# Characterizing Assumption of Rationality by Incomplete Information

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#### **Abstract**

We characterize common assumption of rationality of 2-person games within an incomplete information framework. We use the lexicographic model with incomplete information and show that a belief hierarchy expresses common assumption of rationality within a complete information framework if and only if there is a belief hierarchy within the corresponding incomplete information framework that expresses common full belief in caution, rationality, every good choice is supported, and prior belief in the original utility functions.

Keywords: Epistemic game theory, Lexicographic belief, Assumption of rationality, Incomplete information

### 1. Introduction

Assumption of rationality is a concept in epistemic game theory introduced by Brandenburger et al. Brandenburger et al. (2008) and studied in Perea Perea (2012) by using lexicographic belief. A lexicographic belief is said to assume the opponents' rationality means that a "good" choice always occurs in front of a "bad" one. Here by good we mean a choice of the opponent can be supported by a cautious belief of him, that is, a belief that does not exclude any choice of the opponents; by bad we mean it cannot be supported by any such belief.

Like other concepts in epistemic game theory such as permissibility (Brandenburger Brandenburger (1992)) and proper rationalizability (Schuhmacher Schuhmacher (1999), Ascheim Asheim (2001)), iterative admissibility is defined partly to alleviate the tension between caution and rationality (Blume et al. Blume et al. (1991), Brandenburger Brandenburger (1992), Börgers Börgers (1994), Samuelson Samuelson (1992), Börgers and Samuelson Börgers and Samuelson (1992)) by sacrificing rationality. Caution requires that every choice, be it rational or not, should appear in a belief; assumption of rationality only requires that those rational choices should occur in front of those irrational ones but cannot exclude the irrational ones. On the other hand, since rationality is a basic assumption on human behavior in game theory, it seems desirable to find an approach to have a "complete" rationality while to keep the definition of iterative admissibility.

One approach is to use an incomplete information framework introduced by Perea and Roy Perea and Roy (2017) which defined standard probabilistic epistemic model with incomplete information and used it to characterized  $\varepsilon$ -proper rationalizability. Following their approach, Liu Liu (2017) defined lexicographic epistemic models with incomplete information, constructed a mapping between them and models with complete in-

formation, and characterized permissibility and proper rationalizability. In this paper, we still use the construction in Liu Liu (2017) and characterize assumption of rationality. We show that a choice is optimal for a belief hierarchy which expresses common assumption of rationality within a complete information framework if and only if it is optimal for a belief hierarchy within the corresponding incomplete information framework that expresses common full belief in caution, rationality, every good choice is supported, and prior belief in the original utility functions.

This paper is organized as follows. Section 2 gives a survey of assumption of rationality in epistemic models with complete information and the lexicographic epistemic models with incomplete information. Section 3 gives the characterization result and their proofs. Section 4 gives some concluding remarks on the relationship between the result of this paper and characterization of permissbility in Section 4.6 of Liu Liu (2017).

# 2. Models

# 2.1. Complete information model

In this subsection, we give a survey of lexicographic epistemic model with complete information and define iterative admissibility within it. We adopt the approach of Perea Perea (2012), Chapters 5 and 7. See Brandenburger et al. Brandenburger et al. (2008) for an alternative approach.

Consider a finite 2-person static game  $\Gamma = (C_i, u_i)_{i \in I}$  where  $I = \{1, 2\}$  is the set of players,  $C_i$  is the finite set of choices and  $u_i : C_1 \times C_2 \to \mathbb{R}$  is the utility function for player  $i \in I$ . In the following we sometimes denote  $C_1 \times C_2$  by C. We assume that each player has a lexicographic belief on the opponent's choices, a lexicographic belief on the opponent's lexicographic belief on her, and so on. This belief hierarchy is described by a lexicographic epistemic model with types.

Definition 2.1 (Epistemic model with complete information).

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Consider a finite 2-person static game  $\Gamma = (C_i, u_i)_{i \in I}$ . A finite lexicographic epistemic model for  $\Gamma$  is a tuple  $M^{co} = (T_i, b_i)_{i \in I}$  where

- (a)  $T_i$  is a finite set of types, and
- (b)  $b_i$  is a mapping that assigns to each  $t_i \in T_i$  a lexicographic belief over  $\Delta(C_j \times T_j)$ , i.e.,  $b_i(t_i) = (b_{i1}, b_{i2}, ..., b_{iK})$  where  $b_{ik} \in \Delta(C_j \times T_j)$  for k = 1, ..., K.

Consider  $t_i \in T_i$  with  $b_i(t_i) = (b_{i1}, b_{i2}, ..., b_{iK})$ . Each  $b_{ik}$  (k = 1, ..., K) is called  $t_i$ 's level-k belief. For each  $(c_j, t_j) \in C_j \times T_j$ , we say  $t_i$  deems  $(c_j, t_j)$  possible iff  $b_{ik}(c_j, t_j) > 0$  for some  $k \in \{1, ..., K\}$ . We say  $t_i$  deems  $t_j \in T_j$  possible iff  $t_i$  deems  $(c_j, t_j)$  possible for some  $c_j \in C_j$ . For each  $t_i \in T_i$ , we denote by  $T_j(t_i)$  the set of types in  $T_j$  deemed possible by  $t_i$ . A type  $t_i \in T_i$  is cautious iff for each  $c_j \in C_j$  and each  $t_j \in T_j(t_i)$ ,  $t_i$  deems  $(c_j, t_j)$  possible. That is,  $t_i$  takes into account each choice of player j for every belief hierarchy of j deemed possible by  $t_i$ .

