JMP Analysis *(Cont.)*

- **Step 7: Construct a mathematical model for Brand A.**

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	64.25	0.456435	140.76	<.0001*
Temp(160,180)	11.5	0.456435	25.20	0.0001*
Time(5,15)	-2.5	0.456435	-5.48	0.0120*
Oil[Brand A]	-0.75	0.456435	-1.64	0.1989
Temp*Oil[Brand A]	-5	0.456435	-10.95	0.0016*

- Hardness = 64.25 + 11.50(Temp) – 2.5(Time)

  - 0.75 – 5.00 (Temp*Oil Type)

Where Temp=-1,1 Time=-1,1 For Brand A

JMP Analysis *(Cont.)*

- **Construct a mathematical model for Brand B. Change the sign on the coefficients that contain Oil Type**

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	64.25	0.456435	140.76	<.0001*
Temp(160,180)	11.5	0.456435	25.20	0.0001*
Time(5,15)	-2.5	0.456435	-5.48	0.0120*
Oil Type[Brand A] B	+0.75	0.456435	-1.64	0.1989
Temp*Oil Type[Brand A] B	+5	0.456435	-10.95	0.0016*

- Hardness = 64.25 + 11.50(Temp) – 2.5(Time)

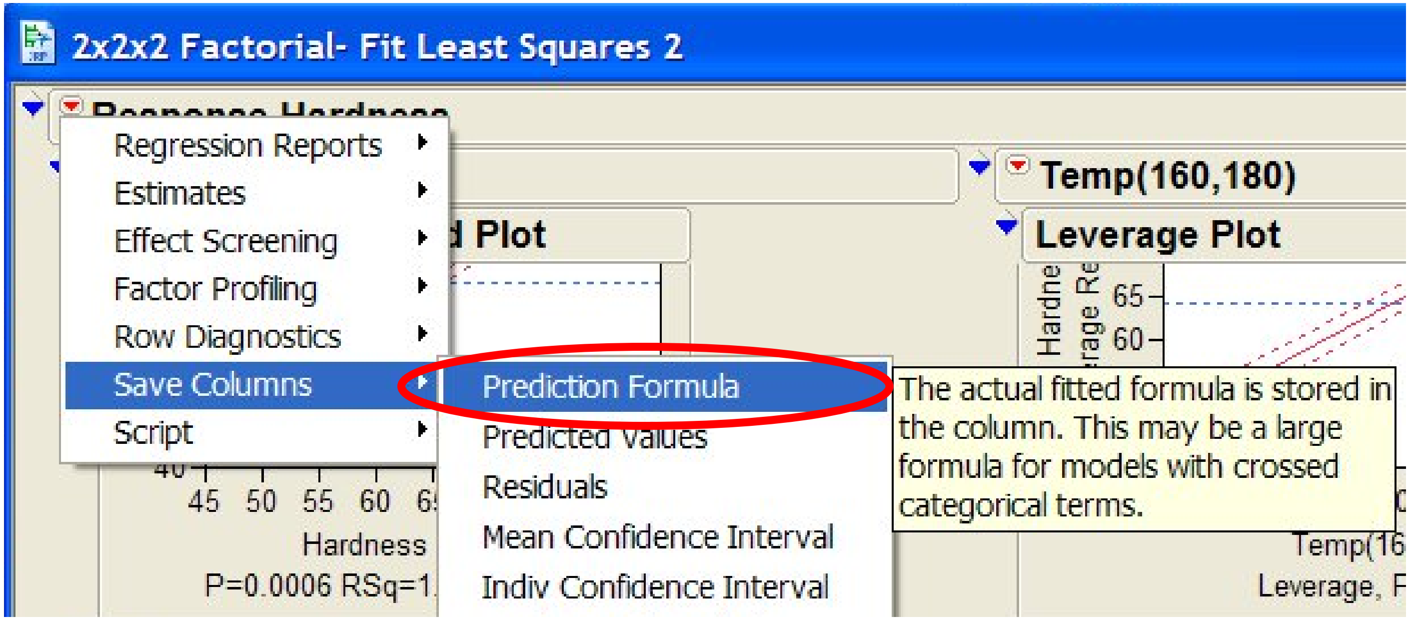
  + 0.75 + 5.00 (Temp*Oil Type)

  Where Temp=-1,1 Time=-1,1 For <u>Brand B</u>

Questions

- **Question 1:** What is the hardness when Temp = 1, Time=1 and Oil=Brand A?
- **Question 2:** What does that value represent?
- **Question 3:** What is the hardness when Temp = 1, Time=1 and Oil=Brand B?
- Check your answers using JMP.

JMP Analysis *(Cont.)*



- **Save Columns>Prediction Formula**

JMP Analysis *(Cont.)*

2x2x2 Factorial

2x2x2 Factorial
Design 2x2x2 Factorial
Screening
Model

Columns (7/0)
Pattern
Temp
Time
Oil
Hardness
Predicted Hardness

	Pattern	Temp	Time	Oil	Hardness	Predicted Hardness	Pred Formula Hardness
1	−−1	160	5	Brand A	60	59.5	59.5
2	+−1	180	5	Brand A	72	72.5	72.5
3	−+1	160	15	Brand A	54	54.5	54.5
4	++1	180	15	Brand A	68	67.5	67.5
5	−−2	160	5	Brand B	52	51	51
6	+−2	180	5	Brand B	83	84	84
7	−+2	160	15	Brand B	45	46	46
8	++2	180	15	Brand B	80	79	79

- **Maximum Hardness: Temp = 180, Time = 5, Oil Brand B**

Conclusions

- **Step 8**: Translate the mathematical model into process terms. Formulate conclusions and recommendations.
  - Conclusions: Temperature has the biggest effect on Hardness and will be a controlling factor in determining the hardness levels. Hardness can be maximized using Oil Type Brand B at Temp = 180°C and Time = 5 sec.
- **Step 9**: Replicate optimum conditions. Plan the next experiment or institutionalize the change.

Adding Center Points to 2^K Factorials

- There is always a risk in 2-level designs of missing a curvilinear relationship by only including two levels of the input variable.
- The addition of “Center Points” is an efficient way to test for curvature without adding a large number of experimental runs.

Example with Center Points

- A process engineer wants to improve the yield for two different die-castings. There are two inputs of interest: pressure and temperature. The engineer decides to conduct the experiment using a 2x2 design augmented with five center points to estimate experimental error and curvature.

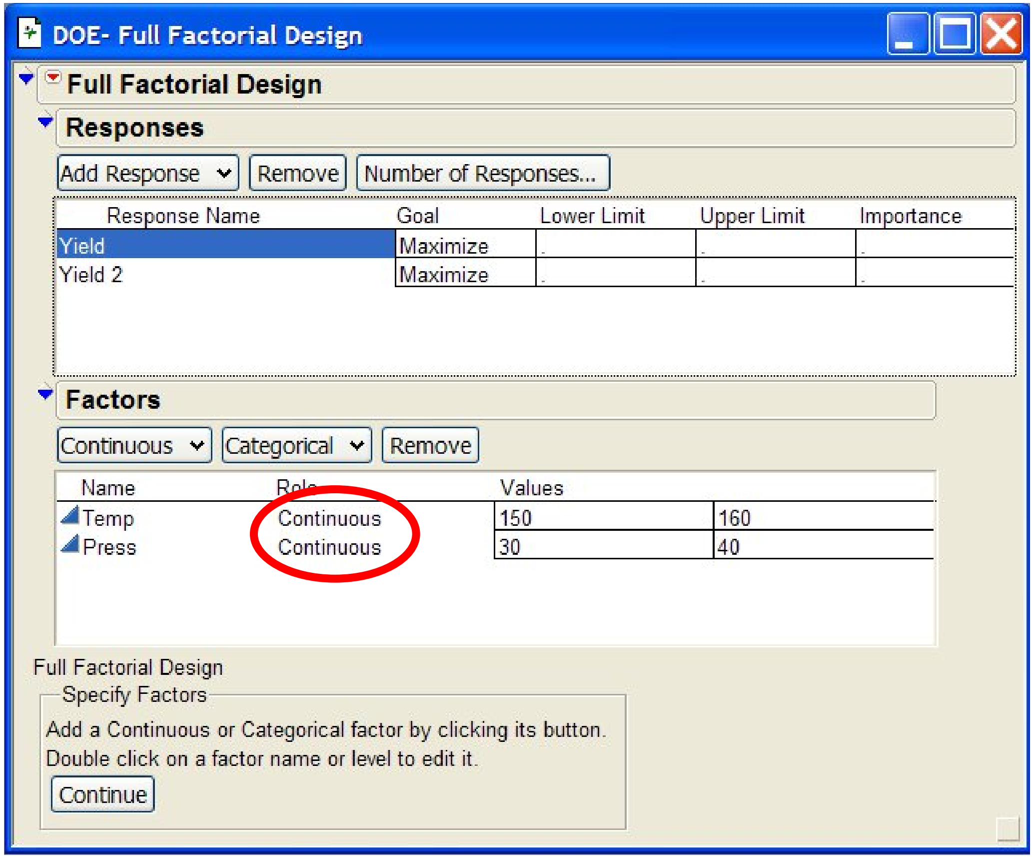
Example with Center Points

- State the problem: A process engineer wants to improve yield for two different die-castings. There are two inputs of interest: pressure and temperature.
- State the factors and levels of interest, and create a JMP experimental data sheet.
  - Temperature: 150, 160
  - Pressure: 30, 40

Design Matrix – Using JMP

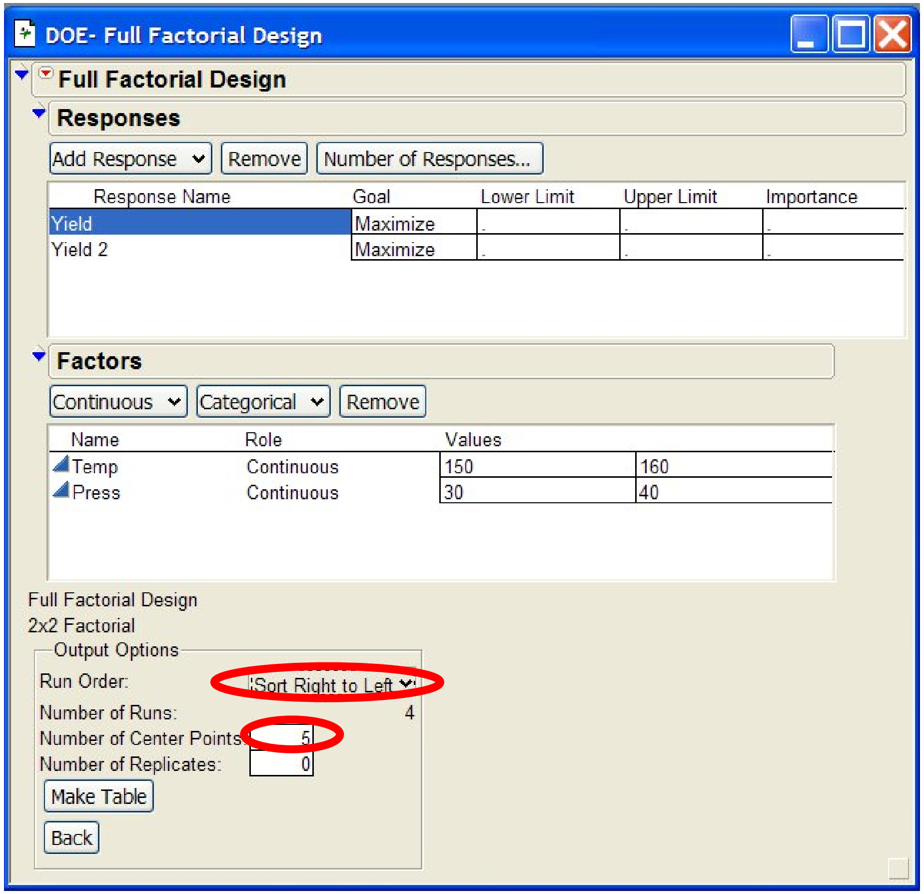
- **JMP>DOE>Classical>Full Factorial Design**

In order to insert center points into JMP, the variables are selected as continuous.



JMP DOE Design *(Cont.)*

- Change the **Number of Center Points** to *5.*
- Change **Randomize** to **Sort Right to Left** for teaching purposes.
- Select **Make Table**.



JMP DOE Design *(Cont.)*

2x2 Factorial

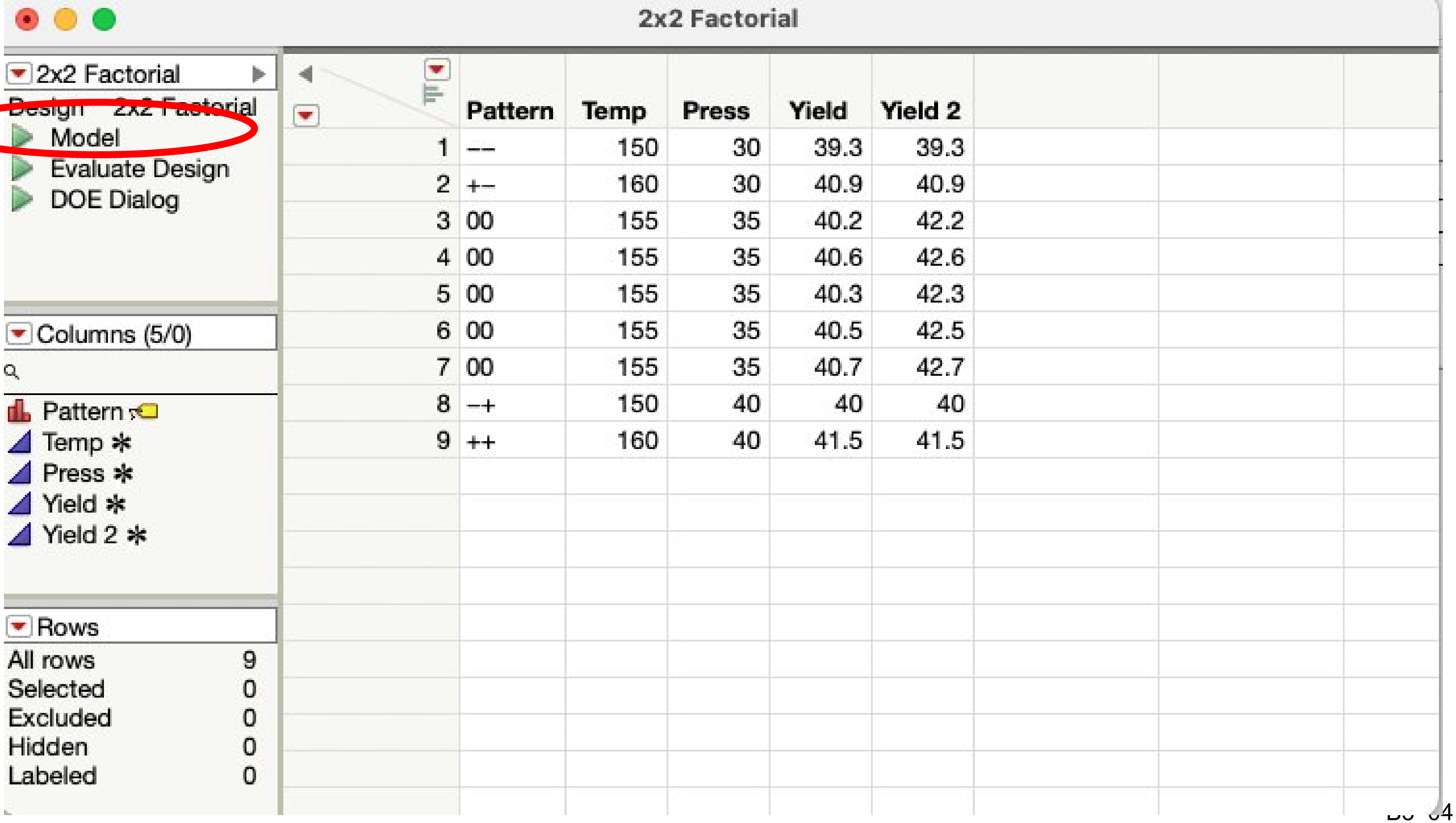
2x2 Factorial
Design 2x2 Factorial
Screening
Model

Columns (5/0)
Pattern
Temp *
Press *
Yield *
Yield 2 *

	Pattern	Temp	Press	Yield	Yield 2
1	−−	150	30	39.3	39.3
2	+−	160	30	40.9	40.9
3	00	155	35	40.2	42.2
4	00	155	35	40.6	42.6
5	00	155	35	40.3	42.3
6	00	155	35	40.5	42.5
7	00	155	35	40.7	42.7
8	−+	150	40	40	40
9	++	160	40	41.5	41.5

Analysis With JMP *(Cont.)*

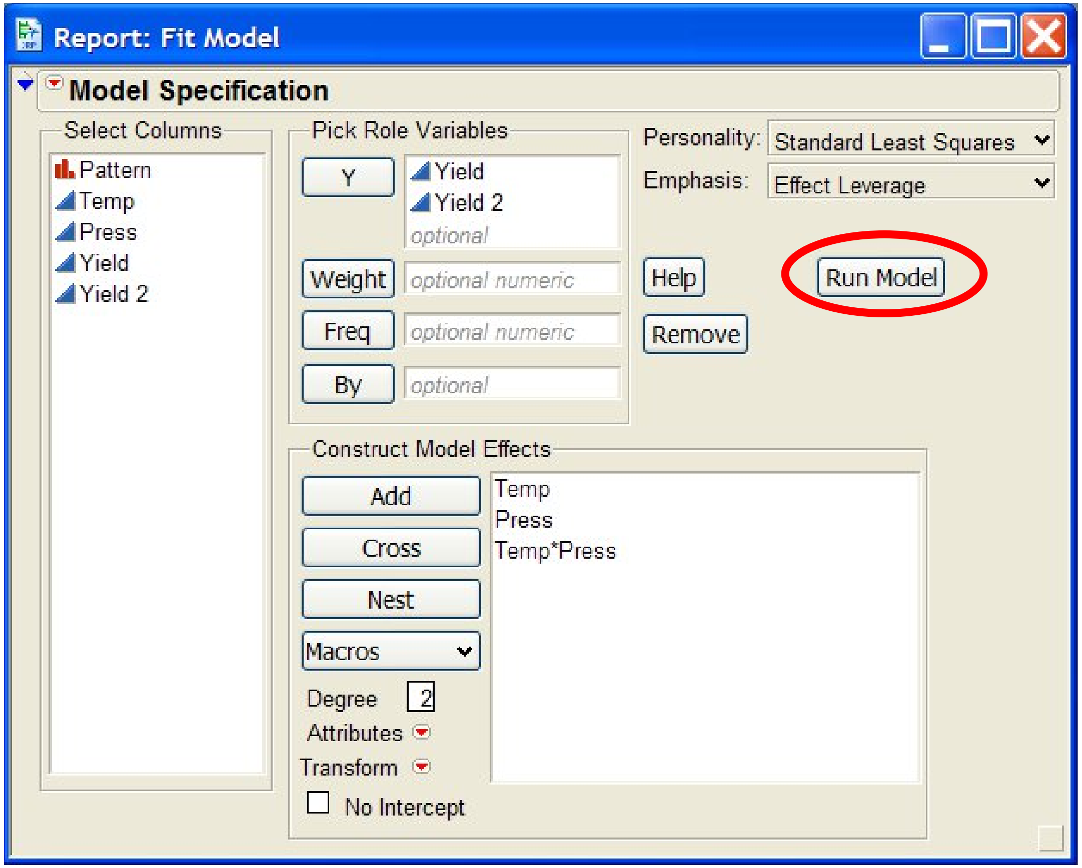
- From the **Model select the Green Triangle**



