Convolutional and Physics-Informed Neural Networks for Structural Mechanics Problems

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Background

Structural Safety

Hidden flaws such as cracks and wall loss corrosion affect structural integrity

- Failure to detect and characterize $\implies$ **catastrophic failure**
- Environmental contamination, economic impacts and death

Two key **failure mechanisms** in large structures:

- Crack-driven fracture
- Plasticity-based collapse

Figure 1. Embedded flaws and improper loading can result in catastrophic failure. Credit: EVstudio and Insider.

Non-Destructive Testing (NDT)

Evaluation of structures without damaging the test object.

The use of ultrasounds allows for the imaging and detection of embedded features.

- **A-scan** = Time–amplitude ultrasound signal
- **Efficiently scan** large structures, but requires human interpretation

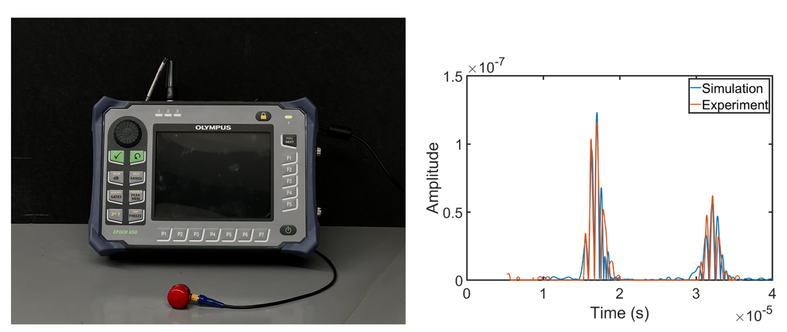
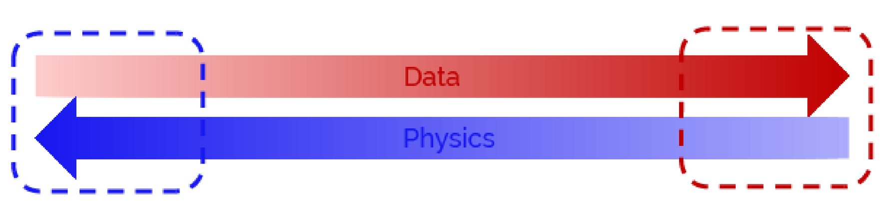


Figure 2. (Left) Olympus Epoch 650 Ultrasound NDT Flaw Detector with straight-beam and single element transducer. (Right) Experimental and simulation ultrasound signals for a 12.7 mm thick smooth sheet of HDPE [1].

Application of Machine Learning to Structural Mechanics

Machine learning can **reduce** costs and **increase** accuracy for solving problems within structural mechanics.

- Finite-element trained convolutional neural network (CNN) for accurate hidden flaw quantification [1, 2, 3, 4]
- Physics-informed neural network (PINN) for boundary value finite strain plasticity problems [5]



Finite-strain Elasto-plasticity Constitutive Model

- Kröner-Lee Decomposition: $\mathbf{F} = \frac{\delta \mathbf{x}}{\delta \mathbf{X}} = \mathbf{F}^{\mathbf{e}}\mathbf{F}^{p}$
- Plastic distortion and stretching: $\mathbf{L}^{p} = \dot{\mathbf{F}}^{p}\mathbf{F}^{p-1}$, $\mathbf{D}^{p} = \frac{1}{2}\left(\mathbf{L}^{p} + \mathbf{L}^{pT}\right)$
- Free energy: $\psi = G\left|\mathbf{E}_{\mathbf{0}}^{\mathbf{e}}\right|^{2} + \frac{K}{2}\left(\mathrm{tr}\mathbf{E}^{\mathbf{e}}\right)^{2}$
- Elastic Hencky strain: $\mathbf{E}^{\mathbf{e}} = \log\left(\mathbf{U}^{\mathbf{e}}\right)$, $\mathbf{F}^{\mathbf{e}} = \mathbf{R}^{\mathbf{e}}\mathbf{U}^{\mathbf{e}}$
- Mandel stress: $\mathbf{M}^{\mathbf{e}} = 2G\mathbf{E}_{\mathbf{0}}^{\mathbf{e}} + K\left(\mathrm{tr}\mathbf{E}^{\mathbf{e}}\right)\mathbf{1}$
- Cauchy stress: $\boldsymbol{\sigma} = J^{-1}\mathbf{R}^{\mathbf{e}}\mathbf{M}^{\mathbf{e}}\mathbf{R}^{\mathbf{eT}}$
- 1^{st} Piola-Kirchoff stress: $\mathbf{P} = J\boldsymbol{\sigma}\mathbf{F}^{-\mathbf{T}}$