For each  $c_i \in C_i$ , let  $u_i(c_i,t_i) = (u_i(c_i,b_{i1}),...,u_i(c_i,b_{iK}))$  where for each k = 1,...,K,  $u_i(c_i,b_{ik}) := \sum_{(c_j,t_j)\in C_j\times T_j}b_{ik}(c_j,t_j)u_i(c_i,c_j)$ , that is, each  $u_i(c_i,b_{ik})$  is the expected utility for  $c_i$  over  $b_{ik}$  and  $u_i(c_i,t_i)$  is a vector of expected utilities. For each  $c_i,c_i'\in C_i$ , we say that  $t_i$  prefers  $c_i$  to  $c_i'$ , denoted by  $u_i(c_i,t_i)>u_i(c_i',t_i)$ , iff there is  $k\in\{0,...,K-1\}$  such that the following two conditions are satisfied:

(a) 
$$u_i(c_i, b_{i\ell}) = u_i(c_i', b_{i\ell})$$
 for  $\ell = 0, ..., k$ , and

(b)  $u_i(c_i, b_{i,k+1}) > u_i(c'_i, b_{i,k+1}).$ 

We say that  $t_i$  is indifferent between  $c_i$  and  $c'_i$ , denoted by  $u_i(c_i, t_i) = u_i(c'_i, t_i)$ , iff  $u_i(c_i, b_{ik}) = u_i(c'_i, b_{ik})$  for each k = 1, ..., K. It can be seen that the preference relation on  $C_i$  under each type  $t_i$  is a linear order.  $c_i$  is rational (or optimal) for  $t_i$  iff  $t_i$  does not prefer any choice to  $c_i$ .

For  $(c_j, t_j)$ ,  $(c'_j, t'_j) \in C_j \times T_j$ , we say that  $t_i$  deems  $(c_j, t_j)$  infinitely more likely than  $(c'_j, t'_j)$  iff there is  $k \in \{0, ..., K-1\}$  such that the following two conditions are satisfied:

(a) 
$$b_{i\ell}(c_j, t_j) = b_{i\ell}(c'_i, t'_j) = 0$$
 for  $\ell = 1, ..., k$ , and

(b) 
$$b_{i,k+1}(c_j,t_j) > 0$$
 and  $b_{i,k+1}(c'_i,t'_i) = 0$ .

**Definition 2.2 (Assumption of rationality)** A cautious type  $t_i \in T_i$  assumes the j's rationality iff the following two conditions are satisfied:

- (A1) for all of player j's choices  $c_j$  that are optimal for some cautious belief,  $t_i$  deems possible some type  $t_j$  for which  $c_j$  is optimal;
- (A2)  $t_i$  deems all choice-type pairs  $(c_j, t_j)$  where  $t_j$  is cautious and  $c_j$  is optimal for  $t_j$  infinitely more likely than any choice-type pairs  $(c'_j, t'_j)$  that does not have this property.

Informally speaking, assumption of the opponent's rationality is that  $t_i$  puts all "good" choices in front of those "bad" choices. The following definition extends assumption of rationality inductively into n-fold for any  $n \in \mathbb{N}$ .

**Definition 2.3** (*n*-fold assumption of rationality) Consider a finite lexicographic epistemic model  $M^{co} = (T_i, b_i)_{i \in I}$  for a game  $\Gamma = (C_i, u_i)_{i \in I}$ . A cautious type  $t_i \in T_i$  expresses 1-fold assumption of rationality iff it assumes j's rationality. For any  $n \in \mathbb{N}$ , we say that a cautious type  $t_i \in T_i$  expresses (n + 1)-fold

assumption of rationality iff the following two conditions are satisfied:

- (nA1) whenever a choice  $c_j$  of player j is optimal for some cautious type (not necessarily in  $M^{co}$ ) that expresses up to n-fold assumption of rationality,  $t_i$  deems possible some cautious type  $t_j$  for player j which expresses up to n-fold assumption of rationality for which  $c_j$  is optimal;
- (**nA2**)  $t_i$  deems all choice-type pair  $(c_j, t_j)$ , where  $t_j$  is cautious and expresses up to n-fold assumption of rationality and  $c_j$  is optimal for  $t_j$ , infinitely more likely than any choice-type pairs  $(c'_i, t'_i)$  that does not satisfy this property.

We say that  $t_i$  expresses common assumption of rationality iff it expresses n-fold assumption of rationality for every  $n \in \mathbb{N}$ .

### 2.2. Incomplete information model

In this subsection, we give a survey of lexicographic epistemic model with incomplete information defined in Liu Liu (2017) which is the counterpart of the probabilistic epistemic model with incomplete information introduced by Battigalli Battigalli (2003) and further developed in Battigalli and Siniscalchi Battigalli and Siniscalchi (2003), Battigalli and Siniscalchi (2007), and Dekel and Siniscalchi Dekel and Siniscalchi (2015). We also define some conditions on types in such a model.

**Definition 2.4 (Lexicographic epistemic model with incomplete information)**. Consider a finite 2-person static game form  $G = (C_i)_{i \in I}$ . For each  $i \in I$ , let  $V_i$  be the set of utility functions  $v_i : C_1 \times C_2 \to \mathbb{R}$ . A finite lexicographic epistemic model for G with incomplete information is a tuple  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  where

- (a)  $\Theta_i$  is a finite set of types,
- (b)  $w_i$  is a mapping that assigns to each  $\theta_i \in \Theta_i$  a utility function  $w_i(\theta_i) \in V_i$ , and
- (c)  $\beta_i$  is a mapping that assigns to each  $\theta_i \in \Theta_i$  a lexicographic belief over  $\Delta(C_j \times \Theta_j)$ , i.e.,  $\beta_i(\theta_i) = (\beta_{i1}, \beta_{i2}, ..., \beta_{iK})$  where  $\beta_{ik} \in \Delta(C_j \times \Theta_j)$  for k = 1, ..., K.

Concepts such as " $\theta_i$  deems  $(c_j, \theta_j)$  possible" and " $\theta_i$  deems  $(c_j, \theta_j)$  infinitely more likely than  $(c'_j, \theta'_j)$ " can be defined in a similar way as in Section 2.1. For each  $\theta_i \in \Theta_i$ , we use  $\Theta_j(\theta_i)$  to denote the set of types in  $\Theta_j$  deemed possible by  $\theta_i$ . For each  $\theta_i \in \Theta_i$  and  $v_i \in V_i$ ,  $\theta_i^{v_i}$  is the auxiliary type satisfying that  $\beta_i(\theta_i^{v_i}) = \beta_i(\theta_i)$  and  $w_i(\theta_i^{v_i}) = v_i$ .

