Fluids

Fluid :

→ A Substance which can flow (liquid / gases) is known as fluid

→ Study of fluids at rest wrt container is called Fluid statics

Density :

→ Mass per unit volume is called density.

$$\boxed{\rho = \frac{\text{mass}}{\text{Volume}}}$$

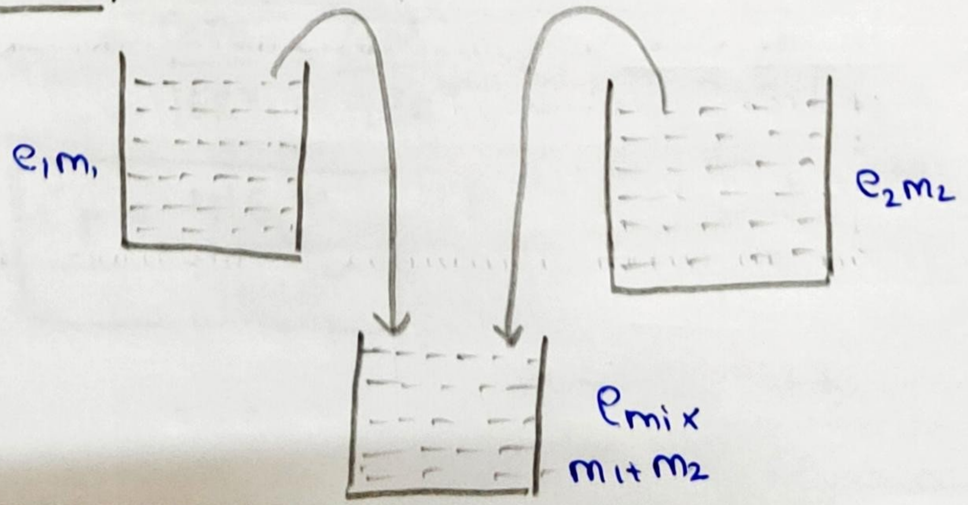
→ S.I unit of density is kg/m^3

⇒ $1\ g/cc = 1000\ kg/m^3$

→ $\rho_{water} = 1\ g/cc$

Density of the mixture :

Case - i :



$\rho_{mix} = \frac{\text{Total mass}}{\text{Total volume}}$

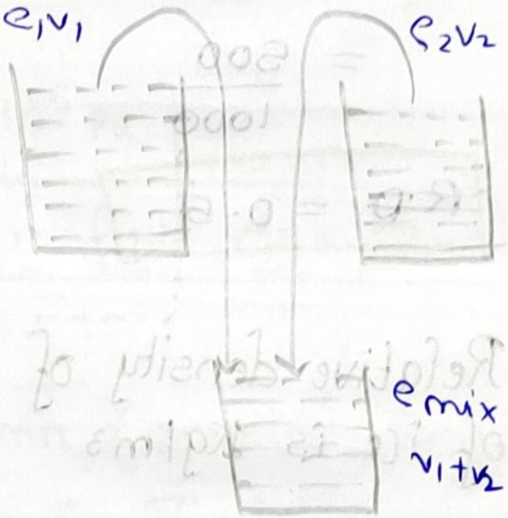
$$\Rightarrow \boxed{\rho_{mix} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}}$$

Note: If $m_1 = m_2 = m$, then.

$$\boxed{\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}}$$

Case-ii :

$$\boxed{\rho_{mix} = \frac{\rho_1 V_1 + \rho_2 V_2}{\rho_1 + \rho_2}}$$



Note: If $V_1 = V_2 = V$

$$\boxed{\rho_{mix} = \frac{\rho_1 + \rho_2}{2}}$$

Relative density :

→ The ratio of density of the substance and density of the water at 4°c is called Relative density of the substance.

$$\boxed{R.d = \frac{\text{Density of the substance}}{\text{Density of water at } 4^\circ C}}$$

→ It's a dimensionless quantity.

→ Since density of water is 1 g/cc; relative density is numerically equal to density of the substance in cgs units.

Ex: Relative density of mercury is 13.6 therefore density of mercury is 13.6 g/cc

(Q) Density of wood is $500 \, kg/m^3$. Find relative density of the wood.

