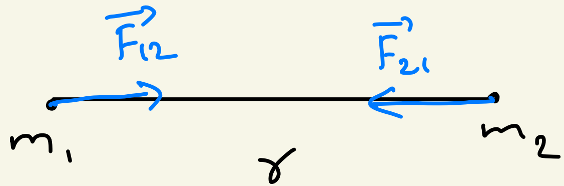
# GRAVITATION

P.AK

# Universal law of gravitation :- ( Inverse square law )




$F \propto m_1 m_2$

$F \propto \frac{1}{r^2}$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

$r \uparrow \quad F \downarrow$

$r \downarrow \quad F \uparrow$

$F = \frac{G m_1 m_2}{r^2}$

$G = \frac{F r^2}{m_1 m_2}$

$G = \frac{N\text{-}m^2}{kg^2}$

G - Universal gravitational constant

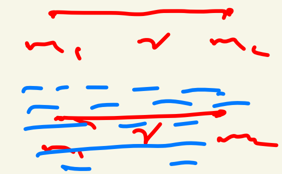
$G = 6.67 \times 10^{-11} \ N\text{-}m^2/kg^2$

F - Gravitational force b/w two particles

$m_1, m_2$ - mass of particle 1 & 2

$r$ - Distance b/w two particles

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}} \Rightarrow \boxed{|\vec{F}_{12}| = |\vec{F}_{21}|}$$




$\vec{F}_{12}$ - Gravitational force on particle 1 due to 2

$\vec{F}_{21}$ - " " " 2 due to 1

## Important Points:

→ Gravitational force is always attractive in nature

→ " " is independent on presence of medium

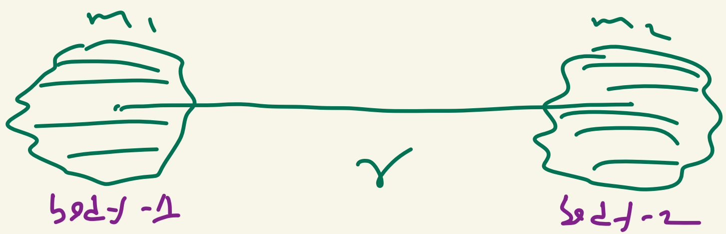
→ Gravitational force b/w two particles always along line joining b/w the particles

→ Gravitational force b/w two particles forms Action - reaction pair i.e

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}} \quad \boxed{|\vec{F}_{12}| = |\vec{F}_{21}|}$$

→ Universal law of gravitation is applicable for point masses. (can not be applied for bodies)


$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$



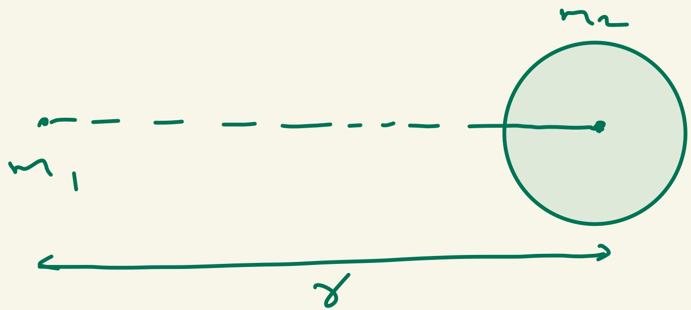

$$\boxed{F \neq \frac{G m_1 m_2}{r^2}}$$

→ But there is exemption for spherical bodies. for spherical bodies we can apply U.L.G by assuming whole mass is concentrated at its center.


$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

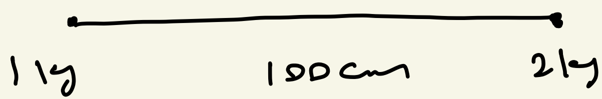
r - Distance b/w com of two spheres

(or)




$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

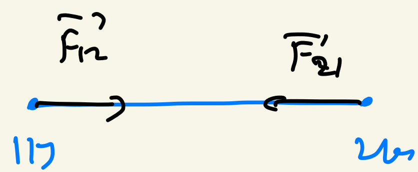
problem:-




Find magnitude of gravitational force b/w two particles?

sol:-

$$F = \frac{G m_1 m_2}{r^2}$$



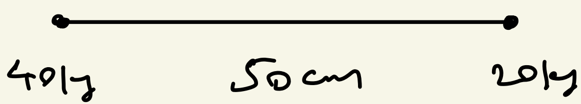

$$F = \frac{6.67\times 10^{-11} \times 1 \times 2}{(100\times 10^{-2})^2} \ N$$

$$F = 6.67 \times 10^{-11} \times 2$$

$$\boxed{F = 13.34 \times 10^{-11}\,N}$$

$$\boxed{|\vec{F}_{12}| = |\vec{F}_{21}| = 13.34 \times 10^{-11}\,N}$$

Problem:-




Find magnitude of gravitational force b/w two particles

Sol:-

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 40 \times 20}{(0.5)^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 8 \times 100 \times 100}{0.5 \times 0.5 \times 100}$$

$$F = \frac{6.67 \times 10^{-11} \times 8 \times 10^{-2} \times 1\cancel{08}4}{\cancel{28}}$$

$$F = 6.67 \times 32 \times 10^{-9}\ N$$

$$\boxed{F = 213.44 \times 10^{-9}\ N}$$

$$\boxed{F = 2.13 \times 10^{-7}\ N}$$

Problem :-

80g ———— 20cm ———— 20g

Find magnitude of Gravitational force

Sol :-

$$F = G\frac{m_1 m_2}{r^2}$$

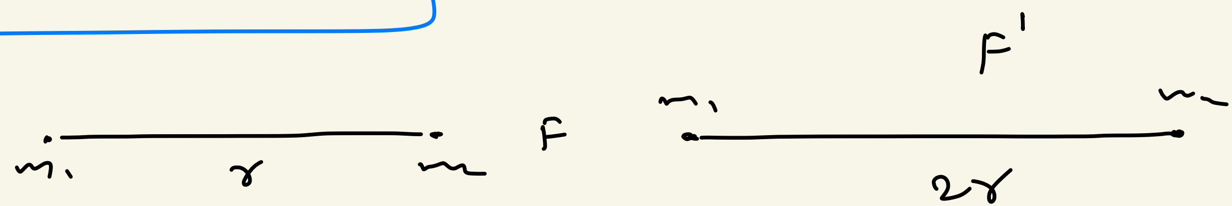
$$F = \frac{6.67\times10^{-11}\times(80\times10^{-3})\times(20\times10^{-3})}{(20\times10^{-2})^2}$$

$$F = \frac{6.67\times10^{-11}\times \overset{2}{\cancel{80}}\times \cancel{20}\times \overset{10^{-2}}{\cancel{10^{-6}}}}{\cancel{400}\times \cancel{10^{-4}}}$$

$$F = 6.67\times4\times10^{-13}\ N$$

$$\boxed{F = 26.68\times10^{-13}\ N}$$

Problem:




Sol:-

$$F = \frac{G m_1 m_2}{r^2} \qquad F' = \frac{G m_1 m_2}{4r^2}$$

$$\boxed{F' = F/4}$$

$$\boxed{\begin{array}{cc} r \uparrow & F \downarrow \\ r \downarrow & F \uparrow \end{array}}$$

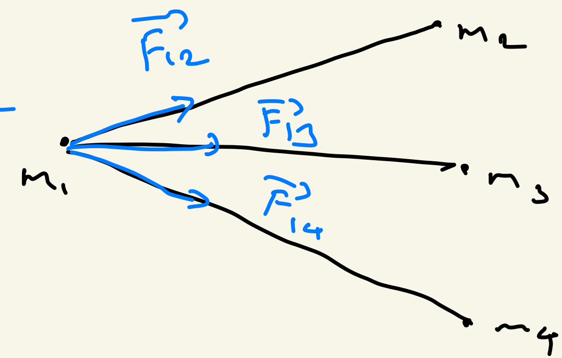
## Gravitational force on one particle due to many particles

(superposition principle)

$\vec{F}_{12}$ – Force on particle 1 due to 2

$\vec{F}_{13}$ – " " " 1 " 3

$\vec{F}_{14}$ – " " " " " 4



→ The resultant force on particle 1 due to remaining particles is equal to vector sum of the gravitational force due to individual particles

$$\boxed{\vec{F}_{res\,1} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots\cdots}$$

(principle of superposition)

→ principle of superposition also states that gravitational force b/w two particles is independent of presence of others particles near by it.

Problem:

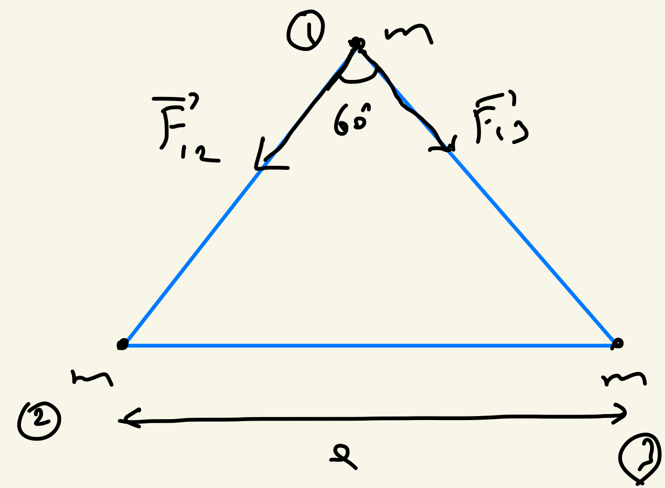
Three particles each having mass m is kept on the vertices of equilateral triangle of side length "a". Find magnitude of gravitational force on any one of the particles due to remaining two particles.

