MSAI Probability Home Assignment 1
deadline: 29/10/2023 23:59 **A**nywhere **O**n **E**arth

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable. This homework includes a python task, which can be solved in Google Colab or in a local Jupyter Notebook. It is thus handy to solve everything (both LaTeX and code) in a single Jupyter Notebook.

Problem 1. (3 points) There are 15 chocolate bars and 10 children. In how many ways can the chocolate bars be distributed to the children, in each of the following scenarios? In each scenario, write if we are sampling with or without replacement, and if order matters or not.

- (1 point) The chocolate bars are all the same
- (1 point) The chocolate bars are all the same, and each child must receive at least one
- (1 point) The chocolate bars are **not** all the same, meaning that it matters which child receives which chocolate bars

Problem 2^* **.** (2 bonus points) Additional scenario for the previous problem: The chocolate bars are **not** all the same, and each child must receive at least one. Hint: use result from the last subproblem and inclusion-exclusion formula to remove extra cases.

Problem 3. (3 points) A basket contains m balls, out of which m_1 are white and m_2 are black ($m_1 + m_2 = m$). We extract n balls from this basket **with replacement** and note their colors. Find the probability that out of these n balls exactly r were white.

Problem 4. (2 points) A fair die is rolled n times. On the webinar, we got the answer for the probability that at least 1 of the 6 values never appears. Use webinar notes and python to create a plot of this probability, and note after which n the probability becomes less than 1%.

Problem 5^* **.** (2 bonus points) The cloakroom of a theater has randomly permuted all n visitors' hats. Find the probability that at least one visitor gets his hat. Give the formula answer with derivation. Given $n = 4$, give a number answer. Hint: use inclusion-exclusion formula.