Mathematics

PART - A

SINGLE ANSWER CORRECT:

1. Let $f(x)=||x|-1|$ then $f'(2)$ equals

a) 0 b) 1 c) 2 d) – 1

1. B

Concept code: M120501

Sol: $f(x)=|x-1|=x-1$ when x is near 2.

2. The equation of tangent to the curve $y=\begin{cases} x^2\sin\frac{1}{x} & ,x\neq 0 \\ 0 & ,x=0 \end{cases}$ at the origin is

a) $x=0$ b) $y=0$ c) $x+y=0$ d) $x-y=0$

2. B

Concept code: M120501

Sol: $f'(0)=\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{h^2\sin\frac{1}{h}}{h}$

$=\lim\limits_{h\to 0}\sin\left(\dfrac{1}{h}\right)=0$

$\Rightarrow y-0=0(x-0)\Rightarrow y=0$

3. Let $f(x)=\sin x$. Let $g(x)=\dfrac{f(x)-f(x_1)}{x-x_1}$. Then $\lim\limits_{x\to x_1} g(x)$ cannot be

a) 0 b) 1 c) -1 d) 2

3. D

Concept code: M120501

Sol: $g(x)=f'(x_1)=\cos x_1\in[-1,1]$

4. If $f(x)+f(2x)+f(3x)+.....+f(10x)=x\ \forall\ x\in R$. Then $f'(0)$ equals

a) 1 b) 55 c) $\dfrac{1}{55}$ d) 0

4. C

Concept code: M120501

Sol: $f'(0)+2f'(0)+....+10f'(0)=1$

$\Rightarrow f'(0)=\frac{1}{55}$

5. The function $f(x)=\frac{x}{2}+\frac{2}{x}$ has a local minimum at

a) $x=2$ b) $x=-2$ c) $x=0$ d) $x=1$

5.a

Concept code: M120605

Sol. We have $f(x)=\frac{x}{2}+\frac{2}{x}$

$\therefore\ f'(x)=\frac{1}{2}-\frac{2}{x^2}$ and $f''(x)=\frac{4}{x^3}$

Now, $f'(x)=0$ or $x^2=4$ or $x=\pm 2$

$\therefore\ f''(x)>0$ for $x=2$

Therefore, f has local minima at $x=2$

6. The maximum value of the function $f(x)=\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left(0,\frac{\pi}{2}\right)$ occurs at

a) $\frac{\pi}{12}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

6.a

Concept code: M120605

Sol. $f(x)=\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$

$=\sqrt{2}\sin\left(x+\frac{\pi}{6}+\frac{\pi}{4}\right)$

$=\sqrt{2}\sin\left(x+\frac{5\pi}{12}\right)$

Its maximum value is $\sqrt{2}$ when $x+\frac{5\pi}{12}=\frac{\pi}{2}$,

i.e ., when $x=\frac{\pi}{2}-\frac{5\pi}{12}=\frac{6\pi-5\pi}{12}=\frac{\pi}{12}$

7. Let $f(x)=\begin{cases} x+2, & -1\le x<0 \\ 1, & x=0 \\ \dfrac{x}{2}, & 0<x\le 1 \end{cases}$

Then on $[-1,1]$, this function has

a) a minimum
b) a maximum
c) either a maximum or a minimum
d) neither a maximum nor a minimum

7.d

Concept code: M120605

Sol. $f(0)>f(0^+)$ and $f(0)<f(0^-)$. Hence, $x=0$ is neither a maximum nor a minimum

8. Let $f(x)=\cos\pi x+10x+3x^2+x^3,\ -2\le x\le 3$. The absolute minimum value of $f(x)$ is

a) 0
b) – 15
c) $3-2\pi$
d) 15

8.b

Concept code: M120605

Sol. $f'(x)=-\pi\sin\pi x+10+6x+3x^2$

$=3(x+1)^2+7-\pi\sin\pi x>0$ for all x

Thus, $f(x)$ is increasing in $-2\le x\le 3$

So, absolute minimum = $f(-2)=1-20+12-8$

9. A real valued function $f(x)=C\log_e|x|+Dx^3+x,\ x\ne 0$, where C and D are constants, has critical points at $x=-1$ and $x=2$. Then the ordered pair (C,D) is

a) $\left(\frac{2}{3},-\frac{1}{9}\right)$
b) $\left(\frac{1}{9},-\frac{2}{3}\right)$
c) $\left(-\frac{2}{3},\frac{1}{9}\right)$
d) $\left(-\frac{1}{9},\frac{2}{3}\right)$

9.a

Concept code: M120606

Sol. $f'(x)=\frac{C}{x}+3Dx^2+1$

$f'(-1)=0$

$-C+3D+1=0$

$f'(2)=0$

$\frac{C}{2}+12D+1=0$

From (1) & (2)

$C = \frac{2}{3},\ D = \frac{-1}{9}$

10. The area (in sq. units) of the region bounded by the curve $\sqrt{x} + \sqrt{y} = 1,\ x, y \geq 0$ and the tangent to it at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ is

a) $\frac{1}{36}$ b) $\frac{1}{8}$ c) $\frac{1}{12}$ d) $\frac{1}{24}$

10.b

Concept code: M120605

Sol. $\sqrt{x} + \sqrt{y} = 1$

$m = \left(\frac{dy}{dx}\right)_{\left(\frac{1}{4}, \frac{1}{4}\right)} = -1$

Tangent equation is $\frac{x}{1/2} + \frac{y}{1/2} = 1$

Area = $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

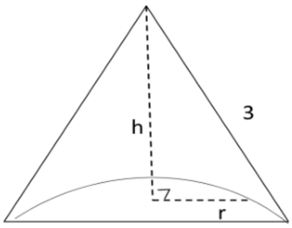
11. The maximum volume (in Cu.m) of the right circular cone having slant height 3m is

a) 6π b) $2\sqrt{3}\pi$ c) $\frac{4\pi}{3}$ d) $3\sqrt{3}\pi$

11.b

Concept code: M120606

Sol.



Volume of the circular cone (v) = $\frac{1}{3}\pi r^2 h$

$r^2 + h^2 = 9$

Volume $v = f(h) = \frac{\pi h}{3}(9 - h^2)$

$\frac{dv}{dh} = (3\pi - \pi h^2) = 0 \rightarrow h = \pm\sqrt{3}$

$\left.\frac{d^2 v}{dh^2}\right]_{h=\sqrt{3}} < 0$

$\therefore$ maximum volume of the cone

$f(\sqrt{3}) = 2\sqrt{3}\pi$

12. The shortest distance between the point $\left(\frac{3}{2},0\right)$ and the curve $y=\sqrt{x},(x>0)$ is

a) $\frac{\sqrt{5}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{3}{2}$ d) $\frac{5}{4}$

12.a

Concept code: M120605

Sol. Let any point on the curve
$y=\sqrt{x}$ is $P\left(t^2,t\right)$

Let distance between $\left(t^2,2\right),\left(\frac{3}{2},0\right)$ is $f(t)=\sqrt{\left(t^2-\frac{3}{2}\right)^2+t^2}$

$f'(t)=0$ then $t^2=1$

Minimum distance $f(t)=\sqrt{\left(1-\frac{3}{2}\right)^2+1}=\frac{\sqrt{5}}{2}$

13. Let $f:R\to R$ be differentiable at $c\in R$ and $f(c)=0$. If $g(x)=\left|f(x)\right|$, then at $x=c,g$ is

a) not differentiable
b) differentiable if $f'(c)\neq 0$
c) differentiable if $f'(c)=0$
d) not differentiable if $f'(c)=0$