Convolutional Neural Networks

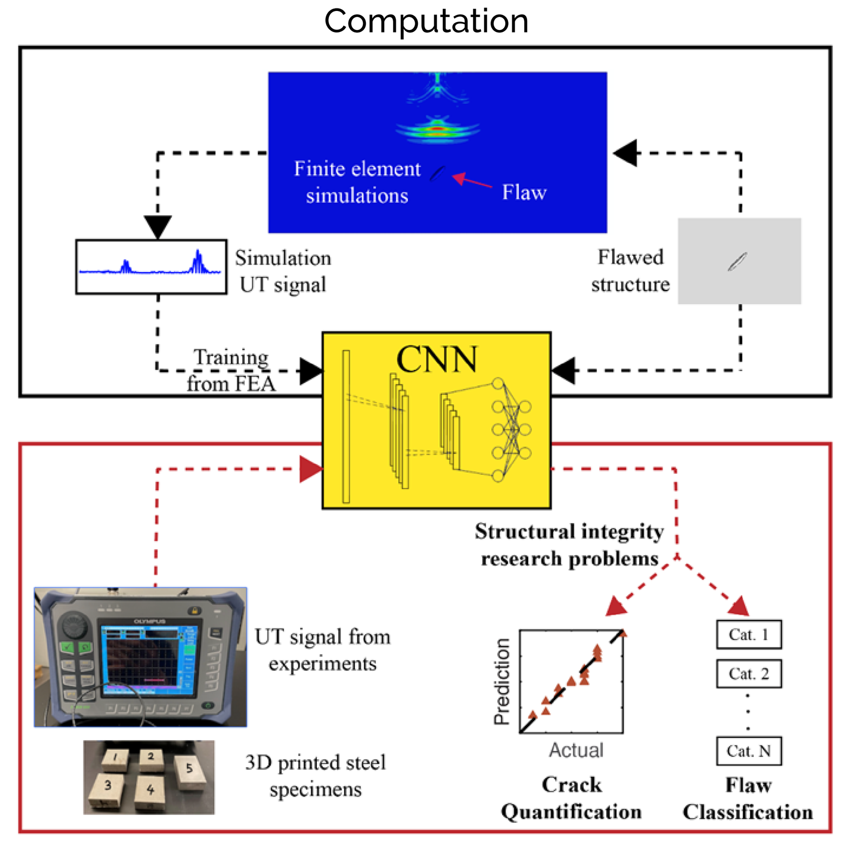
Motivation

The purpose of this study is to apply machine learning to **A-scan** ultrasonic measurements **generated** using finite element analysis for accurately characterizing and quantifying embedded flaws present in **inorganic** structures.

- Length and position and orientation of **penny-shaped embedded cracks** in steel structures

Methods

Computation



Experiment

Figure 3. An overview of the proposed method. A CNN is fully trained using 3D simulation data (top). The simulation trained CNN is then applied to predict length, location, and orientation of embedded cracks from independent experimental ultrasonic signals (bottom) [3].

