Minimal Assumptions Refinement for GR(1) Specifications

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Abstract—Reactive synthesis is concerned with finding a correct-by-construction controller from formal specifications, typically expressed in Linear Temporal Logic (LTL). The specifications describe assumptions about an environment and guarantees to be achieved by the controller operating in that environment. If a controller exists, given the assumptions, the specification is said to be realizable.

This paper focuses on finding a minimal set of assumptions that guarantee realizability in the context of counterstrategy-guided assumption refinement procedures. Specifically, we introduce the notion of minimal assumptions refinements and provide an algorithm that provably computes these with little time overhead. We show experimentally, using common benchmarks, that embedding our algorithm in state-of-the-art approaches for assumption refinement results in consistently shorter solutions than without such embedding, and allows to explore a higher number of candidate solutions. We also propose a hybrid variant for dealing with the higher sparsity of solutions in the space of minimal refinements and show that its application speeds up the identification of a solution.

I. INTRODUCTION

Reactive synthesis is concerned with the automatic construction of provably correct controllers from specifications expressed in some formal language, typically Linear Temporal Logic (LTL). Though LTL synthesis is doubly exponential, more recent advances in the field (such as [1], [2], [3]) have focused on synthesis problems for a subset of LTL called *generalized reactivity of rank 1* (GR(1) for short) which has a comparatively low, polynomial time complexity [4].

GR(1) specifications have an assume-guarantee form: a set of assumptions about the environment and a set of guarantees for the controller. Synthesis aims is to find a controller for which all guarantees are met in every environment that satisfies all the assumptions. If no such controller exists, the specification is said to be *unrealizable*.

Unrealizability often arises when assumptions about the environment are too weak—allowing the environment too many behaviours some of which force any controller to violate its guarantee. Assumptions refinement aims at identifying sets of sufficient assumptions that restrict the environment from exhibiting such violating behaviour, thus making the specification realizable. In its simplest form, it defines a search problem in an intractably large search space. To deal with this, state-of-the-art approaches (e.g., [5], [6], [7], [8]) devise an incremental approach that makes use of counterstrategies, examples of

environment behaviours that force a guarantee violation. In brief, given an unrealizable specification, a counterstrategy c is first computed. From this a set of alternative assumptions $\{\psi_i\}$ is generated, each of which eliminates the counterstrategy c from the set of allowed environment behaviours. Each ψ_i is added in turn to the original assumptions, and tested for realizability. If the new specification is still unrealizable, a new counterstrategy c' is then computed from which a new set of alternative assumptions $\{\psi_j'\}$ is computed and so on until a set of sufficient assumptions is found.

Such incremental approaches have a number of disadvantages. First, they are susceptible to finding solutions that are too restrictive (as they tend to over approximate violating behaviour). For instance, an assumption ψ_j' may be sufficient alone to eliminate both c and c'; in this sense making ψ_i in $\psi_i \wedge \psi_j'$ redundant. Secondly, they are prone to exploring solutions that are lengthy in the number of assumptions added. This increases the computation overhead since counterstrategy computation is linear in the number of assumptions, as for Theorem 3 in [3]. It has also been argued [8] that larger sets of assumptions may negatively affect readability. In this paper, we make the following contributions:

- We formalize the notion of redundancy of assumptions with respect to observed counterstrategies;
- We provide a refinement minimization algorithm that provably removes any redundant assumption from a refinement.
- We enhance the classical FIFO search criterion with duplicate checks to reduce the time needed to find at least one solution;
- We show through experiments that, by integrating the minimization algorithm in counterstrategy-guided approaches, the search explores shorter solutions, in particular some that not found by existing methods alone within a given alloted time; and
- We propose a hybrid refinement generation method that compensates for cases where fewer solutions are found in the allotted time due to the minimization check.

II. MOTIVATING EXAMPLE

Consider the request-grant protocol described in [6]. This protocol consists of two input variables, *req* and *cl*, that the environment uses, respectively, to request access to some

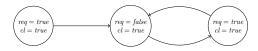


Fig. 1. Simplified counterstrategy for the request-grant example. The output variables and the memory variables are omitted for clarity

resource and to clear that resource; and two output variables, gr and val, which, respectively, grants access to the resource and signals whether it is in a valid state.

We consider specification of request-grant protocols to be represented in GR(1), which extends the classical Boolean logic with the operators ${\bf G}$ ("always"), ${\bf F}$ ("eventually"), and ${\bf X}$ ("next"); formalized later in Section III. The assumption $\phi^{\cal E}={\bf G}{\bf F}\neg req$ ensures that the environment does not request access continuously. The guarantees $\phi_1^{\cal S}={\bf G}(req\to {\bf X}{\bf F}gr)^1$, $\phi_2^{\cal S}={\bf G}((cl\vee gr)\to {\bf X}\neg gr),\ \phi_3^{\cal S}={\bf G}(cl\to \neg val),\ \phi_4^{\cal S}={\bf G}{\bf F}(gr\wedge val)$ guarantees that (1) a request be eventually followed by a grant; (2) whenever a clear or a grant is active at some time step, the grant be unset at the following step; (3) whenever a clear is issued, the valid flag be unset; and (4) the environment can access valid data infinitely many times. This specification is unrealizable since an environment that keeps the cl to true prevents any controller from fulfilling at the same time the guarantees $\phi_3^{\cal S}$ and $\phi_4^{\cal S}$ (Fig. 1).

Assumptions refinement procedures search for a solution in a potentially infinite space of possible assumptions (or doubly exponentially large in the number of variables, if logical equivalence is taken into account). For this reason, a selection criterion, we refer to as an (inductive) *bias*, must be employed to decide which subset of assumptions to consider. Examples of such biases are given in [5], [6], [7].

Consider, for instance a bias based on the interpolation mechanism of [7]. After a counterstrategy is computed (see Fig. 1), the first assumption found is $\psi = \mathbf{GF} \neg cl$, which forces the environment to set cl to false infinitely many times. This is not sufficient to achieve realizability, since an environment can force the violation of the guarantees by setting cl to true at any time following a time step where val is false. This way ϕ_4^S can never be satisfied as gr and val are never satisfied together. Therefore a new counterstrategy is computed and $\psi_2 = \mathbf{G}(\neg val \rightarrow \mathbf{X} \neg cl)$ is generated. The refinement $\psi = \psi_1 \wedge \psi_2$ is sufficient, and is returned as a solution. However even the assumption ψ_2 alone (not found by [7]) is sufficient, deeming ψ_1 redundant, and provides a weaker constraint on the environment as oppose to ψ .

