Mathematics

PART - A

SINGLE ANSWER CORRECT:

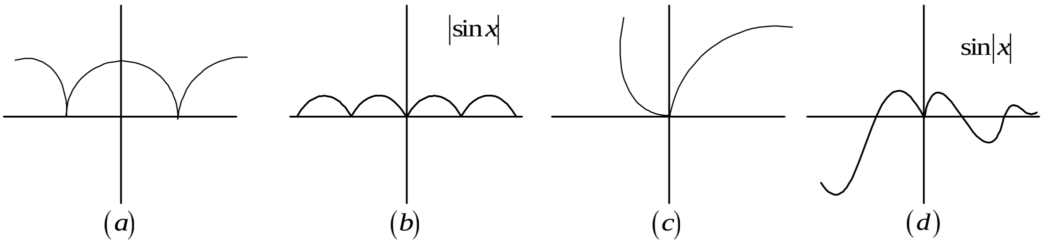
1. Which of the following function is differentiable at $x=0$?

a) $|\cos x|$ b) $|\sin x|$ c) $|x^3|$ d) $\sin|x|$

1. a

Concept code: M120405

Sol:



2. If the equation of tangent to the curve $y^2 = ax^3 + b$ at $(2,3)$ is $y = 4x - 5$. Then

a) $a=0$ b) $a=1$ c) $b=1$ d) $b=-7$

2. d

Concept code: M120601

Sol: $y = 4x-5,\ \left.\dfrac{dy}{dx}\right|_{(2,3)} = 2a \Rightarrow a = 2$

$(2,3)$ is on curve. $9 = 16 + b \rightarrow b = -7$

3. If $f(x) = \begin{cases} \dfrac{\sin(a+2)x + \sin x}{x}; & x<0 \\ b\ ; & x=0 \\ \dfrac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}}, & x>0 \end{cases}$ is continuous at $x=0$, then $a+2b$ is equal to

a) – 2 b) 1 c) 0 d) – 1

3.c

Concept code: M120401

Sol. $\lim_{x\to 0^+} \frac{(1+3x)^{1/3}-1}{x} = 1$

$\lim_{x\to 0^-} \frac{\sin(a+2)x}{x} + \frac{\sin x}{x} = a+3$

$b = a+3 = 1,\ a = -2,\ b = 1$

$a+2b = 0$

4. If a function $f(x)$ defined by $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$ be continuous for some $a,b,c \in R$

and $f^1(0) + f^1(2) = e$, then the value of a is

a) $\frac{1}{e^2-3e+13}$ b) $\frac{e}{e^2-3e-13}$ c) $\frac{e}{e^2+3e+13}$ d) $\frac{e}{e^2-3e+13}$

4.d

Concept code: M120406

Sol. $ae + \frac{b}{e} = c$ (1)

$9c = 9a + 6c \rightarrow c = 3a$ (2)

$f'(0) + f'(2) = e$

$a - b + 4c = e$ (3)

From (1), (2), (3) $a = \frac{e}{e^2-3e+13}$

5. The equation of tangent to the curve $y = \frac{4}{x^2} + x$ which is parallel to the x – axis is

a) $y=8$ b) $y=0$ c) $y=3$ d) $y=2$

5. C

Concept code: M120601

Sol: $\frac{dy}{dx} = 0 \Rightarrow x = 2$. So equation is $y - 3 = 0(x-2)$

$\Rightarrow y = 3$

6. Let $f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x \in R$, where a,b and d are non-zero real constants. Then

a) f is an increasing function of x

b) f is an decreasing function of x

c) f is neither increasing nor decreasing function of x

d) f^1 is not a continuous function of x

6. a

Concept code: M120604

Sol. $f'(x)=\dfrac{a^2}{\left(a^2+x^2\right)^{3/2}}+\dfrac{b^2}{\left(b^2+(d-x)^2\right)^{3/2}}>0\ \forall\ x\in R$

7. Let $f:R\to R$ be a differential function such that $f(u+v)=f(u)+2v^2+4uv$ for all $u,v\in R$. If $f(1)=3$, then the equation of the normal to the curve $y=f(x)$ at the point $\left(\frac{1}{2},f\left(\frac{1}{2}\right)\right)$ is

a) $2x+4y=7$ b) $3x-y=0$ c) $3x+y=3$ d) $2x-4y=-5$

7.a

Concept code: M120407

Sol. $f(u+v)=f(u)+2v^2+4uv$

Put $u=0$,

$f(v)=f(0)+2v^2$

Put $v=1,\ f(1)=f(0)+2$

$f(0)=1$

$\therefore\ f(v)=1+2v^2$

$f(x)=1+2x^2,$ for all $x\in R$

Normal equation at $\left(\frac{1}{2},\frac{3}{2}\right)$ is $y-\frac{3}{2}=-\frac{1}{2}\left(x-\frac{1}{2}\right)$

$2x+4y=7$

8. If the tangent to the curve $y=\dfrac{x}{x^2-3}$, $x\in R$, $\left(x\neq\pm\sqrt{3}\right)$, at a point $(\alpha,\beta)\neq(0,0)$ on it is parallel to the line $2x+6y-11=0$, then

a) $|6\alpha+2\beta|=19$ b) $|2\alpha+6\beta|=11$ c) $|2\alpha+6\beta|=19$ d) $|6\alpha+2\beta|=9$

8.a

Concept code: M120601

Sol. $m=\left(\dfrac{dy}{dx}\right)_{(\alpha,\beta)}=\dfrac{-3-\alpha^2}{\left(\alpha^2-3\right)^2}$

Tangent is parallel to the line

$2x+6y-11=0$

$\dfrac{-3-\alpha^2}{\left(\alpha^2-3\right)^2}=-\dfrac{1}{3}$

$3\left(3+\alpha^2\right)=\left(\alpha^2-3\right)^2$

Let $\alpha^2=a$

$a^2-9a=0 \to a=0 \ or \ a=9$
$\alpha^2=9(\therefore \alpha \neq 0)$
$\alpha=\pm 3$
If $\alpha=3,\ \beta=\dfrac{\alpha}{\alpha^2-3}=\dfrac{1}{2}$
$|6\alpha+2\beta|=19$

9. The function $f(x)=e^{x+1}(4x^2-16x+11)$ is

a) decreasing in $\left(-\infty,-\frac{5}{2}\right)\cup\left(\frac{1}{2},\infty\right)$
b) decreasing in $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{5}{2},\infty\right)$
c) increasing in $(-\infty,-2)\cup(2,\infty)$
d) increasing in $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

9.d

Concept code: M120604

Sol. $f'(x)=e^{x+1}(2x+1)(2x-5)$

10. The number of normals to the curve $x^2=4y$ passing through point $(1,2)$ is

a) 1 b) 2 c) 3 d) 0

10. a

Concept code: M120601

Sol: $\dfrac{dy}{dx}=\dfrac{x}{2} \Rightarrow \dfrac{y_1-2}{x_1-2}=\dfrac{-2}{x_1} \Rightarrow x_1=2,\ y_1=1$
So, one normal.

<u>COMPREHENSION</u>

Consider the curve $x=1-3t^2,\ y=t-3t^3$. A tangent at point $(1-3t^2, t-3t^3)$ is inclined at an angle θ to the positive x-axis and another tangent at point $P(-2,2)$ cuts the curve again at Q.

