

# Constant-delay Enumeration for Lorem Ipsum

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## Abstract

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**2012 ACM Subject Classification** Theory of computation → Database theory

**Keywords and phrases** Streams, query evaluation, enumeration algorithms.

## 1 Introduction

Write an introduction here.

## 2 Preliminaries

**Documents, alphabet, and spans.** We fix a finite alphabet  $\Sigma$ . A **document** is a finite string  $d = a_1 \dots a_n \in \Sigma^*$ . A **span** is a half-open interval  $[i, j)$  with  $1 \leq i \leq j \leq |d| + 1$ , it denotes the substring  $d[i, j) = a_i \dots a_{j-1}$ . The sets of all spans of  $d$  is written  $\text{Spans}(d)$ . (Fagin-Document Spanners...)

**Variables, markers and mappings.** Let  $V$  be a finite set of **variables**. For each  $x \in V$  we use two **variable markers**  $[x$  open and  $x)$  close;  $\text{Markers}_V = \{[x, x) | x \in V\}$ . (3361451). A **mapping** or valuation is a partial function.

$$\mu : \text{dom}(\mu) \subseteq V \rightarrow \text{Spans}(d).$$

Two mappings are **compatible** when they agree on their common variables. Mappings constitute the basic tuples produced by our operators.

**Document spanners.** A document spanner  $P$  associates to every string  $d$  a finite set of mappings over some variable set  $V = \text{SVars}(P)$ . (Fagin-Document Spanners...) Intuitively, a spanner "extracts" all matches of a pattern as span relations.

**Extended Variable Automata (eVA).** REmatch compiles each REQL query to an extended variable-set automaton

$$E = (Q, q_0, F, \delta),$$

whose transitions are quadruples  $(q, a, S, q')$  with  $a \in \Sigma \cup \{\#\}$  and a (possibly empty) set of markers  $S$ . While reading the  $i$ -th symbol, the automaton outputs the pair  $(S, i)$ ; if  $S = \emptyset$  nothing is produced. A run sequence

$$q_0 \xrightarrow{b_0/S_0} q_1 \xrightarrow{b_1/S_1} \dots \xrightarrow{b_n/S_n} q_{n+1}$$

that alternates variable transitions and letter transitions and respects marker nesting. The mapping defined by a valid accepting run is obtained by pairing every  $[x$  with the corresponding  $x)$ . Determinisation via a subset construction yields a deterministic eVA guaranteeing at

## 2 Constant-delay Enumeration for Lorem Ipsum

most one accepting run per output sequence, a key invariant for output-linear enumeration. (Fagin-Document Spanners...)

**Determinisation of extended VA.** an eVA  $E = (Q, q_0, F, \delta)$  is deterministic when

$$\delta : Q \times (\Sigma \cup (2^{Markers_V} \{\emptyset\})) \rightarrow Q$$

is a partial function: for every state  $q$  and pail  $(a, S)$  there is at most one outgoing transition  $(q, a, S, q')$ . Determinism guarantees that, for any document  $d$  and any output sequence, at most one accepting run produces it – a key property to avoid duplicates during enumeration.(paper). Every eVA can be turned into an equivalent deterministic eVA via the classical powerset method.

**Variable-inclusion order.** let  $V$  be a finite set of variables, for two mappings  $\mu, \nu$  over the same document we write

$$\mu \preceq_{varinc} \nu$$

If for all  $x \in V$ , if  $\mu(x)$  is defined, then  $\nu(x)$  is defined and  $\mu(x) = \nu(x)$ .

**MAX operator.** given a spanner  $P$  we define

$$\llbracket MAX(P) \rrbracket_d = \{\mu \in P(d) \mid \text{there is no } \nu \in P(d) \text{ with } \mu \preceq_{varinc} \nu\}$$

That is, we keep only those mappings that are maximal under  $\preceq_{varinc}$ . (GREZ)

### 3 Main results

**Deterministic eVA.** Let  $E = (Q, q_0, F, \delta)$  be an eVA. Its pair-based subset construction yields the deterministic eVA

$$E_{det} = (Q_{det}, X_0, F_{det}, \Delta),$$

with

$$\begin{aligned} Q_{det} &= 2^Q, \\ X_0 &= \{q_0\}, \\ F_{det} &= \{X \subseteq Q \mid X \cap F \neq \emptyset\}, \\ \Delta(X, a, S) &= \{q' \mid \exists q \in X : (q, a, S, q') \in \delta\}. \end{aligned}$$

**Selection strategy MAX.** Given a deterministic extended variable automaton (eVA)  $E_{det} = (2^Q, X_0, F_{det}, \Delta)$ , we define a new eVA

$$E_{max} = (Q_{max}, (R_0, W_0), F_{max}, \Delta_{max})$$

that accepts exactly the mappings that are maximal under variable inclusion, where  $Q_{max} = 2^Q \times 2^Q$  and

$$Q_{max} = \{(R, W) \mid R, W \subseteq Q_{det} \text{ and } R \cap W = \emptyset\}.$$

**Initial state.**  $(R_0, W_0) = (X_0, \emptyset)$ .

**Transition function.**  $R$  represents a set of "current" states of  $E_{det}$  and  $W$  represents the set of states of  $E_{det}$  having a run that dominates the current run under variable inclusion. For  $(R, W) \in Q_{max}$ , letter  $a \in \Sigma \cup \{\#\}$ , and marker set  $S$ :

$$\Delta_{\max}((R, W), a, S) = (R', W')$$

where

$$R' = \Delta(R, a, S) \setminus W' \quad \text{and} \quad W' = \begin{cases} \Delta(W, a, S) \cup \bigcup_{S \neq \emptyset} \Delta(R, a, S') & \text{if } S' = \emptyset, \\ \Delta(W, a, S) \cup \bigcup_{S \subset S'} \Delta(R, a, S') & \text{if } S \neq \emptyset. \end{cases}$$

