Fall 2024 Quantitative Management Modeling
Assignment Instructions: Module 2 - The LP Model

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Question1

a. Decision Variables:

Let: x1 = the number of Collegiate backpacks produced per week. x2 = the number of Mini backpacks produced per week.

b. Objective Function:

The objective of the company is to maximize profit. The profit per Collegiate backpack is $32, and the profit per Mini backpack is $24. Therefore, the objective function is:

- Maximize Z = 32x1 + 24x2

c. Constraints:

1. Nylon Fabric Constraint: The total amount of nylon fabric available is 5000 square feet. Each Collegiate requires 3 square feet, and each Mini requires 2 square feet. Thus, the constraint on the fabric is:
    a. 3x1 + 2x2 <= 5000
2. Labor Time Constraint: Back Savers has 35 laborers, each providing 40 hours (or 2400 minutes) of labor per week, giving a total labor availability of 35 * 2400 = 84,000 minutes. Each Collegiate requires 45 minutes, and each Mini requires 40 minutes. Thus, the constraint on labor time is:
    a. 45x1 + 40x2 <= 84000
3. Sales Limit for Collegiate Backpacks: The maximum sales for Collegiate backpacks is forecasted to be 1000 units per week:
    a. x1 <= 1000
4. Sales Limit for Mini Backpacks: The maximum sales for Mini backpacks is forecasted to be 1200 units per week:
    a. x2 <= 1200
5. Non-Negativity Constraint: The number of backpacks produced cannot be negative, so:
    a. x1 >= 0, x2 >= 0

d. Full Mathematical Formulation:

Maximize $Z = 32x1 + 24x2$

Subject to:

- $3x1 + 2x2 <= 5000$
- $45x1 + 40x2 <= 84000$
- $x1 <= 1000$
- $x2 <= 1200$
- $x1 >= 0, x2 >= 0$

Question2

a. Decision Variables:

Let

- x11 = number of large units produced at plant 1
- x12 = number of large units produced at plant 2
- x13 = number of large units produced at plant 3
- x21 = number of medium units produced at plant 1
- x22 = number of medium units produced at plant 2
- x23 = number of medium units produced at plant 3
- x31 = number of small units produced at plant 1
- x32 = number of small units produced at plant 2
- x33 = number of small units produced at plant 3

b. Objective Function:

The objective is to maximize profit. The profit for each size is:

- $420 per large unit
- $360 per medium unit
- $300 per small unit

The total profit can be expressed as:

Maximize $Z = 420(x11 + x12 + x13) + 360(x21 + x22 + x23) + 300(x31 + x32 + x33)$

c. Constraints:

1. Production Capacity Constraints:

Each plant has a maximum capacity, regardless of the size or combination of products.

- Plant 1 can produce a maximum of 750 units:

    - $x11 + x21 + x31 <= 750$
- Plant 2 can produce a maximum of 900 units:
    - $x12 + x22 + x32 <= 900$
- Plant 3 can produce a maximum of 450 units:
    - $x13 + x23 + x33 <= 450$

2. Storage Space Constraints:

Each plant has a limited amount of in-process storage, and each unit of each size requires a specific amount of space (20 square feet for large, 15 square feet for medium, and 12 square feet for small).

- Plant 1 has 13,000 square feet available:
    - $20 * x11 + 15 * x21 + 12 * x31 <= 13000$
- Plant 2 has 12,000 square feet available:
    - $20 * x12 + 15 * x22 + 12 * x32 <= 12000$
- Plant 3 has 5,000 square feet available:
    - $20 * x13 + 15 * x23 + 12 * x33 <= 5000$

3. Sales Demand Constraints:

There are sales forecasts indicating the maximum number of units that can be sold per day for each size of the product.

- Large units sales forecast:
    - $x11 + x12 + x13 <= 900$
- Medium units sales forecast:
    - $x21 + x22 + x23 <= 1200$
- Small units sales forecast:
    - $x31 + x32 + x33 <= 750$

4. Equal Percentage of Capacity Usage:

The plants must use the same percentage of their available production capacity to avoid layoffs. Let p represent the percentage of each plant’s capacity used. This implies:

- For Plant 1 (capacity 750 units):
    - $x11 + x21 + x31 = 750p$
- For Plant 2 (capacity 900 units):
    - $x12 + x22 + x32 = 900p$

- For Plant 3 (capacity 450 units):
  - x13 + x23 + x33 = 450p

5. Non-Negativity Constraints:

All decision variables must be non-negative:

xij >= 0 for all i, j

d. Full Mathematical Formulation:

Objective:

Maximize Z = 420(x11 + x12 + x13) + 360(x21 + x22 + x23) + 300(x31 + x32 + x33)

Subject to:

Production capacity constraints:

x11 + x21 + x31 <= 750

x12 + x22 + x32 <= 900

x13 + x23 + x33 <= 450

Storage space constraints:

20 * x11 + 15 * x21 + 12 * x31 <= 13000

20 * x12 + 15 * x22 + 12 * x32 <= 12000

20 * x13 + 15 * x23 + 12 * x33 <= 5000

Sales demand constraints:

x11 + x12 + x13 <= 900

x21 + x22 + x23 <= 1200

x31 + x32 + x33 <= 750

Equal capacity usage constraints:

x11 + x21 + x31 = 750p

x12 + x22 + x32 = 900p

x13 + x23 + x33 = 450p

Non-negativity constraints:

xij >= 0 for all i, j