13.c

Concept code: M120501

Sol. f is differentiable at $x=a$

If $\underset{h\to 0}{lt}\frac{f(x+h)-f(x)}{h}$ exist finetly

$g'(c)=\lim\limits_{h\to 0}\frac{\left|f(c+h)\right|-\left|f(c)\right|}{h}$

$=\lim\limits_{h\to 0}\frac{\left|f(c+h)\right|}{h}$

$=\lim\limits_{h\to 0}\left|\frac{f(c+h)-f(c)}{h}\right|\frac{|h|}{h}$

$=\lim\limits_{h\to 0}\left|f'(c)\right|\frac{|h|}{h}$

0 *if* $f'(c)=0$

$\therefore$ g is differentiable at $x=c$ if $f'(c)=0$

14. Let $f(x)=e^x-x$ and $g(x)=x^2-x,\ \forall\ x\in R$. Then the set of all $x\in R$, where the function $h(x)=(fog)(x)$ is increasing, is

a) $\left[-\frac{1}{2},0\right]\cup[1,\infty)$
b) $[0,\infty)$
c) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$
d) $\left[-1,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$

14.c

Concept code: M120604

Sol. $h(x)=f\left(g(x)\right)$

$h'(x)=$

$=\left(e^{x^2-x}-1\right)(2x-1)\geq 0$

Case 1 : $e^{x^2-x}\geq 1$ and $2x-1\geq 0$

$\rightarrow x\in[1,\infty)$

Case 2 : $e^{x^2-x}\leq 1$ and $2x-1\leq 0$

$\rightarrow x\in\left[0,\frac{1}{2}\right]$

$\therefore\ x\in\left[0,\frac{1}{2}\right]\cup[1,\infty)$

15. Let $f(x)$ be a polynomial of degree 5 such that $x=\pm 1$ are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$, then which one of the following is not true?

a) $f(1)-4f(-1)=4$
b) $x=1$ is a point of maxima and $x=-1$ is a point of minimum of f
c) f is an odd function
d) $x=1$ is a point of minima and $x=-1$ is a point of maxima of f

15.b

Concept code: M120605

Sol. $f'(x)=a(x+1)(x-1)x^2$

$=a\left(x^2-1\right)x^2$

$f'(x)=ax^4-x^2$

$f(x)=\frac{ax^5}{5}-\frac{x^3}{3}+c$

$f(0)=0\rightarrow c=0$

$\lim_{x\to 0}\frac{f(x)}{x^3}=2\rightarrow a=-6$

$f(x)=\frac{-6}{5}x^5+2x^3$

16. The value of c in the Lagrange's mean value theorem for the function $f(x)=x^3-4x^2+8x+11$, where $x\in[0,1]$ is

a) $\frac{4-\sqrt{7}}{3}$ b) $\frac{2}{3}$ c) $\frac{\sqrt{7}-2}{3}$ d) $\frac{4-\sqrt{5}}{3}$

16.a

Concept code: M120607

Sol. $f(x)=x^3-4x^2+8x+11,\ x\in[0,1]$

Using LMVT

$$f'(c)=\frac{f(1)-f(0)}{1-0}$$

$$3c^2-8c+8=\frac{16-11}{1-0}$$

$$3c^2-8c+3=0$$

$$c=\frac{4-\sqrt{7}}{3}\in(0,1)$$

17. If c is a point at which Rolle's theorem holds for the function, $f(x)=\log_e\left(\frac{x^2+a}{7x}\right)$ in the interval $[3,4]$, where $a\in R$ then $f''(c)$ is equal to

a) $-\frac{1}{24}$ b) $-\frac{1}{12}$ c) $\frac{\sqrt{3}}{7}$ d) $\frac{1}{12}$

17.d

Concept code: M120607

Sol. $f(3)=f(4)$

$$\log\left(\frac{9+\alpha}{21}\right)=\log\left(\frac{16+\alpha}{28}\right)$$

$$\frac{9+\alpha}{16+\alpha}\cdot\frac{4}{3}=1$$

$$\to\alpha=12$$

Applying Rolle's theorem $c\in(3,4)$ such that $f'(c)=0$

$$c^2=12$$

Now, $f(x)=\log_e(x^2+12)-\log_e 7x$

$$f''(x)=\frac{4x^2}{(x^2+12)^2}+\frac{2}{x^2+12}+\frac{1}{x^2}$$

$$f''(c)=\frac{1}{12}$$

18. Let $f(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$ and $g(x)=|f(x)|+f(|x|)$.

Then, in the interval $(-2,2)$, g is

a) differentiable at all points
b) not continuous
c) not differentiable at two points
d) not differentiable at one point

18.d

Concept code: M120605

Sol. $f(|x|)=x^2-1$ if $x\in[-2,2]$

$$|f(x)|=\begin{cases}1, & if\ x\in[-2,0)\\ 1-x^2, & if\ x\in[0,1)\\ x^2-1, & if\ x\in[1,2]\end{cases}$$

$$g(x)=\begin{cases}x^2 & if\ -2\le x<0\\ 0 & if\ 0\le x<1\\ 2(x^2-1) & if\ 1\le x\le 2\end{cases}$$

Clearly $g(x)$ is not differentiable at $x=1$

19. Let the function $f:(-1,3)\to R$ be defined as $f(x)=\min\{x[x],|x[x]-2|+2\}$, where $[x]$ denotes the greatest integer $\le x$. Then f is

a) neither continues nor differentiable at exactly 3 points
b) not continues at only one point and not differentiable at 3 points
c) neither continues nor differentiable at exactly 2 points
d) not continues at two points and not differentiable at 3 points

19.d

Concept code: M120405

Sol. Clearly from the graph not continuous at 2 points, not differentiable at 3 points

20. The tangent to the curve $y=xe^{x^2}$ passing through the point $(1,e)$ also passes through the point

a) $(2,3e)$
b) $\left(\frac{4}{3},2e\right)$
c) $\left(\frac{5}{3},2e\right)$
d) $(3,6e)$

20.b

Concept code: M120601

Sol. $m=\left(\frac{dy}{dx}\right)_{(1,e)}=3e$

Tangent equation $y-e=3e(x-1)$

PART - D

NUMERICAL ANSWER TYPE:

1. If $y = x^3 + x^2 + x + 1$. Then slope of tangent at $x = 0$ is

1. 1

Concept code: M120601

Sol: $\frac{dy}{dx} = 3x^2 + 2x + 1 = 1$ at $x = 0$

2. The line $x + y = 2$ touches the curve $xy = a^2$ then $|a|$ equals

2. 1

Concept code: M120601

Sol: $\frac{dy}{dx} = -\frac{y}{x}$ and $\frac{dy}{dx} = -1 \Rightarrow y = x$ gives $(x, y) \equiv (1,1)$

Hence $|a| = 1$

3. The maximum value of $f(x) = \frac{x}{1 + 4x + x^2}$ is k then $12k$ =

3.2

Concept code: M120605

Sol. $f'(x) = \frac{(1 + 4x + x^2)1 - x(4 + 2x)}{(1 + 4x + x^2)^2} = \frac{1 - x^2}{(1 + 4x + x^2)^2}$

For maximum or minimum, $f'(x) = 0$ *or* $x = \pm 1$

For $x = 1$, $f'(x)$ changes sign from positive to negative as x passes through 1

Therefore, $f(x)$ is maximum for $x = 1$, and maximum value

$= \frac{1}{1 + 4 + 1} = \frac{1}{6}$

4. The maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 27$ is

4.12

Concept code: M120605

Sol. $y = -x^3 + 3x^2 + 9x - 27$

$\therefore \dfrac{dy}{dx} = -3x^2 + 6x + 9$

Let the slope of tangent to the curve at any point be m (say).