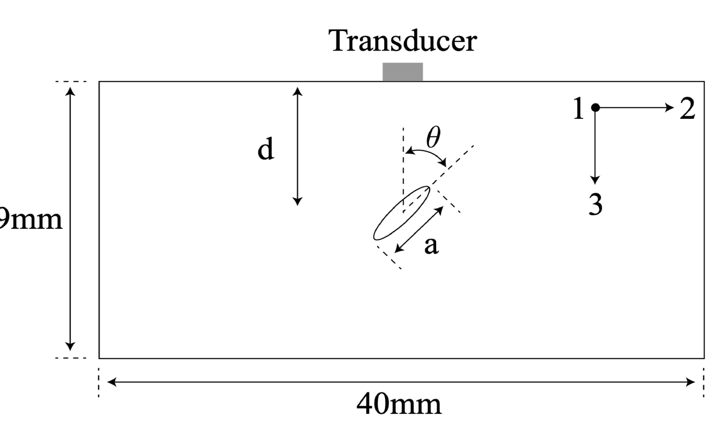


Figure 4. Cross-sectional view illustrating crack length *a*, the crack position *d*, and the crack orientation *θ* of the steel block on its symmetry plane.

Results

Table 1. Percentage errors for selected embedded crack properties measured during validation NDT.

Crack Feature	Length	Location	Orientation
% Error	5.7%	5.6%	8.4%

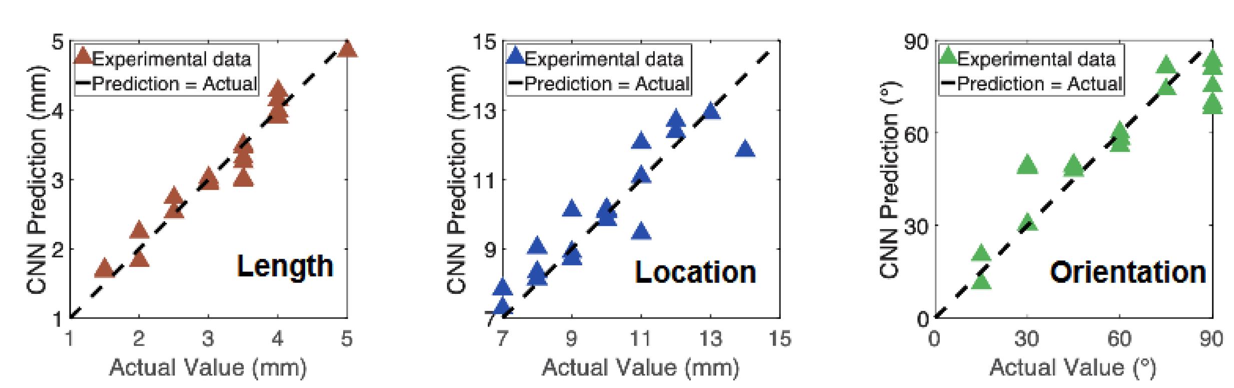


Figure 5. CNN predictive performance on ultrasound NDT experimental data. Points that are closer to dashed 45° line indicate more accurate predictions.

Physics-informed Neural Networks

Motivation

This study also addresses overcoming sparsity in training data from experiments using PINNs. PINNs explicitly encode known **partial differential equations** (PDEs) into data-driven models.

- Solve governing PDEs in boundary value problems (BVP) quickly
- Easier to formulate and solve **forward** and **inverse** problems within solid mechanics
- **Less** computationally intensive due to meshfree nature

