Shared Birthdays

Advanced

This is a great puzzle, and you get to learn a lot about probability along the way ...

> There are 30 people in a room ... what is the chance that any two of them celebrate their birthday on the same day? Assume 365 days in a year.

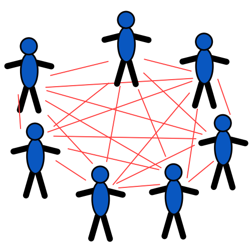
Some people may think:

"there are 30 people, and 365 days, so 30/365 sounds about right.
Which is 30/365 = 0.08..., so about **8% maybe?**"

But no!

The probability is much higher.

It is actually **likely** there are people who share a birthday in that room.



Because you should compare everyone to everyone else.

And with 30 people that is **435 comparisons**.

But you also have to be careful not to over-count the chances.

I will show you how to do it ... starting with a smaller example:

Friends and Random Numbers

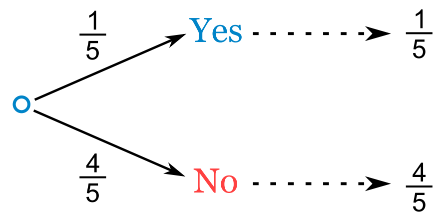
> **4 friends** (Alex, Billy, Chris and Dusty) each choose a random number between 1 and 5. What is the chance that any of them chose the same number?

We will add our friends one at a time ...

First, what is the chance that Alex and Billy have the same number?

Billy compares his number to Alex's number. There is a 1 in 5 chance of a match.

As a tree diagram:



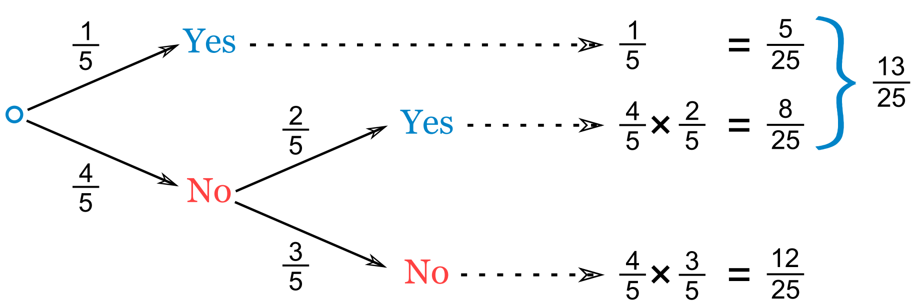
Note: "Yes" and "No" together make 1
(1/5 + 4/5 = 5/5 = 1)

Now, let's include Chris ...

But there are now two cases to consider (called "Conditional Probability"):

- If Alex and Billy **did** match, then Chris has only **one number** to compare to.
- But if Alex and Billy **did not** match then Chris has **two numbers** to compare to.

And we get this:



For the top line (Alex and Billy **did** match) we already have a match (a chance of 1/5).

But for the "Alex and Billy **did not** match" case there are **2 numbers** that Chris could match with, so there is a **2/5** chance of Chris matching (against both Alex and Billy). And a 3/5 chance of not matching.

And we can work out the combined chance by **multiplying the chances** it took to get there:

> Following the *"No, Yes"* path ... there is a 4/5 chance of No, followed by a 2/5 chance of Yes:

(4/5) × (2/5) = 8/25

Following the *"No, No"* path ... there is a 4/5 chance of No, followed by a 3/5 chance of No:

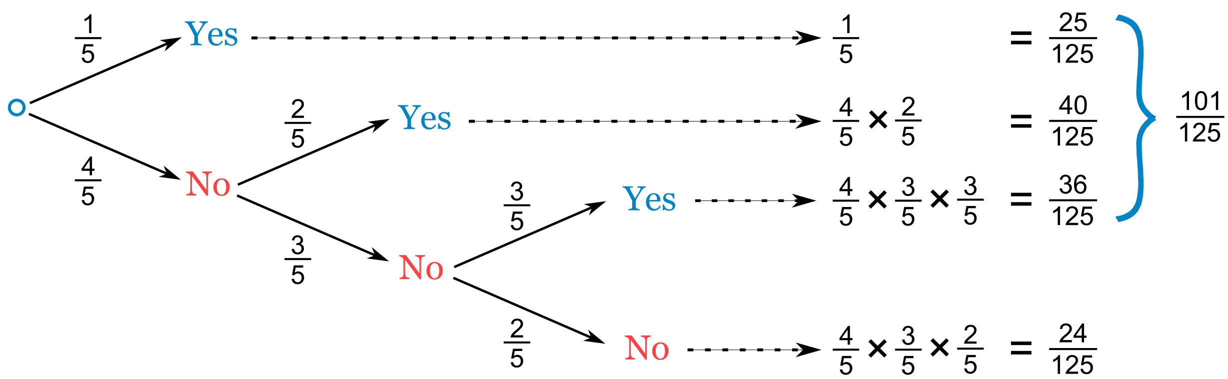
(4/5) × (3/5) = 12/25

Also notice that adding all chances together is **1** (a good check that we haven't made a mistake):

(5/25) + (8/25) + (12/25) = 25/25 = 1

Now what happens when we include Dusty?

