From Scenarios to Optimally Allocated Timed Automata

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Objectives of the Research

Our main focus of the research is:

- 1. To synthesize a timed automaton from a set of scenarios, and
- 2. To optimally allocate clocks in the constructed timed automaton.

Background

Modeling Time

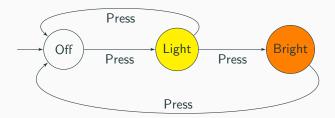
There are three approaches for modeling time:

- <u>Discrete time model</u>: Time is considered as discrete and monotonically increasing sequence of integers. Limits the preciseness: in real-time systems, the events do not occur at integer times
- <u>Fictitious-clock model</u>: It is similar to that of discrete time model except that it assumes sequence of times to be non decreasing integers. Limits accuracy, as the exact time values at which the events occur are not considered.
- <u>Dense time model</u>: In this model, the domain of time is a set of real numbers, and the sequence of times increase monotonically without any limit.

Timed Automata

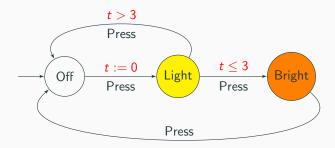
- A timed automaton is a finite state automaton extended with a finite set of real-valued clocks.
- Upon an input, the selection of next state is based not only on the input symbol but also on the time of the current symbol with respect to the formerly read symbols.

A simple light control



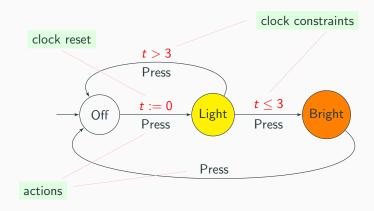
Desired behavior: if Press occurs twice quickly then the light gets brighter, otherwise the light turns off.

A simple light control



Solution: finite state machines augmented with real-valued clocks.

Timed automata



Timed Automata: syntax

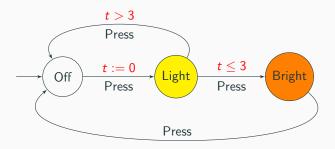
For a set V of clock variables:

- the set $\Phi(V)$ includes *clock constraints* of the form $t \sim a$, where
 - $t \in V$,
 - $\sim \in \{ \leq, \geq, <, >, = \}$,
 - a is a constant in the set of rational numbers, \mathbb{Q} .

A timed automaton is a tuple $A = \langle \Sigma, Q, Q_0, V, E \rangle$, where

- Σ is a finite set of labels/actions (alphabet)
- Q is a finite set of states
- $Q_0 \subseteq Q$ is a set of initial states
- V is a finite set of clocks
- $E \subseteq Q \times Q \times \Sigma \times 2^{V} \times 2^{\Phi(V)}$ is a finite set of edges of the form $(q, q', \sigma, \lambda, \phi)$, where
 - $\lambda \subseteq V$ is the clocks to be reset with this transition,
 - ϕ is a set of clock constraints over V.

Timed automata



```
The language accepted by the automaton:
:
(press, 5)(press, 6.5)(press, 20)(press, 50)(press, 65)...
(press, 3)(press, 9)(press, 15)(press, 19)(press, 100)...
:
```

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Timed Automata: Undecidable Problem

- The number of clocks in a given timed automaton has a direct impact on verification of the system.
- Given a timed automaton \mathcal{A} , the problem of deciding whether there exists another timed automaton \mathcal{B} that accepts the same language as that of \mathcal{A} but with fewer number of clocks is undecidable.

Motivation

Motivation

- Model-based design is a very effective method for designing real-time systems.
- Modeling a system formally can help us to understand the desired and undesired behaviours of the system.
- Building formal models for systems is challenging because of the lack of good formal requirements specifications.

Motivation

- To construct a formal model of a system, the following questions are to be answered first:
 - 1. How the requirements should be expressed formally, and
 - 2. How the formal model of the system can be constructed from requirements.
- The formal model that we build is timed automata.