In general, an assumptions refinement procedure has to trade off between the replacement and concatenation of assumptions when exploring potential solutions. Concatenation typically leads to more constrained environments where realizability can be more easily achieved. Instead, replacement generates weaker refinements. This paper presents a heuristic criterion to balance between the two.

¹Notice that this guarantee does not strictly abide by the GR(1) syntax. However, the work in [9] shows that such formulae can be converted to pure GR(1) via the addition of auxiliary variables

III. BACKGROUND

In the following we describe the basic syntax of the GR(1) specification language, give a formal definition of counterstrategy, and define the problem of assumptions refinement.

A. LTL and GR(1)

LTL is an extension of propositional logic used in software engineering for describing requirements of systems [10], [11], [12]. Let $\mathcal V$ be a set of Boolean variables, and $\Sigma=2^{\mathcal V}$ the set of all possible assignments/valuations to $\mathcal V$. We denote by Σ^ω the set of infinite sequences of elements in Σ , and for every $\pi \in \Sigma^\omega$ we denote by π_i the i-th element of the sequence. An LTL formula is an expression ϕ defined by the grammar (in Backus-Naur form)

$$\phi ::= p \mid \neg \phi \mid \phi \land \phi \mid \mathbf{X} \phi \mid \phi \mathbf{U} \phi .$$

where $p \in \mathcal{V}$. We say that a sequence $\pi \in \Sigma^{\omega}$ satisfies ϕ (in symbols $\pi \models \phi$) if and only if one the following holds: (1) $\phi = p$ and $p \in \pi_1$, (2) $\phi = \neg \phi'$ and $\pi \not\models \phi'$, (3) $\phi = \phi_1 \land \phi_2$ and $\pi \models \phi_1, \pi \models \phi_2$, (4) $\phi = \mathbf{X}\phi'$ and $\pi' = \pi_2\pi_3 \dots \models \phi'$, (5) $\phi = \phi_1 \mathbf{U}\phi_2$, and there exists an index k such that $\pi_k \pi_{k+1} \dots \models \phi_2$ and for every $i < k \pi_i \pi_{i+1} \dots \models \phi_1$. The other Boolean operators $(\lor, \to, \leftrightarrow)$ are defined as usual by using (1) and (2). The minimal set of temporal operators is extended with $\mathbf{F}\phi \equiv true\mathbf{U}\phi$ and $\mathbf{G}\phi \equiv \neg\mathbf{F}\neg\phi$.

In this context, we are interested in a subset of LTL called GR(I) [5]. This includes formulae of the kind $\phi^{\mathcal{E}} \to \phi^{\mathcal{S}}$, where both $\phi^{\mathcal{E}}$ and $\phi^{\mathcal{S}}$ are conjunctions of three kinds of LTL formulae: (1) *initial conditions*, purely Boolean formulae $B(\mathcal{V})$ over the variables in \mathcal{V} not containing any temporal operator, constraining the initial state of a system; (2) *invariants*, of the form $GB(\mathcal{V} \cup X\mathcal{V})$, temporal formulae containing only an outer G operator and G operators such that no two G are nested; these formulae encode one-step transitions allowed in the system; (3) *fairness conditions* of the form $GFB(\mathcal{V})$, expressing a Boolean condition GG holding infinitely many times in a system execution. Each conjunct in G is called *assumption*, and each one in G is called *guarantee*.

B. GR(1) Games and Counterstrategies

GR(1) formulae express properties of two-agent systems, where one of the agents is called *environment* and the other controller. The environment's state is characterized by a subset of variables $\mathcal{X} \in \mathcal{V}$ called *input variables*, while the controller's state is determined by the complementary subset $\mathcal{Y} = \mathcal{V} \setminus \mathcal{X}$ of output variables. The two agents compete in a game. Game states are valuations of $\mathcal V$ and transitions are consistent with assumptions and guarantees. At each step of an execution, the environment selects values for the input variables in order to fulfill the assumptions and forcing the violation of a guarantee. The controller responds by selecting a valuation of the output variables in order to satisfy the guarantees, thus taking the game to a new state. The controller synthesis problem is formalized into identifying a winning strategy for the controller in this game: a *strategy* is a mapping from the history of past valuations in the game and the current input choice by the environment, to an output choice ensuring that the controller satisfy the guarantees. Formal definitions of games and strategies, and of the algorithm to compute controller strategies, are given in [3].

If such a controller strategy does not exist, the GR(1) formula is said to be *unrealizable*. In this case there is a strategy, called a *counterstrategy*, for the environment that forces the violation of at least one guarantee [13]. Counterstrategies and, symmetrically to strategies, map histories of executions and the current state onto the next input choice by the environment. There are several formal definitions of counterstrategies [13], [14], [7], all having in common its representation as a transition system whose states and/or transitions are labeled with valuations of variables. In our context, this is the only aspect of interest, therefore we give a simpler view of counterstrategies.

A counterstrategy is a transition system $c=(Q_c,\delta_c,Q_0,\lambda_c)$, where Q_c is a set of states, $\delta_c\subseteq Q_c\times Q_c$ is a set of transitions between states, $Q_0\subseteq Q$ is a set of initial states, and $\lambda_c:Q\to 2^{\mathcal{V}\cup\mathcal{M}}$ is a labeling function for states. The labeling function assigns Boolean values to the system's variables in \mathcal{V} and to a set of additional "memory" variables \mathcal{M} . These variables encode the history of visited states and affect the environment's next move from each state. They are added automatically by the algorithm in [13] and are should not appear anywhere in the assumptions or the guarantees.

A *run* of the counterstrategy is any infinite sequence of states $q_1q_2\ldots$ with $q_1\in Q_0$ and $(q_i,q_{i+1})\in \delta_c$ for every $i\in \mathbb{N}$. Given a run, the associated *play* is the infinite sequence of the labellings of its states $\pi=\lambda_c(q_1)\lambda_c(q_2)\ldots$. By definition, all of its plays satisfy the assumptions and violate at least one guarantee (in the sense of Section III-A). In the following we say that a counterstrategy c satisfies the LTL formula c0 ($c \models c$ 0) if and only if all of its plays satisfy c0. Therefore, a counterstrategy satisfies c0 and violates c0.

C. Counterstrategy-Guided Assumptions Refinement

If a GR(1) specification $\phi^{\mathcal{E}} \to \phi^{\mathcal{S}}$ is unrealizable, one way to fix it is to strengthen the existing assumptions so that the environment cannot force a guarantee violation under the new assumptions. A *refinement* ψ is a set of assumptions ψ_1, \ldots, ψ_k each of which is either an initial condition, an invariant, or a fairness condition. With a slight abuse of notation, we will refer to ψ both as a set of assumptions $\{\psi_1, \ldots, \psi_k\}$ and as their conjunction $\psi_1 \wedge \cdots \wedge \psi_k$. The problem of assumptions refinement consists in finding one or more refinements ψ such that $\phi^{\mathcal{E}} \wedge \psi \to \phi^{\mathcal{S}}$ is a new GR(1) formula that is realizable. We call the solutions to the assumptions refinement problem sufficient refinements.