Then, $m = -3x^2 + 6x + 9$ *or* $\dfrac{dm}{dx} = -6x + 6$

$\dfrac{d^2m}{dx^2} = -6 < 0$ for all x

Therefore, m is maximum when $\dfrac{dm}{dx} = 0$, i.e., when $x = 1$

Therefore, maximum slope = $-3 + 6 + 9 = 12$

5. Let $f(x) = 5 - |x-2|$ and $g(x) = |x+1|$, $x \in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim\limits_{x \to -\alpha\beta} \dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to

5.0.5

Concept code: M120605

Sol. $f(x)$ attains maximum value at $\alpha = 2$, $g(x)$ attains minimum value at $\beta = -1$

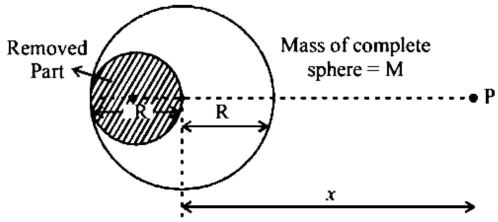
$\lim\limits_{x \to 2} \dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-2)} = \dfrac{1}{2}$

Physics

PART - A

SINGLE ANSWER CORRECT:

1. The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown, has been 'removed out'), at a very far off point, P located as shown, would be (nearly) : ($x >> R$)



a) $\frac{5}{6}\frac{GM}{x^2}$ b) $\frac{8}{9}\frac{GM}{x^2}$ c) $\frac{7}{8}\frac{GM}{x^2}$ d) $\frac{6}{7}\frac{GM}{x^2}$

1. c

CONCEPT CODE : P110901

Sol. Let mass of smaller sphere (which has to be removed) is m

Radius $= \frac{R}{2}$ (from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere

$$M' = M - \frac{M}{8} = \frac{7}{8}M$$

Therefore gravitational field due to the left over part of the sphere

$$= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$$

2. The height h at which the weight of a body will be the same as that at the same depth h from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) :

a) $\frac{\sqrt{5}}{2}R - R$ b) $\frac{R}{2}$ c) $\frac{\sqrt{5}R - R}{2}$ d) $\frac{\sqrt{3}R - R}{2}$

2. c

CONCEPT CODE : P110901

Sol. The acceleration due to gravity at a height h is given by

$$g = \frac{GM}{(R+h)^2}$$

Here, G = gravitational constant
M = mass of earth

The acceleration due to gravity at depth h is

$$g' = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$$

Given, $g = g'$

$$\therefore \quad \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1-\frac{h}{R}\right)$$

$$\therefore \quad R^3 = (R+h)^2(R-h) = (R^2+h^2+2hR)(R-h)$$

$$\Rightarrow \quad R^3 = R^3 + h^2R + 2hR^2 - R^2h - h^3 - 2h^2R$$

$$\Rightarrow \quad h^3 + h^2(2R-R) - R^2h = 0$$

$$\Rightarrow \quad h^3 + h^2R - R^2h = 0 \quad \Rightarrow \quad h^2 + hR - R^2 = 0$$

$$\Rightarrow \quad h = \frac{-R \pm \sqrt{R^2 + 4(1)R^2}}{2}$$

$$= \frac{-R \pm \sqrt{5}R}{2} = \frac{\left(\sqrt{5}-1\right)}{2}R$$

3. Planet A has mass M and radius R. Planet B has half the mas and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is

a) 4 b) 1 c) 2 d) 3

3. a

CONCEPT CODE : P110904

Sol. Escape velocity of the planet A is $v_A = \sqrt{\frac{2GM_A}{R_A}}$ where M_A and R_A be the mass and radius of the planet A.

According to given problem

$$M_B = \frac{M_A}{2}, \quad R_B = \frac{R_A}{2}$$

$$\therefore \quad v_B = \sqrt{\frac{2G\frac{M_A}{2}}{\frac{R_A}{2}}} \qquad \therefore \quad \frac{v_A}{v_B} = \sqrt{\frac{\frac{2GM_A}{R_A}}{\frac{2GM_A/2}{R_A/2}}} = \frac{n}{4} = 1$$

$$\Rightarrow \quad n = 4$$

4. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass 2M. The gravitational field at distance $3a$ from the centre will be

a) $\frac{2GM}{9a^2}$ b) $\frac{GM}{9a^2}$ c) $\frac{GM}{3a^2}$ d) $\frac{2GM}{3a^2}$

4. c

CONCEPT CODE : P110901

Sol. $E_g = \dfrac{GM}{(3a)^2} + \dfrac{G(2M)}{(3a)^2} = \dfrac{GM}{3a^2}$

5. A satellite is revolving in a circular orbit at a height h from the earth's surface (radius of earth R; $h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atomisphere)

a) $\sqrt{\dfrac{gR}{2}}$ b) $\sqrt{gR}\left(\sqrt{2}-1\right)$ c) $\sqrt{2gR}$ d) $\sqrt{gR}$

5. b

<u>CONCEPT CODE</u> : P110904

Sol. For $h << R$, the orbital velocity is $\sqrt{gR}$

Escape velocity $= \sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\left(\sqrt{2}-1\right)$

6. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is :

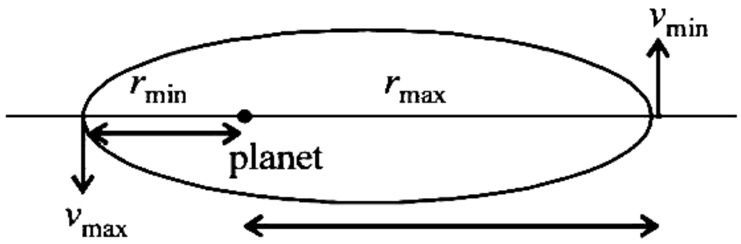
a) 1 : 6 b) 1 : 3 c) 1 : 2 d) 3 : 4

6. a

<u>CONCEPT CODE</u> : P110905

Sol. By angular momentum conservation

$m r_{\min} v_{\max} = m r_{\max} v_{\min}$



Given, $v_{\min} = \dfrac{v_{\max}}{6}$ $\therefore \dfrac{r_{\min}}{r_{\max}} = \dfrac{v_{\min}}{v_{\max}} = \dfrac{1}{6}$

7. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

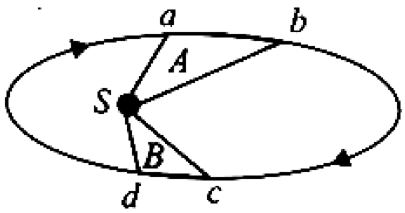
a) zero b) $-\dfrac{4Gm}{r}$ c) $-\dfrac{6Gm}{r}$ d) $-\dfrac{9Gm}{r}$

7. d

<u>CONCEPT CODE</u> : P110903

Sol. Gravitational potential $= \frac{-Gm}{r}$

8. Figure shows the motion of a planet around the Sun S in an elliptical orbit with the Sun at the focus. The shaded areas A and B are also shown in the figure which can be assumed to be equal. If t_1 and t_2 represent the time taken for the planet to move from *a* to *b* and *c* to *d*, respectively, then



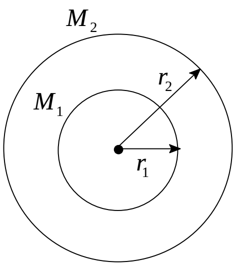
a) $t_1 < t_2$
b) $t_1 > t_2$
c) $t_1 = t_2$
d) from the given information the relation between t_1 and t_2 cannot be determined

