ELASTICITY

P-AK

Elasticity:

The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces is called elasticity.

Terms Related to Elasticity:-

Stress:-

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

→ S.I unit of stress is N/m^2 (or) Pa

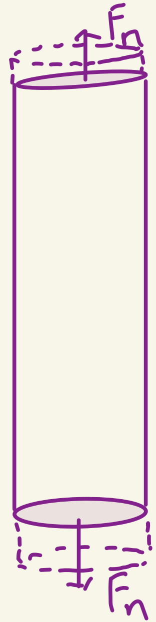
Strain:

$$\text{strain} = \frac{\text{change in configuration (length or volume)}}{\text{original configuration (length or volume)}}$$

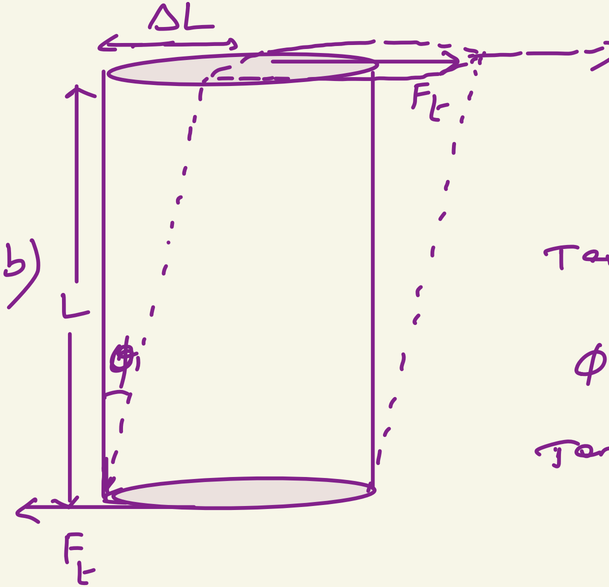
→ No unit

→ There are three ways by which we can change the dimension of the body by applying forces on the body.

a)



b)

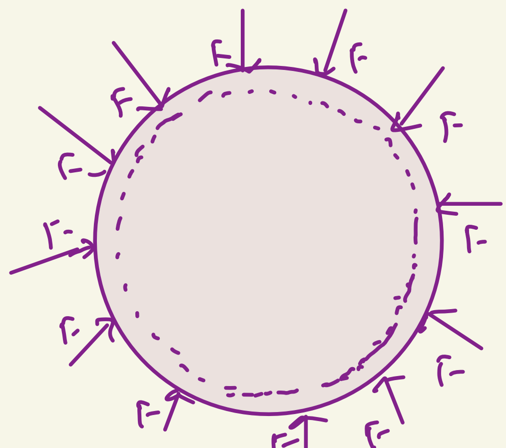


$$\tan\phi = \frac{\Delta L}{L}$$

$\phi \rightarrow$ is very small

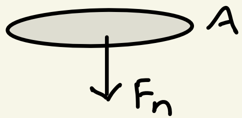
$$\tan\phi \simeq \phi$$

c)



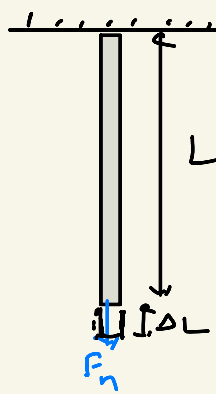
→ Stress corresponding to Figure (a) is called Longitudinal stress

$$\boxed{\text{Longitudinal stress} = \frac{F_n}{A}}$$



→ Strain corresponding to Figure (a) is called Longitudinal strain

$$\boxed{\text{Longitudinal strain} = \frac{\Delta L}{L}}$$



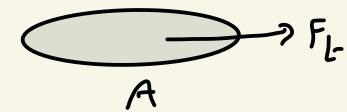
ΔL - change in length of the wire (or) rod

L - original " " " "

