1. A company is considering whether to launch a new product. The sum of £170,000 should be spent now on new equipment. It is estimated that there will be a market for the new product for three years and the product will bring £60,000 of profit per year (assume for simplicity that the yearly profit is received at the end of the year as a single payment). Should this project be pursued if the current risk-free interest rate is 5%?

   –
   Sum : 170, 000 (New equipment)
   Time : 3 Years
   Ppy(Profit per year) : 60, 000 Single payments
   Interest rate : 5%

   Formula : $\frac{price}{(1+r)^T}$

$$\sum_{n=1} \frac{price}{(1+0.05)^n} \tag{1}$$
$$51830.255811 + 54421.768 + 57142.857 = 163395. \tag{2}$$
$$163395 - 170000 = -6605.12 \tag{3}$$

   Because this is negative this project should not be pursued. As the expected is less than the risk free investment.

2. A bond with the face value of £200 matures in 3 years and makes yearly coupon payments at the coupon rate of 15the end of each year.

   Solution. –

   - Face Value : £200
   - maturity : 3 years
   - Yearly CP Payment : 0.15 (End of each year)
   - List all the payments (With Dates and amount that the bonds makes)

$$YearlyPayment = 0.15 \cdot 200 = X = 30 \tag{4}$$
$$ThreeYearPayment = X \cdot 3 = 30 \cdot 3 = 30 + 30 + 230 \tag{5}$$

   Such that:
   Year 1 : Payment of £ 30
   Year 2 : Payment of £ 30
   Year 3 : Payment of £ 230

- Assuming the effective interest rate of 10 % PA , Calculate the value of the bond. - In this part, you would have to calculate the PV value. This can be done with the following formula :

$$\sum (cr \cdot fv) \cdot ((1+r)^t)^{-1} + fv \cdot ((1+r)^N)^{-1} \tag{6}$$

$$\implies cr \cdot fv \cdot \frac{1-(1+r)^{-N}}{r} + \frac{fv}{(1+r)^N} \tag{7}$$

Such that :

$$(\sum_{x=1}^{3}(\frac{0.15 \cdot 200}{(1+0.1)^x})) + \frac{200}{(1+0.1)^3} \tag{8}$$

$$or \tag{9}$$

$$\frac{30}{(1+0.1)^1} + \frac{30}{(1+0.1)^2} + \frac{230}{(1+0.1)^3} \tag{10}$$

$$((0.15 \cdot 200) \cdot \frac{1-(1+0.1)^{-3}}{0.1} \tag{11}$$

$$= 224.8685 \tag{12}$$

■

3. European call option with the strike of 40p maturing in half a year on a share is worth 13p. Suppose that the share is currently worth 45p and the risk-free interest rate (with continuous compounding) is 10Calculate the price eliminating arbitrage opportunities of a European put option with the same strike of 40p maturing in half a year on the same share. You do not need to give a complete proof.

Solution. – Information

- Strike price : 40 p
- Matures in 1/2 a year
- Can be calculated with the put call parity

Formula:

$$C + Xe^{-rT} = P + S_0 \tag{13}$$

$$therefore \tag{14}$$

$$P = C + Xe^{-rT} - S_0 \tag{15}$$

$$P = 6.0491769p \tag{16}$$

$$\tag{17}$$

Therefore the price of the Eu put option with the strike price of 40 P maturing in half a year on the share is 6.0491769, eliminating any arbitrage opportunities that May occur.