	Pattern	Temp	Press	Yield	Yield 2
1	−−	150	30	39.3	39.3
2	+−	160	30	40.9	40.9
3	00	155	35	40.2	42.2
4	00	155	35	40.6	42.6
5	00	155	35	40.3	42.3
6	00	155	35	40.5	42.5
7	00	155	35	40.7	42.7
8	−+	150	40	40	40
9	++	160	40	41.5	41.5

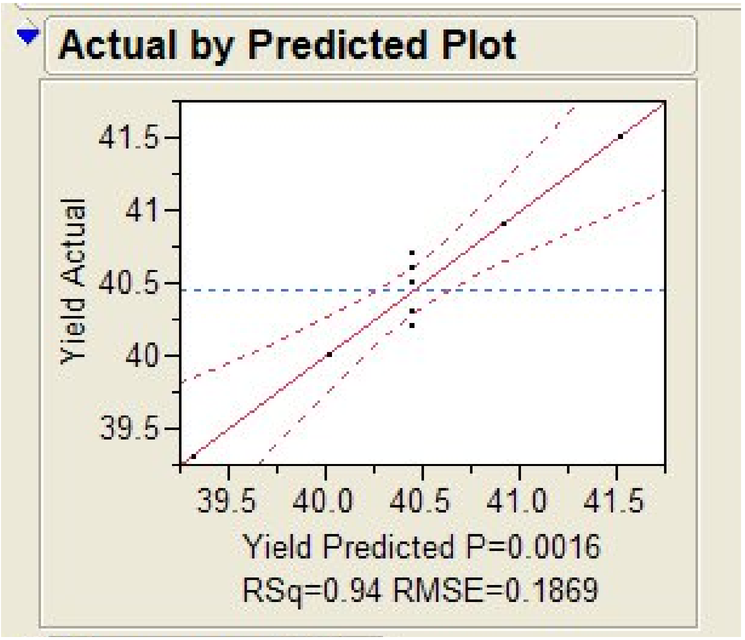
JMP Analysis *(Cont.)*

- Click **Run**

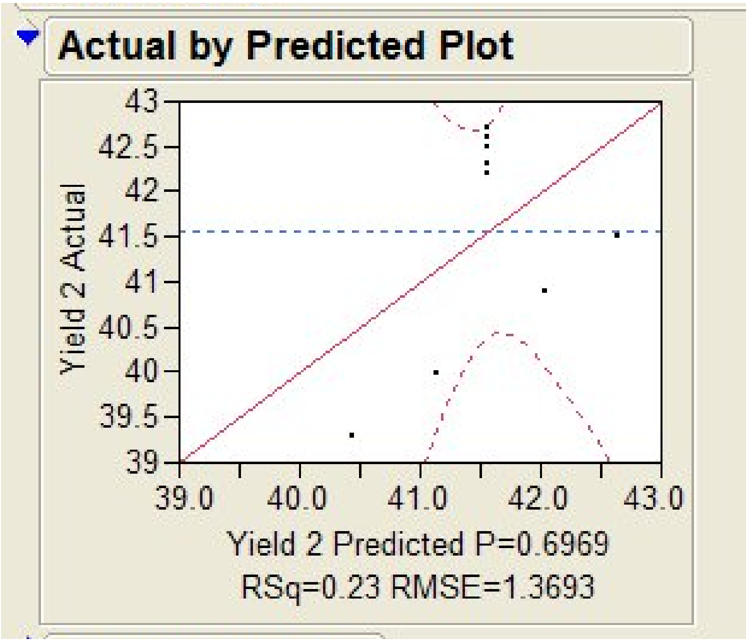


JMP Analysis *(Cont.)*

Yield



Yield 2



JMP Analysis *(Cont.)*

Yield

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	40.444444	0.062311	649.07	<.0001*
Temp(150,160)	0.775	0.093467	8.29	0.0004*
Press(30,40)	0.325	0.093467	3.48	0.0177*
Temp*Press	-0.025	0.093467	-0.27	0.7998

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Temp(150,160)	1	1	2.4025000	68.7520	0.0004*
Press(30,40)	1	1	0.4225000	12.0906	0.0177*
Temp*Press	1	1	0.0025000	0.0715	0.7998

Yield 2

Parameter Estimates

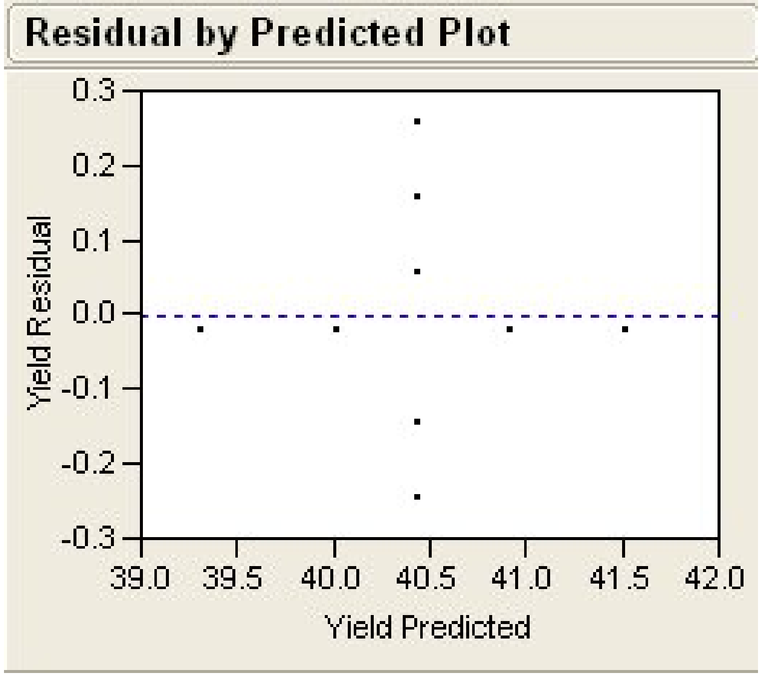
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	41.555556	0.456429	91.05	<.0001*
Temp(150,160)	0.775	0.684643	1.13	0.3090
Press(30,40)	0.325	0.684643	0.47	0.6550
Temp*Press	-0.025	0.684643	-0.04	0.9723

Effect Tests

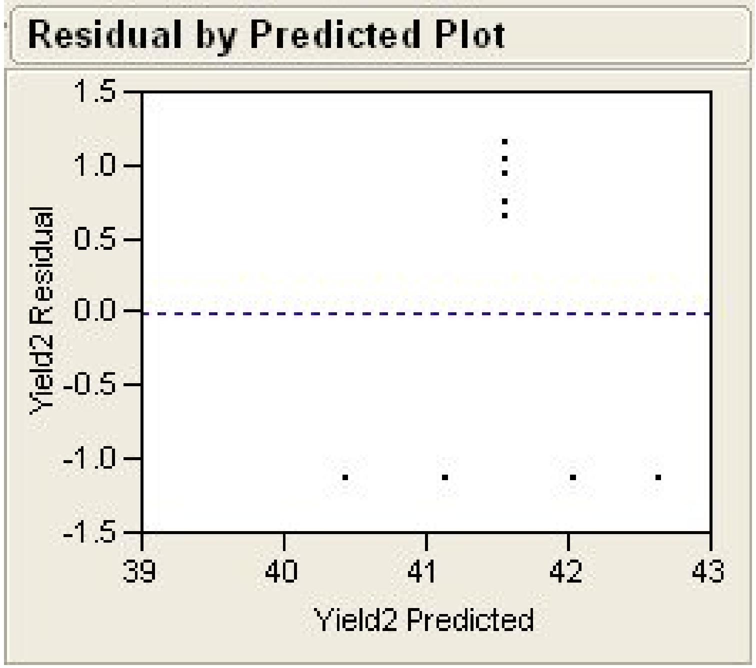
Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Temp(150,160)	1	1	2.4025000	1.2814	0.3090
Press(30,40)	1	1	0.4225000	0.2253	0.6550
Temp*Press	1	1	0.0025000	0.0013	0.9723

JMP Analysis *(Cont.)*

Yield

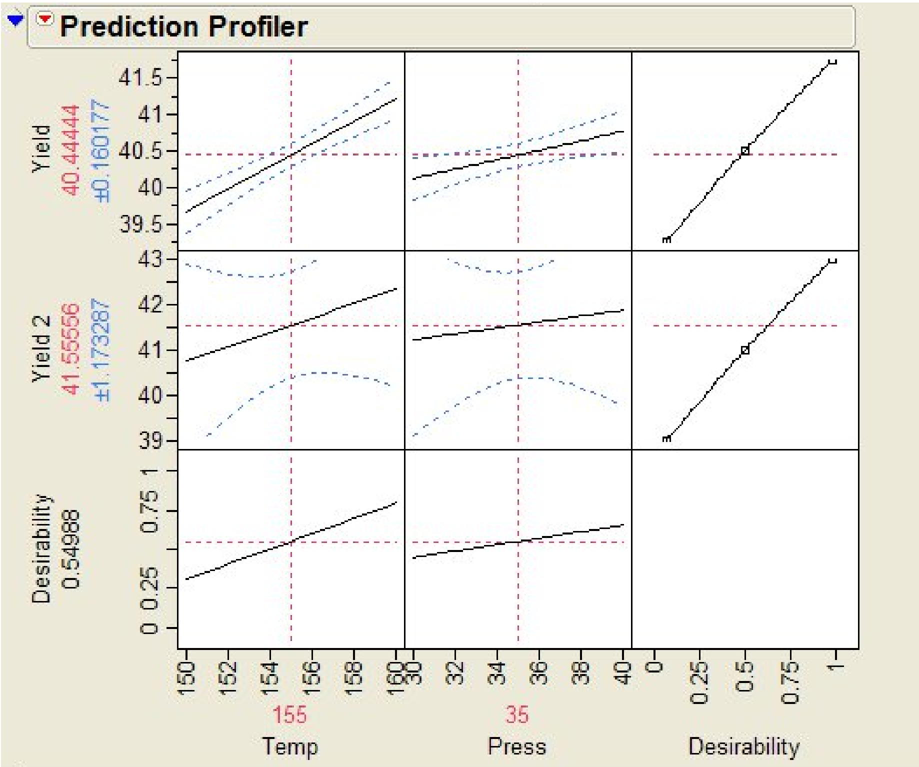


Yield 2



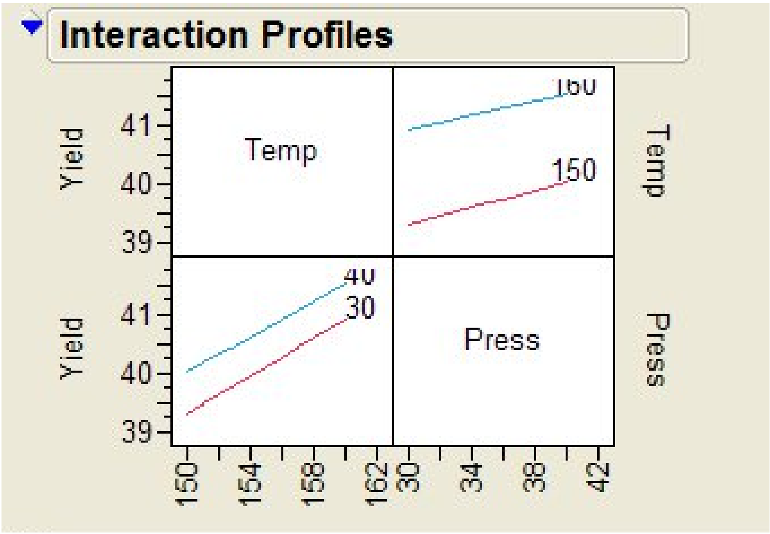
JMP Analysis *(Cont.)*

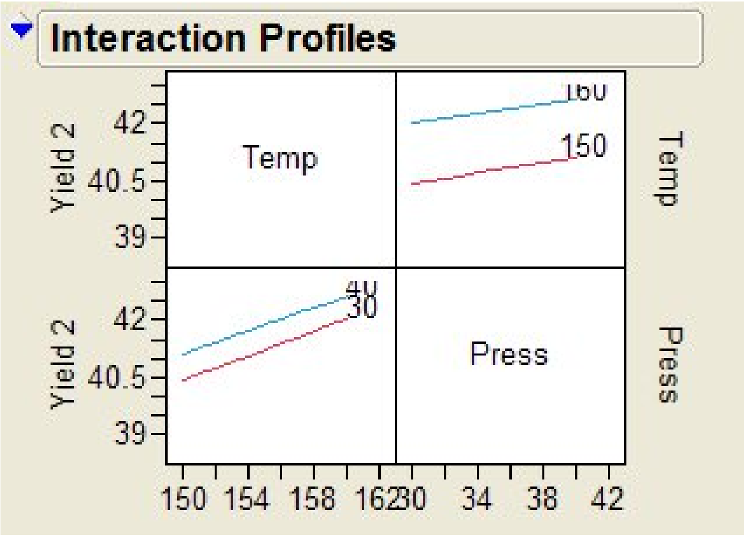
- From the **Response Yield Red Triangle,** select **Factor Profiling>Profiler**



JMP Analysis *(Cont.)*

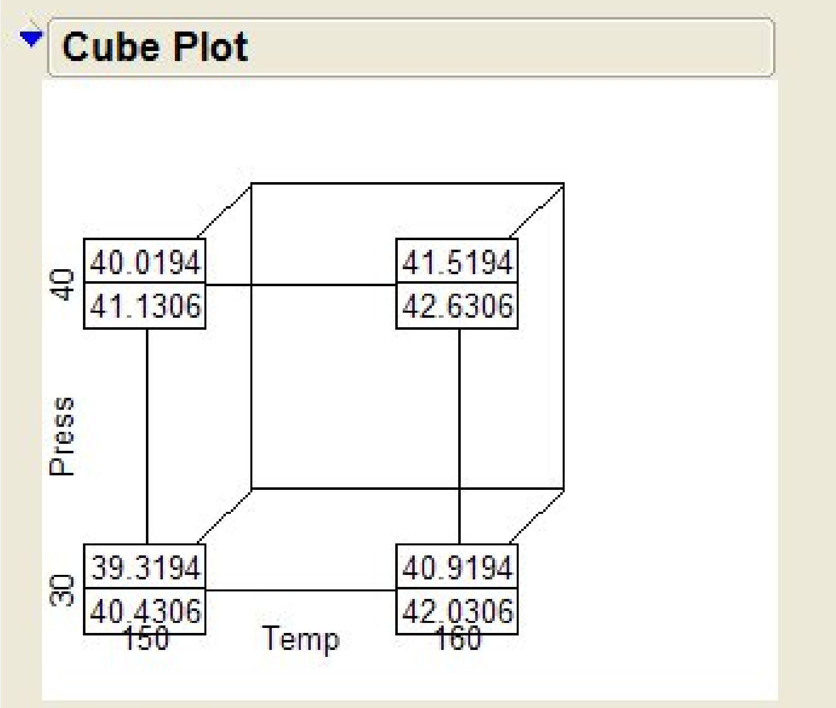
- From the **Response Yield Red Triangle**, select **Factor Profiling>Interaction Plots**





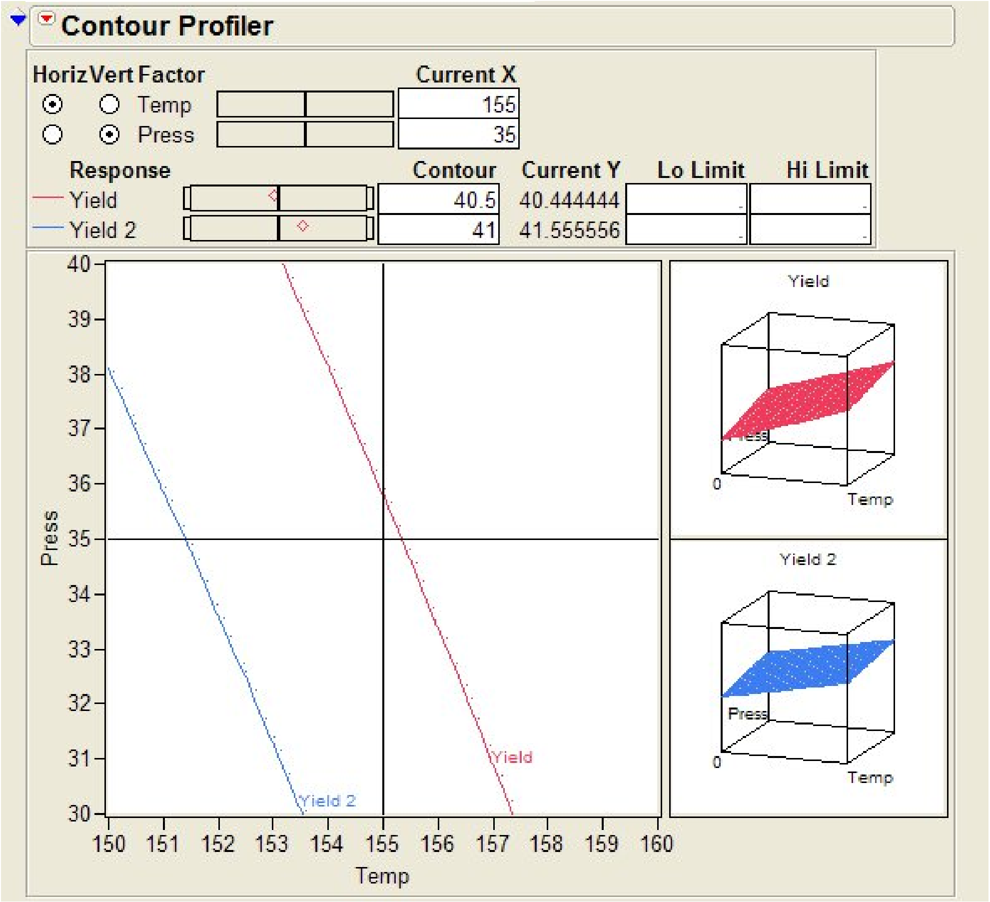
JMP Analysis *(Cont.)*

- From the **Response Yield Red Triangle**, select **Factor Profiling>Cube Plots**