11. The value of $\tan\theta+\sec\theta$ is equal to

a) 3t b) t c) $t-t^2$ d) t^2-2t

11.a

Concept code: M120602

Sol. $\dfrac{dy}{dx}=\dfrac{1-9t^2}{-6t}=\tan\theta$
Or $9t^2-6\tan\theta.t-1=0$
Or $3t=\tan\theta\pm\sec\theta$
Or $\tan\theta+\sec\theta=3t$

12. The point Q will be

a) $(1,-2)$ b) $\left(-\frac{1}{3},-\frac{2}{3}\right)$ c) $(-2,1)$ d) $\left(\frac{1}{3},\frac{2}{3}\right)$

12.b

Concept code: M120602

Sol. $P \equiv (-2,2)$ or $t=-1$ or $\left.\frac{dy}{dx}\right|_{t=-1} = -\frac{4}{3}$

Equation of the tangent is $y-2=-\frac{4}{3}(x+2)$. Therefore,

$t-3t^3-2=-\frac{4}{3}(1-3t^2+2)$

Or $(3t-2)(t+1)^2=0$

Or $t=\frac{2}{3}$

Or $Q \equiv \left(-\frac{1}{3},-\frac{2}{3}\right)$

13. The angle between the tangents at P and Q will be

a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{3}$

13.c

Concept code: M120602

Sol. $\left(\frac{dy}{dx}\right)_{t=\frac{2}{3}} = \frac{3}{4}$

$m_P \cdot m_Q = -1$

$\theta = \frac{\pi}{2}$

<u>COMPREHENSION</u>

$f(x)=x^2+xg'(1)+g''(2)$ and $g(x)=f(1)x^2+x\,f'(x)+f''(x)$

14. The value of $f(3)$ is

a) 1 b) 0 c) – 1 d) – 2

14.b

Concept code: M120503

15. The value of $g(0)$ is

a) 0 b) – 3 c) 2 d) – 2

15.c

Concept code: M120503

16. The domain of the function $\sqrt{\dfrac{f(x)}{g(x)}}$ is

a) $(-\infty,1]\cup(2,\ 3]$ b) $(-2,0]\cup(1,\infty)$ c) $(-\infty,0]\cup(2/3,3]$ d) $[2,\ 3]$

16.c

Concept code: M120503

Sol. Let $g'(1)=a,\ g''(2)=b$ (1)

$f(x)=x^2+ax+b,\ f(1)=1+a+b\ \ or\ \ f'(x)=2x+a,\ f''(x)=2$

$g(x)=(1+a+b)x^2+(2x+a)x+2=x^2(3+a+b)+ax+2$

Hence, $g'(1)=2(3+a+b)+a$ (2)

$g''(2)=2(3+a+b)$ (3)

From (1), (2) and (3), we have

$a=2(3+a+b)+a$ and $b=2(3+a+b)$

Or $3+a+b=0$ and $b+2a+6=0$

Hence, $b=0$ and $a=-3$, So, $f(x)=x^2-3x$ and $g(x)=-3x+2$

$\sqrt{\dfrac{f(x)}{g(x)}}$

$\dfrac{x^2-3x}{-3x+2}\geq 0\ \ \Rightarrow x\in(-\infty,0]\cup(2/3,3]$

PART - B

MATCH THE MATRIX:

1. Match the following

Column – I (Curves)		Column – II (Angle between the curves)	
(1)	$y^2=4x\ and\ x^2=4y$	(A)	90^0
(2)	$2y^2=x^3\ and\ y^2=32x$	(B)	Any one of $Tan^{-1}\dfrac{3}{4}$ or $Tan^{-1}\left(16^{1/3}\right)$
(3)	$xy=a^2$ and $x^2+y^2=2a^2$	(C)	0^0
(4)	$y^2=x$ and $x^3+y^3=3xy$ at other than origin	(D)	$Tan^{-1}\dfrac{1}{2}$

1.1 – A,B ; 2 – A,D; 3 – C; 4 – B

Concept code: M120602

Sol.1. $y^2 = 4x$ and $x^2 = 4y$ intersect at points $(0,0)$ and $(4,4)$

$C_1 : y^2 = 4x$ $\qquad$ $C_2 : x^2 = 4y$

$\left.\frac{dy}{dx}\right|_{0,0} = \infty$ $\qquad$ $\left.\frac{dy}{dx}\right|_{0,0} = 0$

Hence, $\tan\theta = 90^0$ at point $(0,0)$

$\left.\frac{dy}{dx}\right|_{(4,4)} = \frac{1}{2}$ $\qquad$ $\left.\frac{dy}{dx}\right|_{(4,4)} = 2$

$$\tan\theta = \left|\frac{2-\frac{1}{2}}{1+2\times\frac{1}{2}}\right| = \frac{3}{4}$$

2. Solving $I : 2y^2 = x^3$ and $II : y^2 = 32x$, we get $(0,0),(8,16)$, *and* $(8,-16)$

At $(0,0)$, $\left.\frac{dy}{dx}\right|_{(0,0)} = 0$ *for* I

At $(0,0)$, $\left.\frac{dy}{dx}\right|_{(0,0)} = \infty$ *for* II

Hence, angle = 90^0

Now, $\left.\frac{dy}{dx}\right|_{(8,16)} = \frac{3x^2}{4y} = \frac{3}{4}\frac{64}{16} = 3$ *for* I

$\left.\frac{dy}{dx}\right|_{(8,16)} = \frac{32}{2y} = \frac{16}{16} = 1$ *for* II

$\therefore\ \tan\theta = \frac{3-1}{1+3} = \frac{2}{4} = \frac{1}{2}$

Therefore, angle between the two curves at the origin is 90^0

3. The two curves are

$xy = a^2$ $\qquad$ (1)

$x^2 + y^2 = 2a^2$ $\qquad$ (2)

Solving (1) and (2), the points of intersection are (a,a) and $(-a,-a)$

Differentiating $(1), dy/dx = -y/x = m_1(say)$

Differentiating $(2), dy/dx = -x/y = m_2(say)$

At both points, $m_1 = -1 = m_2$

Hence, the two curves touch each other.

4. $y^2 = x,\ x^3 + y^3 = 3xy$

For the first curve, $2y\frac{dy}{dx} = 1$ *or* $\left.\frac{dy}{dx}\right|_p = \frac{1}{2y_1}$

Again for the second curve, $\left.\frac{dy}{dx}\right|_p = \frac{y_1 - x_1^2}{y_1^2 - x_1}$

Solving $y^2 = x$ and $x^3 + y^3 = 3xy$

$\therefore\ y_1 = 2^{1/3}$ and $\therefore\ x_1 = 2^{2/3}$

Now, $m_1 = \frac{1}{2^{\frac{4}{3}}}$; $m_2 = \infty$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{1 - \frac{m_1}{m_2}}{\frac{1}{m_2} + m_1}\right| = \left|\frac{1}{m_1}\right| = 2^{\frac{4}{3}} = 16^{\frac{1}{3}}$$

$$\therefore \ \theta = Tan^{-1}\left(16^{\frac{1}{3}}\right)$$

2. Length of normal, sub-normal, tangents and sub-tangents are to be matched at the given points for the function given

	Column – I		**Column – II**
(1)	Value of 'k' so that length of sub-normal at any point of $y = 2^{1-k}x^k$ is constant	(A)	$\frac{1}{\log_e 2}$
(2)	Length of sub-tangent, at any point of the curve $y = 2^x$ is	(B)	2
(3)	Length of sub-normal at any point on the curve $y^2 = 4x$ is	(C)	$\frac{\sqrt{17}}{4}$
(4)	Length of tangent on $y = x^3 + 3x^2 + 4x - 1$ at $x = 0$ is	(D)	$\frac{1}{2}$

