MSAI Probability Home Assignment 7
deadline: 24/12/2023 23:59 AOE

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable. This homework includes a python task, which can be solved in Google Colab or in a local Jupyter Notebook. It is thus handy to solve everything (both LaTeX and code) in a single Jupyter Notebook.

Problem 1. (2 points) Find the skewness of exponential distribution.

Problem 2. (2 points) Find the kurtosis of the $Be(\frac{1}{2})$ distribution

Problem 3. Let

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix} \right)$$

Find:

1. (2 points) All three marginal distributions

2. (2 points) Conditional PDF $f_{X_2|X_1=5}$

Problem 4*. (3 bonus points) In Python, visualize the joint PDF and the marginals on the same 3D plot.

Problem 5*.

1. (1 bonus point) Compute the moment-generating function of $Geom(p)$.

2. (1 bonus point) Use the MGF to find expectation.

3. (1 bonus point) Use the MGF to find variance.

Problem 6*. Let X and Y be i.i.d. $Geom(p)$, and $N = X + Y$.

1. (2 bonus points) Find the joint PMF of X, Y, N.

2. (1 bonus point) Find the joint PMF of X and N.

3. (2 bonus points) Find the conditional PMF of X given $N = n$.