

Testing Digital Camera by Splitting Mizar

Mizar is a famous star in the constellation Ursa Major (Great Bear, Fig. 1), it is the second star from the end of the Big Dipper's handle of an apparent magnitude 2.23, designated ζ . Its faint companion called Alcor is of magnitude 4.01 and is 11.8 arcminutes to the east. The ability to see Alcor with the naked eye has been a traditional test of good eyesight. The human eye can resolve objects separated by approximately 1 arcminute.

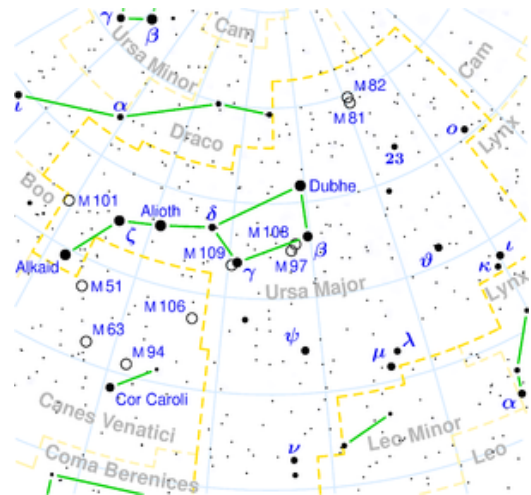


Fig 1. Ursa Major as depicted in Urania's Mirror c. 1825 and in a modern star chart (from Wiki).

It is less known that Mizar itself is a double star which can be split in a small telescope. Mizar A of magnitude 2.27 and Mizar B of magnitude 3.95 are 14.4 arcseconds apart. It is the latter pair that we would like to resolve by a digital camera.

Canon T6i (750D) digital camera has APS-C CMOS sensor 22.3x14.9 mm, 6000x4000 pixels and the full matrix of 24.2 Megapixels (6096x4056). We will use 75-300 mm/4-5.6 Zoom Lens often coming with this camera.

The camera's field of view with $f=300$ mm is
 $\arctg(22.3 \text{ mm}/300 \text{ mm}) \cdot 180/\pi = 4.25 \text{ deg}$
 $\arctg(14.9 \text{ mm}/300 \text{ mm}) \cdot 180/\pi = 2.84 \text{ deg}$
 (with $f=75$ mm the field of view is 16.56 deg x 11.24 deg).

The pixel size is $22.3\text{mm}/6000 = 3.716 \mu\text{m}$, $14.9\text{mm}/4000 = 3.725 \mu\text{m}$, thus the angular resolution $3.72 \mu\text{m}/300\text{mm} (180 \cdot 60 \cdot 60)/\pi = 2.558 \text{ arcsec}$.

The diffraction limit (Rayleigh Criterion) or the radius (the first minimum, the first dark ring) of the Airy disk of a point source

$$\theta = 1.22 \lambda / D,$$

where the coefficient $1.22 = v_{11}/\pi = 3.8317/\pi$, v_{11} is the first zero of the Bessel function J_1 , λ is the green light wavelength $0.5 \mu\text{m}$, D is the lens diameter, $D = f/N = 300 \text{ mm}/5.6$, $N = f/D = 5.6$ is the f-number. For our lens

$$\theta = 1.22 \cdot 0.5 \mu\text{m} \cdot 5.6/300 \text{ mm} = 1.14 \times 10^{-5} \text{ rad} = 2.35 \text{ arcsec}; \quad \theta \cdot 2 = 4.70 \text{ arcsec}.$$

It is usually the diameter of Airy disk

$$d = 2 * \theta * f = 2 * 1.22 * \lambda * N = 2 * 1.22 * 0.5 \mu\text{m} * 5.7 = 6.954 \mu\text{m}$$

that is compared with the pixel size in order to characterize the system performance as being diffraction or instrument limited. In our case we have diffraction limited system:

the pixel size $3.72 \mu\text{m} < d = 6.954 \mu\text{m}$, which is great - we have room for a better lens.

