MSAI Probability Home Assignment 3
deadline: 15/11/2023 23:59 AOE

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable. This homework includes a python task, which can be solved in Google Colab or in a local Jupyter Notebook. It is thus handy to solve everything (both LaTeX and code) in a single Jupyter Notebook.

Problem 1. (1 point) Prove that

$$\sum_{j=0}^{k} \binom{n}{j} \binom{m}{k-j} = \binom{m+n}{k}$$

Problem 2. (2 points) A basket contains m balls, out of which m_1 are white and m_2 are black $(m_1 + m_2 = m)$. We extract n balls from this basket **with replacement** and note their colors. Find the probability that out of these n balls exactly r were white. What is the name of this distribution?

Problem 3. (2 points) If $X \sim Bi(n, p)$ and $Y \sim Bi(m, p)$, and $X \perp Y$ (independent), what is $\mathbb{P}(X \mid X + Y = r)$, the PMF of the conditional distribution of X given $X + Y = r$? What is the name of this distribution?

Problem 4* . (3 bonus points) Two players are playing a game. The first player says a number p_1 between 0 and 1. The second player, knowing the number of the first player, says a number p_2 between 0 and 1. Then, with probability p_1 the first number becomes zero, and with probability p_2 the second number becomes zero. The player whose number is greater, wins.

- (2 bonus points) What is the optimal winning strategy of player two?
- (1 bonus point) What is the optimal winning strategy of player one, if player two follows strategy from the previous point?

Hint: simply consider different cases in which the second player has to make the choice. From the cases, the solution is easy to get.

Problem 5* . (2 bonus points) Find the PMF of distribution of a random variable X, which is equal to the number of failures in a series of Bernoulli trials with success probability p, which are carried out **not** a fixed number of times, but instead until there are r successes.

Problem 6* . (1 bonus points) Use python package SCIPY.STATS to plot the PMF of this distribution (Negative Binomial, NBINOM) for different values of parameter p.