# Mathematics

## PART - A

### ONE OR MORE THAN ONE CORRECT:

1. Let $[x]$ denote the greatest integer less than or equal to $x$. If $f(x)=[x\sin\pi x]$, then $f(x)$ is

a) Continuous at $x=0$ b) Continuous in $(-1,0)$

c) Differentiable at $x=1$ d) Differentiable in $(-1,1)$

1. A,B,D

**Concept code:** M120401

Sol: By the definition of $[x]$, we have $f(x)=[x\sin\pi x]=0$ for $-1\le x\le 1$

$[\because 0\le x\sin\pi x\le 1]$

Also, $f(x)=[x\sin\pi x]=-1$ for $1<x<1+h$ for some small $h>0$, because $\sin\pi x$ is negative and $\ge -1$ for $1<x<1+h$

Thus, $f(x)$ is constant and equal to 0 in the interval $(-1,1)$ and so it is continuous and differentiable in $(-1,1)$

In particular, $f(x)$ is continuous at $x=0$ and in the interval $(-1,0)$

At $x=1$, $\lim\limits_{x\to 1^+} f(x)=-1$ and $\lim\limits_{x\to 1^-} f(x)=0$.

Hence $f$ is not continuous at $x=1$ and, in particular, not differentiable at $x=1$.

2. If $f(x)=\begin{cases} x^2\sin\dfrac{1}{x} & \text{for } x\neq 0 \\ 0 & \text{for } x=0 \end{cases}$, then

a) $f$ and $f'$ are continuous at $x=0$

b) $f$ is derivable at $x=0$

c) $f$ is derivable at $x=0$ and $f'$ is not continuous at $x=0$

d) $f$ is derivable at $x=0$ and $f'$ is continuous at $x=0$

2. B,C

**Concept code:** M120401

Sol: For $x\neq 0$, we have

$$f'(x)=2x\sin\frac{1}{x}+x^2\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)1$$

$$=2x\sin\frac{1}{x}-\cos\frac{1}{x}$$

And for $x\neq 0$, we have

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x}=\lim_{x\to 0}x\sin\frac{1}{x}\Rightarrow f'(0)=0$$

Thus $f$ is derivable at $x=0$. Also,

$$\lim_{x\to 0}f'(x)=\lim_{x\to 0}\left(2x\sin\frac{1}{x}-\cos\frac{1}{x}\right)$$

$$=\lim_{x\to 0}2x\sin\left(\frac{1}{x}\right)-\lim_{x\to 0}\cos\frac{1}{x}$$

Now, $\lim_{x\to 0}2x\sin\left(\frac{1}{x}\right)=0$

But $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exists

Hence $f'$ is not continuous at $x=0$.

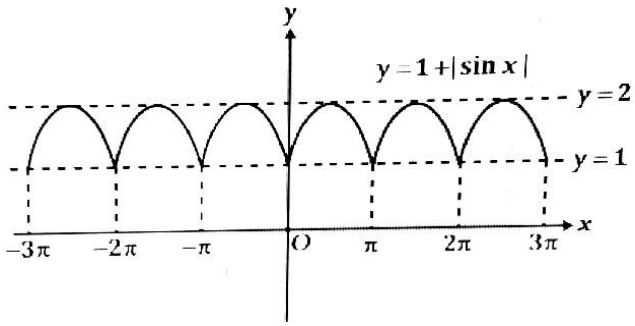
3. The function $f(x)=1+|\sin x|$ is

a) Continuous nowhere
b) Continuous everywhere
c) Differentiable nowhere
d) Not differentiable at an infinite number of points

3. B,D

**Concept code:** M120401

Sol: By plotting the graph of $f$, we can see that $f$ is continuous everywhere




Also, we can write it as

$$f(x)=\begin{cases}1+\sin x & \text{for}\sin x\geq 0\\ 1-\sin x & \text{for}\sin x<0\end{cases}$$

So $f$ is differentiable everywhere except at the points where $\sin x=0$, i.e., $x=n\pi$, $n\in Z$.

Hence $f$ is also not differentiable at $x=0$.

4. Let $h(x)=\min\{x, x^2\}\ \forall\ x$. Then,

a) $h$ is continuous for all $x$
b) $h$ is differentiable for all $x$
c) $h'(x)=1$, for all $x>1$
d) $h$ is not differentiable at two values of $x$

4. A,C,D

**Concept code:** M120401

Sol: $h(x)=\begin{cases} x, & -\infty < x \le 0 \\ x^2, & 0 < x \le 1 \\ x, & 1 \le x < \infty \end{cases}$

Thus $h$ is differentiable except possibly at $x = 0,1$

$h'(0^+)=\lim\limits_{k\to 0^+}\dfrac{h(0+k)-h(0)}{k}=\lim\limits_{k\to 0^+}\dfrac{k^2}{k}=0$

$h'(0^-)=\lim\limits_{k\to 0^-}\dfrac{h(0+k)-h(0)}{k}=\lim\limits_{k\to 0^-}\dfrac{k}{k}=1$

Hence, $h$ is not differentiable at $x = 0$

Similarly, $h'(1^+)=1$ and $h'(1^-)=2$

Also, $\lim\limits_{x\to 0} h(x)=0=h(0)=\lim\limits_{x\to 0^-} h(x)$ and $\lim\limits_{x\to 1^+} h(x)=1=h(1)=\lim\limits_{x\to 1^-} h(x)$,

Hence $h$ is a continuous function but not differentiable at $x = 0,1$

Alternatively, same conclusions can also be made by drawing graph of the function $h(x)$.

5. The equations of the tangents to the curve $y = x^4$ from the point $(2,0)$ not on the curve, are given by

a) $y = 0$

b) $y-1=5(x-1)$

c) $y-\dfrac{4098}{81}=\dfrac{2048}{27}\left(x-\dfrac{8}{3}\right)$

d) $y-\dfrac{32}{243}=\dfrac{80}{81}\left(x-\dfrac{2}{3}\right)$

5. A,C

**Concept code:** M120601

Sol: Let $(x_0, x_0^4)$ be the point of tangency.

Then the equation of the tangent will be $y-x_0^4=y'(x_0)(x-x_0)$.