JMP Analysis *(Cont.)*

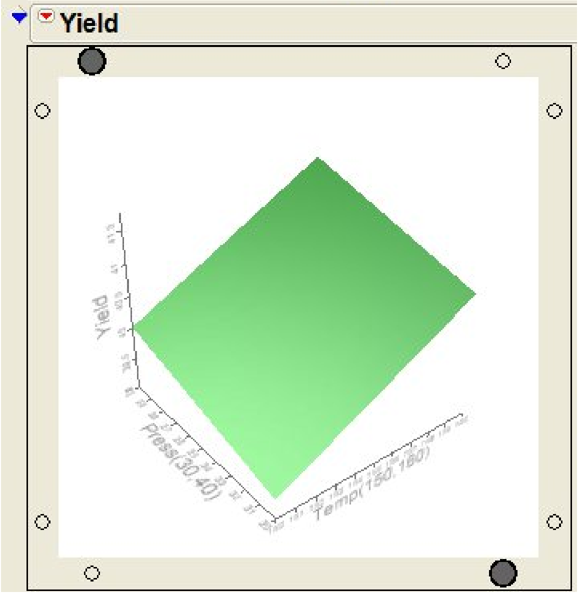
- From the **Response Yield Red Triangle,** select **Factor Profiling>Contour Profiler**



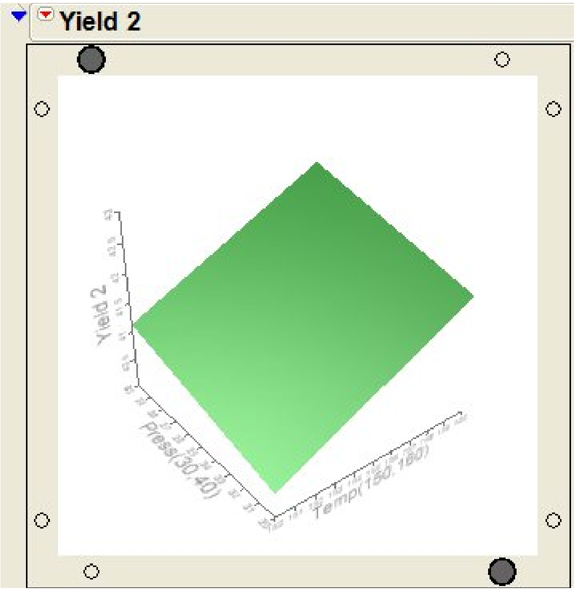
JMP Analysis *(Cont.)*

- From the **Response Yield Red Triangle**, select **Factor Profiling>Surface Profiler**

Yield



Yield 2



Mathematical Model

- State the mathematical model obtained. A simpler model is always preferable (Why?)
  - For the yield of product 1 we can reduce the model because the interaction is not significant.
  - For the yield of product 2 (yield 2), we haven't yet obtained a working model (no main effects or interactions are significant). At this point, we simply know that curvature exists.

- Product 1:

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	40.444444	0.062311	649.07	<.0001
Temp	0.775	0.093467	8.29	0.0004
Pressure	0.325	0.093467	3.48	0.0177
Temp *Pressure	-0.025	0.093467	-0.27	0.7998

$$\text{Yield} = 40.44 + .775{*}\text{Temp} + .325{*}\text{Press}$$

  - Note: The above equation is for coded variables only!

- Product 2:
  - Step 4 shows that Curvature is significant. Stop there!
  - Re-do the DOE. Change your level settings.

Conclusions

- Translate the mathematical model into process terms. Formulate conclusions and recommendations.
  - For product 1: To increase yield, set the current process to run at 160 degrees and 40 psi.
    - We also recommend a follow-up investigation to see if yield can be further improved at factor levels not tested in this experiment.
  - For product 2: Conduct additional experiments to model curvature.
- Replicate optimum conditions. Plan the next experiment or institutionalize the change.

Adding A Block To 2^K Factorials

- **Blocking Variable:** A factor in an experiment that has undesired influence as a source of variability is called a “block.” A block can be a batch of material or a set of conditions likely to produce experimental runs that are more homogenous within the block (or batch) than between blocks.

Run	A	B	C	Block
1	-1	-1	-1	I
2	+1	-1	-1	I
3	-1	+1	-1	I
4	+1	+1	-1	I
5	-1	-1	+1	II
6	+1	-1	+1	II
7	-1	+1	+1	II
8	+1	+1	+1	II

Adding A Block To 2^K Factorials

- Suppose we wanted to run a 2x2x2 factorial. We would like to run the experiment under as homogeneous conditions as possible. But, we find that two batches of raw material are needed to run the entire experiment.

Run	A	B	C	Block
1	-1	-1	-1	I
2	+1	-1	-1	I
3	-1	+1	-1	I
4	+1	+1	-1	I
5	-1	-1	+1	II
6	+1	-1	+1	II
7	-1	+1	+1	II
8	+1	+1	+1	II

Adding A Block To 2^K Factorials

- It's easy to see that if we ran the first four runs with Batch 1 of raw material and the second four runs with Batch 2, we would completely "**confound**" Factor C. We could not distinguish between the effect due to Factor C and the effect of variation in the raw material.

Run	A	B	C	Block
1	-1	-1	-1	I
2	+1	-1	-1	I
3	-1	+1	-1	I
4	+1	+1	-1	I
5	-1	-1	+1	II
6	+1	-1	+1	II
7	-1	+1	+1	II
8	+1	+1	+1	II

Adding A Block To 2^K Factorials

- We must figure out a way to "spread" the raw material effect across the experiment so the differences in material batches are "seen" by all the main effects. Recall the expanded design matrix for a 2^3 factorial experiment showing all contrasts.

Adding A Block To 2^K Factorials

- In general, we make the assumption that "higher order interactions" are not significant (p-value > 0.05). Here we can use the contrast for the 3-way interactions to define our blocking variable. The new design would look like:

Run	A	B	C	A*B	A*C	B*C	A*B*C	Block
1	-1	-1	-1	1	1	1	-1	I
2	1	-1	-1	-1	-1	1	1	II
3	-1	1	-1	-1	1	-1	1	II
4	1	1	-1	1	-1	-1	-1	I
5	-1	-1	1	1	-1	-1	1	II
6	1	-1	1	-1	1	-1	-1	I
7	-1	1	1	-1	-1	1	-1	I
8	1	1	1	1	1	1	1	II

Adding A Block To 2^K Factorials

- We would run runs 1, 4, 6 and 7 with Batch 1 and runs 2, 3, 5 and 8 with Batch 2. This experiment would not allow us to test for the significance of the 3-way interaction, but it would allow us to investigate the main effects and 2-way interactions without worrying about confounding these effects with the Raw Material batch.

Note: In an actual experiment, you would randomize the runs within each Block. JMP will do that for you.

Run	A	B	C	Block
1	-1	-1	-1	I
2	1	-1	-1	II
3	-1	1	-1	II
4	1	1	-1	I
5	-1	-1	1	II
6	1	-1	1	I
7	-1	1	1	I
8	1	1	1	II

Run	A	B	C	Block
1	-1	-1	-1	I
4	1	1	-1	I
6	1	-1	1	I
7	-1	1	1	I
2	1	-1	-1	II
3	-1	1	-1	II
5	-1	-1	1	II
8	1	1	1	II

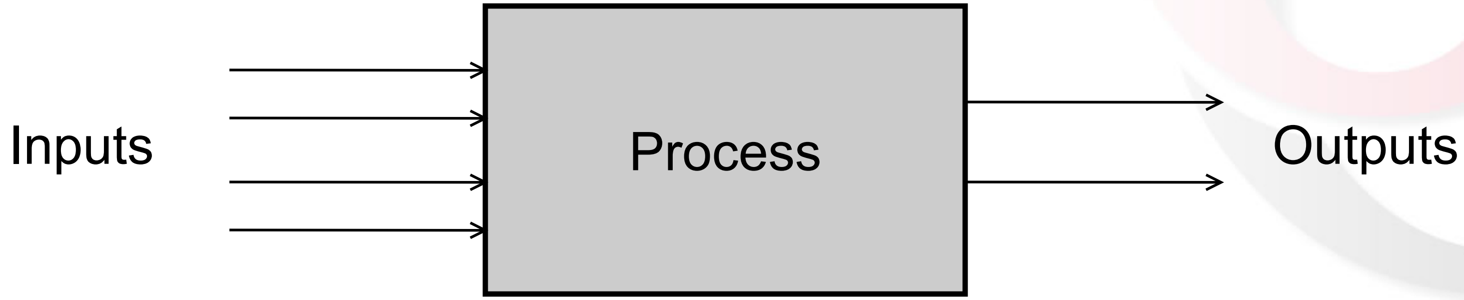
DESIGN OF EXPERIMENTS

Objectives

- Understand why designed experiments are used during the Improve stage of a project
- Define design of experiments (DOE) terms
- Design and analyze a simple experiment

Design of Experiments

Designed experiments allow six sigma teams to systematically determine the effect changing the values of process inputs has on the value of the process outputs.



Design of Experiments

You experiment to:

- Explore relationships in a process
- Predict process performance
- Confirm findings of earlier experiments and hypothesis testing

Steps to Experimentation

1. Define the problem
2. Establish the objective
3. Select the response variables (Output, Y)
4. Select the independent variables (Inputs, X's)
5. Choose the variable levels
6. Select the experimental design
7. Collect data
8. Analyze data
9. Draw statistical conclusions
10. Replicate results
11. Draw practical solutions
12. Implement solutions

Basic DOE Terms

Basic terms include:

- Factor
- Factor level
- Treatment
- Response
- Effect
- Experimental unit
- Run
- Replication

Fundamental Principles

3 Basic Principles of experimental design:

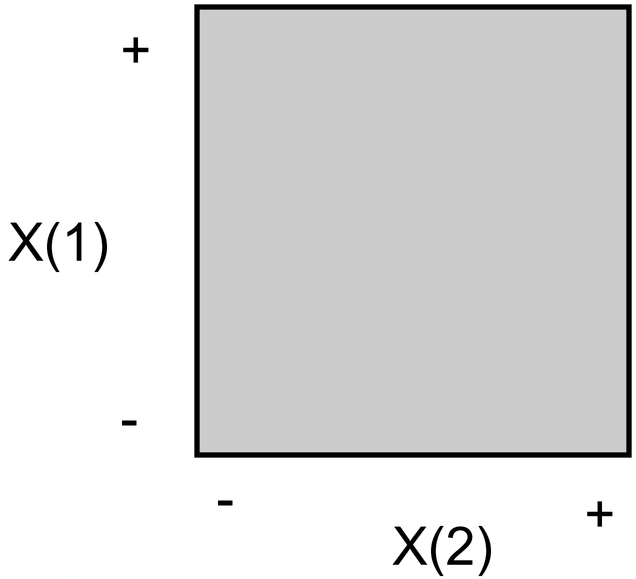
1. Replication or repetition of the basic experiment provides a better estimate of experimental error and allows the experimenter to get better estimates of effects.
2. Randomization, where both the allocation of experimental materials and the order of runs are randomly determined, typically makes the independence assumption valid and helps to average out the effect of nuisance factors.
3. Blocking, a technique that reduces the variability due to nuisance factors, improves the precision of the estimates of factors of interest.

Basic Designs

- Full factorial designs
- Screening designs
- Response surface designs

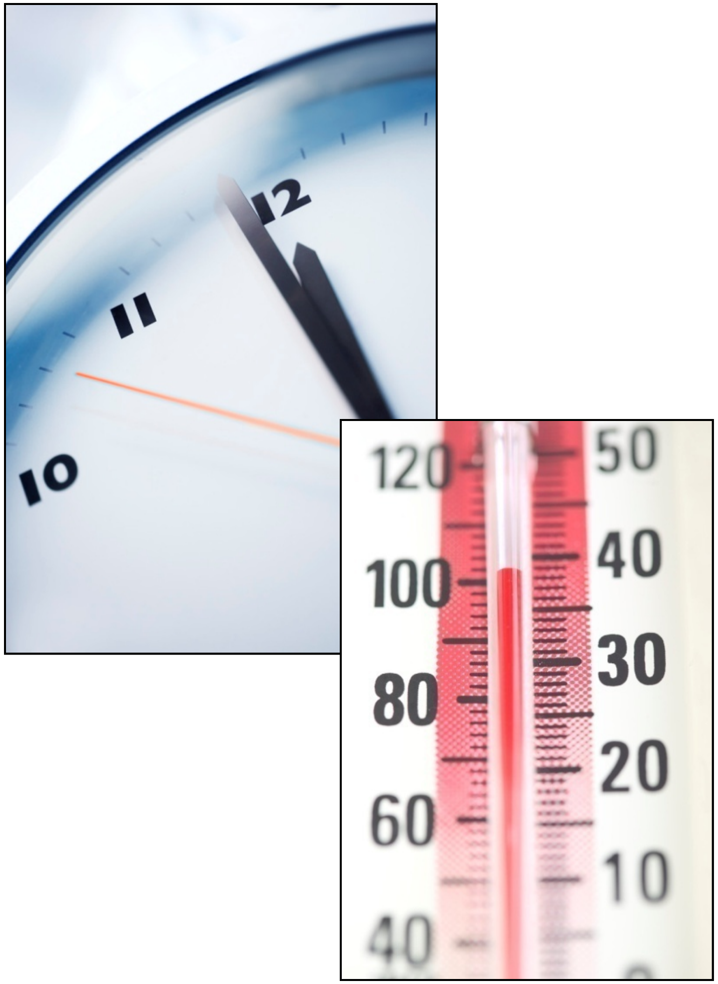
Example: Full Factorial Design

In a Full Factorial design, at least one run is conducted for each treatment or combination of factors and factor levels.



Treatments:
(+, +)
(+, -)
(-, +)
(-, -)

Example: Full Factorial Design



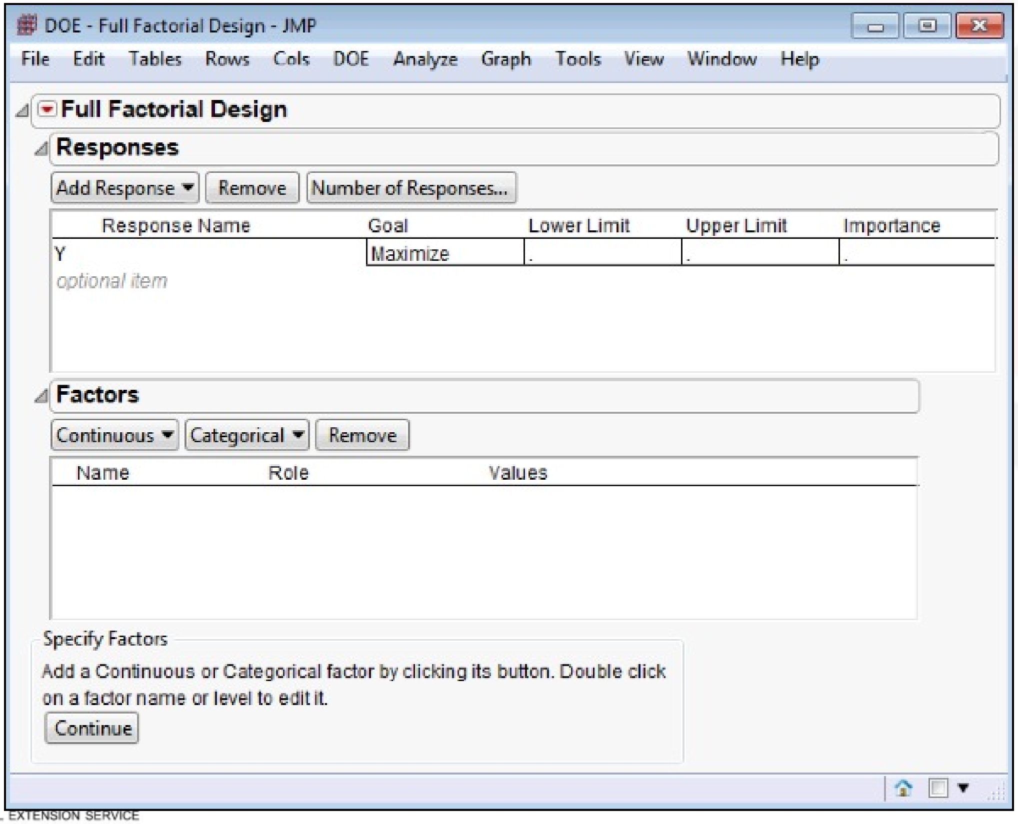
A quality engineer is studying the effect of two factors, cycle time and temperature, on the color fastness of a cotton polyester fabric.

Two levels of cycle time (40 and 60 minutes) and two levels of temperature (300°F and 350°F) were selected for the experiment.

A fabric sample from each run was compared to a standard and assigned a numerical colorfastness score.

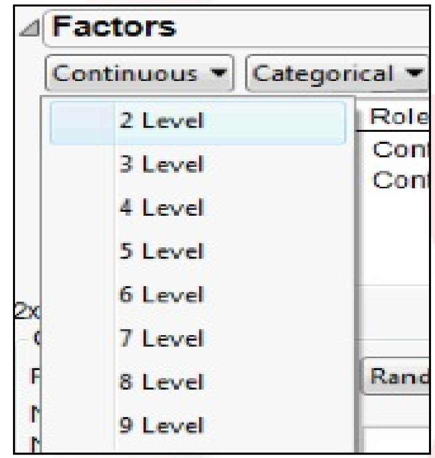
Example: Full Factorial Design

1. Open **JMP**.
2. Select **DOE → Classical → Full Factorial Design**.



Example: Full Factorial Design

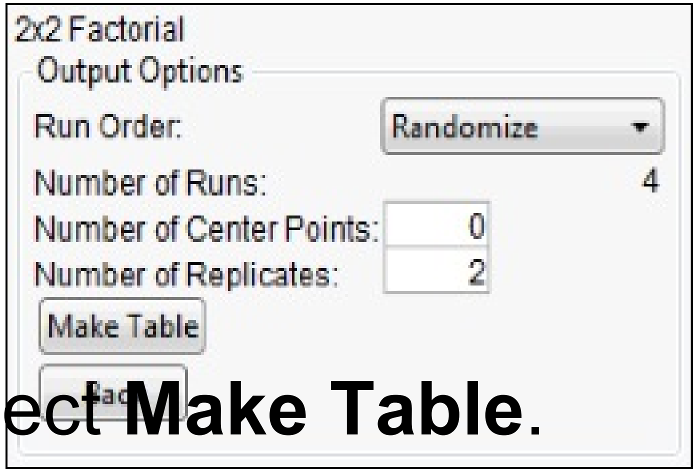
3. Select **Continuous** → **2 Level**.
4. Double Click **X1**.
5. Type **Cycle Time**.
6. Change -1 to **40**.
7. Change +1 to **60**.
8. Select **Continuous** → **2 Level**.
9. Double Click **X2**.
10. Type **Temperature**.
11. Change -1 to **300** and +1 to **350**.