**Final states.**

$$F_{\max} = \{ (R, W) \in Q_{\max} \mid R \cap F_{\det} \neq \emptyset \text{ and } W \cap F_{\det} = \emptyset \}.$$

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## 4 Conclusions

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References

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## A Proofs from Section 2

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## B Proofs of Section 3

### B.1 Proof of Lemma ??

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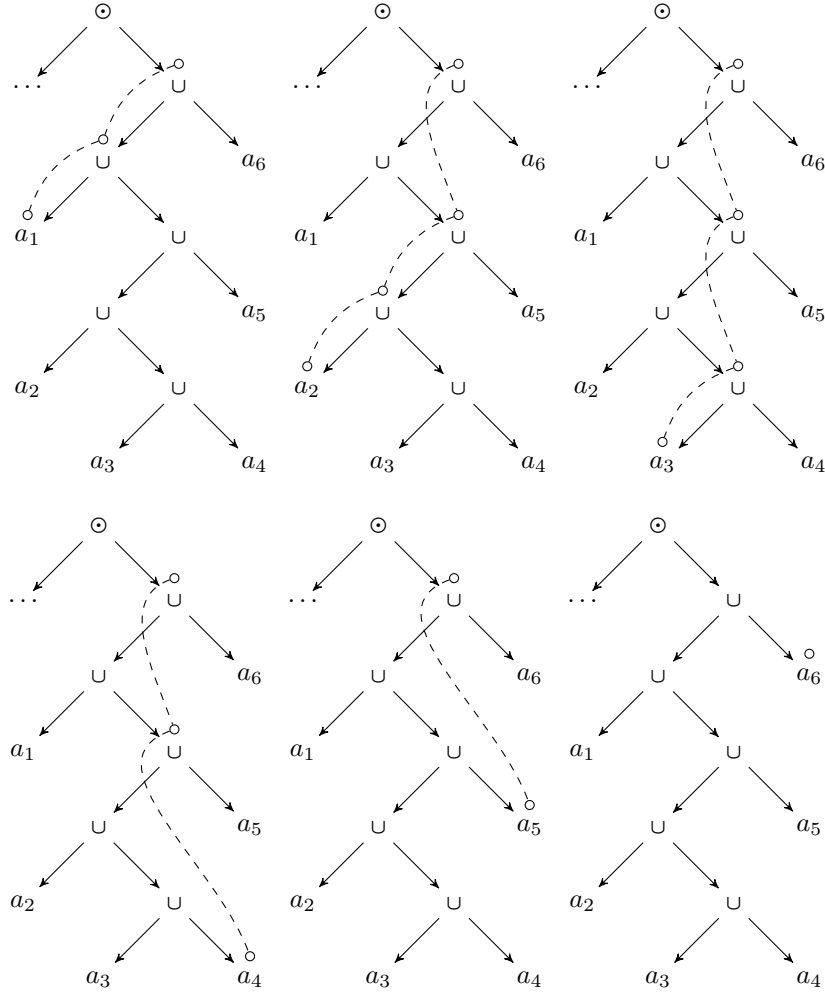
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### B.2 Proof of Theorem ??

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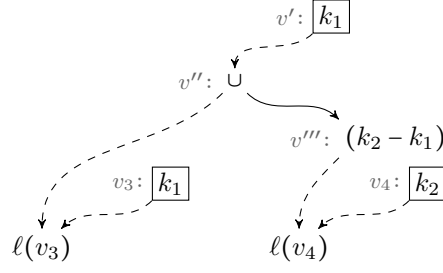


■ **Figure 1** An example iteration of `trav` and `move`. The sequences of nodes joined by dashed lines represent a stack `St`, where the first one was obtained after calling `trav` over the topmost union node, and the following five are obtained by repeated applications of `move(St)`.

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■ **Figure 2** Gadget used in Theorem ??.

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### 130 B.3 Proof of Proposition ??

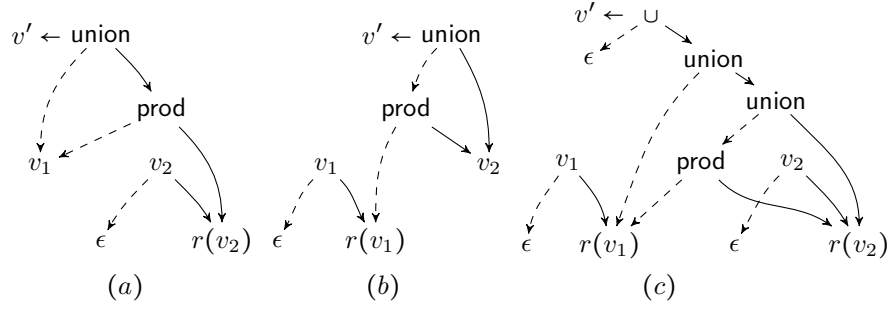
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136 ▷ **Claim 1.** Fix  $k \in \mathbb{N}$ . Let  $\mathcal{C}_k$  be the class of all duplicate-free and  $k$ -bounded  $D$  that satisfy  
 137 the  $\epsilon$  condition. Then one can solve the problem  $\text{Enum}[\mathcal{C}_k]$  with output-linear delay and  
 138 without preprocessing (i.e. constant preprocessing time).

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■ **Figure 3** Gadgets for product as defined for an  $\mathcal{D}$  with the  $\epsilon$ -node.

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