Usually, when no other information on the environment but the assumptions $\phi^{\mathcal{E}}$ is input to the problem, an additional goal is finding a sufficient refinement that is the *weakest* possible, in the sense of being the least constraining [5], [15], [16]. This way the controller synthesized from the refined specification is guaranteed to meet the specification in the widest possible set of environments. Moreover, since the new assumptions are

to be understood and assessed by engineers, it is desirable that they be as concise as possible.

Typical assumptions refinement approaches are based on alternating between counterstrategy computations and assumption generation, thus called *counterstrategy-guided* [5], [14], [6], [7]. Given an unrealizable specification with assumptions ϕ , a counterstrategy c_0 is first computed by using the algorithm in [13], and one or more alternative assumptions ψ_1, \ldots, ψ_h inconsistent with c_0 are generated, using a bias as discussed in Section II. Then, for each ψ_i , if $\phi \wedge \psi_i$ is still not sufficient, a new counterstrategy c_i is generated, and again a set of assumptions $\psi_{i,1}, \dots \psi_{i,h'}$ inconsistent with c_i is generated. Each of these assumptions is conjoined in turn to $\phi \wedge \psi_i$ and if the resulting specifications are still unrealizable, new counterstrategies are computed in the same fashion. The result is tree of refinements that is explored in a breadth-first fashion, from shorter refinements to lengthier ones, and whose leaves are solutions to the realizability problem [7], [6], [5], [8], [14]. We denote this approach as breadth-first search (BFS).

The BFS approach ensures termination, since there are only finitely many non-equivalent logical conditions and the bias ensures only a small subset of them is actually explored. At every level of the refinement tree a finite number of assumptions is generated, and along a branch of the tree the assumptions are progressively strengthened by adding new conjuncts, until either they become inconsistent or a sufficient solution is formed [7]. This however presents a problem. Suppose the search finds a sufficient refinement $\psi = \{\psi_1, \psi_2, \dots, \psi_k\}$; suppose also that a subset of these assumptions is sufficient, that is $\psi' \subset \psi$ is also sufficient to achieve realizability. This solution would not be checked during the incremental search, since there is no backtracking along the refinement tree.

In the following, we will refer to collections of *sets* of counterstrategies $C = \{C_1, \ldots, C_k\}$. We will denote their union as $\bigcup C = \bigcup_{i=1}^k C_i$.

IV. MINIMAL ASSUMPTIONS SETS

A sufficient refinement is a set of assumptions such that any counterstrategy is inconsistent with their conjunction. The goal of counterstrategy-guided approaches is to increementally progress towards a sufficient refinement by proposing at each step a candidate refinement that eliminates at least one new counterstrategy with respect to the previous candidate. If the new assumption alone removes one or more of the counterstrategies that were eliminated by the previous candidate, the corresponding assumptions generated at the previous steps may be redundant in order to form a solution. We formalize the notion of redundant assumption with respect to a set of counterstrategies.

Definition 1 (Redundant assumptions): Given a set of assumptions $\psi = \{\psi_1, \dots, \psi_n\}$ and a set of counterstrategies $C = \{c_1, \dots, c_n\}$, an assumption ψ_i is redundant with respect to ψ and C if and only if for all $c_j \in C$, $c_j \not\models \bigwedge_{\psi_h \in \psi, h \neq i} \psi_h$.

Notice that the definition of redundant assumption is well-formed: a counterstrategy $c \in C$ is inconsistent with a refinement $\psi = \psi_1 \wedge \cdots \wedge \psi_n$ if and only if it is inconsistent

with at least one ψ_h alone. It is not the case that two or more assumptions are needed together to eliminate a counterstrategy c, so, when a redundant assumption ψ_j is removed from ψ , there exists an assumption $\psi_h \neq \psi_j$ that eliminates c alone.

In the following we will just say that an assumption $\psi_i \in \psi$ is redundant, without specifying the sets ψ and C when those are clear from the context.

Definition 2 (Minimal assumptions): A set of assumptions ψ is minimal with respect to a set of counterstrategies C if none of its assumptions is redundant with respect to ψ and C.

In other words, an assumption ψ_i in a refinement ψ is redundant with respect to a set of counterstrategies C if all the counterstrategies in C are still eliminated when one removes ψ_i from ψ . So, in order to construct a sufficient solution given some observed counterstrategies, ψ_i may be safely deleted. A refinement without a redundant assumption is called minimal.

V. MINIMAL REFINEMENTS SEARCH

We first describe the minimization algorithm MINIMALRE-FINEMENT. Then we present the changes to the classical FIFO refinement algorithm [6], [7] to speed up the convergence towards a solution.

A. The function MINIMALREFINEMENT

The core of our proposal is the function MINIMALRE-FINEMENT, shown in Algorithm 1. This function adds a new assumption ψ' to an insufficient refinement ψ by ensuring the resulting refinement is minimal with respect to all the counterstrategies observed so far.

Specifically, suppose an unrealizable specification $\phi^{\mathcal{E}} \to \phi^{\mathcal{S}}$ has been refined with the refinement $\psi = \{\psi_1, \dots, \psi_n\}$, that was computed incrementally from the counterstrategies $C = \{c_1, \dots, c_n\}$. Suppose ψ is not a sufficient refinement. Hence an additional counterstrategy c' is computed, and a new assumption ψ' is generated to eliminate c'. In general, each $\psi_i \in \psi$ and ψ' together may eliminate more than one counterstrategy in $C \cup \{c'\}$, and thus by adding ψ' , some ψ_i may become redundant with respect to $C \cup \{c'\}$ and $\psi \cup \{\psi'\}$.

In order to obtain minimality in the sense of Definition 2, the algorithm needs to know the counterstrategies that each assumption eliminates. The relationship between the assumptions in a refinement and the counterstrategies they eliminate can be pictured as a bipartite graph whose vertices are the elements $\psi_i \in \psi \cup \{\psi'\}$ and $c_j \in C \cup \{c'\}$, such that there is an edge (ψ_i, c_j) if and only if $c_j \not\models \psi_i$.

Example 1: In Figure 2, ψ_1 eliminates c_1 only, ψ_2 removes c_2 and c_3 , and ψ_3 removes c_3 and c_4 . When adding ψ' , which removes c_1 , c_3 , c_4 and c', the assumptions ψ_1 and ψ_3 become redundant.