Name	Role	Values	
Cycle Time	Continuous	40	60
Temperature	Continuous	300	350

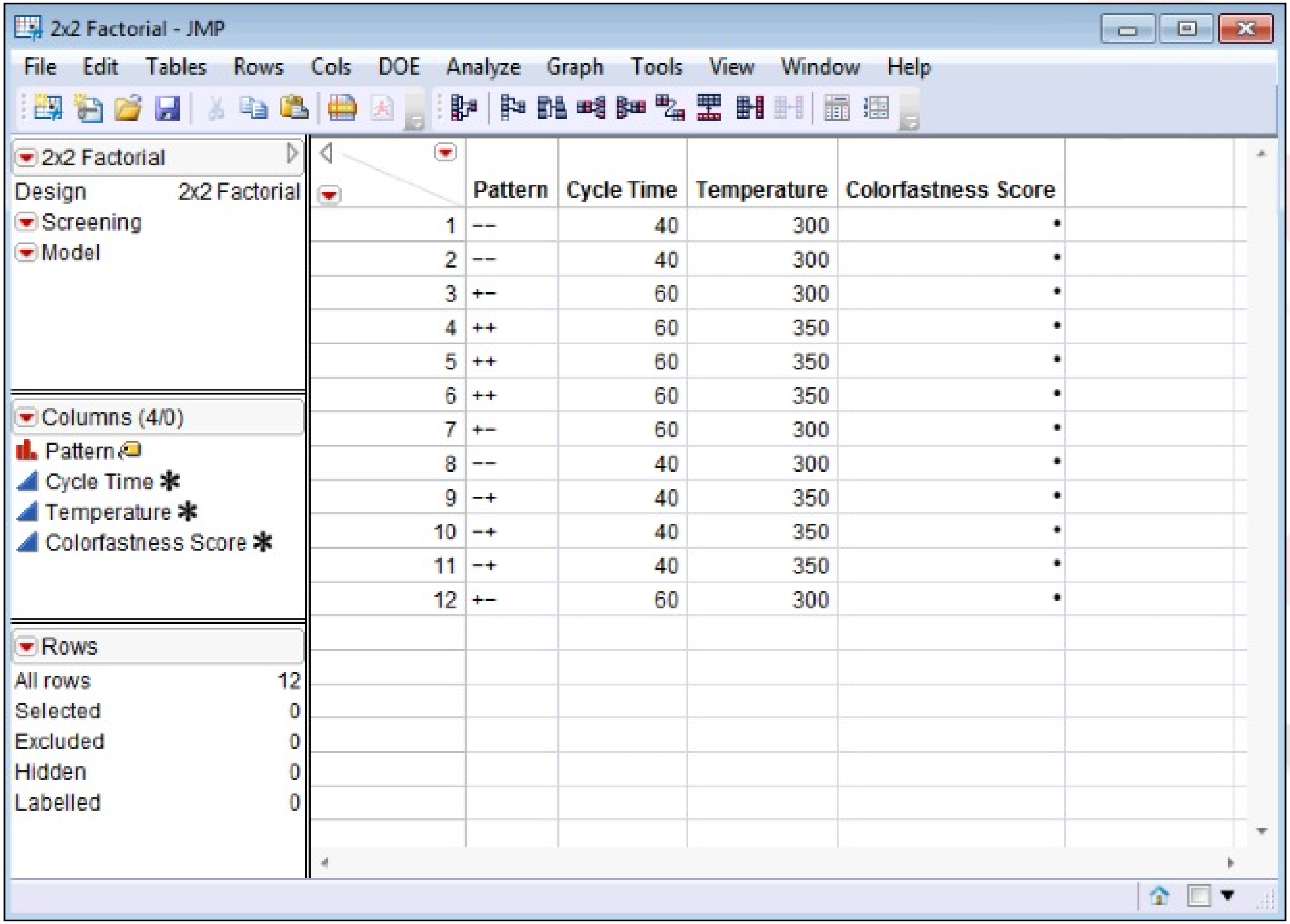
Example: Full Factorial Design

12. At the top, double click **Y** and type **Colorfastness Score**. Leave on Maximize.
13. Select **Continue**.
14. Change the number of replicates to **2**.



12. Select **Make Table**.

Example: Full Factorial Design



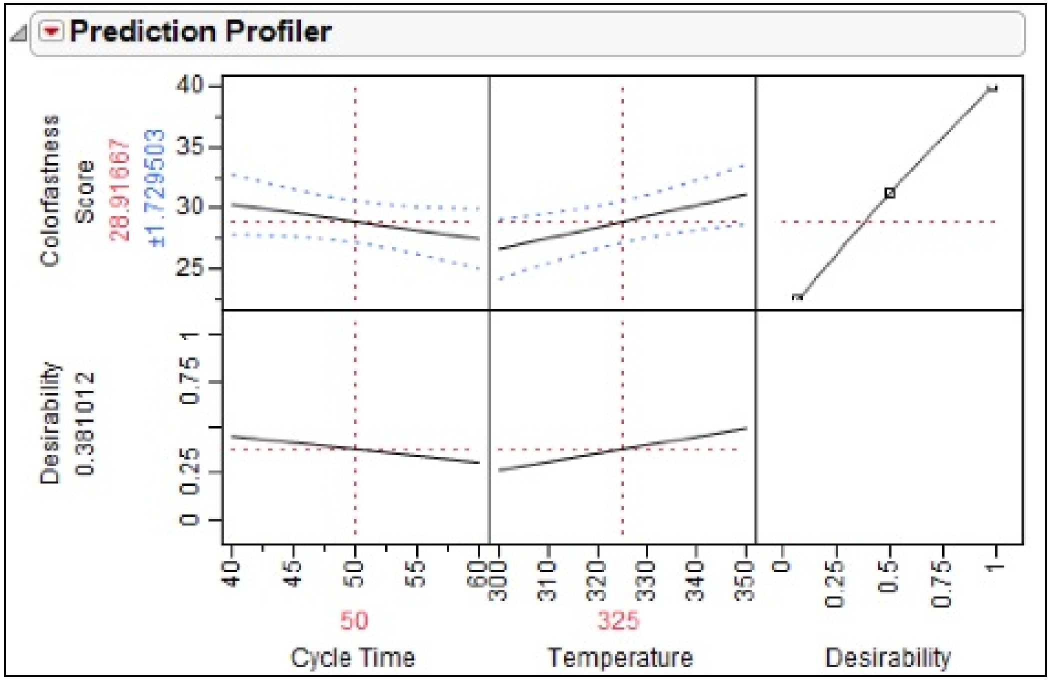
	Pattern	Cycle Time	Temperature	Colorfastness Score
1	−−	40	300	•
2	−−	40	300	•
3	+−	60	300	•
4	++	60	350	•
5	++	60	350	•
6	++	60	350	•
7	+−	60	300	•
8	−−	40	300	•
9	−+	40	350	•
10	−+	40	350	•
11	−+	40	350	•
12	+−	60	300	•

Example: Full Factorial Design

1. Open **Colorfastness_DOE.jmp**. From Moodle
2. Click the green triangle next to **Model**.
3. Check to see if all factors are in Model: Cycle Time, Temperature, Cycle Time*Temperature
4. Make sure the Y is Colorfastness Score
5. Click Run

Example: Full Factorial Design

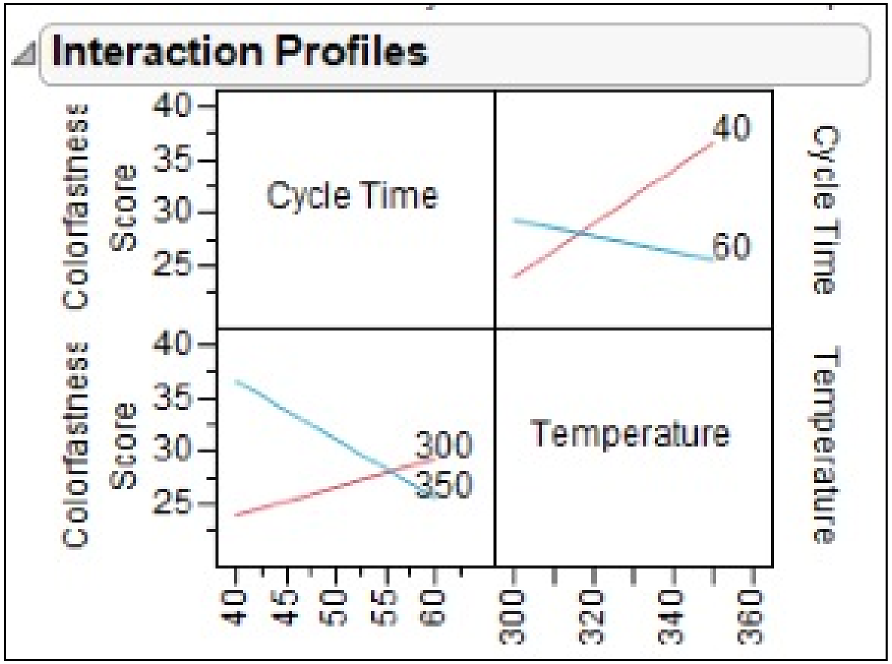
5. Click the red triangle next to **Response Colorfastness Score** and select **Factor Profiling → Profiler**.



The Prediction Profiler in JMP can be used to identify significant factors – the Xs that do impact the output. The steeper the trace, the more important an individual factor is to the output.

Example: Full Factorial Design

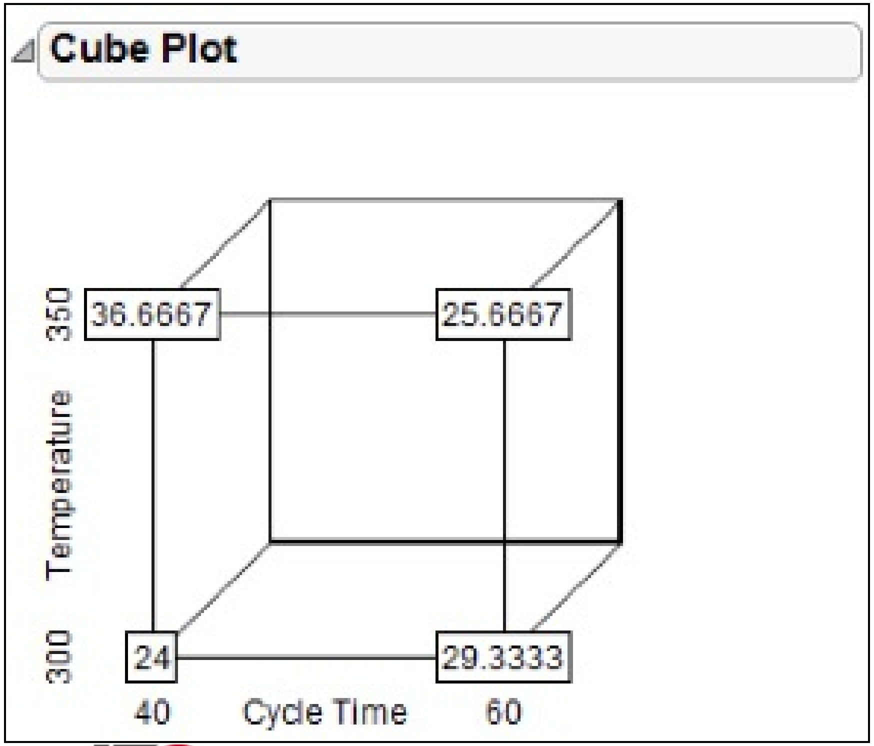
6. Click the red triangle next to **Response Colorfastness Score** and select **Factor Profiling → Interaction Plots**.



If the traces on an interaction plot are not parallel, the factors are interacting: the effect of one factor depends on the level of another factor.

Example: Full Factorial Design

7. Click the red triangle next to **Response Colorfastness Score** and select **Factor Profiling → Cube Plots**.



Based on the cube plot, which combination of factors and levels will result in the highest colorfastness rating?

DESIGN OF EXPERIMENTS

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Design of Experiments

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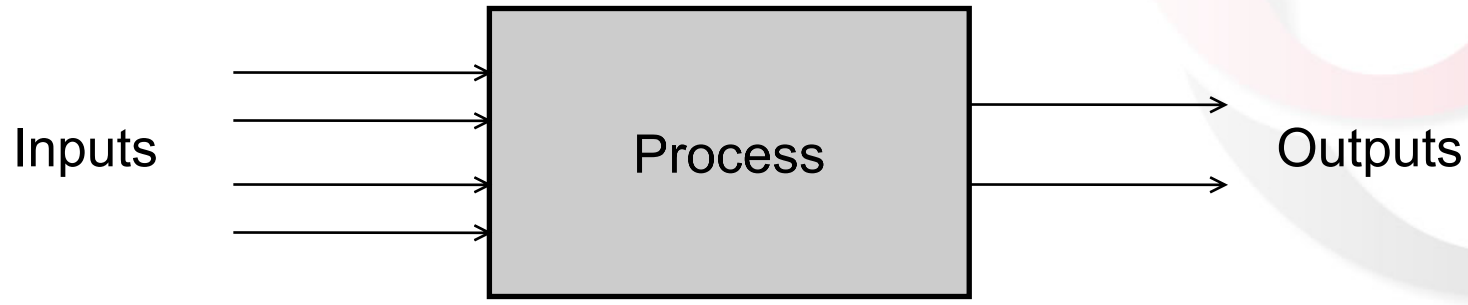


Design of Experiments

You experiment to:

- Explore relationships in a process
- Predict process performance
- Confirm findings of earlier experiments and hypothesis testing

Steps to Experimentation

1. Define the problem
2. Establish the objective
3. Select the response variables (Output, Y)
4. Select the independent variables (Inputs, X's)
5. Choose the variable levels
6. Select the experimental design
7. Collect data
8. Analyze data
9. Draw statistical conclusions
10. Replicate results
11. Draw practical solutions
12. Implement solutions

Basic DOE Terms

Basic terms include:

- Factor
- Factor level
- Treatment
- Response
- Effect
- Experimental unit
- Run
- Replication

Fundamental Principles

3 Basic Principles of experimental design:

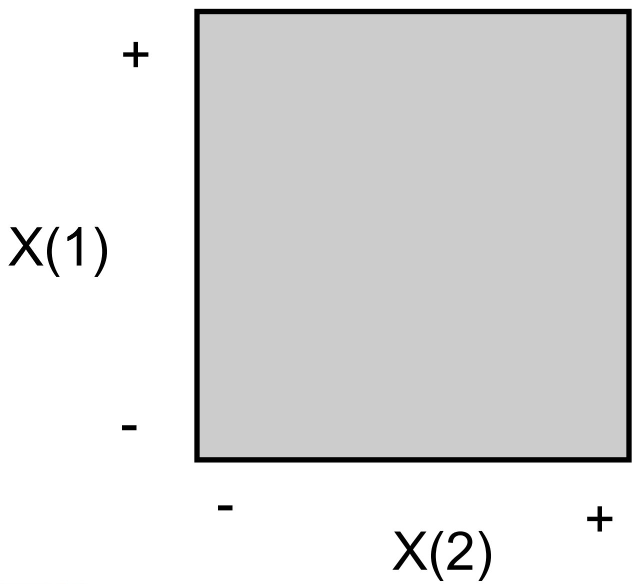
1. Replication or repetition of the basic experiment provides a better estimate of experimental error and allows the experimenter to get better estimates of effects.
2. Randomization, where both the allocation of experimental materials and the order of runs are randomly determined, typically makes the independence assumption valid and helps to average out the effect of nuisance factors.
3. Blocking, a technique that reduces the variability due to nuisance factors, improves the precision of the estimates of factors of interest.

Basic Designs

- Full factorial designs
- Screening designs
- Response surface designs

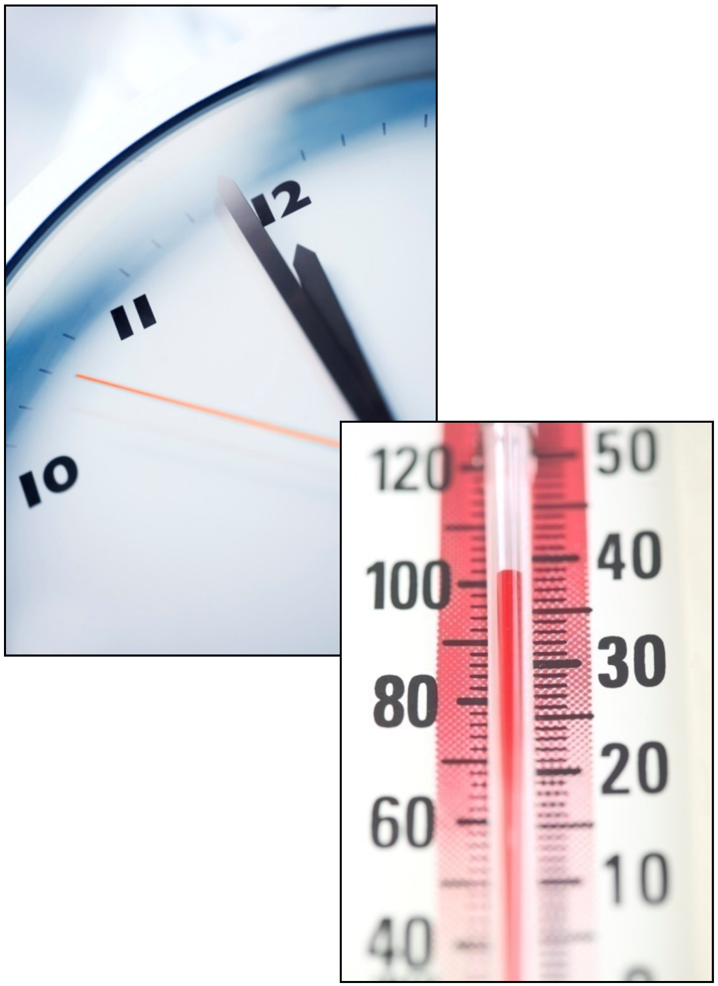
Example: Full Factorial Design

In a Full Factorial design, at least one run is conducted for each treatment or combination of factors and factor levels.