SOL $R.D = \frac{\rho_{wood}}{\rho_{water}}$

$= \frac{500}{1000}$

$\boxed{R.D = 0.5}$

(Q) Relative density of ice is 0.9. Find density of ice is kg/m^3

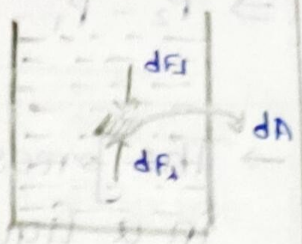
SOL $0.9 = \frac{\rho_{ice}}{1000}$

$\Rightarrow \boxed{\rho_{ice} = 900 \, kg/m^3}$

Pressure in a fluid:

→ When fluid is at rest, it exerts a force perpendicular to any surface in contact with it. Such as a container wall or body immersed in a fluid

$$\boxed{P = \frac{dF_\perp}{dA}}$$



If pressure is uniform at all points at a finite plane surface of area A then pressure on the surface of the fluid is given by :

$$\boxed{P = \frac{F_\perp}{A}}$$

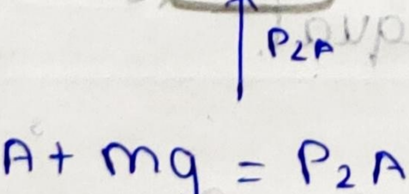
→ S.I unit of pressure is N/m^2 or pascal (Pa)

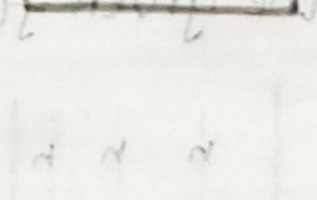
→ $P_{atm} = 1.01 \times 10^5 \, Pa \simeq 10^5 \, Pa$

Variation of pressure in a fluid due to depth :

Consider a cylindrical column (area A, height h) in a liquid of density ρ. The pressure difference b/w the bottom and top layer of the cylindrical column is given by

F.B.D of cylindrical column :





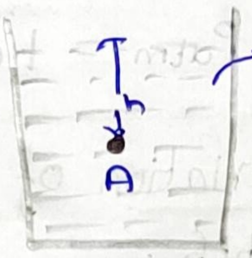
$$P_1A + mg = P_2A$$

$$\Rightarrow P_1\cancel{A} + \rho \cancel{A} hg = P_2 \cancel{A}$$

$\Rightarrow \boxed{P_2 - P_1 = \rho g h}$

→ The pressure difference b/w two points in a liquid depends on density of the liquid, height difference b/w the two points and accelaration due to gravity.

→ From the above equation we can conclude that the pressure at a point inside the liquid at point A is given by is given by



$$\boxed{P_A = P_0 + h\rho g}$$

(or)

$$\boxed{P_A = P_{atm} + h\rho g}$$

P_0 - atmoshpheric pressure

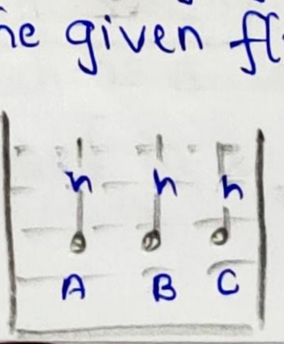
P_A - pressure at point A

h - depth of poin A from the surface.

g - accelaration due to gravity.

__Important points__ :

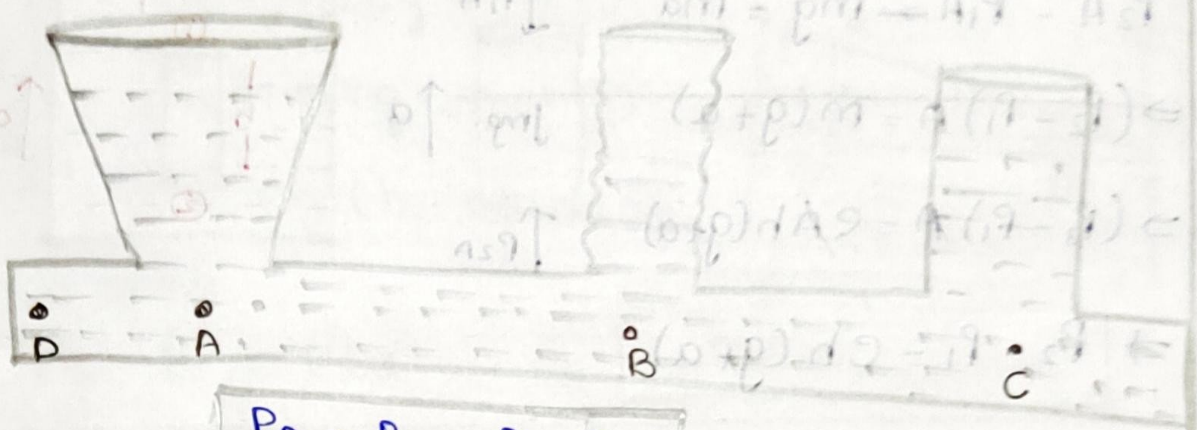
① In the case of stationary fluids pressure at all points lying on same horizontal plane in the given fluid are equal.