Since this tangent passes through the point $(2,0)$, we have

$-x_0^4=4x_0^3(2-x_0)\Rightarrow 3x_0^4-8x_0^3=0$

$\Rightarrow x_0=0$ or $x_0=\dfrac{8}{3}$

So that the points of tangency are $(0,0)$ or $\left(\dfrac{8}{3},\dfrac{4096}{81}\right)$

Therefore, the equation of the tangents are $y = 0$ and $y-\dfrac{4096}{81}=\dfrac{3048}{27}\left(x-\dfrac{8}{3}\right)$

6. Let the function $f(x)=\sin x+\cos x$ be defined in $[0, 2\pi]$ then $f(x)$

a) increases in $\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$

b) decreases in $\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$

c) increases in $\left[0,\dfrac{\pi}{4}\right]\cup\left[\dfrac{5\pi}{4}, 2\pi\right]$

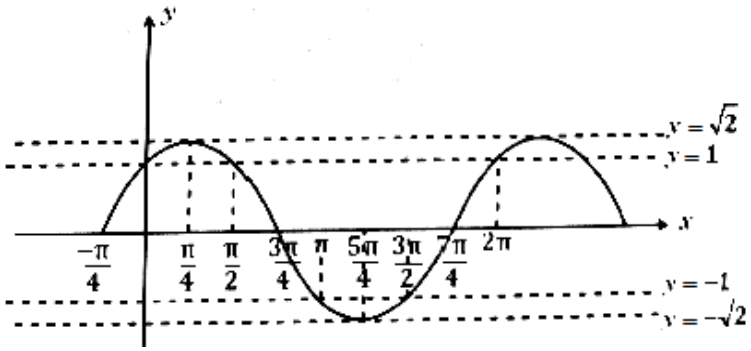
d) decreases in $\left[0,\dfrac{\pi}{4}\right]\cup\left[\dfrac{\pi}{2}, 2\pi\right]$

6. B,C

**Concept code:** M120604

Sol: $f(x)=\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

Draw the graph of $f(x)=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$




7. If $f'(x)=g(x)(x-a)^2$, where $g(a)\neq 0$ and $g$ is continuous at $x=a$ then

a) $f$ is increasing near $a$ if $g(a)>0$ b) $f$ is increasing near $a$ if $g(a)<0$

c) $f$ is decreasing near $a$ if $g(a)>0$ d) $f$ is decreasing near $a$ if $g(a)<0$

7. A,D

**Concept code:** M120604

Sol: Since $g$ is continuous at $x=a$,

If $g(a)>0$, then there exist an open interval $I$ containing $a$ so that

$g(x)>0\ \forall\ x\in I\Rightarrow f'(x)\geq 0\ \forall\ x\in I$

Therefore, $f$ is increasing near $a$.

Similarly $f$ is decreasing near $a$ if $g(a)<0$.

8. In a triangle ABC, let $AB=\sqrt{23}$, $BC=3$ and $CA=4$. Then the value of $\frac{\cot A+\cot C}{\cot B}=$

a) 2 b) 4 c) 3 d) 5

8.a

**Concept code:** M111501

Sol. $\dfrac{\frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}}{\frac{\cos B}{\sin B}}$

$=\dfrac{\sin(A+C)\sin B}{\sin A\sin C\cos B}=\dfrac{\sin B\sin B}{\sin A\sin C\cos B}=\dfrac{2b^2}{a^2+c^2-b^2}=2$

9. In a triangle ABC

a) $b=3c,\ C-B=90^0$ then $\tan B=3$

b) $A=60^0,\ b:c=\left(\sqrt{3}+1\right):2$ then $B-C=60^0$

c) $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$ then $a+c=2b$

d) $(a+b+c)(a-b+c)=3ac$ then $B=60^0$

9. a,c,d

**Concept code:** M111501

Sol. a) $\frac{b}{\sin B}=\frac{c}{\sin C}$

$\frac{\sin B}{\sin C}=\frac{b}{c}$

$\frac{\sin B}{\sin(90+B)}=3$

$\tan B=3$

b) $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$

$\tan\left(\frac{B-C}{2}\right)=2-\sqrt{3}\rightarrow B-C=30^0$

c) $a(1+\cos C)+c(1+\cos A)=3b$

$a+c+b=3b\rightarrow 2b=a+c$

d) $(a+c)^2-b^2=3ac$

$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{1}{2},\ B=60^0$

10. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are $a,b,c$ respectively, then

a) $b^2,c^2,a^2$ are in A.P

b) $c^2,a^2,b^2$ are in A.P

c) $b^2-a^2=a^2+c^2$

d) $a^2,\ b^2,\ c^2$ are in A.P

10.a

**Concept code:** M111501

Sol. $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$

$\frac{\sin(C+B)}{\sin(A+C)}=\frac{\sin(A-C)}{\sin(C-B)}$

$\sin^2 C-\sin^2 B=\sin^2 A-\sin^2 C$

$2\sin^2 C=\sin^2 A+\sin^2 B$

$2c^2=a^2+b^2$

## PART - C

**INTEGER ANSWER TYPE:**

1. The number of points of discontinuity for $f(x)=\text{sgn}(\sin x), x\in[0,4\pi]$ is

1. 5

**Concept code:** M120401

Sol: $f(x)=\text{sgn}(\sin x)$ is discontinuous when $\sin x=0$ or $x=0,\pi,2\pi,3\pi,4\pi$.

2. Let $F(x)=f(x)g(x)h(x)$ for all real $x$, where $f(x),g(x)$ and $h(x)$ are differentiable functions. At some point $x_0$, $F'(x_0)=3F(x_0)$, $f'(x_0)=4f(x_0)$, $g'(x_0)=-7g(x_0)$ and $h'(x_0)=k\,h(x_0)$ where $k\in R$. Then $k$ is equal to

2. 6

**Concept code:** M120401

Sol: $F'(x)=f'(x)g(x)h(x)+f(x)g'(x)h(x)+f(x)g(x)h'(x)$

so $3F(x_0)=F'(x_0)=f'(x_0)g(x_0)h(x_0)+f(x_0)g'(x_0)h(x_0)+f(x_0)g(x_0)h'(x_0)$

$=4f(x_0)g(x_0)h(x_0)-7f(x_0)g(x_0)h(x_0)+k\,f(x_0)g(x_0)h(x_0)$

$=(-3+k)f(x_0)g(x_0)h(x_0)$

$=(-3+k)F(x_0)$

Hence, $3=-3+k\Rightarrow k=6$.

