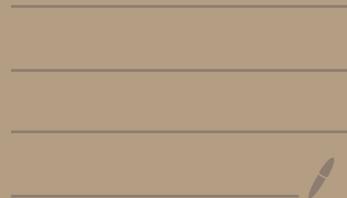
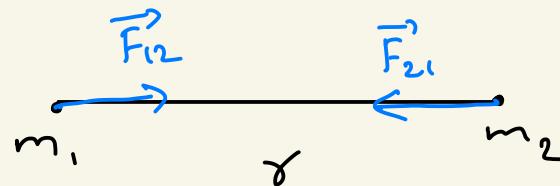


GRAVITATION



Universal law of gravitation :- (Inverse square law)



$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

G - Universal gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$$

F - Gravitational force b/w two particles

m_1, m_2 - mass of particle 1 & 2

γ - Difference b/w two particles

$$\vec{F}_{12} = -\vec{F}_{21}$$

\vec{F}_{12} - Gravitational force on particle 1 due to 2
..... 2 due to 1

$$\vec{F}_{21} = \text{..}$$

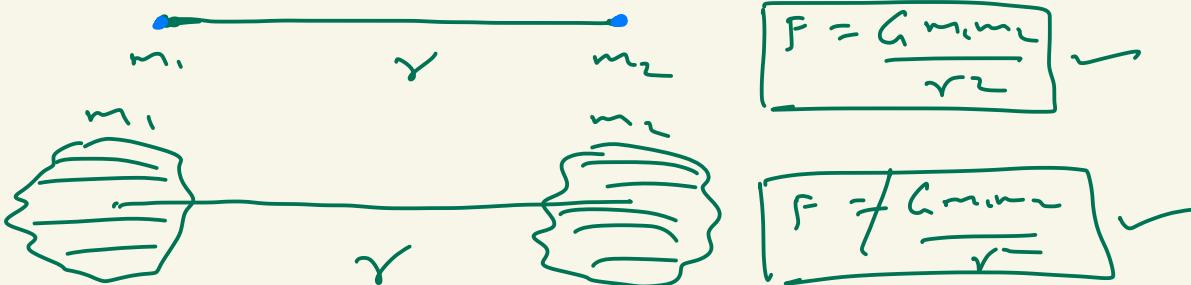
Important Points:

- Gravitational force is always attractive in nature
- is independent on presence of medium
- Gravitational force b/w two particles always along line joining b/w the particles
- Gravitational force b/w two particles forms Action-reaction pair i.e.

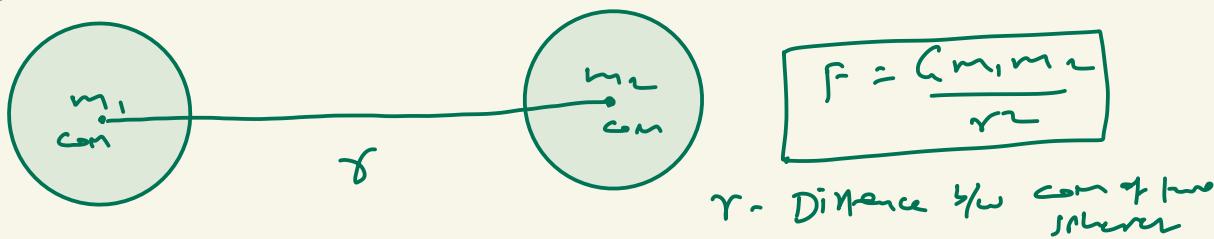
$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

$$\boxed{|\vec{F}_{12}| = |\vec{F}_{21}|}$$

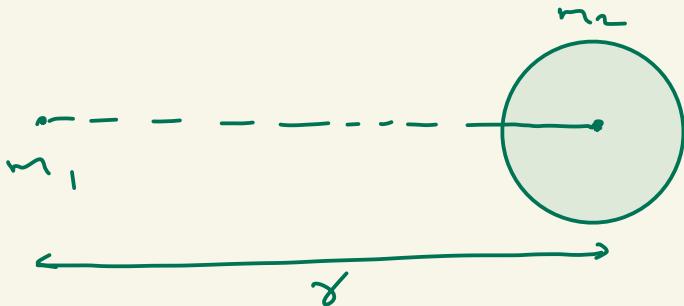
→ Universal law of gravitation is applicable for point masses. (can not be applied for bodies)



→ But there is exception for spherical bodies. For spherical bodies we can apply U.L.G by assuming whole mass is concentrated at its center.

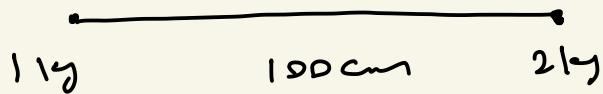


(or)



$$F = \frac{Gm_1m_2}{r^2}$$

problem:-

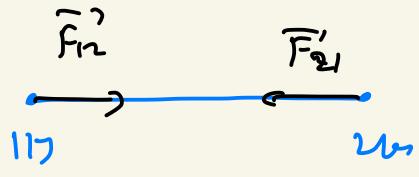


Find magnitude of
gravitational force b/w
two masses?

sol:-

$$F = \frac{Gm_1m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 1 \times 2}{(100 \times 10^{-2})^2} \text{ N}$$

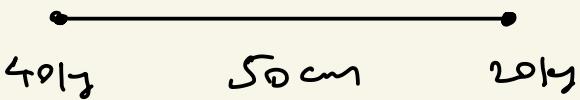


$$F = 6.67 \times 10^{-11} \times 2$$

$$F = 13.34 \times 10^{-11} N$$

$$|\vec{F}_{12}| = |\vec{F}_{21}| = 13.34 \times 10^{-11} N$$

problem:-



Find magnitude of
gravitational force
b/w two particles

Sol:-

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 40 \times 20}{(0.5)^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 8 \pi \textcircled{00} \times \textcircled{100}}{0.5 \times 0.5 \times 100}$$

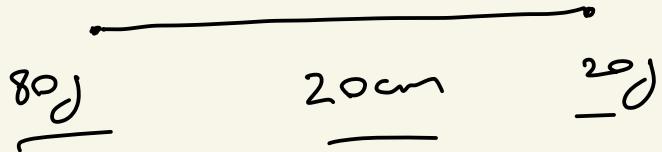
$$F = \frac{6.67 \times 10^{-11} \times 8 \times 10^7 \times 1984}{28}$$

$$F = 6.67 \times 12 \times 10^{-9} N$$

$$F = 213.44 \times 10^{-1} N$$

$$F = 2.13 \times 10^{-7} N$$

problem :-



Find magnitude
of gravitational
force

sol:-

$$F = G \frac{m_1 m_2}{r^2}$$

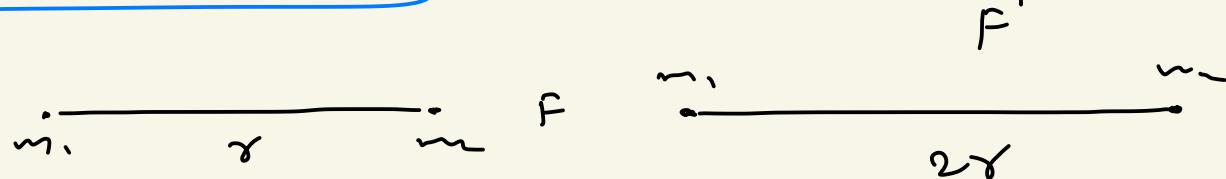
$$F = \frac{6.67 \times 10^{-11} \times (80 \times 10^3) \times (20 \times 10^3)}{(20 \times 10^{-2})^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 80 \times 20 \times 10^{-6}}{400 \times 10^{-4}}$$

$$F = 6.67 \times 4 \times 10^{-13} N$$

$$F = 26.68 \times 10^{-13} N$$

problem:-

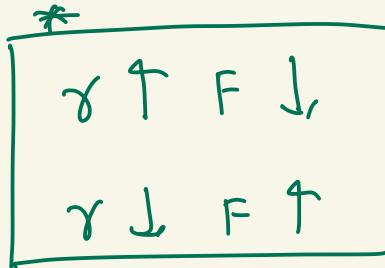


Sol:-

$$F = \frac{C_{min}}{r^2}$$

$$F' = \frac{C_{max}}{4r^2}$$

$$r = r = F/4$$

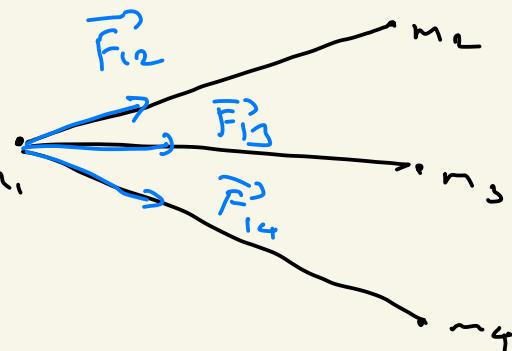


Gravitational force on one particle due to many particles
(superposition principle)

\vec{F}_{12} - Force on particle 1 due to 2

\vec{F}_{13} - - 1 .. 3 m_1

\vec{F}_{14} - - - - 4



→ The resultant force on particle 1 due to remaining particles is equal to **vector sum** of the gravitational force due to individual particles

$$\vec{F}_{\text{ver}\ 1} = \vec{F}'_{12} + \vec{F}'_{13} + \vec{F}'_{14} + \dots$$

(Principle of
superposition)

→ Principle of superposition also states that gravitational force b/w two particles is independent of presence of other particles near by it.

problem:

Three particles each having mass m is kept vertices of equilateral triangle of side length $'a'$. Find magnitude of gravitational force on any one of the particles due to remaining two particles.

