

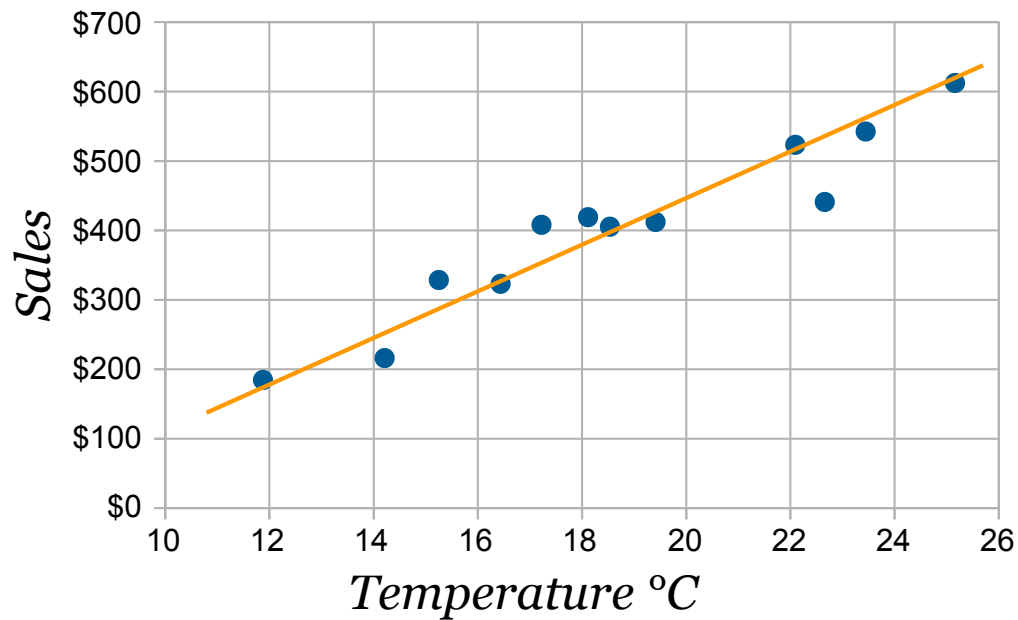


# Least Squares Regression

## Advanced

### Line of Best Fit

Imagine you have some points, and want to have a **line** that best fits them like this:



We can place the line "by eye": try to have the line as close as possible to all points, and a similar number of points above and below the line.

But for better accuracy let's see how to calculate the line using **Least Squares Regression**.

### The Line

Our aim is to calculate the values **m** (slope) and **b** (y-intercept) in the [equation of a line](#) :

$$y = mx + b$$

Where:

- **y** = how far up
- **x** = how far along
- **m** = Slope or Gradient (how steep the line is)
- **b** = the Y Intercept (where the line crosses the Y axis)

## Steps

To find the line of best fit for **N** points:

**Step 1:** For each (x,y) point calculate  $x^2$  and  $xy$

**Step 2:** Sum all  $x$ ,  $y$ ,  $x^2$  and  $xy$ , which gives us  $\Sigma x$ ,  $\Sigma y$ ,  $\Sigma x^2$  and  $\Sigma xy$  ( $\Sigma$  means "sum up")

**Step 3:** Calculate Slope **m**:

$$m = \frac{N \Sigma(xy) - \Sigma x \Sigma y}{N \Sigma(x^2) - (\Sigma x)^2}$$

(N is the number of points.)

**Step 4:** Calculate Intercept **b**:

$$b = \frac{\Sigma y - m \Sigma x}{N}$$

**Step 5:** Assemble the equation of a line

$$y = mx + b$$

Done!

## Example

Let's have an example to see how to do it!

Example: Sam found how many **hours of sunshine** vs how many **ice creams** were sold at the shop from Monday to Friday:

"x" Hours of Sunshine	"y" Ice Creams Sold
2	4
3	5
5	7
7	10
9	15



Let us find the best **m** (slope) and **b** (y-intercept) that suits that data

$$y = mx + b$$

**Step 1:** For each (x,y) calculate  $x^2$  and  $xy$ :

<b>x</b>	<b>y</b>	<b><math>x^2</math></b>	<b>xy</b>
2	4	4	8
3	5	9	15
5	7	25	35
7	10	49	70
9	15	81	135

**Step 2:** Sum x, y,  $x^2$  and xy (gives us  $\Sigma x$ ,  $\Sigma y$ ,  $\Sigma x^2$  and  $\Sigma xy$ ):

<b>x</b>	<b>y</b>	<b><math>x^2</math></b>	<b>xy</b>
2	4	4	8
3	5	9	15
5	7	25	35
7	10	49	70
9	15	81	135
<b><math>\Sigma x</math>: 26</b>	<b><math>\Sigma y</math>: 41</b>	<b><math>\Sigma x^2</math>: 168</b>	<b><math>\Sigma xy</math>: 263</b>

Also **N** (number of data values) = **5**

**Step 3:** Calculate Slope **m**:

$$\begin{aligned}
 m &= \frac{N \sum(xy) - \sum x \sum y}{N \sum(x^2) - (\sum x)^2} \\
 &= \frac{5 \times 263 - 26 \times 41}{5 \times 168 - 26^2} \\
 &= \frac{1315 - 1066}{840 - 676} \\
 &= \frac{249}{164} = 1.5183...
 \end{aligned}$$