Sol:-

$$\vec{F}_{net\,1} = \underbrace{\vec{F}_{12} + \vec{F}_{13}}_{\text{vector sum}}$$

$$|\vec{F}_{12}| = \frac{Gm^2}{a^2}$$

$$|\vec{F}_{13}| = \frac{Gm^2}{a^2}$$



$$|\vec{F}_{12}| = |\vec{F}_{13}| = \frac{Gm^2}{a^2} = F$$

$$|\vec{F}_{ver\,1}| = \sqrt{F^2 + F^2 + 2F^2\cos 60°}$$

$$|\vec{F}_{ver\,1}| = \sqrt{F^2 + F^2 + \cancel{2}F^2 \times \frac{1}{\cancel{2}}}$$

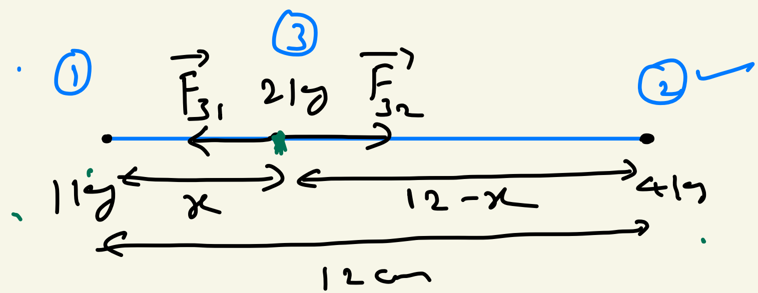
$$|\vec{F}_{ver\,1}| = \sqrt{3}\,F$$

$$\boxed{|\vec{F}_{ver\,1}| = \sqrt{3}\,\frac{Gm^2}{a^2}}$$

Problem:-

2 particles of mass 1kg and 4kg are kept at a separation distance of 12cm. At what distance from 1kg mass along line joining b/w two particles, third particle of mass 2kg should be placed to get resultant force on third particle is zero due to remaining two particles

Sol:-



$$|\vec{F}_{31}| = |\vec{F}_{32}|$$

$$|\vec{F}_{31}| = \frac{2G}{x^2}$$

$$|\vec{F}_{32}| = \frac{8G}{(12-x)^2}$$

$$\frac{\cancel{2}\cancel{G}}{x^2} = \frac{\overset{4}{\cancel{8}}\cancel{G}}{(12-x)^2}$$

$$\left(\frac{12-x}{x}\right)^2 = 4$$

$$\frac{12-x}{x} = 2$$

$$\frac{G m_1 \cancel{m_3}}{x^2} = \frac{G \cancel{m_3} m_2}{(12-x)^2}$$

$$12 - x = 2x$$

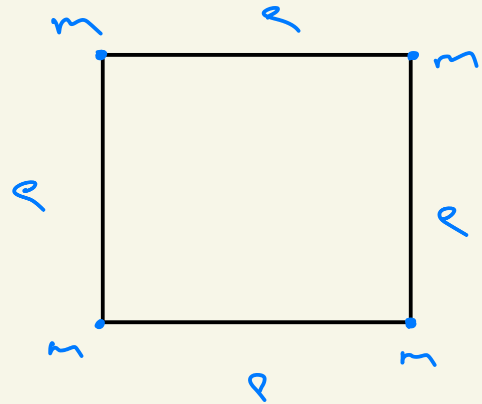
$$12 = 3x$$

$$\boxed{x = 4\,cm}$$

NOTE: The net force on the third particle is zero for any value of mass

Problem:-

Find magnitude of gravitational force on any one particle due to remaining three particles?



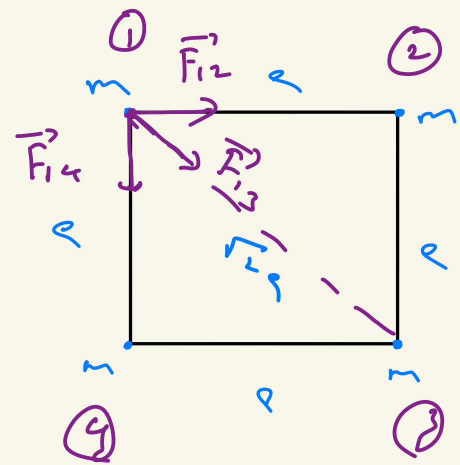

Sol:-

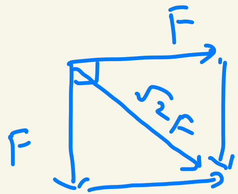
$$\vec{F}_{res1} = \underbrace{\vec{F}_{12} + \vec{F}_{14}} + \vec{F}_{13}$$

$$|\vec{F}_{12}| = \frac{Gm^2}{a^2} = F$$

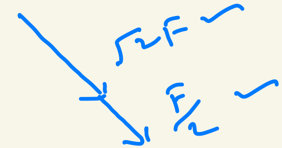
$$|\vec{F}_{14}| = \frac{Gm^2}{a^2} = F$$

$$|\vec{F}_{13}| = \frac{Gm^2}{2a^2} = F/2$$





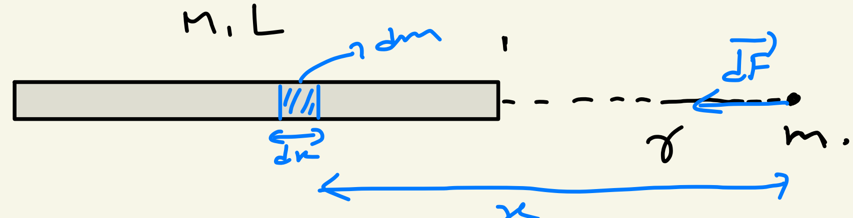

$\sqrt{2}\,F$




$$|\vec{F}_{res1}| = \sqrt{2}\,F + F/2$$

$$\boxed{|\vec{F}_{res1}| = \left(\sqrt{2} + \frac{1}{2}\right)\frac{Gm^2}{a^2}}$$

# Force b/w rod and particle system:-




dF – Force b/w small element of rod and particle

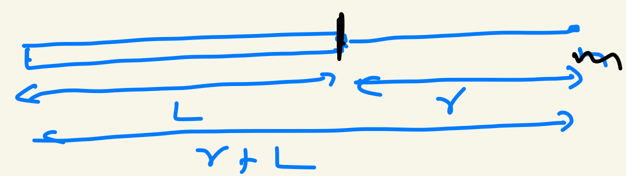
$$dF = \frac{G\,dm\,m}{x^2}$$

$$dm = \frac{M}{L}dx$$

L – M

dx – ?

$$\int dF = \int_{r}^{r+L} \frac{G\left(\frac{M}{L}\right) m\,dx}{x^2}$$



$$F = \frac{GMm}{L} \int_{r}^{r+L} \frac{dx}{x^2}$$

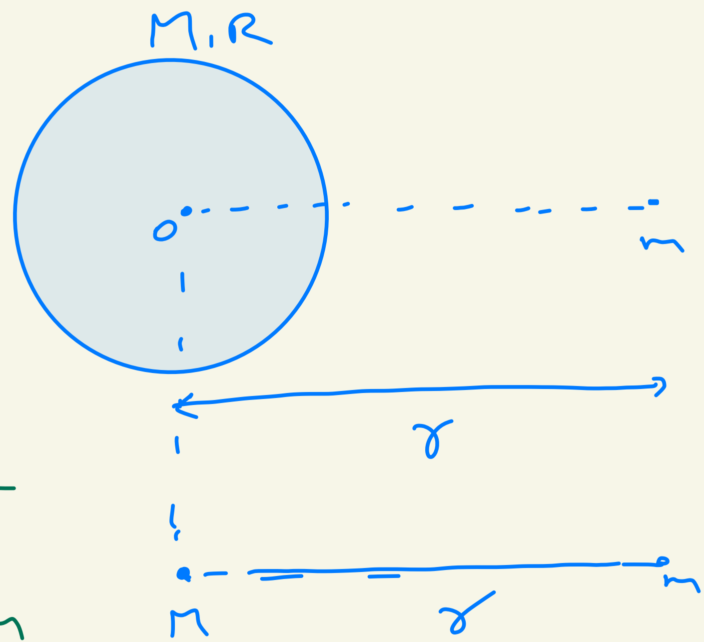
$$F = \frac{GMm}{L} \left[ -\frac{1}{x} \right]_{r}^{r+L}$$

*

$$\boxed{F = \frac{GMm}{r(r+L)}}$$

# Force b/w spherical object and particle

$$\boxed{F = \frac{GMm}{r^2}}$$



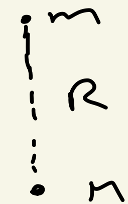

**NOTE:** For spherical objects we can assume that whole mass of the spherical objects is concentrated at its COM and we can apply universal law of gravitation.

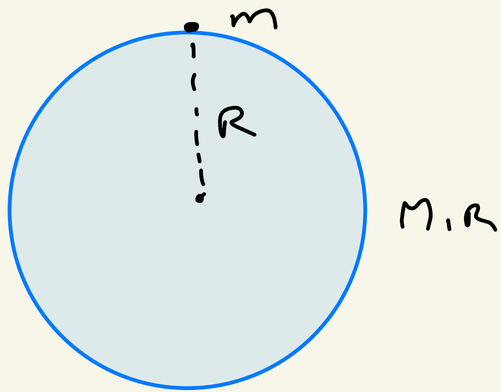
# Acceleration due to gravity ($g$) :-

→ Acceleration due to gravity varies from place to place and planet to planet. (i.e For Different planets acceleration due to gravity is different)

From universal law of gravitation

$$F = \frac{GMm}{R^2} \quad — ①$$




From Newton II$^{nd}$ law

$$F = mg \quad — ②$$

From ① & ②

$$\frac{GM\cancel{m}}{R^2} = \cancel{m}g$$

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$$\boxed{g = \frac{GM}{R^2}}$$ relation b/w g & G

G - Universal gravitational constant
M - Mass of the planet
R - Radius .

NOTE:- ① Acceleration due to gravity is different for different planets i.e it depends on mass of the planet and radius of the planet

② Acceleration due to gravity on the surface of the earth is equal to $\simeq 9.8\, m/s^2$

$G = 6.67 \times 10^{-11}\, Nm^2/kg^2$

$M_e = 6 \times 10^{24}\, kg$

$R_e = 6400\, km = 6400 \times 10^3\, m$

Case i, Acceleration due to gravity at a height 'h'

$$g_h = \frac{GM}{(R+h)^2} \quad \text{—— ①}$$

$$g = \frac{GM}{R^2} \quad \text{--- } (2)$$

Equation $\frac{(1)}{(2)}$

$$\frac{g_h}{g} = \frac{\cancel{GM}}{(R+h)^2} \times \frac{R^2}{\cancel{GM}}$$

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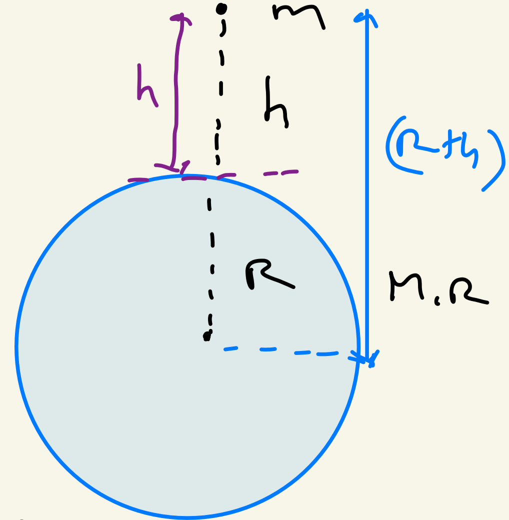
$$\boxed{g_h = g\left[\frac{R}{R+h}\right]^2}$$

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$$\boxed{h\uparrow \; g_h\downarrow}$$

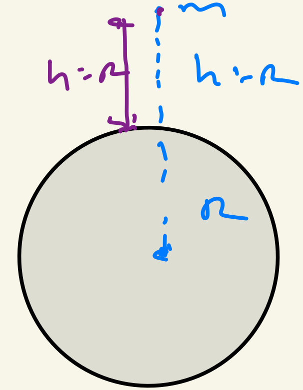
$g$ - A.d.g on the surface of the planet

$R$ - radius of the planet

$h$ - height from the surface of the planet

$g_h$ - A.d.g at height 'h' from the surface of the planet

Eg :- Find A.d.g at a height $h = R$ from the surface of the earth. given that A.d.g on the surface of the earth 'g'.




