP Analysis *(Cont.)*

- Translate the statistical conclusion into process terms. Formulate conclusions and recommendations.
  - To maximize % reacted we should use 2% catalyst at 180 °C. We should also set the concentration to 3%. Select Feed Rate and Agitation Rate to minimize overall cost.
- **Replicate optimum conditions. Plan the next experiment and/or institutionalize the change.**

Goal of DOE


UCL
LCL
6s
UCL
LCL
Performance
Time
Measure
Analyze
Improve
Control

Lean Six Sigma

Fractional Factorial Experiments

Introduction

Objective: To “screen” the list of remaining input variables to isolate those significant factors affecting the response variable.

Deliverables: Vital Few input variables, DOE Report, Updated FMEA

Juran 47.52

Why Do Fractional Factorial Experiments?

- As the number of factors of interest increases, so does the number of runs required to run a full 2^K factorial experiment. Recall (without repeats or replicates):
  - 2 Factors, 2^2 Factorial = 4 runs
  - 3 Factors, 2^3 Factorial = 8 runs
  - 4 Factors, 2^4 Factorial = 16 runs
  - 5 Factors, 2^5 Factorial = 32 runs, etc...
- If the experimenter **assumes higher-order interactions are negligible**, it is possible to do a fraction of the full factorial and still get good estimates of lower-order interactions and main effects.

Why Do Fractional Factorial Experiments?

- Fractional factorials are referred to as “screening experiments.” This means investigating a relatively large number of factors in a relatively small number of runs.
- Screening experiments are usually done in the early stages of the process-improvement phase.

Fractional Factorial Vocabulary

- **Screening (Fractional) Experiments:**
  - Experiments that allow you to investigate main effects and/or lower- order interactions without having to run full-factorial experiments.
- **Half Fraction:**
  - Experiments that allow you to investigate main effects and/or lower-order interactions in half the runs required by a full factorial.
- **Quarter Fractions:**
  - Experiments that allow you to investigate main effects and/or lower-order interactions in one fourth the runs required by a full factorial.

Fractional Factorial Vocabulary

- **Aliased or Confounded:**
  - The inability to determine which main effect or interaction is causing the true effect. One or more effects that cannot unambiguously be attributed to a single factor or interaction.
- **Design Resolution:**
  - A Roman numeral notation which allows you to describe the "worst case" confounding scheme associated with a design.
- **Fold Over:**
  - The ability to add sequential fractional experiments to an existing fractional experiment with the intention of estimating specific main effects or interactions free of particular confounding patterns.

Juran 47.53 - 57

Fractional Factorial Notation

- The general notation to designate a fractional factorial design:

$$2_R^{k-p}$$

  - Where

    k = number of factors to be investigated

    2^{k-p} = number of runs

    R = design resolution (III, IV, V)

  - Note:

    If p = 1, then half-fraction factorial

    If p = 2, then quarter-fraction factorial

Half-Fraction Design

- Recall the expanded representation of a 2^3 full-factorial design. But suppose we wanted to investigate four input variables instead of three. Since all the contrasts are independent (orthogonal) we can select any interaction as the contrast to represent the fourth variable. **Usually we select the highest-order interaction to represent the fourth factor. In this case, the AxBxC interaction can now represent the levels of factor D.**
- When we replace the AxBxC Interaction with Factor D, we say the ABC is **aliased** with D. Obviously, ABC can no longer be estimated.

Half-Fraction Design

Factor D

A	B	C	AXB	AXC	BXC	AXBXC
-1	-1	-1	1	1	1	-1
1	-1	-1	-1	-1	1	1
-1	1	-1	-1	1	-1	1
1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	-1	1
1	-1	1	-1	1	-1	-1
-1	1	1	-1	-1	1	-1
1	1	1	1	1	1	1

The Design Generator is D = ABC.

Half-Fraction Design

- Why is it called a "half" fraction? We would call this a **half-fraction** since a full 2^4 factorial design would take 16 runs to complete. In a half fraction we can estimate **4 factors** in only **8 runs.** There is a cost - we lose the higher-order interaction.

Half-Fraction Design

	A	B	C	D	A X B	A X C	B X C	A X B X C
1	-1	-1	-1	-1	1	1	1	-1
	1	-1	-1	-1	-1	-1	1	1
	-1	1	-1	-1	-1	1	-1	1
4	1	1	-1	-1	1	-1	-1	-1
	-1	-1	1	-1	1	-1	-1	1
6	1	-1	1	-1	-1	1	-1	-1
7	-1	1	1	-1	-1	-1	1	-1
	1	1	1	-1	1	1	1	1
	-1	-1	-1	1	1	1	1	-1
2	1	-1	-1	1	-1	-1	1	1
3	-1	1	-1	1	-1	1	-1	1
	1	1	-1	1	1	-1	-1	-1
5	-1	-1	1	1	1	-1	-1	1
	1	-1	1	1	-1	1	-1	-1
	-1	1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1	1

Half-Fraction Exercise

- Use this matrix as a starting point to design a half-fraction experiment to estimate 5 main effects in only 16 runs.
  - What is the design generator?
  - Which 16 of the 32 runs will be used for the fractional factorial experiment?
- Let's investigate the alias structure of this half-fraction experiment.

Run	A	B	C	D	E	ABCD
1	-1	-1	-1	-1	-1	
2	1	-1	-1	-1	-1	
3	-1	1	-1	-1	-1	
4	1	1	-1	-1	-1	
5	-1	-1	1	-1	-1	
6	1	-1	1	-1	-1	
7	-1	1	1	-1	-1	
8	1	1	1	-1	-1	
9	-1	-1	-1	1	-1	
10	1	-1	-1	1	-1	
11	-1	1	-1	1	-1	
12	1	1	-1	1	-1	
13	-1	-1	1	1	-1	
14	1	-1	1	1	-1	
15	-1	1	1	1	-1	
16	1	1	1	1	-1	
17	-1	-1	-1	-1	1	
18	1	-1	-1	-1	1	
19	-1	1	-1	-1	1	
20	1	1	-1	-1	1	
21	-1	-1	1	-1	1	
22	1	-1	1	-1	1	
23	-1	1	1	-1	1	
24	1	1	1	-1	1	
25	-1	-1	-1	1	1	
26	1	-1	-1	1	1	
27	-1	1	-1	1	1	
28	1	1	-1	1	1	
29	-1	-1	1	1	1	
30	1	-1	1	1	1	
31	-1	1	1	1	1	
32	1	1	1	1	1	

Half-Fraction Exercise

- Multiply the signs in the AxBxCxD columns to get the signs in the ABCD column.
  - Ex: Run 1
  - -1x-1x-1x-1= 1 (ABCD)
- Select the 16 rows where the signs in columns E and ABCD match. Factor E is confounded with the ABCD interaction.
- These are the runs for the ½ Fraction Design.

Run	A	B	C	D	E	ABCD
1	-1	-1	-1	-1	-1	1
2	1	-1	-1	-1	-1	-1
3	-1	1	-1	-1	-1	-1
4	1	1	-1	-1	-1	1
5	-1	-1	1	-1	-1	-1
6	1	-1	1	-1	-1	1
7	-1	1	1	-1	-1	1
8	1	1	1	-1	-1	-1
9	-1	-1	-1	1	-1	-1
10	1	-1	-1	1	-1	1
11	-1	1	-1	1	-1	1
12	1	1	-1	1	-1	-1
13	-1	-1	1	1	-1	1
14	1	-1	1	1	-1	-1
15	-1	1	1	1	-1	-1
16	1	1	1	1	-1	1
17	-1	-1	-1	-1	1	1
18	1	-1	-1	-1	1	-1
19	-1	1	-1	-1	1	-1
20	1	1	-1	-1	1	1
21	-1	-1	1	-1	1	-1
22	1	-1	1	-1	1	1
23	-1	1	1	-1	1	1
24	1	1	1	-1	1	-1
25	-1	-1	-1	1	1	-1
26	1	-1	-1	1	1	1
27	-1	1	-1	1	1	1
28	1	1	-1	1	1	-1
29	-1	-1	1	1	1	1
30	1	-1	1	1	1	-1
31	-1	1	1	1	1	-1
32	1	1	1	1	1	1

Half-Fraction Exercise

- What is confounded with AB? What is confounded with AE?
- Complete the matrix for the indicated interactions (arrows) to determine the confounded factor(s)/interaction(s).

