MSAI Probability Home Assignment 2
deadline: 08/11/2023 23:59 AOE

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable. This homework includes a python task, which can be solved in Google Colab or in a local Jupyter Notebook. It is thus handy to solve everything (both LaTeX and code) in a single Jupyter Notebook.

Problem 1. (2 points) There are two baskets. The first basket contains one white ball, the second basket contains one black ball. One basket is chosen randomly and a white ball is put into the chosen basket. The balls in this basket are shuffled. Then one ball is extracted from this basket. This ball turns out to be white. What is the posterior probability that the second ball drawn from this basket is also white?

Problem 2. (2 points) If you get a positive result on a COVID test that only gives a false positive with probability 0.001 (true positive with probability 0.999), what's the chance that you've actually got COVID, if

1. (1 point) The prior probability that a person has COVID is 0.01
2. (1 point) The prior probability that a person has COVID is 0.0001

Problem 3. (1 point) Show that if $\mathbb{P}(A|B) = \mathbb{P}\left(A|\overline{B}\right)$ for two non-zero events A and B, then $\mathbb{P}(AB) = \mathbb{P}(A)\,\mathbb{P}(B)$ (i.e. A and B are independent).

Problem 4^* **.** (2 bonus points) You have a coin. You believe initially that this coin is fair, i.e. the probability of each side is equal. You begin to toss this coin. After each toss, you calculate the posterior probability of side heads. Implement the calculation of posterior probability for each toss in python. Find the probability of side heads, if you did 7 tosses and each time the result was heads.

Problem 5^* **.** (4 bonus points) You are the contestant on the Monty Hall show. Monty is trying out a new version of his game, with rules as follows. You get to choose one of three doors. One door has a car behind it, another has a computer, and the other door has a goat (with all permutations equally likely). Monty, who knows which prize is behind each door, will open a door (but not the one you chose) and then let you choose whether to switch from your current choice to the other unopened door.

1. (2 bonus points) Suppose for this part only that Monty always opens the door that reveals your less preferred prize out of the two alternatives, e.g., if he is faced with the choice between revealing the goat or the computer, he will reveal the goat. Monty opens a door, revealing a goat (this is again for this part only). Given this information, should you switch? If you do switch, what is your probability of success in getting the car?
2. (2 bonus points) Now suppose that Monty reveals your less preferred prize with probability p, and your more preferred prize with probability $q = 1 - p$. Monty opens a door, revealing a computer. Given this information, should you switch (your answer can depend on p)? If you do switch, what is your probability of success in getting the car (in terms of p)?