Treatments:
- (+, +)
- (+, -)
- (-, +)
- (-, -)

Example: Full Factorial Design



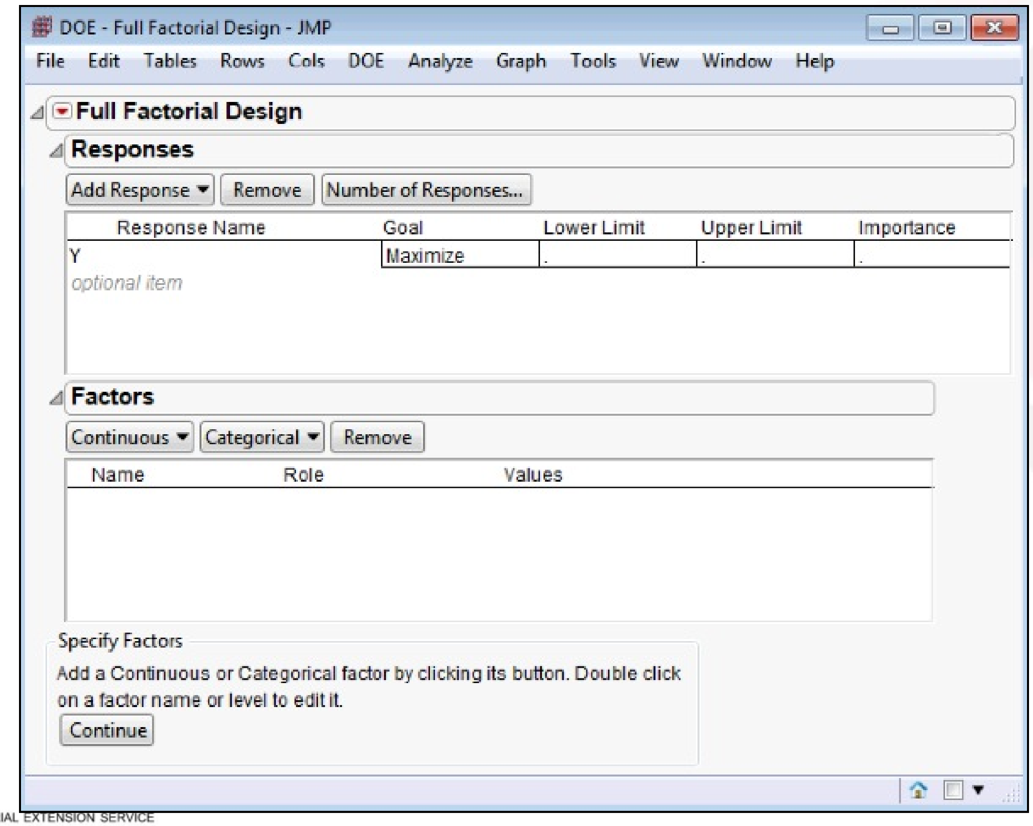
A quality engineer is studying the effect of two factors, cycle time and temperature, on the color fastness of a cotton polyester fabric.

Two levels of cycle time (40 and 60 minutes) and two levels of temperature (300°F and 350°F) were selected for the experiment.

A fabric sample from each run was compared to a standard and assigned a numerical colorfastness score.

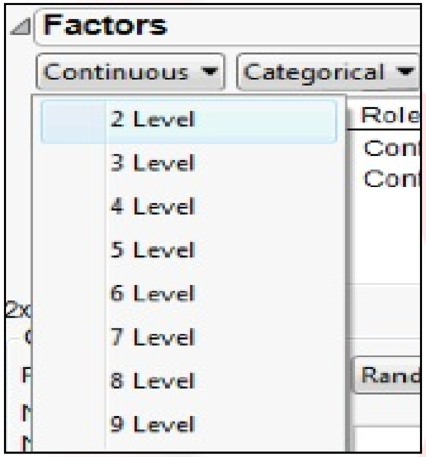
Example: Full Factorial Design

1. Open **JMP**.
2. Select **DOE → Classical → Full Factorial Design**.



Example: Full Factorial Design

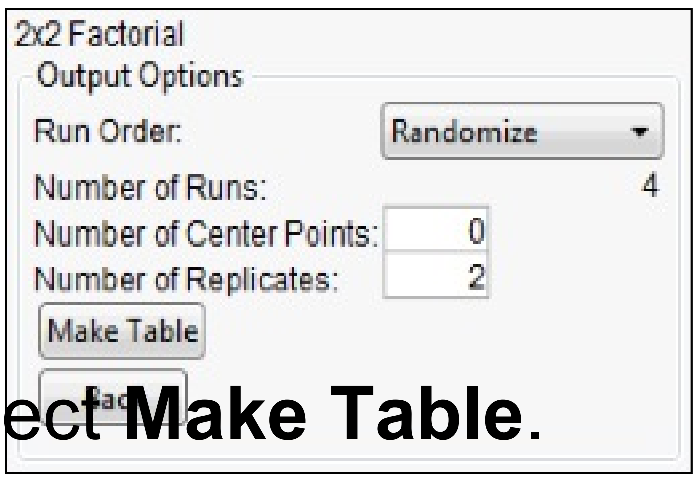
3. Select **Continuous → 2 Level**.
4. Double Click **X1**.
5. Type **Cycle Time**.
6. Change -1 to **40**.
7. Change +1 to **60**.
8. Select **Continuous → 2 Level**.
9. Double Click **X2**.
10. Type **Temperature**.
11. Change -1 to **300** and +1 to **350**.



Name	Role	Values	
Cycle Time	Continuous	40	60
Temperature	Continuous	300	350

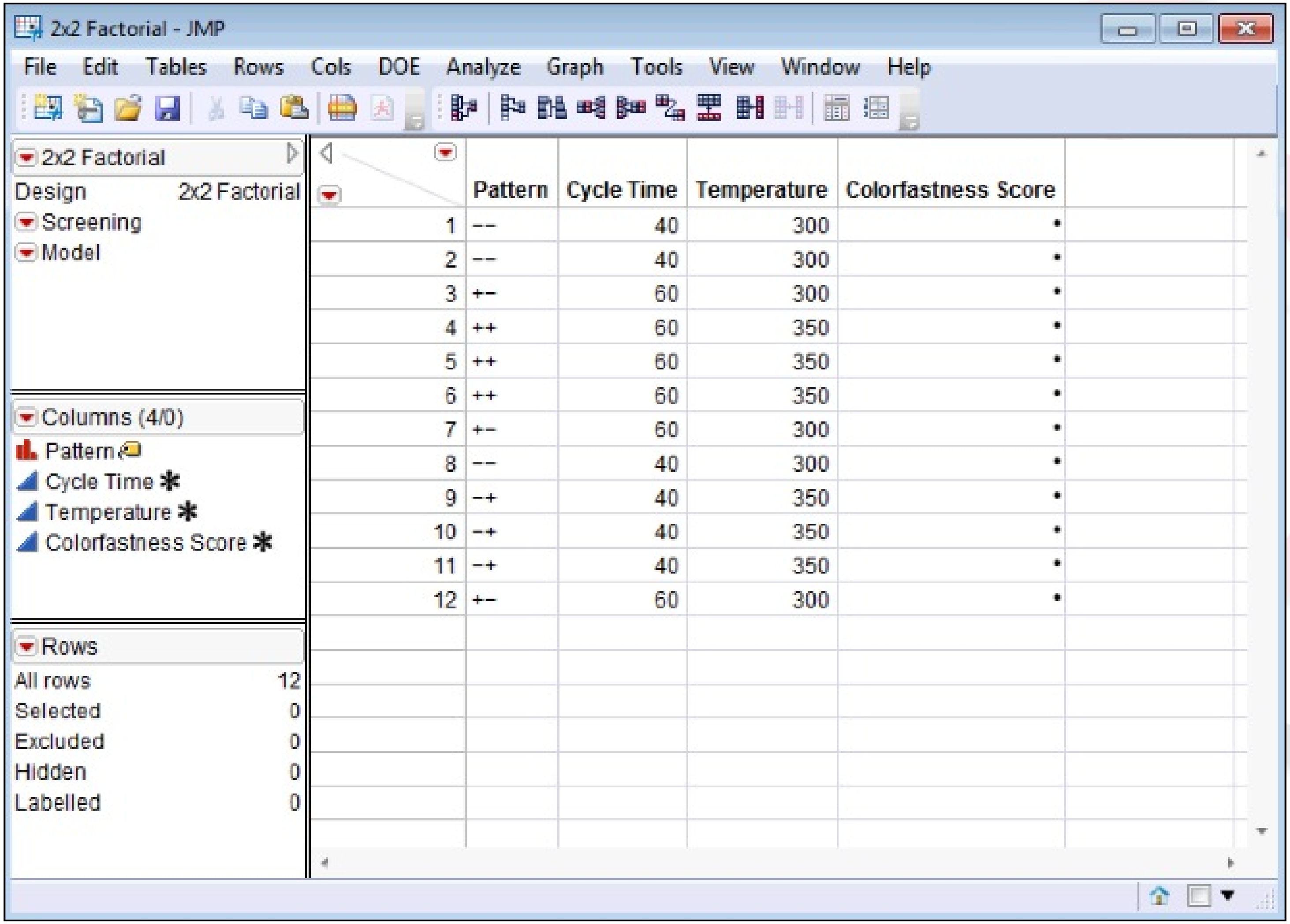
Example: Full Factorial Design

12. At the top, double click **Y** and type **Colorfastness Score**. Leave on Maximize.
13. Select **Continue**.
14. Change the number of replicates to **2**.



12. Select **Make Table**.

Example: Full Factorial Design



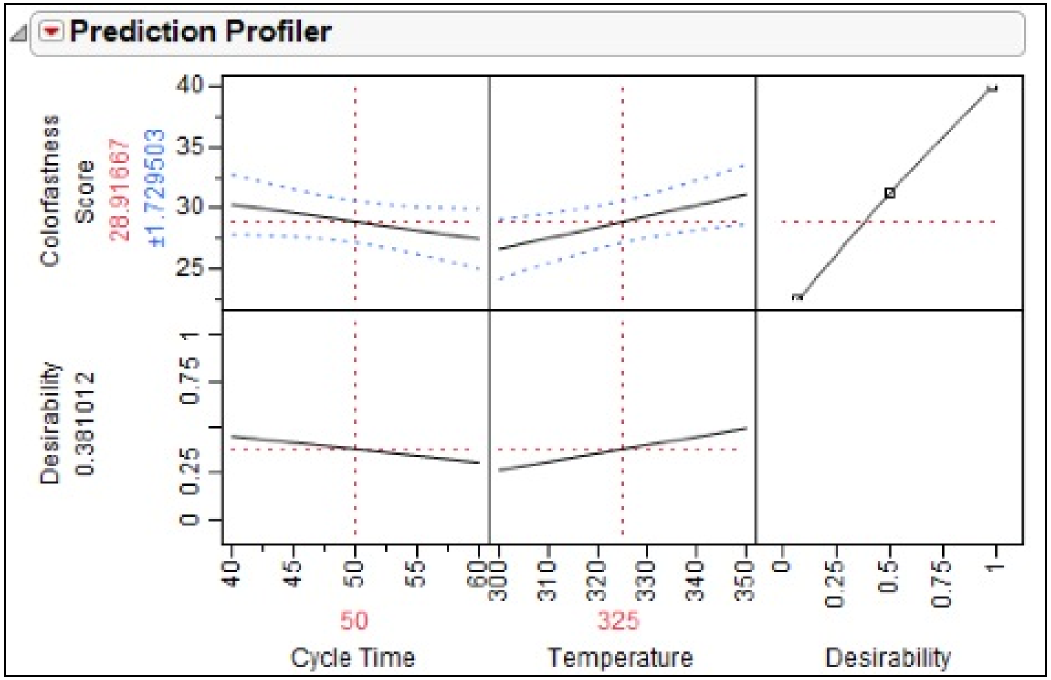
	Pattern	Cycle Time	Temperature	Colorfastness Score
1	−−	40	300	•
2	−−	40	300	•
3	+−	60	300	•
4	++	60	350	•
5	++	60	350	•
6	++	60	350	•
7	+−	60	300	•
8	−−	40	300	•
9	−+	40	350	•
10	−+	40	350	•
11	−+	40	350	•
12	+−	60	300	•

Example: Full Factorial Design

1. Open **Colorfastness_DOE.jmp**. From Moodle
2. Click the green triangle next to **Model**.
3. Check to see if all factors are in Model: Cycle Time, Temperature, Cycle Time*Temperature
4. Make sure the Y is Colorfastness Score
5. Click Run

Example: Full Factorial Design

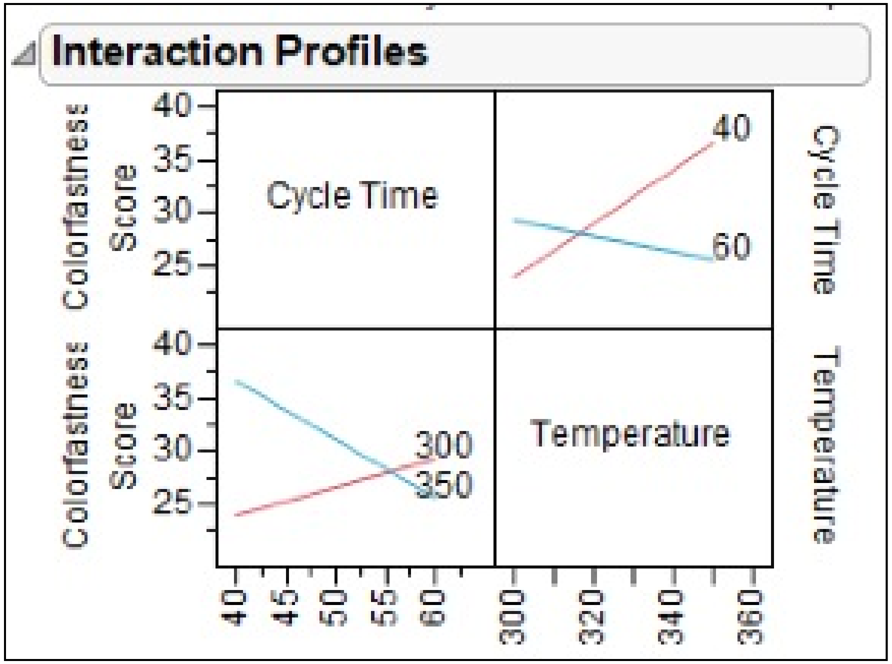
5. Click the red triangle next to **Response Colorfastness Score** and select **Factor Profiling → Profiler**.



The Prediction Profiler in JMP can be used to identify significant factors – the Xs that do impact the output. The steeper the trace, the more important an individual factor is to the output.

Example: Full Factorial Design

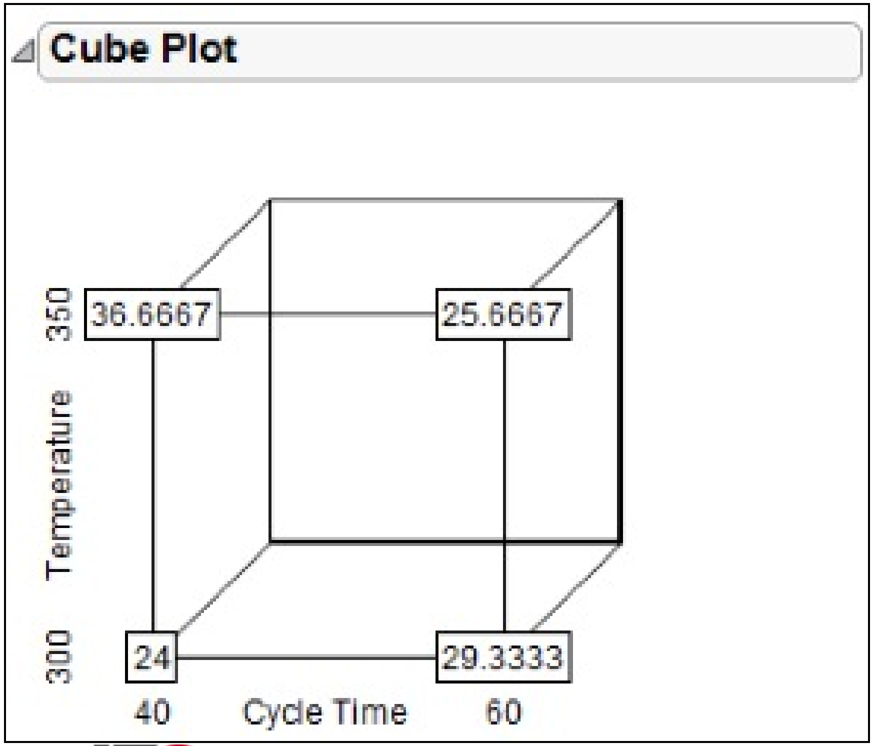
6. Click the red triangle next to **Response Colorfastness Score** and select **Factor Profiling → Interaction Plots**.



If the traces on an interaction plot are not parallel, the factors are interacting: the effect of one factor depends on the level of another factor.

Example: Full Factorial Design

7. Click the red triangle next to **Response Colorfastness Score** and select **Factor Profiling → Cube Plots**.



Based on the cube plot, which combination of factors and levels will result in the highest colorfastness rating?

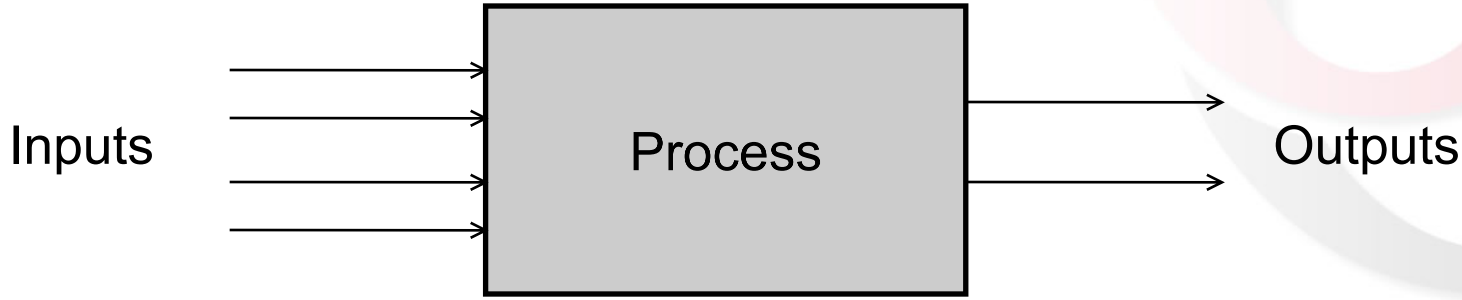
DESIGN OF EXPERIMENTS

Objectives

- Understand why designed experiments are used during the Improve stage of a project
- Define design of experiments (DOE) terms
- Design and analyze a simple experiment

Design of Experiments

Designed experiments allow six sigma teams to systematically determine the effect changing the values of process inputs has on the value of the process outputs.