8. c

<u>CONCEPT CODE</u> : P110905

Sol. $\frac{dA}{dt} = \text{constant}$

9. Two concentric shells of masses M_1 and M_2 are having radii r_1 and r_2. Which of the following is the correct expression for the gravitational field on a mass *m*?



a) $F = \frac{G(M_1 + M_2)}{r^2}$, for $r < r_1$
b) $F = \frac{G(M_1 + M_2)}{r^2}$, for $r < r_2$
c) $F = \frac{GM_2}{r^2}$, for $r_1 < r < r_2$
d) $F = \frac{GM_1}{r^2}$, for $r_1 < r < r_2$

9. d

<u>CONCEPT CODE</u> : P110901

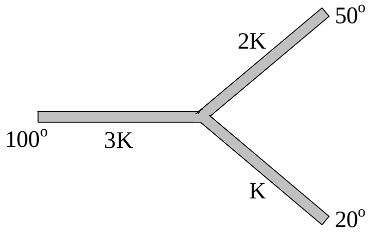
10. A metal disc having circular hole at its centre is heated. If the metal expands on heating the diameter of the hole will
a) increase
b) decrease
c) remain unchanged
d) increases or decreases depending upon the metal

10. a

<u>CONCEPT CODE</u> : P111207

Sol. Subjective

11. Three rods of the same dimension have thermal conductivities 3K, 2K and K. They are arranged as shown in figure. Given below, with their ends at $100^{o}C$, $50^{o}C$ and $20^{o}C$. The temperature of their junction is

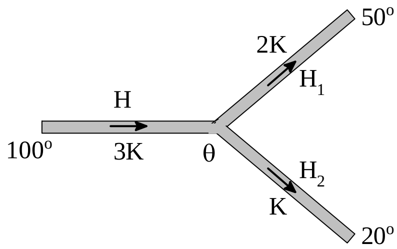


a) $60^{o}C$ b) $70^{o}C$ c) $50^{o}C$ d) $35^{o}C$

11. b

<u>CONCEPT CODE</u> : P111207

Sol. Let the temperature of junction be q then according to following figure.



$H = H_1 + H_2$

$$\frac{3K \times A \times (100-\theta)}{l} = \frac{2K(\theta-50)}{l} + \frac{KA(\theta-20)}{l}$$

$\Rightarrow$ $300 = 3q = 3q?$

$120q = 70^{o}C$

12. Three discs A, B and C having radii 2m, 4m and 6m respectively are coated with carbon black on their other surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively

a) Q_A is maximum b) Q_B is maximum
c) Q_C is maximum d) $Q_A = Q_B = Q_C$

12. b

<u>CONCEPT CODE</u> : P111208

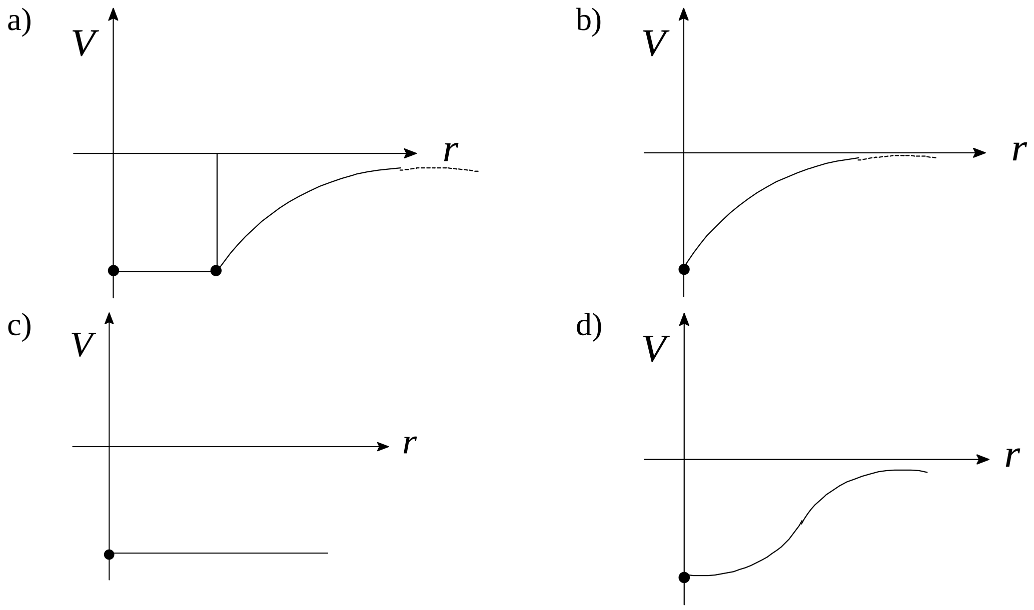
Sol. Radiated power $P = A\varepsilon\sigma T^4$ $P \propto T^4$

From Wein's law,

$\lambda_m T =$ constant $T \propto \frac{1}{\lambda_m}$ $P \propto \frac{A}{(\lambda_m)^4} \propto \frac{r^2}{(\lambda_m)^4}$

$Q_A : Q_B : Q_C = \frac{2^2}{(300)^4} : \frac{4^2}{(400)^4} : \frac{6^2}{(500)^4}$ Q_B will be maximum.

13. Which of the following graph shows the variation of gravitational potential with distance from centre of the hollow sphere.



13. a

<u>CONCEPT CODE</u> : P110903

Sol. Subjective

14. The heat is following through two cylindrical rods of same material. The diameters of the rods are in the ratio 1 : 2 and their lengths are in the ratio 2 : 1. If the temperature difference between their ends is the same, the ratio of rate of flow of heat through them will be

a) 1 : 1 b) 2 : 1 c) 1 : 4 d) 1 : 8

14. d

<u>CONCEPT CODE</u> : P111207

Sol. $\frac{dQ}{dt} = \frac{KA\Delta T}{\ell}$

15. The coefficient of thermal conductivity of conductor depends upon

a) Temperature difference across the ends of the conductor
b) Area of the conductor
c) Thickness of the conductor
d) Material of the conductor

15. d

<u>CONCEPT CODE</u> : P111207

Sol. Coefficient of thermal conductivity depends on native of material.

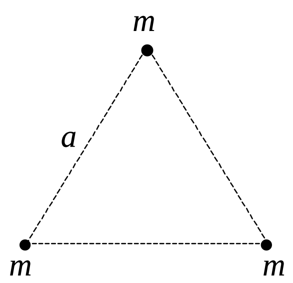
16. Three particles each of mass m are located at the corners of an equilateral triangle of side a. Find minimum work to be done on the system to place the particles at the coners of an equilateral triangle of side $2a$

a) $\frac{-3Gm^2}{a}$ b) $\frac{3}{2}\frac{Gm^2}{a}$ c) $\frac{-3Gm^2}{a}$ d) zero

16. b

CONCEPT CODE : P110903

Sol.



$$U_i = \frac{-3Gm^2}{a} \qquad \frac{1}{2}:\frac{4}{1}$$

$$U_f = \frac{-3Gm^2}{2a} \qquad 1:8$$

$$\text{Minimum work done} = \frac{3}{2}\frac{Gm^2}{a}$$

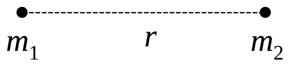
17. Three particles each of mass m are placed at the corners of an equilateral triangle of side a. If we consider the gravitational potential energy of the system is zero when separated by ∞ distance then the gravitational potential energy of the system is

a) $\frac{3Gm^2}{a}$ b) $\frac{-3Gm^2}{a}$ c) $\frac{3Gm^2}{a^2}$ d) $\frac{-3Gm}{a}$