■

4. Suppose that the price of a share on the spot market is 50p, while the price of a European put option on this share that matures in one month and has the strike price of 55p is 3p. Assume that the risk-free interest rate (with continuous compounding) is 5% pa and the share will not be paying any dividends during the coming two months. This situation implies an arbitrage opportunity. Describe an arbitrage strategy and show that it brings profit

   Solution. – Yes the situation implies an arbitrage opportunity, mainly due to the price option is underpriced. This can be seen with the following steps :

   - Buy the share on the spot market for 50 p
   - You also buy the put for 3p [Known as the initial Investment]
   - Your Debt will increase with X with respect to T with $53p$ total
   - $53 \cdot e^{0.05 \cdot \frac{1}{12}} = 53.221294$
   - We wait till maturity Such that when $T(1/12)$ The stock should be sold, atleast the one that you bought for 50p with respect to the strike price at 55 p
   - The remaining amount should be paid off which should be 53.221294
   - you will have a difference factor : $55 - 53.221294 = 1.7787p$ Profit

   Profiting on the mispricing of the European put option, this method removes the arbitrage possibility and generates a profit. ■

5. Suppose that the underlying asset in Question 4 above is a metal rather than a share. Let all the prices be the same as above: the spot price per ounce of the metal be 50p and the price of a European put option on an ounce that matures in one month and has the strike price of 55p be 3p. The metal can be bought or sold with no overhead costs, but its storage incurs the cost of 2p per ounce per month payable up front. Does the arbitrage strategy you described in your answer to Question 4 above still bring profit?

   Solution. –

   (a) S_0 Spot price is of 50 p
   (b) X_θ Strike price is 55 p
   (c) r Interest rate is of 5%
   (d) T Time to maturity is 1/12
   (e) p Put price is 3p

(f) storage cost is 2 p per month

Such that from question 4, the recalulation would have to account for the initial storage cost - which would change the arbitrage.

- You buy the stock and put for 3 p. *Including the storage of 2p*
- Where your amount due will be $(50 + 3 + 2) \cdot e^{0.05 * \frac{1}{12}}$
- this evaluates to 55.286 p

Due to 55.286 being larger than 55 p there is zero profit to be gained.

■

6. Suppose that the option from Questions 4 and 5 is American. Is there an arbitrage opportunity? If so, describe an arbitrage strategy and show that it brings profit in the case when

Solution. –

An American option gives the holder the right to exercise the option at any time before its expiration date. This means that the holder can choose to exercise the option early if it is profitable to do so.

Formula: Let S be the spot price of the underlying asset, K be the strike price of the American option, t be the time to expiration, r be the risk-free interest rate, and V be the price of the American option.

$$V > max(S - Ke^{rt}, 0)$$

- the underlying asset is a share as in Question 4; [4 marks]
  - Buy the american option for 3p
  - sell the shares on the spot market for 50p
  - invest the rest from selling the shares at risk free rate : 5% for 1 month
  - $50 \cdot e^{.05 * \frac{1}{12}} = 50.208$
  - compare the value of the underlying asset with the strike price
  - if the strike price K, if $S \geq K$ Exercise the option and sell the underlying asset for 55 p
  - else $S < K$ do not exercise the option
  - repay any debt and calculate the profit : $3p + max(50.208 - 55p, 0)$ Such that you would get a profit $\sim 5p$

  Further information / Using the stratergy based on question 4 :

  - There would still be an arbitrage opportunity
  - The strategy would be similar to that described in Question 4: buy the share on the spot market for 50p and buy the put for 3p (total initial investment of 53p)

  - Since this is an American option, you could choose to exercise it early if it becomes profitable to do so before maturity
  - If you wait until maturity and sell the stock at the strike price of 55p, after paying off your debt (which has increased due to interest), you will have a profit
  - For example, if the price of the share falls below the strike price of 55p before maturity, you could exercise your put option early and sell your share at the strike price of 55p. After paying off your debt (which has increased due to interest), you would have a profit.

  Allthough there are two different strategies, both methods would work . As you follow the the basis of when to exercise your right.