Let the *degree* of a vertex be the number of edges incident to that vertex. Then an assumption is redundant if and only if all the counterstrategies to which it is connected have a degree at least 2. Indeed, even if such an assumption were eliminated, there would be another assumption eliminating all the counterstrategies connected to it. The algorithm then scans all the ψ_i 's and removes all redundant assumptions based on

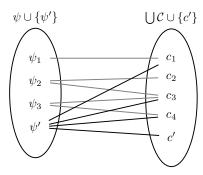


Fig. 2. Bipartite graph of assumptions and counterstrategies

the degree of their connected counterstrategies. The resulting refinement is minimal.

Example 2: In Fig. 2, the addition of ψ' makes ψ_1 and ψ_3 redundant. In fact, all counterstrategies that are linked to them have degree of 2 or above. So, those two assumptions can be safely removed from the refinement.

In order to construct the bipartite graph, the algorithm needs to determine for each pair (ψ_i, c_j) whether $c_j \models \psi_i$. This is a classical model checking problem, and can be solved with standard model checking algorithms [17]. However, in this setting there is no need to perform all the model checking operations for every pair (ψ_i, c_j) . MINIMALREFINEMENT is called as part of the loop in Algorithm 2 (see Section V-B): thus at each call, part of the graph is already constructed.

The inputs ψ and \mathcal{C} correspond to the bipartite graph produced by previous iterations. \mathcal{C} is a collection of counterstrategy sets C_i such that ψ_i eliminates all the counterstrategies in C_i ; that is, there is an edge between ψ_i and each of the counterstrategies in C_i . ψ' and c' are the counterstrategy and the assumption produced in one iteration of the counterstrategy-guided approach (see Algorithm 2); hence, by hypothesis $c' \not\models \psi'$.

The function consists of two blocks. In the first (lines 1-4), the algorithm model checks all counterstrategies in $\bigcup \mathcal{C}$ against ψ' and constructs the set $C' = \{c \in \bigcup \mathcal{C} \cup \{c'\} \mid c \not\models \psi'\}$; by construction $c' \in C'$ (see line 1).

The second block (lines 5-16) analyzes the bipartite graph and identifies the redundant assumptions. First, it builds the new refinement $\psi_{new} = \psi \wedge \psi'$ to be minimized and the collection of counterstrategy sets \mathcal{C}_{new} by extending \mathcal{C} accordingly with C' (lines 5-8). Then for every counterstrategy $c \in \bigcup \mathcal{C}_{new}$, the function COUNTSETS counts how many sets in \mathcal{C}_{new} contain c. This corresponds to the degree of the counterstrategy in the bipartite graph. Finally, lines 11-16 remove any ψ_i from ψ_{new} such that every counterstrategy in C_i has a degree greater or equal to 2.

Example 3: Let us consider again the motivating example in Section II. In this case, $\psi_1 = \mathbf{GF} \neg cl$ eliminates the first counterstrategy generated, call it c_1 . So $C_1 = \{c_1\}$ and $\mathcal{C} = \{C_1\}$. At the first call of MINIMALREFINEMENT, the bipartite graph has a single assumption vertex and a single counterstrategy vertex; so, no minimization occurs.

Algorithm 1: MINIMALREFINEMENT function

```
Input: \psi = (\psi_1, \dots, \psi_k): insufficient refinement of
             cardinality k
   Input: C: sequence of sets of eliminated
             counterstrategies with cardinality k
   Input: \psi': assumption to be added to \psi
   Input: c': a counterstrategy consistent with \psi and
             eliminated by \psi'
   Output: (\psi_{new}, \mathcal{C}_{new}): pair containing a minimal
               refinement \psi \wedge \psi' and the corresponding set of
               eliminated counterstrategies
1 C' \leftarrow \{c'\};
2 foreach c \in \bigcup \mathcal{C} do
        if c \not\models \psi' then
            C' \leftarrow C' \cup \{c\};
5 \psi_{new} \leftarrow \psi;
6 \psi_{new}.append(\psi');
7 C_{new} \leftarrow C;
8 C_{new}.append(C');
9 foreach c \in \bigcup \mathcal{C}_{new} do
    c.degree = CountSets(C_{new}, c);
11 for i = 1 to k do
        if \forall c \in C_i \ c.degree \geq 2 then
12
             \psi_{new}.remove(\psi_i);
13
             C_{new}.remove(C_i);
14
             foreach c \in C_i do
15
                  c.degree \leftarrow c.degree - 1;
16
17 return (\psi_{new}, \mathcal{C}_{new})
```

After the second counterstrategy is produced, $\psi_2 = \mathbf{G}(\neg val \rightarrow \mathbf{X} \neg cl)$ is generated. MINIMALREFINEMENT checks whether ψ_2 is also inconsistent with c_1 and finds that it does. So $C_2 = \{c_1, c_2\}$.

Now c_1 has degree 2 and ψ_1 is only connected to c_1 . Therefore, the algorithm finds that ψ_1 is redundant and it removes it when constructing the new refinement ψ_{new} .

B. Search Algorithm

BFS approaches to assumptions refinement [6], [7], [5], [8] keep a FIFO queue of partial refinements—iteratively, the first element of the queue is extracted, and if it is not sufficient a new set of alternative further refinements is generated and appended to the queue. If a refinement of size n is extracted, the appended refinements have size n+1, and eventually no refinement of size n remains in the queue. Therefore, even if the same refinements appears in the queue more than once, each of them is explored a finite number of times.

On the contrary using minimization could lead to the same refinement being added to the queue and explored infinitely many times. We call a refinement already explored that is added more than once to the queue a *duplicate refinement*. Exploring duplicate refinements more than once can lead to

useless repeated computations, hence delaying the identification of a solution, as the example below shows.

Example 4: Suppose that $\psi_1 \wedge \psi_2'$ is an insufficient refinement and the bias component of the algorithm (see Section II) generates the additional assumption ψ_3 , which makes ψ_2' redundant w.r.t. the observed counterstrategies. Suppose also that $\psi_1 \wedge \psi_2''$ is insufficient and again the bias generates ψ_3 that makes ψ_2'' insufficient. Then $\psi_1 \wedge \psi_3$ is added twice to the queue of candidate refinements. All the further refinements being extensions of $\psi_1 \wedge \psi_3$ will also be explored twice, hence delaying reaching any solution.

One simple modification to the search procedure is keeping track of the refinements (only) which have already been explored and avoiding their extension again. However, this may prevent progress towards a solution. The following example serves to argue why we should also keep track of counterstrategies eliminated by a refinement when checking for duplicates.

Example 5: Given an initial specification $\phi^{\mathcal{E}} \to \phi^{\mathcal{S}}$, suppose $\phi^{\mathcal{E}} \wedge \psi_1 \wedge \psi_2 \wedge \psi_3$ is a solution to the realizability problem. The following table describes an instance of the steps within a minimal assumptions refinement search procedure.