17. b

CONCEPT CODE : P110903

Sol.



$$U_g = \frac{-Gm_1m_2}{r}$$

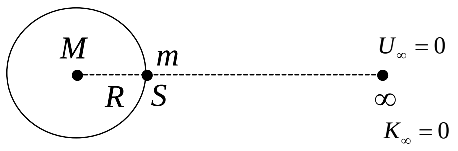
18. A particle of mass m is located on the surface of a planet of mass M and radius R. Find with what minimum speed the particle must be projected so that the particle escapes the gravitational of the planet

a) $2\sqrt{\frac{GM}{R}}$ b) $\sqrt{\frac{3GM}{R}}$ c) $\sqrt{\frac{GM}{R}}$ d) $\sqrt{\frac{2GM}{R}}$

18. d

CONCEPT CODE : P110904

Sol.



$$U_S + K_S = U_\infty + K_\infty$$

$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = 0$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{R}$$

$$v^2 = \frac{2GM}{R}$$

$$v = \sqrt{\frac{2GM}{R}}$$

19. A black body at temperature of 1640 K has the wavelength corresponding to maximum emission equal to 1.75 μm . Assuming the moon to be a perfectly black body, the temperature of the moon, if the wavelength corresponding to maximum emission is 14.35 μm is

a) 100 K b) 150 K c) 200 K d) 250 K

19. c

<u>CONCEPT CODE</u> : P111208

Sol. $\frac{T_2}{T_1} = \frac{\lambda_{m_1}}{\lambda_{m_2}} = \frac{1.75}{14.35} \Rightarrow T_2 = \frac{1.75}{14.35} \times 1640 = 200\ K$

20. Two spheres of the same material have radii 1m and 4m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is

a) 1 : 4 b) 1 : 1 c) 2 : 1 d) 3 : 4

20. b

<u>CONCEPT CODE</u> : P111208

Sol. $P = \frac{d\theta}{dt} = \sigma A T^4$

$$\frac{P_1}{P_2} = \frac{r_1^2 T_1^4}{r_2^2 T_2^4} = \frac{(1)^2 (4000)^4}{(4)^2 (2000)^4} = \frac{1}{1}$$

PART - D

NUMERICAL ANSWER TYPE:

1. The earth (mass $=10^{24}$ kg) revolves round the Sun with an angular velocity 2×10^{-7} rad s^{-1} in a circular orbit of radius 1.5×10^{8} km. The force exerted by the Sun on the earth is $n\times10^{21}$ N. The value of n is

1. 6.00

CONCEPT CODE : P110902

Sol. $F = mr\omega^2$

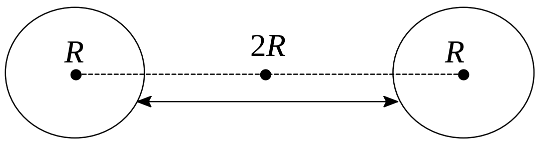
2. Two conductors of same dimensions are connected in series combination. Coefficient of thermal conductivity of the conductors are K and 2K respectively. The equivalent thermal conductivity of the combination is nK. The value of n is

2. 1.33

CONCEPT CODE : P111207

Sol. $$\frac{2}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$
$$K_e = \frac{4K}{3}$$

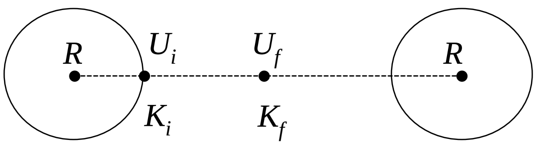
3. Two solid spheres each of mass M and radius R are separated by a distance 2R as shown in figure. The minimum KE of the particle of mass m on surface of sphere 1 to reach sphere 2 is $n\cdot\frac{GMm}{R}$. The value of n is



3. 0.33

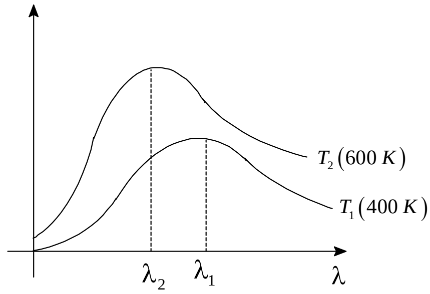
CONCEPT CODE : P110903

Sol.



$$U_i + K_i = U_f + K_f$$

4. Intensity, wavelength of a black body radiation at different temperature is as shown in figure. The rate of E_λ. Then $\frac{\lambda_1}{\lambda_2}$ is



4. 1.5

<u>CONCEPT CODE</u> : P111208

Sol. Wien's displacement law

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2} = \frac{600}{400} = \frac{3}{2}$$

5. Temperature of a black body is doubled. The fractional change in radiation power of the black body is

5. 15

<u>CONCEPT CODE</u> : P111208

Sol. $\frac{dQ}{dt} \propto T^4$

Chemistry

PART - A

SINGLE ANSWER CORRECT:

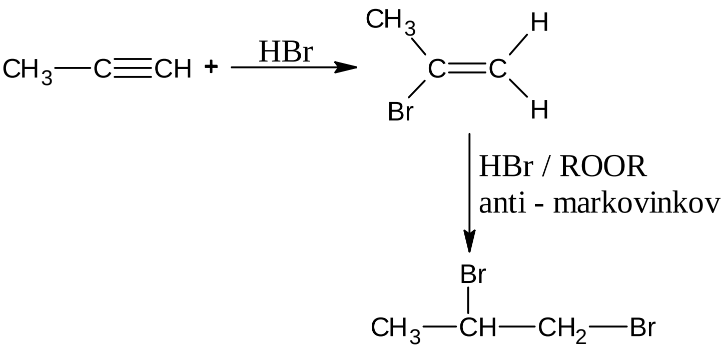
1. Consider the following sequence of reaction $CH_3C \equiv CH \xrightarrow{HBr\,(1eq)} I \xrightarrow[ROOR]{HBr} II$. The products (I) and (II) are respectively.

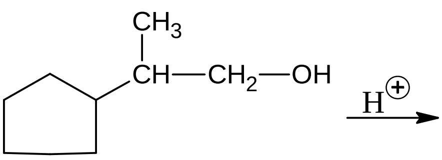
a) trans $CH_3CH = CHBr,\ CH_3CH_2CHBr_2$ b) $CH_3CBr = CH_2,\ CH_3CHBrCH_2Br$

c) cis $CH_3CH = CHBr,\ CH_3CHBrCH_2Br$ d) trans $CH_3CH = CHBr,\ CH_3CHBrCH_2Br$

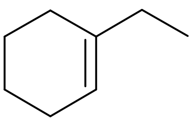
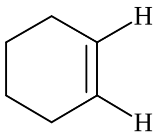
1. B

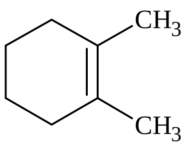
Concept code: C111712

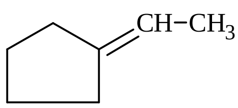
Sol: 

2. 