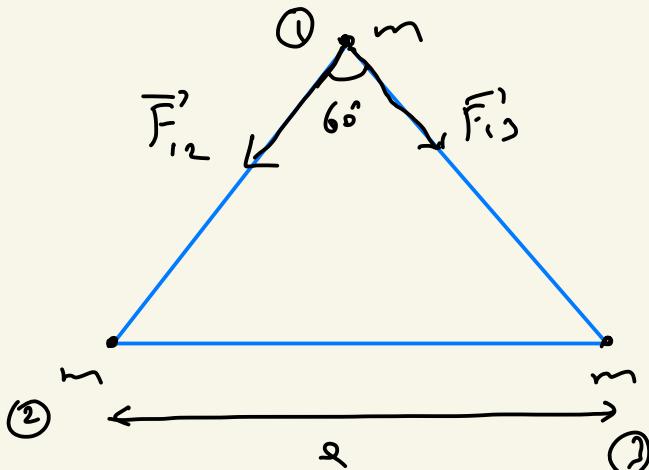
sph:-

$$\vec{F}_{\text{grav}} = \vec{F}_{12} + \vec{F}_{13}$$

vector sum

$$|\vec{F}_{12}| = \frac{Gm^2}{a^2}$$

$$|\vec{F}_{13}| = \frac{Gm^2}{a^2}$$



$$|\vec{F}_{12}| = |\vec{F}_{13}| = \frac{Gm^2}{r^2} = F$$

$$|\vec{F}_{\text{ver}\perp}| = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$$

$$|\vec{F}_{\text{ver}\parallel}| = \sqrt{F^2 + F^2 + 2F^2 \times \frac{1}{2}}$$

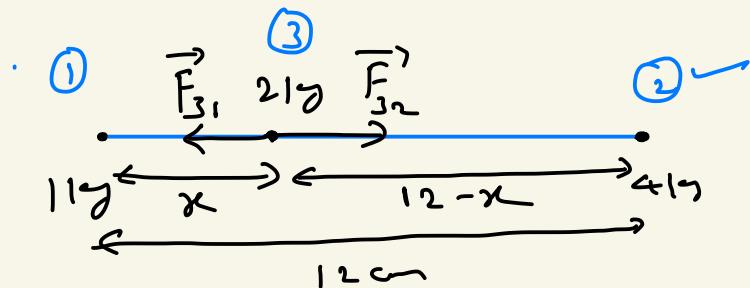
$$|\vec{F}_{\text{ver}\perp}| = \sqrt{3} F$$

$$|\vec{F}_{\text{ver}\parallel}| = \sqrt{3} \frac{Gm^2}{r^2}$$

problem:-

2 particles of mass $11g$ and $41g$ are kept at a separation distance of 12cm . At what distance from $11g$ mass along fine joining b/w two particles, third particle of mass $21g$ should be placed to get resultant force on third particle is zero due to remaining two particles?

Sol:-



$$|\vec{F}_{31}| = |\vec{F}_{32}|$$

$$|\vec{F}_{31}| = \frac{2G}{n^2}$$

$$|\vec{F}_{32}| = \frac{8G}{(12-n)^2}$$

$$\frac{2G}{n^2} = \frac{8G}{(12-n)^2}$$

~~$\frac{6m \cdot 73}{n^2}$~~ = $\frac{44Gm}{(12-n)^2}$

$$12-n = 2n$$

$$12 = 3n$$

$$n = 4 \text{ cm}$$

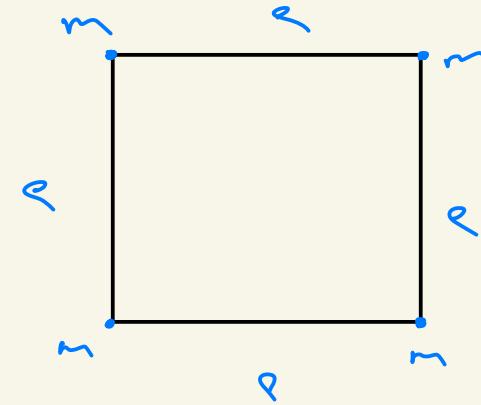
$$\left(\frac{12-n}{n}\right)^2 = 4$$

$$\frac{12-n}{n} = 2$$

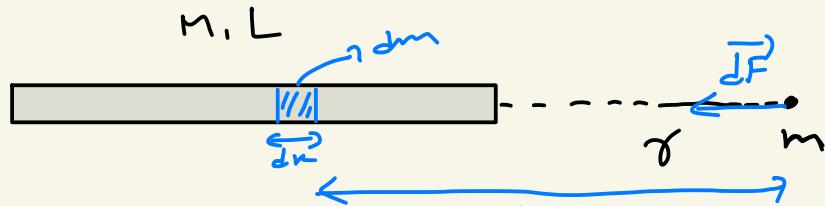
Note: The net force on the third particle is zero for any value of n .

problem:-

Find magnitude of gravitational force on any one particle due to remaining three particles?



Force b/w rod and particle system:-



dF - Force b/w small element of rod and particle

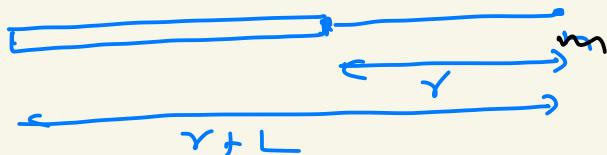
$$dF = \frac{G dm m}{r^2}$$

$$dm = \frac{M}{L} dr$$

$$dF = \int_{r}^{r+L} \frac{G (M/L) m dr}{r^2}$$

$$L = M$$

$$dr = ?$$



$$F = \frac{GMm}{L} \int_{r}^{r+L} \frac{dx}{x^2}$$

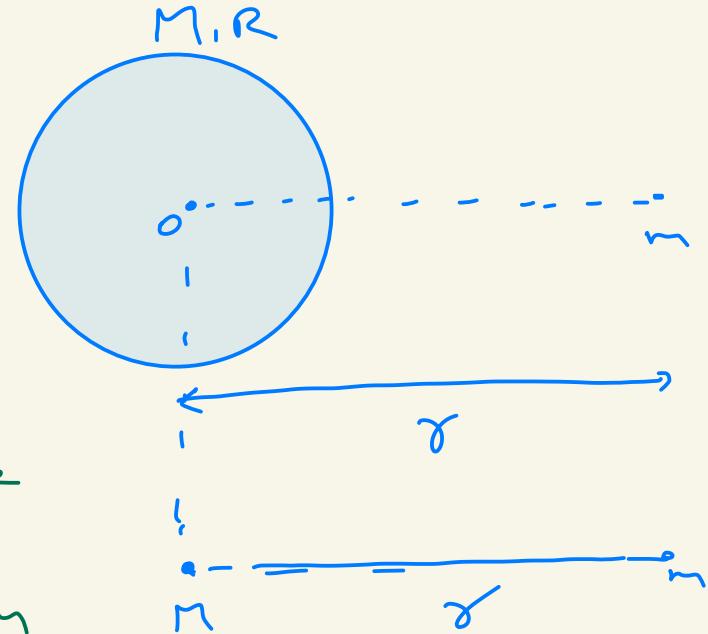
$$F = \frac{GMm}{L} \left[-\frac{1}{x} \right]_r^{r+L}$$

*

$$F = \frac{GMm}{r(r+L)}$$

Force b/w spherical object and particle

$$F = \frac{GMm}{r^2}$$



NOTE: For spherical objects
we can assume that whole
mass of the spherical objects
is concentrated at its com
and we can apply universal
law of gravitation.