The column $\{(\psi_i, \bigcup \mathcal{C}_i)\}$ represents the FIFO queue, such that the element at row i is the head of the queue at step i. At each search step, the first element in the queue (left most) is extracted, a counterstrategy c is computed and a set Ψ' of alternative assumptions that eliminate c is generated. The column MINREF shows the minimal refinement generated when adding the assumption in column Ψ' to the refinement extracted from the queue given the observed counterstrategies in $\bigcup \mathcal{C}_i$ and c.

At the beginning only the empty refinement is in the queue, the counterstrategy c_1 is produced and two assumptions ψ_1 and ψ_2 are generated, which are minimized and added to the queue in steps 2 and 3. Then in turn, the refinement in step 2 is extracted from the queue, c_2 is computed, ψ_2 is generated by the bias and after minimizing $\psi_1 \wedge \psi_2$ w.r.t. the counterstrategies c_1 and c_2 only ψ_2 remains.

Notice that ψ_2 is generated twice: once in step 1 as a singleton refinement, and once in step 2 as the product of minimization. Therefore, it is popped twice from positions 3 and 4. At step 4, finally ψ_2 is not redundant and therefore it does not disappear after minimization. If the refinement ψ_2 were not explored twice, the refinement $\psi_2 \wedge \psi_1$ would not have been added to the queue and the solution above could not be reached. The difference between the first and the second time ψ_2 is explored is in the counterstrategies it eliminates.

The above suggests a different way for managing the queue and checking for duplicates, as shown in Algorithm 2. As in existing iterative search algorithms, the function COM-

Algorithm 2: REFINEMENTSEARCH function

```
Input: \phi^{\mathcal{E}}: set of initial assumptions
    Input: \phi^{S}: set of guarantees
    Output: refs \subseteq \{ \psi \mid \langle \phi^{\mathcal{E}} \wedge \psi, \phi^{\mathcal{S}} \rangle \text{ is realizable} \}
 1 refs \leftarrow \varnothing:
 2 candidateRefQueue \leftarrow \{\emptyset\};
3 exploredRefs \leftarrow \emptyset;
4 repeat
          (\psi, \mathcal{C}) \leftarrow candidateRefQueue.dequeue();
5
          if \neg(\psi, || |\mathcal{C}|) \in exploredRefs then
6
                if not ISREALIZABLE\langle \phi^{\mathcal{E}} \wedge \psi, \phi^{\mathcal{S}} \rangle then
7
 8
                        Compute Counters trategy (\phi^{\mathcal{E}} \wedge \psi, \phi^{\mathcal{S}});
                       \Psi' \leftarrow \text{APPLYBIAS}(c', \phi^{\mathcal{E}} \land \psi, \phi^{\mathcal{S}});
                      foreach \psi' \in \Psi' do
10
                             (\psi_{new}, \mathcal{C}_{new}) \leftarrow
11
                              MINIMALREFINEMENT(\psi, C, \psi', c');
                             candidateRefQueue.enqueue((\psi_{new}, \mathcal{C}_{new}));
12
                            if hybrid then
13
                                  C' \leftarrow \text{last set in } \mathcal{C}_{new};
14
                                  candidateRefQueue.enqueue((\psi \cup
15
                                    \{\psi'\}, C \cup \{C'\});
16
                else if IsSatisfiable(\phi^{\mathcal{E}} \wedge \psi) then
17
                      refs.append(\psi);
18
                exploredRefs.append((\psi, | | | \mathcal{C}|));
19
          end
20
21 until candidateRefQueue = \emptyset;
22 return refs
```

VI. HYBRID REFINEMENT GENERATION

Our minimization procedure leads to a more thorough exploration of shorter refinements within a search space compared to BFS. This allows for solutions to be found that would not have been without minimization. This however may delays the exploration of longer refinements and hence in some cases the convergence towards a solution. This is a problem, in particular, if the specification of interest does not have short solutions. Moreover, an approach that solely explores

minimal refinements might miss shorter solutions which may only be found by minimizing long candidate refinements, as the example below shows.

Example 6: In the AMBA04 case study (see Section VII) a solution is $\psi = \mathbf{G}(\neg hready \rightarrow \mathbf{X}hready)$, containing only one assumption. Interpolation does not compute this assumption until some partial refinement of length 3 is given as input to APPLYBIAS. This partial refinement is found by BFS in fewer iterations than Algorithm 2.

To soften this effect, our approach can be easily hybridized with BFS. It is sufficient to add to the queue the refinements explored by BFS together with the minimal refinements generated by Algorithm 1, as done in lines 14-15 of Algorithm 2. In this way longer candidate refinements are added to the FIFO queue in earlier iterations of the search, and longer solutions can be identified in shorter time.

VII. EXPERIMENTS

We present the results of an experiment conducted on a benchmark of six case studies of unrealizable specifications. The experiment is devised so as to show the impact of adding MINIMALREFINEMENT in terms of the number and the length of both the partial refinements explored and the final solutions. The experiment assumes the work shown in [18] as a baseline. There, the authors apply a pure BFS approach to the six case studies over 24 hours and record the number of results over time, reporting also statistics of the explored refinement tree like the length. Accordingly, we executed Algorithm 2 for 24 hours, collecting the relevant measurements on the explored refinements. The first run makes no use of the hybrid approach described in Section VI. Then we repeated the experiment on all the case studies with the hybrid approach, in order to analyze its effect on the number of refinements explored and solutions found within the 24 hours. For each case study, [18] reports results for two alternative versions of APPLYBIAS, which they call interpolation and multivarbias; at the time of the experiment, these are the only approaches for which data is available on the number of solutions found over a long run of the search. In our experiment, to get a stronger baseline, we selected the bias that found more solutions in BFS.

Our implementation uses the synthesis tool RATSY [19] to compute counterstrategies, and our Python implementation builds on the algorithm of [17] to check counterstrategies against assumptions. All approaches were executed on the same machine (Ubuntu, Intel Core i7 CPU and 16 GiB of RAM). The implementation and data are available at [20].

A. Case Studies

The six case studies are three versions of the popular AMBA-AHB protocol specification and three case studies developed in [21] for testing Justice Violation Transition Systems (see Section VIII), called JVTS case studies for short. The AMBA-AHB specification [3], [6], [5], [22] describes the requirements of an on-chip communication protocol developed by ARM. It provides a number of masters (the environment) that can initiate a communication on a shared bus via raising

an $hbusreq_i$ signal, and an arbiter (the controller to synthesize) granting access to the bus via the signal $hgrant_i$. An environment variable hready is set to true when the bus is ready for a switch of masters. The three versions of the protocol we use are the ones with respectively 2, 4 and 8 masters.