Contribution 1

- We use scenarios to build a formal model of a real-time system.
- We use mode graphs to specify the legal events that can occur in the system.

We synthesize a *minimal*, *acyclic* and *deterministic* timed automaton given a set of scenarios and a mode graph.

Contribution 2

- We propose a method to optimally allocate clocks in a timed automaton.
- We perform liveness analysis of clocks in a timed automaton and use that information to minimize the number of clocks and optimally allocate them.

Contributions

Synthesis of Timed Automata from Scenarios

Our method of synthesizing a timed automaton model of a real-time system from scenarios involves two steps:

- 1. Constructing a time annotated graph from a set of scenarios, and
- 2. Transforming this graph to the final timed automaton.

Scenarios

- A scenario is a partial description of the behaviour of a system.
- A scenario not only describes the events, but also the timing relations among the events.
- A set of scenarios can capture the behaviour of a real-time system.
- We use mode graphs to specify the legal events that can occur in the system.
- We propose Timed Event Sequences (TES) to describe the scenarios formally.

Mode Graph

A mode graph is a deterministic state machine in which the states are called modes and the transitions triggered by the events in the system. It is a tuple $\mathcal{M}=(M,m_0,m_f,\Sigma,T)$ where,

- *M* is a finite set of modes,
- m_0 is the initial mode,
- m_f is the final mode,
- \bullet Σ is a set of events, and
- $T: M \times \Sigma \to M$ is a transition function.

Mode Graph Example

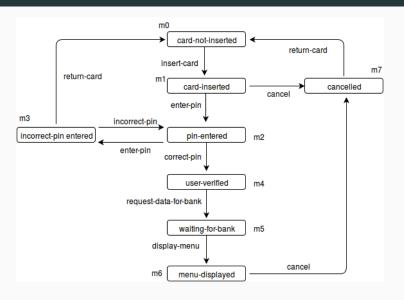


Figure 1: Mode Graph for ATM

Timed Event Sequences (TES)

A Timed Event Sequence ξ contains:

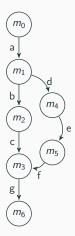
- 1. The initial mode of the scenario,
- 2. The final mode of the scenario,
- A set of events and their corresponding time annotations.

TES of ATM scenario

Dominance Assumption

- Given two modes m_i and m_j , m_i is said to be the dominating mode of m_j iff all the paths to m_j from the initial mode in the mode graph pass through m_i . We call this the *Dominance* relation and denote it as m_i *DOM* m_i .
- Dominance assumption ensures that time variables are well defined.

Dominance Assumption Example



- m_0 and m_1 are dominating modes of m_3 ,
- Transition g is dominated by all the modes that dominate m_3 ,
- t_0 and t_1 , on transition g refer to the time of leaving m_0 and m_1 .

Constructing a Time Annotated Graph from Scenarios

Given a mode graph \mathcal{M} and a set of Timed Event Sequences $\Xi = \{\xi_1, \xi_2, ..., \xi_k\}$ as inputs, we propose an algorithm for synthesizing a time annotated graph (TAG) G. Initially we start with an empty graph, G_0 and perform the following steps:

- 1. Build a partial graph G_1 using the first scenario ξ_1 ,
- 2. The algorithm repeatedly takes a partially built graph G_k , and a scenario ξ_{k+1} (1 < k < n) and then generates a new partial graph G_{k+1} .

Decision on whether to create new states and transitions is resolved with the help of state labels (modes). A new state s is created and labelled with a mode m_j if there is an event e from state q such that $L(q) = m_i$ and $(m_i, e, m_j) \in \mathcal{T}$.

Properties of Constructed Time Annotated Graph

The graph constructed by our algorithm has the following properties:

- 1. It is acyclic,
- 2. It is connected,
- 3. By construction, two states have the same label only if one is a predecessor of the other,
- 4. It is finite,
- 5. It is deterministic,
- 6. It is minimal,
- 7. After construction, every scenario is a partial run of the constructed graph, and
- 8. For every path in the constructed graph there is a corresponding path in the mode graph.