Acceleration due to gravity (g):-

→ Acceleration due to gravity varies from place to place and planet to planet. (i.e. For different planets acceleration due to gravity is different)

From universal law of gravitation

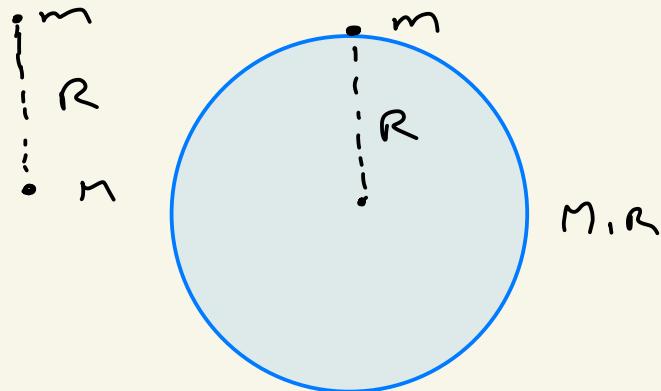
$$F = \frac{G M m}{r^2} \quad \text{--- (1)}$$

From Newton's 2nd law

$$F = m g \quad \text{--- (2)}$$

From (1) & (2)

$$\frac{G M m}{r^2} = m g$$



*
$$g = \frac{GM}{R^2}$$
 relating g & G

G - Universal gravitational constant
 M - mass of the planet
 R - radius

NOTE :- ① Acceleration due to gravity is different for different planets i.e. it depends on mass of the planet and radius of the planet.

② Acceleration due to gravity on the surface of the earth is equal to $\approx 9.8 \text{ m/s}^2$

$$g = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \rightarrow$$

$$M_e = 6 \times 10^{24} \text{ kg}$$

$$R_e = 6400 \text{ km} = 6400 \times 10^3 \text{ m}$$

Case :- Acceleration due to gravity at a height 'h'

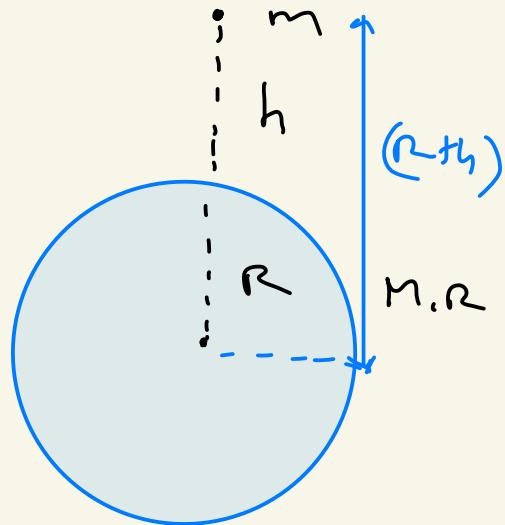
$$g_h = \frac{GM}{(R+h)^2} \quad \text{--- } ①$$

$$g = \frac{GM}{r^2} - ②$$

Equation $\frac{D}{I}$

$$\frac{d_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

*
$$d_h = g \left[\frac{R}{R+h} \right]^2$$



g - Accel. on the surface of the planet

R - Radius of the planet

h - height from the surface of the planet

d_h - Accel. at height 'h' from the surface of the planet

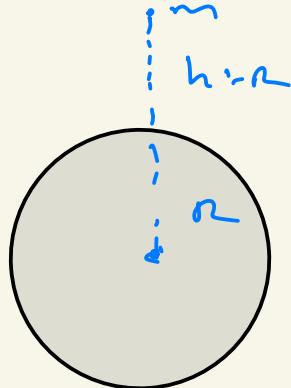
Fq:- Find A.d.j at a height $h=R$ from the surface of the earth . Given that A.d.j on the surface of the earth 'j'.

Sol:-

$$d_h = \sigma \left[\frac{R}{R+h} \right]^2$$

$$d_h = \sigma \left[\frac{1}{2} \right]^2$$

$$d_h = \frac{\sigma}{4}$$



NOTE:- ① If $h \uparrow \sigma \downarrow$ vice versa

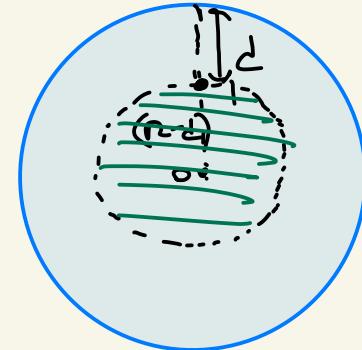
** ② If $h \ll R \Rightarrow d_h = \sigma \left[1 - \frac{2h}{R} \right]$ ($h \ll R$)

Acceleration due to gravity at depth "d"

*
$$g_d = g \left(1 - \frac{d}{R}\right)$$

d - dist from surface of planet

g_d - A.d.g or depth "d"



M.R

Proof:-

$$g_d = \frac{GM'}{(R-d)^2}$$

$$\frac{4}{3}\pi r^3 - M$$

$$\frac{4}{3}\pi (R-d)^3 - ?$$

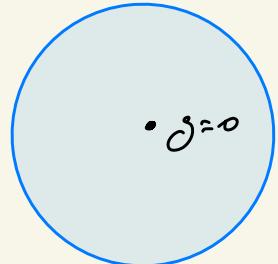
$$M' = \frac{(R-d)^3}{r^3} \times M$$

$$g_d = \left(\frac{GM}{R^2} \right) \left(\frac{R-d}{R} \right)$$

*

$$g_d = g \left(1 - \frac{d}{R} \right)$$

NOTE:- ① If $d=R \Rightarrow g_d=0 \Rightarrow \text{Weight}_{\text{(center)}}=0$



② $d \uparrow g_d \downarrow$, vice versa

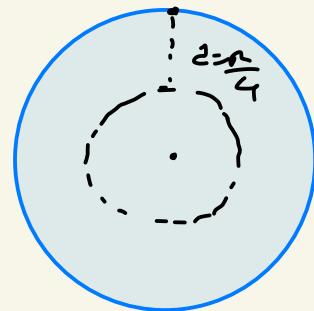
③ As we move above the surface of the planet (or)
below the surface of the planet A. & j decrease.

Ex:- Find Acceleration due to gravity at depth $d = \frac{R}{4}$
Given that A.G. on the surface of the
planet is \mathcal{J}

Sol:-

$$g_d = g \left(1 - \frac{d}{R} \right)$$

$$\boxed{g_d = \frac{3g}{4}}$$



Variation A.d.g Due to rotation of the earth

$$g' = g - r\omega^2 \cos^2 \theta \quad -$$

At poles

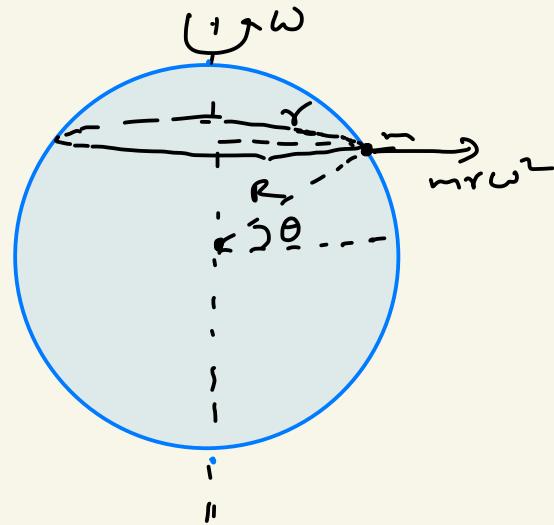
$$\theta = 90^\circ$$

$$g' = g$$

At equator

$$\theta = 0^\circ$$

$$g' = g - R\omega^2$$



ω - Angular velocity of earth about its axis

$$\omega = \frac{2\pi}{T} \Rightarrow \omega = 2\pi f$$

$$T = 24 \text{ hours}$$

Problem

A) what height from the surface of the earth
 $g \cdot d \cdot g$ is $\frac{1}{4}$ th of the $g \cdot d \cdot g$ on the surface of the
 earth? ($R_e = 6400 \text{ km}$)

Sol:-

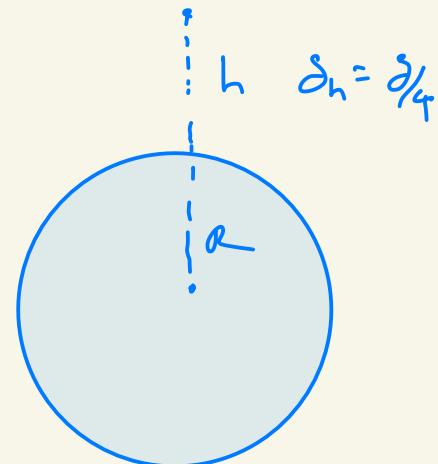
$$g_h = g \left(\frac{R}{R+h} \right)^2$$

$$\frac{1}{4} = \left(\frac{R}{R+h} \right)^2$$

$$\left(\frac{R}{R+h} \right)^2 = \frac{1}{4}$$

$$\frac{R}{R+h} = \frac{1}{2} \Rightarrow 2R = R+h \Rightarrow h=R$$

$$h = 6400 \text{ km}$$



Prblm:-

The weight of the object on the surface of the earth is 200N. What is the weight of the same object at a height $h = \frac{3}{4}R$? R - Radius of the earth.