For each  $c_i \in C_i, v_i \in V_i$ , and  $\theta_i \in \Theta_i$  with  $\beta_i(\theta_i) = (\beta_{i1}, \beta_{i2}, ..., \beta_{iK})$ , let  $v_i(c_i, \theta_i) = (v_i(c_i, \beta_{i1}), ..., v_i(c_i, \beta_{iK}))$  where for each k = 1, ..., K,  $v_i(c_i, \beta_{ik}) := \sum_{(c_j, \theta_j) \in C_j \times \Theta_j} \beta_{ik}(c_j, \theta_j) v_i(c_i, c_j)$ . For each  $c_i, c_i' \in C_i$  and  $\theta_i \in \Theta_i$ , we say that  $\theta_i$  prefers  $c_i$  to  $c_i'$  iff  $w_i(\theta_i)(c_i, \theta_i) > w_i(\theta_i)(c_i', \theta_i)$ . As in Section 2.1, this is also the lexicographic comparison between two vectors.  $c_i$  is rational (or optimal) for  $\theta_i$  iff  $\theta_i$  does not prefer any choice to  $c_i$ .

**Definition 2.5 (Caution).**  $\theta_i \in \Theta_i$  is *cautious* iff for each  $c_j \in C_j$  and each  $\theta_j \in \Theta_j(\theta_i)$ , there is some utility function  $v_j \in V_j$  such that  $\theta_i$  deems  $(c_j, \theta_j^{v_j})$  possible.

This is a faithful translation of Perea and Roy Perea and Roy (2017)'s definition of caution in probabilistic model (p.312)

into lexicographic model. It is the counterpart of caution defined within the complete information framework in Section 2.1; the only difference is that in incomplete information models we allow different utility functions since  $c_j$  will be required to be rational for the paired type.

**Definition 2.6 (Belief in rationality).**  $\theta_i \in \Theta_i$  believes in j's rationality iff  $\theta_i$  deems  $(c_j, \theta_j)$  possible only if  $c_j$  is rational for  $\theta_j$ .

The following lemma shows that caution and a belief of full rationality can be satisfied simultaneously in an incomplete information model because each type is assigned with a belief on the opponent's choice-type pairs as well as a payoff function. The consistency of caution and full rationality is the essential difference between models with incomplete information and those with complete information.

**Lemma 2.1 (Belief in rationality can be satisfied).** Consider a static game form  $G = (C_i)_{i \in I}$ ,  $C'_i \in C_i$ , and  $\beta_i = (\beta_{i1}, \beta_{i2}, ..., \beta_{iK})$  such that  $\beta_{ik} \in \Delta(C_j)$  for each k = 1, ..., K. Then there is  $v_i \in V_i$  such that each  $c_i \in C'_i$  is optimal in  $v_i$  for  $\beta_i$ .

**Proof.** There are various way to construct such a  $v_i$ . Here we provide a simple one. For each  $c \in C$ , let

$$v_i(c) = \begin{cases} 1 \text{ if } c_i \in C_i' \text{ and } c_j \in \text{supp}\beta_{i1}; \\ 0 \text{ otherwise} \end{cases}$$

It can be seen that each  $c_i \in C'_i$  is optimal in  $v_i$  for  $\beta_i$ . //

Caution and belief in rationality can be extended into k-fold for any  $k \in \mathbb{N}$  as follows. Let P be an arbitrary property of lexicographic beliefs. We define that

(CP1)  $\theta_i \in \Theta$  expresses 0-fold full belief in P iff  $\theta_i$  satisfies P;

(CP2) For each  $n \in \mathbb{N}$  with  $n \ge 2$ ,  $\theta_i \in T_i$  expresses *n*-fold full belief in *P* iff  $\theta_i$  only deems possible *j*'s types that express *n*-fold full belief in *P*.

 $\theta_i$  expresses common full belief in P iff it expresses n-fold full belief in P for each  $n \in \mathbb{N}$ . By replacing P with "caution" or "rationality" we can obtain common full belief in caution or in rationality.

The following two conditions are important in characterizing assumption of rationality.

**Definition 2.7 (Every good choice is supported)**. Consider a static game form  $G = (C_i)_{i \in I}$ , a lexicographic epistemic model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  for G with incomplete information, and a pair  $u = (u_i)_{i \in I}$  of utility functions. A cautious type  $\theta_i \in \Theta_i$  believes in that every good choice of j is supported iff for each  $c_j$  that is optimal for some cautious type of j (may not be in  $M^{in}$ ) with  $u_j$  as its assigned utility function,  $\theta_i$  deems possible a cautious type  $\theta_j \in \Theta_j$  such that  $w_j(\theta_j) = u_j$  and  $c_j$  is optimal for  $\theta_j$ .

**Definition 2.8** (**Prior belief in** u). Consider a static game form  $G = (C_i)_{i \in I}$ , a lexicographic epistemic model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  for G with incomplete information, and a pair  $u = (u_i)_{i \in I}$  of utility functions.  $\theta_i \in \Theta_i$  priorly believes in u iff for any  $(c_j, \theta_j)$  with  $\theta_j$  cautious deemed possible by  $\theta_i$  satisfying that  $w_j(\theta_j) = u_j$ , then  $\theta_i$  deems  $(c_j, \theta_j)$  infinitely more likely than any pair does not satisfy that property.

Common full belief in that every good choice is supported and prior belief in u is different from that in caution or rationality. We have the following definition.

**Definition 2.9** (*n*-fold belief in that every good choice is supported and prior belief in *u*) Consider a static game form  $G = (C_i)_{i \in I}$ , a lexicographic epistemic model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  for G with incomplete information, and a pair  $u = (u_i)_{i \in I}$  of utility functions.  $\theta_i \in \Theta_i$  express 1-fold belief in that every good choice is supported and prior belief in u iff it believes that every good choice of j is supported and has prior belief in u. For any  $n \in \mathbb{N}$ , we say that a cautious type  $\theta_i \in \Theta_i$  expresses (n+1)-fold belief in prior belief in that every good choice is supported and prior belief in u iff the following two conditions are satisfied:

(**nP1**) whenever a choice  $c_j$  of player j is optimal for some cautious type (not necessarily in  $M^{in}$ ) with  $u_j$  as its assigned utility function that expresses up to n-fold belief in that every good choice is supported,  $\theta_i$  deems possible some cautious type  $\theta_j$  with  $w_j(\theta_j) = u_j$  for player j which expresses up to n-fold belief in that every good choice is supported and prior belief in u for which  $c_j$  is optimal.