Constructing a Timed Automaton from Time Annotated Graph

After constructing the time annotated graph, we need to transform it to the target timed automaton. For that, the following steps should be performed in order:

- 1. Determine the required number of clocks,
- 2. Generate clock resets and clock constraints, and
- 3. Add the clock resets and constraints to the appropriate transitions in the time annotated graph.

Example: If there is a time annotation $W - t_0 > 5$ on a transition, then the clock constraint $c_0 > 5$ is added to that transition and clock c_0 is reset on all transitions from the state labelled with mode m_0 .

Example of Synthesis Method

```
minitial: card-not-inserted
                                                     minitial: card-not-inserted
(insert-card, {})
( enter-pin, \{W - t_0 > 5, W - t_0 < 60\})
(incorrect-pin, {})
                                                     (insert-card, {})
( re-enter-pin, \{W - t_0 \ge 5, W - t_0 \le 60\})
                                                     ( enter-pin, \{W - t_0 \ge 5, W - t_0 \le 60\})
(correct-pin, {})
                                                     ( correct-pin, {})
( request-data-from-bank, {})
                                                     ( request-data-from-bank, {})
(display-menu, \{W - t_4 < 5\})
                                                     (display-menu, \{W - t_4 < 5\})
mfinal: menu-displayed
                                                     mfinal: menu-displayed
             TFS of Scenario 1
                                                                 TES of Scenario 2
```

Figure 2: Timed Event Sequences of the ATM

Example (Cont.)

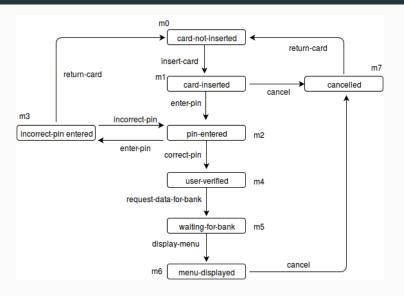
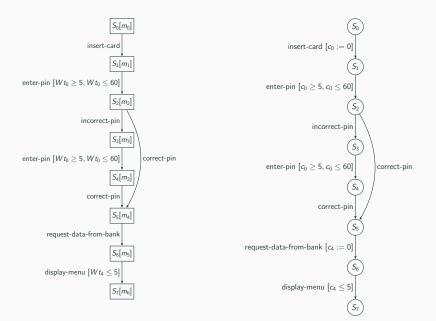


Figure 3: Mode Graph for ATM

Example (Synthesized Graph and Timed Automaton)



Class of Timed Automata

The timed automaton constructed as a result of our synthesis method belongs to the class of timed automata that satisfies these properties:

- 1. The automaton is connected and has a unique initial state s_0 ,
- 2. A clock constraint on a transition 'r' can only refer to the times of transitions from states that dominate the transition 'r', we call this the *dominance assumption*,
- 3. A clock t_j can only be reset on a transition leaving a state s, where label is j, that is L(s) = j.

Optimal Clock Allocation of Timed Automata

To optimally allocate clocks in a timed automaton A that belongs to our class of timed automata, we need to perform the following steps in order:

- 1. Determine the liveness ranges of clocks in the timed automaton A,
- 2. Determine the minimum number of clocks required,
- 3. Replace the original clocks in ${\cal A}$ with a set of new clocks,
- 4. Rewrite the clock constraints and clock resets in ${\cal A}$ in terms of new clock variables.

Liveness Range Analysis

Liveness Ranges of all the clocks in a timed automaton helps us determine if a particular clock is needed on these transitions.

- Let $A = (E, Q, \{q^0\}, Q_f, V, R, L)$ be the timed automaton and $r = (s, s', e, \lambda_r, \phi_r) \in R$ be a transition.
- Let N = {j | t_j ~ a ∈ φ ∨ t_j ∈ λ, where (s, s', e, λ_r, φ_r) ∈ R}, be a set of *clock numbers* used to denote subscripts of the clocks on all the transitions in R.

