Home assignment № 1

The solutions to the following tasks can be submitted in the hand-written form scanned in PDF format. However, in this case the student is responsible for readability of the submitted text. The preferable way to prepare solutions is LaTeX or MS Word or any other tools for nice representation of equations. The following template `https://www.overleaf.com/read/vknkchxdwsmk` and tutorial `https://www.overleaf.com/learn/latex/Tutorials` can help in preparing solutions in LaTeX.

1. (1 pts) Consider the following function $f(x) = |x \cos x|$. Mark the correct claims about this function and explain your decisions.

   ☐ It has 5 extremum in segment $[-4; 4]$

   ☐ The global minimum is zero

   ☐ The global maximum is unique

   ☐ The number of local minimum is finite

   ☐ The number of local maximum is countable

   ☐ This function is smooth

   ☐ This function is continuous

2. (3 pts) Verify the convexity of the following sets or explain why some of them are not convex.

   (a) $\{\mathbf{a} \in \mathbb{R}^k \mid p(0) = 1, \ |p(t)| \leq 1, \ \alpha \leq t \leq \beta\}$, where $p(t) = a_1 + a_2 t + \ldots + a_k t^{k-1}$
   (b) $\{(x, y) \in \mathbb{R}_+^2 \mid xy \leq 1\}$
   (c) $\{\mathbf{X} \in \mathbf{S}_+^n \mid X_{ii} = 1, \ X_{ij} \geq 0\}$
   (d) $\{\mathbf{x} \in \mathbb{R}^n \mid \min\limits_{i=1,\ldots,n} x_i = 1\}$
   (e) $\{\mathbf{x} \in \mathbb{R}^n \mid \max\limits_{i=1,\ldots,n} x_i \geq 1\}$
   (f) $\{\mathbf{X} \in \mathbf{S}^n \mid rank \ \mathbf{X} = 1\}$

3. (2 pts) In the class we discussed that convexity of set is preserved under linear and perspective maps. Surprisingly, inverses of the aforementioned maps also preserve convexity. Formally, prove that set $f^{-1}(C)$ is convex if C is convex and f is linear or perspective map. Be definition, $f^{-1}(C) = \{\mathbf{x} \in \mathrm{dom} f \mid f(\mathbf{x}) \in C\}$

4. (2 pts) Show that $C \subseteq \mathbb{R}^n$ is convex if and only if $(\alpha + \beta)C = \alpha C + \beta C$ for all nonnegative α, β.