(**nP2**)  $\theta_i$  deems all choice-type pair  $(c_j, \theta_j)$ , where  $\theta_j$  is cautious and expresses up to n-fold belief in that prior belief in u and every good choice is supported and satisfies  $w_j(\theta_j) = u_j$ , infinitely more likely than any choice-type pairs  $(c'_j, \theta'_j)$  that does not satisfy this property.

We say that  $t_i$  expresses common full belief in that every good choice is supported and prior belief in u iff it expresses n-fold belief in that every good choice is supported and prior belief in u for every  $n \in \mathbb{N}$ .

# 3. Characterization

So far we have introduced two different groups of concepts for static games: one includes assumption of rationality within a complete information framework, the other contains some conditions on types within an incomplete information framework. In this section we will show that there is correspondence between them.

Theorem 3.1 (Characterization of iterative admissibility). Consider a finite 2-person static game  $\Gamma = (C_i, u_i)_{i \in I}$  and the corresponding game form  $G = (C_i)$ .  $c_i^* \in C_i$  is optimal to some type expressing common full belief in caution and common assumption of rationality within some finite epistemic model with complete information if and only if there is some finite epistemic model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  with incomplete information for G and some  $\theta_i^* \in \Theta_i$  with  $w_i(\theta_i^*) = u_i$  such that

(a)  $c_i^*$  is rational for  $\theta_i^*$ , and

(b)  $\theta_i^*$  expresses common full belief in caution, rationality, that every good choice is supported, and prior belief in u.

To show Theorem 3.1, we construct the mappings between finite lexicographic epistemic models with complete information and those with incomplete information. First, consider  $\Gamma = (C_i, u_i)_{i \in I}$  and a finite lexicographic epistemic model  $M^{co} = (T_i, b_i)_{i \in I}$  with complete information for  $\Gamma$ . We first define types in a model with incomplete information in the following two

steps:

**Step 1.** For each  $i \in I$  and  $t_i \in T_i$ , let  $\Pi_i(t_i) = (C_{i1}, ..., C_{iL})$ be the partition of  $C_i$  defined in Lemma 2.1, that is,  $\Pi_i(t_i)$  is the sequence of equivalence classes of choices in  $C_i$  arranged from the most preferred to the least preferred under  $t_i$ . We define  $v_{i\ell}(t_i) \in V_i$  for each  $\ell = 1, ..., L$ . We let  $v_{i1}(t_i) = u_i$ . By Lemma 2.1, for each  $C_{i\ell}$  with  $\ell > 1$  there is some  $v_{i\ell}(t_i) \in V_i$ such that each choice in  $C_{i\ell}$  is rational at  $v_{i\ell}(t_i)$  under  $t_i$ .

**Step 2.** We define  $\Theta_i(t_i) = \{\theta_{i1}(t_i), ..., \theta_{iL}(t_i)\}$  where for each  $\ell = 1, ..., L$ , the type  $\theta_{i\ell}(t_i)$  satisfies that (1)  $w_i(\theta_{i\ell}(t_i)) = v_{i\ell}(t_i)$ , and (2)  $\beta_i(\theta_{i\ell}(t_i))$  is obtained from  $b_i(t_i)$  by replacing every  $(c_i, t_i)$  with  $c_i \in C_{ir} \in \Pi_i(t_i)$  for some r with  $(c_i, \theta_i)$  where  $\theta_i = \theta_{ir}(t_i)$ , that is,  $w_i(\theta_i)$  is the utility function among those corresponding to  $\Pi_i(t_i)$  in which  $c_i$  is the rational for  $t_i$ .

For each  $i \in I$ , let  $\Theta_i = \bigcup_{t_i \in T_i} \Theta_i(t_i)$ . Here we have constructed a finite lexicographic epistemic model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  for the corresponding game form  $G = (C_i)_{i \in I}$  with incomplete information. In the following example we show how this construction goes.

**Example 3.2.** Consider the following game  $\Gamma$  (Perea Perea (2012), p.188):

$u_1 \setminus u_2$	C	D
A	1,0	0, 1
В	0,0	0, 1

and the lexicographic epistemic model  $M^{co} = (T_i, b_i)_{i \in I} \Gamma$  where  $T_1 = \{t_1\}, T_2 = \{t_2\}, \text{ and }$ 

$$b_1(t_1) = ((D, t_2), (C, t_2)), b_2(t_2) = ((A, t_1), (B, t_1)).$$

We show how to construct a corresponding model  $M^{in}$  =  $(\Theta_i, w_i, \beta_i)_{i \in I}$ . First, by Step 1 it can be seen that  $\Pi_1(t_1) =$  $({A}, {B})$  and  $\Pi_2(t_2) = ({D}, {C})$ . We let  $v_{11}(t_1) = u_1$  where A is rational for  $t_1$  and  $v_{12}(t_1)$  where B is rational for  $t_1$  as follows. Similarly, we let  $v_{21}(t_2) = u_2$  where D is rational under  $t_2$ and  $v_{22}(t_2)$  where C is rational under  $t_2$  as follows:

$v_{12}(t_1)$	C	D
A	1	0
В	0	1

$v_{22}(t_2)$	С	D
A	2	1
В	0	1

Then we go to Step 2. It can be seen that  $\Theta_1(t_1) =$  $\{\theta_{11}(t_1), \theta_{12}(t_1)\}\$ , where

$$w_1(\theta_{11}(t_1)) = v_{11}(t_1), \ \beta_1(\theta_{11}(t_1)) = ((D, \theta_{21}(t_2)), (C, \theta_{22}(t_2)))$$
  
 $w_1(\theta_{12}(t_1)) = v_{12}(t_1), \ \beta_1(\theta_{12}(t_1)) = ((D, \theta_{21}(t_2)), (C, \theta_{22}(t_2)))$ 

Also,  $\Theta_2(t_2) = \{\theta_{21}(t_2), \theta_{22}(t_2)\}\$ , where

$$w_2(\theta_{21}(t_2)) = v_{21}(t_2), \ \beta_2(\theta_{21}(t_2)) = ((A, \theta_{11}(t_1)), (B, \theta_{12}(t_1)))$$
  