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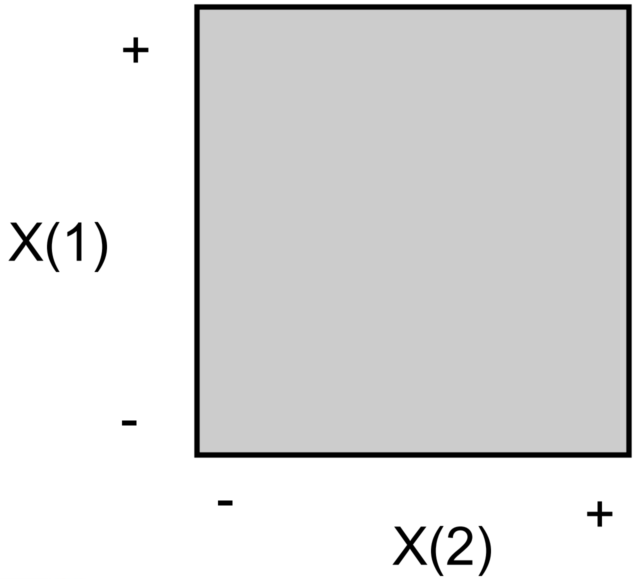

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(+, -)
(-, +)
(-, -)

Example: Full Factorial Design

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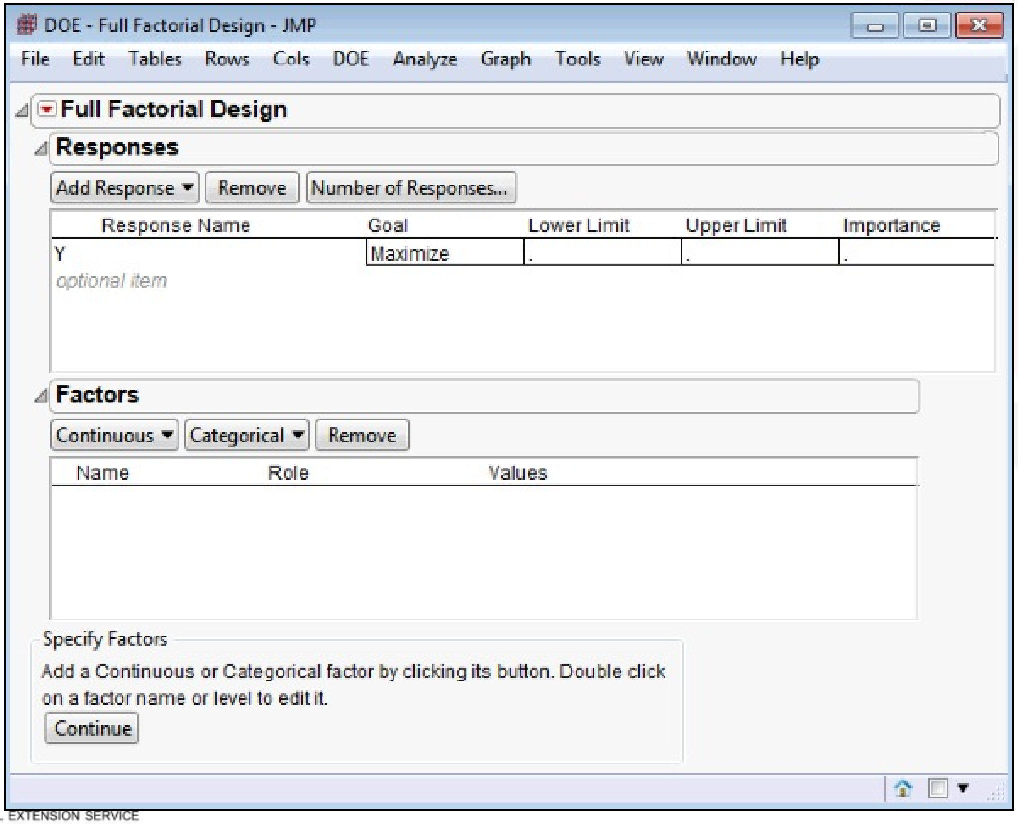

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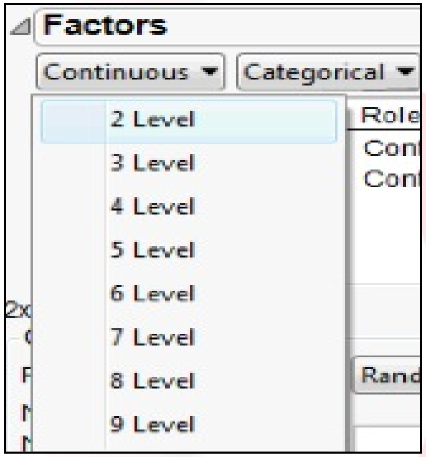

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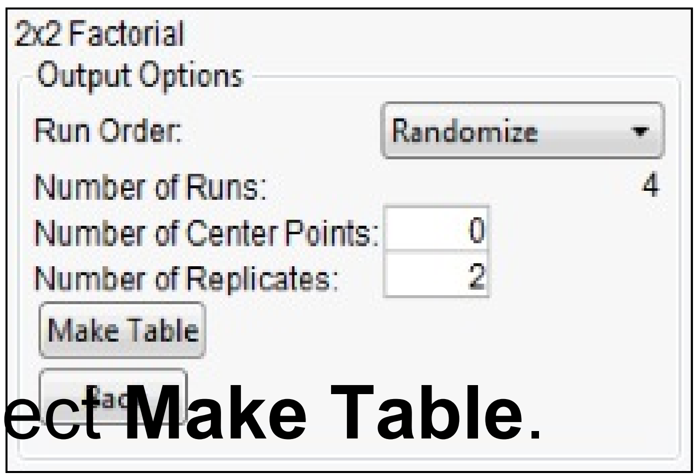

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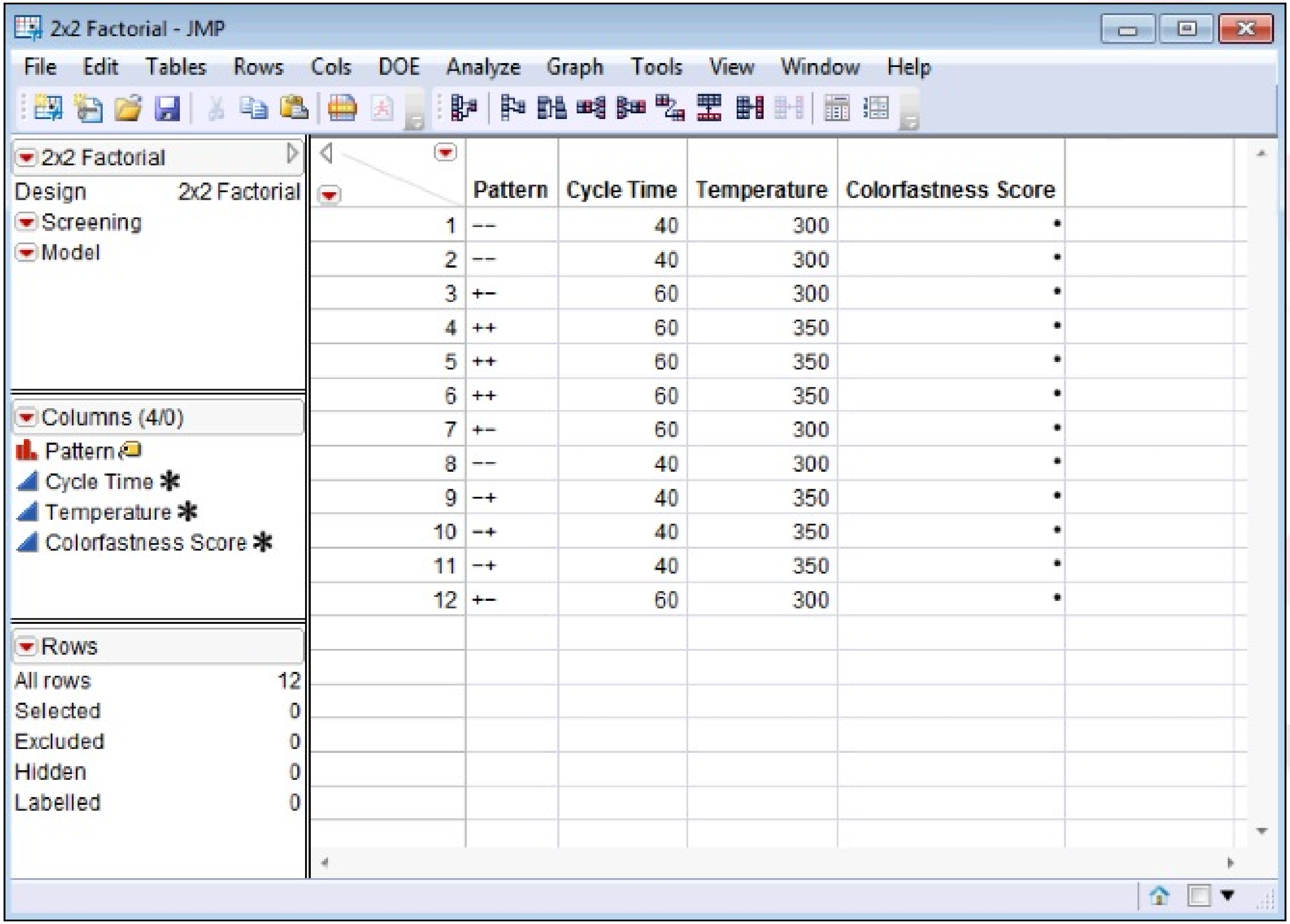

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8	−−	40	300	•
9	−+	40	350	•
10	−+	40	350	•
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12	+−	60	300	•

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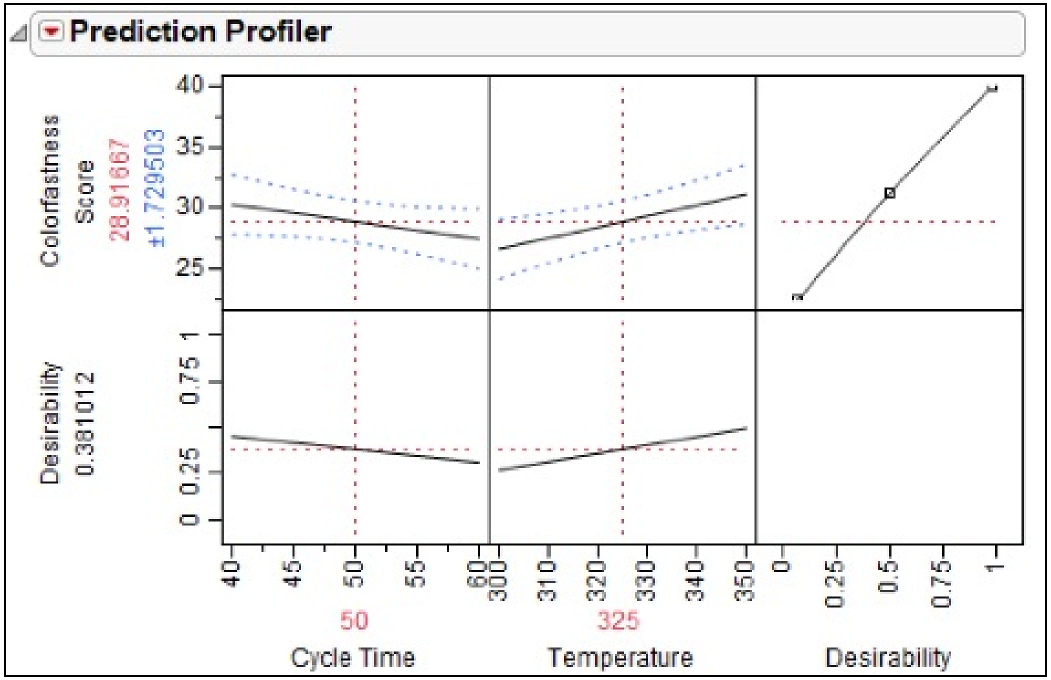

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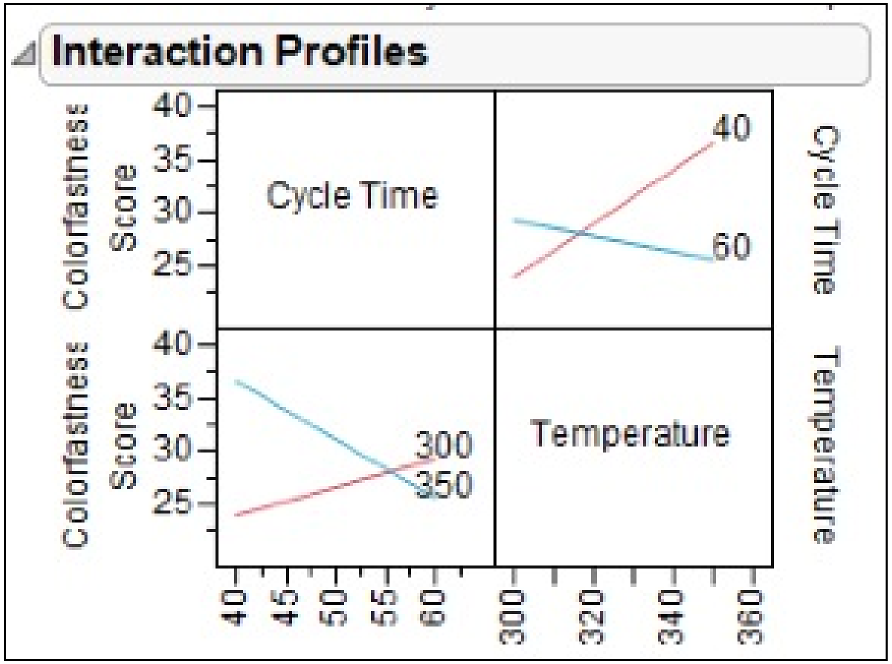

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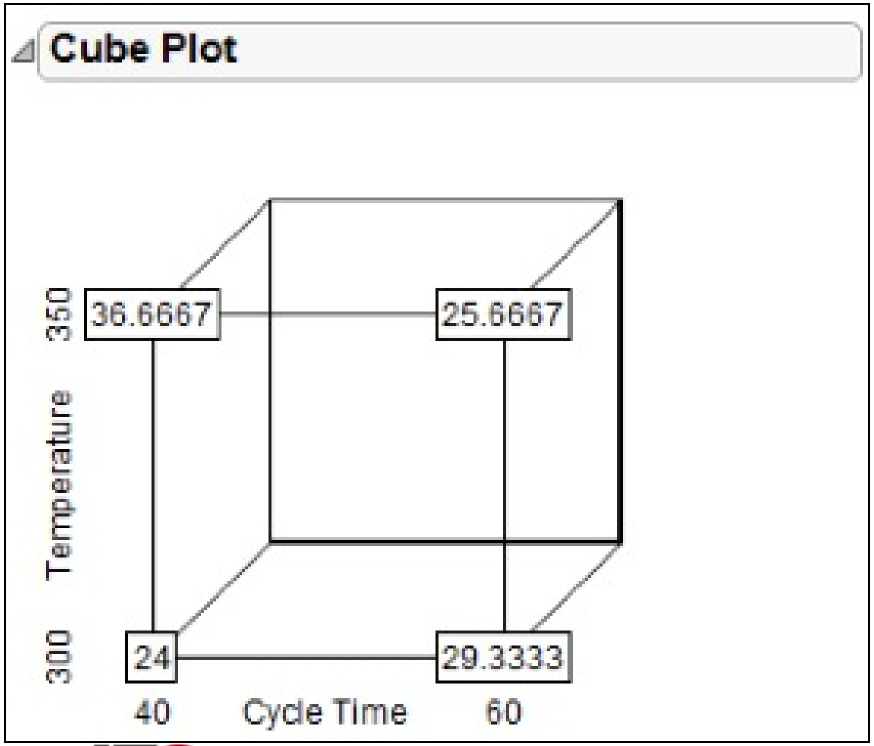

Based on the cube plot, which combination of factors and levels will result in the highest colorfastness rating?

Lean Six Sigma

Fractional Factorial Experiments

Introduction

Objective: To "screen" the list of remaining input variables to isolate those significant factors affecting the response variable.

Deliverables: Vital Few input variables, DOE Report, Updated FMEA

Juran 47.52

Why Do Fractional Factorial Experiments?

- As the number of factors of interest increases, so does the number of runs required to run a full 2^K factorial experiment. Recall (without repeats or replicates):
  - 2 Factors, 2^2 Factorial = 4 runs
  - 3 Factors, 2^3 Factorial = 8 runs
  - 4 Factors, 2^4 Factorial = 16 runs
  - 5 Factors, 2^5 Factorial = 32 runs, etc...
- If the experimenter **assumes higher-order interactions are negligible**, it is possible to do a fraction of the full factorial and still get good estimates of lower-order interactions and main effects.

Why Do Fractional Factorial Experiments?

- Fractional factorials are referred to as “screening experiments.” This means investigating a relatively large number of factors in a relatively small number of runs.
- Screening experiments are usually done in the early stages of the process-improvement phase.

Fractional Factorial Vocabulary

- **Screening (Fractional) Experiments:**
  - Experiments that allow you to investigate main effects and/or lower- order interactions without having to run full-factorial experiments.
- **Half Fraction:**
  - Experiments that allow you to investigate main effects and/or lower-order interactions in half the runs required by a full factorial.
- **Quarter Fractions:**
  - Experiments that allow you to investigate main effects and/or lower-order interactions in one fourth the runs required by a full factorial.

Fractional Factorial Vocabulary

- **Aliased or Confounded:**
  - The inability to determine which main effect or interaction is causing the true effect. One or more effects that cannot unambiguously be attributed to a single factor or interaction.
- **Design Resolution:**
  - A Roman numeral notation which allows you to describe the “worst case” confounding scheme associated with a design.
- **Fold Over:**
  - The ability to add sequential fractional experiments to an existing fractional experiment with the intention of estimating specific main effects or interactions free of particular confounding patterns.

Juran 47.53 - 57

Fractional Factorial Notation

- **The general notation to designate a fractional factorial design:**

$$2_R^{k-p}$$

  - **Where**

    k = number of factors to be investigated

    2^{k-p} = number of runs

    R = design resolution (III, IV, V)

  - **Note:**

    If p = 1, then half-fraction factorial

    If p = 2, then quarter-fraction factorial

Juran 47.54

Half-Fraction Design

- Recall the expanded representation of a 2^3 full-factorial design. But suppose we wanted to investigate four input variables instead of three. Since all the contrasts are independent (orthogonal) we can select any interaction as the contrast to represent the fourth variable. **Usually we select the highest-order interaction to represent the fourth factor. In this case, the AxBxC interaction can now represent the levels of factor D.**
- When we replace the AxBxC Interaction with Factor D, we say the ABC is **aliased** with D. Obviously, ABC can no longer be estimated.