major product on dehydrationgives

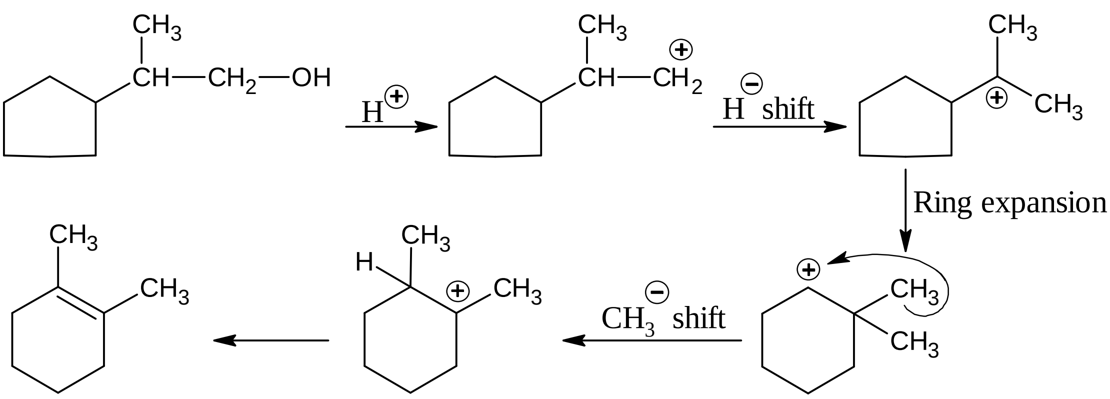
a)  b) 

c) 

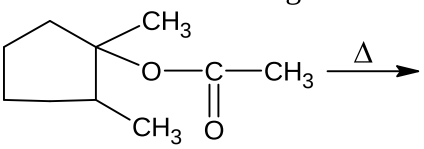
d) 

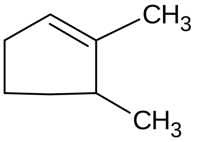
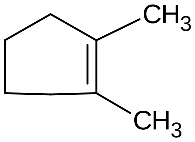
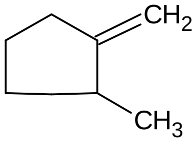
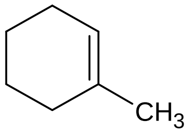
2. C

Concept code: C111708

Sol: 

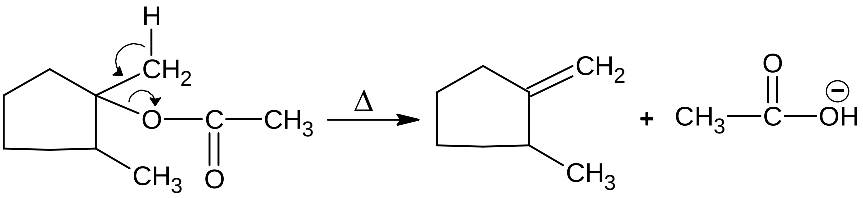
3. Which of the following is the major product for the given reaction ?

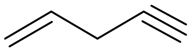


a)  b)  c)  d) 

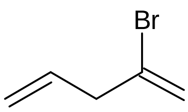
3. C

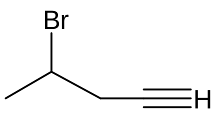
Concept code:C111705

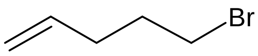
Sol: 

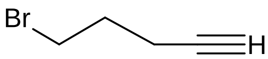
4. Identify the product(s) in the following reaction  $\xrightarrow{\text{1 eq HBr}}$ (B) .

(A)
pent-1-en-4-yne

a) 

b) 

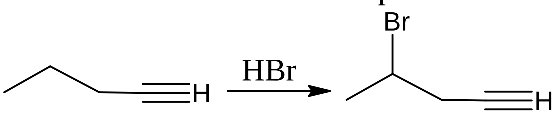
c) 

d) 

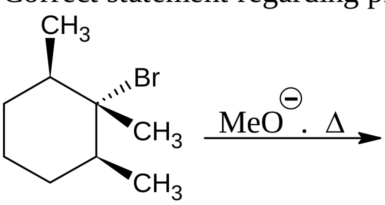
4. B

Concept code:C111707

Sol: In the given compound, electrophilic addition of 1 eq of *HBr* takes place at double bond as double bond is more reactive than triple bond.



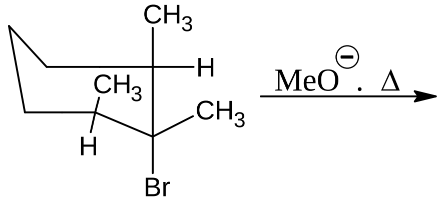
5. Correct statement regarding product ?

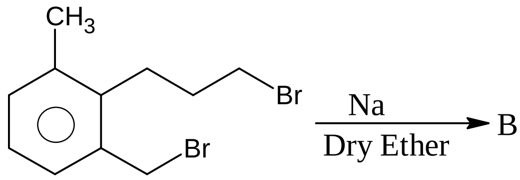


a) only one alkene is produced
b) none – resolvable major product
c) major product shows geometrical isomerism
d) no elimination product is formed

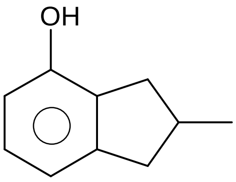
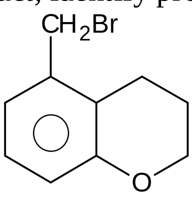
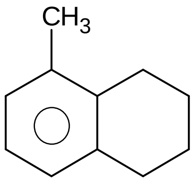
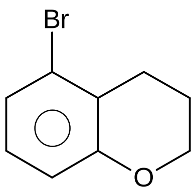
5. D

Concept code:C111705

Sol:  No product as E_2 is anti elimination.

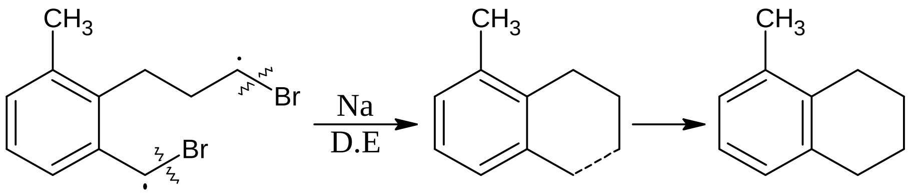
6. 

B is Intramolecular product, identify product B.

a)  b)  c)  d) 

6. C

Concept code:C111701

Sol: 

Intramolecular wurtz reaction

7. $$A \xleftarrow[HgSO_4.H_2O]{H_2SO_4} H-C\equiv C-CH_2-CH_3 \xrightarrow[H_2O_2/OH^-]{BH_3.THF} B$$

A and B are

a) Positional isomers b) Metamers c) Functional isomers d) Homologues

7. C

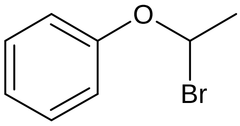
Concept code:C111712

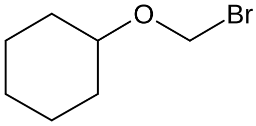
Sol: $H_3C-CH_2-\overset{\overset{O}{||}}{C}-CH_3$ (A)

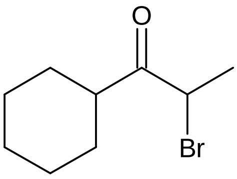
$H_3C-CH_2-CH_2-\overset{\overset{O}{||}}{C}-H$ (B)

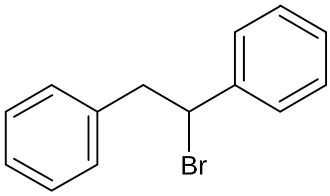
A and B are functional isomers.

