Algorithms and Data Structures

Assignment 3

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1 Problem 3.1

1.1 $f(n) = 9n$, $g(n) = 5n^3$

1. $f \in O(g)$
   In order for $f \in O(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   $\lim_{x\to\infty} \frac{9n}{5n^3} = 0 < \infty$
   $\therefore f \in O(g)$

2. $f \in \Omega(g)$
   In order for $f \in \Omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} > 0$ must hold.
   $\lim_{x\to\infty} \frac{9n}{5n^3} = 0$, it is not > 0
   $\therefore f \notin \Omega(g)$

3. $f \in \Theta(g)$
   In order for $f \in \Theta(g)$, $0 < \lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   $\Theta(g) = O(g) \cap \Omega(g)$
   It was proven that $f \notin \Omega(g)$
   $\therefore f \notin \Theta(g)$

4. $f \in o(g)$
   In order for $f \in o(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{9n}{5n^3} = 0$
   $\therefore f \in o(g)$

5. $f \in \omega(g)$
   In order for $f \in \omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = \infty$ must hold.
   $\lim_{x\to\infty} \frac{9n}{5n^3} = 0 \neq \infty$
   $\therefore f \notin \omega(g)$

6. $g \in O(f)$
   As claimed previously $f \notin \Omega(g)$
   According to the transpose symmetric property $f(n) = O(g(n)) \Leftrightarrow g(n) = \Omega(f(n))$
   $\therefore g \notin O(f)$

7. $g \in \Omega(f)$
   As claimed previously $f \in O(g)$
   According to the transpose symmetric property $f(n) = \Omega(g(n)) \Leftrightarrow g(n) = O(f(n))$
   $\therefore g \in \Omega(f)$

8. $g \in \Theta(f)$
   It was previously proven that $g \notin O(f)$
   $\Theta(f) = O(f) \cap \Omega(g)$
   $\therefore g \notin \Theta(g)$

9. $g \in o(f)$
   In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{5n^3}{9n} = \infty$
   $\therefore g \notin o(f)$

10. $g \in \omega(f)$
    In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = \infty$ must hold.
    $\lim_{x\to\infty} \frac{5n^3}{9n} = \infty$
    $\therefore g \in \omega(f)$

1.2 $f(n) = 9n^{0.8} + 2n^{0.3} + 14log(n)$, $g(n) = n^{0.5}$

1. $f \in O(g)$
   In order for $f \in O(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   $\lim_{x\to\infty} \frac{9n^{0.8}+2n^{0.3}+14log(n)}{n^{0.5}} = \infty$
   $\therefore f \notin O(g)$

2. $f \in \Omega(g)$
   In order for $f \in \Omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} > 0$ must hold.
   $\lim_{x\to\infty} \frac{9n^{0.8}+2n^{0.3}+14log(n)}{n^{0.5}} = \infty > 0$
   $\therefore f \in \Omega(g)$

3. $f \in \Theta(g)$
   In order for $f \in \Theta(g)$, $0 < \lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   $\Theta(g) = O(g) \cap \Omega(g)$
   It was proven that $f \notin O(g)$
   $\therefore f \notin \Theta(g)$

4. $f \in o(g)$
   In order for $f \in o(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{9n^{0.8}+2n^{0.3}+14log(n)}{n^{0.5}} = \infty$
   $\therefore f \notin o(g)$

5. $f \in \omega(g)$
   In order for $f \in \omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = \infty$ must hold.
   $\lim_{x\to\infty} \frac{f9n^{0.8}+2n^{0.3}+14log(n)}{n^{0.5}} = \infty$
   $\therefore f \in \omega(g)$

6. $g \in O(f)$
   As claimed previously $f \in \Omega(g)$
   According to the transpose symmetric property $f(n) = O(g(n)) \Leftrightarrow g(n) = \Omega(f(n))$
   $\therefore g \in O(f)$

7. $g \in \Omega(f)$
   As claimed previously $f \notin O(g)$
   According to the transpose symmetric property $f(n) = \Omega(g(n)) \Leftrightarrow g(n) = O(f(n))$
   $\therefore g \notin \Omega(f)$

8. $g \in \Theta(f)$
   It was previously proven that $g \notin \Omega(f)$
   $\Theta(f) = O(f) \cap \Omega(g)$
   $\therefore g \notin \Theta(g)$

9. $g \in o(f)$
   In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{n^{0.5}}{9n^{0.8}+2n^{0.3}+14log(n)} = 0$
   $\therefore g \in o(f)$

10. $g \in \omega(f)$
    In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = \infty$ must hold.
    $\lim_{x\to\infty} \frac{n^{0.5}}{9n^{0.8}+2n^{0.3}+14log(n)} = 0$
    $\therefore g \notin \omega(f)$