2. 1 – D, 2 – A, 3 – B, 4 – C

Concept code: M120601

Sol. Using the formulas

Length of the tangent $\left|\frac{y_1\sqrt{1+m^2}}{m}\right|$

Length of the normal $\left|y_1\sqrt{1+m^2}\right|$

Length of the subtangent $\left|\frac{y_1}{m}\right|$

Length of the subnormal $\left|y_1 m\right|$

Physics

PART - A

SINGLE ANSWER CORRECT:

1. The radiation power of a black body at temperature 200K is 544 watt. Its surface area is $\left(\sigma = 5.67\times10^{8} wm^{-2}k^{-4}\right)$
   a) 6×10^{-2} m^2 b) 6 m^2 c) 6×10^{-6} m^2 d) 6×10^{2} m^2

1. b

CONCEPT CODE : P111208

Sol. $\rho = \sigma AeT^4$

2. The temperature of the sun is about 6000 k and maximum intensity is emitted at 4800A^0 from it. If the sun cools to 3000k, then the maximum intensity would occur at
   a) 4800A^0 b) 2400A^0 c) 9600A^0 d) 19200A^0

2. c

CONCEPT CODE : P111208

Sol. λt = constant

3. An ideal gas is heated at a constant pressure. The fraction of heat supplied used for external work is (γ - adiabatic exponent)
   a) $\frac{1}{\gamma}$ b) $1-\frac{1}{\gamma}$ c) $\gamma - 1$ d) $1-\frac{1}{\gamma^2}$

3. b

CONCEPT CODE : P111205

Sol. $\frac{dw}{dr} = \frac{dr - du}{dr}$

4. The pressure (P) of an ideal gas is changing with its volume (V) $P = xV$, where x is a positive constant. The work done by the gas when its volume is changed from $2V_0$ to $4V_0$ is
   a) $(2V_0)x$ b) $\left(6V_0^2\right)x$ c) zero d) $\left(4V_0^2\right)x$

4. b

CONCEPT CODE : P111205

Sol. $w = \int P\,dv$

5. A metal rod having a coefficient of linear expansion of 2 x 10^{-5} /^{0} C has a length of 100cm at 20^0 C. The temperature at which it is shortened by 10^{-3} m is
a) – 40^0 C b) – 30^0 C c) – 20^0 C d) – 10^0 C

5. b

CONCEPT CODE : P111201

Sol. $\Delta\ell = \ell\alpha\Delta\theta$

6. Four point masses each equal to *m* are arranged at the four corners of a square of side *l*. Force acting on any one mass due to other masses is
a) $\frac{Gm^2}{l^2}\left(\sqrt{2}+\frac{1}{2}\right)$ b) $\frac{Gm^2}{l^2}\left(\sqrt{2}\right)$ c) $\frac{Gm^2}{l^2}\frac{1}{2}$ d) zero

6. a

CONCEPT CODE : P110901

Sol. $F_{net} = \sqrt{2}\frac{Gm^2}{l^2} + \frac{Gm^2}{2l^2}$

7. A number of point masses each equal to *m* are arranged along x-axis at $x = 1\,m$, $x = 2\,m$, $x = 4\,m$ and so on. Another mass M is arranged at the origin. Force experienced by M due to infinite point masses is
a) $GMm\frac{4}{3}$ b) $GMm\frac{3}{4}$ c) $\frac{GMm}{3}$ d) GMm

7. a

CONCEPT CODE : P110901

Sol. $F = F_1 + F_2 + F_3 + ...$

8. Two identical spheres of same radius R and same density ρ are in contact. Force between the spheres is
a) $\frac{4}{9}G\pi^2R^4\rho^2$ b) $G\pi^2R^4\rho^2$ c) $\frac{1}{9}G\pi^2R\rho^2$ d) $GR\rho$

8. a

CONCEPT CODE : P110901

Sol. $F = \frac{Gm_1m_2}{4R^2}$

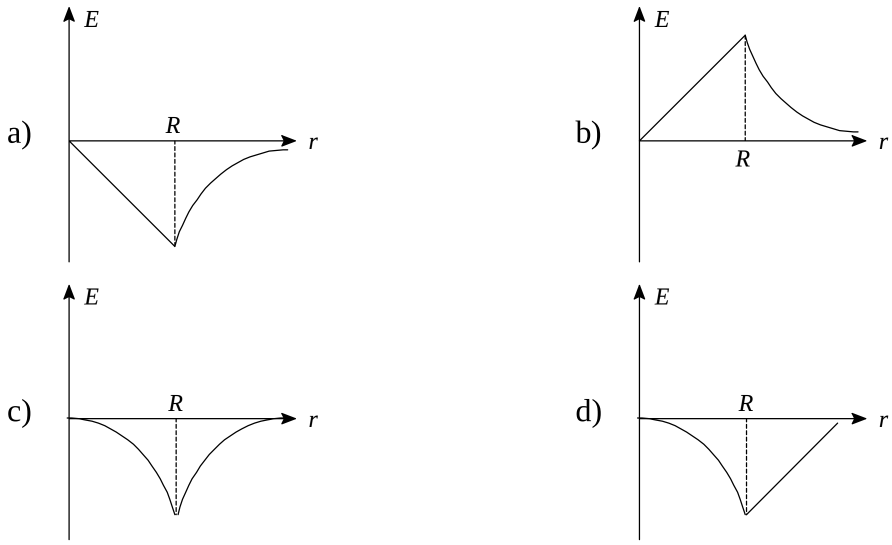
9. Mass M = 1 unit is divided into two parts X and (1 – X). For a given separation the value of X for which the gravitational force between them becomes maximum is
a) $\frac{1}{2}$ b) $\frac{3}{5}$ c) 1 d) 2

9. a

CONCEPT CODE : P110901

Sol. $F = \dfrac{G \times m(1-x)mx}{R^2}$ is max when $x = \dfrac{1}{2}$

10. The magnitude of gravitation field strength due to a solid sphere (mass M, radius R) varies with distance r from centre as :



10. b

CONCEPT CODE : P110901

Sol. $\vec{g}' = -\dfrac{g\vec{r}}{R}$ for $r \le R$ and $g' = \dfrac{g}{(1+r/R)^2}$ for $r \ge R$

COMPREHENSION

Three identical particles each of mass m, are placed at the three corners of an equilateral triangle of side 'a'

11. Magnitude of force exerted by this system on another particle of mass m placed at the midpoint of any side of the triangle is

a) $\dfrac{Gm^2}{a^2}$ b) $\dfrac{2}{3}\dfrac{Gm^2}{a^2}$ c) $\dfrac{3}{4}\dfrac{Gm^2}{a^2}$ d) $\dfrac{4}{3}\dfrac{Gm^2}{a^2}$

11. d

CONCEPT CODE : P110901

Sol.