Sol :-

$$g_h = g\left[\frac{R}{R+R}\right]^2$$

$$g_h = g\left[\frac{1}{2}\right]^2$$

$$\boxed{g_h = \frac{g}{4}}$$

NOTE :- ① If $h \uparrow$ $g \downarrow$ vice versa

*** ② If $h << R \Rightarrow \boxed{g_h = g\left[1 - \frac{2h}{R}\right]}$ $(h << R)$

special case

## Acceleration due to gravity at depth 'd'

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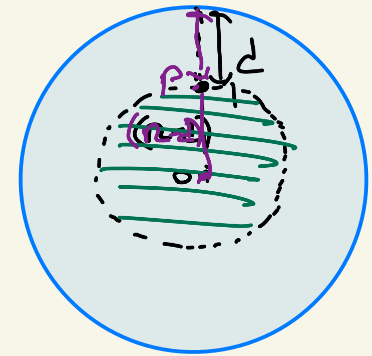
$$\boxed{g_d = g\left(1 - \frac{d}{R}\right)}$$




M, R

$g$ – A.d.g on the surface of the planet

$d$ – depth from surface of planet

$g_d$ – A.d.g at depth 'd'

**Proof:**

$$g_d = \frac{GM'}{(R-d)^2}$$

$$\frac{4}{3}\pi R^3 \;-\; M$$

$$\frac{4}{3}\pi (R-d)^3 \;-\; ?$$

$$M' = \frac{(R-d)^3}{R^3} \times M$$

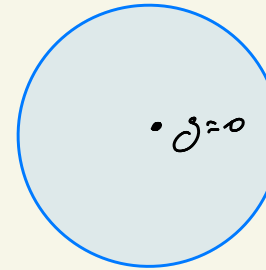
$$g_d = \frac{GM(R-d)^{\cancel{3}\,1}}{R^3\,\cancel{(R-d)^2}}$$

$$g_d = \frac{GM}{R^2}\left(\frac{R-d}{R}\right)$$

$$* \quad \boxed{g_d = g\left(1 - \frac{d}{R}\right)}$$

**NOTE:-**

1. If $d = R \Rightarrow g_d = 0 \Rightarrow \text{Weight}_{(\text{center})} = 0$




2. $d \uparrow \; g_d \downarrow$, vice versa

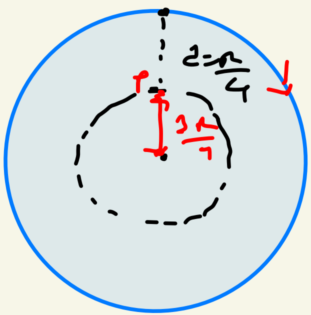
3. As we move above the surface of the planet (or) below the surface of the planet A.d.g decreases.

Ex:- Find Acceleration due to gravity at depth $d = r/4$ given that A.d.g on the surface of the planet is $g$

Sol:-

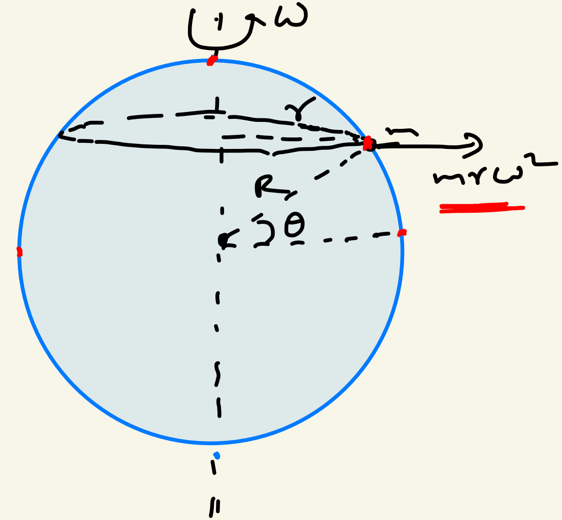
$$g_d = g\left(1 - \frac{\cancel{r}}{4(\cancel{r})}\right)$$

$$\boxed{g_d = \frac{3g}{4}}$$



## Variation A.d.g Due to rotation of the earth

$$\boxed{g' = g - R\omega^2 \cos^2\theta}$$




**At poles**

$\theta = 90°$

$$\boxed{g' = g}$$

**At equator**

$\theta = 0°$

$$\boxed{g' = g - R\omega^2}$$

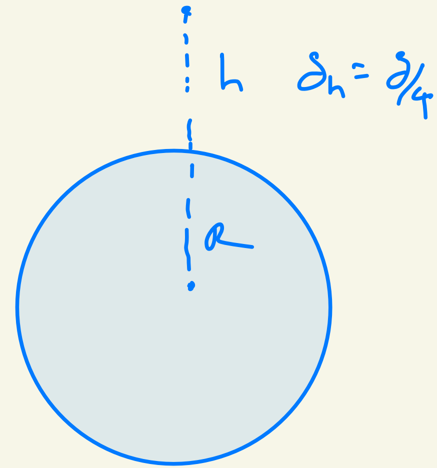
$\omega$ - Angular velocity of earth about its axis

$$\boxed{\omega = \frac{2\pi}{T}} \Rightarrow \boxed{\omega = 2\pi f}$$

$$\boxed{T = 24 \text{ hours}}$$

Problem

At what height from the surface of the earth the a.d.g is $\frac{1}{4}$th of the A.d.g on the surface of the earth? ( $R_e = 6400$ km)




Sol:-

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

$$\frac{g}{4} = g\left(\frac{R}{R+h}\right)^2$$

$$\left(\frac{R}{R+h}\right)^2 = \frac{1}{4}$$

$$\frac{R}{R+h} = \frac{1}{2} \Rightarrow 2R = R+h \Rightarrow h = R$$

$$\boxed{h = 6400\text{ km}}$$

Problem:- The weight of the object on the surface of the earth is 200N. What is the weight of the same object at a height $h = \frac{3}{4}R$? R - Radius of the earth.

Sol:-

$$mg_h = mg\left(\frac{R}{R+h}\right)^2$$

$$\underline{W'} = W\left(\frac{R}{R+h}\right)^2$$

$$W' = 200\left(\frac{R}{R+\frac{3}{4}R}\right)^2$$

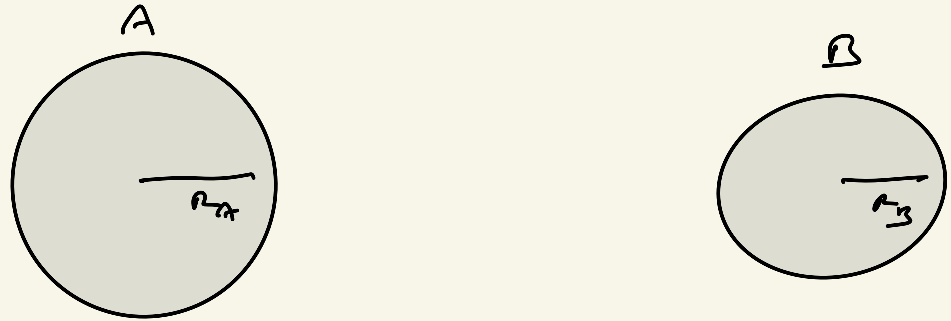
$$W' = 200\left(\frac{4}{7}\right)^2$$

$$W' = 200 \times \frac{16}{49}$$

$$W' = \frac{3200}{49}\,N$$

$$\boxed{W' = 65.3\,N}$$

problem:




A.d.g of planet A is 'g' what is A.d.g of planet B where density is 8 times the density of planet A and mass is same as planet A.

sol:-

$$g_A = g = \frac{GM}{R_A^2} \qquad g_B = \frac{GM}{R_B^2}$$

$$\rho_B = 8\rho_A$$

$$\frac{\cancel{M}}{\cancel{\frac{4}{3}\pi} R_B^3} = 8\,\frac{\cancel{M}}{\cancel{\frac{4}{3}\pi} R_A^3}$$

$$R_A^3 = 8 R_B^3$$

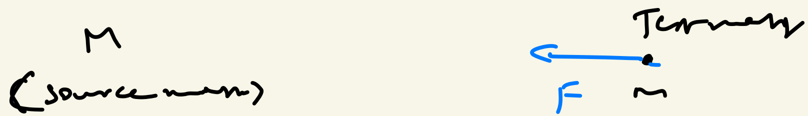
$$\boxed{R_A = 2R_B}$$

$$g_A = g = \frac{GM}{R_A^2}$$

$$g_B = \frac{GM \times 4}{R_A^2}$$

$$\boxed{g_B = 4g}$$

# Gravitational field Intensity ($\vec{E}$)

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$$\boxed{\vec{E} = \frac{\vec{F}}{m}}$$

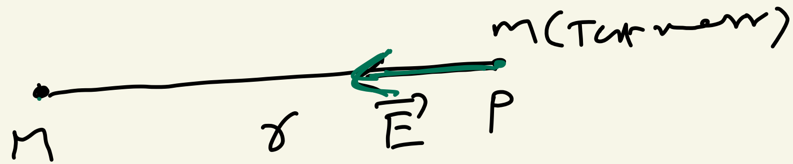
→ Gravitational field is a vector quantity. and its direction is always in the direction of gravitational force (It's direction is always towards the mass)

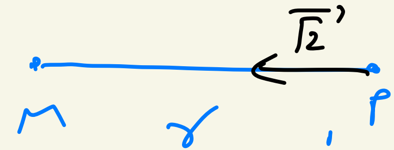
→ S.I unit of G. field is N/kg

→ If we rearrange above equation we get

$$\boxed{\vec{F} = m\vec{E}}$$

# Gravitational field due to a point mass

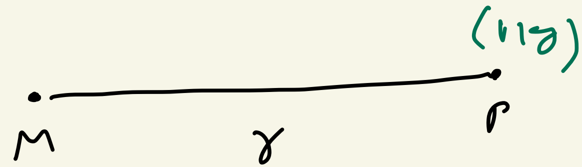


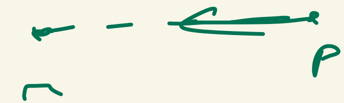




$$|\vec{E}| = \frac{\frac{GMm}{r^2}}{m}$$

$$\boxed{|\vec{E}| = \frac{GM}{r^2}}$$

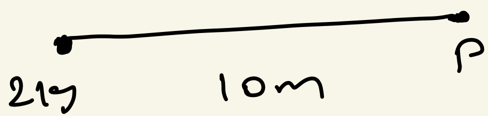
→ Gravitational field due to point mass at a point is numerically equal to force experienced by the unit mass (1kg) placed at that point




$$\boxed{|\vec{F}| = \frac{GM(1)}{r^2} = |\vec{E}|}$$

Eg:- Find magnitude G. field due to 2kg mass at point P.