1.3 $f(n) = \frac{n^2}{log(n)}$, $g(n) = nlog(n)$

1. $f \in O(g)$
   In order for $f \in O(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   We simplify our initial given functions to $\lim_{x\to\infty} \frac{n}{(log(n))^2} = \infty$
   $\therefore f \notin O(g)$

2. $f \in \Omega(g)$
   In order for $f \in \Omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} > 0$ must hold.
   $\lim_{x\to\infty} \frac{n}{(log(n))^2} = \infty > 0$
   $\therefore f \in \Omega(g)$

3. $f \in \Theta(g)$
   In order for $f \in \Theta(g)$, $0 < \lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   $\Theta(g) = O(g) \cap \Omega(g)$
   It was proven that $f \notin O(g)$
   $\therefore f \notin \Theta(g)$

4. $f \in o(g)$
   In order for $f \in o(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{n}{(log(n))^2} = \infty$
   $\therefore f \notin o(g)$

5. $f \in \omega(g)$
   In order for $f \in \omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = \infty$ must hold.
   $\lim_{x\to\infty} \frac{n}{(log(n))^2} = \infty$
   $\therefore f \in \omega(g)$

6. $g \in O(f)$
   As claimed previously $f \in \Omega(g)$
   According to the transpose symmetric property $f(n) = O(g(n)) \Leftrightarrow g(n) = \Omega(f(n))$
   $\therefore g \in O(f)$

7. $g \in \Omega(f)$
   As claimed previously $f \notin O(g)$
   According to the transpose symmetric property $f(n) = \Omega(g(n)) \Leftrightarrow g(n) = O(f(n))$
   $\therefore g \notin \Omega(f)$

8. $g \in \Theta(f)$
   It was previously proven that $g \notin \Omega(f)$
   $\Theta(f) = O(f) \cap \Omega(g)$
   $\therefore g \notin \Theta(g)$

9. $g \in o(f)$
   In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{(log(n))^2}{n} = 0$
   $\therefore g \in o(f)$

10. $g \in \omega(f)$
    In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = \infty$ must hold.
    $\lim_{x\to\infty} \frac{(log(n))^2}{n} = 0$
    $\therefore g \notin \omega(f)$

1.4 $f(n) = (log(3n))^3$, $g(n) = 9log(n)$

1. $f \in O(g)$
   In order for $f \in O(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   We simplify our initial given functions to $\lim_{x\to\infty} \frac{(log(3n))^3}{9log(n)} = \infty$
   $\therefore f \notin O(g)$

2. $f \in \Omega(g)$
   In order for $f \in \Omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} > 0$ must hold.
   $\lim_{x\to\infty} \frac{(log(3n))^3}{9log(n)} = \infty > 0$
   $\therefore f \in \Omega(g)$

3. $f \in \Theta(g)$
   In order for $f \in \Theta(g)$, $0 < \lim_{x\to\infty} \frac{f(n)}{g(n)} < \infty$ must hold.
   $\Theta(g) = O(g) \cap \Omega(g)$
   It was proven that $f \notin O(g)$
   $\therefore f \notin \Theta(g)$

4. $f \in o(g)$
   In order for $f \in o(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{(log(3n))^3}{9log(n)} = \infty$
   $\therefore f \notin o(g)$

5. $f \in \omega(g)$
   In order for $f \in \omega(g)$, $\lim_{x\to\infty} \frac{f(n)}{g(n)} = \infty$ must hold.
   $\lim_{x\to\infty} \frac{(log(3n))^3}{9log(n)} = \infty$
   $\therefore f \in \omega(g)$

6. $g \in O(f)$
   As claimed previously $f \in \Omega(g)$
   According to the transpose symmetric property $f(n) = O(g(n)) \Leftrightarrow g(n) = \Omega(f(n))$
   $\therefore g \in O(f)$

7. $g \in \Omega(f)$
   As claimed previously $f \notin O(g)$
   According to the transpose symmetric property $f(n) = \Omega(g(n)) \Leftrightarrow g(n) = O(f(n))$
   $\therefore g \notin \Omega(f)$

8. $g \in \Theta(f)$
   It was previously proven that $g \notin \Omega(f)$
   $\Theta(f) = O(f) \cap \Omega(g)$
   $\therefore g \notin \Theta(g)$

9. $g \in o(f)$
   In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = 0$ must hold.
   $\lim_{x\to\infty} \frac{9log(n)}{(log(3n))^3} = 0$
   $\therefore g \in o(f)$

10. $g \in \omega(f)$
    In order for $f \in o(f)$, $\lim_{x\to\infty} \frac{g(n)}{f(n)} = \infty$ must hold.
    $\lim_{x\to\infty} \frac{9log(n)}{(log(3n))^3} = 0$
    $\therefore g \notin \omega(f)$