Half-Fraction Exercise

Run	A	B	C	D	E	AB	AC	AD	AE	BC	BD	BE	CD	CE	DE	ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE	ABCD
1	1	-1	-1	-1	-1																					-1
2	-1	1	-1	-1	-1																					-1
3	-1	-1	1	-1	-1																					-1
4	1	1	1	-1	-1																					-1
5	-1	-1	-1	1	-1																					-1
6	1	1	-1	1	-1																					-1
7	1	-1	1	1	-1																					-1
8	-1	1	1	1	-1																					-1
9	-1	-1	-1	-1	1																					1
10	1	1	-1	-1	1																					1
11	1	-1	1	-1	1																					1
12	-1	1	1	-1	1																					1
13	1	-1	-1	1	1																					1
14	-1	1	-1	1	1																					1
15	-1	-1	1	1	1																					1
16	1	1	1	1	1																					1

Half-Fraction Exercise

A	B	C	D	E	AB	AC	AD	AE	BC	BD	BE	CD	CE	DE	ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE	ABCD	ABCE	ABDE	ACDE	BCDE
1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1
-1	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1
1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	1
-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	-1	1	-1	-1	-1
1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1
1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1
-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1
-1	-1	-1	-1	1	1	1	1	-1	1	1	-1	1	-1	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1	-1
1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	1
1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	-1
1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	1
-1	1	-1	1	1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	1	-1
-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1
1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Check the 12 =345 or AB = CDE

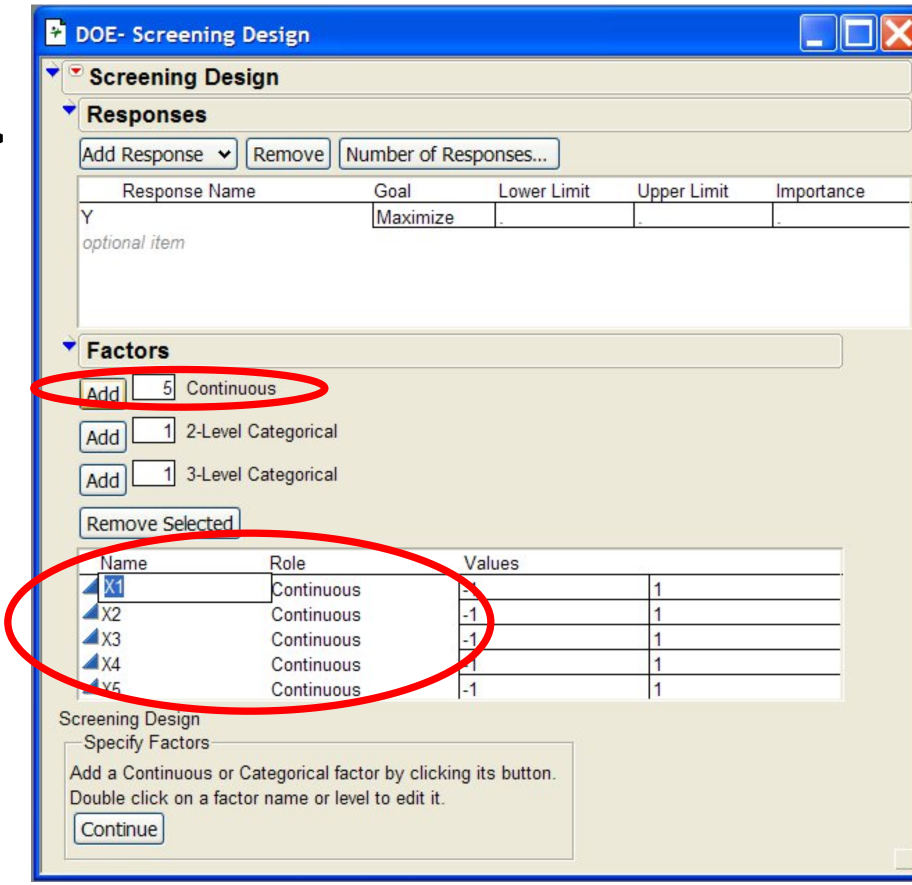
And the 15 = 234 or AE = BCD Alias Structure

Half-Fraction Exercise

- Use a half-fraction experiment to estimate 5 main effects in only 16 runs. What is the alias structure?
- Use JMP to confirm your results.

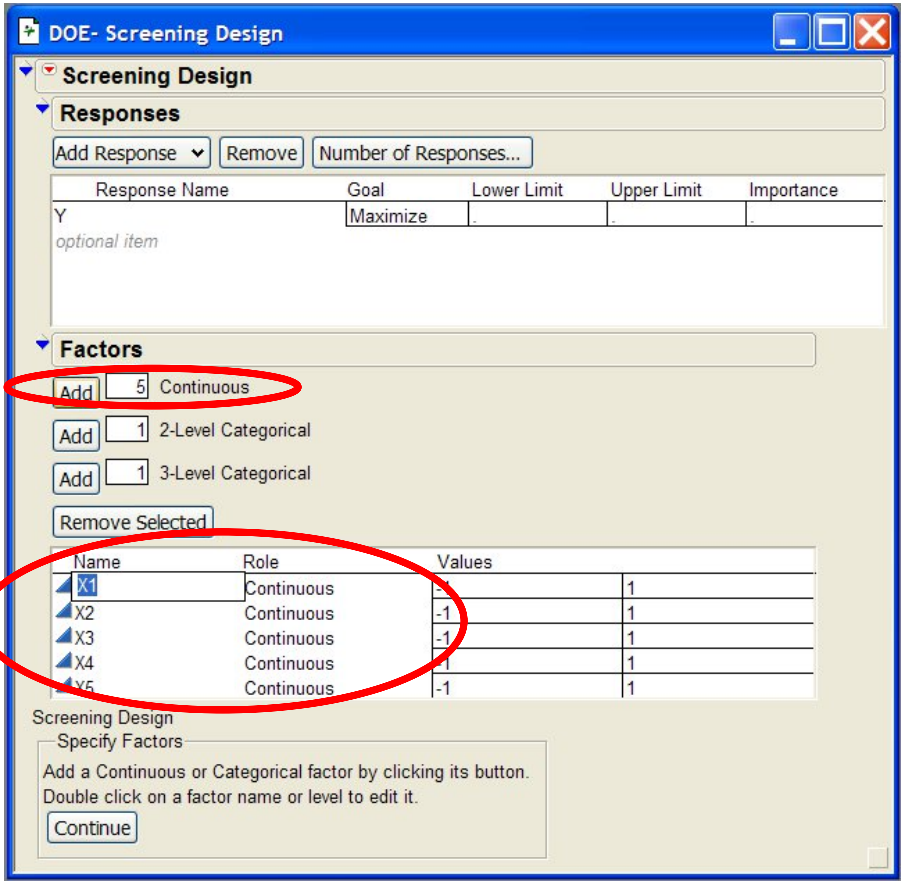
Confounding and Alias Structure

- **JMP>DOE>Classical>**
- **>Two Level Screening > Screening Design**
- Add 5 continuous factors
- Select Continue