 $w_2(\theta_{22}(t_2)) = v_{22}(t_2), \ \beta_2(\theta_{22}(t_2)) = ((A, \theta_{11}(t_1)), (B, \theta_{12}(t_1)))$ 

Let  $M^{co} = (T_i, b_i)_{i \in I}$  and  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  be constructed from  $M^{co}$  by the two steps above. We have the following obser-

**Observation 3.1 (Redundancy).** For each  $t_i \in T_i$  and each  $\theta_i, \theta_i' \in \Theta_i(t_i), \beta_i(\theta_i) = \beta_i(\theta_i').$ 

**Observation 3.2 (Rationality).** Eeach  $\theta_i \in \Theta_i(t_i)$  believes in j's rationality.

We omit their proofs since they hold by construction. Observation 3.1 means that the difference between any two types in a  $\Theta_i(t_i)$  is in the utility functions assigned to them. Observation 3.2 means that in an incomplete information model constructed from one with complete information, each type has (full) belief in the opponent's rationality. This is because in the construction, we requires that for each pair  $(c_i, t_i)$  occurring in a belief, its counterpart in the incomplete information replaces  $t_i$  by the type in  $\Theta_i(t_i)$  with the utility function in which  $c_i$  is optimal for  $b_i(t_i)$ . It follows from Observation 3.2 that each  $\theta_i \in \Theta_i(t_i)$ expresses common full belief in rationality.

The following lemma shows that caution is preserved in this construction.

**Lemma 3.1** (Caution<sup>co</sup>  $\rightarrow$  Caution<sup>in</sup>). Let  $M^{co} = (T_i, b_i)_{i \in I}$ and  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  be constructed from  $M^{co}$  by the two steps above. If  $t_i \in T_i$  expresses common full belief in caution, so does each  $\theta_i \in \Theta_i(t_i)$ .

**Proof.** We show this statement by induction. First we show that if  $t_i$  is cautious, then each  $\theta_i \in \Theta_i(t_i)$  is also cautious. Let  $c_i \in C_i$  and  $\theta_i \in \Theta_i(\theta_i)$ . By construction, it can be seen that the type  $t_i \in T_i$  satisfying the condition that  $\theta_i \in \Theta_i(t_i)$  is in  $T_i(t_i)$ . Since  $t_i$  is cautious,  $t_i$  deems  $(c_j, t_j)$  possible. Consider the pair  $(c_i, \theta'_i)$  in  $\beta_i(\theta_i)$  corresponding to  $(c_i, t_i)$ . Since both  $\theta_i$  and  $\theta'_i$  are in  $\Theta_i(t_i)$ , it follows from Observation 3.1 that  $\beta_i(\theta_i) = \beta_i(\theta_i')$ . Hence  $(c_j, \theta_i^{w_j(\theta_j')})$  is deemed possible by  $\theta_i$ . Here we have shown that  $\theta_i$  is cautious.

Suppose we have shown that, for each  $i \in I$ , if  $t_i$  expresses *n*-fold full belief in caution then so does each  $\theta_i \in \Theta_i(t_i)$ . Now suppose that  $t_i$  expresses (n + 1)-fold full belief in caution, i.e., each  $t_i \in T_i(t_i)$  expresses *n*-fold full belief in caution. By construction, for each  $\theta_i \in \Theta_i(t_i)$  and each  $\theta_i \in \Theta_i(\theta_i)$  there is some  $t_i \in T_i(t_i)$  such that  $\theta_i \in \Theta_i(t_i)$ , and, by inductive assumption, each  $\theta_i \in \Theta_i(\theta_i)$  expresses *n*-fold full belief in caution. Therefore, each  $\theta_i \in \Theta_i(t_i)$  expresses (n+1)-fold full belief in caution.

We also need a mapping from epistemic models with incomplete information to those with complete information. Consider a finite 2-person static game  $\Gamma = (C_i, u_i)_{i \in I}$ , the corresponding game form  $G = (C_i)_{i \in I}$ , and a finite epistemic model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  for G with incomplete information. We  $w_1(\theta_{11}(t_1)) = v_{11}(t_1), \ \beta_1(\theta_{11}(t_1)) = ((D, \theta_{21}(t_2)), (C, \theta_{22}(t_2)))$  construct a model  $M^{co} = (T_i, b_i)_{i \in I}$  for  $\Gamma$  with complete information  $w_1(\theta_{12}(t_1)) = v_{12}(t_1), \ \beta_1(\theta_{12}(t_1)) = ((D, \theta_{21}(t_2)), (C, \theta_{22}(t_2)))$  ion as follows. For each  $\theta_i \in \Theta_i$ , we define  $E_i(\theta_i) = \{\theta_i' \in \Theta_i: \theta_i \in \Theta_i\}$  $\beta_i(\theta_i') = \beta(\theta_i)$ . In this way  $\Theta_i$  is partitioned into some equivalence classes  $\mathbb{E}_i = \{E_{i1}, ..., E_{iL}\}$  where for each  $\ell = 1, ..., L$ ,  $E_{i\ell} = E_i(\theta_i)$  for some  $\theta_i \in \Theta_i$ . To each  $E_i \in \mathbb{E}_i$  we use  $t_i(E_i)$  $w_2(\theta_{21}(t_2)) = v_{21}(t_2), \ \beta_2(\theta_{21}(t_2)) = ((A, \theta_{11}(t_1)), (B, \theta_{12}(t_1)))_{to \text{ represent a type.}}$  We define  $b_i(t_i(E_i))$  to be a lexicographic  $w_2(\theta_{22}(t_2)) = v_{22}(t_2), \beta_2(\theta_{22}(t_2)) = ((A, \theta_{11}(t_1)), (B, \theta_{12}(t_1)))$  belief which is obtained from  $\beta_i(\theta_i)$  by replacing each occurrence of  $(c_i, \theta_i)$  by  $(c_i, t_i(E_i(\theta_i)))$ ; in other words,  $b_i(t_i(E_i))$  has the same distribution on choices at each level as  $\beta_i(\theta_i)$  for each  $\theta_i \in E_i$ , while each  $\theta_j \in \Theta_j(\theta_i)$  is replaced by  $t_j(E_j(\theta_j))$ . For each  $i \in I$ , let  $T_i = \{t_i(E_i)\}_{E_i \in \mathbb{E}_i}$ . We have constructed from  $M^{in}$ a finite epistemic model  $M^{co} = (T_i, b_i)_{i \in I}$  with complete information for  $\Gamma$ .