Liveness Range Analysis

The following are a set of functions used to calculate the liveness ranges:

- **clock_ref**: *clock_ref*(*r*) is the set of clocks which are referred to in the clock constraints on *r*.
- born: born(r) identifies a clock that is reset on r whose value can be used on some transition reachable from r.
- active: active(r) identifies clocks that are "alive" on r (i.e., their values may be subsequently used). Notice that $born(r) \subseteq active(r)$.
- **needed**: Maps transition r to $active(r) \cup clock_ref(r)$.

Liveness Analysis Algorithm

Algorithm 1: Building the liveness ranges for clocks

```
Input: A timed automaton A = \langle E, Q, \{q^0\}, Q_f, V, R, L \rangle.
Output: An extended timed automaton
          A_e = \langle E, Q, \{g^0\}, Q_f, V, R_e, L \rangle, where R_e is the set of
          extended transitions.
R_{\circ} := \emptyset:
foreach transition r = (s, q, e, \phi) \in R in A do
   born(r) := active(r) := \emptyset;
repeat
   foreach transition r = (s, q, e, \lambda, \phi) \in R in A do
        foreach r_0 \in out(q) do
           active(r) :=
         active(r) \cup ((active(r_o) \cup clock\_ref(r_o)) \setminus born(r_o));
       if L(s) = i and i \in active(r) then
      until there were no changes;
```

Liveness Range Analysis Example

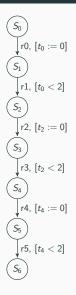


Table 1: born and active functions

Transition	Born	Active
r_0	{0}	{0}
r_1	ϕ	ϕ
r_2	{2}	{2}
r_3	ϕ	ϕ
r_4	{4 }	{4 }
<i>r</i> ₅	ϕ	ϕ

Figure 4: A simple timed automaton

Clock Allocation

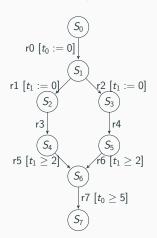
Our liveness analysis algorithm calculates the liveness ranges of clocks and generates extended transitions of the form (r, born(r), active(r)).

Our method to optimally allocate the clocks revolves around the idea that:

- A clock can be reused if the active range of the clock has ended,
- The clock cannot be reused on a transition if the transition belongs to active range of that clock.

Clock Allocation

Definition: Given a timed automaton \mathcal{A} with the set R of (extended) transitions and the set N of clock numbers, a *clock allocation* for \mathcal{A} is a relation $alloc \subset R \times P_0 \times N$ such that $(r, c, j) \in alloc \Rightarrow j \in active(r)$. Where, P_0 is the pool of new clock variables.



alloc =
$$\{(r_0, c_0, 0), (r_1, c_0, 0), (r_2, c_0, 0), (r_3, c_0, 0), (r_4, c_0, 0), (r_5, c_0, 0), (r_6, c_0, 0), (r_1, c_1, 1), (r_2, c_1, 1), (r_3, c_1, 1), (r_4, c_1, 1)\}$$

Problematic States

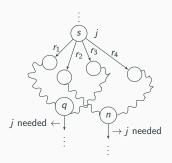


Figure 5: A timed automaton with problematic states

- Clock t_i is born on all outgoing transitions of state s.
- r_1 , r_3 meet at state q and r_2 , r_4 meet at state n.

States q and n are the *problematic* states. So, r_1 , r_3 should be assigned the same clock and r_2 , r_4 should be assigned the same clock.

Handling the Problematic States

To handle the problematic states, we partition the transitions into mothers and others.

- mothers: $\{r \in out(s) \mid j \in born(r)\}$
- others: $\{r \in out(s) \mid born(r) = \emptyset\}$

We use the following functions:

- reachable : $Q \rightarrow 2^Q$ maps state q to the set of states that are reachable from q by some non-empty path.
- $reachable_from: Q \rightarrow 2^Q$ maps state q to the set of states from which it can be reached by some non-empty path.