Confounding and Alias Structure

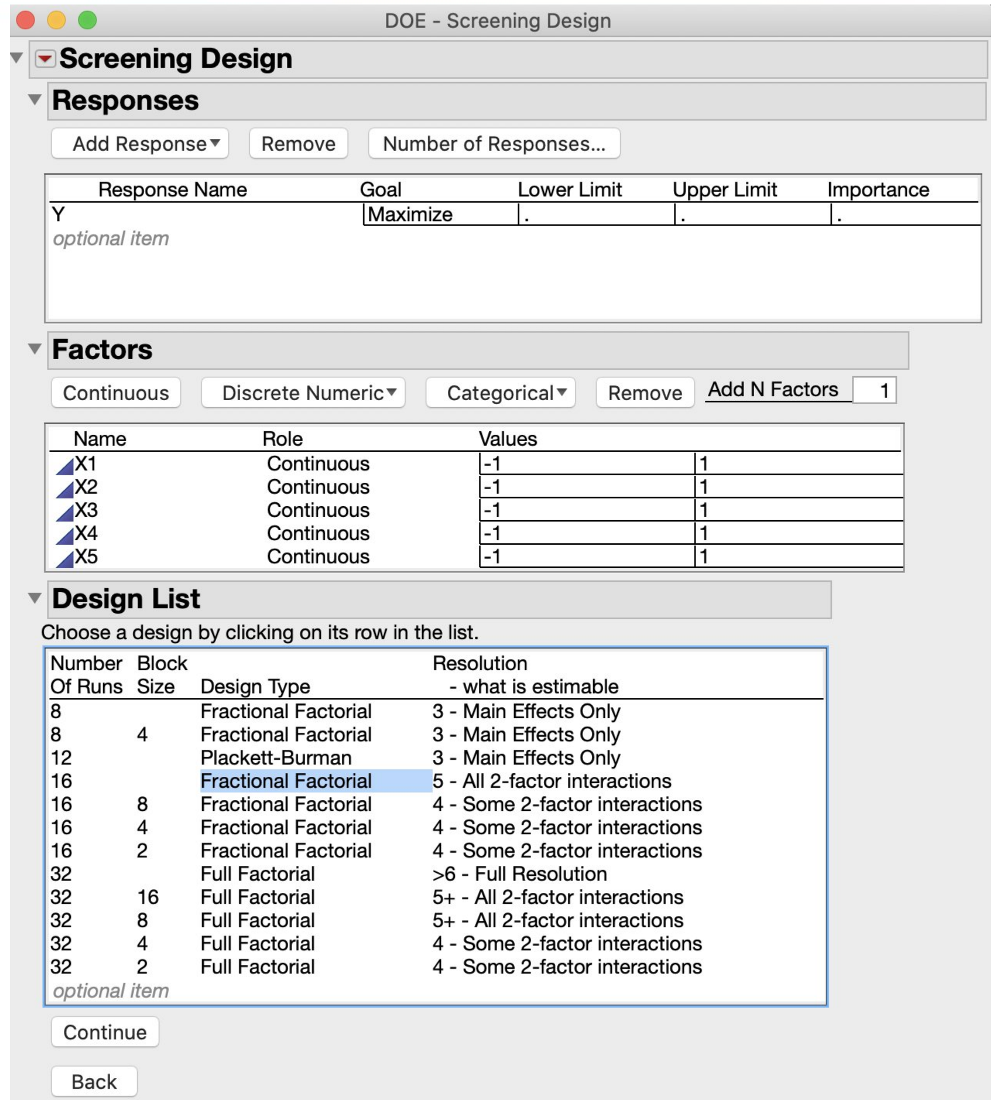
- **Select Choose from a list of fractional factorial designs**
- Select Continue



Confounding and Alias Structure

- Select Fractional Factorial with Resolution 5

- Select Continue



Confounding and Alias Structure

- Select Aliasing of Effects
- Then select the Red Triangle and show confounding order, and for the aliases to what order, enter 5
- Select OK

Confounding and Alias Structure

Confounding Pattern

Confounding P...

Columns (2/0)
Effect Names
Alias Names

Rows
All rows 32
Selected 0
Excluded 0
Hidden 0
Labelled 0

	Effect Names	Alias Names
1	C	= 1 2 3 4 5
2	1	= 2 3 4 5
3	2	= 1 3 4 5
4	1 2	= 3 4 5
5	3	= 1 2 4 5
6	1 3	= 2 4 5
7	2 3	= 1 4 5
8	1 2 3	= 4 5
9	4	= 1 2 3 5
10	1 4	= 2 3 5
11	2 4	= 1 3 5
12	1 2 4	= 3 5
13	3 4	= 1 2 5
14	1 3 4	= 2 5
15	2 3 4	= 1 5
16	1 2 3 4	= 5
17	5	= 1 2 3 4
18	1 5	= 2 3 4
19	2 5	= 1 3 4
20	1 2 5	= 3 4
21	3 5	= 1 2 4
22	1 3 5	= 2 4
23	2 3 5	= 1 4
24	1 2 3 5	= 4
25	4 5	= 1 2 3
26	1 4 5	= 2 3
27	2 4 5	= 1 3
28	1 2 4 5	= 3
29	3 4 5	= 1 2
30	1 3 4 5	= 2
31	2 3 4 5	= 1
32	1 2 3 4 5	= C

Confounding and Alias Structure

Examine the Alias Structure:

The second order 12 interaction is confounded with the third order 345 interaction.

12 = 345

What is the 15 interaction aliased with?

Confounding Pattern

Confounding Pattern

Columns (2/0)
Effect Names
Alias Names

Rows
All rows 32
Selected 0
Excluded 0
Hidden 0
Labelled 0

	Effect Names	Alias Names
1	C	= 1 2 3 4 5
2	1	= 2 3 4 5
3	2	= 1 3 4 5
4	1 2	= 3 4 5
5	3	= 1 2 4 5
6	1 3	= 2 4 5
7	2 3	= 1 4 5
8	1 2 3	= 4 5
9	4	= 1 2 3 5
10	1 4	= 2 3 5
11	2 4	= 1 3 5
12	1 2 4	= 3 5
13	3 4	= 1 2 5
14	1 3 4	= 2 5
15	2 3 4	= 1 5
16	1 2 3 4	= 5
17	5	= 1 2 3 4
18	1 5	= 2 3 4
19	2 5	= 1 3 4
20	1 2 5	= 3 4
21	3 5	= 1 2 4
22	1 3 5	= 2 4
23	2 3 5	= 1 4
24	1 2 3 5	= 4
25	4 5	= 1 2 3
26	1 4 5	= 2 3
27	2 4 5	= 1 3
28	1 2 4 5	= 3
29	3 4 5	= 1 2
30	1 3 4 5	= 2
31	2 3 4 5	= 1
32	1 2 3 4 5	= C

Design Resolution

- Example: Here we are investigating five factors will be in $2^{5-1} = 16$ runs.
- This design is a resolution V or 5.
- See the Design List to check this.

Design Resolution *(Cont.)*

- **Resolution V Designs:**
  - **No main effect aliased with another main effect.**
  - **No main effect aliased with any two-factor interactions.**
  - **No two-factor interaction aliased with another two factor-interaction.**
  - **Two factor-interactions aliased with three factor-interactions.**

Design Resolution

- **Example: The designation here means four factors will be investigated in 2^{4-1} = 8 runs.**

  This design is a resolution IV.

$$2_{IV}^{4-1}$$

Design Resolution *(Cont.)*

- **Resolution IV Designs:**
  - **No main effect aliased with another main effect.**
  - **No main effect aliased with any two-factor interactions.**
  - **At least one two-factor interaction aliased with another two-factor interaction.**

Design Resolution

- **Resolution III Designs:**
  - **No main effects are aliased with other main effects.**
  - **At least one main effect will be aliased with a two-factor interaction.**

Aliasing Structure

- For this example:
  - I, the Identity element, equals 1 (for all practical purposes)
  - I = ABCDE, the defining relation, can be used to determine any confounding information
  - Question: Why is $A^2 = I$?
  - Question: What is A, AE, CD confounded with?