Sol:-

$$mg_h = mg \left(\frac{R}{R+h} \right)^2$$

$$\underline{W'} = W \left(\frac{R}{R+h} \right)^2$$

$$W' = 200 \times \frac{16}{49}$$

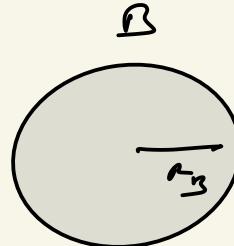
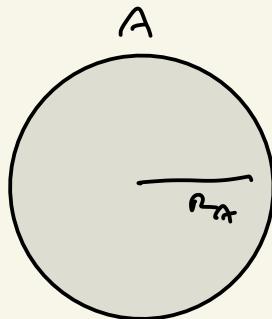
$$W' = 200 \left(\frac{R}{R + \frac{3}{4}R} \right)^2$$

$$W' = \frac{3200}{49} N$$

$$W' = 200 \left(\frac{4}{7} \right)^2$$

$$\boxed{W' = 65.3 N}$$

problem:



Angular velocity of planet A is ω . What is angular velocity of planet B where density is 8 times the density of planet A and mass is same as planet A.

Sol:-

$$\omega_A = \omega = \frac{GM}{R_A^3}$$

$$\omega_B = \frac{GM}{R_B^3}$$

$$P_B = 8P_A$$

$$\frac{M}{4\pi r_s^3} = \frac{8M}{4\pi r_A^3}$$

$$r_A^3 = 8 r_s^3$$

$$R_A = 2r_s$$

$$d_A = d = \frac{C M}{r_A^2}$$

$$d_B = \frac{C M \times 4}{r_A^2}$$

$$d_B = 4d$$

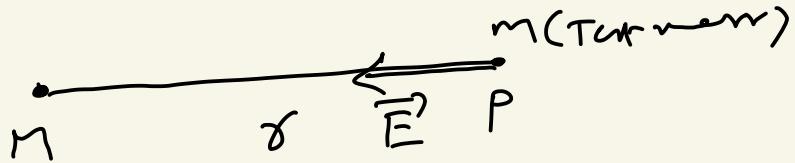
Gravitational Field Intensity (\vec{E})

$$\boxed{\vec{E} = \frac{\vec{F}}{m}}$$

Terminology
 $\sim \vec{F}$

- Gravitational field is a vector quantity and its direction is always in the direction of gravitational force (Its direction is always towards the mass)
- S.I unit of G-field is N/kg
- If we rearrange above equation under
 - $\boxed{\vec{F} = m\vec{E}}$

Gravitational field due to a Point mass



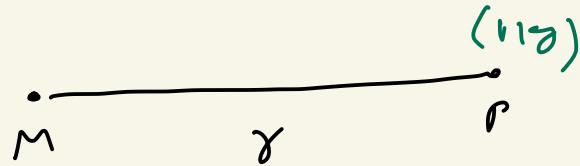
$$|\vec{E}| = \frac{GMm}{r^2}$$

$$|\vec{F}| = \frac{GM}{r^2}$$

at a point -

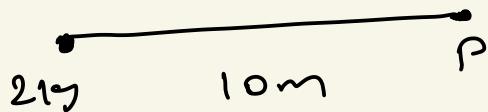
→ Gravitational field due to Point mass - is numerically equal to Force experienced by the

unit vector (\hat{r}) placed at point P



$$|\vec{F}| = \frac{GM(1)}{r^2} = |\vec{F}_z|$$

Ex:-



Sol:-

$$|\vec{F}| = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 2}{(10)^2}$$

$$|\vec{E}| = 13.39 \times 10^{-11} > 10^{-2} \text{ N/C}$$

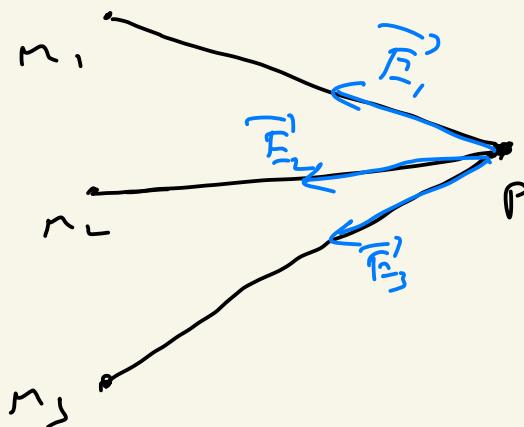
$$|\vec{E}| = 13.39 \approx 10^{-3} \text{ N/C}$$

1221,
1, 2, 3, 4

Gravitational field due to many point masses

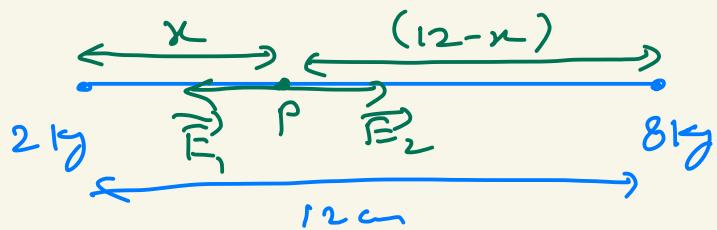
$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

(vector sum)



Eg:- Two particles of 2kg mass and 8kg are kept at a separation distance 12 cm. Find where gravitational field due to two masses is zero?

Sol:-



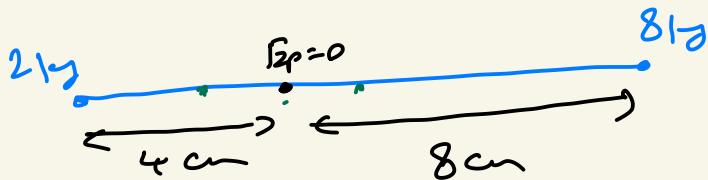
$$|\vec{F}_1| = \frac{2G}{x^2} \quad |\vec{F}_2| = \frac{8G}{(12-x)^2}$$

$$\vec{F}_r = 0 \Rightarrow |\vec{F}_1| = |\vec{F}_2|$$

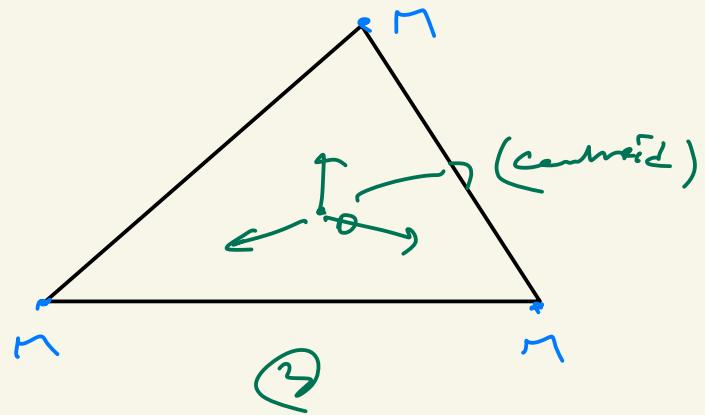
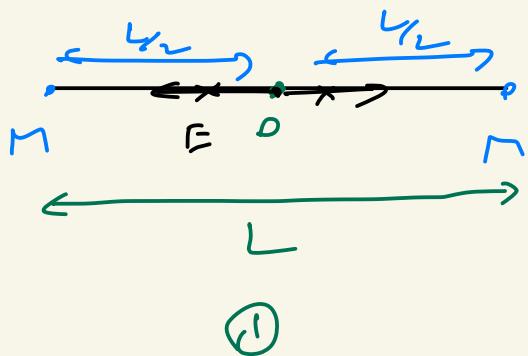
$$\frac{2G}{x^2} = \frac{8G}{(12-x)^2}$$

