

Volatility Smile

- The implied volatility of an option is the volatility for which the Black-Scholes price equals the market price, and it may depend on the strike.
- The volatility smile is the graph showing the dependency of implied volatility on strike price, and it is often minimal for strike prices close to the current stock price (i.e., for at-the-money options).
- However, the shape of the volatility smile can also be skewed or even frown-shaped, and there is no commonly accepted theoretical model explaining this phenomenon.
- Possible explanations for the volatility smile include deviations of the stock price from the log-normal distribution and liquidity issues.
- To account for the effect of time to maturity, the volatility surface shows the dependence of volatility on both the strike price and time to maturity.

Market Efficiency

Market efficiency is the idea that market prices reflect all relevant information.
Forms of Market Efficiency

- There are different forms of the efficiency hypothesis, including weak efficiency, semi-strong efficiency, and strong efficiency.

Weak Efficiency

- Weak efficiency corresponds to the Markov property of random processes, where future values are independent of past values given the current value.

Semi-Strong Efficiency

- Semi-strong efficiency means that no investor can earn excess returns from trading rules based on any publicly available information.

Strong Efficiency

- Strong efficiency means that no investor can earn excess returns using any information, even insider information.

Debates and Consensus

- The near consensus in academia is that there are some inefficiencies in modern financial markets, but they are quickly eliminated.
- Defining "excess returns" can be difficult, as it requires defining what is considered "excess" with respect to a model.
- One practical solution to measure market efficiency is to determine if one can beat the index, which is a portfolio made up of shares of top companies

in proportion to their capitalization and is supposed to reflect the behavior of the market in general.

To better understand these notes, it may be helpful to review the concepts of implied volatility and the Black-Scholes model for pricing options. The volatility smile is a phenomenon observed in the market where the implied volatility of at-the-money options is lower than the implied volatility of options that are further in- or out-of-the-money. This phenomenon is difficult to explain with existing theoretical models and may be due to factors such as market liquidity and non-normality of stock price distributions.