→ Stress corresponding to Figure (b) is called Lateral stress

Tangential (or) parallel

$$\boxed{\text{Lateral stress} = \frac{F_t}{A}}$$



→ Strain corresponding to Figure (b) is called Lateral strain

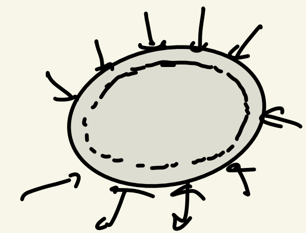
$$\boxed{\text{Lateral strain} = \frac{\Delta L}{L}}$$

ΔL - Displacement of upper layer w.r.to lower layer

L - Distance b/w two layers

→ Stress corresponding to Figure (c) is called
Bulk stress

$$\boxed{\text{Bulk stress} = \frac{F_\perp}{A} = \text{Pressure}}$$



→ strain corresponding to Figure (c) is called
Bulk strain

$$\boxed{\text{Bulk strain} = \frac{\Delta V}{V}}$$

ΔV - change in volume
V - original volume

Hooke's law :-

within the elastic limit, stress is directly proportional to strain i.e

stress $\propto$ strain

$$\boxed{\text{stress} = K \text{ strain}}$$

K - Modulus of Elasticity

Figure (a) - Young's modulus of Elasticity

Figure (b) - Shear modulus of Elasticity

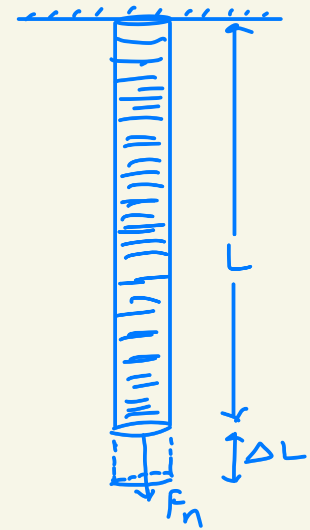
Figure (c) - Bulk modulus of Elasticity

Young's modulus of elasticity (Y)

$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$Y = \frac{F_n / A}{\Delta L / L} = \frac{F_n L}{A \Delta L}$$

$$\boxed{Y = \frac{FL}{A \Delta L}}$$



$A = \pi r^2$

$$A = \frac{\pi d^2}{4}$$

d – Diameter of the wire

→ S.I unit of Y.M.E is N/m^2

Shear modulus of elasticity (or) modulus of rigidity (η)

$$\boxed{\eta = \frac{\text{shearing stress}}{\text{shearing strain}}} \quad \text{(or)} \quad \boxed{\eta = \frac{\text{Lateral stress}}{\text{Lateral strain}}}$$

$$\eta = \left(\frac{\frac{F_{Tangential}}{A}}{\phi} \right) = \frac{F_{Tangential}}{A\phi}$$

$$\tan\phi \simeq \phi = \frac{\Delta L}{L} \text{ — shearing strain}$$

→ S.I units of of S.M.E N/m^2

Bulk modulus of elasticity 'K' or B

With in the elastic limit the ratio of the volume stress and the volume strain is called B.M.E.

$$K \text{ (or) } B = \frac{\text{volume stress}}{\text{volume strain}}$$

$$\boxed{K \text{ (or) } B = \frac{F/A}{-\Delta V/V} = \frac{\Delta P}{-\Delta V/V}}$$

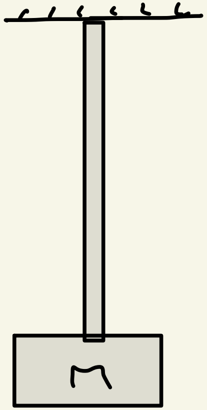
→ The minus sign indicates a decrease in volume with an increase in stress

→ $\Delta V/V$ - fractional change in volume

→ ΔP - change in pressure

NOTE:- reciprocal of Bulk modulus is called compressibility (C) (i.e $C = \frac{1}{B}$)

problem:- A block of mass M is suspended by a thin copper wire of length L and radius 'r' as shown in the figure. Find elongation in the wire due to weight of the block? Y - Young's of the copper, g - acceleration due to gravity?