$$P_A = P_B = P_C$$

② Hydro static paradox :-

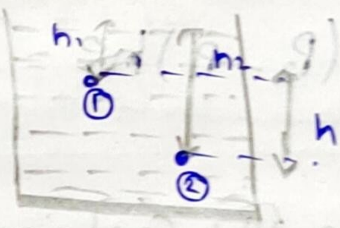
The pressure at a point in the fluid doesnot depend on shape of the container on amount of fluid above the point.



$$\boxed{P_A = P_B = P_C = P_D}$$

③ The pressure difference b/w two points depends on the height (vertical) b/w two points.

$$\boxed{P_2 - P_1 = \rho g h}$$

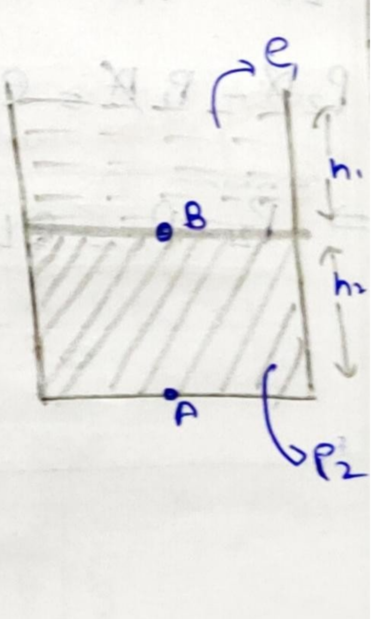


④ Pressure ~~due~~ at a point in two immesible liquids

$P_B = P_0 + h_1 \rho_1 g$

$P_A - P_B = h_2 \rho_2 g$

$$\Rightarrow \boxed{P_A = P_0 + h_1 \rho_1 g + h_2 \rho_2 g}$$



Variation of pressure in accelarating Container :

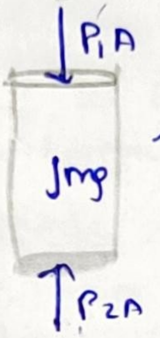
Case - I : Container moving upward with accelaration

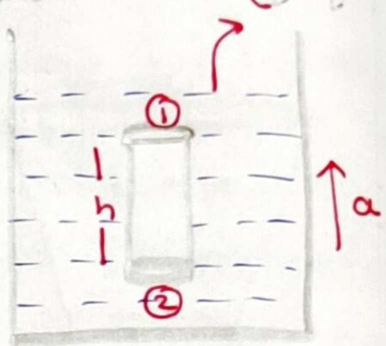
$P_2 A - P_1 A - mg = ma$

$\Rightarrow (P_2 - P_1) A = m(g + a)$

$\Rightarrow (P_2 - P_1) A = \rho A h (g + a)$

$\Rightarrow \boxed{P_2 - P_1 = \rho h (g + a)}$



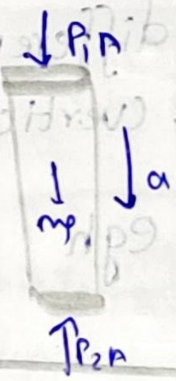


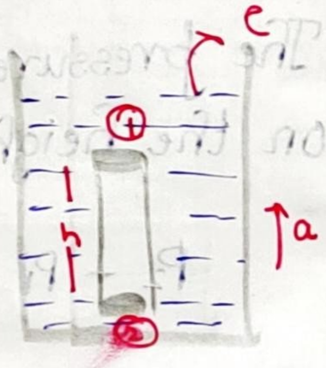
Case - II : Container accelarating downward