Soln:- $|\vec{E}| = \frac{GM}{r^2} = \frac{6.67\times 10^{-11}\times 2}{(10)^2}$

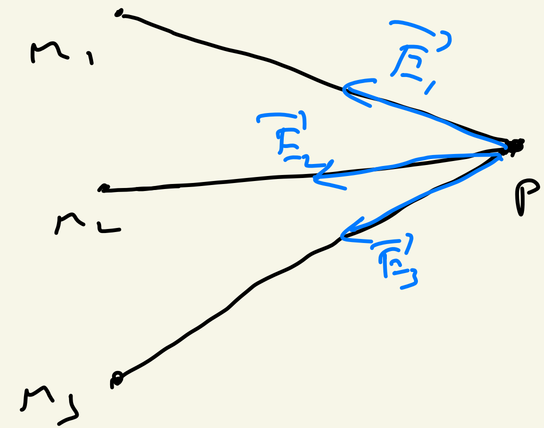
$$|\vec{E}| = 13.39 \times 10^{-11} \times 10^{-2} \ N/kg$$

$$\boxed{|\vec{E}| = 13.39 \times 10^{-13} \ N/kg}$$

## Gravitational field due to many point masses

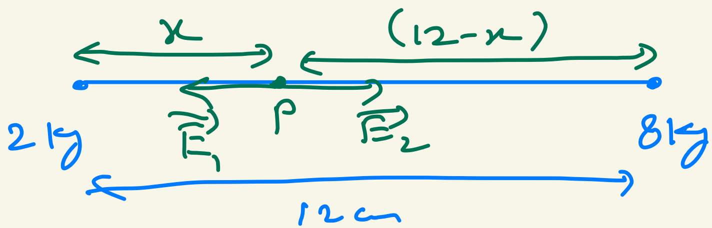
$$\boxed{\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots}$$

(vector sum)



Eg:- Two particles of 2kg mass and 8kg are kept at a seperation distance 12 cm. Find where gravitational field due to two masses is zero?

Sol:-



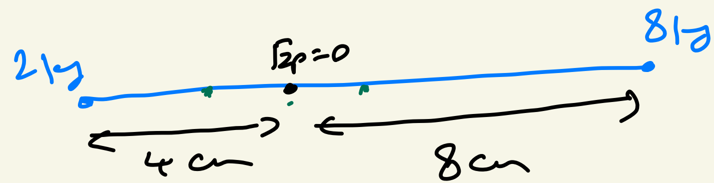

$$|\vec{E}_1| = \frac{2G}{x^2} \qquad |\vec{E}_2| = \frac{8G}{(12-x)^2}$$

$$\vec{E}_P = 0 \Rightarrow |\vec{E}_1| = |\vec{E}_2|$$

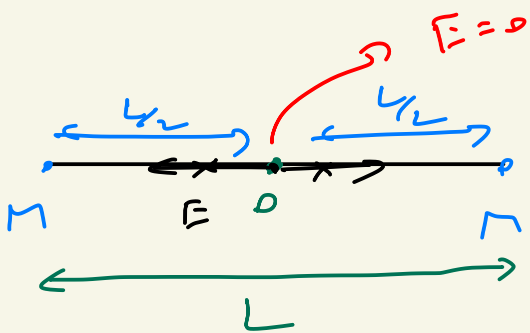
$$\frac{2\cancel{G}}{x^2} = \frac{8\cancel{G}}{(12-x)^2}$$

$$\left(\frac{12-x}{x}\right)^2 = 4$$

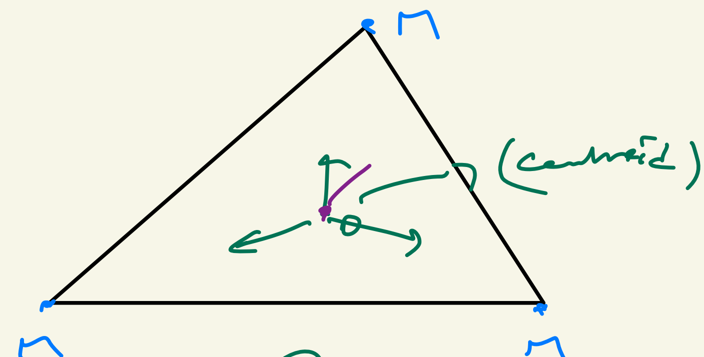
$$\frac{12-x}{x} = 2 \Rightarrow \boxed{x = 4\text{ cm}}$$



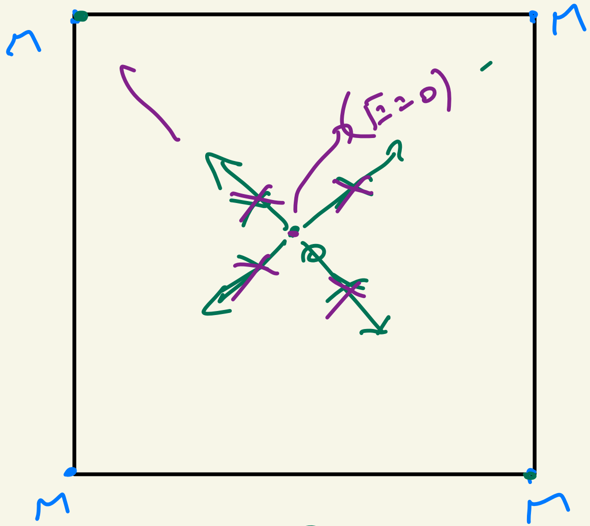

Eg :-



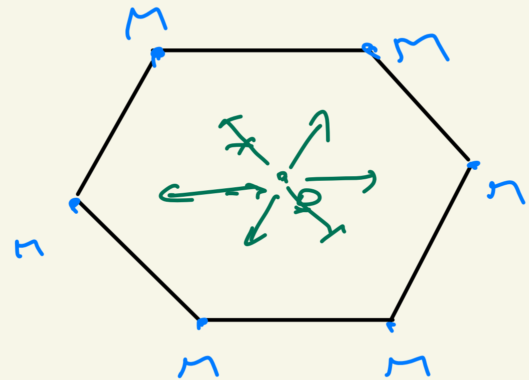

①




③




③

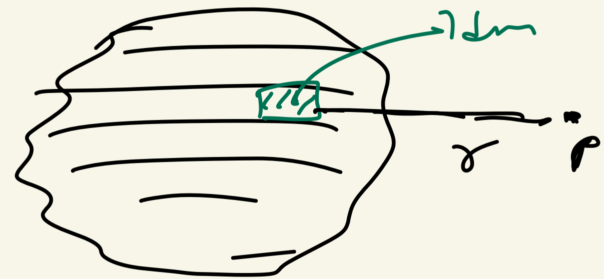



④

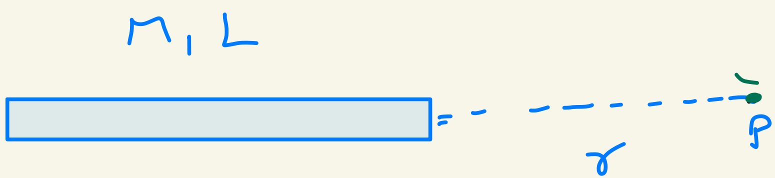
The gravitational field at point 'O' in all the above cases is zero

## Gravitational field due to continuous mass Distribution

$$\boxed{dE = \int \frac{G\,dm}{r^2}}$$




## Gravitational field due to Thin rod




$$\boxed{|\vec{E}| = \frac{GM}{r(r+L)}}$$

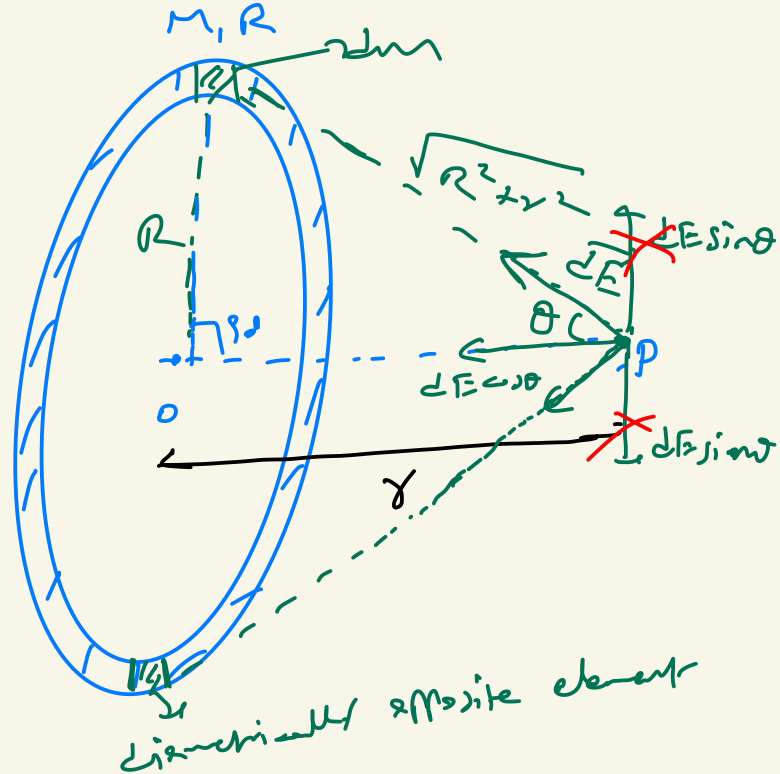
Field due to ring :-

$$dE = \frac{G\,dm}{\left(\sqrt{R^2+r^2}\right)^2}$$

$$dE_x = dE\cos\theta$$

$$\int dE_x = \int \frac{G\,dm}{(R^2+r^2)}\left(\frac{r}{(R^2+r^2)^{1/2}}\right)$$

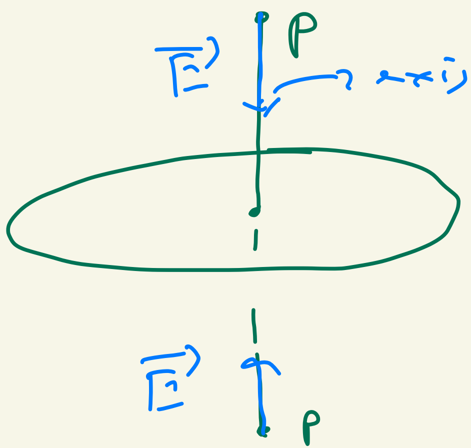
$$E_x = \frac{Gr}{(R^2+r^2)^{3/2}}\int dm$$

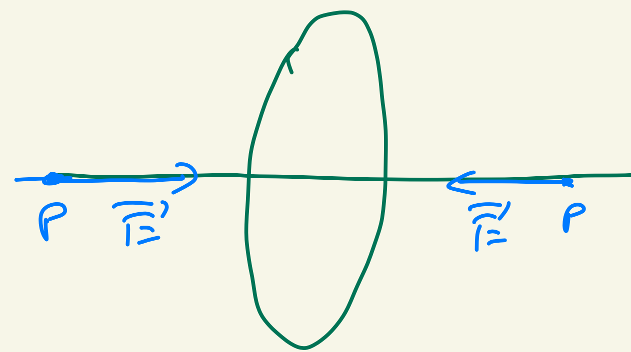


***

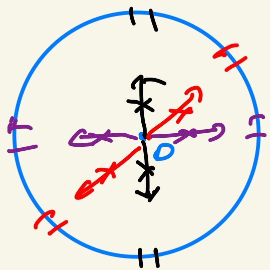
$$\boxed{|\vec{E}_P| = \frac{GMr}{(R^2+r^2)^{3/2}}}$$

→ G.F. direction due to ring at any point on the axis of the ring is towards along the axis of the ring and towards center of the ring.



→ At the centre of the ring G.F is zero.