PINN Architecture

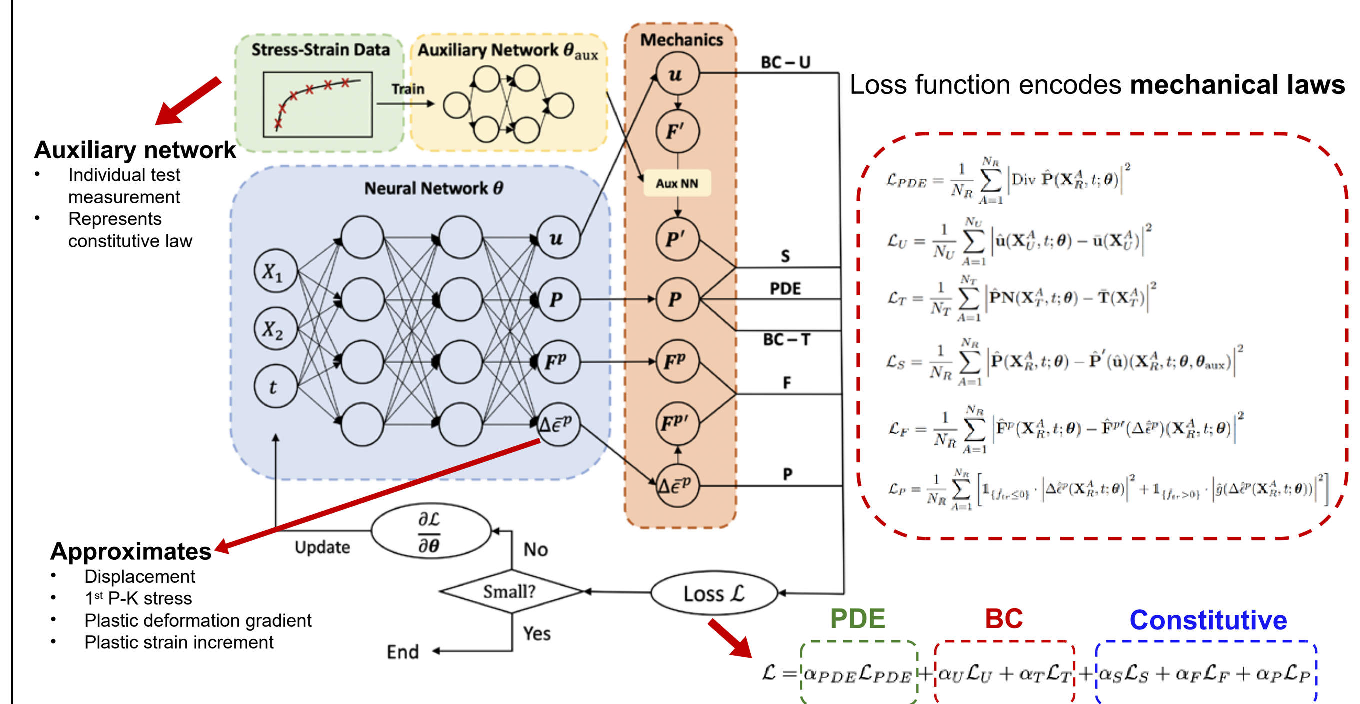


Figure 6. Architecture of the PINN for finite-strain elasto-plasticity. The setup is based on the plane-strain formulation with multiple (pseudo)-time steps. The loss function is formulated based on the PDE, boundary conditions and constitutive relations.

Numerical Example

Consider a *plane-strain*, *hole-in-a-plate* square geometry subjected to 3-step **uniaxial tensile loading** (Figure 7).

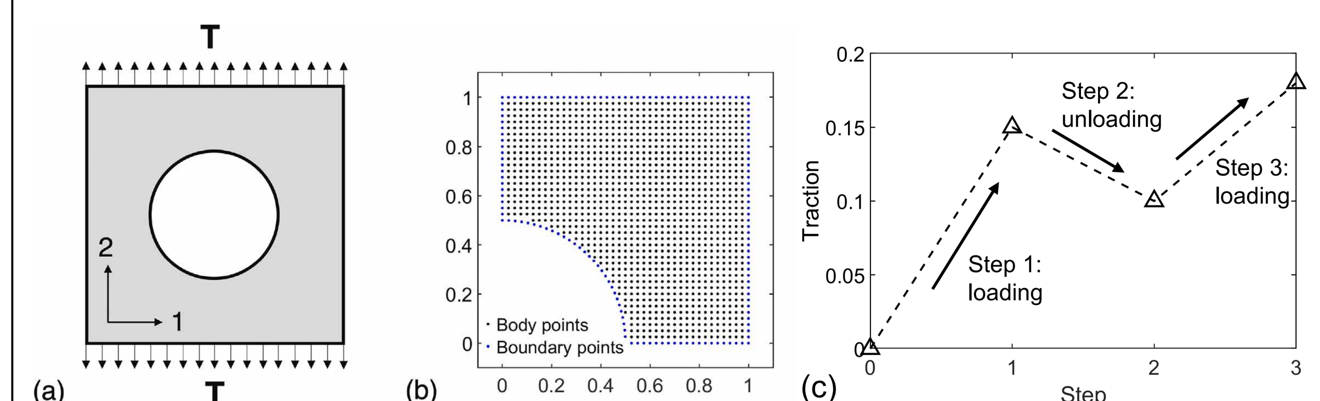


Figure 7. (a) Schematic of the numerical example in which a plate with a hole is subject to uniaxial tensile loading in the 2-direction. (b) Distribution of the residual points in the reference configuration for a quarter of the geometry. (c) Three step loading diagram for the geometry in uniaxial tension.