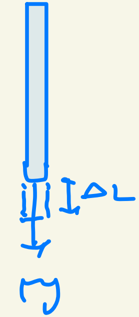
Sol:-

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{\Delta L A}$$

$$\Delta L = \frac{FL}{AY}$$

$$\boxed{\Delta L = \frac{MgL}{\pi r^2 Y}}$$

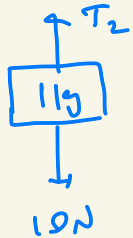
Ans

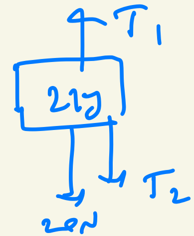


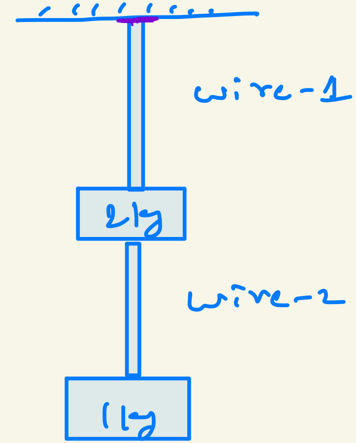
problem:-

One end of the wire is fixed to ceiling and a load of 2kg hangs from other end. A similar wire is attached to bottom of the load and another load of 1kg hangs from this lower wire. Area of cross section of each wire is $0.005\ cm^2$ and $Y = 2\times10^{11}\ N/m^2$, $g = 10\ m/s^2$. Find longitudinal strain in two wires?

Soln:







$T_2 = 10N$ $\qquad$ $T_1 = 30N$

$$\text{Longitudinal strain (1)} = \frac{F}{AY}$$

$$\text{Longitudinal strain (1)} = \frac{T_1}{AY}$$

$$\text{Longitudinal strain (1)} = \frac{\cancel{30}\,15}{0.005\times10^{-4}\times\cancel{2}\times10^{11}}$$

$$\boxed{\text{L. strain (1)} = \frac{15}{5\times10^{4}} = 3\times10^{-4}}$$

$$L \cdot \text{Strain}(2) = \frac{T_2}{A Y_1}$$

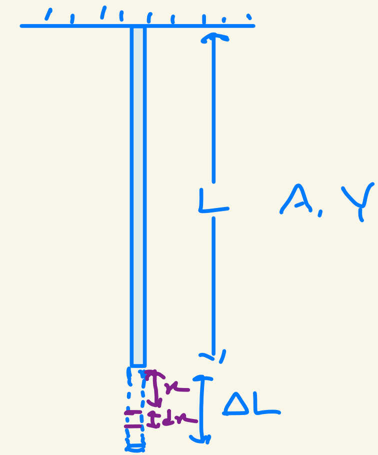
$$'' = \frac{10}{0.005\times10^{-4}\ 2\times10^{11}}$$

$$= \frac{10}{5\times10^{4}\times2}$$

$$= 10\times10^{-5}$$

$$\boxed{L \cdot \text{Strain}(2) = 10^{-4}}$$

Elastic energy stored in stretched wire:-



$$Y = \frac{FL}{xA}$$

$$F(x) = \left(\frac{YA}{L}\right)x$$

$$dW = F(x)\,dx$$

$$\int dW = \int_0^{\Delta L} \left(\frac{YA}{L}\right) x\,dx$$

$$W = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^{\Delta L} \Rightarrow \frac{YA}{L}\left(\frac{\Delta L^2}{2}\right)$$

$$U = \frac{1}{2}\frac{YAL}{L^2}\Delta L^2 \Rightarrow \frac{1}{2}Y(AL)\left(\frac{\Delta L}{L}\right)^2$$

$$U = \frac{1}{2} \times \frac{\text{stress}}{\cancel{\text{strain}}} (AL)(\text{strain})^{\cancel{2}}$$

$$\boxed{U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}}$$

$$U = \frac{1}{2} \times (\text{strain})^2 \times \text{volume}$$

$$U = \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume}$$

→ Elastic energy stored per unit volume (u)

$$\boxed{u = \frac{U}{\text{volume}} = \frac{1}{2} \times \text{stress} \times \text{strain}}$$