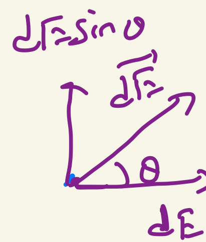


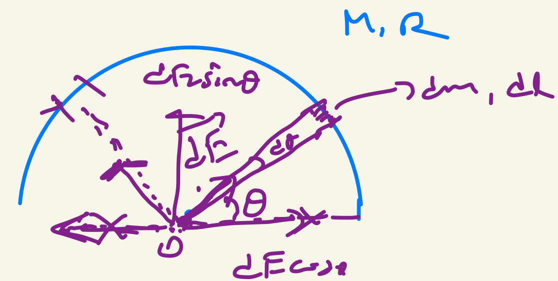

$$\boxed{E_0 = 0}$$

$\because \; r = 0 \Rightarrow \boxed{E_0 = 0}$

## Gravitational field due to semicircular wire

$$dE = \frac{G\,dm}{R^2}$$



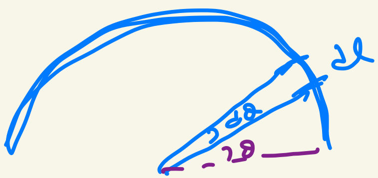




→ Gravitational field due all x-component cancel each other Therefore due to x-component

G. Field is zero

$$dE_y = dE \sin\theta$$

$$dE_y = \frac{G\,dm}{r^2}\sin\theta \quad \text{--- (1)}$$




$\pi R$ — $M$

$dl$ — $?$

$$\boxed{dl = R\,d\theta}$$

$$dm = \frac{M}{\pi R}\,dl$$

$$dm = \frac{M}{\pi R}(R\,d\theta) \quad \text{--- (2)}$$

$$\int dE_y = \int \frac{GM\,(\cancel{R}\,d\theta)}{\pi \cancel{R}\; r^2}\sin\theta$$

$$F_y = \frac{GM}{\pi r^2}\int_0^{\pi} \sin\theta \, d\theta$$

$$F_y = \frac{GM}{\pi R^2}\Big[-\cos\theta\Big]_0^{\pi}$$

$$F_y = \frac{GM}{\pi r^2}\Big[-\cos\pi - (-\cos 0)\Big]$$

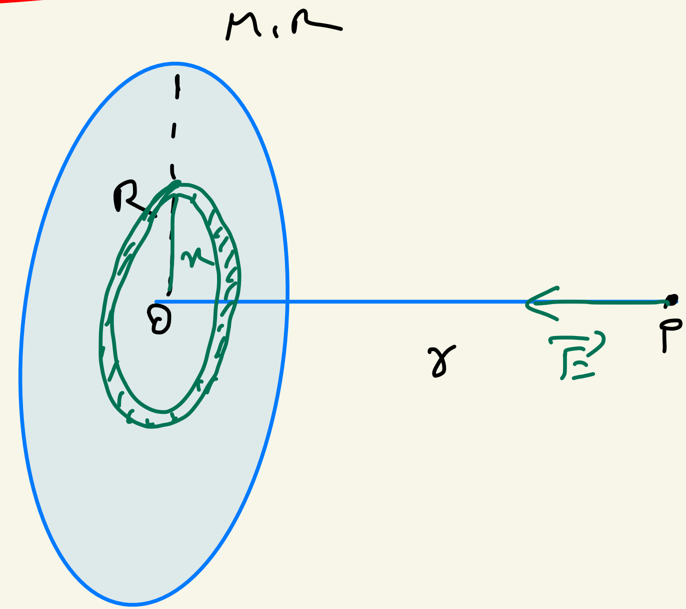
$$F_y = \frac{GM}{\pi r^2}\big[1+1\big]$$

$$\boxed{F_y = \frac{2GM}{\pi r^2}}$$

# Gravitational field due to Disc

***

$$\boxed{E = \frac{2GM}{R^2}\left[1 - \frac{r}{\sqrt{R^2+r^2}}\right]}$$




NOTE: Gravitational field direction Due to Disc at a point P on the axis of Disc is always along the axis of the Disc and towards centre of the Disc.

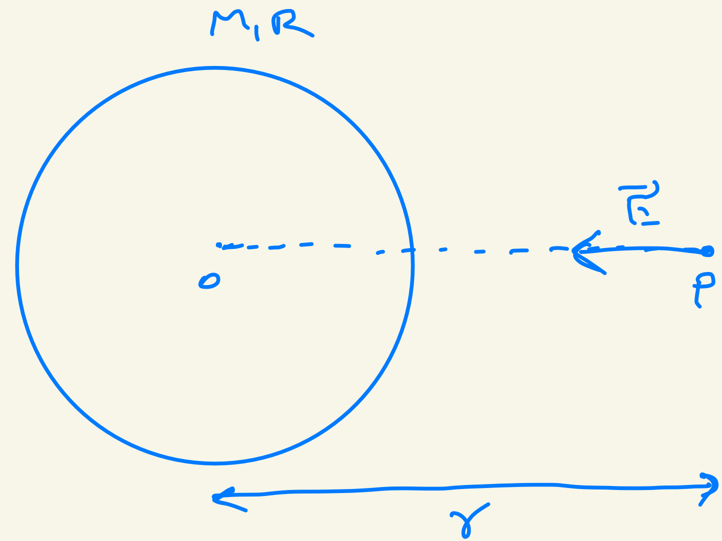
# Gravitational field due to hollow sphere (or) (spherical shell)

**Case 1** $r > R$ (outside)

$$\boxed{|\vec{E}| = \frac{GM}{r^2}} \Rightarrow E \propto \frac{1}{r^2}$$

$$\Downarrow$$

$$r \uparrow \; E \downarrow$$

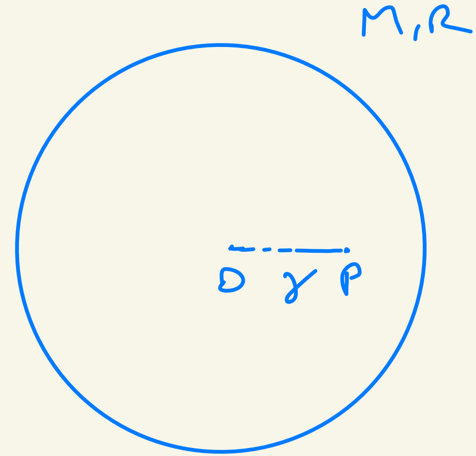



NOTE: we can assume that whole mass of the sphere concentrated at its centre and we can apply Gravitational field to point mass Definition (only for outside points)

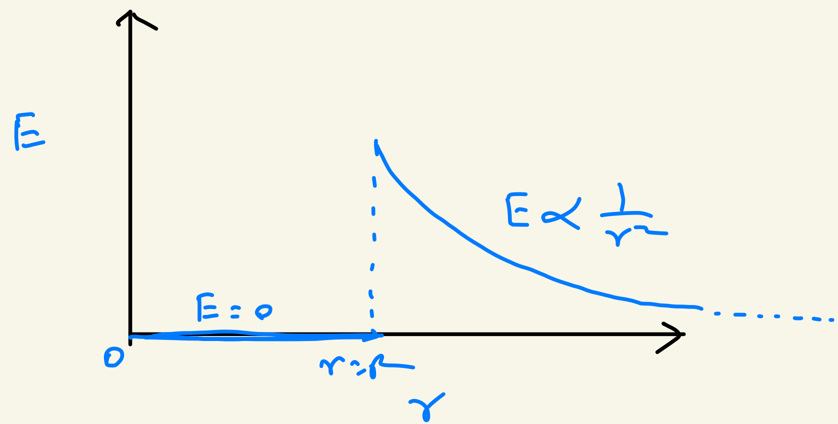
Case II $r < R$

$$\boxed{E = 0}$$

NOTE:- Gravitational field due to spherical shell at any point inside the spherical shell is equal to zero.




Graph b/w $E$ vs $r$



## Gravitational field due to solid sphere (spherical volume of mass distribution)

case (i) $r \geq R$

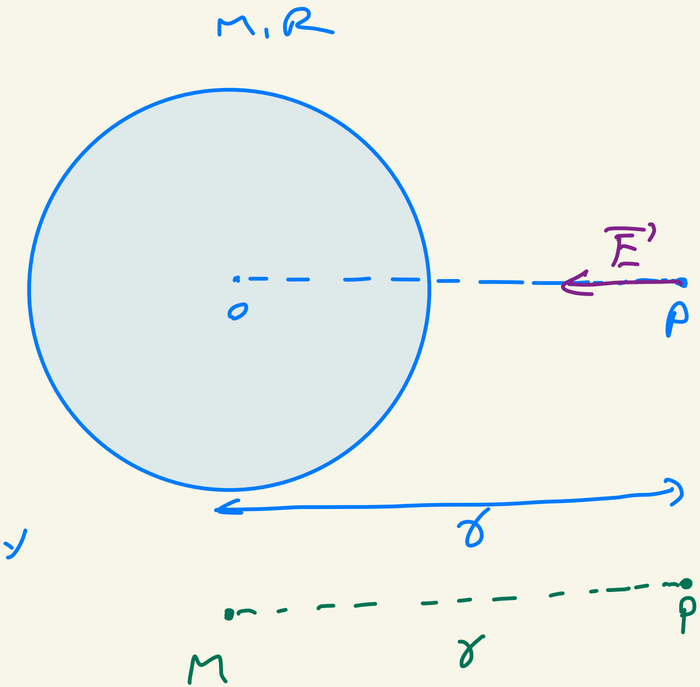
$$\boxed{|\vec{E}| = \frac{GM}{r^2}} \Rightarrow E \propto \frac{1}{r^2}$$

At $r = R$

$$\boxed{|\vec{E}| = \frac{GM}{R^2} = g} \rightarrow \text{Acceleration due to gravity}$$

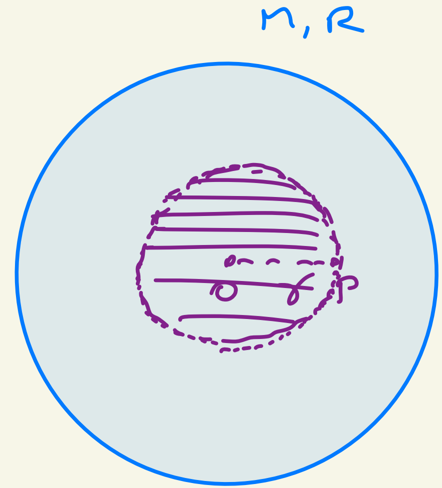
M - mass of the sphere

r - Distance from centre of the sphere to point 'P'



case ii $r \leq R$

Solid sphere – combination of spherical shells




$$|\vec{E}| = \frac{GM'}{r^2}$$

$$\frac{4}{3}\pi R^3 \ — \ M$$

$$\frac{4}{3}\pi r^3 \ — \ ?$$

$$M' = \frac{M}{\cancel{\frac{4}{3}\pi} R^3} \cancel{\frac{4}{3}\pi} r^3$$

$$|\vec{E}| = \frac{GM r^{\cancel{3}}}{R^3 \cancel{r^2}}$$

*

$$\boxed{|\vec{E}_p| = \frac{GMr}{R^3}} \Rightarrow |\vec{E}| \propto r$$

$$\Downarrow$$

$r \uparrow \ E \uparrow$

NOTE :- (1) At the center of the solid sphere Field is zero

$$\boxed{E_0 = 0} \qquad \boxed{\because \; r = 0}$$

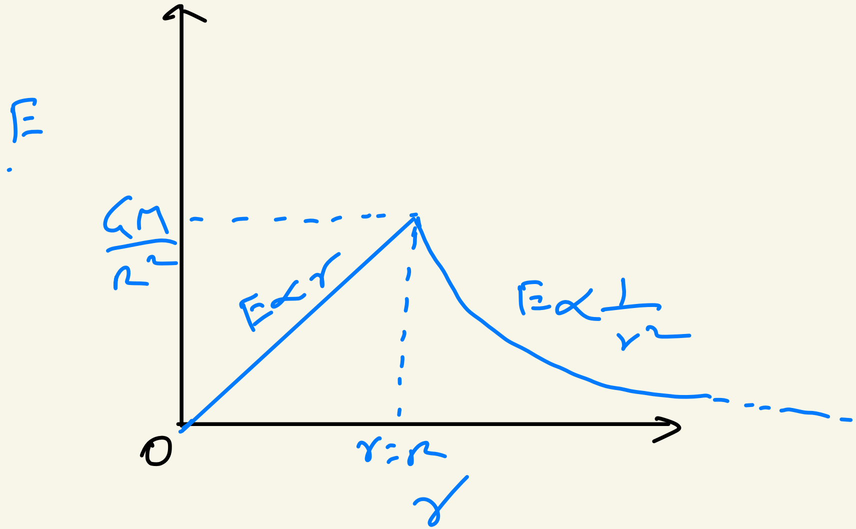
(2) In terms density Gravitational field inside the sphere.

