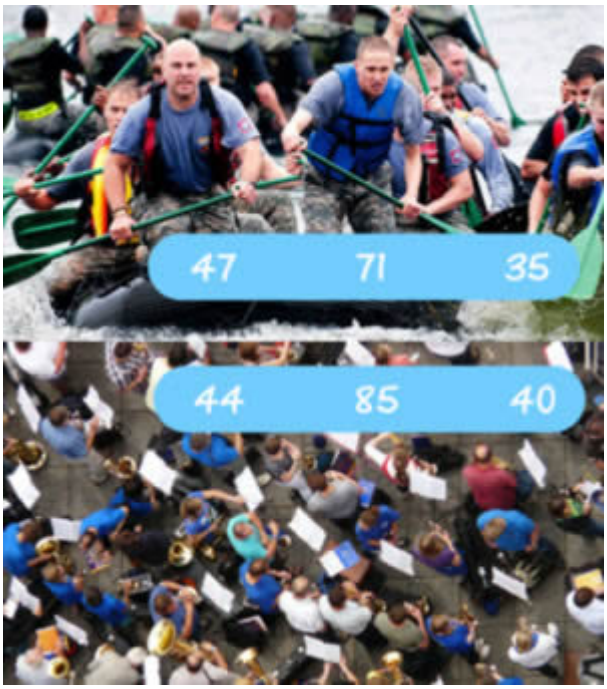
Chi-Square Test

Groups and Numbers *Advanced*



You research two groups and put them in categories single, married or divorced:

The numbers are definitely different, but ...

- Is that just random chance?
- Or have you found something interesting?

The **Chi-Square Test** gives a "p" value to help you decide!

Example: "Which holiday do you prefer?"

	Beach	Cruise
Men	209	280
Women	225	248

Does Gender affect Preferred Holiday?

If Gender (Man or Woman) **does** affect Preferred Holiday we say they are **dependent**.

By doing some special calculations (explained later), we come up with a "p" value:

p value is 0.132

Now, **p < 0.05** is the usual test for dependence. In this case **p is greater than 0.05**, so we believe the variables are **independent** (ie not linked together).

In other words Men and Women probably do **not** have a different preference for Beach Holidays or Cruises.

Understanding "p" Value

"p" is the probability the variables are **independent**.

Imagine that the previous example was in fact two random samples of **Men** each time:

Men:
Beach 209, Cruise 280

Men:
Beach 225, Cruise 248

Is it **likely** you would get such different results surveying Men each time?

Well the "p" value of **0.132** says that it really could happen every so often.

Surveys are random after all. We expect slightly different results each time, right?

So most people want to see a **p** value less than **0.05** before they are happy to say the results show the groups have a different response.

Let's see another example:

Example: "Which pet do you prefer?"

	Cat	Dog
Men	207	282
Women	231	242

By doing the calculations (shown later), we come up with:

P value is 0.043

In this case **p < 0.05**, so this result is thought of as being "significant" meaning we think the variables are **not** independent.

In other words, because **0.043 < 0.05** we think that Gender is linked to Pet Preference (Men and Women have different preferences for Cats and Dogs).

*Just out of interest, notice that the numbers in our two examples are similar, but the resulting p-values are very different: **0.132** and **0.043**. This shows how sensitive the test is!*

Why p<0.05 ?

It is just a choice! **Using p<0.05 is common**, but we could have chosen p<0.01 to be even more sure that the groups behave differently, or any value really.

Calculating P-Value

So how do we calculate this p-value? We use the Chi-Square Test!

Chi-Square Test

Note: **Chi** Sounds like "Hi" but with a **K**, so say Chi-Square like "**Ki** square"

And Chi is the greek letter X, so we can also write it X^2

Important points before we get started:

- This test only works for **categorical** data (data in categories), such as Gender {Men, Women} or color {Red, Yellow, Green, Blue} etc, but **not numerical** data such as height or weight.
- The numbers must be large enough. Each entry must be **5** or more. In our example we have values such as 209, 282, etc, so we are good to go.

Our first step is to state our **hypotheses**:

Hypothesis: A statement that might be true, which can then be tested.

The two **hypotheses** are.

- Gender and preference for cats or dogs are **independent**.
- Gender and preference for cats or dogs are **not independent**.

Lay the data out in a table:

	Cat	Dog

Men	207	282
Women	231	242

Add up rows and columns:

	Cat	Dog	
Men	207	282	489
Women	231	242	473
	438	524	962

Calculate "Expected Value" for each entry:

Multiply each row total by each column total and divide by the overall total:

	Cat	Dog	
Men	489×438/962	489×524/962	489
Women	473×438/962	473×524/962	473
	438	524	962

Which gives us:

	Cat	Dog	
Men	222.64	266.36	489
Women	215.36	257.64	473
	438	524	962

Subtract expected from actual, square it, then divide by expected:

	Cat	Dog	
Men	$\frac{(207-222.64)^2}{222.64}$	$\frac{(282-266.36)^2}{266.36}$	489
Women	$\frac{(231-215.36)^2}{215.36}$	$\frac{(242-257.64)^2}{257.64}$	473
	438	524	962

Which is:

	Cat	Dog	
Men	1.099	0.918	489

Women	1.136	0.949	473
	438	524	962

Now add up those values:

$$1.099 + 0.918 + 1.136 + 0.949 = 4.102$$

Chi-Square is 4.102

From Chi-Square to p

To get from Chi-Square to p-value is a difficult calculation, so either look it up in a table, or use the Chi-Square Calculator.

But first you will need a "Degree of Freedom" (DF)

Calculate Degrees of Freedom

Multiply (rows − 1) by (columns − 1)

Example: DF = (2 − 1)(2 − 1) = 1×1 = 1

Result

The result is:

$$p = 0.04283$$

Done!

Chi-Square Formula

This is the formula for Chi-Square:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

- O = the **Observed** (actual) value
- E = the **Expected** value

Question 1 Question 2 Question 3 Question 4 Question 5 Question 6
Question 7 Question 8 Question 9 Question 10