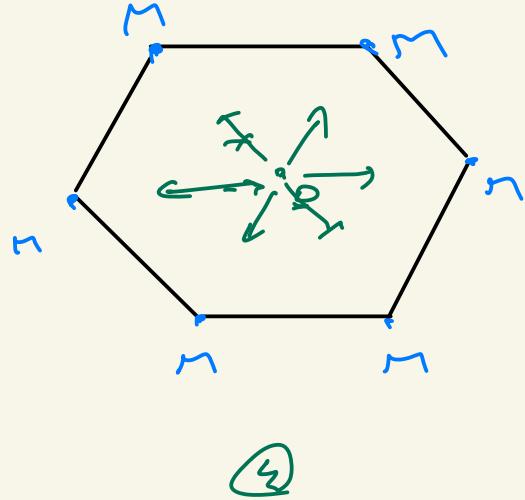
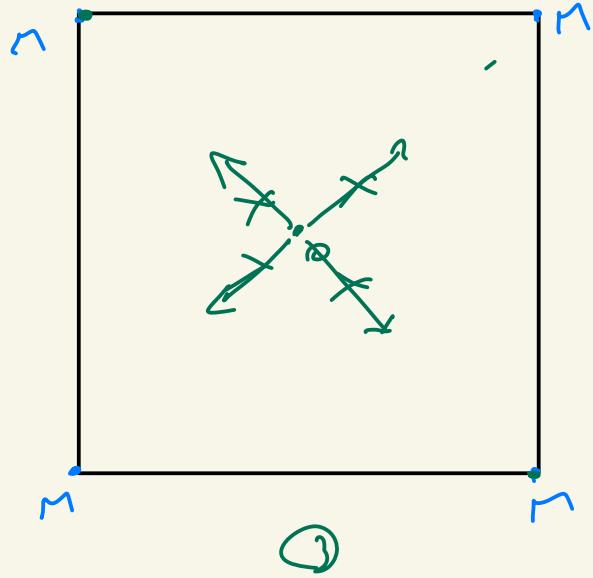
$$\left(\frac{12-n}{n}\right)^2 = 4$$

$$\frac{12-n}{n} = 2 \Rightarrow n = 4 \text{ cm}$$



Ej :-

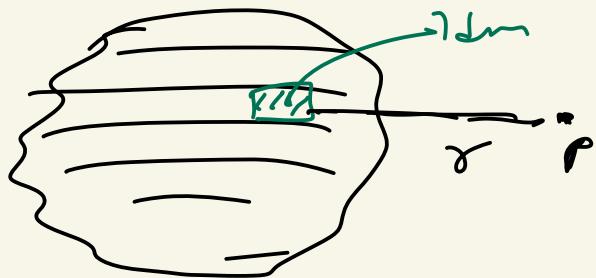




The gravitational field at point 'o' in all the above cases is zero

Gravitational field due to continuous mass distribution

$$d\vec{E} = \int \frac{G dm}{r^2} \hat{r}$$



Gravitational field due to Thin rod -

m, L



$$|\vec{E}| = \frac{GM}{r(r+L)}$$

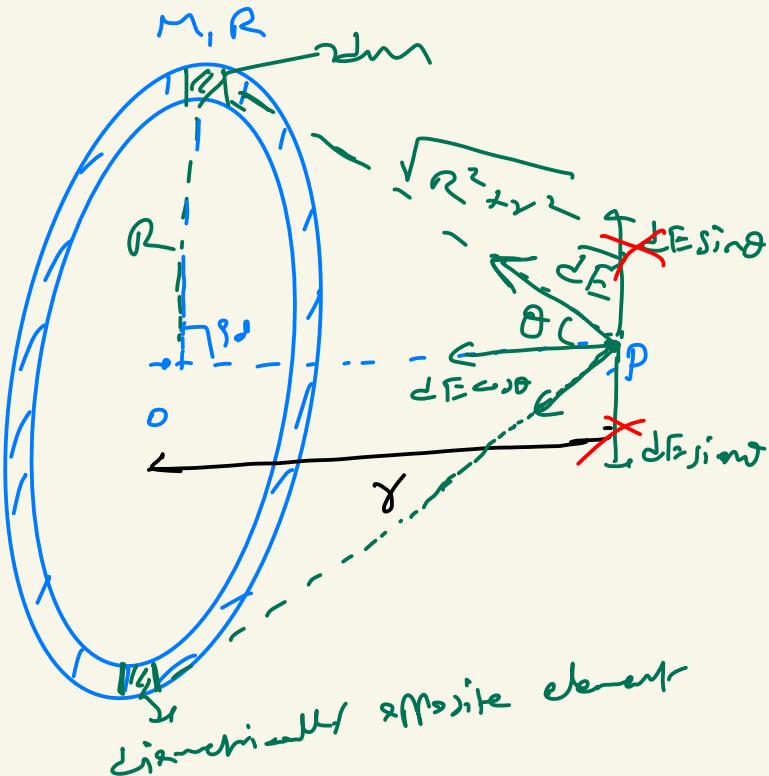
Field due to ring:-

$$dE = \frac{G dm}{(r^2 + r^2)^{1/2}}$$

$$dE_n = dE \cos \theta$$

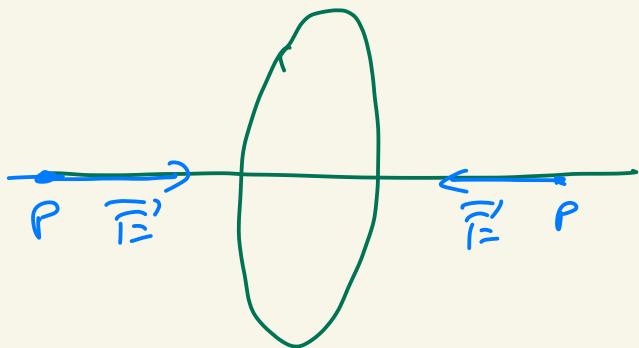
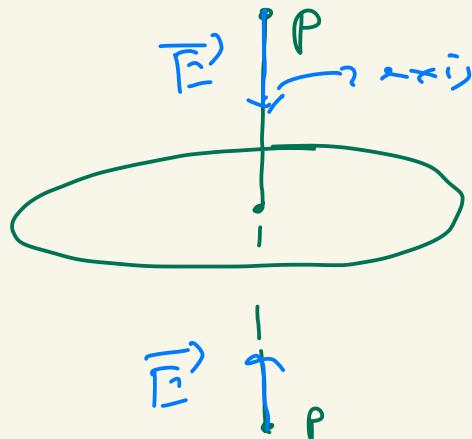
$$\int dE_n = \int \frac{G dm}{(r^2 + r^2)^{1/2}} \left(\frac{\gamma}{(r^2 + r^2)^{1/2}} \right)$$

$$E_n = \frac{G \gamma}{(a^2 + r^2)^{3/2}} \int dm$$

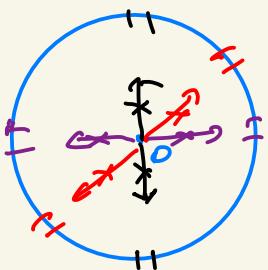


$$|\vec{E}_P| = \frac{GM\gamma}{(n^2 r^2)^{3/2}}$$

→ direction
 C.F. due to ring at any point on the axis
 of the ring is always along the axis of the
 ring and towards center of the ring.



→ At the centre of the ring G.F. is zero.



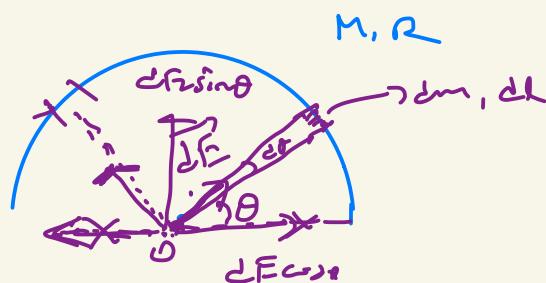
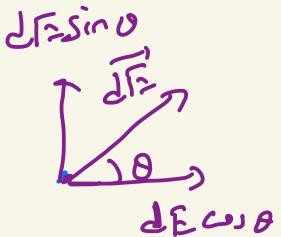
$$\bar{E}_o = 0$$

$$\therefore \gamma = 0 \Rightarrow$$

$$\bar{E}_o = 0$$

Gravitational field due to semicircular wire

$$d\bar{E} = \frac{G dm}{R^2}$$



→ Gravitational field due all n-components cancel each other. Therefore due to n-components