It can be seen that this is the reversion of the previous construction. That is, let  $M^{co} = (T_i, b_i)_{i \in I}$  satisfying that  $b_i(t_i) \neq b_i(t_i')$  for each  $t_i, t_i' \in T_i$  with  $t_i \neq t_i'$ , and  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  be constructed from  $M^{co}$  by the previous two steps. Then  $\mathbb{E}_i = \{\Theta_i(t_i)\}_{t_i \in T_i}$  and  $t_i(\Theta_i(t_i)) = t_i$  for each  $i \in I$ .

In the following example we show how this construction goes.

**Example 3.3.** Consider the game  $\Gamma$  in Example 3.2 and the model  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  for the corresponding game form where  $\Theta_1 = \{\theta_{11}, \theta_{12}\}, \Theta_2 = \{\theta_{21}, \theta_{22}\}$ , and

$$w_1(\theta_{11}) = u_1, \, \beta_1(\theta_{11}) = ((D, \theta_{21}), (C, \theta_{22})),$$

$$w_1(\theta_{12}) = v_1, \, \beta_1(\theta_{12}) = ((D, \theta_{21}), (C, \theta_{22})),$$

$$w_2(\theta_{21}) = u_2, \, \beta_2(\theta_{21}) = ((A, \theta_{11}), (B, \theta_{12})),$$

$$w_2(\theta_{22}) = v_2, \, \beta_2(\theta_{22}) = ((A, \theta_{11}), (B, \theta_{12})).$$

where  $v_1 = v_{12}(t_1)$  and  $v_2 = v_{22}(t_2)$  in Example 3.2. It can be seen that  $\mathbb{E}_1 = \{\{\theta_{11}, \theta_{12}\}\}$  since  $\beta_1(\theta_{11}) = \beta_1(\theta_{12})$  and  $\mathbb{E}_2 = \{\{\theta_{21}, \theta_{22}\}\}$  since  $\beta_2(\theta_{21}) = \beta_2(\theta_{22})$ . Corresponding to those equivalence classes we have  $t_1(\{\theta_{11}, \theta_{12}\})$  and  $t_2(\{\theta_{21}, \theta_{22}\})$ , and

$$b_1(t_1(\{\theta_{11}, \theta_{12}\})) = ((D, t_2(\{\theta_{21}, \theta_{22}\})), (C, t_2(\{\theta_{21}, \theta_{22}\}))), b_2(t_2(\{\theta_{21}, \theta_{22}\})) = ((A, t_1(\{\theta_{11}, \theta_{12}\})), (B, t_1(\{\theta_{11}, \theta_{12}\}))).$$

We have the following lemmas.

**Lemma 3.2** (Caution<sup>in</sup>  $\rightarrow$  Caution<sup>co</sup>). Let  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  and  $M^{co} = (T_i, b_i)_{i \in I}$  be constructed from  $M^{in}$  by the above approach. If  $\theta_i \in \Theta_i$  expresses common full belief in caution, so does  $t_i(E_i(\theta_i))$ .

**Proof.** We show this statement by induction. First we show that if  $\theta_i$  is cautious, then  $t_i(E_i(\theta_i))$  is also cautious. Let  $c_j \in C_j$  and  $t_j \in T_j(t_i(E_i(\theta_i)))$ . By construction,  $t_j = t_j(E_j)$  for some  $E_j \in \mathbb{E}_j$ , and there is some  $\theta_j \in E_j$  which is deemed possible by  $\theta_i$ . Since  $\theta_i$  is cautious, there is some  $\theta'_j$  with  $\beta_j(\theta'_j) = \beta_j(\theta_j)$ , i.e.,  $\theta'_j \in E_j$ , such that  $(c_j, \theta'_j)$  is deemed possible by  $\theta_i$ . By construction,  $(c_j, t_j)$  is deemed possible by  $t_i(E_i(\theta_i))$ .

Suppose we have shown that, for each  $i \in I$ , if  $\theta_i$  expresses n-fold full belief in caution then so does  $t_i(E_i(\theta_i))$ . Now suppose that  $\theta_i$  expresses (n+1)-fold full belief in caution, i.e., each  $\theta_j \in \Theta_j(\theta_i)$  expresses n-fold full belief in caution. Since, by construction, for each  $t_j \in T_j(t_i(E_i(\theta_i)))$ , there is some  $\theta_j \in \Theta_j(\theta_i)$  such that  $t_j = t_j(E_j(\theta_j))$ , by inductive assumption  $t_j$  expresses n-fold full belief in caution. Therefore,  $t_i(E_i(\theta_i))$  expresses (n+1)-fold full belief in caution. //

Lemma 3.3 (Assumption of rationality  $\longleftrightarrow$  every good choice is supported prior + belief in u). Let  $M^{co} = (T_i, b_i)_{i \in I}$  and  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  be constructed from  $M^{co}$ . If  $t_i \in T_i$  expresses common assumption of rationality, then each  $\theta_i \in \Theta_i(t_i)$  expresses common full belief in that every good choice is supported and prior belief in u.

On the other hand, let  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  and  $M^{co} = (T_i, b_i)_{i \in I}$  be constructed from  $M^{in}$ . If  $\theta_i \in \Theta_i$  expresses common full belief in that every good choice is supported and prior belief in u, then  $t_i(E_i(\theta_i))$  expresses common full assumption of rationality.