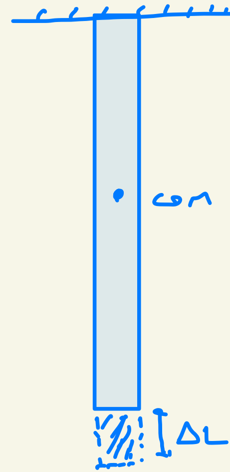
→ $$u = \frac{1}{2} \times (\text{strain})^2 \times Y$$

→ $$u = \frac{1}{2} \times \frac{(\text{stress})^2}{Y}$$

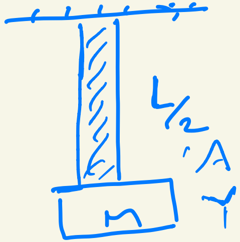
Elongation in a uniform rod due to its own weight:-

$$Y = \frac{FL}{A\,\Delta L}$$

$$\Delta L = \frac{FL}{AY}$$



$M, L, Y, A \Rightarrow$

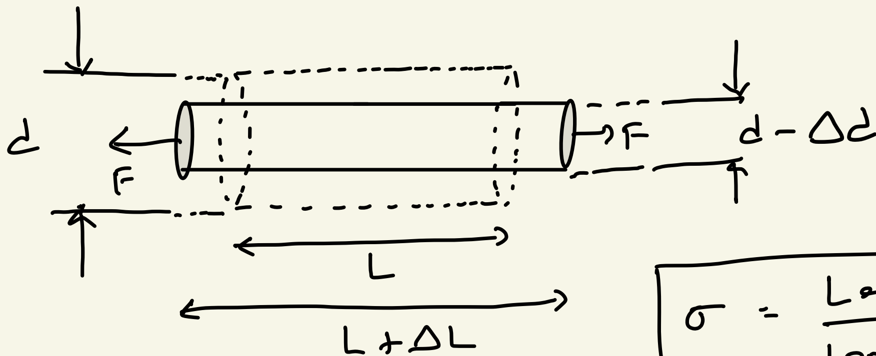


$$\Delta L = \frac{mg\,(L/2)}{AY}$$

$$\boxed{\Delta L = \frac{mgL}{2AY}}$$

Poisson's ratio :- (σ)

It is defined as the ratio of the lateral strain to longitudinal strain and is constant within the limits of proportionality.



$$\boxed{\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}}$$

$$\boxed{\sigma = - \frac{\Delta d / d}{\Delta L / L}}$$

→ negative sign indicates that σ is positive. It is dimension less quantity

→ 'σ' value lies b/w

-1 to 0.5 (Theoritical value)

0 to 0.5 (Experimental value)

problem:-

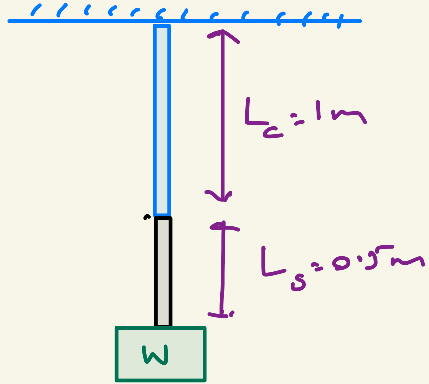
A copper wire of length 1m and a steel wire of length 0.5m having equal cross sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1mm. If the Young's modulii of copper and steel are respectively $1\times10^{11}\ N/m^2$ and $2\times10^{11}\ N/m^2$