Half-Fraction Design

Factor D

A	B	C	AXB	AXC	BXC	AXBXC
-1	-1	-1	1	1	1	-1
1	-1	-1	-1	-1	1	1
-1	1	-1	-1	1	-1	1
1	1	-1	1	-1	-1	-1
-1	-1	1	1	-1	-1	1
1	-1	1	-1	1	-1	-1
-1	1	1	-1	-1	1	-1
1	1	1	1	1	1	1

The Design Generator is D = ABC.

Half-Fraction Design

- Why is it called a “half” fraction? We would call this a **half-fraction** since a full 2^4 factorial design would take 16 runs to complete. In a half fraction we can estimate **4 factors** in only **8 runs.** There is a cost - we lose the higher-order interaction.

Half-Fraction Design

	A	B	C	D	A X B	A X C	B X C	A X B X C
1	**-1**	**-1**	**-1**	**-1**	**1**	**1**	**1**	**-1**
	1	-1	-1	-1	-1	-1	1	1
	-1	1	-1	-1	-1	1	-1	1
4	**1**	**1**	**-1**	**-1**	**1**	**-1**	**-1**	**-1**
	-1	-1	1	-1	1	-1	-1	1
6	**1**	**-1**	**1**	**-1**	**-1**	**1**	**-1**	**-1**
7	**-1**	**1**	**1**	**-1**	**-1**	**-1**	**1**	**-1**
	1	1	1	-1	1	1	1	1
	-1	-1	-1	1	1	1	1	-1
2	**1**	**-1**	**-1**	**1**	**-1**	**-1**	**1**	**1**
3	**-1**	**1**	**-1**	**1**	**-1**	**1**	**-1**	**1**
	1	1	-1	1	1	-1	-1	-1
5	**-1**	**-1**	**1**	**1**	**1**	**-1**	**-1**	**1**
	1	-1	1	1	-1	1	-1	-1
	-1	1	1	1	-1	-1	1	-1
8	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

Half-Fraction Exercise

- Use this matrix as a starting point to design a half-fraction experiment to estimate 5 main effects in only 16 runs.
  - What is the design generator?
  - Which 16 of the 32 runs will be used for the fractional factorial experiment?
- Let's investigate the alias structure of this half-fraction experiment.

Run	A	B	C	D	E	ABCD
1	-1	-1	-1	-1	-1	
2	1	-1	-1	-1	-1	
3	-1	1	-1	-1	-1	
4	1	1	-1	-1	-1	
5	-1	-1	1	-1	-1	
6	1	-1	1	-1	-1	
7	-1	1	1	-1	-1	
8	1	1	1	-1	-1	
9	-1	-1	-1	1	-1	
10	1	-1	-1	1	-1	
11	-1	1	-1	1	-1	
12	1	1	-1	1	-1	
13	-1	-1	1	1	-1	
14	1	-1	1	1	-1	
15	-1	1	1	1	-1	
16	1	1	1	1	-1	
17	-1	-1	-1	-1	1	
18	1	-1	-1	-1	1	
19	-1	1	-1	-1	1	
20	1	1	-1	-1	1	
21	-1	-1	1	-1	1	
22	1	-1	1	-1	1	
23	-1	1	1	-1	1	
24	1	1	1	-1	1	
25	-1	-1	-1	1	1	
26	1	-1	-1	1	1	
27	-1	1	-1	1	1	
28	1	1	-1	1	1	
29	-1	-1	1	1	1	
30	1	-1	1	1	1	
31	-1	1	1	1	1	
32	1	1	1	1	1	

Half-Fraction Exercise

- Multiply the signs in the AxBxCxD columns to get the signs in the ABCD column.
  - Ex: Run 1
  - -1x-1x-1x-1= 1 (ABCD)
- Select the 16 rows where the signs in columns E and ABCD match. Factor E is confounded with the ABCD interaction.
- These are the runs for the ½ Fraction Design.

Run	A	B	C	D	E	ABCD
1	-1	-1	-1	-1	-1	1
2	1	-1	-1	-1	-1	-1
3	-1	1	-1	-1	-1	-1
4	1	1	-1	-1	-1	1
5	-1	-1	1	-1	-1	-1
6	1	-1	1	-1	-1	1
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13	-1	-1	1	1	-1	1
14	1	-1	1	1	-1	-1
15	-1	1	1	1	-1	-1
16	1	1	1	1	-1	1
17	-1	-1	-1	-1	1	1
18	1	-1	-1	-1	1	-1
19	-1	1	-1	-1	1	-1
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21	-1	-1	1	-1	1	-1
22	1	-1	1	-1	1	1
23	-1	1	1	-1	1	1
24	1	1	1	-1	1	-1
25	-1	-1	-1	1	1	-1
26	1	-1	-1	1	1	1
27	-1	1	-1	1	1	1
28	1	1	-1	1	1	-1
29	-1	-1	1	1	1	1
30	1	-1	1	1	1	-1
31	-1	1	1	1	1	-1
32	1	1	1	1	1	1

Half-Fraction Exercise

- What is confounded with AB? What is confounded with AE?
- Complete the matrix for the indicated interactions (arrows) to determine the confounded factor(s)/interaction(s).

Half-Fraction Exercise

Run	A	B	C	D	E	AB	AC	AD	AE	BC	BD	BE	CD	CE	DE	ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE	ABCD
1	1	-1	-1	-1	-1																					-1
2	-1	1	-1	-1	-1																					-1
3	-1	-1	1	-1	-1																					-1
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1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1
-1	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1
1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	-1	-1	1	1	-1	-1	1	1	1
-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	-1	1	-1	-1	-1
1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1
1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1
-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1
-1	-1	-1	-1	1	1	1	1	-1	1	1	-1	1	-1	-1	-1	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1	-1
1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	1
1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	-1
1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	1	-1	-1	1
-1	1	-1	1	1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	1	1	-1	1	-1
-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1
1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

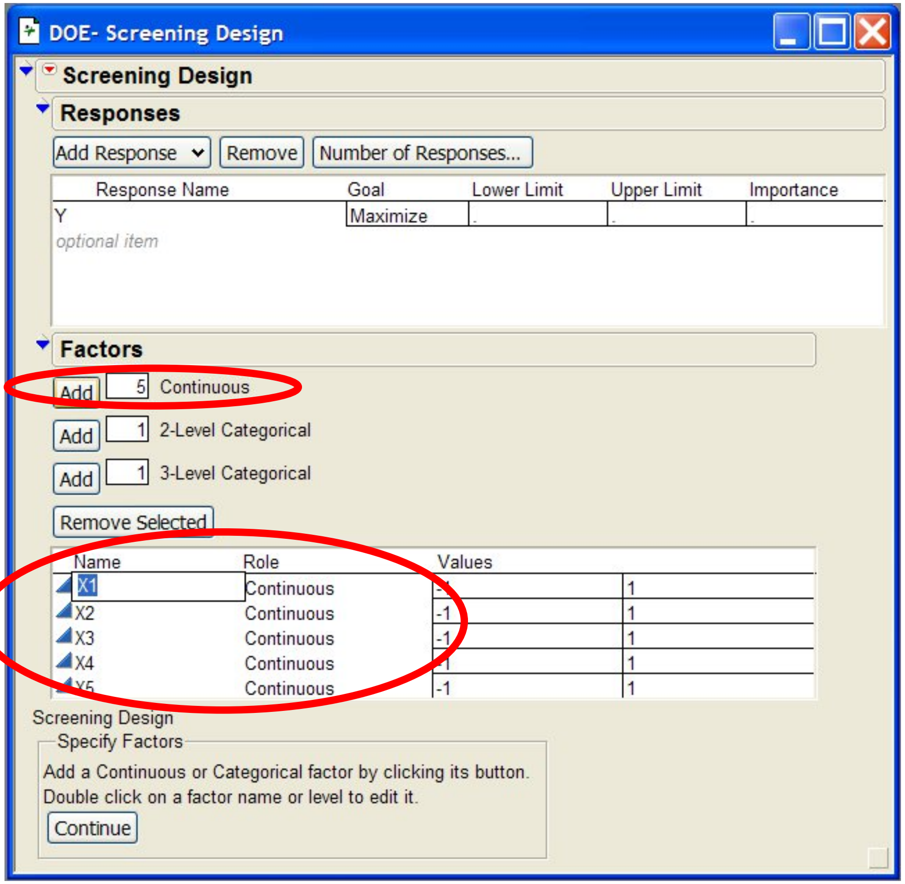
Check the 12 =345 or AB = CDE
And the 15 = 234 or AE = BCD Alias Structure

Half-Fraction Exercise

- Use a half-fraction experiment to estimate 5 main effects in only 16 runs. What is the alias structure?
- Use JMP to confirm your results.

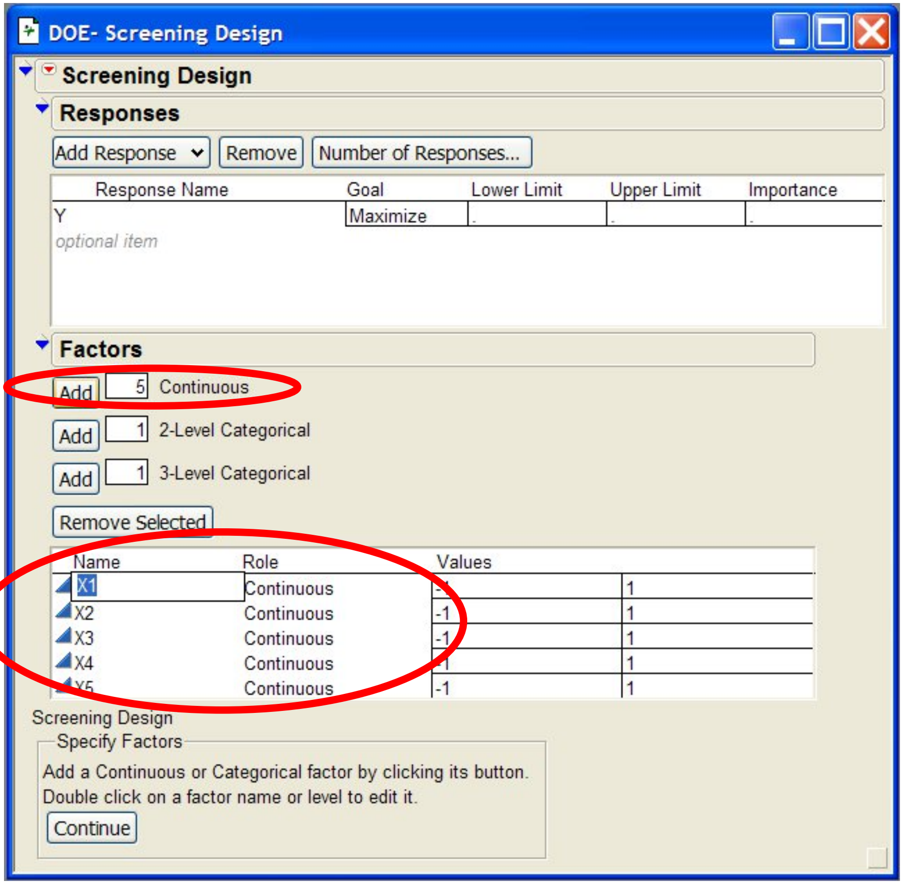
Confounding and Alias Structure

- **JMP>DOE>Classical>**
- **>Two Level Screening > Screening Design**
- Add 5 continuous factors
- Select Continue



Confounding and Alias Structure

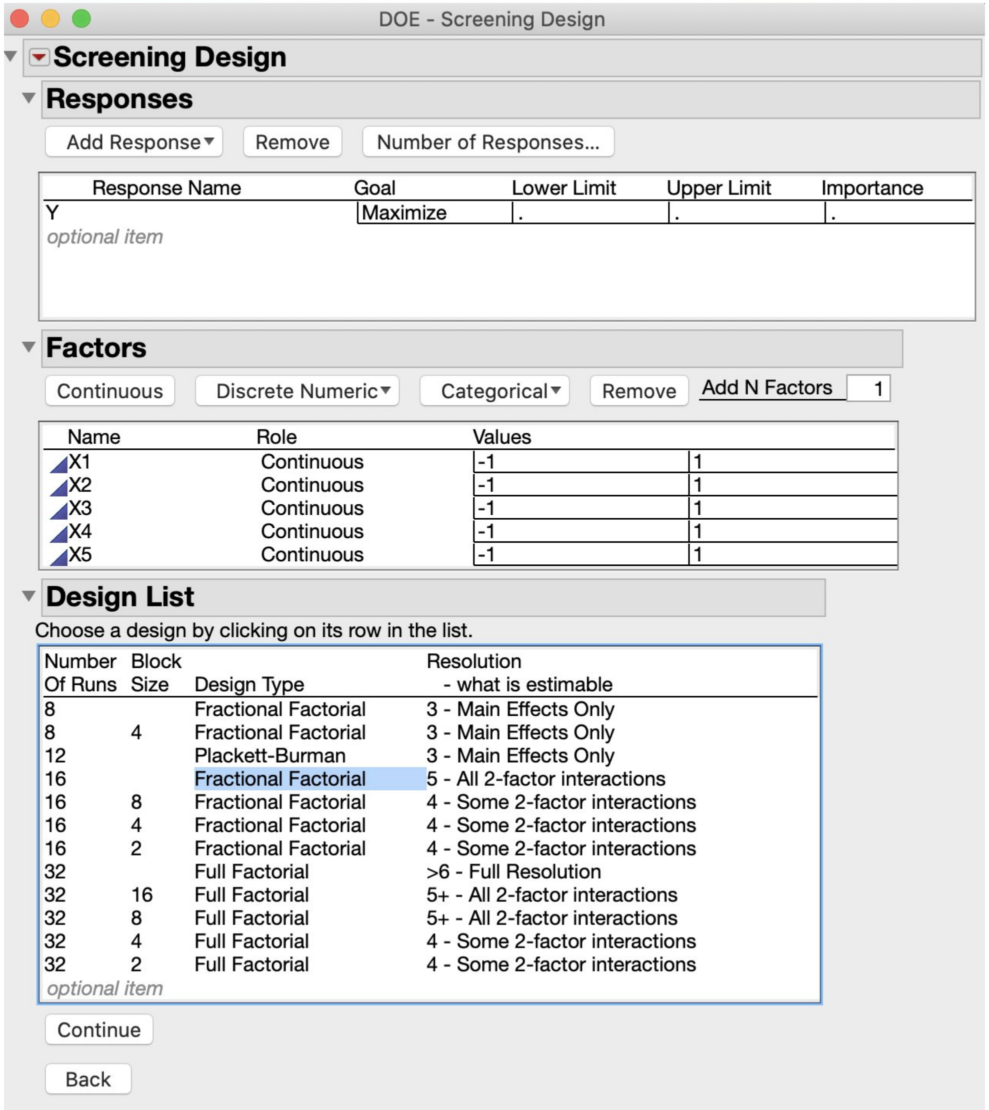
- **Select Choose from a list of fractional factorial designs**
- Select Continue



Confounding and Alias Structure

- Select Fractional Factorial with Resolution 5

- Select Continue



Confounding and Alias Structure

- Select Aliasing of Effects
- Then select the Red Triangle and show confounding order, and for the aliases to what order, enter 5
- Select OK

Confounding and Alias Structure

Confounding Pattern

Confounding P…

Columns (2/0)
Effect Names
Alias Names

Rows
All rows 32
Selected 0
Excluded 0
Hidden 0
Labelled 0

	Effect Names	Alias Names
1	C	= 1 2 3 4 5
2	1	= 2 3 4 5
3	2	= 1 3 4 5
4	1 2	= 3 4 5
5	3	= 1 2 4 5
6	1 3	= 2 4 5
7	2 3	= 1 4 5
8	1 2 3	= 4 5
9	4	= 1 2 3 5
10	1 4	= 2 3 5
11	2 4	= 1 3 5
12	1 2 4	= 3 5
13	3 4	= 1 2 5
14	1 3 4	= 2 5
15	2 3 4	= 1 5
16	1 2 3 4	= 5
17	5	= 1 2 3 4
18	1 5	= 2 3 4
19	2 5	= 1 3 4
20	1 2 5	= 3 4
21	3 5	= 1 2 4
22	1 3 5	= 2 4
23	2 3 5	= 1 4
24	1 2 3 5	= 4
25	4 5	= 1 2 3
26	1 4 5	= 2 3
27	2 4 5	= 1 3
28	1 2 4 5	= 3
29	3 4 5	= 1 2
30	1 3 4 5	= 2
31	2 3 4 5	= 1
32	1 2 3 4 5	= C

Confounding and Alias Structure

Examine the Alias Structure:

The second order 12 interaction is confounded with the third order 345 interaction.

12 = 345

What is the 15 interaction aliased with?

Confounding Pattern

Confounding Pattern

Columns (2/0)
- Effect Names
- Alias Names

Rows
- All rows 32
- Selected 0
- Excluded 0
- Hidden 0
- Labelled 0

	Effect Names	Alias Names
1	C	= 1 2 3 4 5
2	1	= 2 3 4 5
3	2	= 1 3 4 5
4	1 2	= 3 4 5
5	3	= 1 2 4 5
6	1 3	= 2 4 5
7	2 3	= 1 4 5
8	1 2 3	= 4 5
9	4	= 1 2 3 5
10	1 4	= 2 3 5
11	2 4	= 1 3 5
12	1 2 4	= 3 5
13	3 4	= 1 2 5
14	1 3 4	= 2 5
15	2 3 4	= 1 5
16	1 2 3 4	= 5
17	5	= 1 2 3 4
18	1 5	= 2 3 4
19	2 5	= 1 3 4
20	1 2 5	= 3 4
21	3 5	= 1 2 4
22	1 3 5	= 2 4
23	2 3 5	= 1 4
24	1 2 3 5	= 4
25	4 5	= 1 2 3
26	1 4 5	= 2 3
27	2 4 5	= 1 3
28	1 2 4 5	= 3
29	3 4 5	= 1 2
30	1 3 4 5	= 2
31	2 3 4 5	= 1
32	1 2 3 4 5	= C

Design Resolution

- Example: Here we are investigating five factors will be in $2^{5-1} = 16$ runs.
- This design is a resolution V or 5.
- See the Design List to check this.