**Proof.** We show this statement by induction. Let  $\theta_i \in \Theta_i(t_i)$ . First we show that if  $t_i$  assumes in j's rationality,  $\theta_i$  believes that every good choice is supported and prior belief in u. Let  $c_j \in C_j$  be optimal for some cautious type of j whose assigned utility function is  $u_j$  within an epistemic model with incomplete information. It is easy to see that  $c_j$  is optimal for its corresponding type, which is also cautious by Lemma 3.2, in any complete information model constructed from the one with incomplete information by our approach above. Since  $t_i$  assumes j's rationality,  $t_i$  deems possible a cautious type  $t_j$  for which  $c_j$  is optimal. By construction, some  $\theta_j \in \Theta_j(t_j)$  is deemed possible by  $\theta_i$ . Since  $t_i$  is cautious,  $(c_j, t_j)$  is deemed possible by  $t_i$ , and, by construction  $(c_j, \theta_{j1}(t_j))$  is deemed possible by  $\theta_i$ . Since  $w_j(\theta_{j1}(t_j)) = u_j$  and  $c_j$  is optimal for  $\theta_{j1}(t_j)$ , it follows that  $\theta_i$  believes in that every good choice is supported.

Let  $(c_j, \theta_j)$  with  $\theta_j$  cautious deemed possible by  $\theta_i$  satisfying  $w_j(\theta_j) = u_j$  and  $(c'_j, \theta'_j)$  a pair which does not satisfy that condition. Let  $(c_j, t_j)$  and  $(c'_j, t'_j)$  be the pairs occuring in the belief of  $t_i$  corresponding to  $(c_j, \theta_j)$  and  $(c'_j, \theta'_j)$ . Since  $c_j$  is rational to  $\theta_j$  and  $w_j(\theta_j) = u_j$ , it follows that  $c_j$  is optimal for  $t_j$ . On the other hand,  $c'_j$  is not optimal for  $t'_j$ . Since  $t_i$  assumes j's rationality,  $t_i$  deems  $(c_j, t_j)$  infinitely more likely than  $(c'_j, t'_j)$ . By construction,  $\theta_i$  deems  $(c_j, \theta_j)$  infinitely more likely than  $(c'_j, \theta'_j)$ . Here we have shown that  $\theta_i$  priorly believes in u.

Now we show the other direction: suppose that if  $\theta_i \in \Theta_i$  believes in that every good choice is supported and priorly believes in u, we prove that  $t_i(E_i(\theta_i))$  assumes j's rationality. Suppose that  $c_j$  is optimal for some cautious type within some epistemic model with complete information. It can be seen by construction that  $c_j$  is optimal for some cautious type with  $u_i$  as its assigned utility function within some epistemic model with incomplete information which corresponds to that complete information model. Since  $\theta_i$  believes in that every good choice is supported,  $\theta_i$  deems possible a cautious type  $\theta_j$  such that  $w_j(\theta_j) = u_j$  and  $c_j$  is optimal for  $\theta_j$ . By construction it follows that  $t_i(E_i(\theta_i))$  deems  $t_j(E_j(\theta_j))$  possible for which  $c_j$  is optimal.

Let  $(c_j, t_j)$  with  $t_j$  cautious be a pair which is deemed possible by  $t_i(E_i(\theta_i))$  satisfying that  $c_j$  is optimal for  $t_j$ , and  $(c'_j, t'_j)$  be a pair deemed possible by  $t_i(E_i(\theta_i))$  which does not satisfy that condition. Let  $(c_j, \theta_j)$  and  $(c'_j, \theta'_j)$  be the corresponding pairs occuring in the belief of  $\theta_i$ . Since  $\theta_i$  believes in rationality, by construction it follows that  $u_j(\theta_j) = u_j$  while  $u_j(\theta'_j) \neq u_j$ . Since  $\theta_i$  priorly believes in u,  $\theta_i$  deems  $(c_j, \theta_j)$  infinitely more likely than  $(c'_j, \theta'_j)$ . It follows that  $t_i(E_i(\theta_i))$  deems  $(c_j, t_j)$  infinitely more likely than  $(c'_j, t'_j)$ . Here we have shown that  $t_i(E_i(\theta_i))$  assumes j's rationality.

Suppose that, for some  $n \in \mathbb{N}$ , we have shown that for each  $k \le n$ ,

(n1) if  $t_i \in T_i$  expresses k-fold assumption of rationality, then each  $\theta_i \in \Theta_i(t_i)$  expresses k-fold full belief in that every good choice is supported and prior belief in u;

(n2) If  $\theta_i \in \Theta_i$  expresses k-fold full belief in that every good choice is supported and prior belief in u, then  $t_i(E_i(\theta_i))$  expresses k-fold assumption of rationality.

Now we show that these two statements hold for n + 1. First, suppose that  $t_i \in T_i$  expresses (n + 1)-fold assumption of ratio-

nality. Let  $c_j \in C_j$  be a choice of j optimal for some cautious type whose assigned utility function is  $u_j$  that expresses up to n-fold belief in that every good choice is supported. Then it is easy to see that (1) by inductive assumption, in the constructed complete information model the corresponding type expresses n-fold assumption of rationality, and (2)  $c_j$  is optimal for that type. Since  $t_i$  expresses (n+1)-fold assumption of rationality,  $t_i$  deems possible a cautious type  $t_j$  that expresses up to n-fold assumption of rationality and for which  $c_j$  is optimal. By construction, it follows that  $\theta_i$  deems possible some  $\theta_j \in \Theta_j(t_j)$ . By inductive assumption it follows that each  $\theta_j \in \Theta_j(t_j)$  expresses n-fold belief in that every good choice is supported. Since  $\theta_i$  expresses common belief in caution and rationality it follows that  $\theta_i$  deems  $(c_j, \theta_{j1})$  for  $\theta_{j1} \in \Theta_j(t_j)$  (that is,  $w_j(\theta_{j1}) = u_j$ ).

Let  $(c_j, \theta_j)$  with  $\theta_j$  cautious deemed possible by  $\theta_i$  satisfying that  $\theta_j$  expresses up tp n-fold belief in prior belief in u and that every good choice is supported and  $w_j(\theta_j) = u_j$  and  $(c'_j, \theta'_j)$  a pair which does not satisfy those conditions. Let  $(c_j, t_j)$  and  $(c'_j, t'_j)$  be the pairs occuring in the belief of  $t_i$  corresponding to  $(c_j, \theta_j)$  and  $(c'_j, \theta'_j)$ . Since  $c_j$  is rational for  $\theta_j$  and  $w_j(\theta_j) = u_j$ , it follows that  $c_j$  is optimal for  $t_j$ . Also, by inductive assumption, it follows that  $t_j$  expresses up to n-fold assumption of rationality. On the other hand, it can be seen that  $(c'_j, t'_j)$  does not satisfy these conditions. Since  $t_i$  expresses (n+1)-fold of assumptions of rationality,  $t_i$  deems  $(c_j, t_j)$  infinitely more likely than  $(c'_j, t'_j)$ . By construction,  $\theta_i$  deems  $(c_j, \theta_j)$  infinitely more likely than  $(c'_j, \theta'_j)$ . Here we have shown that  $\theta_i$  expresses (n+1)-fold full belief in that every good choice is supported and prior belief in u.