$P_1 A + mg - P_2 A = ma$

$\Rightarrow (P_2 - P_1) A = m(g - a)$

$\Rightarrow \boxed{(P_2 - P_1)\cancel{h\rho} = h\rho (g - a)}$



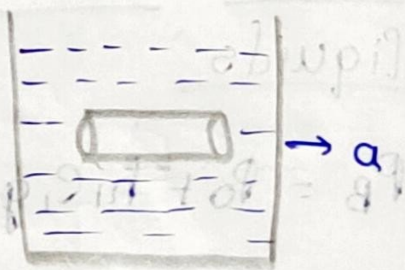


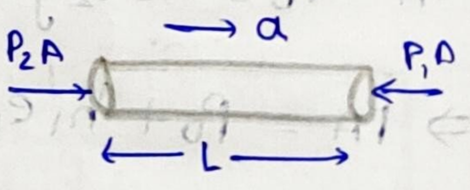
Case - III : Container accelarating horizontally

$P_2 A - P_1 A = ma$

$P_2 \cancel{A} - P_1 \cancel{A} = \rho \cancel{A} L a$

$\Rightarrow \boxed{P_2 - P_1 = \rho L a}$





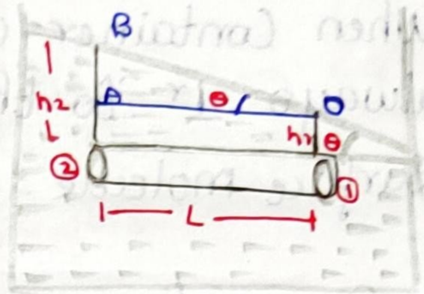
Case-IV :

In case of container accelarating horizontally the pressure due to fluid on points lying on same horizontal surface is different.

Method-1 :

$P_2 = P_0 + h_2 \rho g \longrightarrow ①$

$P_1 = P_0 + h_1 \rho g \longrightarrow ②$



$P_2 - P_1 = \rho g (h_2 - h_1) \longrightarrow ③$

① and ②

$P_2 - P_1 = \rho L a \longrightarrow ④$

From ③ and ④

$\cancel{\rho} g (h_2 - h_1) = \cancel{\rho} L a$

$$\boxed{\frac{a}{g} = \frac{h_2 - h_1}{L}}$$

From ΔOAB

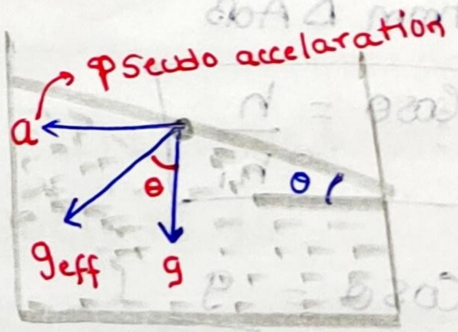
$\tan\theta = \frac{h_2 - h_1}{L}$

$$\Rightarrow \boxed{\frac{a}{g} = \tan\theta}$$

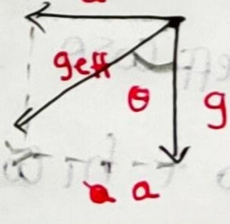
Method-2 :

$g_{eff} = \sqrt{g^2 + a^2}$

$$\Rightarrow \boxed{\tan\theta = \frac{a}{g}}$$



→ a



Note:

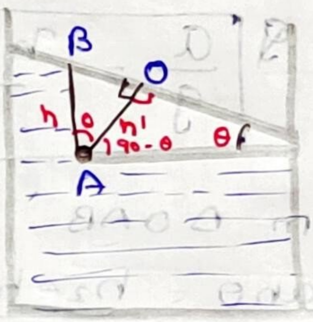
① When Container accelarates free surface of the liquid always $\perp^r$ to g_{eff}

(or)

When Container accelarates a fluid surface always $\perp^r$ to the net force acting on the surface molecule.