Results

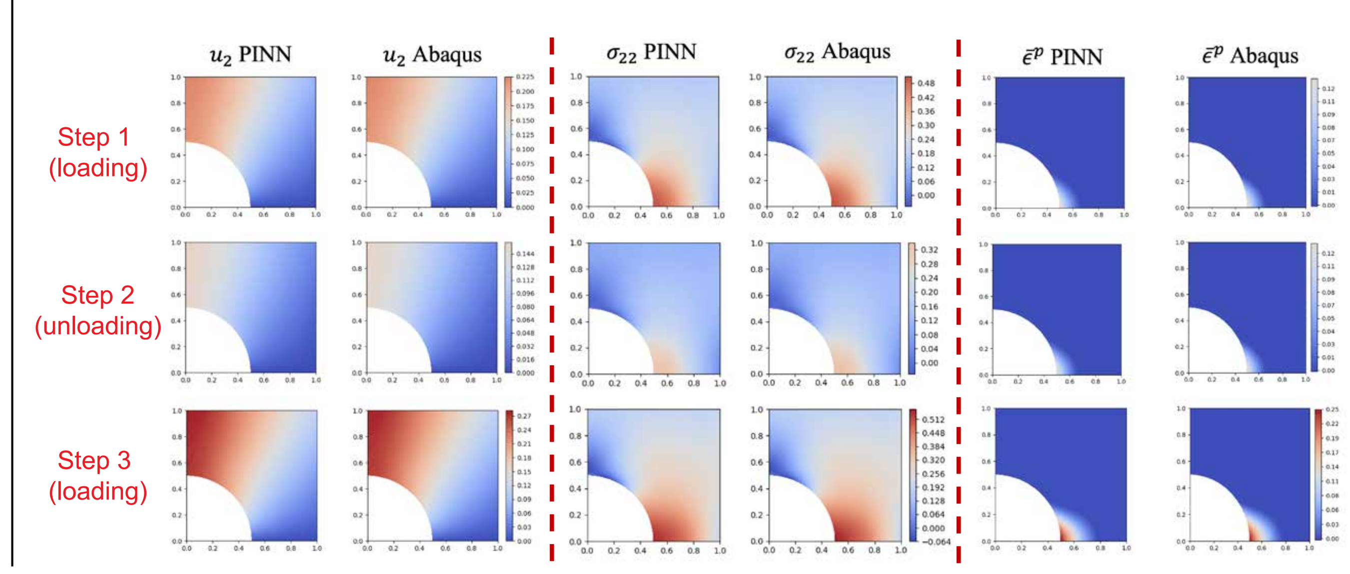


Figure 8. PINN results compared with FEA results for all 3 uniaxial tensile loading steps.

Conclusions

Our finite element trained **CNN** is based on simulation-generated ultrasound time amplitude signal and **not** based on costly image analysis.

- **Fast** scans without any post-processing artifacts
- Feature reflected ultrasound time signal contains all necessary information for **crack characterization**

PINNs can accurately approximate boundary value problem solutions for a generalized loading, unloading and reloading path.

- Quickly captures *plastic deformation*, *elastic unloading* and *stress concentrations*
- Requires **less** training data than *other* machine learning methods
- **Meshfree** nature and ability to **implicitly** approximate functions and their derivatives

Future Directions

Increasing **demand** for structural materials necessitates swift and precise **detection** of crack properties to avert **sudden** catastrophic failures.

- Extend methodology for detecting and characterizing hidden flaws to more diverse materials (polymers, biological tissues) (Figure 9) [1, 4]

Figure 9. Breast Phantoms with Different Tumor Sizes and Elastic Moduli [4].

PINN remains an **active** research area within scientific machine learning.

- **Improve** computational efficiency and **accelerate** training process
- Apply PINN to inverse problems: material and geometry identification

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