Design Resolution *(Cont.)*

- **Resolution V Designs:**
  - **No main effect aliased with another main effect.**
  - **No main effect aliased with any two-factor interactions.**
  - **No two-factor interaction aliased with another two factor-interaction.**
  - **Two factor-interactions aliased with three factor-interactions.**

Design Resolution

- **Example: The designation here means four factors will be investigated in 2^{4-1} = 8 runs.**

  This design is a resolution IV.

$$2_{IV}^{4-1}$$

Design Resolution *(Cont.)*

- **Resolution IV Designs:**
  - **No main effect aliased with another main effect.**
  - **No main effect aliased with any two-factor interactions.**
  - **At least one two-factor interaction aliased with another two-factor interaction.**

Design Resolution

- **Resolution III Designs:**
  - **No main effects are aliased with other main effects.**
  - **At least one main effect will be aliased with a two-factor interaction.**

Aliasing Structure

- For this example:
  - I, the Identity element, equals 1 (for all practical purposes)
  - I = ABCDE, the defining relation, can be used to determine any confounding information
  - Question: Why is $A^2 = I$?
  - Question: What is A, AE, CD confounded with?

$I = ABCDE$

$A = A^2BCDE$

$A = IBCDE$

$A = BCDE$

$$I = ABCDE$$
$$AE\,(I) = AE\,(ABCDE)$$
$$AE = A^2BCDE^2$$
$$AE = BCD$$

$$I = ABCDE$$
$$CD = CD\,(ABCDE)$$
$$CD = ABC^2D^2E$$
$$CD = ABE$$

JMP Example

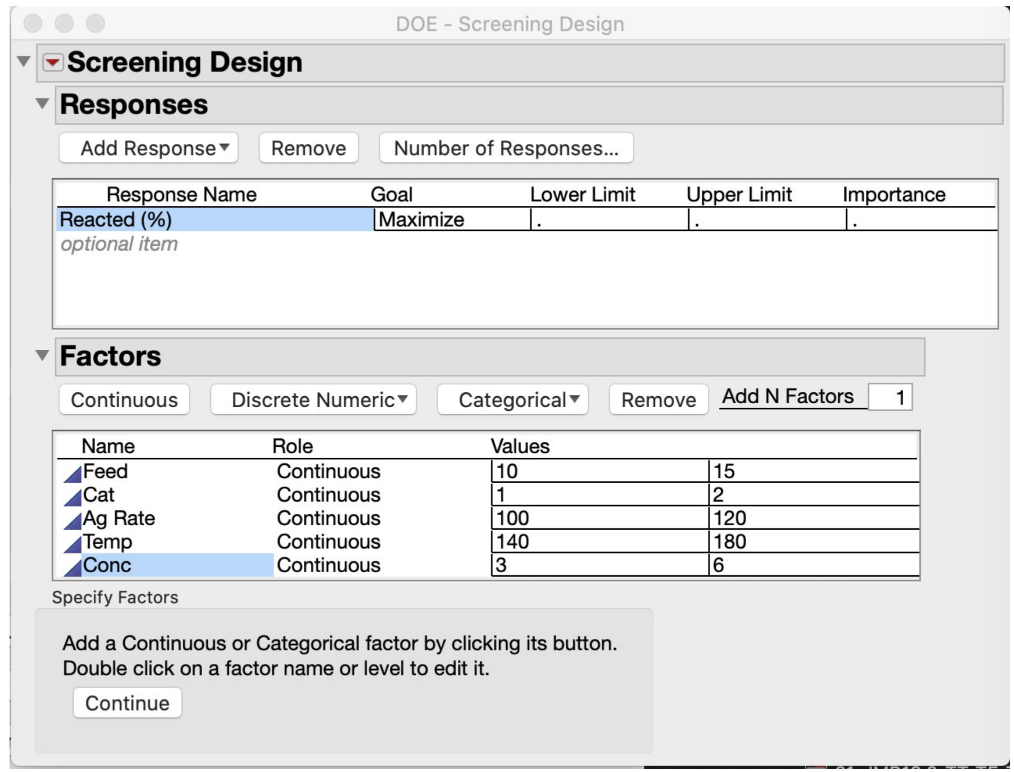
- State the Practical Problem
  - To determine which factors can increase the % reacted of a chemical process.
- State the factors and levels of interest, create a JMP experimental data sheet
  - We only have enough funds for 16 runs of this experiment.

JMP Example *(Cont.)*

- Output Variable:
  - % Reacted
- Input Variables:
  - A: Feed Rate (liters/minute) 10(-1), 15(+1)
  - B: Catalyst (%) 1(-1), 2(+1)
  - C: Agitation Rate (rpm) 100(-1), 120(+1)
  - D: Temperature (°C) 140(-1), 180(+1)
  - E: Concentration (%) 3(-1), 6(+1)

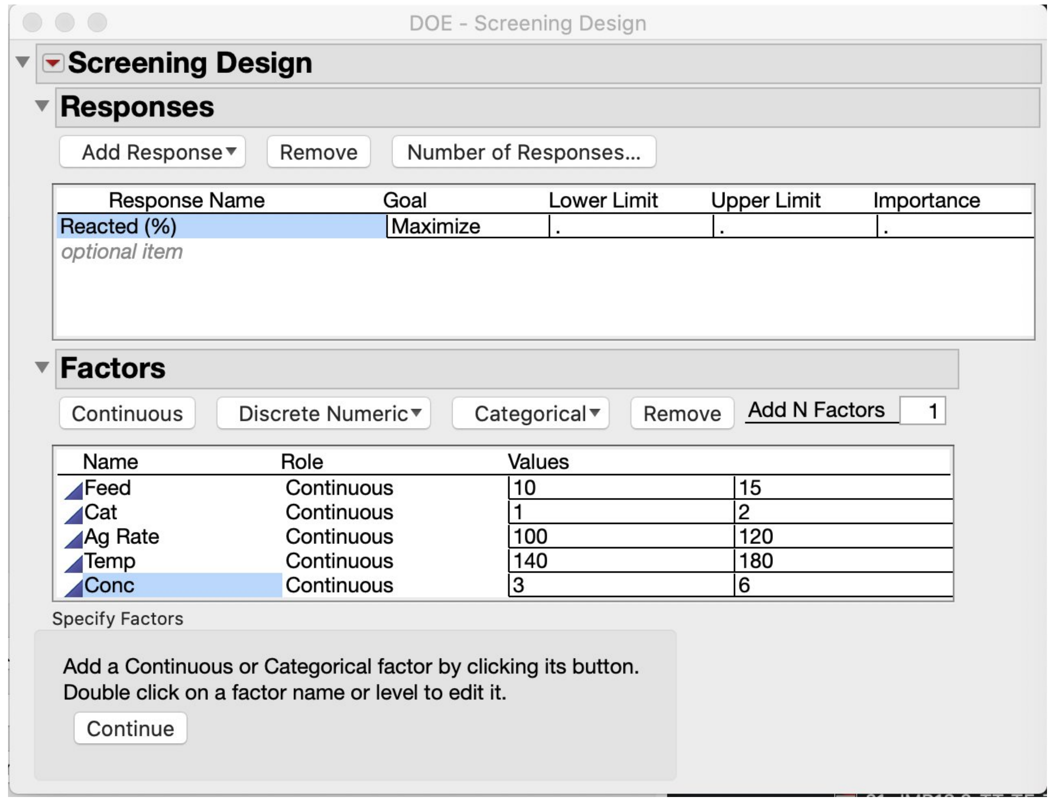
JMP DOE Design

- **JMP>DOE>Classical**
- **>Two Level Screening> Screening Design**
- Next to **Continuous Factors,** change the number from *1* to *5* since there are 5 continuous factors.
- Insert the Factor names and levels.



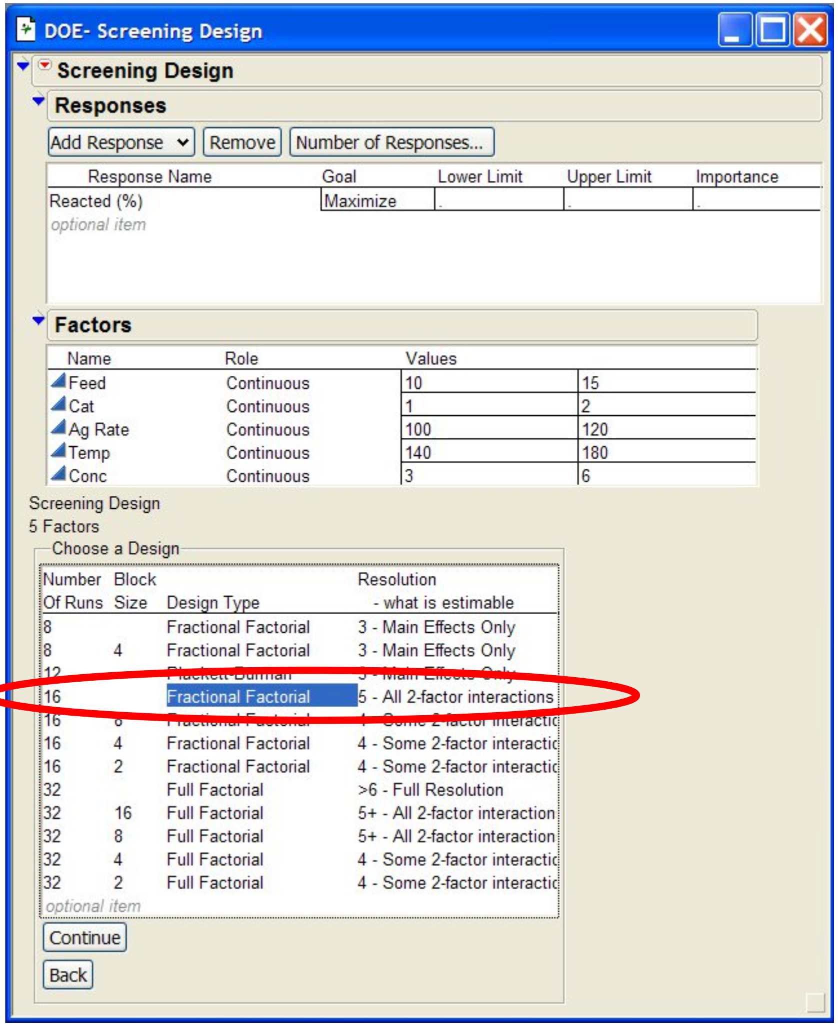
JMP DOE Design

- **JMP>DOE>Classical**
- **>Screening Design**
- Change the Y under Response Name to Reacted (%).
- We want to maximize the response
- Select continue.



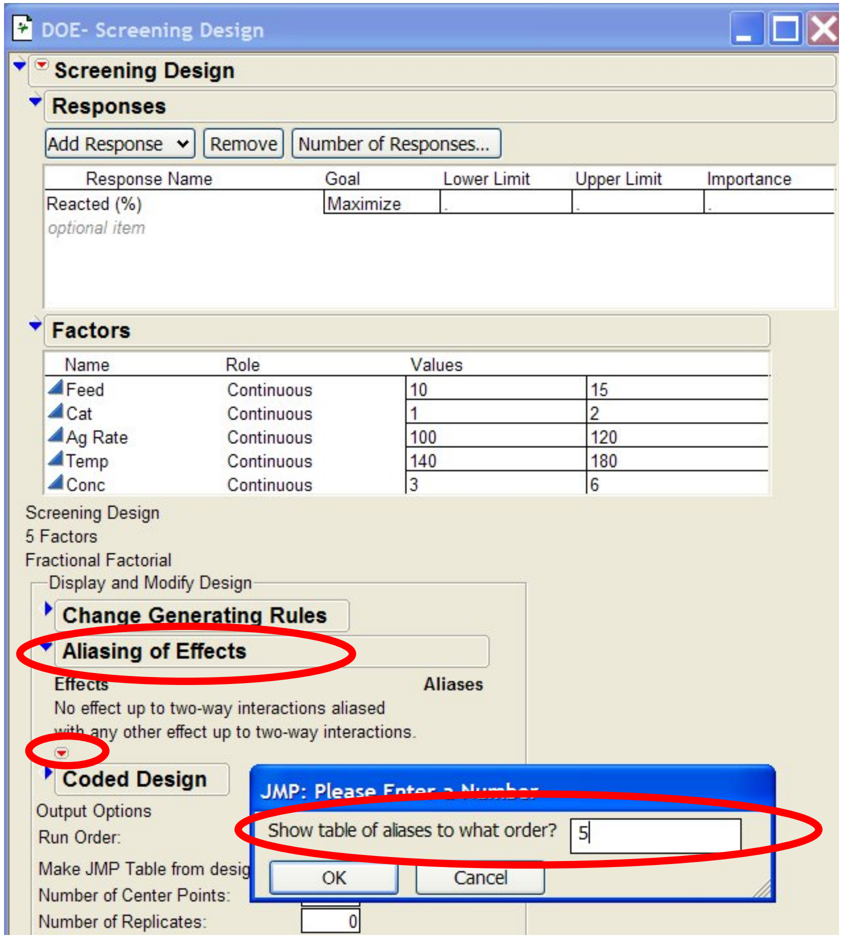
JMP DOE Design *(Cont.)*

- Since we want to conduct a ½ Fractional Factorial with no block effects, select 16.
- Note: $2^5=32$, $2^{5-1}=16$
- Note: This is **Resolution V** Design [Alias Structure Ex. 1=2345, 12=345, 35=_____, 5=_______]
- Select **Continue.**



JMP DOE Design *(Cont.)*

- Select Aliasing of Effects.
- Then quick on the Red Triangle and for the aliases to what order, enter 5.
- Select OK.



JMP DOE Design *(Cont.)*

Note: C=12345 is called the Defining Relation, I

1*12345 = 2345 or

1=2345

12*12345 = 345 or

12 = 345

Use the defining relation to compute the alias structure for:

14*12345 = ___

14 = ___

4*12345 = ____

4 = ____

Confounding Pattern

Confounding Pattern

Columns (2/0)

Effect Names

Alias Names

Rows

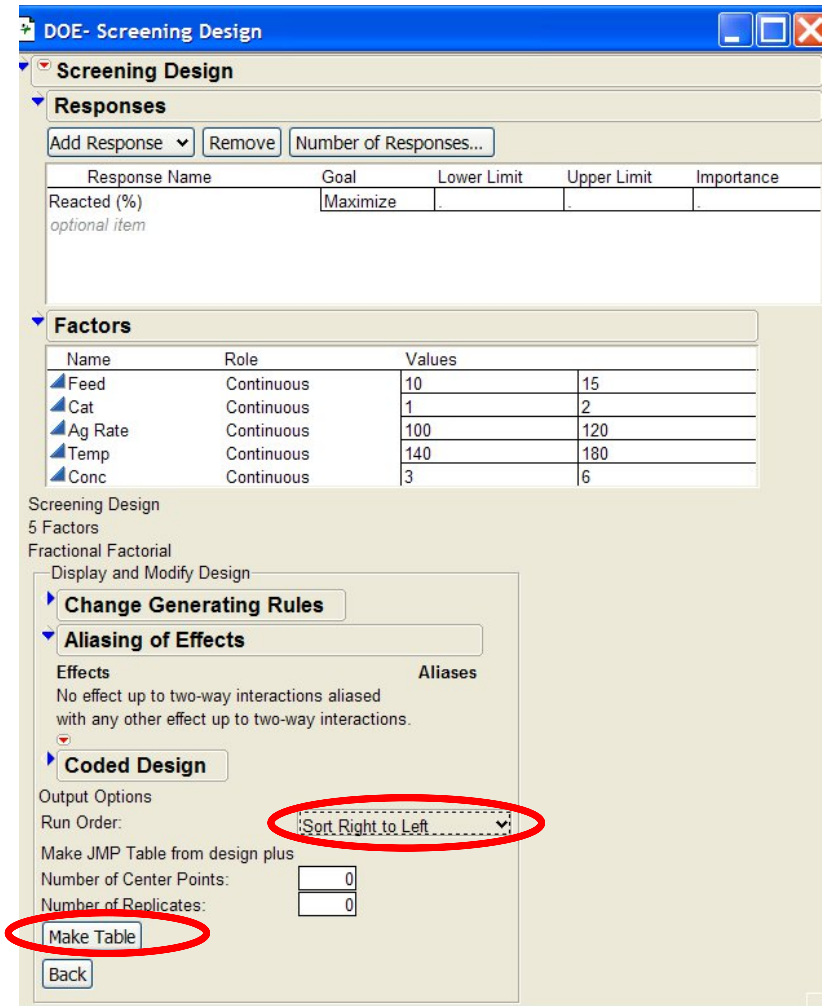
All rows	32
Selected	0
Excluded	0
Hidden	0
Labelled	0

	Effect Names	Alias Names
1	C	= 1 2 3 4 5
2	1	= 2 3 4 5
3	2	= 1 3 4 5
4	1 2	= 3 4 5
5	3	= 1 2 4 5
6	1 3	= 2 4 5
7	2 3	= 1 4 5
8	1 2 3	= 4 5
9	4	= 1 2 3 5
10	1 4	= 2 3 5
11	2 4	= 1 3 5
12	1 2 4	= 3 5
13	3 4	= 1 2 5
14	1 3 4	= 2 5
15	2 3 4	= 1 5
16	1 2 3 4	= 5
17	5	= 1 2 3 4
18	1 5	= 2 3 4
19	2 5	= 1 3 4
20	1 2 5	= 3 4
21	3 5	= 1 2 4
22	1 3 5	= 2 4
23	2 3 5	= 1 4
24	1 2 3 5	= 4
25	4 5	= 1 2 3
26	1 4 5	= 2 3
27	2 4 5	= 1 3
28	1 2 4 5	= 3
29	3 4 5	= 1 2
30	1 3 4 5	= 2
31	2 3 4 5	= 1
32	1 2 3 4 5	= C

- Now **Minimize** the Confounding Pattern Data Table

JMP DOE Design *(Cont.)*

- Under the Output Options line, click on bar next to word Randomize and select **Sort Right to Left.** (This for educational purposes only. You would want to **randomize** the real design.)
- Select **Make Table**



JMP DOE Design *(Cont.)*

- The next step in the experiment is to collect the data.
- Enter the **Reacted (%) data** into the table.

	Pattern	Feed	Cat	Ag Rate	Temp	Conc	Reacted (%)
1	+−−−−	15	1	100	140	3	53
2	−+−−−	10	2	100	140	3	63
3	−−+−−	10	1	120	140	3	53
4	+++−−	15	2	120	140	3	61
5	−−−+−	10	1	100	180	3	69
6	++−+−	15	2	100	180	3	93
7	+−++−	15	1	120	180	3	60
8	−+++−	10	2	120	180	3	95
9	−−−−+	10	1	100	140	6	56
10	++−−+	15	2	100	140	6	65
11	+−+−+	15	1	120	140	6	55
12	−++−+	10	2	120	140	6	67
13	+−−++	15	1	100	180	6	45
14	−+−++	10	2	100	180	6	78
15	−−+++	10	1	120	180	6	49
16	+++++	15	2	120	180	6	82