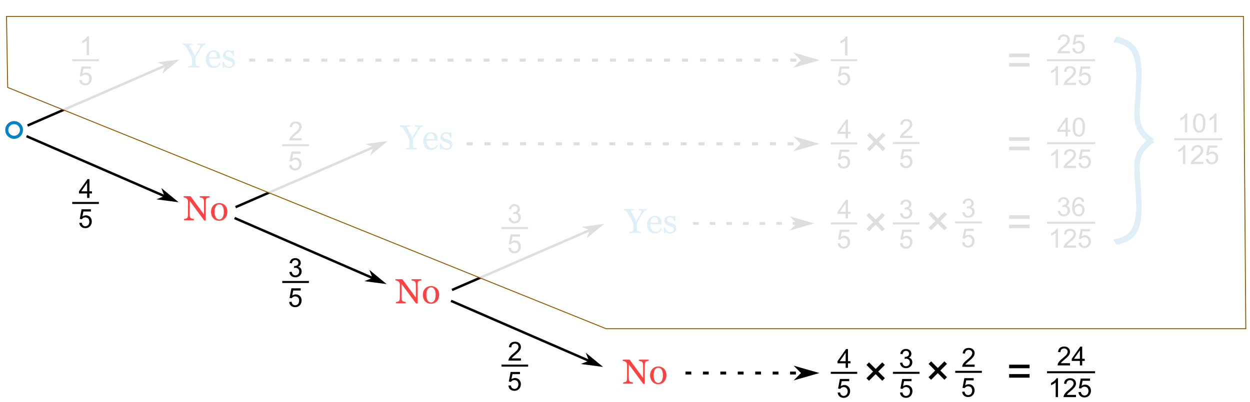
It is the same idea, just more of it:



OK, that is all 4 friends, and the "Yes" chances together make 101/125:

Answer: **101/125**

But here is something interesting ... if we follow the "No" path we can **skip all the other calculations** and make our life easier:



The chances of **not matching** are:

$$(4/5) \times (3/5) \times (2/5) = \mathbf{24/125}$$

So the chances of **matching** are:

$$1 - (24/125) = \mathbf{101/125}$$

(And we didn't really need a tree diagram for that!)

And that is a popular trick in probability:

It is often easier to work out the "No" case
(and subtract from 1 for the "Yes" case)

Example: what are the chances that with 6 people any of them celebrate their Birthday in the same month? (Assume equal months)

The "no match" case for:

- 2 people is 11/12
- 3 people is (11/12) × (10/12)
- 4 people is (11/12) × (10/12) × (9/12)
- 5 people is (11/12) × (10/12) × (9/12) × (8/12)
- 6 people is (11/12) × (10/12) × (9/12) × (8/12) × (7/12)

So the chance of **not matching** is:

$$(11/12) \times (10/12) \times (9/12) \times (8/12) \times (7/12) = 0.22...$$

Flip that around and we get the chance of **matching**:

$$1 - 0.22... = \mathbf{0.78...}$$

So, there is a **78% chance** of any of them celebrating their Birthday in the same month

And now we can try calculating the "Shared Birthday" question we started with:

There are 30 people in a room ... what is the chance that any two of them celebrate their birthday on the same day? Assume 365 days in a year.

It is just like the previous example! But bigger and more numbers:

The chance of **not matching**:

$$364/365 \times 363/365 \times 362/365 \times \ldots \times 336/365 = \mathbf{0.294...}$$

(I did that calculation in a spreadsheet, but there are also mathematical shortcuts)

And the probability of **matching** is **1 − 0.294...** :

The probability of sharing a birthday = 1 − 0.294... = **0.706...**

Or a 70.6% chance, which is **likely**!

So the probability for **30** people is about **70%**.

And the probability for **23** people is about **50%**.

And the probability for **57** people is **99%** (almost certain!)

So, next time you are in a room with a group of people why not find out if there are any shared birthdays?

Footnote: In real life birthdays are not evenly spread out ... more babies are born in July, August, and September. Also Hospitals prefer to work on weekdays, not weekends, so there are more births early in the week. And then there are leap years. But you get the idea.

Question 1 Question 2 Question 3 Question 4 Question 5 Question 6
Question 7 Question 8 Question 9 Question 10