When the particle at the midpoint of a side (at P)

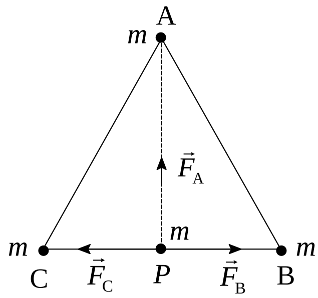
$$\vec{F}_C = -\vec{F}_B$$

And they gaves each other

Hence force experienced by the particle

$$\vec{F} = \vec{F}_A$$

$$|\vec{F}| = |\vec{F}_A| = \frac{Gm^2}{(AP)^2} = \frac{Gm^2}{(a\sin 60)^2} = \frac{4}{3}\frac{Gm^2}{a^2}$$



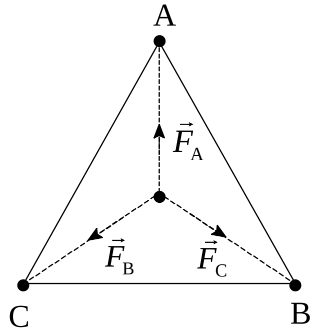
12. The force exerted by this system on another particle of mass m placed at the centre of the triangle

a) zero b) $\sqrt{3}\frac{Gm^2}{a^2}$ c) $\frac{Gm^2}{3a^2}$ d) $\frac{3}{2}\frac{Gm^2}{a^2}$

12. a

<u>CONCEPT CODE</u> : P110901

Sol. If the particle placed at the centre of the triangle, the net force on the particle P due to particles at the corners A, B and C will be zero.



$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C = 0$$

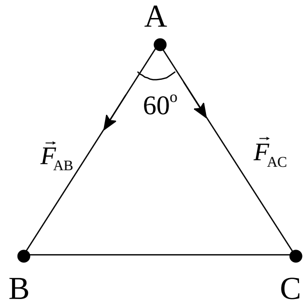
13. The magnitude of gravitational force on any one of the particles due to other two particles

a) $\frac{3}{2}\frac{Gm^2}{a^2}$ b) $\sqrt{3}\frac{Gm^2}{a^2}$ c) $\frac{\sqrt{3}}{4}\frac{Gm^2}{a^2}$ d) $\frac{2}{3}\frac{Gm^2}{a^2}$

13. b

<u>CONCEPT CODE</u> : P110901

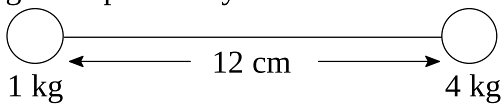
Sol.



$$\vec{F}_{res} = \sqrt{\left(\vec{F}_{AB}\right)^2 + \left(\vec{F}_{AC}\right) + 2\vec{F}_{AB}\vec{F}_{AC}\cos 60} = \sqrt{3}\frac{Gm^2}{a^2}$$

COMPREHENSION

Two particles of mass 1 kg and 4 kg are separated by a distance of 12 cm as shown in figure



14. At what distance from particle 1 kg along the line joining of two particles, the 3rd particle of mass 1 kg should be placed to get resultant gravitational force on the 3rd particle due to remaining 2 particle forces is zero

a) 2 cm b) 4 cm c) 6 cm d) 8 cm

14. b

CONCEPT CODE : P110901

Sol. $\dfrac{G\cdot 1\cdot 1}{x^2} = \dfrac{G\cdot 1\cdot 4}{(12-x)^2}$

$\dfrac{12-x}{x} = 2 \qquad 3x = 12 \;\Rightarrow\; x = 4\text{ cm}$

15. At what distance from 4 kg resultant gravitational field due to two masses, along the line joining between the masses is zero

a) 2 cm b) 4 cm c) 6 cm d) 8 cm

15. d

CONCEPT CODE : P110901

Sol. $\dfrac{G\cdot 4}{x^2} = \dfrac{G\cdot 1}{(12-x)^2}$

$\dfrac{12-x}{x} = \dfrac{1}{4}$

$x = 8\text{ cm}$

16. If mass of the two particles (given 1 kg and 4 kg) reduced to half of them and it's separation is reduced to half of it's original value then force between them becomes

a) zero b) doubled c) Three times d) remains same

16. d

CONCEPT CODE : P110901

Sol. $F' = \dfrac{G\dfrac{m}{2}\dfrac{m}{2}}{\left(\dfrac{d}{2}\right)^2} = F$ (remains same)

PART - B

MATCH THE MATRIX:

1. If M be the mass of the body, R be the Radius of the body, r be the distance of the point.

Column – I		Column – II	
1)	Gravitational field of a ring on its axis at a point of distance r	A)	Zero
2)	Gravitational field due to a uniform thin spherical shall at a point in side the shell	B)	$\frac{GM}{r^2}(r \geq R)$
3)	Gravitational field due to a uniform solid sphere at a point outside of the solid sphere	C)	$\frac{GMr}{R^3}(r < R)$
4)	Gravitational field due to a uniform solid sphere at a point inside of the solid sphere	D)	$\frac{GMr}{(R^2+r^2)^{3/2}}(r > R)$

1. 1 – D; 2 – A; 3 – B; 4 – C.

CONCEPT CODE : P110901

Sol. Conceptual

2. Heat conduction is taking place through the three rods (slabs) each of equal length 'ℓ' and each area of cross section 'A' having thermal conductivity 'K' to each of the rod. If the system is in steady state then

Column – I		Column – II	
1)	If the three rods are in series the effective co-efficient of thermal conductivity	A)	K
2)	If the three rods are in parallel the effective co-efficient of thermal conductivity	B)	$\frac{K}{2}$
3)	If any two rods are in series the effective co-efficient of thermal conductivity	C)	$\frac{K}{3}$
4)	If any two rods are in parallel and the third rod is in series to that parallel combination, the co-efficient of thermal conductivity	D)	$\frac{2}{3}K$

2. 1 – A; 2 – C; 3 – A; 4 – D.

CONCEPT CODE : P111207

Sol. In series

$$\frac{n}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2} + ... + \frac{1}{K_n}$$

In parallel

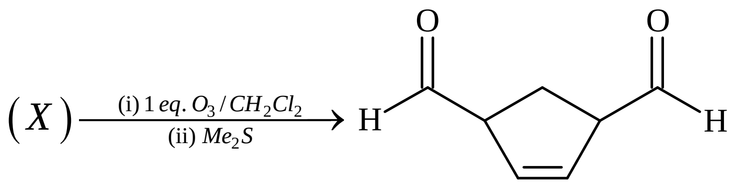
$$K_{eff} = \frac{K_1 + K_2 + ... + K_n}{n}$$

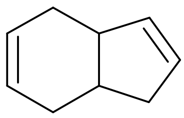
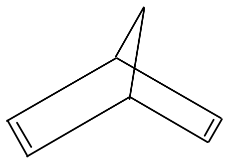
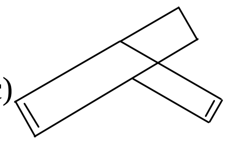
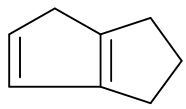
Chemistry

PART - A

SINGLE ANSWER CORRECT:

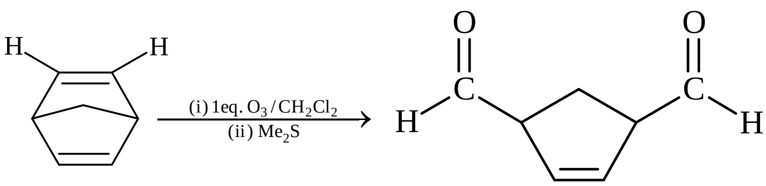
1. Identify the reactant (X) on the given reaction

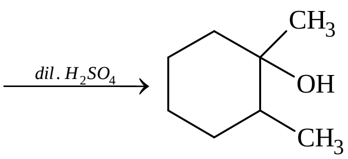


a)  b)  c)  d) 