- The underlying asset is a metal as in Question 5. [4 marks] the arbitrage statergy for the american put option would be the same as the described answer above. But instead the profit calculation would never account for the cost of storage, which is 2p .And is another payable upfront
  - Buy american option for 3p
  - Sell the metal for 50p
  - invest the rest from selling the metal at the risk free rate of 5%
  - At expiration, calculate the value of the underlying asset: $s = 50 \cdot e^{0.05 \cdot \frac{1}{12}} = 50.209$
  - compare the value of the underlying asset with the strike price . With respect to K
  - if $S \geq K$ exercise the option and sell the underlying asset for 55 p
  - else do not exercise the option
  - repay the deby and you would have to calculate the following $3p + max(50.208 - 55, 0) - 2$ What this would yield is the following $5 - 3 - 2 = 0$
  - In any scenario here, you would always end up in the negative, so it does not make sense. Even with using the previous options listed.

  Further information :

  - In the case where the underlying asset is a metal as in Question 5 and the option is American, there would not be an arbitrage opportunity
  - After accounting for storage costs and interest on your initial investment of 55p (buying metal on spot market for 50p + buying put for 3p + storage cost of 2p), your debt would increase to more than the strike price of 55p
  - Since this is greater than what you would receive from exercising the option at maturity or earlier, there would be no profit to be gained from this strategy
  - Even though this is an American option and you could choose to exercise it early if it becomes profitable to do so before maturity, it is unlikely that exercising early would result in a profit
  - This is because the storage cost of 2p per ounce per month payable up front has already been incurred and cannot be recovered

- Additionally, since your debt has increased due to interest on your initial investment, it would be difficult to make a profit even if you were able to exercise the option early.

■

7. A generalised Wiener process x(t) starts from the point $x(0) = 10$ and satisfies the stochastic differential equation $dx = 5dt + 3dz$, where z is the Wiener process.

   A: What is the distribution of the increment $x(4) - x(0)$? Calculate its parameters.

$$x(4) - x(0) = \int_0^4 5dt + \int_0^3 3dz \tag{18}$$

$$\tag{19}$$

With the

- first integral evaluating to $dt = 20$
- Second integral evaluates to $\mu = 0$
- $\sigma^2 = 3^2 \cdot 4 = 36$
- $SD = \sqrt{\sigma^2}$

Such that

$$x(4) - x(0) \sim N(20, 36) \tag{20}$$

$$\tag{21}$$

Its variance is equal to the square of the diffusion coefficient multiplied by the time increment: $(3^2) * (4 - 0) = 36$ Proof : Because it is the weiner process you would have a zero mean and an equal varience with respect to the time interval.

$$\mathbb{E}[x(4) - x(0)] = \mathbb{E}[\int_0^4 5dt] + \mathbb{E}[\int_0^4 3dz] = 20 \tag{22}$$

$$Var[x(4) - x(0)] = Var[\int_0^4 5dt] + Var[\int_0^4 3dz] = 36 \tag{23}$$

$$\tag{24}$$

What this means is that $x(4) - x(0)$is distrubted with a $\mu = 20$ (mean) and $\sigma^2 = 36$ (varience)

B: What is the probability that $x(4) < 0$? Express the probability in terms of $N(x)$, the cumulative distribution function for $\varphi(0,1)$, and then use Python or any other tool to calculate it.

$$x(4) = x(0) + \int_0^4 (5dt + 3dz) \tag{25}$$

Hence x(4) is $10 + 20 = 30$ mean : and its varience is 36 : which implies that the $x(4) < 0$: it can be standardized into the following format:

$$\phi(30, 36) \tag{26}$$
$$P(x(4) < 0) => x(4); \mu = 30; \sigma = 6 \tag{27}$$

Now with the following Z value formula, we can compute the following

$$\frac{x(4) - 30}{6} \sim \phi(0, 1) \tag{28}$$
$$x(4) = 6z + 30 \tag{29}$$
$$\tag{30}$$

Breaking this down you would get the following

$$pr(x(4) < 0) \tag{31}$$
$$pr(6z + 30 < 0) \tag{32}$$
$$pr(6z < -30) \tag{33}$$
$$pr(z < -5) \tag{34}$$
$$\tag{35}$$

This allows for $N(-5) = 2.8665... \cdot 10^{-7}$