② When Container accelarates along horizontal direction the pressure at a point inside the fluid is given by

$$P_A = P_0 + h\rho g_{eff}$$



$$\boxed{P_A = P_0 + h\rho\sqrt{g^2+a^2}}$$

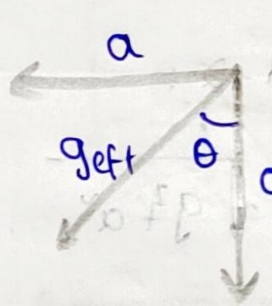
(or)

$$\boxed{P_A = P_0 + h'\rho g}$$

Proof:

$$P_A = P_0 + h\rho\sqrt{g^2+a^2}$$

From ΔAOB



$$\boxed{\cos\theta = \frac{h}{h_1}}$$

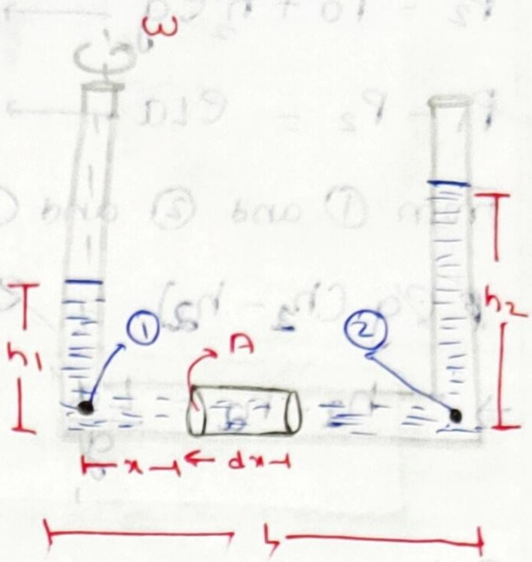
$$\cos\theta = \frac{g}{g_{eff}}$$

$\Rightarrow g = g_{eff}\cos\theta$ and $h_1\cos\theta = h$

$$P_A = P_0 + h_1\cos\theta\,\rho\, g_{eff}$$

$$\boxed{P_A = P_0 + h_1 \rho g}$$

Case-V :



$$dP = \rho\, dx\, a$$

$$\int_{P_1}^{P_2} P = \int_0^L \rho\, dx\, x \omega^2$$

$$\Rightarrow P_2 - P_1 = \rho \omega^2 \left[\frac{x^2}{2}\right]_0^L$$

$$\Rightarrow P_2 - P_1 = \frac{\rho \omega^2 L^2}{2} \longrightarrow (4)$$

$$P_1 = P_0 + h_1 \rho g \longrightarrow (1)$$

$$P_2 = P_0 + h_2 \rho g \longrightarrow (2)$$

From (1) and (2)

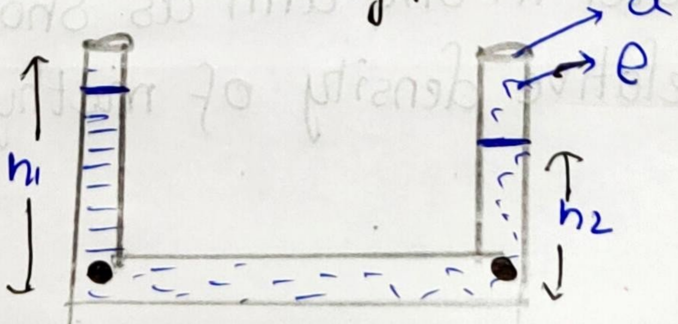
$$P_2 - P_1 = \rho g (h_2 - h_1) \longrightarrow (3)$$

From (3) and (4)

$$\Rightarrow \cancel{\rho} g (h_2 - h_1) = \frac{\cancel{\rho} \omega^2 L^2}{2}$$

$$\Rightarrow \boxed{h_2 - h_1 = \frac{\omega^2 L^2}{2g}}$$

(Q) Find the height difference of the liquid in the two limbs of U-tube when it is accelarated horizontally.