Clock Allocation Algorithm

```
Procedure compute-allocation(timed automaton \mathcal{A}_e, set of clocks P_0)

Input: An extended timed automaton \mathcal{A}_e = \langle E, Q, \{q^0\}, Q_f, V, R_e, L \rangle and the initial pool of available clocks, P_0.

Output: An extended timed automaton \mathcal{A}'_e = \langle E, Q_e, \{q^0\}, Q_f, V, R_e, L \rangle, where Q_e = Q \times 2^{P_0} \times 2^{P_0 \times N}.

foreach state s \in Q do

Set the status of s to Unseen; annotate (q^0, P_0, \emptyset); visit(q^0);
```

```
Procedure annotate(state q, set of clocks p, set of assignments \dashv)
// Invoked only when status of q is Unseen.
pool(q) := p;
assignments(q) := \dashv;
Set the status of q to Seen;
Procedure visit(state q)
// Invoked only when the status of q is Seen or Visited.
if status of q is not Visited then
   Set the status of q to Visited:
   annotate-immediate-successors-of(q);
   foreach r \in out(q) do
       visit(target(r));
```

Procedure annotate-immediate-successors-of(state q) Partition out(q) into mothers and others; foreach $r \in others$ do **if** status of target(r) is Unseen **then** propagate(q, r, \emptyset); // Otherwise target(r) is already properly annotated if $mothers \neq \emptyset$ then Groups := partition-into-a-set-of-groups(q, mothers);**foreach** *group* ∈ *Groups* **do** c := find-clock(q, group);**foreach** $r \in group$ **do** // The target of r is *Unseen* (by the dominance assumption). $propagate(q, r, \{c\});$

Procedure propagate(state q, transition r, set of clocks sc)

```
//q is the source of r. Propagate pool(q) and assignments(q) to
 target(r), taking into account that some clock ranges
// may end on r. If sc is not empty, it must be a singleton: in that case
 assign its member to clock number L(q).
// Invoked only when the target of r is Unseen.
freed\_assignments := \{(d, j) \mid (d, j) \in assignments(q) \land j \notin active(r)\};
freed\_clocks := \{d \mid (d, j) \in freed\_assignments\};
tmp\_pool := pool(q) \cup freed\_clocks;
tmp\_assignments := assignments(q) \setminus freed\_assignments;
if sc \neq \emptyset then
    tmp\_pool := tmp\_pool \setminus sc;
   tmp\_assignments := tmp\_assignments \cup \{(c, L(q))\}, where c \in sc;
annotate(target(r), tmp_pool, tmp_assignments);
```

```
Procedure partition-into-a-set-of-groups(state q, set of transitions
mothers)
```

```
mother\_targets := \{target(r) \mid r \in mothers\};
// Initially, each mother is in its own group.
Groups := \emptyset;
foreach r \in mothers do
    Groups := Groups \cup \{r\};
PP := \emptyset; // potentially problematic states
foreach r \in mothers do
   foreach s \in reachable(target(r)) do
       if L(q) \in active(r'), where r' is an arbitrary transition of in(s)
        then
         PP := PP \cup \{s\};
// Those states in PP that can be reached from more than one mother
 are the problematic states.
```

foreach $s \in PP$ do

```
targets := reachable\_from(s) \cap mother\_targets;
Merge those members of Groups that contain transitions whose
 target is in targets;
```

return Groups:

Procedure find-clock(state q, set of transitions group)

```
// Find a clock for L(q) on transitions in group. live\_on\_entry := \{j \mid (c,j) \in assignments(q)\}; dying\_all := \bigcap_{r \in group}(live\_on\_entry \setminus active(r)); // The set of clocks whose liveness ranges end in all the transitions in group: released\_all := \{c \mid (c,j) \in assignments(q) \land j \in dying\_all\}; available := released\_all \cup pool(q); Return the clock variable with the smallest number in available;
```

Example: Optimal Clock Allocation

$$S_0$$
 \downarrow r0, $[t_0 := 0]$
 S_1
 \downarrow r1, $[t_0 < 2]$
 S_2
 \downarrow r2, $[t_2 := 0]$
 S_3
 \downarrow r3, $[t_2 < 2]$
 S_4
 \downarrow r4, $[t_4 := 0]$
 S_5
 \downarrow r5, $[t_4 < 2]$
 S_6