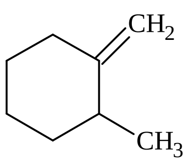
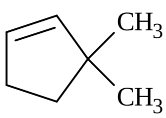
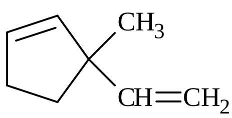
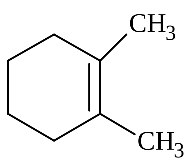
1. B

CONCEPT CODE: C111708

Sol: 

2. Compound (A) 

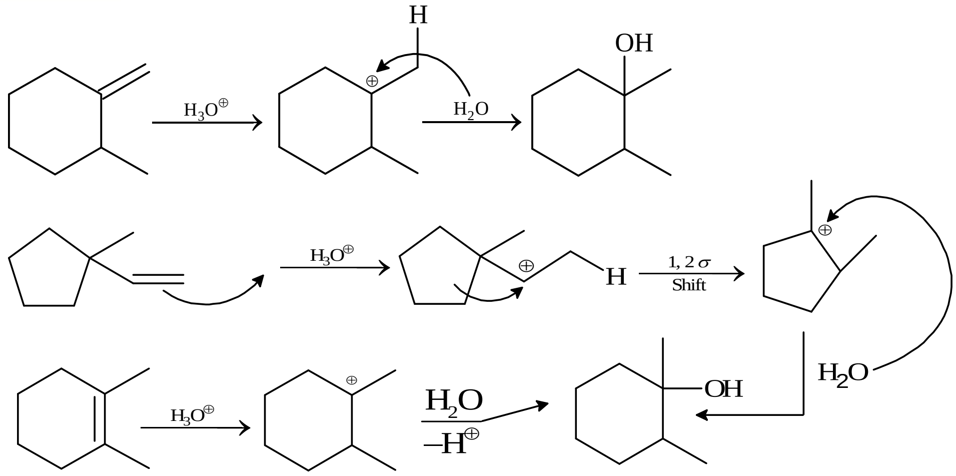
Compound (A) cannot be

a)  b)  c)  d) 

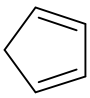
2. B

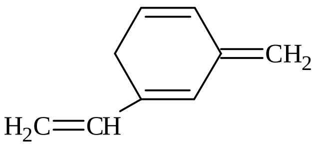
CONCEPT CODE: C111707

Sol:



3. Which of the following does not give glyoxal as one of the product on ozonolysis?

a) 

b) 

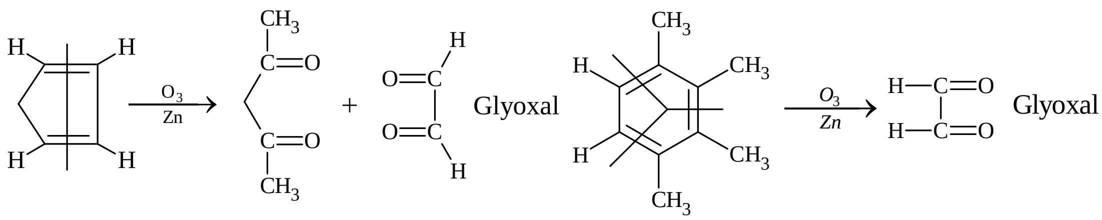
c) $H_2C{=}CH{-}CH{=}CH_2$

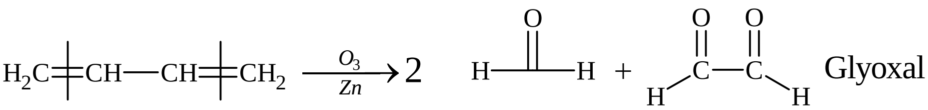
d) Benzene

3. B

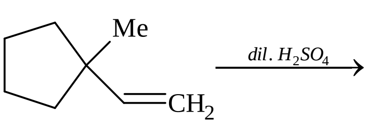
<u>CONCEPT CODE:</u> C111708

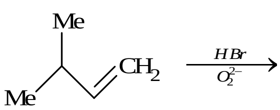
Sol:

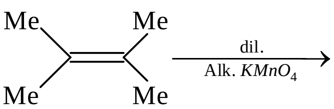


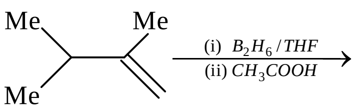


4. In which of the following reaction the rearrangement of carbocation is involved?

a) 

b) 

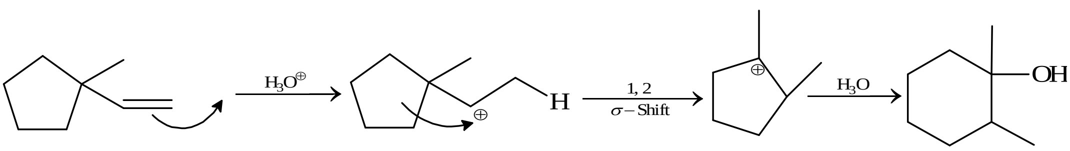
c) 

d) 

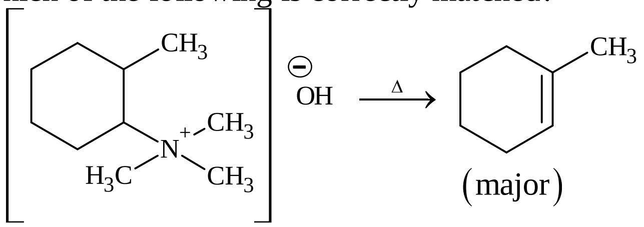
4. A

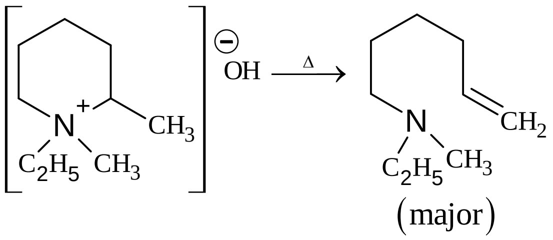
<u>CONCEPT CODE:</u> C111707

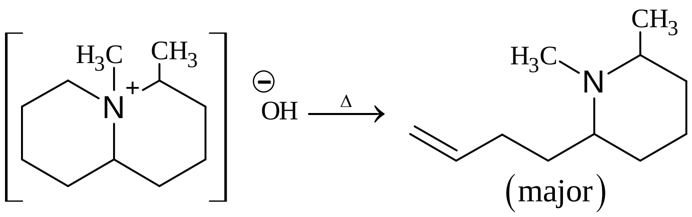
Sol:



5. Which of the following is correctly matched?

a) 

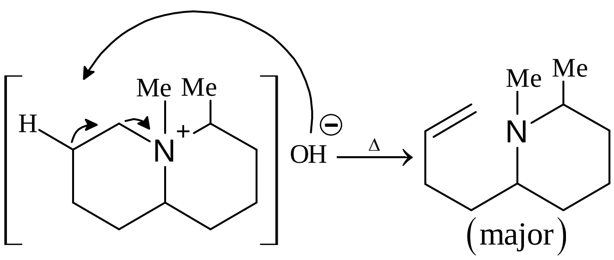
b) 

c) 

d) All of these

5. C

<u>CONCEPT CODE:</u> C111708

Sol: 