**Step 4:** Calculate Intercept **b**:

$$\begin{aligned}
 b &= \frac{\sum y - m \sum x}{N} \\
 &= \frac{41 - 1.5183 \times 26}{5} \\
 &= 0.3049...
 \end{aligned}$$

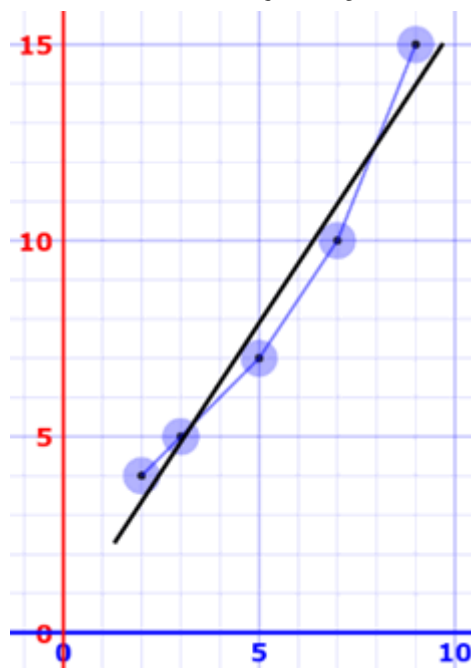
**Step 5:** Assemble the equation of a line:

$$\begin{aligned}
 y &= mx + b \\
 y &= 1.518x + 0.305
 \end{aligned}$$

Let's see how it works out:

<b>x</b>	<b>y</b>	<b>y = 1.518x + 0.305</b>	<b>error</b>
2	4	3.34	-0.66
3	5	4.86	-0.14
5	7	7.89	0.89
7	10	10.93	0.93
9	15	13.97	-1.03

Here are the (x,y) points and the line **y = 1.518x + 0.305** on a graph:



Nice fit!

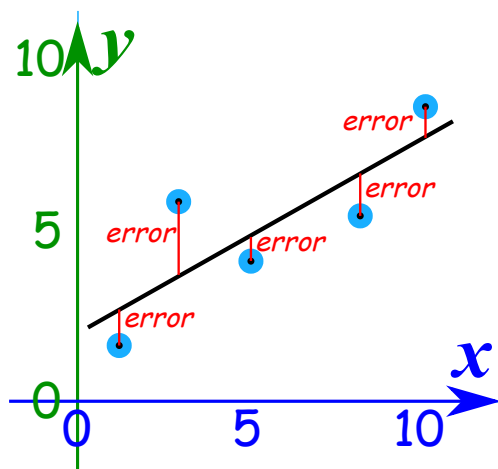
Sam hears the weather forecast which says "we expect 8 hours of sun tomorrow", so he uses the above equation to estimate that he will sell

$$y = 1.518 \times 8 + 0.305 = 12.45 \text{ Ice Creams}$$

Sam makes fresh waffle cone mixture for 14 ice creams just in case. Yum.

## How does it work?

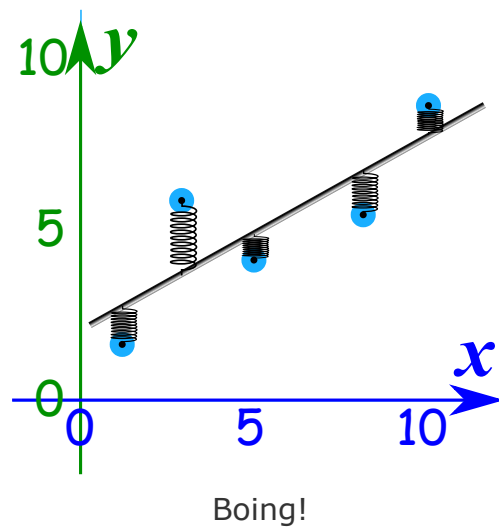
It works by making the total of the **square of the errors** as small as possible (that is why it is called "least squares"):



The straight line minimizes the sum of squared errors

So, when we square each of those errors and add them all up, the total is as small as possible.

You can **imagine** (but not accurately) each data point connected to a straight bar by springs:



## Outliers

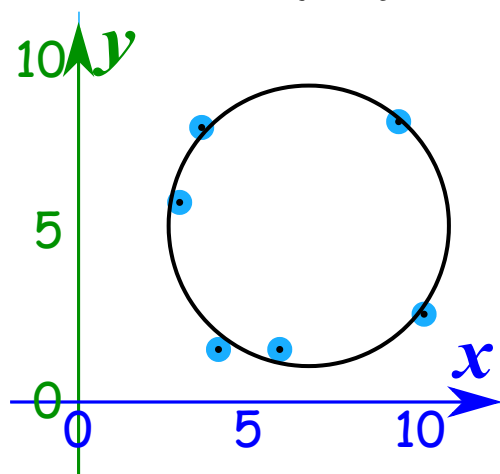
Be careful! Least squares is sensitive to outliers. A strange value will pull the line towards it.

## Use the App

Have a play with the [Least Squares Calculator](#)

## Not Just For Lines

This **idea** can be used in many other areas, not just lines.



A "circle of best fit"

But the formulas (and the steps taken) will be very different!

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