3. A function $f$, defined for all positive real numbers, satisfies the equation $f(x^2)=x^3$ for every $x>0$. Then the value of $f'(4)$ is

3. 2

**Concept code:** M120401

Sol: $2xf'(x^2)=3x^2$ or $4f'(4)=12$ or $f'(4)=3$.

4. The volume of a cube is increasing at a rate of 9 cubic centimeters per second, and the rate at which surface area is increasing when the length of the edge is 10cm is $k\ cm^2/\text{sec}$ then $\frac{10k}{4}=$

4. 9

**Concept code:** M120604

Sol: Let $x$ be the length of the edge of the cube, $V$ be its volume and $S$ be its surface area. Then $V = x^3$ and $S = 6x^2$. Given that rate of change of volume is $9\,cm^3/\sec$.

Therefore, $\frac{dV}{dt} = 9\,cm^3/\sec$.

Now differentiating $V$ w.r.t. $t$, we get,

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 9 = 3x^2\frac{dx}{dt}$$

i.e., $\frac{dx}{dt} = \frac{3}{x^2}$.

Differentiating $S$ w.r.t. $t$, we get

$$\frac{dS}{dt} = 12x \times \frac{dx}{dt}$$

$$= 12x \times \frac{3}{x^2} = \frac{36}{x}.$$

Hence, when $x = 10\,cm$, $\frac{dS}{dt} = \frac{36}{10} = 3.6\ cm^2/\sec$.

5. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \left\{x \middle| x^2 + 20 \le 9x\right\}$ is

5.5

**Concept code:** M120606

Sol. $f'(x) = 6x^2 - 30x + 36$ on A, where $A = [4,5]$

$$= 6(x-2)(x-3)$$

$f'(x) > 0 \not\forall\ x \in [4,5]$

$f$ is increasing on $[4,5]$

$\therefore$ The minimum value of $f$ is $f(5)$

6. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

6.2

**Concept code:** M120606

Sol. $g(f(x)) = x$

$g'(f(x))\,f'(x) = 1$

$g'(f(x)) = \frac{1}{f'(x)}$, $\quad f(x) = 1 \to x = 0$

$$f'(x) = 3x^2 + e^{x/2} \cdot \frac{1}{2}$$

$$g'(1) = \frac{1}{f'(0)} = 2$$

7. With the usual notation, in $\Delta ABC$, if $\angle A+\angle B=120^0$, $a=\sqrt{3}+1$ and $b=\sqrt{3}-1$, then the ratio $\angle A:\angle B$ is $k:1$ then $k=$

7. 7

**Concept code:** M111501

Sol. $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}=\frac{2}{2\sqrt{3}}\times\sqrt{3}$

$\rightarrow \frac{A-B}{2}=45^0 \rightarrow A-B=90^0$

$A+B=120^0$

$\rightarrow A=105^0,\ B=15^0$

8. If a, b and c (all distinct) are the sides of a triangle ABC opposite to the angles A, B and C respectively, then $\frac{c\sin(A-B)}{a^2-b^2}-\frac{b\sin(C-A)}{c^2-a^2}$ is equal to

8. 0

**Concept code:** M111501

Sol. $\frac{2R\sin(A+B)\sin(A-B)}{4R^2(\sin^2A-\sin^2B)}-\frac{2R\sin(C+A)\sin(C-A)}{4R^2(\sin^2C-\sin^2A)}$

$=\frac{1}{2R}-\frac{1}{2R}=0$

9. Suppose a differential function $f(x)$ satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$, for all real $x$ and $y$. If $\lim_{x\to0}\frac{f(x)}{x}=1$, then $f'(3)-2$ is equal to

9. 8

**Concept code:** M120407

Sol. $f(x+y)=f(x)+f(y)+xy^2+x^2y$

Put $x=y=0$

$f(0)=2f(0)\rightarrow f(0)=0$

$\lim_{x\to0}\frac{f(x)}{x}=1$

$\lim_{x\to0}\frac{f'(x)}{1}=1$

$f'(0)=1$

Partial diff. w.r.t $x$

$f'(x)=f'(0)+x^2$

$f'(x)=1+x^2$

$f'(3)=10$

10. In $\Delta ABC$, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ then $\frac{\angle C}{10^0}=$

10. 6

**Concept code:** M111501

Sol. $\frac{b+c+a+c}{(a+c)(b+c)}=\frac{3}{a+b+c}$

$ab=a^2+b^2-c^2=2ab\cos C$

$\cos C=\frac{1}{2}\rightarrow \angle C=60^0$

# Physics

## PART - A

### ONE OR MORE THAN ONE CORRECT:

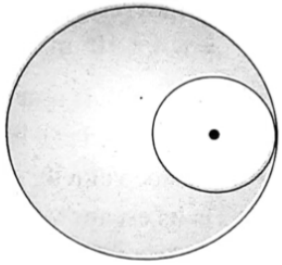
1. If two satellites of different masses are revolving in the same orbit, they have the same
   a) angular momentum  b) energy
   c) time period  d) speed

1. c, d

**CONCEPT CODE : P110905**

Sol. Subjective

2. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)



   a) $\frac{-GM}{2R}$  b) $\frac{-GM}{R}$  c) $\frac{-2GM}{3R}$  d) $\frac{-2GM}{R}$

2. b

**CONCEPT CODE : P110903**

Sol. $V_C = \frac{-3}{2}\frac{GM}{R} + \frac{2GM}{8R}$

$= \frac{-GM}{R}$

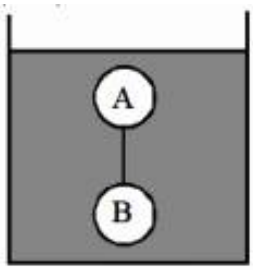
3. An iceberg is floating partially immersed in sea water. The density of sea water is 1.03 $g\,cm^{-3}$ and that of ice is 0.92 $g\,cm^{-3}$. The approximate percentage of total volume of iceberg above the level of sea water is
   a) 8  b) 11  c) 34  d) 89