Figure 6: A simple timed automaton

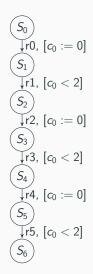


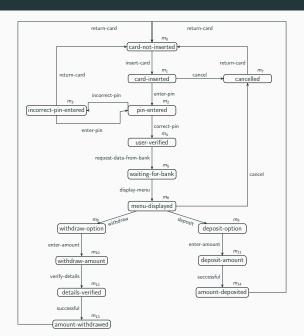
Figure 7: A simple optimally allocated timed automaton

Case Studies

We use an invariant of an ATM:

- 1. Initially, the ATM waits for a user to insert his card,
- 2. User has to enter his PIN within 5 to 60 seconds. User has to enter correct PIN in 3 attempts,
- 3. ATM requests the bank to verify the user's PIN, and
- 4. If the PIN is correct, ATM displays menu to the user within 5 seconds.

Mode Graph of ATM



```
\begin{array}{c} \hline m^{initial} \colon \  \, \text{card-not-inserted} \\ \\ ( \  \, \text{insert-card}, \ \{\}) \\ ( \  \, \text{enter-pin}, \ \{W-t_0 \geq 5, \ W-t_0 \leq 60\}) \\ ( \  \, \text{incorrect-pin}, \ \{\}) \\ ( \  \, \text{re-enter-pin}, \ \{W-t_0 \geq 5, \ W-t_0 \leq 60\}) \\ ( \  \, \text{correct-pin}, \ \{\}) \\ ( \  \, \text{correct-pin}, \ \{\}) \\ ( \  \, \text{request-data-from-bank}, \ \{\}) \\ ( \  \, \text{display-menu}, \ \{W-t_4 \leq 5\}) \\ \hline \\ m^{final} \colon \  \, \text{menu-displayed} \\ \hline \\ \text{TES of Scenario 1} \\ \\ \hline \end{array}
```

```
m^{initial}: card-not-inserted ( insert-card, {}) ( enter-pin, {W-t_0 \geq 5, \ W-t_0 \leq 60}) ( correct-pin, {}) ( request-data-from-bank, {}) ( display-menu, {W-t_4 \leq 5}) m^{final}: menu-displayed
```

Figure 8: Timed Event Sequences of the ATM

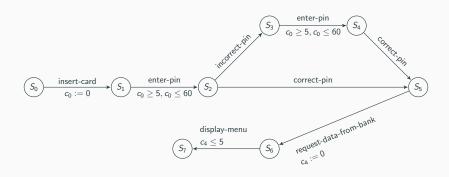


Figure 9: Timed automaton synthesized from Scenario 1 and Scenario 2

Now consider an extended behaviour of the ATM machine. After the menu is displayed, assume that, the user can choose from the two available options:

Deposit:

- User enters the amount to deposit, and
- ATM returns success message if the deposit is successful.

Withdraw:

- 1. User enters the amount to withdraw,
- 2. Bank verifies the user details, and
- Returns success message if the withdraw is successful.

```
m^{initial}: menu-displayed m^{initial}: menu-displayed m^{initial}: menu-displayed m^{initial}: m^{initial}: m^{initial}: m^{initial}: card-not-inserted m^{initial}: card-not-inserted m^{initial}: m^{initial}:
```

```
minitial: menu-displayed
( withdraw, {})
( enter-amount, \{W - t_6 \le 20\})
(verify-details, {})
( successful, \{W - t_{10} < 10\})
( return-card, {})
m<sup>final</sup>: card-not-inserted
        TES of Scenario 4
```

Figure 10: Timed Event Sequences of the ATM with withdraw and deposit option

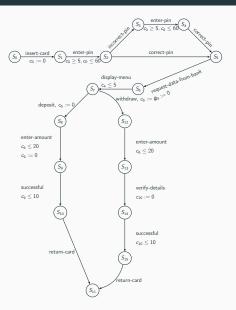


Figure 11: The synthesized timed automaton of the ATM

Light Control System

Consider the variant of the light control system:

- 1. Idle is both the initial and final mode,
- 2. The mode changes from *Idle* to *Light* upon issuing the command *ON*, and
- 3. If *ON* is issued again within 3 units of time, the light will go *Bright*. Else the light goes back to *Idle*.