6. Identify end products A, B and C of the following

$$CH_3CH{=}CH_2 \xrightarrow[\text{(ii) } H_2O]{\text{(i) } D^{\oplus}} A$$

$$CH_3CH{=}CH_2 \xrightarrow[\text{(ii) } D_2O]{\text{(i) } H^{\oplus}} B$$

$$CH_3CH{=}CH_2 \xrightarrow[\text{(ii) } D_2O]{\text{(i) } D^{\oplus}} C$$

a) $CH_3\underset{\underset{OH}{|}}{C}HCH_3$ in all cases

b) $CH_3CH(OH)CH_2D$, $CH_3CH(OD)CH_3$, $CH_3CH(OD)CH_2D$

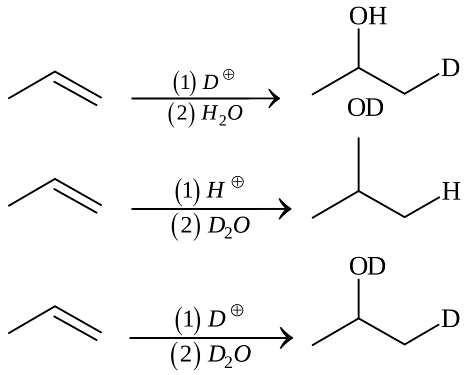
c) $CH_3\underset{\underset{OD}{|}}{C}HCH_3$ in all cases

d) $CH_3\underset{\underset{OD}{|}}{C}HCH_2D$ in all cases

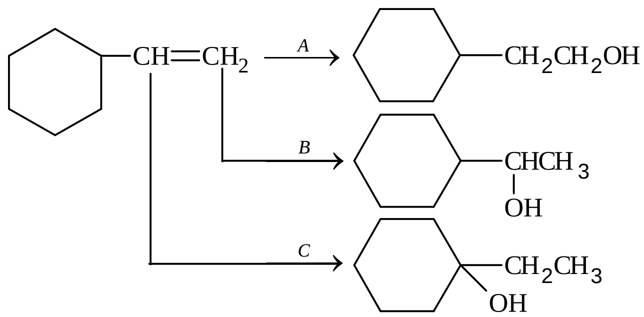
6. B

CONCEPT CODE: C111707

Sol:



7.



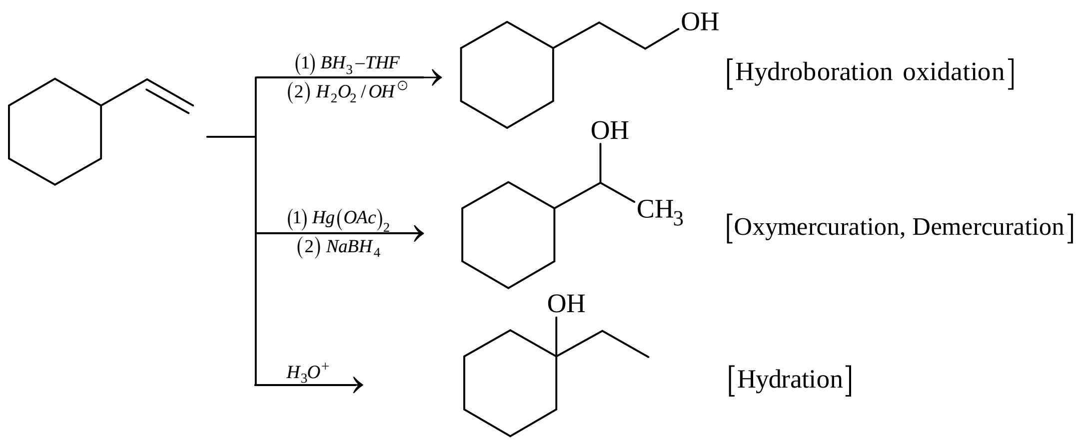
Schemes A, B and C are

a) simple acid catalysed hydration
b) Hydroboration-oxidation, mercuration-demercuration, acid –catalysed hydration
c) acid-catalysed hydration, Hydroboration-oxidation, mercuration-demercuration
d) mercuration-demercuration, acid-catalysed hydration, Hydroboration-oxidation

7. B

CONCEPT CODE: C111708

Sol:



8. Which of the following reactions will not give alkyne?

a) $CH_3CH_2\underset{\mid \atop Cl}{CH}-\underset{\mid \atop Cl}{CH_2} \xrightarrow[H_2O]{NaNH_2}$

b) $CH_3CH_2CHBr_2 \xrightarrow{alc.\ KOH}$

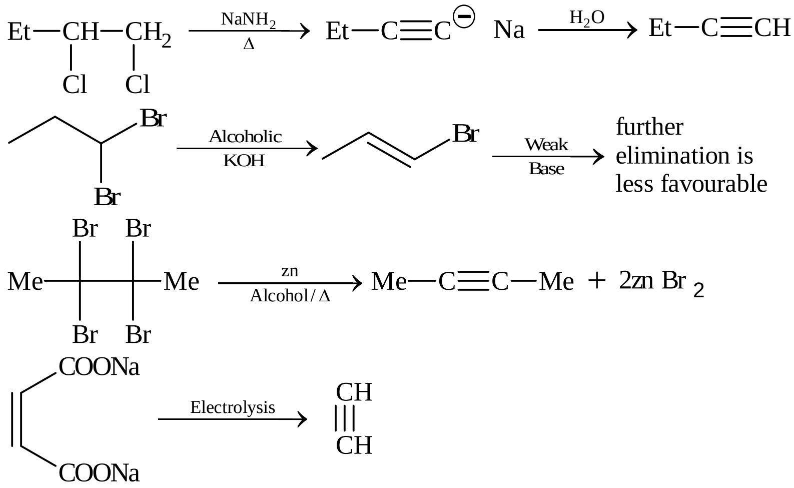
c) $CH_3-\overset{Br \atop \mid}{\underset{\mid \atop Br}{C}}-\overset{Br \atop \mid}{\underset{\mid \atop Br}{C}}-CH_3 \xrightarrow[\Delta]{Zn\,/\,\text{alcohol}}$

d) Potassium maleate $\xrightarrow{\text{Electrolysis}}$

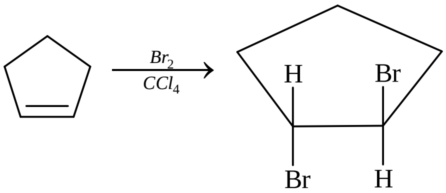
8. B

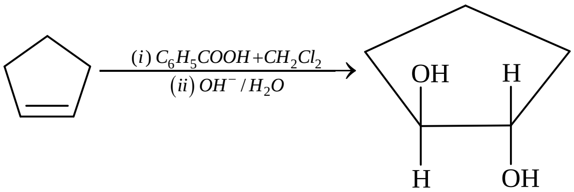
<u>CONCEPT CODE:</u> C111710

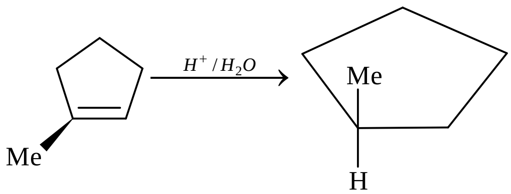
Sol:

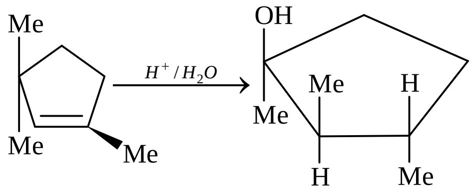


9. Which of the following reactions are correct?

a) 