3. b

**CONCEPT CODE : P111002**

Sol. $\frac{v_i}{v} = \frac{d}{\rho}$

4. Two solid spheres A and B of equal volumes but of different densities $d_A$ and $d_B$ are connected by a string. They are fully immersed in a fluid of density $d_F$. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if




a) $d_A < d_F$ b) $d_B > d_F$ c) $d_A > d_F$ d) $d_A + d_B = 2d_F$

4. a, b, d

**CONCEPT CODE : P111002**

Sol. $F_{net} = \vec{F}_B + \vec{F}_g$ $F_B = V\rho g$

5. A log of wood of mass 120 kg floats in water. The weight that can be put on the raft to make it just sink, should be (density of wood $= 600\ kg/m^3$)

a) 80 kg b) 50 kg c) 60 kg d) 30 kg

5. a

**CONCEPT CODE : P111002**

Sol. Volume of log of wood

$$V = \frac{\text{mass}}{\text{density}} = \frac{120}{600} = 0.2\ m^3$$

Let $x$ weight that can be put on the log of wood.

So weight of the body $= (120 + x) \times 10\ N$

Weight of displaced liquid $V\sigma g = 0.2 \times 10^3 \times 10\ N$.

The body will just sink in liquid if the weight of the body will be equal to the weight of displaced liquid.

$(120 + x) \times 10 = 0.2 \times 10^3 \times 10 \Rightarrow 120 + x = 200$

$x = 80\ kg$.

6. A concrete sphere of radius R has a cavity of radius $r$ which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be

a) 8 : 1 b) 4 : 1 c) 3 : 1 d) 2 : 1

6. b

**CONCEPT CODE : P111002**

Sol. $V = V_C + V_S$ $F_B = (V_C + V_S)\rho g$

7. A sample of metal weighs 210 gm in air, 180 gm in water and 120 gm in liquid. Then specific gravity of

a) Metal is 3 b) Metal is 7

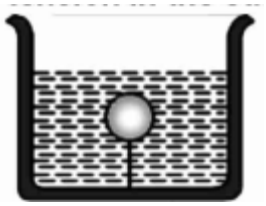
c) Liquid is 3 d) Liquid is $\frac{1}{3}$

7. c

**CONCEPT CODE : P111002**

Sol. Density of metal $=\rho$, Density of liquid $=$ s.
If V is the volume of sample then according to problem
$210 = V\rho g$ …… (i)
$180 = V(\rho - 1)g$ …… (ii)
$120 = V(\rho - \sigma)g$ …… (iii)
By solving (i), (ii) and (iii) we get $\rho = 7$ and $\sigma = 3$

8. A solid sphere of specific gravity $\eta < 1$, is suspended in a water tank by a string tied to its base as shown in figure. If the mass of the sphere is *m* then the tension in the string is given by



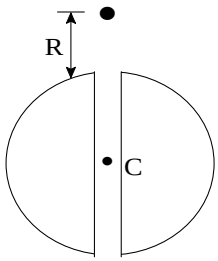
a) $\left(\frac{\eta - 1}{\eta}\right)mg$ b) $\eta mg$ c) $\frac{mg}{\eta - 1}$ d) $\left(\frac{1 - \eta}{\eta}\right)mg$

8. d

**CONCEPT CODE : P111002**

Sol. Tension in spring T $=$ upthrust
Weight of sphere $= V\sigma g - V\rho g$
$= V\frac{1}{\eta}\rho g - V\rho g$ (As $\sigma = \eta\rho$)
$= \left(\frac{1}{\eta} - 1\right)V\rho g = \left(\frac{1}{\eta} - 1\right)mg$

9. A particle is dropped from a height equal to the radius of the earth above the tunnel dug through the earth as shown in the figure. (Radius of earth = R; mass of earth = M)




a) Particle will oscillate through the earth to a height R on both sides
b) Particle will execute simple harmonic motion
c) Motion of the particle is periodic
d) Particle passes the centre of earth with a speed $= \sqrt{\frac{2GM}{R}}$

9. a, c, d

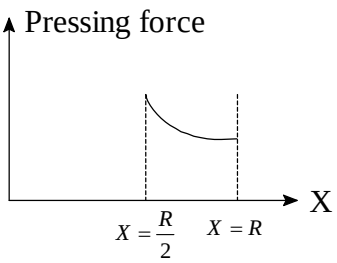
**CONCEPT CODE : P110902**

Sol. From COE (A) is correct.
The force outside the earth varies as inverse square of the distance
∴ Motion is not simple harmonic. However, from symmetry of motion, the motion will be periodic.
∴ (C) is correct.
From COE

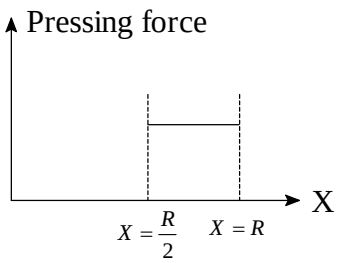
$$\frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{3GMm}{2R}\right).$$

10. A tunnel is dug along a chord of the earth at a perpendicular distance $\frac{R}{2}$ from the earth's centre. The wall of the funnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The passing force by the particle on the wall and the acceleration of the particle varies with × (distance of the particle from the centre) according to :

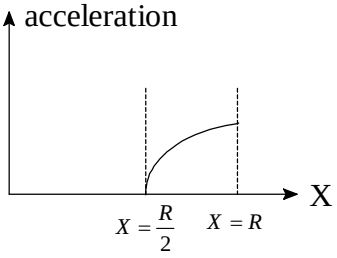
a)



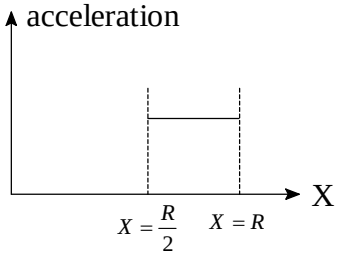

b)




c)




d)