The astrophotographs will be taken from a fixed mount; therefore, the rotating sky will cause stars to leave streaks in the images. Earth turns 360 deg in 24 siderial hours so here are some typical exposure times τ and corresponding arc angles:

1 min -> 15 arcmin

15 sec -> 3.75 arcmin

8 sec -> 2.0 arcmin

1 sec -> 15 arcsec

$\frac{1}{4}$ sec -> 3.75 arcsec

$\frac{1}{8}$ sec -> 1.875 arcsec (this is close to our camera pixel resolution of 2.55 arcsec)

Thus, Mizar A and B would be approximately 6 pixels apart and should be resolvable with $\frac{1}{4}$ sec exposure.

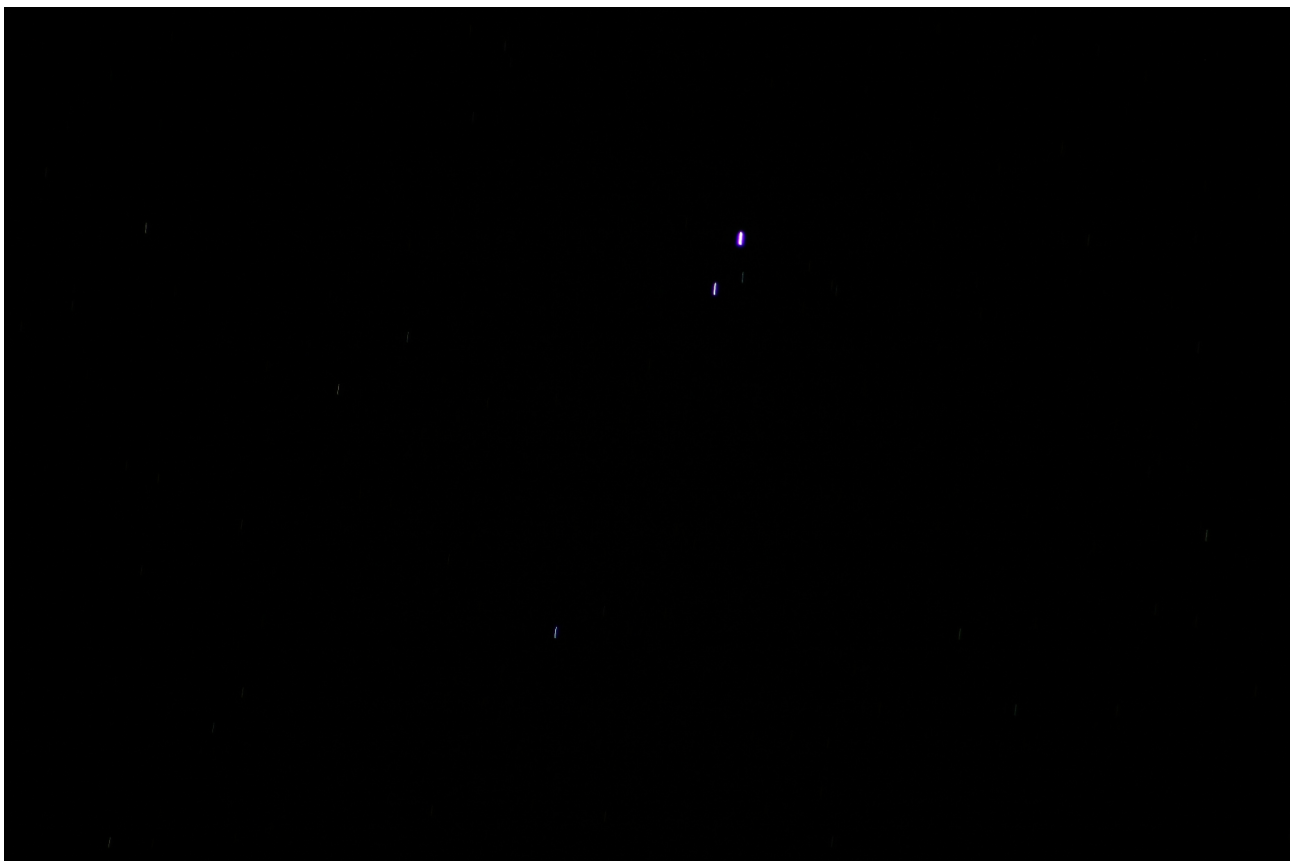


Fig. 1. (IMG_1375) Mizar-Alcor (upper two stars) with fainter Sidus Ludovicianum forming a small triangle. The star in the lower part of the image is AUID 000-BBT-461 of magnitude 5.637. $f=300\text{mm}$, $f/5.6$, ISO-200, exp 15sec. Full field of view.