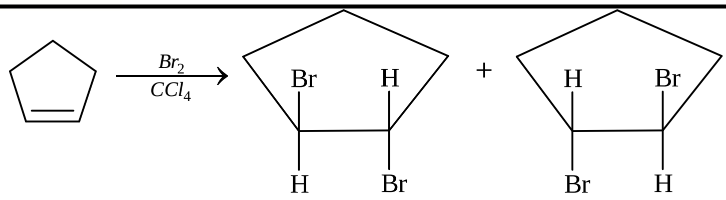
b) 

c) 

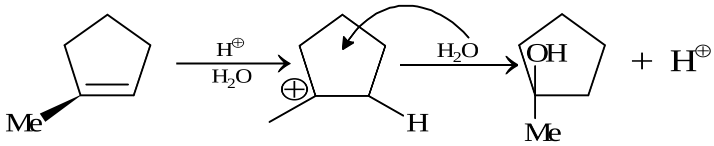
d) 

9. A

<u>CONCEPT CODE:</u> C111708

Sol: 

Anti-addition



10. In which of the following Acetylene is produced?

a) $CaC_2 + H_2O \longrightarrow$ b) $Be_2C + H_2O \longrightarrow$

c) $Al_4C_3 + H_2O \longrightarrow$ d) $Mg_2C_3 + H_2O \longrightarrow$

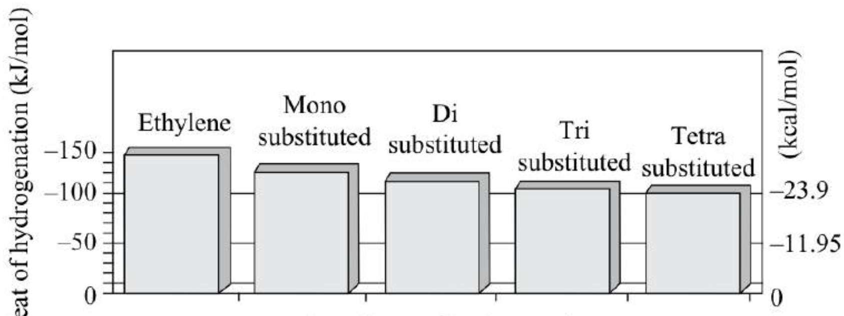
10. A

CONCEPT CODE: C111710

Sol: $CaC_2 + 2H_2O \longrightarrow C_2H_2 + Ca(OH)_2$

COMPREHENSION

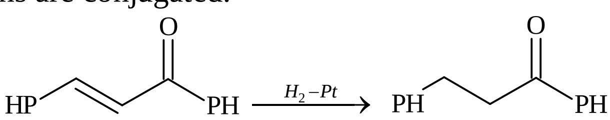
Following figure is given to test analytical ability. Based on it, answer the questions at the end of it.



A plot of heat of hydrogenation versus substitution pattern for alkenes

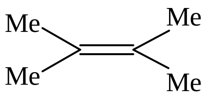
Catalytic hydrogenation is usually a stereospecific reaction called syn addition.

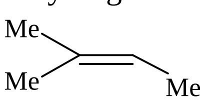
- The $C \equiv C$ bond is reduced more readily than $C \equiv C$ but other unsaturated groups (except nitro and acid chlorides) are reduced less readily. Catalytic hydrogenation can, therefore, be used for the selective reduction of $C \equiv C$ in the presence of aromatic rings and carbonyl groups, whether or not the unsaturated functions are conjugated.

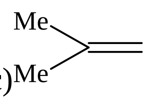


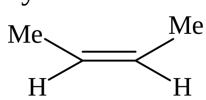
The rate of hydrogenation of olefinic bonds under standard state is $-CH=CH_2 > -CH=CH-$ or a ring double bond.

11. Base on the data of heat of hydrogenation, which has maximum stability ?

a) 

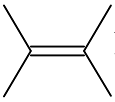
b) 

c) 

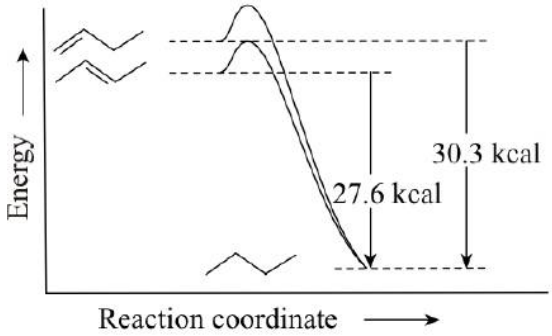
d) 

11. A

CONCEPT CODE: C111708

Sol:  has '12' $\alpha - \text{H}$ and is most stable.

12. Consider following graph



From this, it is clear that

a) cis-2-butene is more stable than 1-butene by 2.7 kcal

b) trans-2-butene is more stable than 1-butene by 2.7 kcal

c) trans-2-butene is more stable than cis-2-butene by 11 kJ

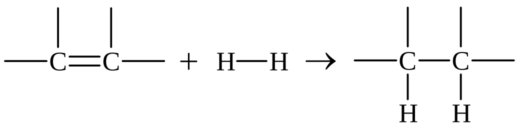
d) trans-2-butene is more stable than 1-butene by 11 kJ

12. B

CONCEPT CODE: C111708

Sol: Trans – 2 – Butene has lower energy than 1 – butene.

13. Bond energies (in kcal mol^{-1}) of different types of bonds have been given as $\underset{\pi}{C - C}$ (π bond $= 40$); $H - H = (104)$ and $C - H = (87)$.



Heat of hydrogenation of the above reaction is

a) 57 kcal mol^{-1} b) -57 kcal mol^{-1} c) -30 kcal mol^{-1} d) 30 kcal mol^{-1}

13. C

CONCEPT CODE: C111708

Sol: $\Delta H^{-} = \text{B.E of } \underset{\pi}{C - C} + \text{B.E of } H - H - 2 \times \text{B.E of } C - H$

$= 40 + 104 - 2 \times 87 \qquad = -30\,\text{k cal mol}^{-1}$

COMPREHENSION

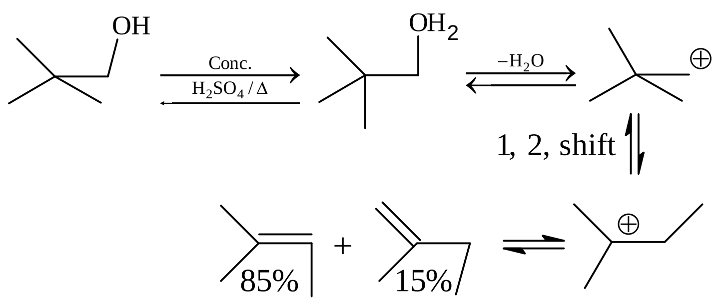
Strictly speaking, then, dehydration is not an $E1$ reaction of the protonated alcohol. In a true $E1$ elimination, the rate of reaction depends only upon heterolysis step, since every carbocation formed goes rapidly on to the product, that is, loss of a proton is much faster than regeneration of substrate. Here that is not the case for carbocations that are formed reversely from the protonated alcohol, and every so often one looses a proton to yield an alkene. Where the structure of alkyl group permits, rearrangement takes place. The initially formed carbocation rearranges to a more stable carbocation. The alkenes obtained are those formed by a loss of proton from this rearranged carbocation as well as from the original one.
When more than one alkene can be formed the preferred product is the more stable one. Another factor comes in here. Since dehydration is reversible, the composition of the product does not necessarily reflect which alkene is formed faster but depending upon hoe nearly reaction approaches equilibrium which alkene is more stable.