$$I = ABCDE$$
$$A = A^2BCDE$$
$$A = IBCDE$$
$$A = BCDE$$

$$I = ABCDE$$
$$AE\,(I) = AE\,(ABCDE)$$
$$AE = A^2BCDE^2$$
$$AE = BCD$$

$$I = ABCDE$$
$$CD = CD\,(ABCDE)$$
$$CD = ABC^2D^2E$$
$$CD = ABE$$

JMP Example

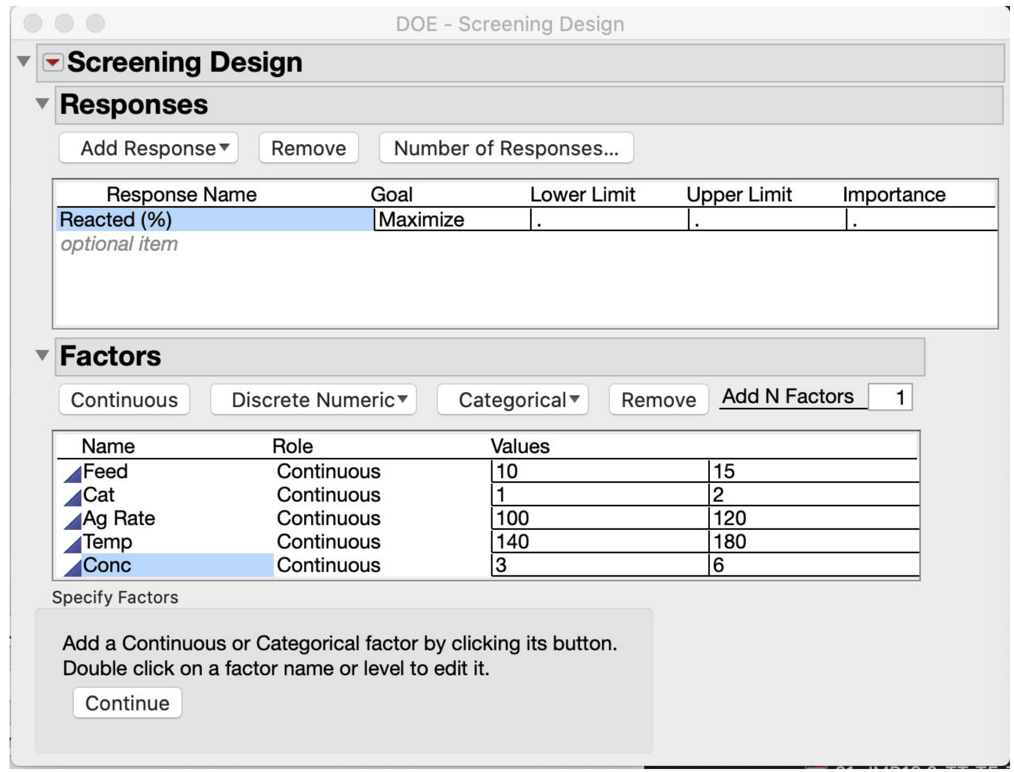
- State the Practical Problem
  - To determine which factors can increase the % reacted of a chemical process.
- State the factors and levels of interest, create a JMP experimental data sheet
  - We only have enough funds for 16 runs of this experiment.

JMP Example *(Cont.)*

- Output Variable:
  - % Reacted
- Input Variables:
  - A: Feed Rate (liters/minute) 10(-1), 15(+1)
  - B: Catalyst (%) 1(-1), 2(+1)
  - C: Agitation Rate (rpm) 100(-1), 120(+1)
  - D: Temperature (°C) 140(-1), 180(+1)
  - E: Concentration (%) 3(-1), 6(+1)

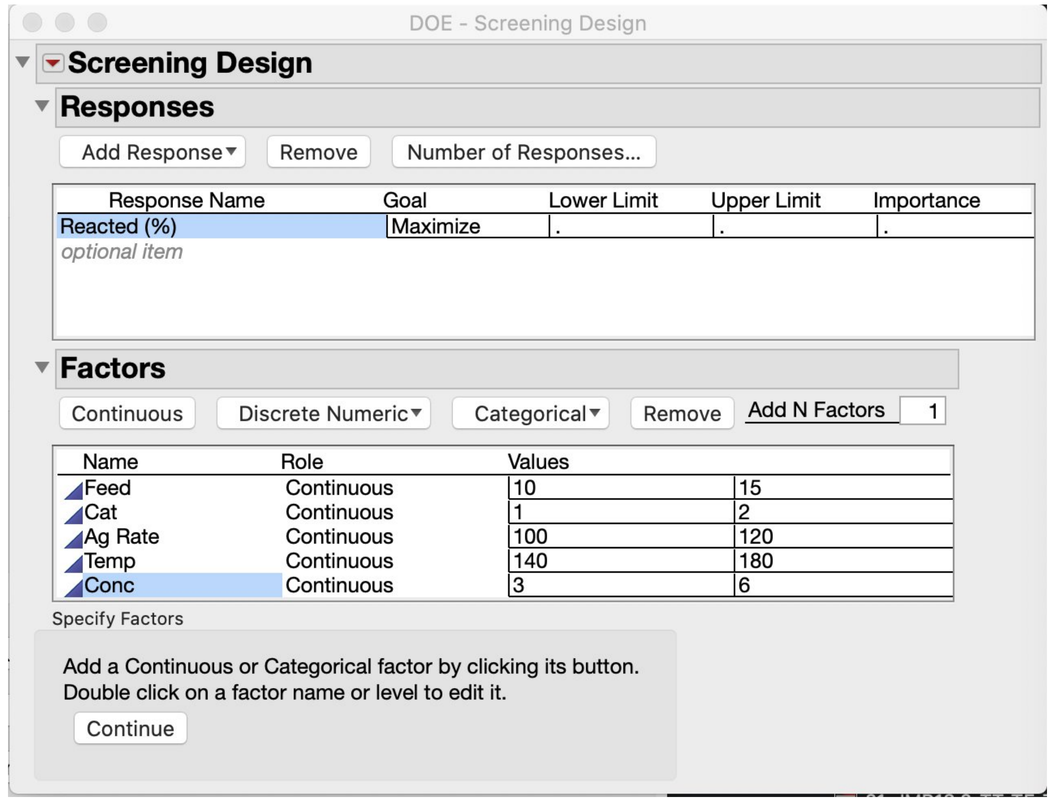
JMP DOE Design

- **JMP>DOE>Classical**
- **>Two Level Screening> Screening Design**
- Next to **Continuous Factors,** change the number from *1* to *5* since there are 5 continuous factors.
- Insert the Factor names and levels.



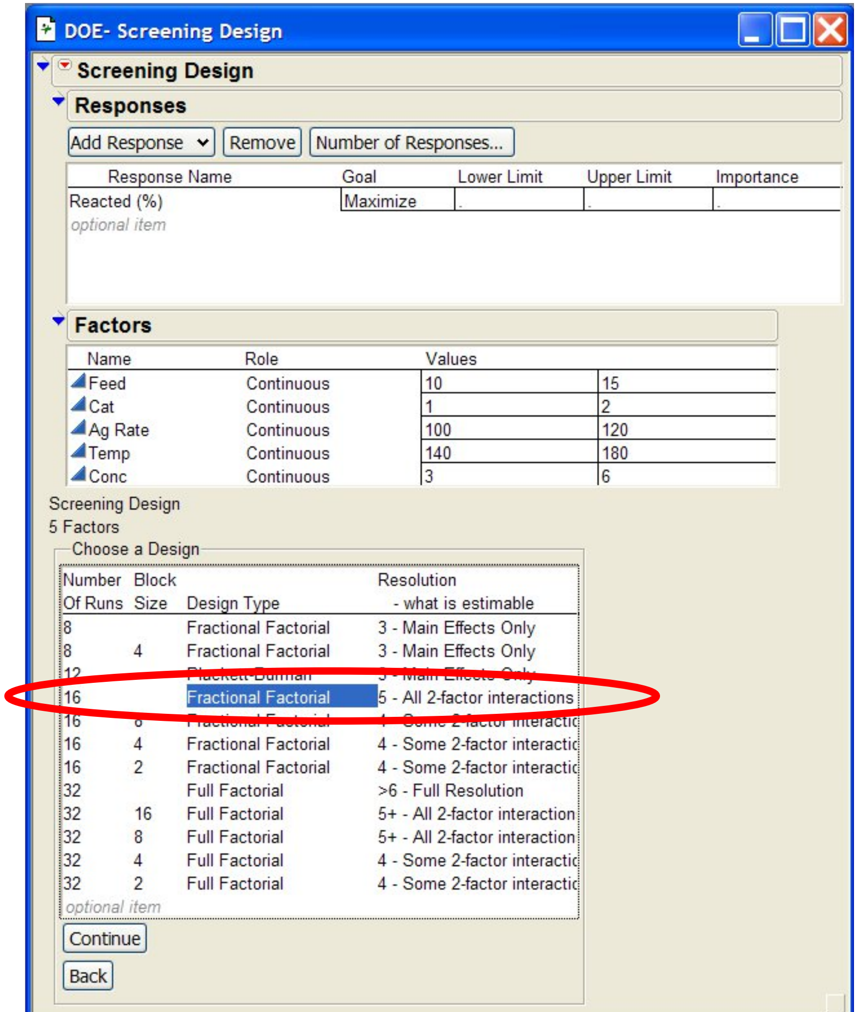
JMP DOE Design

- **JMP>DOE>Classical**
- **>Screening Design**
- Change the Y under Response Name to Reacted (%).
- We want to maximize the response
- Select continue.