10. b, c

**<u>CONCEPT CODE</u> : P110902**

Sol. Net force towards centre of earth $= mg' = \frac{mgx}{R}$

Normal force $N = mg'\sin\theta$

Thus pressing face $N = \frac{mgx}{R}\frac{R}{2x}$

$N = \frac{mg}{2}$ constant and independent of X

Tangential force $F = ma = mg'\cos\theta$

$$a = g'\cos\theta = \frac{gx}{R}\frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$$

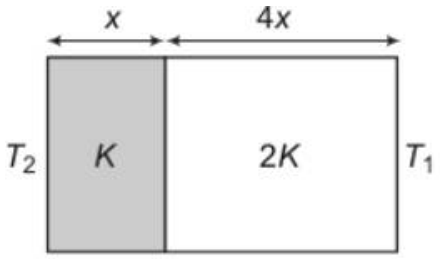
$$a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

Curve is parabolic and at $x = \frac{R}{2}$, $a = 0$.

## PART - C

**INTEGER ANSWER TYPE:**

1. The temperature of the two outer surfaces of a composite slab consisting of two materials having coefficients of thermal conductivity K and 2K and thickness $x$ and $4x$ respectively are $T_2$ and $T_1$ $(T_2 > T_1)$. The rate of heat transfer through the slab in steady state is $\frac{AK(T_2 - T_1)}{nx}$. The value of $n$ is




1. 3

**CONCEPT CODE : P111207**

Sol. $R_{net} = R_1 + R_2 = \left(\frac{X}{KA}\right) + \frac{4X}{2KA} = \frac{3X}{KA}$

Now, $H = \frac{dQ}{dt} = \frac{TD}{R_{net}}$

$= \frac{(T_2 - T_1)}{(3X / KA)} = \left[\frac{KA(T_2 - T_1)}{x}\right]\left(\frac{1}{3}\right)$

2. A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of 2 g of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is $\frac{1}{4}$ that of first, the rate at which ice melts in $\frac{g}{s}$ will be

2. 4

**CONCEPT CODE : P111207**

Sol. $R = \frac{l}{KA}$

$l$ is halved, A is four times and K is $\frac{1}{4}$ times.

$\therefore$ R will become half. Hence, heat current will become two times. Therefore, rate of melting of ice will also become two times or 4 g/s.

3. The earth is assumed to be a sphere of radius R. A platform is arranged at a height 3R from the surface of the earth. The escape velocity of a body from this platform is $\frac{v_e}{f}$, where $v_e$ is its escape velocity from the surface of the earth. Find the value of $f$.

3. 2

**CONCEPT CODE : P110904**

Sol. $v_e = \sqrt{\frac{2GM}{R}}$

$v_e' = \sqrt{\frac{2GM}{4R}} = \frac{1}{2}\sqrt{\frac{2GM}{R}} = \frac{v_e}{f}$

$f = 2$

4. A bullet is fired vertically upwards with velocity *v* from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}$ th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$ , then the value of N is _____ . (lgnore energy loss due to atmosphere)

4. 2

**CONCEPT CODE : P110904**

Sol. $gh = \frac{1}{4}g$

$\frac{GM}{r^2} = \frac{1}{h}\frac{GM}{R^2}$

$r = 2R$

$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{2R}$

$v = \sqrt{\frac{GM}{R}}$

$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2}\sqrt{\frac{GM}{R}}$

$N = 2$

5. Two satellites A and B have masses *m* and 2*m* respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, $\frac{T_A}{T_B}$ is

5. 1

**CONCEPT CODE : P110905**

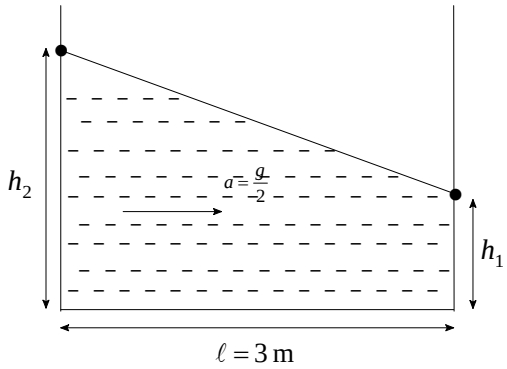
Sol. $V_0 = \sqrt{\frac{GM}{r}}$

$K_A = \frac{1}{2}m\left(\frac{GM}{R}\right)$

$K_B = \frac{1}{2}2m\left(\frac{GM}{2R}\right) = K_A$

$\frac{K_A}{K_B} = \frac{1}{1}$

6. A container felled with water is accelerated horizontally at an acceleration $a = \frac{g}{2}$. ($g$ is magnitude of acceleration due to gravity. If $h_1 = 1\,\text{m}$ then the value of $h_2$ is meters is



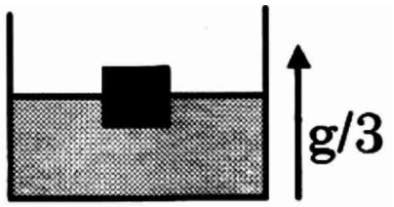

6. 3

**CONCEPT CODE : P111001**

Sol. $\tan\theta = \frac{a}{g} = \frac{h_2 - h_1}{\ell} = \frac{1}{2}$

$h_2 = 3\,\text{m}$

7. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with a net acceleration of $\frac{g}{3}$. The fraction of volume immersed in the liquid is $\frac{1}{n}$, the value of $n$ is




7. 2

**CONCEPT CODE : P111002**

Sol. $\frac{V_{\text{immesed}}}{V_{\text{body}}} = \frac{d}{p}$

8. A ball whose density is $0.4 \times 10^3\,\text{kg/m}^3$ falls into water from a height of 9 cm. To what depth does the ball sink. (in cm)

8. 6

**CONCEPT CODE : P111002**

Sol. The velocity of ball before entering the water surface

$v = \sqrt{2gh} = \sqrt{2g \times 9}$

When ball enters into water, due to upthrust of water the velocity of ball decreases (or retarded).