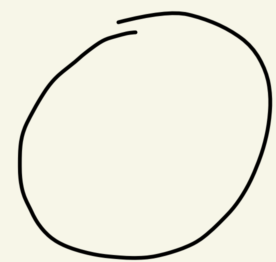
$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow M = \rho \frac{4}{3}\pi R^3$$

$$E = \frac{G\rho \times \frac{4}{3}\pi \cancel{R^3}\, r}{\cancel{R^3}}$$

$$\boxed{E_P = \frac{G\rho 4\pi r}{3}} \Rightarrow E \propto r$$

# Graph b/w E vs r

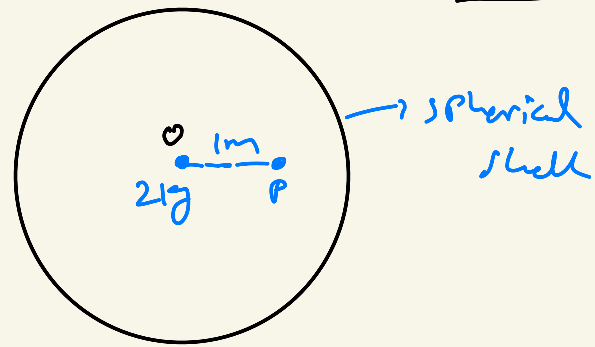


Problem:-

In the figure shown find magnitude of Gravitational field at point P.

$M = 4\,kg$ $\quad R = 2\,m$




Sol:-

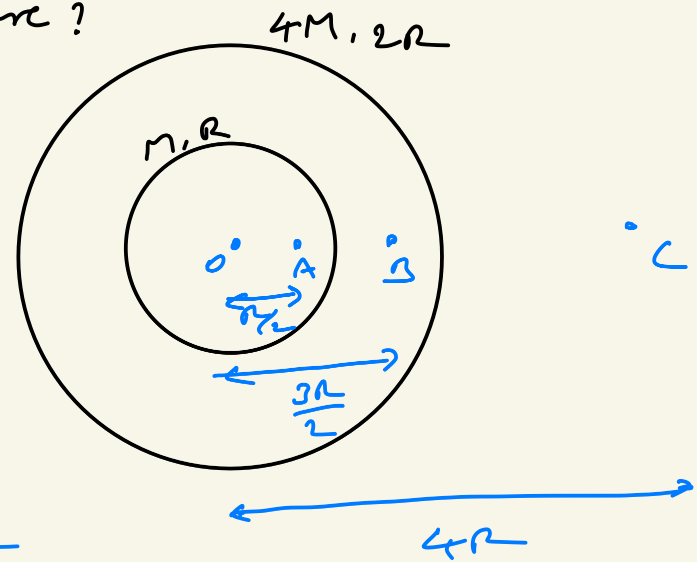
$$|\vec{E}| = \frac{G \times 2}{(1)^2}$$

$$|\vec{E}| = 6.67 \times 10^{-11} \times 2 \ N/kg$$

$$\boxed{|\vec{E}| = 13.34 \times 10^{-11} \ N/kg}$$

$$\begin{array}{r} 6.67 \\ \times\ 2 \\ \hline 13.34 \end{array}$$

**Problem:-** Find Gravitational field (magnitude) due to two concentric spherical shells at point A, B and C as shown in the figure?



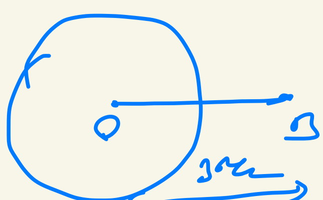

**Sol:-**

**At point A**

$$\boxed{E_A = 0}$$

**At point B**

→ Bigger shell do not produce G.F at point B (point B is Inside point)




$$\boxed{E_B = \frac{GM}{(3R/2)^2} = \frac{4GM}{9R^2}}$$

At Point C

→ Both shells contribute Field at Point 'C'

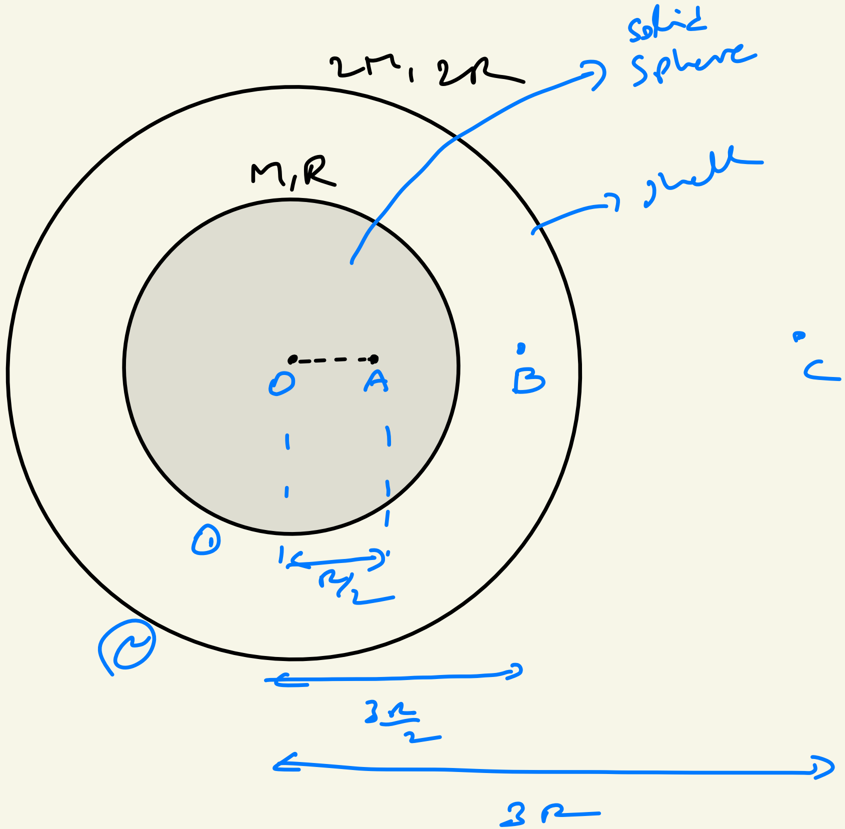
$$|\vec{E}_1| = \frac{GM}{(4r)^2} = \frac{GM}{16r^2}$$

$$|\vec{E}_2| = \frac{G(4M)}{(4r)^2} = \frac{4GM}{16r^2}$$

$$|\vec{E}_P| = E_1 + E_2 = \frac{GM}{16r^2} + \frac{GM}{4r^2}$$

$$\boxed{|\vec{E}_c| = \frac{5GM}{16r^2}}$$

Eg:-




find field at point A, B and C due to sphere and shell

## At Point A

$$F_1 = \frac{GMr}{R^3} = \frac{GM(R/2)}{R^3} = \frac{GM}{2R^2}$$

$$F_2 = 0$$

$$\boxed{F_A = \frac{GM}{2R^2}}$$

## At Point B

$$F_1 = \frac{GM}{(3R/2)^2} = \frac{4GM}{9R^2}$$

$$\boxed{F_B = \frac{4GM}{9R^2}}$$

$$F_2 = 0$$

## At point C

$$E_1 = \frac{GM}{(3r)^2}$$

$$E_2 = \frac{G(2M)}{(3r)^2}$$

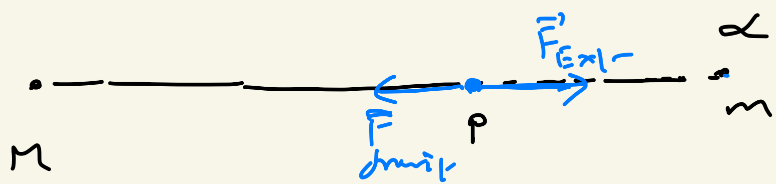
$$E_C = \frac{GM}{9r^2} + \frac{2GM}{9r^2}$$

$$\boxed{E_C = \frac{3GM}{9r^2} = \frac{GM}{3r^2}}$$

# Gravitational Potential (scalar function)

G.P at a point P in gravitational field is defined as work done by the external agent per unit mass to bring the unit mass from infinity to point 'P' very slowly [without changing K.E]

$$\boxed{V_P = \frac{W_{Ext(\infty P)}}{m} = -\frac{W_{gr}}{m}}$$

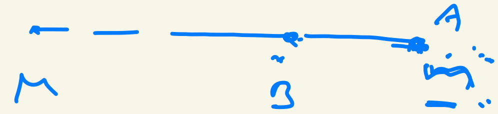



$$\vec{F}_{Ext} + \vec{F}_{gr} = 0 \Rightarrow \vec{F}_{gr} = -\vec{F}_{Ext}$$

→ It is a scalar physical quantity

→ S.I unit is $\frac{J}{kg}$

→ $W_{Ext} = m(V_B - V_A)$




→ Potential at infinity we define it as zero

## Gravitational Potential difference (relation b/w V and E)

Gravitational potential difference b/w two point ($\alpha$ to $\beta$ / A to B) in a gravitational field is defined as negative of line integral of $\vec{E}\cdot\vec{dl}$ from $\alpha$ to $\beta$ or A to B

$$V_\beta - V_\alpha = -\frac{\int_\alpha^\beta \vec{F}\cdot\vec{dl}}{m}$$

*** $\vec{F} = m\vec{E}$

$$\boxed{V_p - V_\alpha = -\int_\alpha^p \vec{E} \cdot \vec{dl}}$$

(relation b/w potential and field)

If two points are A and B (A to B)

$$\boxed{V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dl}}$$

→ The use of this relation we can find P.d from field

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\vec{dl} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

## *** Calculation of Gravitational field from potential:

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$\frac{\partial (\ )}{\partial x}$ partial derivative

$\frac{d(\ )}{dx}$ normal derivative

*** In case of partial Differentiation when you differentiate w.r.t. $x$, $y$ and $z$ variables should be considered as constants