8. Which of the following, upon treatment with tert – BuO^-Na^+ followed by addition of bromine water fails to decolorize the colour of bromine.

a) 

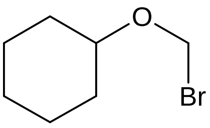
b) 

c) 

d) 

8. B

<u>Concept code:</u>C111708

Sol: 

$\xrightarrow{\text{ter - BuONa}}$ No elimination

9. HBr reacts with $CH_2 = CH - OCH_3$ under anhydrous conditions at room temperature to give. (major product) ________.

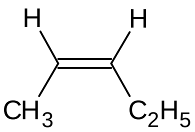
a) $CH_3 - CHBr - OCH_3$ b) $BrCH_2CHO$ and CH_3OH
c) $Br - CH_2 - CH_2 - OCH_3$ d) CH_3CHO and CH_2Br

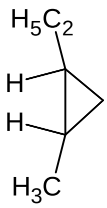
9. A

<u>Concept code:</u>C111708

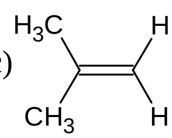
Sol: $H_2C{=}CH{-}OCH_3 \xrightarrow{HBr} H_3C{-}\underset{\displaystyle |\atop \displaystyle Br}{CH}{-}OCH_3$

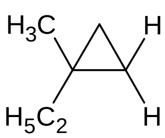
10. $CH_3 - C \equiv C - C_2H_5 \xrightarrow{H_2/Lindalar's\ catalyst} X \xrightarrow[Zn-Cu]{CH_2I_2} Y$;
What are X, Y respectively.
X Y

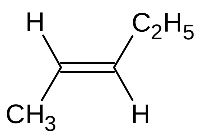
a) 

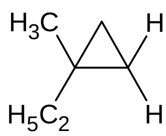


b) $CH_3{-}CH{=}CH{-}NH_2$ $CH_3{-}CH{=}CH{-}CH_2I_2$

c) 



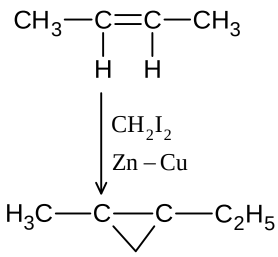
d) 



10. A

Concept code: C111711

Sol: $CH_3 - C \equiv C - C_2H_5 \xrightarrow{H_2/\text{Lindalar's catalyst}}$



11. End product of the following sequence is

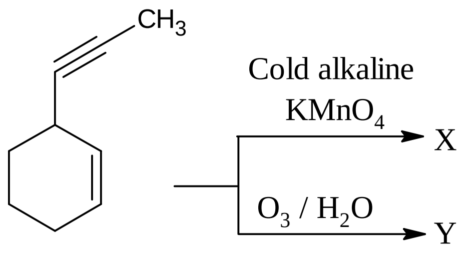
$CaO + C \xrightarrow{Heat} CaC_2 \xrightarrow{H_2O} B \xrightarrow[H_2SO_4]{Hg^{2+}} C$

a) Ethanol b) Ethyl hydrogen sulphate c) Ethanal d) Ethylene slycol

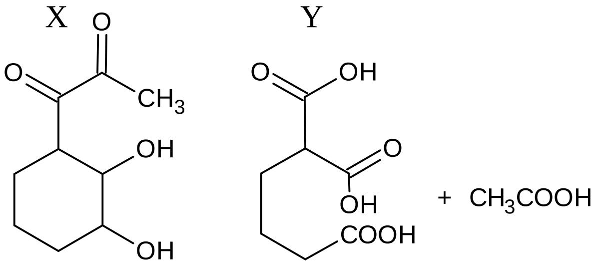
11. C

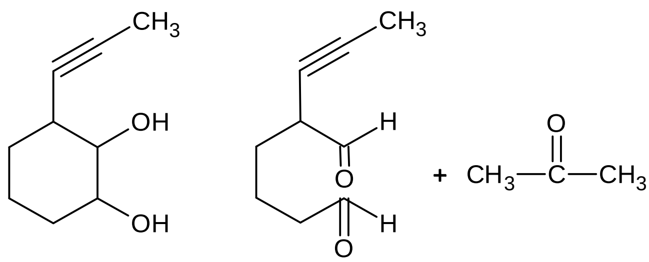
Concept code: C111710

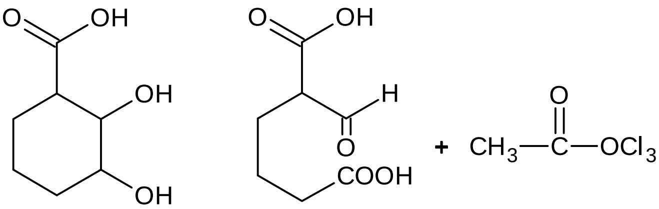
Sol: $CaC_2 \xrightarrow{H_2O} Ca(OH)_2 + H - C \equiv C - H \xrightarrow[H_2SO_4]{Hg^{2+}} CH_3 - \overset{\overset{O}{\|}}{C} - H$

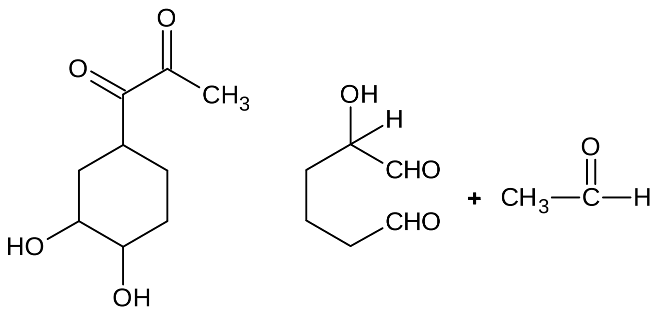
12. 

. Identify X, Y respectively.

a) 

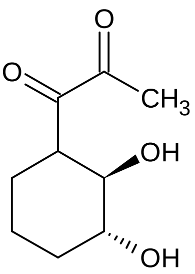
b) 

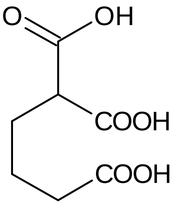
c) 

d) 

12. A

<u>Concept code:</u>C111708

Sol: X =  $KMnO_4$ hydroxylates $C=C$ bond and $C \equiv C$ bond.

y =  + CH_3COH, $C=C$ and $C \equiv C$ bonds are affected due to ozonolysis.

13. Permutit is
a) Hydrated sodium aluminium ortho silicate b) Sodium hexametaphosphate
c) Sodium silicate d) Sodium metal-aluminate

13. A

Concept code:C111003

Sol: Conceptual

14. Hydrogen cannot be obtained by
a) $Zn + dil\ H_2SO_4$ b) $Mg + H_2O$ c) $Zn + dil\ HNO_3$ d) $Mg + dil\ H_2SO_4$

14. C

Concept code:C111001

Sol: Zn reacts to form hydrogen with dilute HNO_3. Before the hydrogen is released, it reduces HNO_3 into an oxide of nitrogen.

15. Which of the following hydrides conducts electricity in fused state ?
a) SiH_4 b) B_2H_6 c) CH_4 d) KH

15. D

Concept code:C111002

Sol: KH is ionic hydride.

16. Which of the following process gives very pure hydrogen (99.9 %).
a) Reaction of salt like hydrides with water
b) Electrolysing warm aqueous barium hydroxide solution
c) Reaction of methane with steam
d) Mixing natural hydrocarbons of high molecular weight

16. B

<u>Concept code:</u>C111002

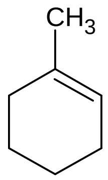
Sol: Highly pure hydrogen is obtained by the electrolysis of water.