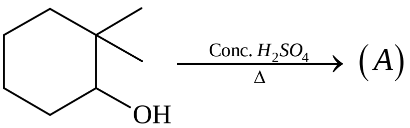
14. When neopentyl alcohol, $(CH_3)_3CCH_2OH$ is heated with an acid, it is slowly converted to a 85:15 mixture of two alkenes of formula C_5H_{10}. The 85 % of these alkene is

a) $H_3C-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH_2$ b) $H_3C-\underset{\displaystyle CH_3}{\underset{|}{C}}-CH=CH_2$ c) $H_3C-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH-CH_3$ d) $H_2C=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_2\text{-}CH_3$

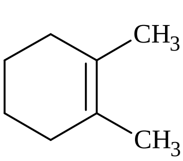
14. C

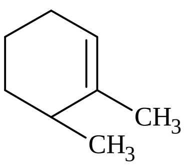
CONCEPT CODE: C111707

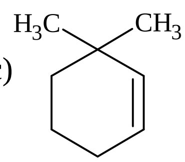
Sol:


15.


The product (A) is

a)


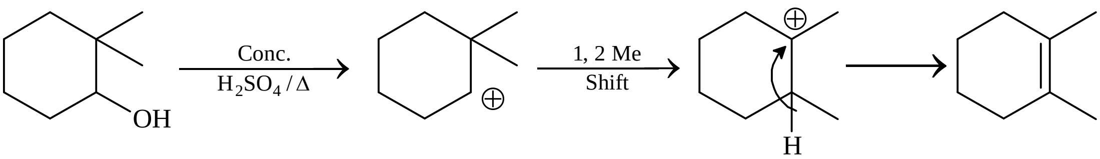
b)


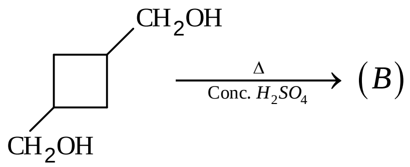
c)


d) None

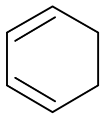
15. A

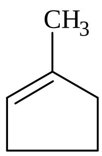
CONCEPT CODE: C111707

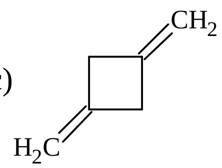
Sol:


16. 

The product (B) is

a) 

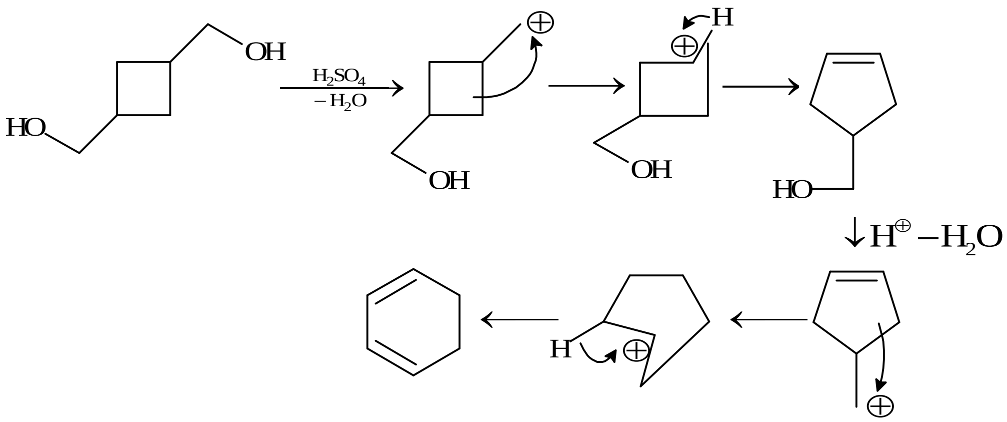
b) 

c) 

d) None

16. A

<u>CONCEPT CODE:</u> C111707

Sol: 

<u>PART - B</u>

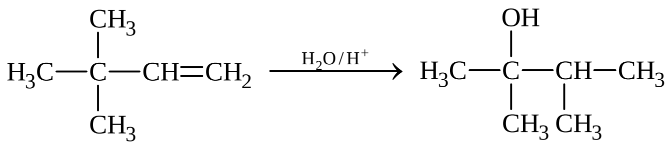
<u>MATCH THE MATRIX:</u>

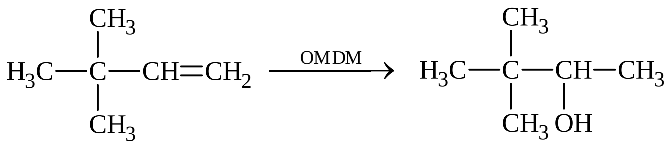
1. Match the following:

	COLUMN – I		**COLUMN - II**
(1)	$C(H_3C)_3-CH=CH_2 \rightarrow (H_3C)_2\underset{\substack{\vert \\ OH}}{C}-CH(CH_3)_2$	(A)	$B_2H_6 + H_2O/OH^-$
(2)	$C(H_3C)_3-CH=CH_2 \rightarrow (H_3C)_3C-\underset{\substack{\vert \\ OH}}{CH}-CH_3$	(B)	$O_3 + Zn/H_2O$
(3)	$H_5C_6-CH=CH_2 \rightarrow C_6H_5-CHO$	(C)	$Hg(OAc)_2/H_2O + NaBH_4/OH^-$
(4)	$H_5C_6-CH=CH_2 \rightarrow C_6H_5-CH_2-CH_2OH$	(D)	H_2O/H^+

1. 1 – D; 2 – C; 3 – B; 4 – A

<u>CONCEPT CODE:</u> C111708

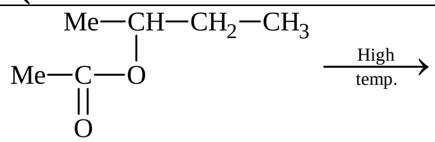
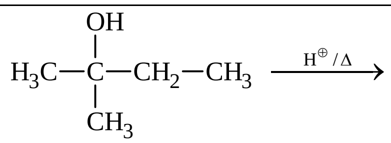
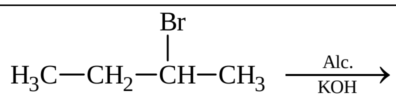
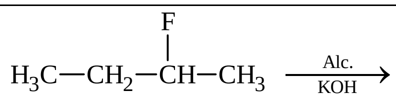
Sol: 1) 

2) 

3) $C_6H_5-CH=CH_2 \xrightarrow[\text{(ii) Zn/H}_2\text{O}]{\text{(i) O}_3} C_6H_5-CH_2-CH_2-OH$

4) $C_6H_5-CH=CH_2 \xrightarrow{\text{H.B.O}} C_6H_5-CH_2-CH_2-OH$

2. Match the following:

COLUMN – I (Substrate for elimination reaction)		COLUMN – II (Type of elimination)	
(1)		(A)	E1
(2)		(B)	E2
(3)		(C)	Pyrolysis
(4)		(D)	Saytzeff product is major
		(E)	Hoffmann product is major

2. 1 – C,E; 2 – A,D; 3 – B,D; 4 – B,E

<u>CONCEPT CODE:</u> C111708

Sol: 1) Pyrolysis of Ester, Hoffmann elimination product is major
2) Acid catalysed dehydration of 3° – Alcohol proceeds E_1 mechanism Saytzeff elimination product is major.
3) Dehydro halogenation of 2^P – Alkyl halide E_2 mechanism and Saytzeff elimination is major
4) Alkyl Fluoride proceeds by E_2 mechanism and Hoffmann elimination is major