The retardation, $a = \frac{\text{apparent weight}}{\text{mass of ball}}$

$$= \frac{V(\rho-\sigma)g}{V_\rho}$$

$$= \left(\frac{\rho-\sigma}{\rho}\right)g = \left(\frac{0.4-1}{0.4}\right)\times g$$

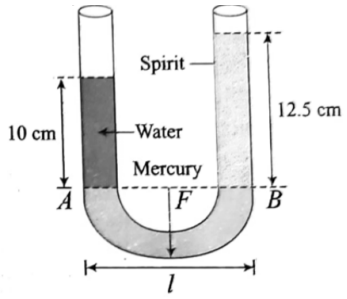
$$= -\frac{3}{2}g\,.$$

If $h$ be the depth upto which ball sink, then,

$$0 - v^2 = 2\times\left(-\frac{3}{2}g\right)\times h$$

$$2g\times 9 = 3gh \qquad h = 6\,cm$$

9. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in the same level with 10 cm of water in one arm and 12.5 cm of spirit in the other. The specific gravity of spirit is $\frac{x}{5}$, the value of $x$ is




9. 4

**CONCEPT CODE : P111001**

Sol. $P_A = P_B$

$$10\times\rho_W\times g = 12.5\times\rho_S\times g$$

$$\frac{\rho_S}{\rho_W} = \frac{10}{12.5} = \frac{4}{5}$$

10. Two solids A and B float in water. It is observed that A floats with $\frac{1}{2}$ of its body immersed in water and B floats with $\frac{1}{4}$ of its volume above the water level. The ratio of the density of A to that of B is $2:n$, the value of $n$ is

10. 3

**CONCEPT CODE : P111002**

Sol. Upthrust = weight of body

For A : $\frac{V_A}{2}\times\rho_w\times g = V_A\times\rho_A\times g = \rho_A$

For B : $\frac{3}{4}V_B\times\rho_w\times g = V_B\times\rho_B\times g = \rho_B$

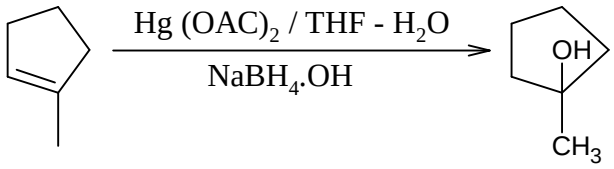
(Since $\frac{1}{4}$ of volume of B is above the water surface)

$$\frac{\rho_A}{\rho_B} = \frac{\rho_w/2}{3/4\rho_w} = \frac{2}{3}.$$

## Chemistry

### PART - A

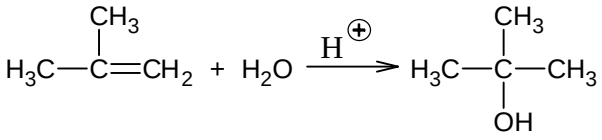
**ONE OR MORE THAN ONE CORRECT:**

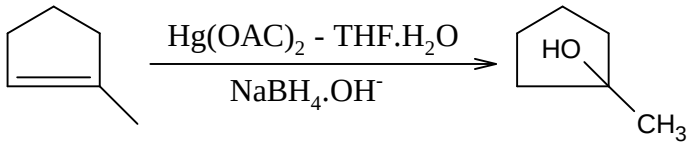
1. Which of the following is/are correct statement ?
   a) Cis – alkenes have higher boiling points than the corresponding trans – isomer
   b) Addition of *HCl* and *HI* in presence of peroxide follows Markonikov's rule
   c) Isobutene react with water in presence of $H_2SO_4$ follows antimarkonikov's addition
   d) 

   In the above reaction final product *OH* comes from $H_2O$ and *H* comes from $NaBH_4 / OH^-$.

1. A, B, D

**Concept code:** C111708

Sol: (a) Cis – alkenes being polar have stronger dipole – dipole interactions while trans – alkene being non- polar.
(b) conceptual
(c) 

ter.butylalcohol
(d) 

1 - methyl cyclo pentanol
In the final product, *OH* comes from $H_2O$ and *H* comes from $NaBH_4 / OH^-$.

2. Which of the following statements is/are correct ?
   a) the ease of dehydration of alcohols is $1° > 2° > 3°$
   b) the dehydrogenation of alkyl halides is an example of 1, 2 elimination and is brought about by action of a base
   c) 1, 2 – elimination reaction involving $E_2$ mechanism does not involve any rearrangement of carbon skeleton
   d) 1, 2 – elimination reaction involving $E_1$ mechanism may involve the rearrangement of carbon skeleton

2. B, C, D

**Concept code:** C111707

Sol: (A) The correct order is $3° > 2° > 1°$

(C) $E_2$ mechanism involves simultaneous elimination of atoms from the neighbouring carbon atoms, no rearrangement is possible

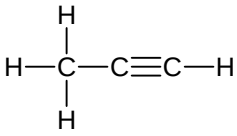
3. Methylacetylene contains

a) Six sigma bonds and two *pi* bonds
b) three sigma and two *pi* bonds
c) one methyl group and one triple bond
d) one sigma and two *pi* bonds

3. A, C

**Concept code:** C111709

Sol:



Methylacetylene $6\sigma$ bonds, and $2\pi$ – bonds

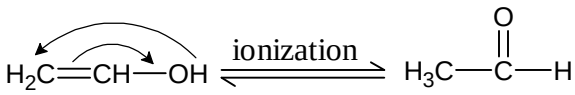
4. Which of the following statements are correct ?

a) vinyl alcohol on isomerization produces ethanal
b) acetylene is produced by the action of water on aluminium carbide
c) acetylene on reacting with *HOCl* produces 1, 2 – dichloro – 1, 2 – ethanediol
d) 1 – pentyne can be distinguished from 2 – pentyne with the help of ammonical $AgNO_3$ solution

4. A, D

**Concept code:** C111712

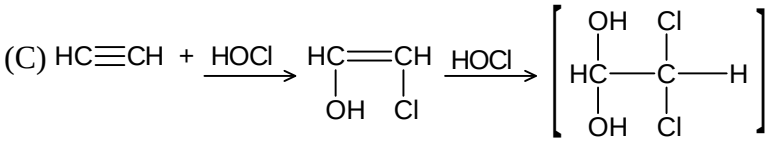
Sol: (A)




(B) Calcium carbide on treating with water gives acetylene :

$$CaC_2 + 2H_2O \rightarrow CH \equiv CH + Ca(OH)_2$$

(C)




$Cl_2HCCHO$

1, 2 - dichloro ethanal

(D) 1 – pentyne have acidic hydrogen where as 2 – pentyne absence of acidic hydrogen.