$\frac{\partial (\ )}{\partial z}$ — $x, y$ are constants

$\frac{\partial (\ )}{\partial x}$ — $y, z$ are constants

$\frac{\partial (\ )}{\partial y}$ — $x, z$ are constants

Example:- Gravitational potential in a particular region as a function of $x, y, z$ is given by $V = xy + yz + zx$ calculate the gravitational field in the same region as a function of $x, y$ and $z$ and also find gravitational field at point $(1, 1, 1)$

Take $x, y, z$ are in m and $V$ is in $\frac{J}{kg}$

sol:- i)

$$E_x = -\frac{dV}{dx}$$

$$\boxed{\frac{d(x^n)}{dx} = n x^{n-1}}$$

$$E_x = -\frac{d}{dx}(xy + yz + zx)$$

constant

$$F_x = -\frac{\partial(xy)}{\partial x} - \frac{\partial(yz)}{\partial x} - \frac{\partial(zx)}{\partial x}$$

$$F_x = -y - 0 - z$$

$$\boxed{F_x = -y - z}$$

$Z$

$$F_y = -\frac{\partial}{\partial y}(xy + yz + zx)$$

$$F_y = -\frac{\partial(xy)}{\partial y} - \frac{\partial(yz)}{\partial y} - \frac{\partial(zx)}{\partial y}$$

$$\boxed{F_y = -x - z}$$

$$F_z = -\frac{\partial}{\partial z}(xy + yz + zx)$$

$$F_z = -\frac{\partial(xy)}{\partial z} - \frac{\partial(yz)}{\partial z} - \frac{\partial(zx)}{\partial z}$$

$$\boxed{F_z = -y - x}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$\boxed{\vec{E} = -(y+z)\hat{i} - (x+z)\hat{j} - (x+y)\hat{k}}$$

ii) Field at $\begin{array}{ccc} x & y & z \\ (1, & 1, & 1) \end{array}$

$$\boxed{\vec{E} = -2\hat{i} - 2\hat{j} - 2\hat{k}}$$

problem

$V = -(xyz)$ Find field in the same region?

Sol:- $E_x = -\dfrac{\partial V}{\partial x}$ $E_y = -\dfrac{\partial V}{\partial y}$ , $E_z = -\dfrac{\partial V}{\partial z}$

$$E_x = -\left[\frac{\partial(-xyz)}{\partial x}\right] \qquad E_y = -\left[\frac{\partial(-xyz)}{\partial y}\right]$$

$$E_x = +yz \qquad E_y = +xz \qquad E_z = +xy$$

$$\boxed{\vec{E} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}}$$

Problem:- The gravitational field due to a mass Distribution is given by $|\vec{E}| = kx^{-3/2}$ along x-direction where k is constant. Taking gravitational potential to be zero at Infinity. Find its value at a distance x ?

sol:-

$$|\vec{F}| = kx^{-3/2}$$

$$V_p - V_\infty = -\int_\infty^p \vec{F}(x)\,dx$$

$$V_p - 0 = -\int_\infty^x kx^{-3/2}\,dx$$

$$V_p = -k\left[-\frac{2}{\sqrt{x}}\right]_\infty^x$$

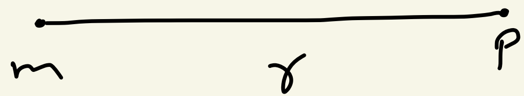
$$\boxed{V_p = \frac{2k}{\sqrt{x}}}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}$$

$$\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}$$

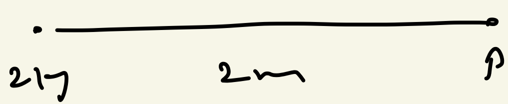
# Gravitational potential due to a point mass




$$\boxed{V_p = -\frac{Gm}{r}}$$

Gravitational

Eg:




Find potential at point 'P'

V – Potential

F – field

Sol:

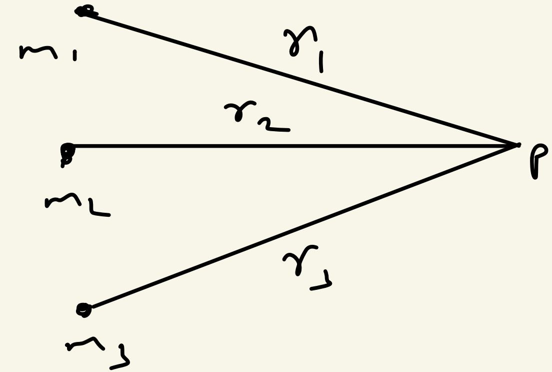
$$V_p = -\frac{G(\cancel{2})}{(\cancel{2})}$$

$$\boxed{V_p = -6.67\times10^{-11}\ \frac{J}{kg}}$$

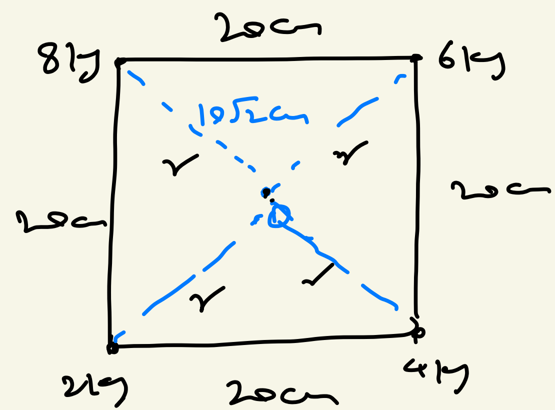
## Due to many point masses (algebraic sum)

$$V_p = V_1 + V_2 + V_3$$

$$\boxed{V_p = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} - \frac{Gm_3}{r_3}}$$




Eg:- Find potential at 'O'




Sol: $V_o = V_1 + V_2 + V_3 + V_4$

$$V_o = -\frac{G}{r}(m_1 + m_2 + m_3 + m_4)$$

$20\sqrt{2}$

$$V_o = \frac{-6.67\times10^{-11}}{(10\sqrt{2}\times10^{-2})}(2+4+6+8)$$

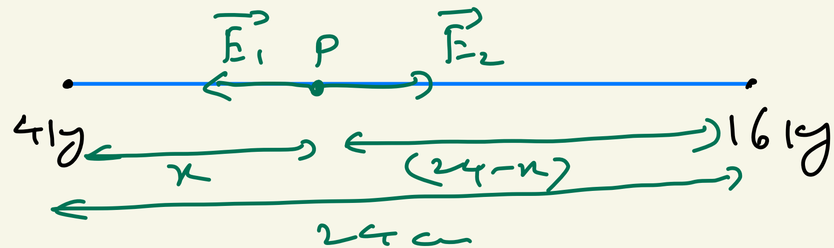
$$V_o = \frac{-6.67\times10^{-10}}{\sqrt{2}}(20)$$

$$V_o = -6.67\times10^{-9}\times\sqrt{2}\ J/kg$$

$$\boxed{V_o = -6.67\sqrt{2}\times10^{-9}\ \frac{J}{kg}}$$

Eg:- Two masses 4 kg and 16 kg are kept at a separation distance 20 cm. Find gravitational potential due to two masses along the line joining between them where gravitational field due to two masses is zero?

soln:




$$|\vec{E}_1| = |\vec{E}_2|$$

$$\frac{\cancel{G}\times \cancel{4}}{x^2} = \frac{\cancel{G}\times \cancel{16}\,4}{(24-x)^2}$$

$$\left(\frac{24-x}{x}\right)^2 = 4$$

$$\frac{24-x}{x} = 2$$

$$24 - x = 2x$$

$$24 = 3x$$

$$\boxed{x = 8\text{ cm}}$$

$$V_p = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$$

$$V_p = -\frac{G\times \cancel{4}}{2\cancel{8}\times 10^{-2}} - \frac{G\times \cancel{16}}{\cancel{16}\times 10^{-2}}$$

$$V_p = -100G\left[\frac{1}{2}+1\right]$$

$$V_p = -\frac{300G}{2} = -150G$$

$$\boxed{V_p = -150\times 6.67\times 10^{-11}\ \text{J/kg}}$$

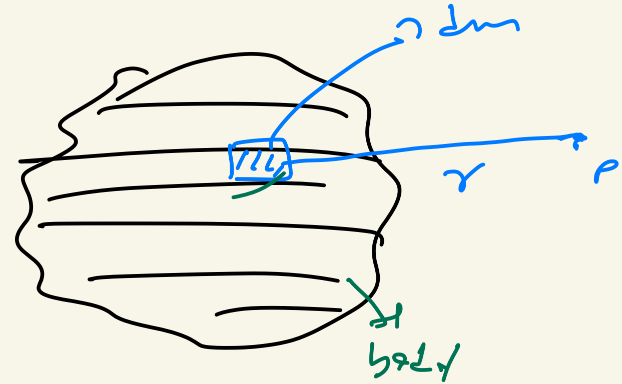
$$V_p = \underline{1000.5\times 10^{-11}}\ \text{J/kg}$$

$$V_p \simeq 10^{-8}\ \text{J/kg}$$

→ If the field is zero at a point Gravitational potential need not be zero

# Gravitational potential due to continuous mass Distribution

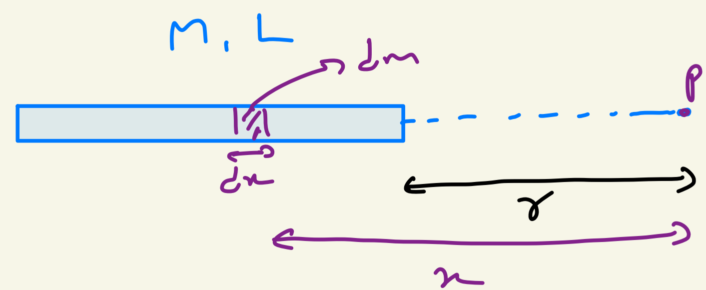
$$\boxed{\int dV = -\int \frac{G\,dm}{r}}$$