A photo with a long exposure of 15 sec is shown in Fig. 1. To minimize vibration the shutter delay was set to 2 sec after pressing the shoot button. Fig. 1b (below) shows an expanded view. Note the length 3.75 arcminutes and the direction of the streaks left by the stars due to natural rotation of the sky.

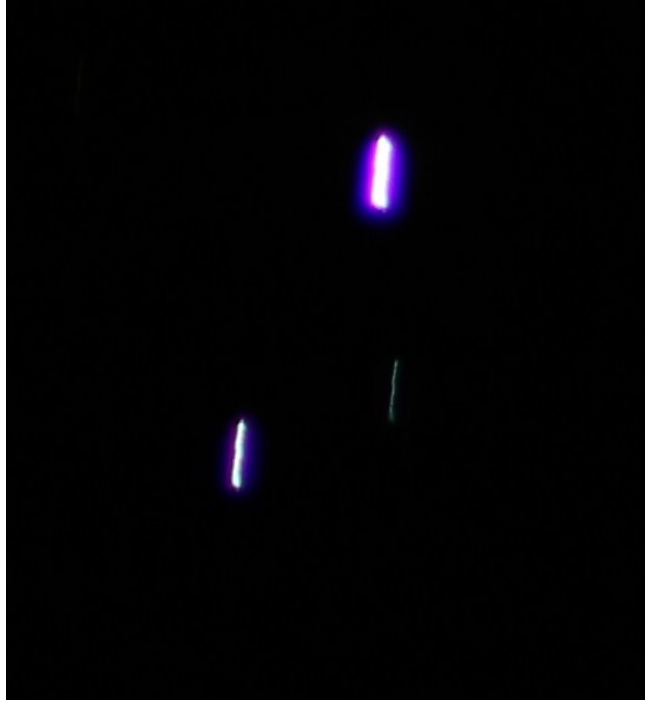


Fig. 1b. (IMG_1375) Expanded view: Mizar (brightest) , Alcor (second), and faint Sidus Ludovicianum leave streaks caused by sky rotation. Mizar saturates the pixels.



Fig. 2 (IMG_1401) $f=300\text{mm}$ $f/5.6$, ISO-800, exp 1/15sec. Expanded view: Mizar and Alcor.

The photo in Fig. 2 (taken next night) has a shorter exposure to eliminate the streaks and pixel saturation. Again the shutter delay was 2 sec to minimize vibration. However, one still can see small streaks in the direction not consistent with the sky rotation (Mizar-Alcor direction is different in this photo). These streaks are caused by vibrations triggered by the mirror movement during the shutter operation.

To eliminate these vibrations the next photo (Fig. 3) was taken with shutter exposure set to 4 sec, but the lens was blocked manually by holding a piece of cardboard, and briefly removing it for $\frac{1}{4}$ sec by hand waving, without touching the camera. Therefore the camera was perfectly still while collecting photons.

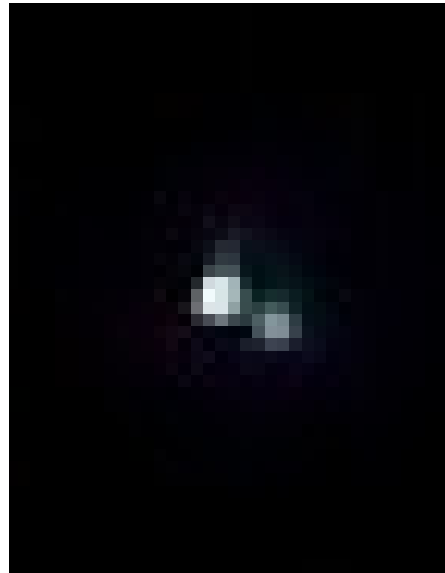


Fig 3. (IMG_1413) $f=300\text{mm}$ $f/5.6$, ISO-100, exp 1/4sec (hand waving). Expanded views: (left panel) Mizar A, B, and Alcor; (right panel) Mizar A and B.

So this is an ultimate shot resolving Mizar A and B. The separation between stars is $dx, dy=5, 2$ pixels in agreement with our estimates.

The above photos were shown in jpeg format. Further details on a pixel scale can be inspected in the Canon Raw (.CR2) format files.

Vitalii Sheremet
vsheremet@whoi.edu