The total extension of the composite wire is

a) 1.75 mm   b) 2 mm   c) 1.5 mm   d) 1.25 mm

sol'n

$$\Delta l_{copper} = \frac{F L_{copper}}{A_{copper} Y_{copper}} \quad — (1)$$



$$\Delta l_{steel} = \frac{F L_{steel}}{A_s Y_s} \quad — (2)$$

$$\frac{1\times10^{-3}}{\Delta l_{steel}} = \left(\frac{L_c}{L_s}\right)\left(\frac{Y_s}{Y_c}\right)$$

$$\frac{1\times10^{-3}}{\Delta l_{steel}} = \left(\frac{1}{0.5}\right)\left(\frac{2\times\cancel{10^{11}}}{1\times\cancel{10^{11}}}\right)$$

$$\Delta l_{steel} = \frac{1 \times 10^{-3}}{4} m$$

$$\Delta l_{steel} = 0.25 mm$$

$$\Delta l_{Total} = \Delta l_{steel} + \Delta l_{c}$$

$$\boxed{\Delta l_{Total} = 1.25 mm}$$

Problem:-

Two wires are made of the same material and have the same volume. However wire 1 has cross sectional area A and wire 2 has cross sectional area 3A. If the length of wire 1 is increases by Δx on applying force F How much force is needed to stretch wire 2 by the same amount ?

a) $6F$ ~~b)~~ $9F$ c) F d) $4F$

sol:-

$$Y_1 = \frac{F_1 L_1}{A_1 \Delta L_1}$$

$$Y_2 = \frac{F_2 L_2}{A_2 \Delta L_2}$$

$$\frac{F_1 L_1}{A_1 \Delta L_1} = \frac{F_2 L_2}{A_2 \Delta L_2}$$

$$A_1 L_1 = A_2 L_2$$

$$A L_1 = 3A L_2$$

$$\frac{L_1}{L_2} = 3$$

$$\frac{F L_1}{A \cancel{\Delta L}} = \frac{F_2 L_2}{3A \cancel{\Delta L}}$$

$$\frac{F}{F_2} = \frac{L_2}{L_1} \times \frac{1}{3}$$

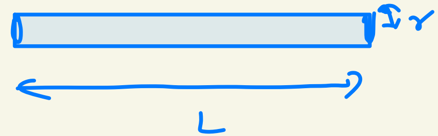
$$\frac{F}{F_2} = \frac{1}{9} \Rightarrow \boxed{F_2 = 9F}$$

Problem: A uniform wire ($Y = 2\times10^{11}\ N/m^2$) is subjected to longitudinal tensile stress of $5\times10^{7}\ N/m^2$. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to ?

a) 1×10^{-4} b) 1.5×10^{-4} c) 0.25×10^{-4} d) 5×10^{-4}

Sol:-

$$V = \pi r^2 L$$



$$\frac{\Delta V}{V} = \left(2\frac{\Delta r}{r}\right) + \left(\frac{\Delta L}{L}\right)$$

$$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain}\left(\frac{\Delta L}{L}\right) = \frac{\text{stress}}{Y} = \frac{5\times10^{7}}{2\times10^{11}}$$

$$\frac{\Delta L}{L} = 2.5\times10^{-4}$$

$$\frac{0.02}{100} = 2\left(\frac{\Delta r}{r}\right) + 2.5\times10^{-4}$$

$$2\frac{\Delta r}{r} = 2\times10^{-4} - 2.5\times10^{-4}$$

$$\frac{F}{A} \cdot \frac{\Delta l}{l}$$

$$2\frac{\Delta r}{r} = -0.5\times10^{-4}$$

$$\frac{\Delta r}{r} = -0.25\times10^{-4}$$

Problem:-

$A = 10^{-6}\,m^2$, $L = 1.5\,m$, $Y = 2\times10^{11}\,N/m^2$, $\Delta L = 4\,mm$

Find Elastic energy stored in the wire?