The JVTS case studies are *ColorSort*, the specification of a robot sorting Lego pieces by color, *GyroAspect*, a robot with self-balancing capabilities, *Humanoid*, a humanoid-shaped robot. For the function APPLYBIAS we use interpolation for AMBA02, AMBA04, AMBA08 and ColorSort, and multivarbias for GyroAspect and Humanoid. The work in [21] contains several versions of these same systems: we randomly selected one version per system and manually translated it into RATSY's syntax for adapting it to our implementation.

B. Results

Table I summarizes the measurements of applying our approach (labeled as MINIMALREFINEMENT) by comparing it with [18] (labeled as BFS). ExploredRefs contains the number of non-duplicate refinements explored, that is the number of nodes for which realizability has been checked. Notice that this value is consistently higher for MINIMALREFINEMENT, in accordance with the fact that computing realizability and counterstrategies of smaller specifications requires sensibly less time: such gains can be observed in the average computation time despite that the length of the minimal refinements is just typically 1 to 3 assumptions less than the nonminimal ones. Min/Max/ModeLength reports the minimum, maximum, and the mode (i.e., the value with the highest relative frequency) of the length of the explored refinements (in terms of the number of assumptions). The use of refinement minimization effectively reduces the number of assumptions in most explored refinements. This is confirmed by the row Min/Max/ModeRedundAssump, reporting the minimum, maximum and mode of the number of redundant assumptions eliminated in each call to MINIMALREFINEMENT. Sol contains the number of distinct solutions; this count is obtained by considering just once all the refinements containing the exact same assumptions, if such refinements are found more than once during the search. However, due to time constraints no semantic check is performed on the distinct solutions: for instance, $\mathbf{GF}(\neg hbusreq1)$ and $\mathbf{GF}(\neg hbusreq1 \land \neg (hbusreq1 \land (hbusreq1 \land \neg (hbusreq1 \land (hbusreq1 \land \neg (hbusreq1 \land ($ $\neg hready$)) are considered as different assumptions, both for BFS and for MINIMALREFINEMENT.

As expected, our approach finds less distinct solutions than BFS (i.e., Sol is lower); this is because minimal refinements are less constraining of the environment, and more steps are required to achieve realizability. However, the solutions found are frequently shorter: this can be read in the row Min/Max/ModeSolLength, containing the minimum, maximum, and most frequent solution length.

To compare the solutions returned by MINIMALREFINE-MENT and Hybrid with the ones returned by BFS, we define a notion of coverage between solutions. We say that the solution ψ_1 covers the BFS solution ψ_2 if $\psi_1 \subseteq \psi_2$; in this case, ψ_2 is bound to contain redundant assumptions

not needed for realizability. For instance, in AMBA04 the solution $\mathbf{G}(\neg hready \rightarrow \mathbf{X}(\neg(\neg hready)))$ identified by MINIMALREFINEMENT covers the BFS solutions $\{\mathbf{G}(\mathbf{F}(\neg(hbusreq3 \land \neg hmaster1))), \mathbf{G}(\mathbf{F}(\neg(\neg hmaster1 \land hbusreq2) \land \neg(hbusreq2 \land \neg hmaster1) \land \neg(hbusreq2 \land hmaster0))\}, \mathbf{G}(\mathbf{F}(\neg(hmaster1 \land hbusreq1))), \mathbf{G}((\neg hready) \rightarrow \mathbf{X}(\neg(\neg hready)))\}$ and $\{\mathbf{G}(\mathbf{F}(\neg(hbusreq3 \land \neg hmaster1))), \mathbf{G}((\neg hmaster1 \land hbusreq2) \rightarrow \mathbf{X}(\neg(hbusreq2))), \mathbf{G}((hmaster1 \land hbusreq1) \rightarrow \mathbf{X}(\neg(hbusreq1)))\}$.

CoveredBFSSol shows the total number of BFS solutions covered by at least one minimal solution. SolMinimalOnly shows the number of distinct minimal solutions that do not cover any BFS solution (and are therefore found by MINIMALREFINEMENT only). SolBFSOnly shows the number of solutions that are not covered by any solution found by MINIMALREFINEMENT. ExpandedRefs shows the number of refinements that have undergone expansion, that is, the functions COMPUTECOUNTERSTRATEGY and APPLYBIAS have been called on them. This number is higher in our approach, meaning that more counterstrategies could be computed within the same time. False contains the number of inconsistent refinements computed during the search (equivalent to the false constant), and DuplicateRefs is the number of generated nodes discarded by our search for being duplicates (see Section V-B).

In summary, although our approach produces fewer (meaningful) solutions than BFS, they are typically smaller and hence less restrictive (also note that our method explores more refinements with fewer assumptions).

Table I also shows the effect of hybridization. SolHybrid-Minimal shows the solutions common to the hybrid approach and MINIMALREFINEMENT, and SolHybridOnly the ones that do not cover any BFS solution nor are returned by MINIMAL-REFINEMENT. In all the cases, both the number of refinements explored and the number of solutions increase. The former can be explained since more alternative assumptions (both minimal and non-minimal) are generated at every single iteration of REFINEMENTSEARCH. The latter is due to the fact that longer solutions are generated earlier in the search. Note that the higher number of solutions allows for covering more BFS solutions, rendering the latter redundant; in AMBA08, one of the solutions covers all the 10 BFS solutions, while we were not able to find any solution by solely exploring minimal refinements. However, some of the solutions returned by Hybrid are not minimal, and therefore the length of those solutions can be higher than the ones of MINIMALREFINEMENT.

VIII. RELATED WORK

As pointed out above, our work follows the lines of [5], [6], [7] in using counterstrategies for generating assumptions. Unlike this work, we do not propose a new bias for computing new assumptions from individual counterstrategies. Our work, instead, focuses on a novel search strategy that mutates the refinements computed using these biases. Furthermore, our strategy for considering new refinements uses information

TABLE I
SUMMARY OF RESULTS OF BFS, MINIMALREFINEMENT, AND HYBRID. CHECK SECTION VII-B FOR A DESCRIPTION OF THE ENTRIES. THE ROWS WITH