$P_1 = P_0 + h_1 \rho g \longrightarrow$ (1)

$P_2 = P_0 + h_2 \rho g \longrightarrow$ (2)

$P_1 - P_2 = \rho L a \longrightarrow$ (3)

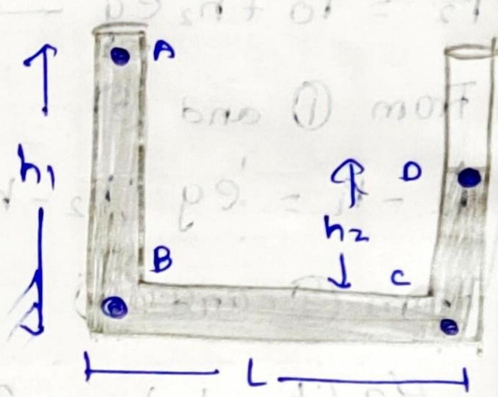
From (1) and (2) and (3)

$\cancel{\rho} g (h_1 - h_2) = \cancel{\rho} L a$

$\Rightarrow \boxed{h_1 - h_2 = \frac{La}{g}}$

II^{nd} method ;

Write the pressure equation from A to B and C to D



$P_A + h_1 \rho g - \rho L a - \rho h_2 g = P_D$

$P_A = P_D = P_0$

$\Rightarrow \cancel{P_0} + h_1 \rho g - \rho L a - h_2 \rho g = \cancel{P_0}$

$\Rightarrow h_1 - h_2 = \frac{La}{g}$

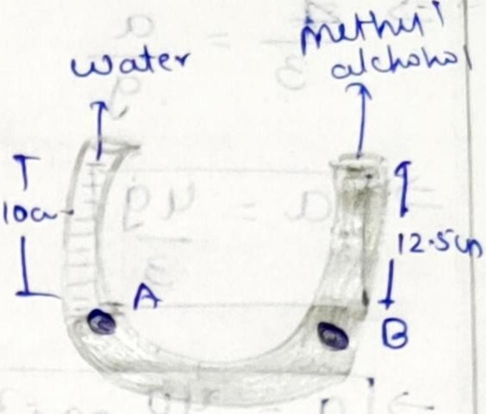
(Q) A utube contains water and methyl alcohol separated by mecury. The mecury columns in the two arms at the same level with 10cm of water in one arm as shown in fig. Find relative density of methyl alcohol.

SoL $P_A = P_B$

$\cancel{P_0} + h_w \rho_w \cancel{g} = \cancel{P_0} + h_a \rho_a \cancel{g}$

$\Rightarrow 10(1) = 12.5\, \rho_a$

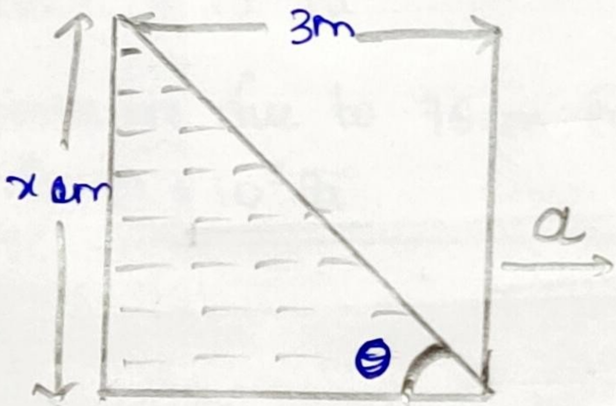
$\Rightarrow \rho_a = 0.8 g/c.c$

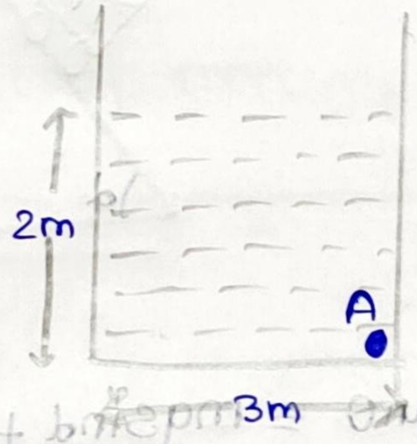


Relative density (R.d) = 0.8

(Q) The container shown in the figure, height of the liquid in it is 2m and length of container 3m, with what minimum accelaration container should accelarate horizontally that A is exposed to atmo-sphere.