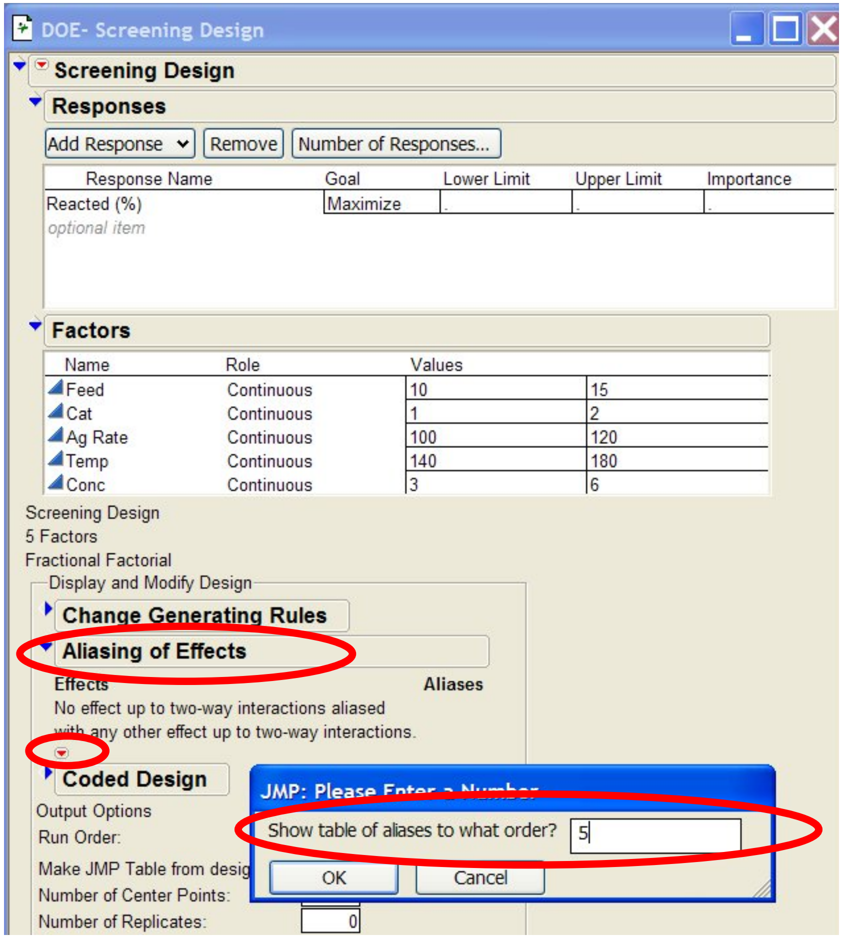
JMP DOE Design *(Cont.)*

- Since we want to conduct a ½ Fractional Factorial with no block effects, select 16.
- Note: $2^5=32$, $2^{5-1}=16$
- Note: This is **Resolution V** Design [Alias Structure Ex. 1=2345, 12=345, 35=_____, 5=_______]
- Select **Continue.**



JMP DOE Design *(Cont.)*

- Select Aliasing of Effects.
- Then quick on the Red Triangle and for the aliases to what order, enter 5.
- Select OK.



JMP DOE Design *(Cont.)*

Note: C=12345 is called the
Defining Relation, I

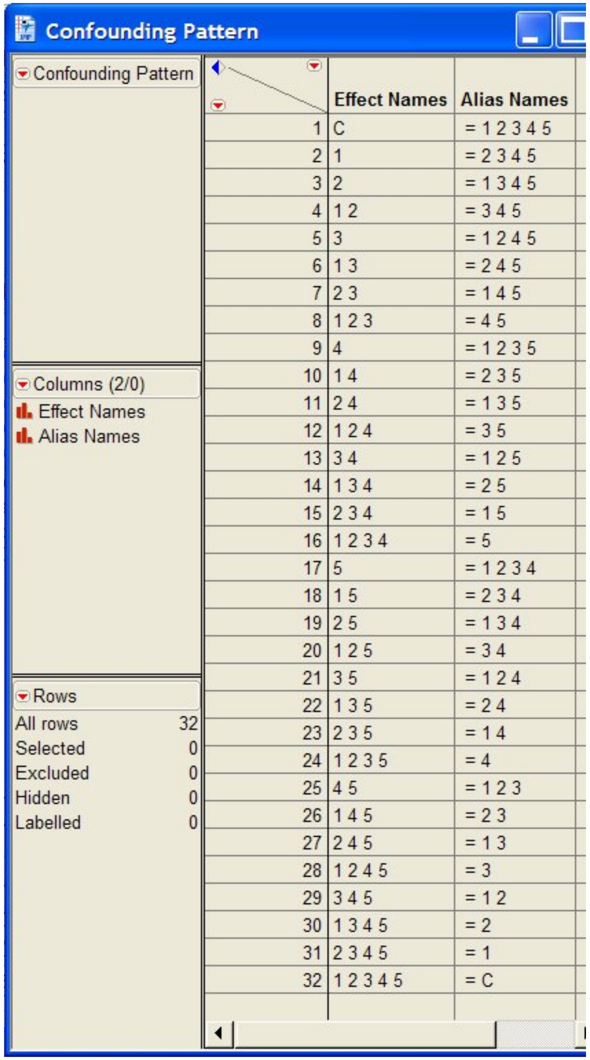
1*12345 = 2345 or
1=2345

12*12345 = 345 or
12 = 345

Use the defining relation to compute the alias structure for:

14*12345 = ____
14 = ____

4*12345 = ____
4 = ____



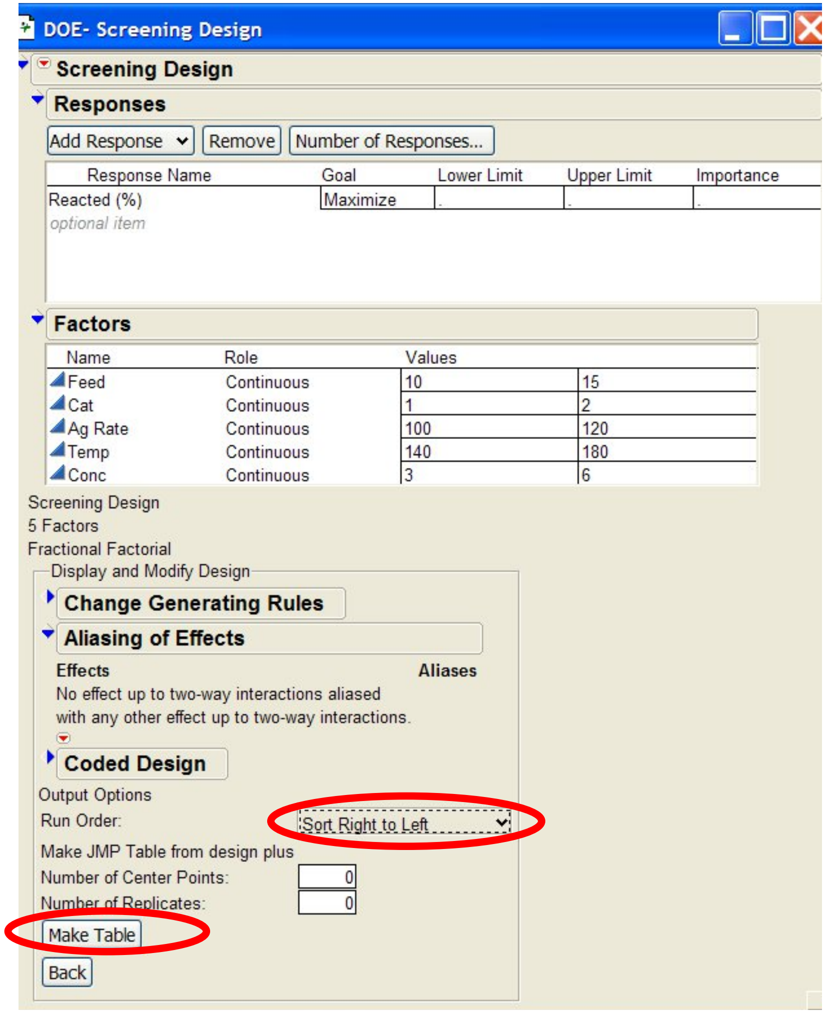
	Effect Names	Alias Names
1	C	= 1 2 3 4 5
2	1	= 2 3 4 5
3	2	= 1 3 4 5
4	1 2	= 3 4 5
5	3	= 1 2 4 5
6	1 3	= 2 4 5
7	2 3	= 1 4 5
8	1 2 3	= 4 5
9	4	= 1 2 3 5
10	1 4	= 2 3 5
11	2 4	= 1 3 5
12	1 2 4	= 3 5
13	3 4	= 1 2 5
14	1 3 4	= 2 5
15	2 3 4	= 1 5
16	1 2 3 4	= 5
17	5	= 1 2 3 4
18	1 5	= 2 3 4
19	2 5	= 1 3 4
20	1 2 5	= 3 4
21	3 5	= 1 2 4
22	1 3 5	= 2 4
23	2 3 5	= 1 4
24	1 2 3 5	= 4
25	4 5	= 1 2 3
26	1 4 5	= 2 3
27	2 4 5	= 1 3
28	1 2 4 5	= 3
29	3 4 5	= 1 2
30	1 3 4 5	= 2
31	2 3 4 5	= 1
32	1 2 3 4 5	= C