$\zeta \cdot \text{Fideis} = \text{zero}$

$$dE_y = dE \sin \theta$$

$$dE_y = \frac{\zeta dm}{n^2} \sin \theta - ①$$

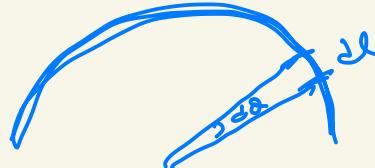
$$\pi R = M$$

$$dl = r$$

$$dm = \frac{M}{\pi R} dl$$

$$dm = \frac{M}{\pi R} (\alpha d\theta) - ②$$

$$\int dE_y = \int \frac{GM(\alpha d\theta)}{\pi R n^2} \sin \theta$$



$$dl = r d\theta$$

$$E_J = \frac{GM}{\pi R^2} \int_0^{\pi} \sin \theta d\theta$$

$$E_J = \frac{GM}{\pi R^2} \left[-\cos \theta \right]_0^{\pi}$$

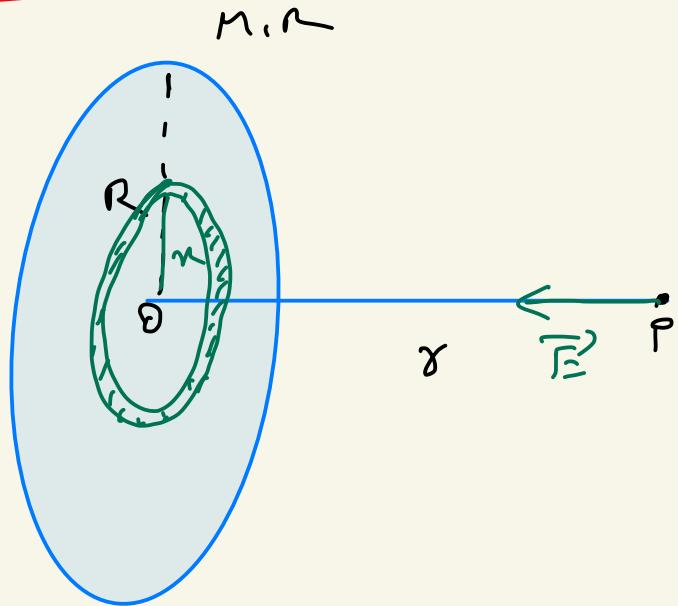
$$E_J = \frac{GM}{\pi R^2} \left[-\cos \pi - (-\cos 0) \right]$$

$$E_J = \frac{GM}{\pi R^2} [1 + 1]$$

$$\boxed{E_J = \frac{2GM}{\pi R^2}}$$

Gravitational field due to Disc

$$E = \frac{2GM}{r^2} \left[1 - \frac{\gamma}{\sqrt{r^2 + r^2}} \right]$$



NOTE: Gravitational field direction

Due to Disc at point P

on the axis of Disc is always

along the axis of the Disc and towards centre of the Disc

Disc

* * * Gravitational field due to hollow sphere (or) (Spherical shell)

Case i) $r > R$ (outside)

$$|\vec{F}_g| = \frac{GM}{r^2} \Rightarrow F_g \propto \frac{1}{r^2}$$

\Downarrow
 $\propto r^2 F_g \downarrow$

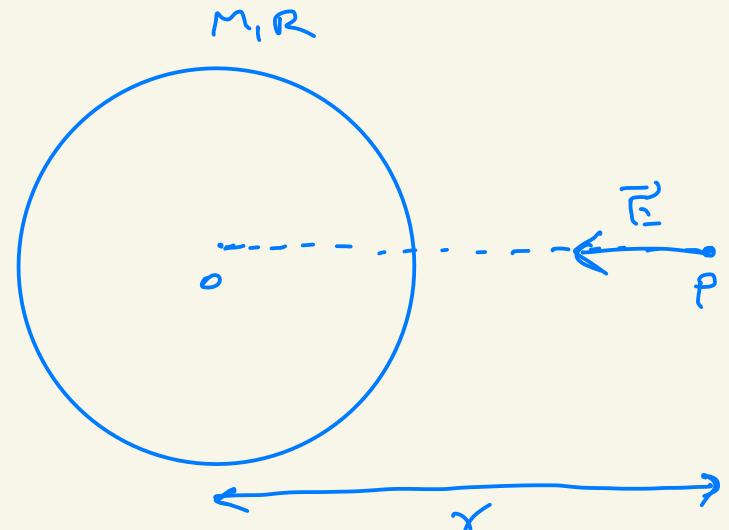
NOTE: we can assume that

whole mass of the sphere

concentrated at its centre

and we can apply gravitational field to point mass definition

(and for outside points)

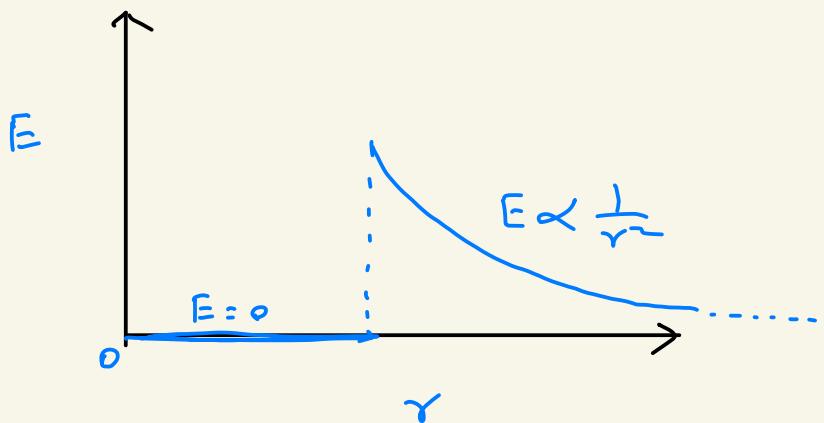
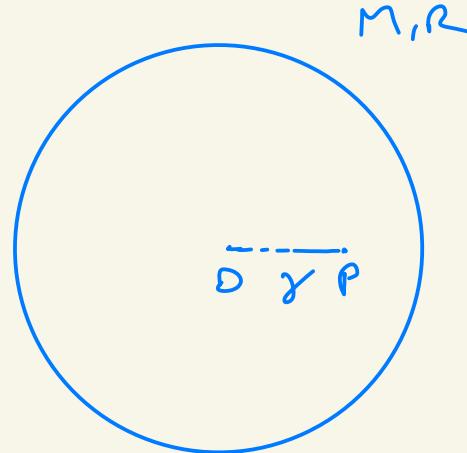


except $r < R$

$$E = 0$$

NOTE: Gravitational field due to spherical shell at any point inside the spherical shell is equal to zero.

Graph b/w E vs r



Gravitational field due to solid sphere (spherical volume of mass distribution)

case ii, $r \geq R$

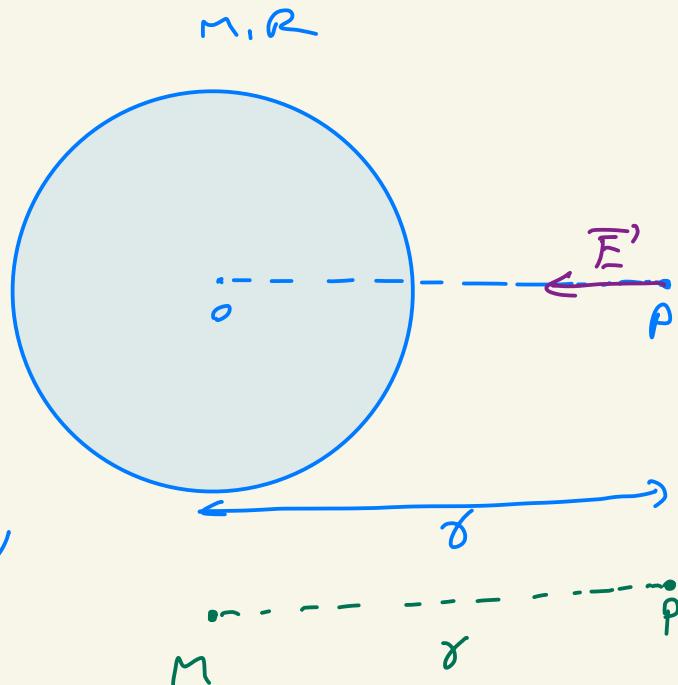
$$|\vec{E}| = \frac{GM}{r^2} \Rightarrow E \propto \frac{1}{r}$$

At $r = R$

$$|\vec{E}| = \frac{GM}{R^2} = g \rightarrow \text{Acceleration due to gravity}$$

M - Mass of the sphere

r - Distance from centre of the sphere to point 'P'



cavii $r \leq R$

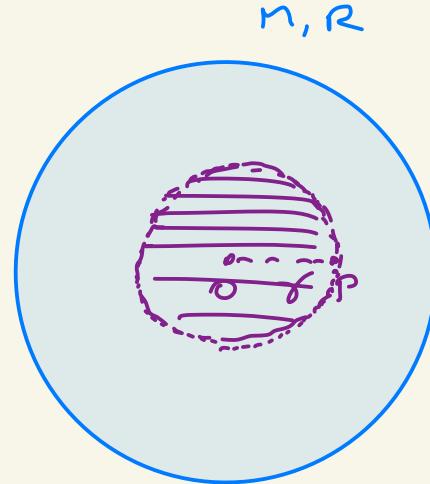
Solid sphere - combination of spherical shells

$$|\vec{E}| = \frac{Cm}{r^2}$$

$$\frac{4}{3}\pi r^3 - M$$

$$\frac{4}{3}\pi r^3 - ?$$

$$m' = \frac{\Delta}{\frac{4}{3}\pi r^3} \cdot \frac{4}{3}\pi r^3$$



$$|\vec{E}| = \frac{CMr^2}{r^3 \cancel{r^2}}$$

* $|\vec{E}_p| = \frac{CMr}{r^2} \Rightarrow |\vec{E}| \propto \frac{1}{r}$

$$\underline{r \uparrow \underline{E} \uparrow}$$

NOTE :- ① At the center of the solid sphere field is zero

$$F_0 = 0$$

$$\therefore r = 0$$

② In terms density gravitational field inside sphere.