5. Which of the following statements are not correct ?

a) The addition of $H_2$ to 2 – butyne using *Na* in liquid $NH_3$ gives cis 1, 2 – dimethyl ethene

b) The addition of $H_2$ to 2 – butyne using $H_2$ and lindalar catalyst gives Cis 1, 2 – dimethylethene

c) The Cis 1 , 2 – dimethylethene is more stable than its trans isomer

d) Cis – trans mixture of 2 – pentene cannot be converted to either cis or trans form

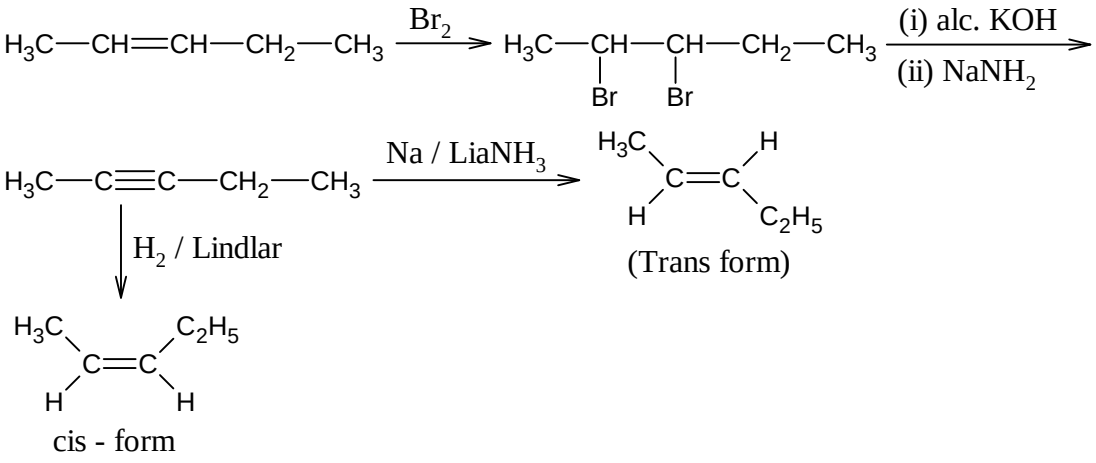
5. A, C, D

**Concept code:** C111708

Sol: (A) the trans isomer is obtained

(C) the trans isomer is more stable than its cis isomer

(D) the conversion may achieved as follows




6. Which of the following statements are incorrect ?

a) potassium superoxide is diamagnetic in nature

b) the thermal stability of hydroxides of group – I decrease on moving down the group

c) *Be*, *Mg* do not respond to flame test

d) the compound $Na_2O_2$ is sodium dioxide

6. A, B, D

**Concept code:** C110803

Sol: Conceptual

7. Which of the following statements are correct ?

a) amongst group – I elements, $Li^+$ ion has the highest enthalpy of hydration

b) $BeH_2$, $MgH_2$ are covalent and polymeric while other hydrides are ionic

c) the metallic oxides of group – I elements become more basic on going down the group

d) the ionic size of $Li^+$ in water is minimum in comparision to other alkali metal ions

7. A, B, C

**Concept code:** C110805

Sol: Conceptual

8. Which of the following is/are correct for lithium ?
a) Lithium is least reactive but the strongest reducing agent among all the alkali metals
b) $LiCl$ is deliquescent and crystallizes as a hydrate $LiCl.2H_2O$
c) Lithium hydrogen carbonates being unstable is not obtained in the solid form
d) Lithium is much heavily hydrated than those of the rest of the group

8. A, B, C, D

**Concept code:** C110806

Sol: Conceptual

9. Which of the following is/are incorrect ?
a) $LiH > NaH > KH > RbH > CsH$ (Stability of hydrides)
b) $H_2O < MgH_2 < NaH$ (Reducing property)
c) Percentage strength of $28V$ $H_2O_2$ is 2.8
d) Zeolites can be regenerated by Conc. $HCl$ solution

9. C, D

**Concept code:** C111003

Sol: (A) Smaller cation stabilised by smaller anion
(B) Ionic hydrides are stronger reducing agents than covalent hydrides order of ionic character and so reducing character is $H_2O < MgH_2 < NaH$
(C) Strength $= \frac{17 \times Vol.strength}{5.6} = \frac{17 \times 28}{5.6} = 85\, g/L$

% Strength $= \frac{85}{1000} \times 100 = 8.5\ \%$

(D) Zeolites can be regenerated by passing brine

10. Which of the following statements is/are correct ?
a) A dilute solution of $H_2O_2$ can be concentrated by heating it under reduced pressure
b) Heavy water is usually prepared by prolonged electrolysis of ordinary water. In fact a dil. solution (0.5 M) of $NaOH$ is taken to improve its electrolytic properties
c) In clark's process water softening is done by calculated amount of slaked lime, $Ca(OH)_2$
d) $H^+$, $D^+$ and $T^+$ differ in all number of neutrons and ionic mass

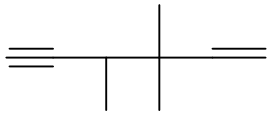
10. A, B, C, D

**Concept code:** C111004

Sol: Conceptual

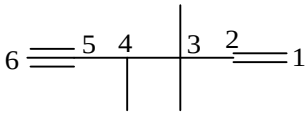
## PART - C

**INTEGER ANSWER TYPE:**

1. In the given compound  , the locant for 'yne' in IUPAC name is _____

1. 5

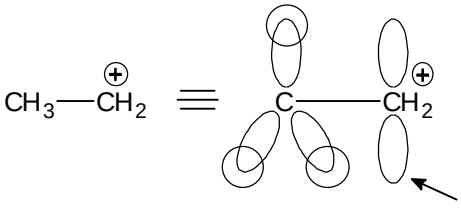
**Concept code:** C111304

Sol:  whenever both double and triple bonds are present at terminal positions, double bond is given preference over the triple bond.

