MSAI Probability Home Assignment 8
deadline: 03/01/2024 23:59 AOE

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable. This homework includes a python task, which can be solved in Google Colab or in a local Jupyter Notebook. It is thus handy to solve everything (both LaTeX and code) in a single Jupyter Notebook.

Problem 1. (2 points) Let $U \sim U[0, \frac{\pi}{2}]$. Find the PDF of $\sin(U)$.

Problem 2. Let X and Y be i.i.d. uniform on $(-b, b)$.

1. (2 points) Find the probability that equation $t^2 + tX + Y = 0$ has real roots.
2. (1 point) Visualize this probability as a function of b in Python.
3. (2 points) Find the limit of this probability as $b \to \infty$.

Problem 3. (2 points) Let X and Y be i.i.d. $Exp(\lambda)$, and $T = \log(X/Y)$. Find the CDF and PDF of T.