- Now **Minimize** the Confounding Pattern Data Table

JMP DOE Design *(Cont.)*

- Under the Output Options line, click on bar next to word Randomize and select **Sort Right to Left.** (This for educational purposes only. You would want to **randomize** the real design.)

- Select **Make Table**



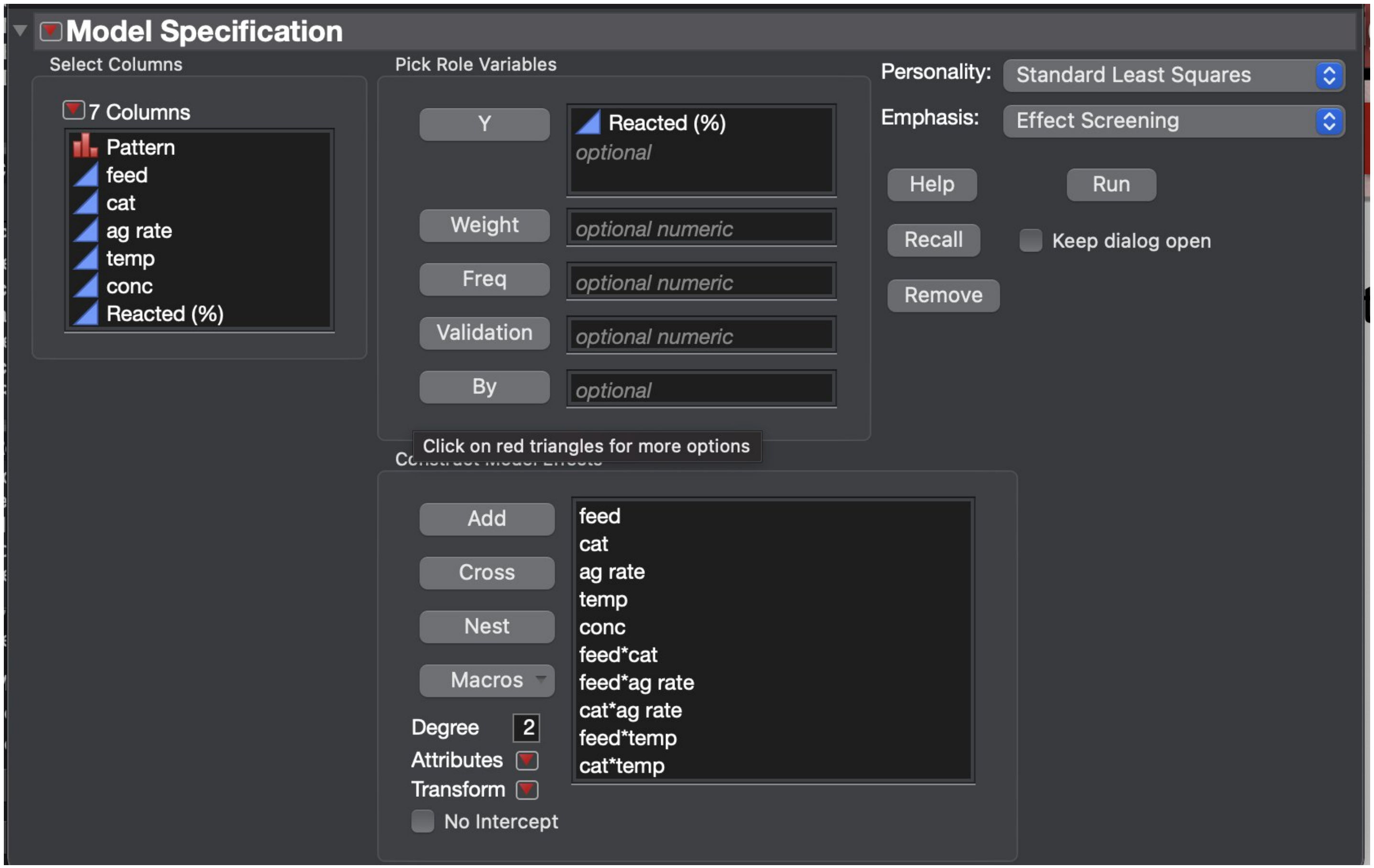
JMP DOE Design *(Cont.)*

- The next step in the experiment is to collect the data.
- Enter the **Reacted (%) data** into the table.

	Pattern	Feed	Cat	Ag Rate	Temp	Conc	Reacted (%)
1	+−−−−	15	1	100	140	3	53
2	−+−−−	10	2	100	140	3	63
3	−−+−−	10	1	120	140	3	53
4	+++−−	15	2	120	140	3	61
5	−−−+−	10	1	100	180	3	69
6	++−+−	15	2	100	180	3	93
7	+−++−	15	1	120	180	3	60
8	−+++−	10	2	120	180	3	95
9	−−−−+	10	1	100	140	6	56
10	++−−+	15	2	100	140	6	65
11	+−+−+	15	1	120	140	6	55
12	−++−+	10	2	120	140	6	67
13	+−−++	15	1	100	180	6	45
14	−+−++	10	2	100	180	6	78
15	−−+++	10	1	120	180	6	49
16	+++++	15	2	120	180	6	82

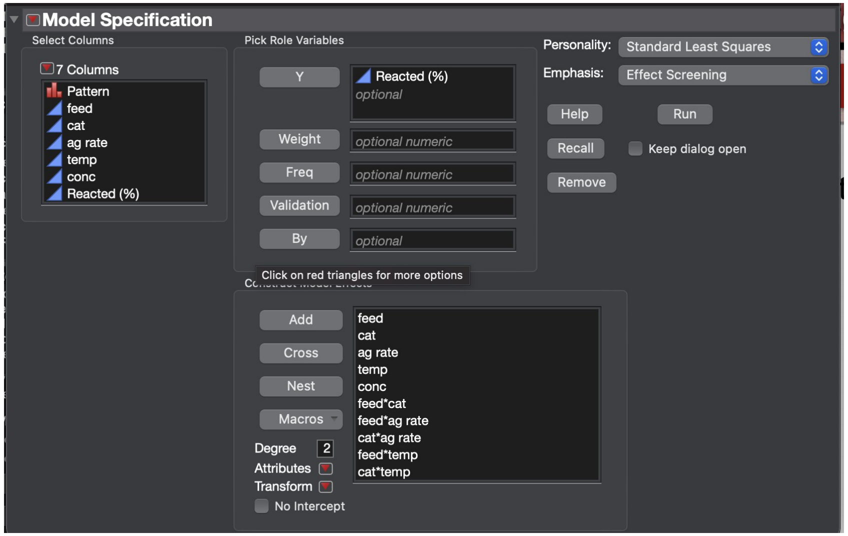
JMP DOE Design *(Cont.)*

- Select **Model** and **Run** when new screen appears.