* DO NOT CONSIDER HYBRID

	Statistics	AMBA02	AMBA04	AMBA08	ColorSort	GyroAspect	Humanoid
BFS	ExploredRefs	279	315	287	723	1980	2734
	Min/Max/ModeLength	0/5/5	0/5/5	0/4/4	0/6/6	0/3/3	0/3/3
	Sol	84	48	10	28	896	150
	Min/Max/ModeSolLength	1/5/4	3/5/4	4/4/4	3/6/5	2/3/3	1/3/3
	SolBFSOnly*	46	30	10	2	853	23
	ExpandedRefs	45	85	49	154	71	102
	False	39	42	10	0	401	19
MINIMAL	ExploredRefs	463	658	455	1251	2139	3558
REFINEMENT	Min/Max/ModeLength	0/4/2	0/3/2	0/3/1	0/4/2	0/3/2	0/2/1
	Min/Max/ModeRedundAssump	0/3/1	0/3/1	0/2/0	0/4/0	0/2/1	0/2/1
	Sol	59	24	0	22	483	36
	Min/Max/ModeSolLength	1/3/2	1/3/3	N/A	1/1/1	1/3/2	1/2/1
	CoveredBFSSol	38	18	0	26	43	127
	SolMinimalOnly*	30	22	0	2	453	9
	ExpandedRefs	169	372	248	651	817	1236
	False	11	2	0	0	173	5
	DuplicateRefs	455	540	468	774	6117	14495
Hybrid	Explored refs	1072	745	791	1962	2197	3494
	Min/Max/ModeLength	0/4/2	0/5/4	0/5/2	0/6/6	0/4/2	0/4/2
	Sol	229	93	3	59	680	164
	Min/Max/ModeSolLength	1/3/2	1/5/4	1/4/4	1/6/6	1/4/2	1/4/2
	CoveredBFSSol	86	30	10	27	40	141
	SolHybridMinimal	7	2	0	5	59	14
	SolHybridOnly	207	81	2	52	620	143
	ExpandedRefs	248	398	234	667	1148	1139
	False	11	10	3	0	272	20

about all the counterstrategies eliminated by that refinement instead of solely the last counterstrategy observed.

The work in [8] proposes an assumption refinement procedure that makes use of a more efficient alternative to counterstrategies, namely Justice Violation Transition Systems (JVTSes) [21]. A JVTS is an abstraction of counterstrategies in which cycle states and transient states are merged into "macrostates". Although the functions COMPUTECOUNTERSTRATEGY and APPLYBIAS in Algorithm 2 assume counterstrategies, these can an be seamlessly applied to JVTSes. This is because our MINIMALREFINEMENT procedure treats counterstrategies as abstract objects, only exploiting their satisfaction relation with assumptions. The main point to consideration is the definition of satisfaction of an LTL formula in a JVTS.

Our focus has been on synthesis from GR(1) specifications for which linear-time synthesis algorithms exist [4], [3]. A different thread of research focuses on different subsets of LTL [23], [24], [25] and reduces the synthesis to planning. However, these approaches use explicit-state representations, and define bounded-time versions of realizability or finite-time versions of LTL to counter the state explosion problem.

The definition of minimal assumptions sets is inspired by the problem of minimum set cover [27], [28]. Given a set of elements C and a collection S of subsets of C, the minimum set cover is a subcollection of subsets $S' \subseteq S$ such that $\bigcup S' = C$ and S' contains the least number of subsets needed to cover C. Notice that our problem of finding minimal refinements is a relaxation of this NP-hard problem, as we are not interested

in minimizing the number of subsets; rather, our notion of minimality corresponds to that of non-redundancy in [28].

Redundancy and minimization in logical formulae has already been studied extensively, such as in [29], [30], [31], [32]. The notion of redundancy we present here is substantially different from the one in the literature. Redundancy in logic is typically related to implication and entailment: a formula or a clause is redundant if another one in the knowledge base entails it. In our case, redundancy is defined with respect to the goal of eliminating a sample of counterstrategies: one assumption may not be entailed by any other in a refinement and still be redundant if all the observed counterstrategies are eliminated by the other assumptions in the refinement.

IX. CONCLUSIONS

In this paper, we presented a new search method for assumptions refinement to solve the unrealizability problem of GR(1) specifications. We introduced a definition of redundancy of assumptions with respect to a set of counterstrategies and hence the concept of minimality of refinements. We proposed an algorithm that returns minimal refinements when embedded in the classical counterstrategy-guided refinement loop. This provides a systematic way for trading off concatenation and replacement of assumptions when constructing refinements. The experiment shows that our approach explores a greater number of shorter refinements and finds shorter, non-redundant solutions. A matter for future work is exploring the effect of minimization on more recent refinement approaches such as

the ones using JVTSes [8], and the combination of minimization and weakness heuristics like the one proposed in [15].

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APPENDIX

A. Algorithm Correctness

We can prove the following three propositions, which formalize the above discussion.

Proposition 1 (Coverage of all counterstrategies): Suppose: (1) for every $\psi_i \in \psi$, $C_i \in \mathcal{C}$ and $c \in \bigcup \mathcal{C}$, $c \in C_i$ if and only if $c \not\models \psi_i$; (2) $c' \not\models \psi'$; (3) $c' \models \psi$.

Then $\bigcup \mathcal{C}_{new} = \bigcup \mathcal{C} \cup \{c'\}.$

Proposition 2 (MINIMALREFINEMENT invariant): Under the same hypotheses on the input as in Proposition 1, $(\psi_{new}, \mathcal{C}_{new})$ satisfies hypothesis (1) by replacing ψ with ψ_{new} and C with \mathcal{C}_{new} .

Proof: This can be easily concluded by observing that lines 1-4 populate C' with each and every $c \in \bigcup \mathcal{C}_{new}$ such that $c \not\models \psi'$; the subsequent lines that add or remove elements from ψ_{new} and \mathcal{C}_{new} are always paired. Therefore, the j-th element of \mathcal{C}_{new} contains each and every counterstrategy in $\bigcup \mathcal{C}_{new}$ that is inconsistent with the j-th element of ψ_{new} .

Proposition 3 (Minimality of output): Under the same hypotheses on the input as in Proposition 1, ψ_{new} is minimal with respect to $\bigcup \mathcal{C}_{new}$.

This result is based on the following lemmas:

Lemma 1: Let a refinement ψ and a collection C of counterstrategy sets fulfil hypothesis (1) of Proposition 3.

Let the bipartite graph (V, E) be such that $V = \psi \cup \bigcup \mathcal{C}$ and $E = \{(\psi_i, c_j) \in \psi \times \bigcup \mathcal{C} \mid c_j \not\models \psi_i\}.$

Then ψ_i is redundant with respect to ψ and $\bigcup \mathcal{C}$ if and only if for every c_j such that $(\psi_i, c_j) \in E$, the degree of c_j is greater or equal to 2.

Proof: Given how the graph is defined, the degree of a counterstrategy is the number of assumptions that are inconsistent with it. So, $\deg(c_j) \geq 2$ if and only if more than two assumptions or more eliminate it. Hence, there exists $\psi_h \neq \psi_i$ such that $\psi_h \not\models c_j$ for any c_j connected to ψ_i . By Definition 1 this means ψ_i is redundant.