SoL





Area's are equal in both cases

Hence

$$3 \times 2 = \frac{1}{2} \times x \times 3$$

$$\Rightarrow \boxed{x = 4m}$$

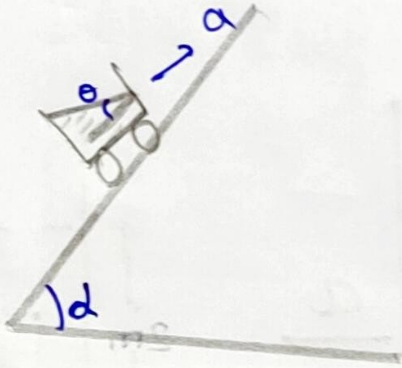
$$\tan\theta = \frac{a}{g}$$

$\Rightarrow \frac{\cancel{4}}{3} = \frac{a}{g}$

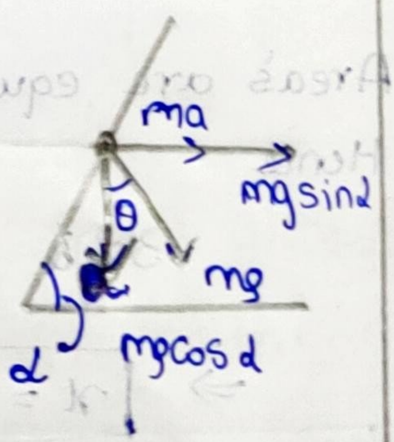
$\Rightarrow \boxed{a = \frac{4g}{3}}$

$\Rightarrow \boxed{a = \frac{40}{3}\, ms^{-2}}$

(Q) A fluid container containing a is accelarating ~~a afor~~ with accelaration a along the liquid of density ρ as shown in the figure. Find the angle of inclination of free surface.



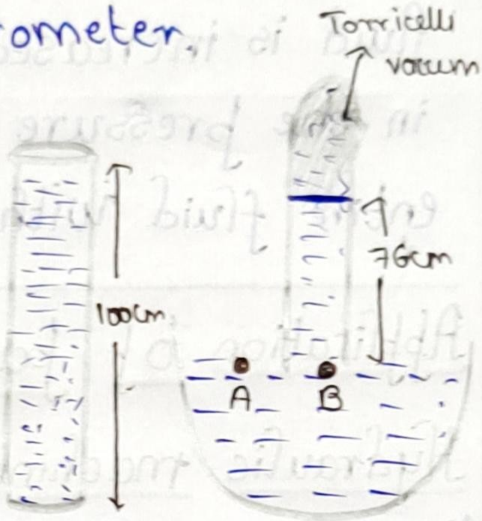
$\tan\theta = \frac{mg\sin\alpha + ma}{mg\cos\alpha}$



$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{a + g\sin\alpha}{g\cos\alpha}\right)}$

Barometer:

→ A device which measure's the atmospheric pressure is known as barometer.



$P_A = P_B$

$P_A = P_{atm}$

$P_{atm} = h_m \rho_m g$

$\Rightarrow P_{atm} = 76 \times 10^{-2} \times 13.6 \times 10^{3} \times 9.8$

$\boxed{P_{atm} = 1.01 \times 10^{5}\ Pa}$

~~Ma~~ $P_{atm} \approx 10^{5}\ Pa$

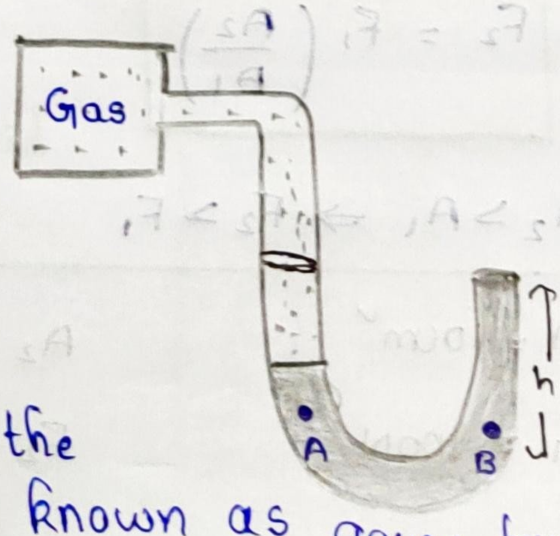
→ $1\ atm = 10^{5}\ Pa$

→ $1\ bar = 10^{5}\ Pa$

→ Pressure due to 76cm height of mercury column $= 10^{5}\ Pa$.