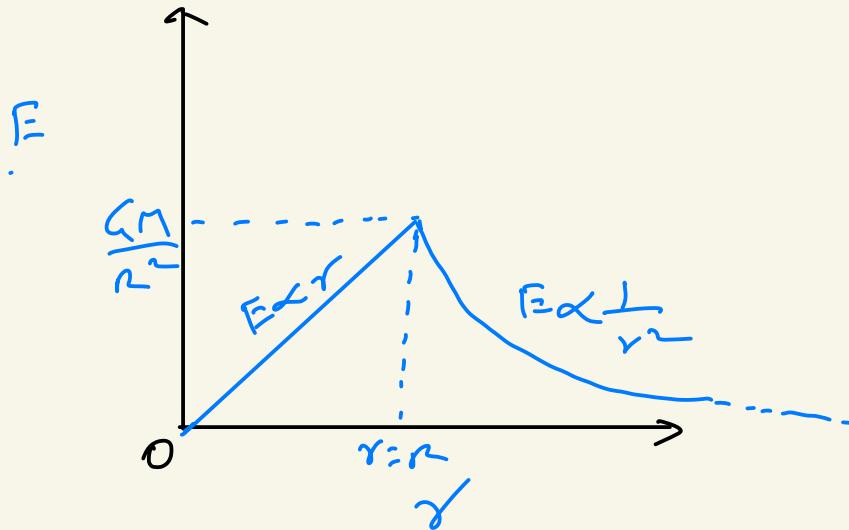
$$\rho = \frac{M}{\frac{4}{3}\pi r^3} \Rightarrow M = \rho \frac{4}{3}\pi r^3$$

$$F = G \frac{\rho \times \frac{4}{3}\pi r^3 \gamma}{r^2}$$



$$F_p = G \frac{\rho \frac{4}{3}\pi r^3 \gamma}{r^2} \Rightarrow F \propto r$$

Graph b/w E vs r



problem:-

In the figure shown find
magnitude of gravitational field
at point P.

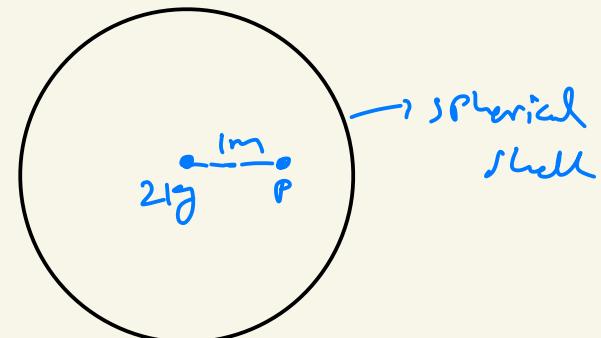
solt:-

$$|\vec{E}| = \frac{G \times 2}{(1)^2}$$

$$|\vec{E}| = 6.67 \times 10^{-11} \times 2 \text{ N/kg}$$

$$|\vec{E}| = 13.34 \times 10^{-11} \text{ N/kg}$$

$$M = 4\pi r^3 \rho / 3 = 4\pi (2)^3 \rho / 3 = 32\pi \rho / 3$$



$$\frac{6.67 \times 10^{-11}}{2^2} \times 4\pi \rho / 3$$

Problem:- Find Gravitational field (magnitude) due to two concentric spherical shells at Point A, B and C or shown in the figure?

Sol:-

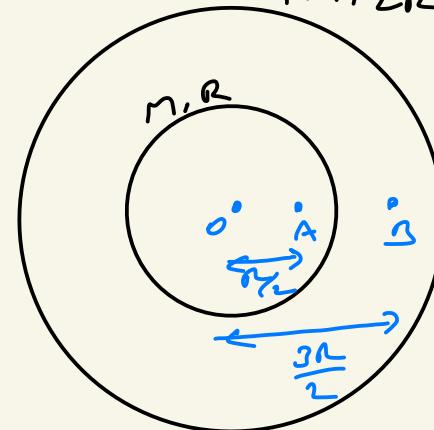
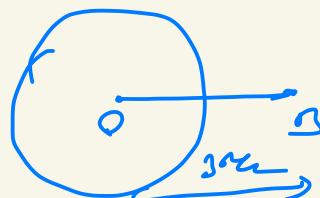
At point A

$$E_A = 0$$

At point B

→ Outer shell does not produce

G.F at point B (Point B is Inside) (B)



$$E_B = \frac{GM}{(3R)^2} = \frac{4GM}{9R^2}$$

At Point C

→ Both shells contribute field at point 'C'

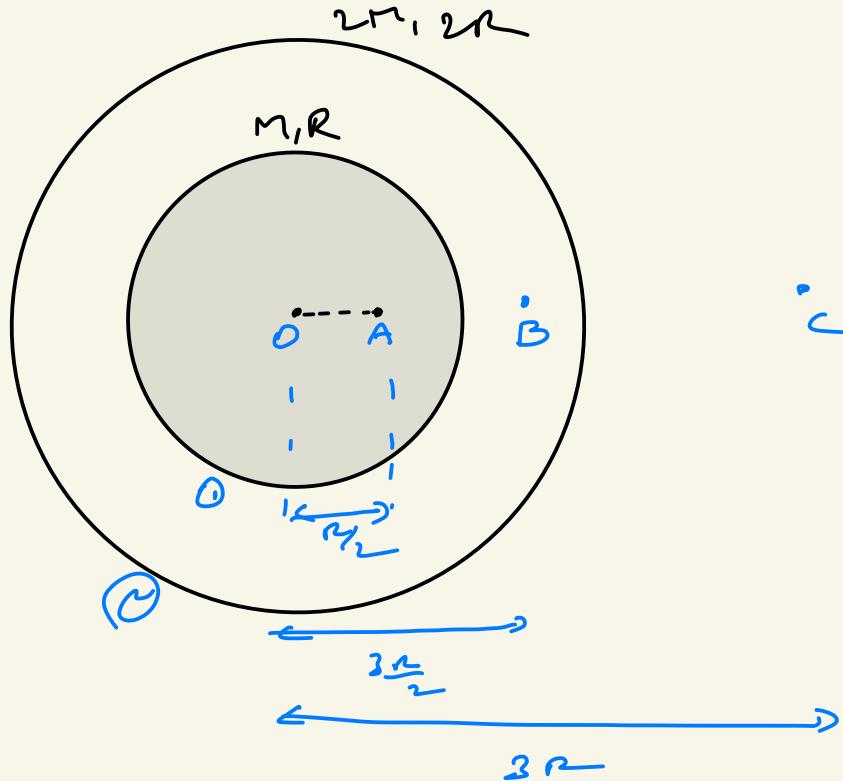
$$|\vec{E}_1| = \frac{eM}{(4n)^2} = \frac{eM}{16n^2}$$

$$|\vec{E}_2| = \frac{e(4m)}{(4n)^2} = \frac{4em}{16n^2}$$

$$|\vec{E}_r| = E_1 + E_2 = \frac{eM}{16n^2} + \frac{eM}{4n^2}$$

$$|\vec{E}_c| = \boxed{\frac{5eM}{16n^2}}$$

Ej:-



At Point A

$$F_1 = \frac{GM\gamma}{r^2} = \frac{GM(\gamma)}{r^2} = \frac{GM}{2r}$$

$$F_2 = 0$$

$$F_4 = \frac{GM}{2r}$$

At Point B

$$F_1 = \frac{GM}{(3r)^2} = \frac{4GM}{9r^2}$$

$$F_2 = 0$$

$$F_3 = \frac{4GM}{9r^2}$$

At point C

$$F_1 = \frac{GM}{(3r)^2}$$

$$F_2 = \frac{G(2M)}{(2r)^2}$$

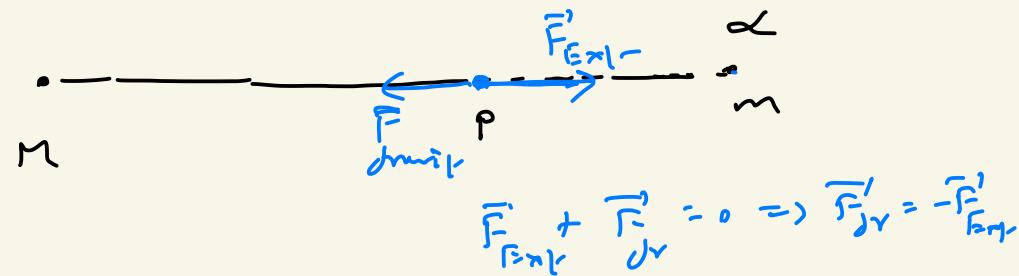
$$F_C = \frac{GM}{r^2} + \frac{2GM}{(2r)^2}$$

$$F_C = \frac{3GM}{r^2} - \frac{GM}{(2r)^2}$$

Gravitational Potential (scalar function)

