MSAI Probability Home Assignment 4
deadline: 22/11/2023 23:59 AOE

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable.

After submission deadline of this homework, we will have a webinar with discussions of HW1-4 solutions. Please make sure to submit all homeworks by then, as any solutions after homework discussion will not be graded.

Problem 1. (1 point) An airline overbooks a flight, selling more tickets for the flight than there are seats on the plane (figuring that it's likely that some people won't show up). The plane has 100 seats, and 110 people have booked the flight. Each person will show up for the flight with probability 0.9, independently. Find the probability that there will be enough seats for everyone who shows up for the flight.

Problem 2. (2 points) A good student solves a problem correctly with probability 0.95, while a bad student — with probability 0.15. What is the minimal number of problems that the test should include so that the probability that good student does not pass the test does not exceed 0.01, and the probability that the bad student passes the test does not exceed 0.1? Passing the test means solving strictly more than half of the problems.

Problem 3. (3 points) Consider two independent random variables $X \sim F_X$ and $Y \sim F_Y$. Find the CDF of random variables $Z_1 = \max(X, Y)$ (it means that for every outcome w we have $Z_1(w) = \max(X(w), Y(w))$, so Z_1 jumps between values of X and Y) and $Z_2 = \min(X, Y)$ (same reasoning applies).