Manometer:

It is a device which is used to measure the pressure of a gas.



$P_A = P_B$

$\Rightarrow P_A = P_{Gas}$

$\Rightarrow \boxed{P_{Gas} = P_0 + h\rho g}$

The pressure excess to the atmospheric pressure is known as gauge pressure

Here $h\rho g$ is the gauge pressure.

Pascal's Law:

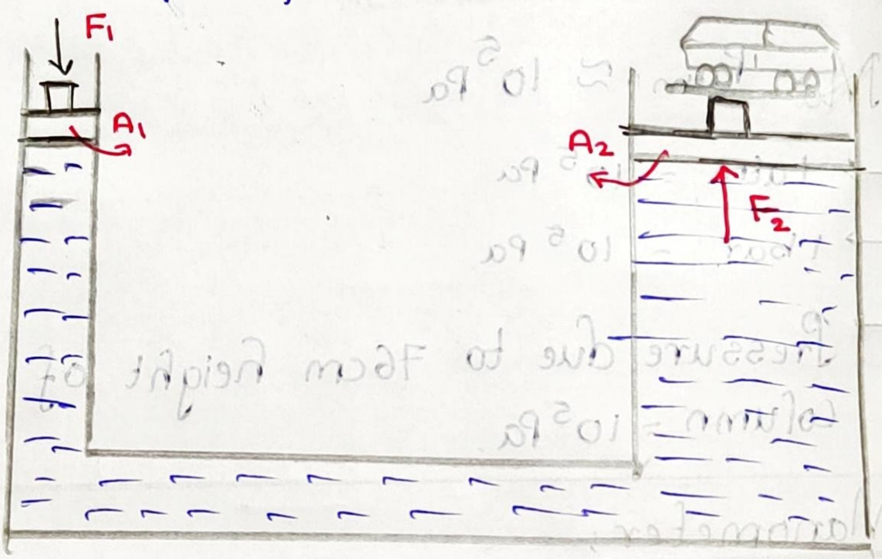
→ It states that "If the pressure in the fluid is increased at a point the increase in the pressure would be transmitted to entire fluid without dimnishing it's value."

Application of pascal's law:

① Hydraulic machine:

It is a force multiplying device.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow \boxed{F_2 = F_1\left(\frac{A_2}{A_1}\right)}$$

$A_2 > A_1 \Rightarrow F_2 > F_1$

(Q) $A_1 = 10cm^2$ $\quad$ $A_2 = 100cm^2$

$F_1 = 100N$ $\quad$ $F_2 = ?$

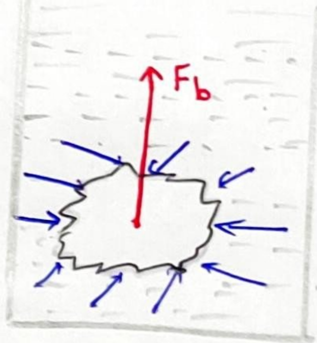
Sol $F_2 = F_1 \left(\frac{A_2}{A_1}\right)$

$\Rightarrow F_2 = 100 \left(\frac{100}{10}\right)$

$\Rightarrow \boxed{F_2 = 1000N}$

Buoyant force (F_b):

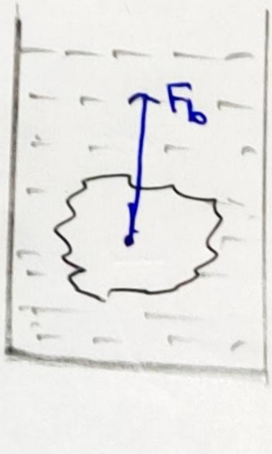
→ The resultant of all the forces exerted by the fluid on the body is called buoyant force.



Archimedes principle:

→ When a body partially / completly immersed in a fluid, fluid exert's an upward force whose magnitude is equal to weight of the fluid displaced.

F_b = Weight of the fluid displaced



$F_b = m_f g$

$\boxed{F_b = \rho_f V_i g}$

ρ_f - density of the fluid

V_i - Volume of immersed part

g - Accelaration due to gravity

F_b - Buoyant force.

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