27/01/22
Thursday

Q) A tudnnel has digged and a particle is projected from $\frac{R}{2}$. Find escape velocity.

$TE = 0$

$K_i + U_i = 0$

$\frac{1}{2}mv^2 - V_p m = 0$

$V_p = \frac{-GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$

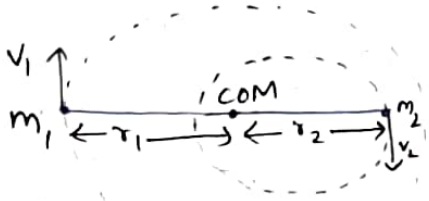
$V_p = -\frac{GM}{2R^3} \times \frac{11R^2}{4}$

$\frac{1}{2}mv_e^2 = \frac{11GMm}{8R}$

$$V_e = \sqrt{\frac{11GM}{4R}}$$

*** Binary star :-

In a binary star system two stars move in a circular path about their common centre of mass due to mutual gravitational force b/w them the speeds of the two stars and time period of the two stars moving in circular path is given by



Star-1:-

$\frac{Gm_1m_2}{d^2} = \frac{m_1v_1^2}{r_1}$ — ①

$r_1 = \frac{m_2 d}{m_1+m_2}$ — ②

from ① & ②

$\frac{Gm_1m_2}{d^2} = \frac{m_1v_1^2}{m_2 d/m_1+m_2}$

$$V_1 = \sqrt{\frac{Gm_2^2}{(m_1+m_2)d}}$$

Star-2:-

$\frac{Gm_1m_2}{d^2} = \frac{m_2v_2^2}{r_2}$ — ①

$r_2 = \frac{m_1 d}{m_1+m_2}$ — ②

from ① & ②.

$$V_2 = \sqrt{\frac{G m_1^2}{(m_1+m_2)d}}$$

Star-1:-

$$V_1 = r_1 \omega_1 .$$

$$\sqrt{\frac{G m_2^2}{(m_1+m_2)d}} = \left(\frac{m_2 d}{m_1+m_2}\right) \omega \frac{2\pi}{T_1} .$$

$$T_1 = 2\pi \left(\frac{m_2 d}{m_1+m_2}\right) \sqrt{\frac{G m_2^2}{(m_1+m_2)d}} .$$

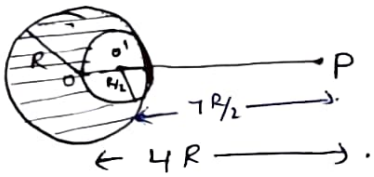
$$T_1 = 2\pi \sqrt{\frac{d^3}{G(m_1+m_2)}}$$

Star-2:-

$$V_2 = r_2 \omega_2 .$$

$$T_2 = 2\pi \sqrt{\frac{d^3}{G(m_1+m_2)}}$$

Q) From a sphere of mass M and Radius R, a smaller sphere of radius $\frac{R}{2}$ is removed from one end of the orginal sphere as shown in the figure. Find the magnitude of gravitational field due to remaining sphere at a distance 4R from the centre of the orginal sphere.



$$E_0 = \frac{GM}{16R^2}$$

$$E_r = \frac{GM'}{r'^2}$$

$$E_r = \frac{GM}{8 \cdot \frac{49R^2}{4}}$$

$$E_r = \frac{GM}{98R^2}$$

$$\frac{4}{3}\pi R^3 - M$$

$$\frac{4}{3}\pi \frac{R^3}{8} - x .$$

$$x = \frac{M}{8} .$$

$$|E_p| = E_0 - E_r$$

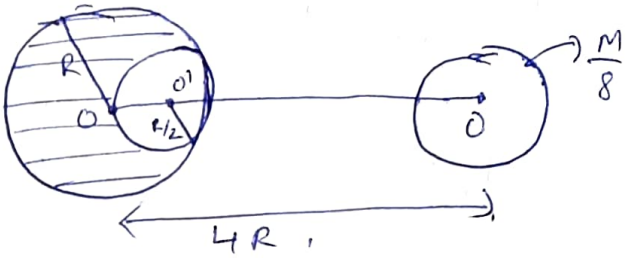
$$= \frac{GM}{16R^2} - \frac{GM}{98R^2}$$

$$= \frac{49M - 8GM}{49 \times 16R^2} = \frac{41GM}{784R^2}$$

$$|E_p| = \frac{41GM}{784R^2}$$

Q) In the above Question find the Magnitude of.

In the above Question the Removed sphere is kept at a distance 4R from the centre of the orginal sphere (The distance b/w centre of the removed sphere and orginal sphere is 4R). Find the magnitude of Gravitational force on the smaller sphere due to remaining sphere.



$$F = \frac{M}{8} \cdot \frac{41GM}{784R^2}$$

$$\boxed{F = \frac{641GM^2}{6272R^2}}$$