17. Water gas is produced by
a) passing steam through a red hot coke bed
b) saturating hydrogen with moisture
c) mixing oxygen and hydrogen in the ratio of 1 : 2
d) heating a mixture of CO_2 and CH_4 in petroleum refineries

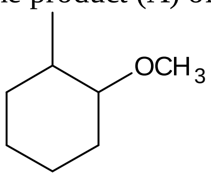
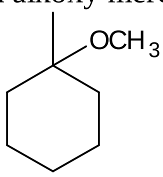
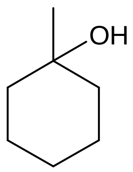
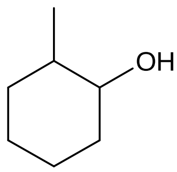
17. A

<u>Concept code:</u>C111003

Sol: $\underset{(Steam)}{H_2O(g)} + \underset{(Red\ hot\ coke)}{C(s)} \xrightarrow[Ni]{1270K} \underset{(Water\ gas)}{H_2(g)+CO(g)}$

18.  $\xrightarrow[\text{(ii) NaBH}_4\text{, OH}^{\ominus}]{\text{(i) Hg(O}_2\text{CCH}_3\text{)}_2\text{.CH}_3\text{OH}}$ A (major)

The product (A) of given alkoxy mercurationdemercuration is

a) 
b) 
c) 
d) 

18. B

<u>Concept code:</u>C111708

Sol: Addition of CH_3OH according to markovnikoffs rule take place.

19. Which of the following is incorrect for isotopes of hydrogen.
a) Melting point and density $\left({}_1^1H < {}_1^2H < {}_1^3H\right)$ b) Bond length $\left(H_2 = D_2 = T_2\right)$
c) Bond dissociation $\left(H_2 > D_2 > T_2\right)$ d) Abundance $\left(H_2 > D_2 > T_2\right)$

19. C

<u>Concept code:</u>C111002

Sol: Bond dissociation is $\left(H_2 < D_2 < T_2\right)$

20. How many coordinating and hydrogen bonded water molecules are associated with $CuSO_4.5H_2O$ respectively.
a) 5 and zero b) zero and 5 c) 4 and 1 d) 1 and 4

20. C

Concept code: C111003

Sol: Only one water molecule, which is not side the brackets (coordination sphere) is hydrogen bonded. The other 4 molecules of water are coordinated.

PART - D

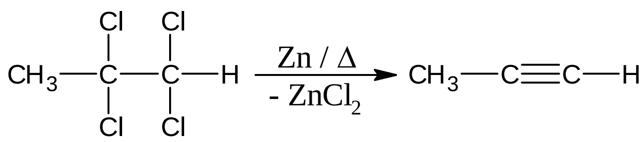
NUMERICAL ANSWER TYPE:

1. 36.4g of 1, 1, 2, 2 –tetrachloropropane was heated with zinc in alcohol and the product was bubbled through ammonical $AgNO_3$. The wight of precipitate (in g) obtained is _________.

1. 29.40

Concept code: C111711

Sol:



(M.wt = 182)

$$CH_3-C\equiv C-H+AgNO_3\xrightarrow[\Delta]{NH_4OH} CH_3-C\equiv C-Ag \;+\; NH_4NO_3+H_2O$$

$$(\text{silver propynide PPT})$$

$$(\text{Mol wt}=147)$$

- 182 g of tetrachloro alkane gives 147 g of PPT.

36.4 g of tetra chloro alkane gives $=\dfrac{147\times 36.4}{182}=29.4g$

2. Calgon is a complex salt for softening water $Na_P\left[Na_Q\left(PO_R\right)_S\right]$ what is $\dfrac{QS}{PR}+12$ is _______.

2. 16.00

Concept code: C111003

Sol: $P=2,\ Q=4,\ R=3,\ S=6$

$\dfrac{4\times 6}{2\times 3}=\dfrac{24}{6}=4+12=16$.

3. One litre of a sample of hard water contains 1mg of $CaCl_2$ and 1mg of $MgCl_2$. Find the total hardness in terms of parts of $CaCO_3$ per 10^6 parts of water by mass. The total hardness is _________ ppm.

3. 1.95

Concept code: C111003

Sol: mol. Mass of $CaCl_2=111$,

mol. mass of $MgCl_2=95$

111 grams of $CaCl_2 = \frac{100}{111} \times 1\text{mg of } CaCO_3 = 0.9\text{mg of } CaCO_3$

95 grams of $MgCl_2 = 100$ grams of $CaCO_3$

1mg of $MgCl_2 = \frac{100}{95} \times 1\text{mg of } CaCO_3$ = 1.05mg of $CaCO_3$

Thus one litre of hard water contains (0.9 + 1.05) = 1.95 mg of $CaCO_3$.

One litre water $= 10^3$ grams $= 10^6\, mg$

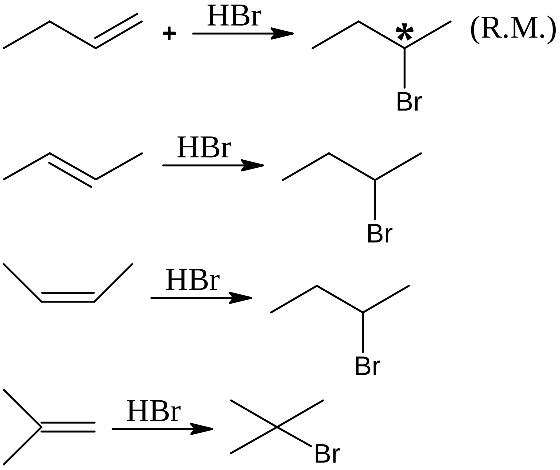
Degree of hardness = 1. 95 ppm.

4. (x) all possible olefins with formula $C_4H_8 \xrightarrow{HBr}$ (y) total number of products. Then find $x \times y$.

4. 12.00

<u>Concept code:</u>C111707

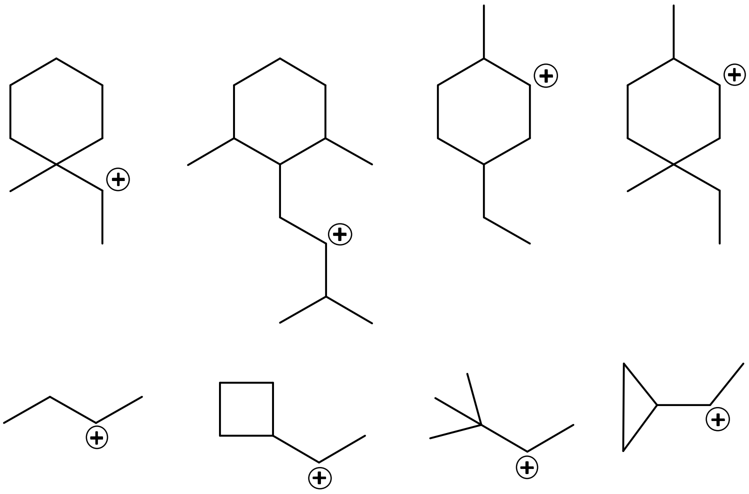
Sol:



$x = 4,\ y = 3;$

$\Rightarrow x \times y = 12$

5.



From the above,'x' no. of species undergo rearrangement from the following. Find the value 3x.

5. 18.00

<u>Concept code:</u> C111708

Sol: 1 – methyl shift 2 – hydride shift 3 – hydride shift
4 – hydride shift 5 – no rearrangement
6 – ring expansion 7 – methyl shift
8 – stable carbocation due to presence of $\sigma-$ resonance so it will not rearrange