JMP DOE Design (Cont.)

- Select **Run**
- Note: We will examine all of the main effects (ex. Feed Rate) and 2nd order interactions (Feed*Cat)
- Click Response Reacted %
- Under Regression Reports
- Select **Parameter Estimates and Summary of Fit**



JMP DOE Design *(Cont.)*

- JMP provides graphical and statistical analysis of the model as well as model predictions of the output, you must click on what you want.
- Since we do not have an estimate of the error in the current model, the statistical analysis cannot be completed.
- Let's continue to explore the results graphically!

Response Reacted (%)

Summary of Fit

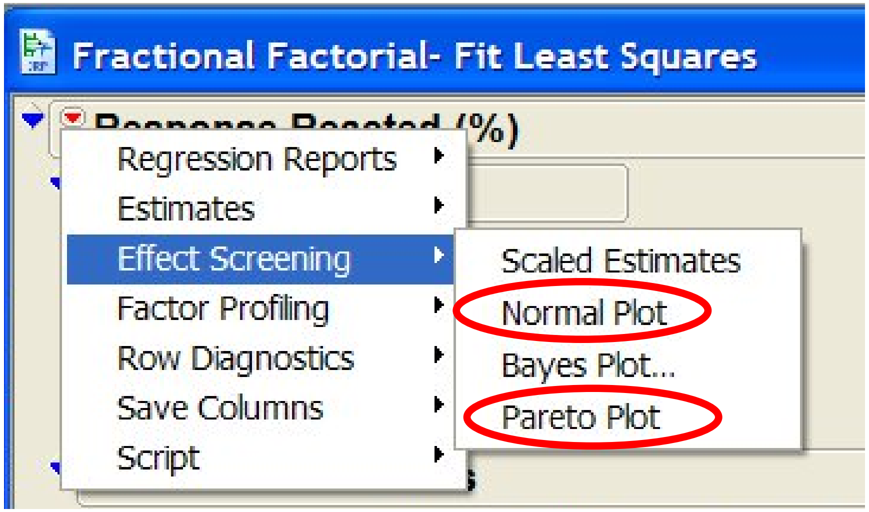
RSquare	1
RSquare Adj	.
Root Mean Square Error	.
Mean of Response	65.25
Observations (or Sum Wgts)	16

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.25	.	.	.
Feed(10,15)	-1	.	.	.
Cat(1,2)	10.25	.	.	.
Ag Rate(100,120)	0	.	.	.
Temp(140,180)	6.125	.	.	.
Conc(3,6)	-3.125	.	.	.
Feed*Cat	0.75	.	.	.
Feed*Ag Rate	0.25	.	.	.
Cat*Ag Rate	0.75	.	.	.
Feed*Temp	-0.375	.	.	.
Cat*Temp	5.375	.	.	.
Ag Rate*Temp	0.125	.	.	.
Feed*Conc	0.625	.	.	.
Cat*Conc	0.625	.	.	.
Ag Rate*Conc	1.125	.	.	.
Temp*Conc	-4.75	.	.	.

JMP DOE Design *(Cont.)*

- Under **Response Reacted (%)** select **Effect Screening>Pareto Plot**
- Next, select **Effect Screening>Normal Plot**



JMP DOE Design *(Cont.)*

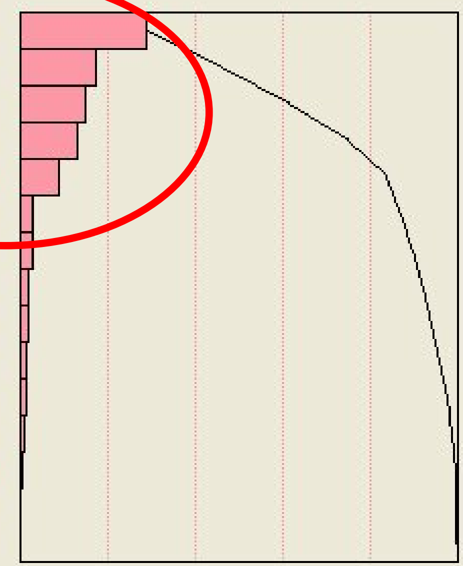
- The Pareto Plot shows us which factors most influence the response.
- Factors and interactions to keep in the model:

  Cat, Temp, Conc,

  Cat*Temp, Temp*Conc
- Note: The estimates are the coefficients of the prediction equation.

Pareto Plot of Estimates

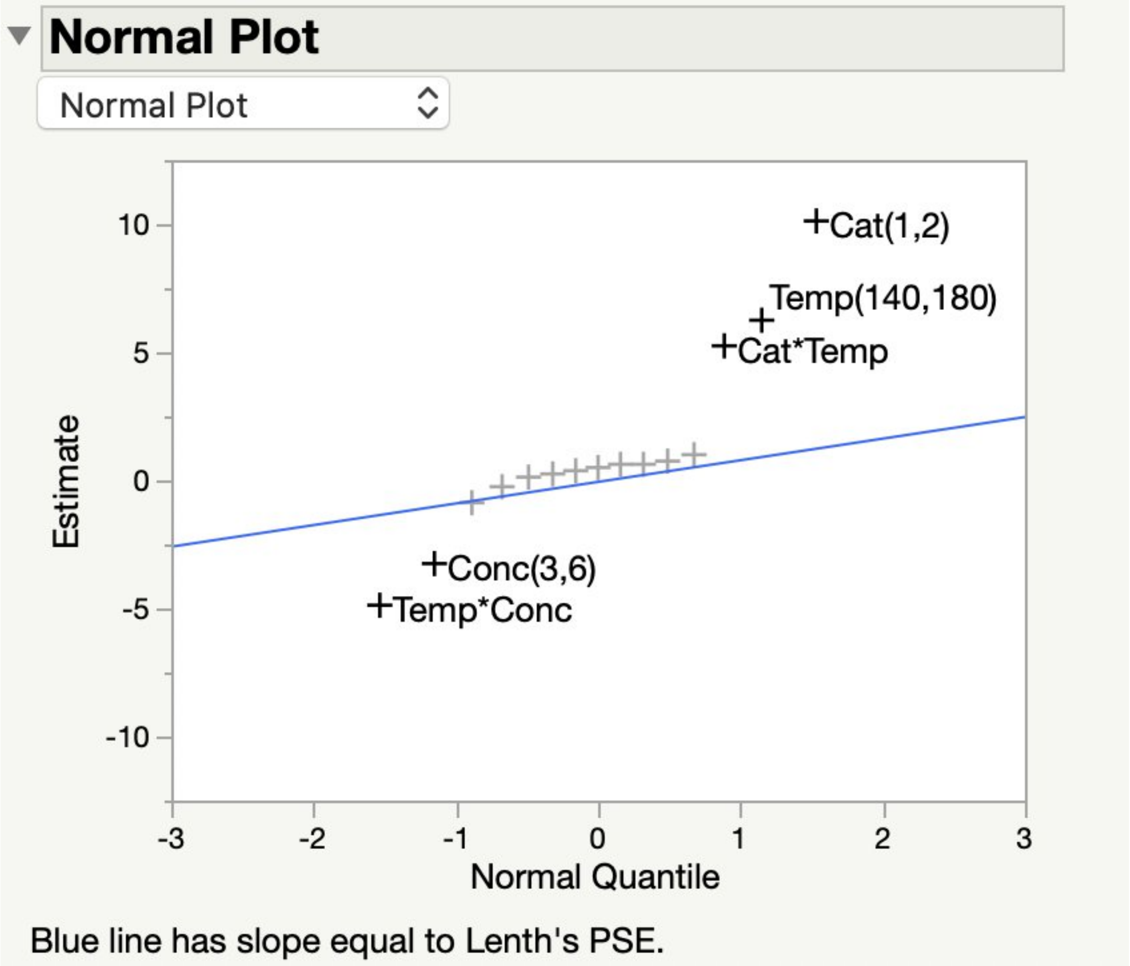
Term	Estimate
Cat(1,2)	10.25000
Temp(140,180)	6.12500
Cat*Temp	5.37500
Temp*Conc	-4.75000
Conc(3,6)	-3.12500
Ag Rate*Conc	1.12500
Feed(10,15)	-1.00000
Feed*Cat	0.75000
Cat*Ag Rate	0.75000
Cat*Conc	0.62500
Feed*Conc	0.62500
Feed*Temp	-0.37500
Feed*Ag Rate	0.25000
Ag Rate*Temp	0.12500
Ag Rate(100,120)	0.00000