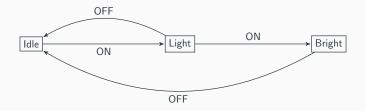


Figure 12: Mode graph of the Light Control System

Light Control System

```
minitial . Idle
                               minitial: Light
                                                               minitial: Bright
(ON, \{\})
(OFF, \{w - t0 > 3\})
                               (ON, \{w-t0 <= 3\})
                                                              (OFF, {})
(ON, \{\})
                              (OFF, {})
(ON, \{w-t0 <= 3\})
                                                              (ON, \{w-t0 \le 3\})
                             (ON, \{\})
                               (OFF, \{w - t0 > 3\})
(OFF, {})
                                                              (OFF, {})
                               m<sup>final</sup>: Idle
                                                              m<sup>final</sup>: Idle
m<sup>final</sup>: Idle
      Scenario 1
                                      Scenario 2
                                                                     Scenario 3
```

Figure 13: Timed Event Sequences of the Light Control System

Light Control System

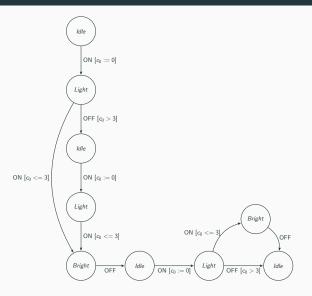


Figure 14: Timed automaton of the Light Control System

Traffic Light

Consider the behaviour of a traffic light.

- The system periodically moves from $Initial \rightarrow Red$, $Red \rightarrow Yellow$, $yellow \rightarrow Green$, and then it is Reset,
- On the transitions turn yellow and turn green a clock constraint is checked and a new clock is born.

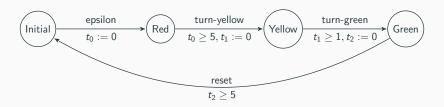


Figure 15: Timed automaton of the Traffic Light

Traffic Light

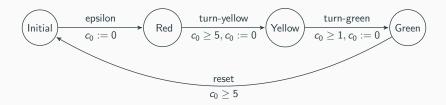


Figure 16: The optimally allocated timed automaton of the Traffic Light

CSMA/CD Protocol

Consider a variant of the CSMA/CD protocol.

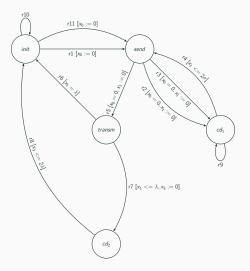


Figure 17: The timed automaton for the sender in CSMA/CD protocol

CSMA/CD Protocol

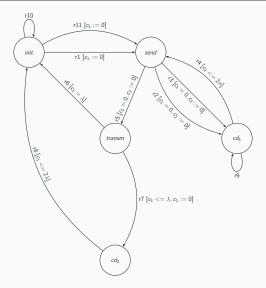


Figure 18: The optimally allocated timed automaton for the sender in CSMA/CD protocol

Conclusion

Conclusion

- Proposed Timed Event Sequences to formally represent scenarios,
- Developed and implemented an algorithm to construct timed automaton from a given set of scenarios expressed as TES and a mode graph,
- The synthesized timed automaton is minimal, deterministic and acyclic,
- The generated timed automaton belongs to a class of timed automata that satisfies the dominance assumption,
- Developed and implemented an optimal clock allocation algorithm to a class of timed automata that satisfies the dominance assumption, and
- Our algorithm, does not change the size of the original timed automaton and its complexity is quadratic in the size of the graph.

THANK YOU!

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