Lemma 2 (Monotonicity w.r.t. ψ of non-redundancy): If ψ_i is non-redundant with respect to ψ and C, it is non-redundant w.r.t. μ and C for any $\mu \subseteq \psi$.

Proof: The intuition is trivial. If ψ_i is the only assumption removing some counterstrategy c_i , then by removing some of the other assumptions in ψ it will still be the only assumption removing c_i .

If ψ_i is non-redundant, by negating Definition 1 there exists a counterstrategy $c_i \in C_i$ such that $c_i \models \bigvee_{\psi_h \in \psi, h \neq i} \psi_h$. Since μ is a subset of ψ , it also holds that $c_i \models \bigvee_{\psi_h \in \mu, h \neq i} \psi_h$. Therefore, ψ_i is non-redundant with respect to μ and C.

Now let us prove Proposition 3.

Proof: (of Proposition 3) To prove minimality, we use a loop invariant over the loop on lines 11-16. Let $\psi_{new}^{[i]}$ be the state of ψ_{new} at the end of the i-th iteration, with the initial $\psi_{new}^{[0]} = \psi \cup \{\psi'\}$ and the output $\psi_{new} = \psi_{new}^{[k]}$. We claim the following loop invariant: at the end of iteration i, for all $h \leq i$, $\psi_h \in \psi_{new}$ if and only if ψ_h is non-redundant with respect to $\psi_{new}^{[i]}$ and $\bigcup \mathcal{C}_{new}$.

As inductive hypothesis, suppose the loop invariant holds at the end of iteration i-1: for all $h \leq i-1$, $\psi_h \in \psi_{new}$ if and only if ψ_h is non-redundant w.r.t. $\psi_{new}^{[i-1]}$ and $\bigcup \mathcal{C}_{new}$. During iteration i, ψ_i is removed if and only if the condition in line 12 holds; by Lemma 1 this condition being true corresponds to ψ_i being redundant w.r.t. $\psi_{new}^{[i-1]}$. Hence, ψ_i is not removed if and only if it is not redundant w.r.t. $\psi_{new}^{[i-1]}$.

Moreover, $\psi_{new}^{[i]} \subseteq \psi_{new}^{[i-1]}$, since at most one removal occurs during an iteration of the loop. Therefore, by Lemma 2 ψ_i is not removed if and only if it is not redundant w.r.t. $\psi_{new}^{[i]}$. This completes the inductive step: we have proved that for all $h \leq i$, $\psi_h \in \psi_{new}$ if and only if ψ_h is non-redundant with respect to $\psi_{new}^{[i]}$.

The property trivially holds for i=1. In this case ψ_1 is removed from $\psi_{new}^{[0]}=\psi$ if and only if it is redundant w.r.t. ψ . And again by lemma 2 this implies that it is kept in $\psi_{new}^{[1]}$ if and only if it is non-redundant w.r.t. $\psi_{new}^{[1]}$.

Hence the loop invariant holds for i=k and every $\psi_i \in \psi_{new}$ is non-redundant w.r.t. ψ_{new} and $\bigcup \mathcal{C}_{new}$. By Definition 2, ψ_{new} is minimal w.r.t. $\bigcup \mathcal{C}_{new}$.

B. Time Complexity

Executing MINIMALREFINEMENT introduces an additional overhead to the generation of each refinement compared with the state-of-the-art BFS strategy. In the following we determine the execution time of a single call to this function.

Let $m_{\mathcal{C}}$ be the number of counterstrategies in $\bigcup \mathcal{C}$ and m_{ψ} the number of assumptions in ψ , and $|Q_c^{max}|$ the maximum number of states in a counterstrategy. Executing lines 1-4 requires $m_{\mathcal{C}}$ LTL model checking operations; each of these involves a counterstrategy and a single GR(1) assumption, which may be either an initial condition, an invariant, or a fairness condition. When using the algorithm in [17], LTL model checking is converted into a problem of Computation Tree Logic (CTL) model checking with fairness conditions: a fairness condition is generated for each **G** and **F** subformula in the formula to check; the full reduction procedure is described in the cited paper. Since all checked formulae contain at most two of these operators, there are at most two fairness conditions in the CTL problem. Solving this problem with a symbolic algorithm requires applying two nested alternating fixpoint operations for each fairness condition (chapter 6 of [33]), each taking a number of iterations upper bounded by the number of states in the model to check (see also [34] regarding complexity of nested fixpoint operations). In summary, solving one of the model checking problems requires $O(|Q_c^{max}|^2)$, and since m_C model checks are performed, the total computation time of lines 1-4 is $O(m_C|Q_c^{max}|^2)$.

The bipartite graph in lines 5-16 has size $O(m_{\psi}m_{\mathcal{C}})$: it requires this amount of computation to be built, and asymptotically as many operations to be minimized. Also notice that in the worst case Algorithm 1 produces as many assumptions as counterstrategies in a refinement: so, $m_{\psi} \leq m_{\mathcal{C}}$, yielding an execution time of $O(m_{\mathcal{C}}^2)$ for the graph operations.

In summary, executing MINIMALREFINEMENT requires an asymptotic time of $T_{MinRef} = O(m_C|Q_c^{max}|^2 + m_c^2)$. Let us

compare this complexity with the time $O(nm_{\psi}|Q|^2)$ for realizability checks and counterstrategy computation (lines 7-8 of Algorithm 2). If assumption minimization was not performed, then $m_{\psi}=m_{\mathcal{C}}$ for every refinement, and counterstrategy computation would take $O(m_{\mathcal{C}}|Q|^2)$. By introducing minimization, a part of this contribution quadratic in |Q| is replaced by using MINIMALREFINEMENT, quadratic in $|Q_c^{max}|$; typically counterstrategies contain significantly fewer states than entire games (which grow exponentially with the number of variables in the system).

For each explored refinement, the minimization function is called as many times as assumptions returned by APPLYBIAS; let us denote this number as $|\Psi'|$. Existing generation methods yield a $|\Psi'|$ in the order of tenths [18]. Putting all together, for small $m_{\mathcal{C}}$, such that the quadratic term in T_{MinRef} is not dominating, we obtain a speedup factor proportional to

$$\frac{m_{\mathcal{C}}|Q|^2}{m_{\psi}|Q|^2+|\Psi|m_{\mathcal{C}}|Q_c^{max}|^2}\approx\frac{m_{\mathcal{C}}}{m_{\psi}}$$

for a single refinement exploration. This can be interpreted this way: minimizing refinements yields the same gain in computation time as one would obtain by just exploring shorter nodes; the overhead induced by the actual minimization operations is negligible.