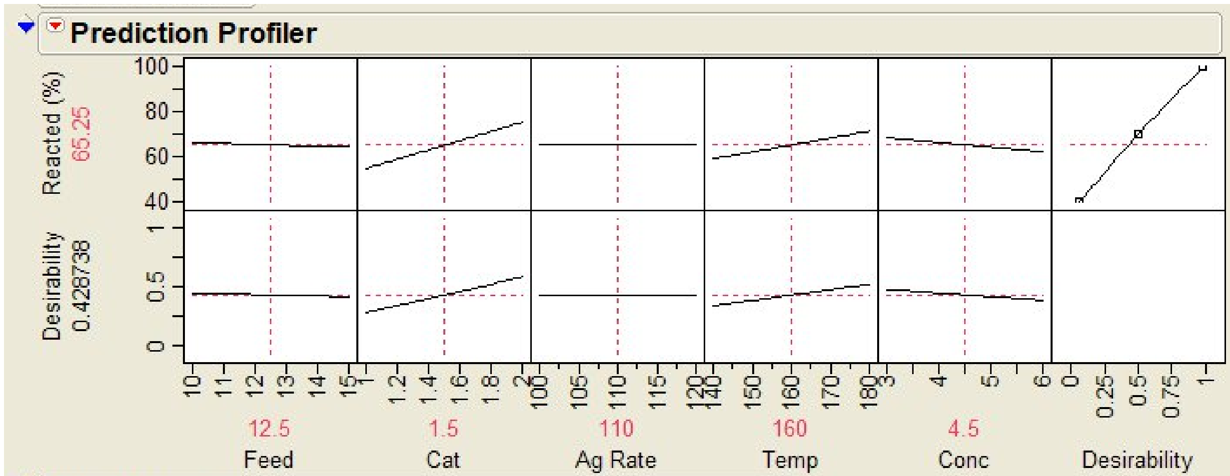
Note: If an interaction is used, then its main factors must be kept in the model [Ex. Temp*Conc, the Temp and Conc must be in the model].

JMP DOE Design *(Cont.)*

- The **Normal Plot** also shows us which factors most influence the response.
- The normal plot graphically shows the "outliers" (the factors that tend to be significantly different from the "random noise" of the experiment).



JMP DOE Design *(Cont.)*



- Before we reduce the model and keep only the factors that have the greatest effect on the response, look at the Prediction Profiler.
- Is there any useful information in the factors that were not considered significant (Feed, Ag rate)? Think about the optimizing the process. For the process to run, a level of each factor must be selected.

JMP DOE Design *(Cont.)*

- Now, let's reduce the model to include only those factors having the most influence on the response.
- Click on the Fit Model Window.

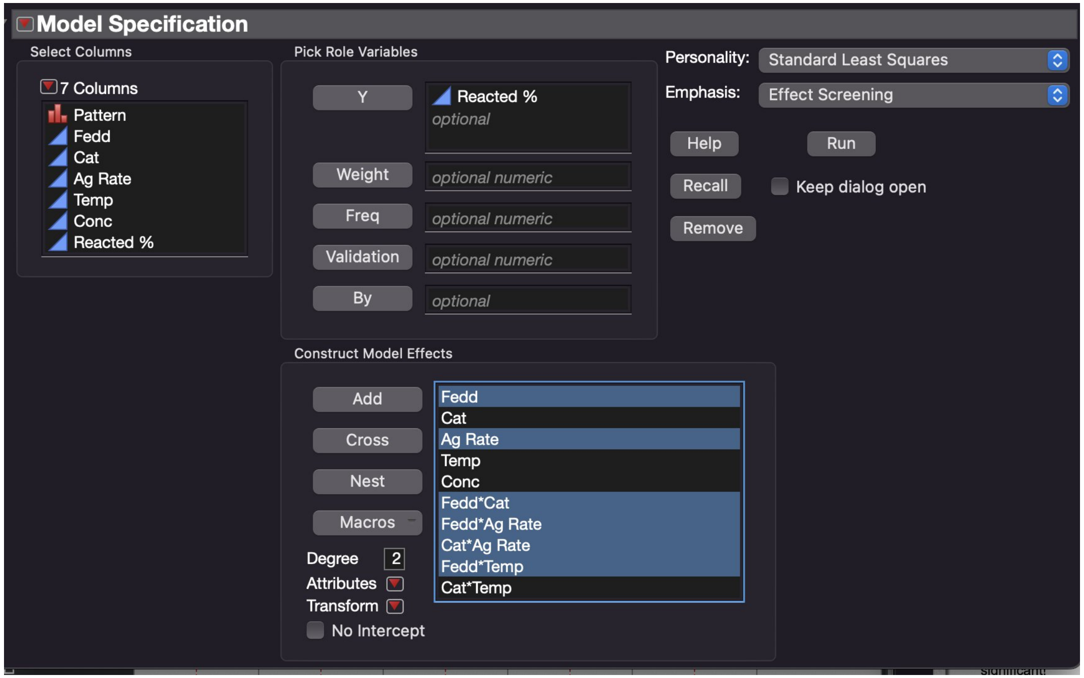
(Model Dialog) Or Under Analyze

- **Remove** all model factors <u>except</u>:

  Cat, Temp, Conc,

  Cat*Temp, Temp*Conc

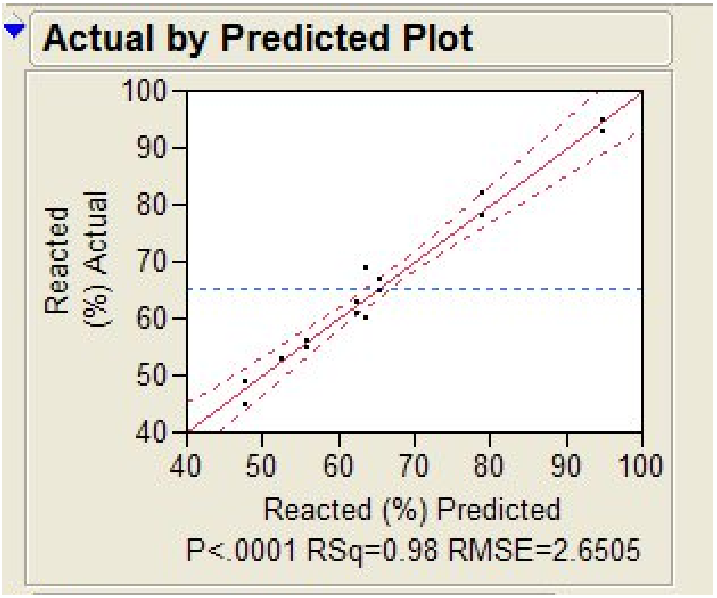
- **Run Model.**



JMP DOE Design *(Cont.)*

Actual by Predicted shows:

1) p<.0001, the model is significant!
2) RSq = 0.98, 98% of the variation can be explained by the model. (This is a good model!)
3) Graphically, the confidence bands are tight around the diagonal solid line (The model is a close approximation of the data).



JMP DOE Design *(Cont.)*

- Expanded Estimates show the Prediction Formula Coefficients.
- Effect Tests show that all of the model factors are significant.

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	65.25	0.662618	98.47	<.0001*
Cat(1,2)	10.25	0.662618	15.47	<.0001*
Temp(140,180)	6.125	0.662618	9.24	<.0001*
Conc(3,6)	-3.125	0.662618	-4.72	0.0008*
Cat*Temp	5.375	0.662618	8.11	<.0001*
Temp*Conc	-4.75	0.662618	-7.17	<.0001*

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Cat(1,2)	1	1	1681.0000	239.2883	<.0001*
Temp(140,180)	1	1	600.2500	85.4448	<.0001*
Conc(3,6)	1	1	156.2500	22.2420	0.0008*
Cat*Temp	1	1	462.2500	65.8007	<.0001*
Temp*Conc	1	1	361.0000	51.3879	<.0001*