## Thin rod :-

$$dV = -\frac{G\,dm}{x}$$

$$dm = \frac{M}{L}\,dx$$



$$\int dv = -\int_{r}^{r+L} \frac{GM\,dx}{Lx}$$

$$v = -\frac{GM}{L}\Big[\ln x\Big]_{r}^{r+L}$$

$$v = -\frac{GM}{L}\Big[\ln(r+L) - \ln(r)\Big]$$

$$\boxed{v = -\frac{GM}{L}\left[\ln\frac{(r+L)}{r}\right]}$$

$$\boxed{v = \frac{GM}{L}\left[\ln\left(\frac{r}{r+L}\right)\right]}$$

$$\boxed{\ln m - \ln n = \ln\frac{m}{n}}$$

$$\boxed{\ln\left(\frac{m}{n}\right) = -\ln\left(\frac{n}{m}\right)}$$

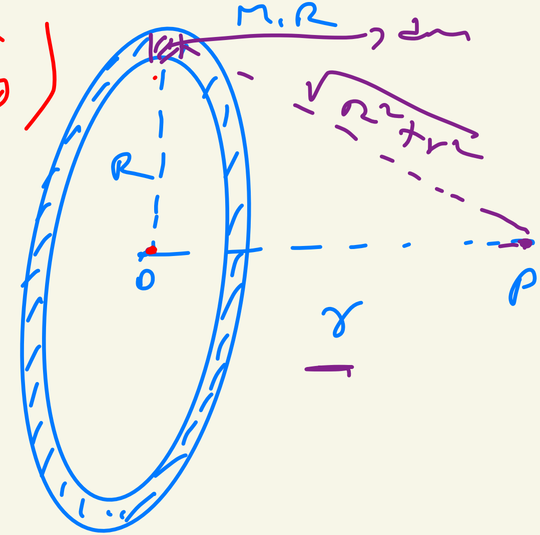
Ring :- (G. Potential due to ring at a point P on the axis of the ring)

$$\int dV = \int -\frac{G\,dm}{\sqrt{R^2+r^2}}$$

$$V_P = -\frac{G}{\sqrt{R^2+r^2}}\int dm$$

*

$$\boxed{V_P = -\frac{GM}{\sqrt{R^2+r^2}}}$$



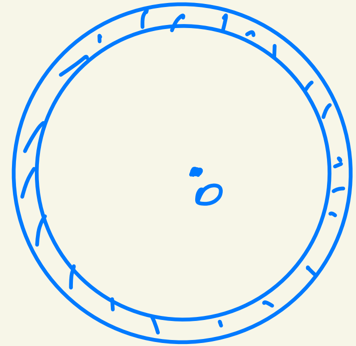

M - Mass of the ring

R - Radius " "

r - Distance from point 'O' to the point P on the axis of the ring

G - universal gravitational constant

NOTE:- (1) Gravitational Potential at the centre of the ring




$$\boxed{r = 0}$$

$$\boxed{V_0 = -\frac{GM}{R}}$$ *

(2) Gravitational potential in terms of Linear mass density ($\lambda$) on the axis of the ring

$$\lambda = \frac{M}{2\pi R}$$

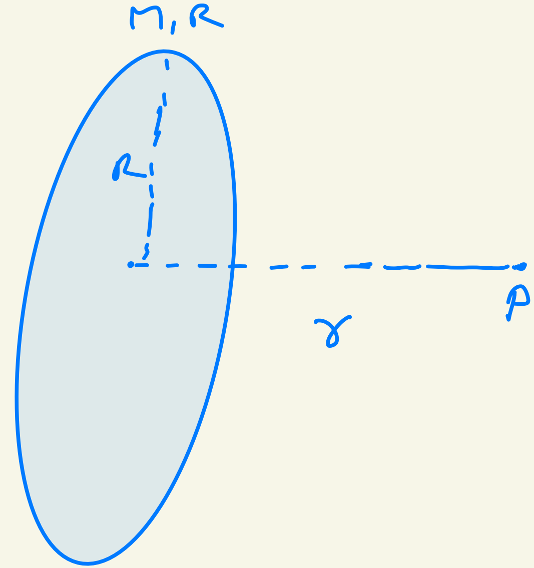
$$\boxed{V_P = -\frac{G\lambda 2\pi R}{\sqrt{R^2+r^2}}}$$

At the center of the ring $r = 0$

$$\boxed{V_0 = -G\lambda 2\pi}$$

## Disc:

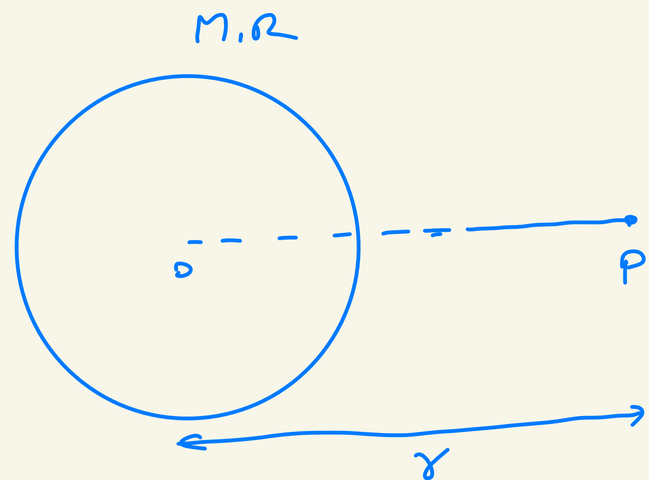
$$\boxed{V_p = -\frac{2GM}{R^2}\left[\sqrt{R^2+r^2} - r\right]}$$



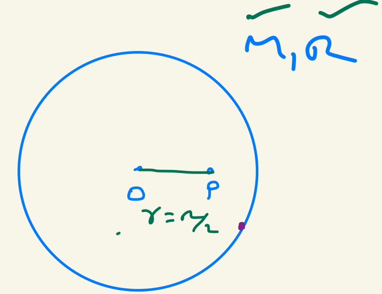

## Hollow sphere: (spherical shell)

case i. $r \geq R$

$$\boxed{V_p = -\frac{GM}{r}}$$



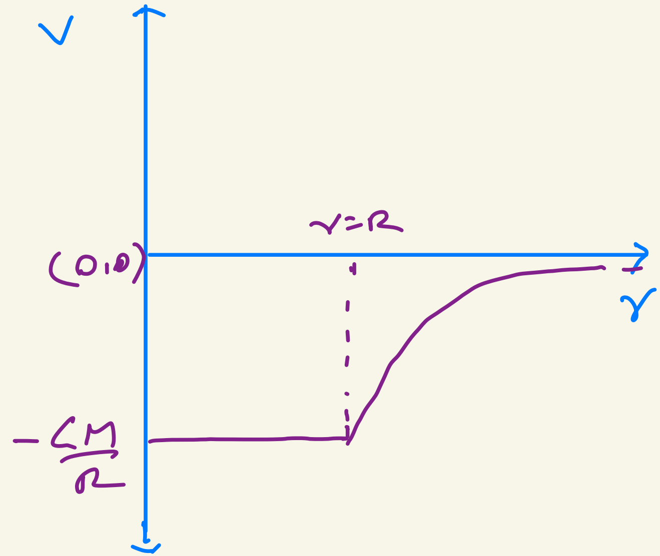
## case ii   $r \le R$




→ Gravitational potential due to spherical shell at any point inside the spherical shell is constant. It is equal potential of the shell.

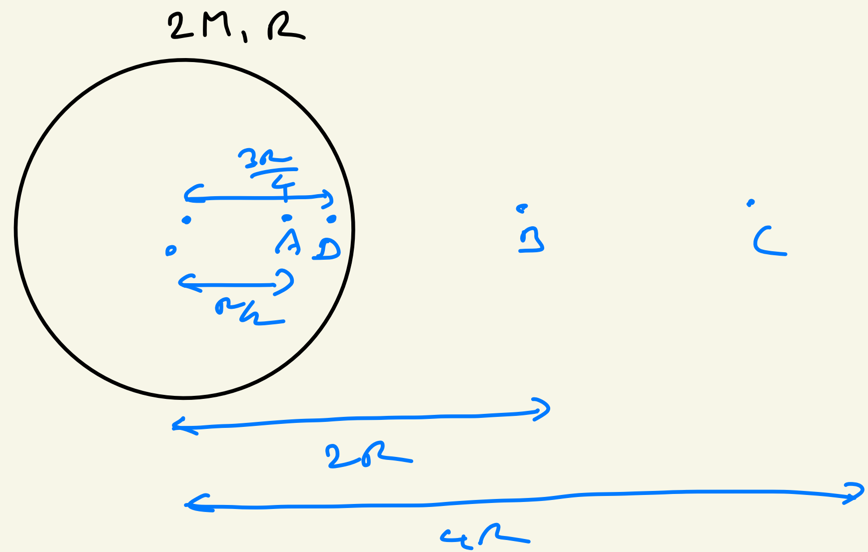
$$\boxed{V_{shell} = -\frac{GM}{R}}$$

$$\boxed{V_{at\ any\ point} = -\frac{GM}{R}}$$




NOTE: > Gravitational Potential difference b/w any two points inside the shell is equal to zero.

Eg:- Find gravitational potential due to spherical shell of mass 2M and radius R at point A, B and C as shown in the figure.




Sol:- At point 'A'

$$V_A = -\frac{2GM}{R}$$

$$\boxed{V_D = -\frac{2GM}{R}}$$

At point B

$$V = -\frac{Gm}{r}$$

$m = 2M$

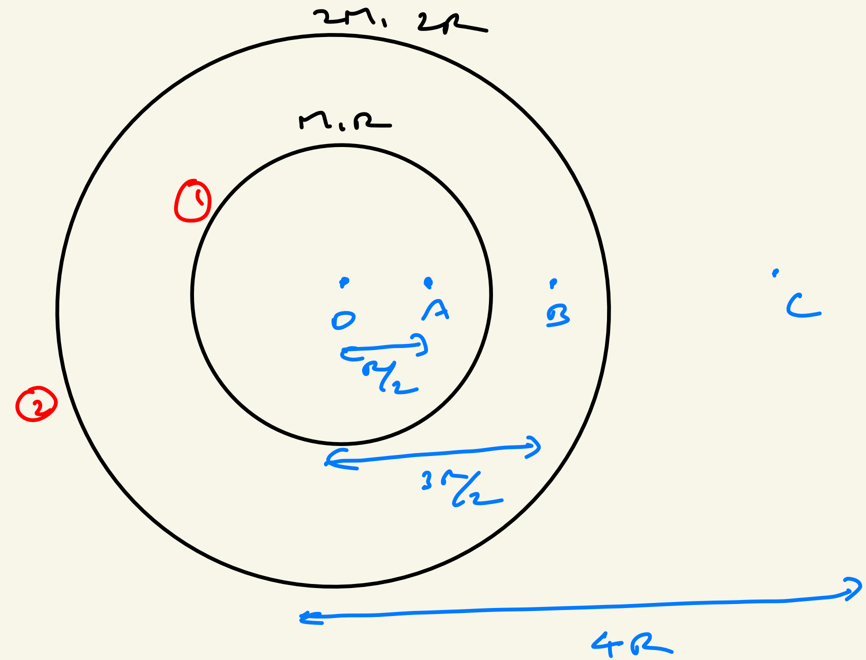
$r = 2R$

$$V_B = -\frac{G(2M)}{2R}$$

$$\boxed{V_B = -\frac{GM}{R}}$$

$$\boxed{V_C = -\frac{G(2M)}{4R}}$$

Eg:- Find potential due to two concentric spherical shells at point A, B and C




Sol:- At point 'A'

$$V_1 = -\frac{GM}{R}$$

$$V_2 = -\frac{2GM}{2R} = -\frac{GM}{R}$$

$$V_A = V_1 + V_2$$

$$V_A = -\frac{GM}{R} - \frac{GM}{R} = -\frac{2GM}{R}$$

## At point B

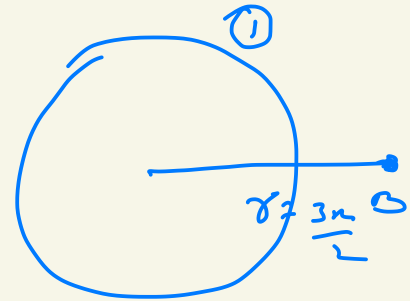
$$V_1 = \frac{-2GM}{3R} = \frac{-2GM}{3r}$$

$$V_2 = \frac{-GM}{r}$$

$$V_B = V_1 + V_2$$

$$= \frac{-2GM}{3r} - \frac{GM}{r}$$

$$\boxed{V_B = \frac{-5GM}{3r}}$$




## At point C