Sol:-

$$U = \frac{1}{2}\times \text{stress} \times \text{strain} \times \text{volume}$$

1.07 Joules

$$U = \frac{1}{2} \times (\text{Strain})^2 \times Y \times \text{volume}$$

$$U = \frac{1}{2} \times \left(\frac{4 \times 10^{-3}}{1.5}\right)^2 \times 2 \times 10^{11} \times 10^{-6} \times 1.5$$

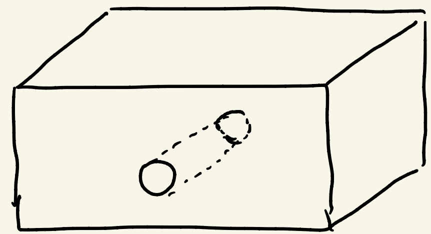
$$U = \frac{1}{\cancel{2}} \times \frac{16 \times \cancel{10^{-6}}}{1.5 \times \cancel{1.5}} \cancel{\times 2} \times \overset{10^{-1}}{\cancel{10^{11}}} \times \cancel{10^{-6}} \times \cancel{1.5}$$

$$U = \frac{16}{1.5} \times 10^{-1} = \frac{16}{15} \text{ Joules}$$

$$15 \overline{)\,16\,}(1.06$$
$$\underline{15}$$
$$100$$

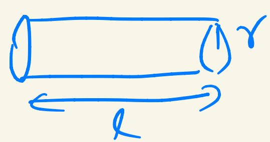
$$\boxed{U = 1.06 \text{ Joules}}$$

Problem: calculate the force needed to punch a 1.46 cm diameter hole in steel plate of 1.27 cm thick shown in figure. The shear stress of steel is 345×10^6 N/m²



Sol:- $$\text{shear stress} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Force} = \text{shear stress} \times \text{Area (surface Area)}$$



Surface Area = $2\pi r l$

Force $= S \cdot S \times 2\pi r \times t$

$=$ shear stress $\times$ π(diameter) $\times$ thickness

Force $= 345 \times 10^{6} \times \pi \times (1.46 \times 10^{-2}) \times 1.27 \times 10^{-2}$

$$\boxed{\text{Force} = 2 \times 10^{5}\ N}$$

Problem:- A steel wire of 4 cm in length is stretched through 2 mm. The cross sectional area of the wire is $2\,mm^2$. $Y_{steel} = 2 \times 10^{11}\ N/m^2$ i) Find energy density of wire

ii) Elastic potential energy stored in the wire sol :-

i) 2.5×10^4 J/m^3

ii) 0.20 Joule

problem:- A fractional change in volume of the body is 1% when a pressure of 2×10^7 N/m^2 is applied on the body. Find bulk modulus and also compressibility?

Sol:-

$$B = -\frac{P}{\Delta V/V}$$

$$\frac{\Delta V}{V} = \frac{1}{100}$$

$$B = \frac{2\times10^7}{\frac{1}{100}}$$

$$\boxed{B = 2\times10^9 \text{ N/m}^2}$$

$$\text{compressibility} = \frac{1}{B} = \frac{1}{2\times10^9} = 0.5\times10^{-9} \text{ m}^2/\text{N}$$

$$= 5\times10^{-10} \text{ m}^2/\text{N}$$

problem: A 3 cm long copper wire is stretched to increase in length by 0.3 cm. Find the lateral strain in the wire, if the poission ratio of the copper is 0.26

sol:

$$\sigma = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

$$\text{Lateral strain} = \sigma \times \text{Longitudinal strain}$$

$$= \sigma \times \frac{\Delta L}{L}$$

$$= 0.26 \times \frac{0.3}{3}$$

$$\boxed{\text{Lateral strain} = 0.026}$$