Vsevolod
Kovalev

Valid: True in all rows
Satisfiable: True in at least 1
Unsatisfiable: Otherwise

1. A.

P	Q	$P \land Q$	$\neg Q$	$(P \land Q) \lor \neg Q$
0	0	0	1	1
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

Satisfiable but not valid

B.

P	Q	R	$P \land Q$	$P \Rightarrow R$	$Q \Rightarrow R$	$(P \land Q) \Rightarrow R$	$(P \Rightarrow R) \lor (Q \Rightarrow$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	0	1	1
0	1	1	0	1	1	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

Full expression $\Rightarrow$ The formula is valid
1
1
1
1
1
1
1
1

2. A. $A \Rightarrow B$ becomes $\neg A \lor B$

So, our sentence becomes:

$\neg(\text{saturday} \lor \text{sunday}) \lor (\neg\text{free} \lor \text{concert})$

Using De Morgan's:

$(\neg\text{saturday} \land \neg\text{sunday}) \lor (\neg\text{free} \lor \text{concert})$

Distribute OR

$(\neg\text{saturday} \lor \neg\text{free} \lor \text{concert}) \land (\neg\text{sunday} \lor$
$\lor \neg\text{free} \lor \text{concert})$

2.B. From the CNF:

($\neg$ saturday $\vee \neg$ free $\vee$ concert)

($\neg$ sunday $\vee \neg$ free $\vee$ concert)

Both are Horn clauses.

Implication form:

(saturday $\wedge$ free) $\Rightarrow$ concert

(sunday $\wedge$ free) $\Rightarrow$ concert

3. A KB:

1. $Y \Rightarrow O$
2. $\neg Y \Rightarrow (\neg O \wedge M)$
3. $(O \vee M) \Rightarrow H$
4. $H \Rightarrow G$

B CNF

1. $Y \Rightarrow O$

$\neg Y \vee O$

2. $\neg Y \Rightarrow (\neg O \wedge M)$

$Y \vee (\neg O \wedge M)$

$(Y \vee \neg O) \wedge (Y \vee M)$

3. $(O \vee M) \Rightarrow H$

$\neg(O \vee M) \vee H$

$(\neg O \wedge \neg M) \vee H$ (De Morgan)

$(\neg O \vee H) \wedge (\neg M \vee H)$

4. $H \Rightarrow G$

$\neg H \vee G$

C. Resolution proof.

Add $\neg G$ to the KB

1. From $\neg H \vee G$ resolve $\neg G \Rightarrow \neg H$
2. Now we have $\neg H$
3. From $(\neg O \vee H)$ resolve $\neg H \Rightarrow \neg O$
4. From $(\neg M \vee H)$ resolve $\neg H \Rightarrow \neg M$

3.C. 5. From $(Y \vee M)$ resolve $\neg M \Rightarrow Y$

6. From $(Y \vee \neg O)$, since Y is true, the clause is satisfied.

7. From $(\neg Y \vee O)$ resolve $Y \Rightarrow O$

Now we have both O and $\neg O$, which is a contradiction.

Hence, G is provable by resolution.

D. Resolution Proof.

Add $\neg H$ to the KB

1. From $(\neg O \vee H)$ resolve $\neg H \Rightarrow \neg O$
2. From $(\neg M \vee H)$ resolve $\neg H \Rightarrow \neg M$
3. From $(Y \vee M)$ resolve $\neg M \Rightarrow Y$
4. From $(\neg Y \vee O)$ resolve $Y \Rightarrow O$

Since now we have both O and $\neg O$ which is a contradiction, H is provable by resolution.