G.P at a point P in gravitational field is defined as work done by the external agent per unit mass to bring the unit mass from infinity to point 'P' very slowly [without changing $\vec{L} \cdot \vec{E}$]

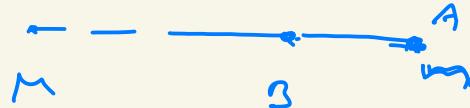
$$V_p = \frac{W_{\text{Extr}}(dP)}{m} = - \frac{W_{dP}}{m}$$



→ It is a scalar physical quantity

→ S.I unit is $\frac{\text{J}}{\text{kg}}$

→ $W_{\text{Ext}} = m(V_B - V_A)$



→ Potential at infinity we define it as zero

Gravitational Potential difference (relation b/w V_A & V_B)

Gravitational Potential difference b/w two points (∞ to A to B)
in a gravitational field is defined as negative
of line integral of $\vec{F}_g \cdot d\vec{r}$ from ∞ to A to B

$$V_p - \infty = - \frac{\int \vec{F}_g \cdot d\vec{r}}{m}$$

$$\vec{F} = m \vec{E}$$

$$V_p - V_\infty = - \int_{\infty}^p \vec{E} \cdot d\vec{l}$$

If two plates are A and B ($A \parallel B$)

$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{l}$$

→ The use of (1) we can find P.d from field

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$d\vec{l} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

* * * Calculation of Gravitational field from potential:

$$\boxed{\begin{aligned} \mathbf{E}_n &= -\frac{\partial V}{\partial n} \\ \mathbf{E}_r &= -\frac{\partial V}{\partial r} \\ \mathbf{E}_\theta &= -\frac{\partial V}{\partial \theta} \end{aligned}}$$

$\frac{\partial(\)}{\partial n}$ partial derivative

$\frac{\partial(\)}{\partial n}$ normal derivative

* * *

In case of partial differentiation when you differentiate w.r.t. n and r variables should be considered as constant

$\frac{\partial(\)}{\partial r}$ - n, r are zero constant
 $\frac{\partial(\)}{\partial n}$ - r, z are constant

$\frac{\partial(\)}{\partial r}$ - n, z are constant

Example: Gravitational Potential in a particular region
as a function of x, y, z is given by $V = x^2 + y^2 + z^2$
calculate the gravitational field in the same
region as a function of x, y and z and
also the gravitational field at point $(1, 1, 1)$
Take x, y, z are in m and V is in $\frac{J}{kg}$

Sol:- i) $E_x = - \frac{\partial V}{\partial x}$

$$E_x = - \frac{\partial (x^2 + y^2 + z^2)}{\partial x}$$

$$E_n = -\frac{d(n\gamma)}{dn} - \frac{d(-\gamma z)}{dn} - \frac{d(zn)}{dn}$$

$$E_n = -\gamma - \rho - 2$$

$$\boxed{E_n = -\gamma - 2} \quad \text{Z}$$

$$E_\gamma = -\frac{d(n\gamma + \gamma z + zn)}{d\gamma}$$

$$E_\gamma = -\frac{d(n\gamma)}{d\gamma} - \frac{d(\gamma z)}{d\gamma} - \frac{d(zn)}{d\gamma}$$

$$\boxed{E_\gamma = -n - 2}$$

$$E_z = -\frac{d(n\gamma + \gamma z + zn)}{dz}$$

$$E_z = -\frac{d(n\gamma)}{dz} - \frac{d(\gamma z)}{dz} - \frac{d(zn)}{dz}$$

$$\boxed{E_z = -\gamma - n}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\vec{E} = -(n+2) \hat{i} - (n+2) \hat{j} - (n+1) \hat{k}$$

ii) Field at $(\frac{n+2}{3}, \frac{n+2}{3}, \frac{n+1}{3})$

$$\vec{E} = -2 \hat{i} - 2 \hat{j} - 2 \hat{k}$$



problem $\nabla = - (n+2)$ find field in the same
region?

Sol:-

$$E_x = -\frac{\partial V}{\partial n} \quad E_y = -\frac{\partial V}{\partial \theta} \quad . \quad E_z = -\frac{\partial V}{\partial \phi}$$

$$F_x = \left[\frac{d(-n+2)}{dx} \right] \quad F_y = \left[\frac{d(-n+2)}{dy} \right]$$

$$F_x = +x^2 \quad F_y = +xz \quad F_z = +x^2$$

$$\boxed{\vec{F} = -x^2\hat{i} + xz\hat{j} + x\hat{k}}$$

problem: The gravitational field due to a mass distribution is given by $|\vec{F}| = k n^{-3/r}$ along n -direction where k is const. . Taking gravitational potential to be zero at infinity. Find its value at a distance r ?

Sol:-

$$|\vec{F}| = kn^{-\frac{3}{2}}$$

$$V_p - V_\infty = - \int_{\infty}^p F(n) dn$$

$$V_p - 0 = - \int_{\infty}^p kn^{-\frac{3}{2}} dn$$

$$V_p = -k \left[-\frac{2}{\sqrt{n}} \right]_{\infty}^p$$

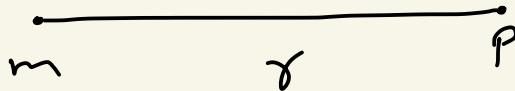
$$V_p = \frac{2k}{\sqrt{p}}$$

$$\int n^n dn = \frac{n^{n+1}}{n+1}$$

$$\frac{n^{-\frac{3}{2}} + 1}{-\frac{1}{2} + 1}$$

$$\frac{n^{-\frac{1}{2}}}{\frac{1}{2}}$$

Gravitational Potential due to a Point mass

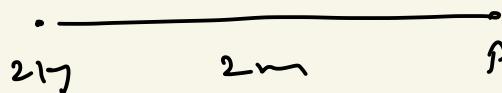


- Gm

$$V_p = -\frac{Gm}{r}$$

Gm
✓

F_g:



Find Potential at Point 'P'

Sol:-

$$V_p = -\frac{G(x)}{(x)}$$

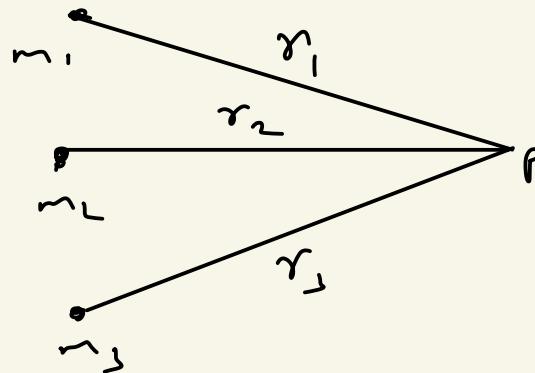
$$V_p = -6.67 \times 10^{-11} \frac{J}{kg}$$

V - Potential
F - Field

Due to many point charges (algebraic sum)

$$V_p = V_1 + V_2 + V_3$$

$$V_p = -\frac{Cm_1}{r_1} - \frac{Cm_2}{r_2} - \frac{Cm_3}{r_3}$$

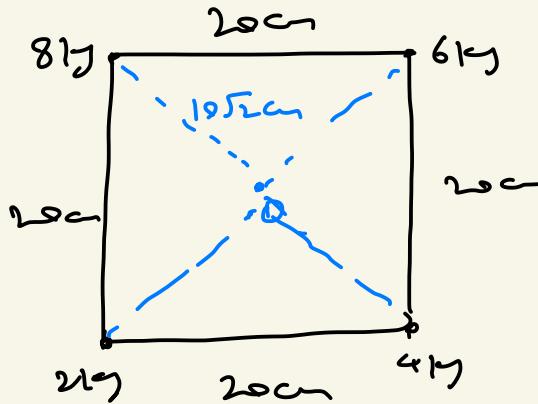


Ex:- Find Potential at 'O'

Sol:

$$V_o = V_1 + V_2 + V_3 + V_4$$

$$V_o = -\frac{C}{\gamma} (m_1 + m_2 + m_3 + m_4)$$



$$V_i = \frac{-6.67 \times 10^{-11}}{(10\sqrt{2} \times 10^{-2})} (2 + 4 + 6 + 8)$$

$$V_o = \frac{-6.67 \times 10^{-10}}{\sqrt{2}} (20)$$

$$V_o = -6.67 \times 10^{-9} \times \sqrt{2} \text{ J/J}$$

$$V_o = -6.67 \sqrt{2} \times 10^{-9} \frac{\text{J}}{\text{J}}$$