Now suppose that  $\theta_i \in \Theta_i$  expresses (n+1)-fold full belief in that every good choice is supported and prior belief in u. Let  $c_j \in C_j$  be a choice of j optimal for some cautious type that expresses to n-fold assumption of rationality. By inductive assumption it follows that the corresponding type within some incomplete information model also expresses n-fold full belief in that every good choice is supported and prior belief in u. It can be seen that  $c_j$  is optimal to the constructed type having  $u_j$  as its utility functionand the type expresses up to n-fold full belief in that every good choice is supported and prior belief in u. Then  $\theta_i$  deems possible a type  $\theta_j$  with  $w_j(\theta_j) = u_j$  for player j which expresses up to n-fold belief in that every good choice is supported for which  $c_j$  is optimal. By inductive assumption it follows that  $t_i(E_i(\theta_i))$  deems possible  $t_j(E_j(\theta_j))$  which expresses n-fold assumption of rationality and for which  $c_j$  is optimal.

Let  $(c_j, t_j)$  be a pair with  $t_j$  cautious deemed possible by  $t_i(E_i(\theta_i))$  where  $t_j$  expresses up to n-fold assumption of rationality and  $c_j$  is optimal for  $t_j$ , and let  $(c'_j, t'_j)$  be a pair that does not satisfy this property. Let  $(c_j, \theta_j)$  and  $(c'_j, \theta'_j)$  be the corresponding pairs occurring in the belief of  $\theta_i$ . By inductive assumption and by construction,  $\theta_j$  is cautious and expresses up to n-fold belief in that prior belief in u and every good choice is supported and  $w_j(\theta_j) = u_j$ , while  $(c'_j, \theta'_j)$  does not satisfy this property. Therefore  $\theta_i$  deems  $(c_j, \theta_j)$  infinitely more likely than  $(c'_j, \theta'_j)$ , which implies that  $t_i(E_i(\theta_i))$  deems  $(c_j, t_j)$  infinitely more likely than  $(c'_j, t'_j)$ . Here we have shown that  $t_i(E_i(\theta_i))$  expresses (n+1)-fold assumption of rationality. //

**Proof of Theorem 3.1.** (**Only-if**) Let  $M^{co} = (T_i, b_i)_{i \in I}$ ,  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$  be constructed from  $M^{co}$  by the two steps above,  $c_i^* \in C_i$  be a permissible choice, and  $t_i^* \in T_i$  be a type expressing common full belief in caution and common assumption of rationality such that  $c_i^*$  is rational for  $t_i^*$ . Let  $\theta_i^* = \theta_{i1}(t_i^*)$ . By definition,  $w_i(\theta_i^*) = u_i$  and  $\beta_i(\theta_i^*)$  has the same distribution on j's choices at each level as  $b_i(t_i^*)$ . Hence  $c_i^*$  is rational for  $\theta_i^*$ . Also, it follows from Observation 3.2, Lemmas 3.1, and 3.3 that  $\theta_i^*$  expresses common full belief in caution, rationality, that a good choice is supported, and prior belief in u.

(If). Let  $M^{in} = (\Theta_i, w_i, \beta_i)_{i \in I}$ ,  $M^{co} = (T_i, b_i)_{i \in I}$  be constructed from  $M^{in}$  by the above approach, and  $c_i^* \in C_i$  be rational for some  $\theta_i^*$  with  $w_i(\theta_i^*) = u_i$  which expresses common full belief in caution, rationality, that a good choice is supported, and prior belief in u. Consider  $t_i(E_i(\theta_i^*))$ . Since  $w_i(\theta_i^*) = u_i$  and  $b_i(t_i(E_i(\theta_i^*)))$  has the same distribution on j's choices at each level as  $\beta_i(\theta_i^*)$ ,  $c_i^*$  is rational for  $t_i(E_i(\theta_i^*))$ . Also, by Lemmas 3.2 and 3.3,  $t_i(E_i(\theta_i^*))$  expresses common full belief in caution and common assumption of rationality.  $f(x_i)$ 

## 4. Concluding Remarks

Assumption of rationality is a refinement of permissibility (See Perea Perea (2012)). This can also be seen within the framework of incomplete information. Comparing our characterization of the former of the characterization of the latter in Section 4.6 in Liu Liu (2017) it can be seen that there is correspondence between the conditions. Section 4.6 in Liu Liu (2017) characterizes permissibility by weak caution, rationality, and primary belief in u within the incomplete information framework. The characterization of assumption of rationality shares rationality with it, while caution and prior belief are stronger than weak caution and primary belief in u, respectively.

An interesting phenomenon is the role of rationality. Liu Liu (2017) provides two ways to characterize permissibility, one with rationality and one without it. The characterization of proper rationality there is a stronger version of the latter, while the characterization in this paper a stronger version of the former. So far, it seems that using or not using rationality in the characterization differentiate the two refinements of permissibility, that is, assumption of rationality and proper rationalizability, within the incomplete information framework. It would be interesting that any future research would confirm this statement or provide any counterexample, that is, show that proper rationalizability can be characterized with rationality while assumption of rationality can be done without it.

On the other hand, as shown in Liu Liu (2017) (and the construction here), it is always possible to construct epistemic models with incomplete information which satisfies rationality as well as all conditions for characterization of proper rationalizability. Further, prior belief in u is voguely a condition between primary belief in u and u-centered belief which is used in Theorem 3.2 of Liu Liu (2017) to characterize proper rationalizability. Those seem to correspond to the fact within the complete information framework that there is always possible to construct belief hierarchy which both assumes the opponent's rationality and respects the opponent's preferences.

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