2. How many carbon – hydrogen bond orbitals are available for overlap with the vacant *p*-orbital in ethyl carbocation ?

2. 3

**Concept code:** C111303

Sol: 


Partial overlapping $\sigma-\pi$ conjugation (Hyper conjugation) or $\sigma$ – bond resonance.

3. *x* no. of moles of $BH_3$ are needed to react completely with 7 mole of 1 – pentene in hydroboration oxidation reaction what is $x\times 3$ is _____

3. 7

**Concept code:** C111708

Sol: 3 mole of alkene reacts with 1 mole of $BH_3$

7 mole of 1 – pentene reacts with 7/3 mole of $BH_3$

$x=\frac{7}{3}$

$$x \times 3 = \frac{7}{3} \times 3 = 7.$$

4. $Na_2O_2$ → (water / 25°C) A + B↑ + C; ($Na_2O_2$ → (water / O°C) A + D)

Calculate sum of bond order between same bonded atoms in B and D compounds.

4. 3

**Concept code:** C110802

Sol: $Na_2O_2$ → ($H_2O$ / 25°C) NaOH (A) + $O_2$ (B) + $H_2O$ (C); ($H_2O$ / O°C) NaOH (A) + $H_2O_2$ (D)

B.O of $O_2 = 2.0$

B.O of $H_2O_2 = 1.0$

Sum of bond order between same bonded atoms in B and D compounds = 1 + 2 = 3.

5. An acidified solution of titanium salt when treated with $H_2O_2$, a yellow or orange colour compound (X) developed and $H_2O_2$ on shaking with acidified $K_2Cr_2O_7$ with little ether blue colour compound (Y) is produced. In compound (X) the oxidation state of Titanium is *P* and in compound *Y* oxidation state of chromium is *Q*. What is $Q - P =$

5. 2

**Concept code:** C111004

Sol: $Ti(SO_4)_2 + H_2O_2 + 2H_2O \rightarrow \underset{(X)}{H_2TiO_4} + 2H_2SO_4$

$4H_2O_2 + K_2Cr_2O_4 + 4H_2SO_4 \rightarrow K_2SO_4 + \underset{(Y)}{2CrO_5} + 5H_2O$

$H_2TiO_4\ (X): Ti\ (+4)$

$CrO_5\ (Y): Cr\ (+6)$

$P = 4;\ \ Q = 6$

$\therefore\ Q - P = 6 - 4 = 2$

6. How many elements in group 2 will show paramagnetic solution in liquid ammonia. (except radioactive element)

6. 3

**Concept code:** C110902

Sol: *Ca*, *Sr*, *Ba*

7. To prepare 1 mole of acetylene we need *x* mole of $CaCO_3$ and *y* moles of Coke / carbon are required and 'Z' moles of silver is heated with iodoform to produce 1 mole of acetylene. What is *xy* + *z* = _____.

7. 9

**Concept code:** C111710

Sol: $CaCO_3 \xrightarrow{\Delta} CaO + CO_2 \uparrow$

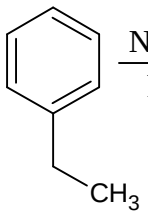
$CaO + 3C \longrightarrow CaC_2 + CO \uparrow$

$CaC_2 + 2H_2O \longrightarrow Ca(OH)_2 + C_2H_2$

$CHI_3 + 6Ag + CHI_3 \xrightarrow{\Delta} H - C \equiv C - H + 6AgI$

$x = 1; \quad y = 3; \quad z = 6$

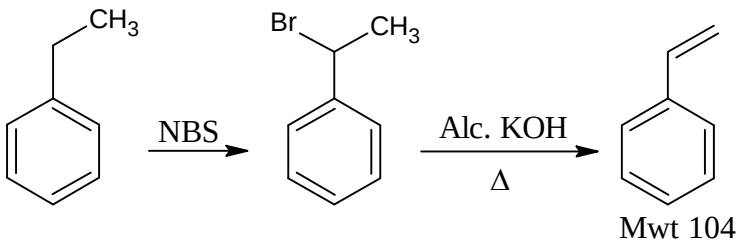
$\Rightarrow (x \times y) + z = (1 \times 3) + 6 = 9$

8.  $\xrightarrow[h\nu]{NBS}$ A $\xrightarrow[KOH, \Delta]{Alcoholic}$ B (major product) than molar mass of B is *X*.

What is $\frac{X}{26} =$

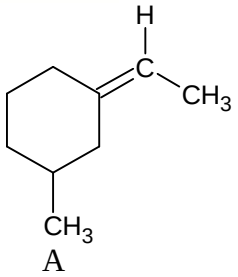
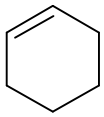
8. 4

**Concept code:** C111708

Sol: 

Mwt 104

$X = 104; \qquad \frac{X}{26} = \frac{104}{26} = 4$

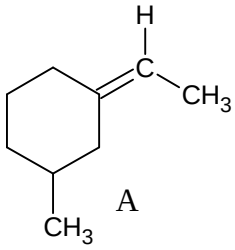
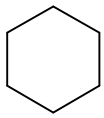
9.  ;  $\xrightarrow{H_2 / Ni}$ (B) major

A

'X' number of stereoisomers possible for (A) and 'Y' is the degree of unsaturation in (B), what is $X + Y$ ?

9. 5

**Concept code:** C111708

Sol:  ; 

A B

S.I = $2^2 = 4$ Degree of unsaturation = 1

X = 4 Y =1

$\therefore\ X + Y = 4 + 1 = 5$

10. The summation of water molecules in microcosmic salt, sodium thiosulphate (Hypo), and glauber's salt is $X$. What is $X - 10$ = _____.

10. 9

**Concept code:** C110808

Sol: Microcosmic salt : $Na(NH_4)HPO_4.4H_2O$

Hypo : $Na_2S_2O_3.5H_2O$

Glauber's salt : $Na_2SO_4.10H_2O$

$4 + 5 + 10 = 19$

$X = 19$

$\therefore\ X - 10 = 19 - 10 = 9$