ONE OR MORE THAN ONE CORRECT:

1. If two satellites of different masses are revolving in the same orbit, they have the same

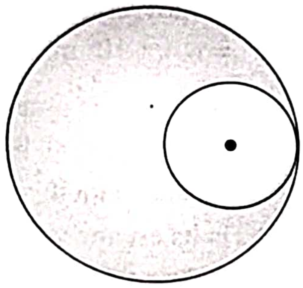
a) angular momentum b) energy
c) time period d) speed

1. c, d

CONCEPT CODE : P110905

Sol. Subjective

2. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V=0$ at $r=\infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)



a) $\frac{-GM}{2R}$ b) $\frac{-GM}{R}$ c) $\frac{-2GM}{3R}$ d) $\frac{-2GM}{R}$

2. b

CONCEPT CODE : P110903

Sol. $V_C = \frac{-3}{2}\frac{GM}{R} + \frac{2GM}{8R}$

$= \frac{-GM}{R}$

3. An iceberg is floating partially immersed in sea water. The density of sea water is $1.03\ \text{g cm}^{-3}$ and that of ice is $0.92\ \text{g cm}^{-3}$. The approximate percentage of total volume of iceberg above the level of sea water is

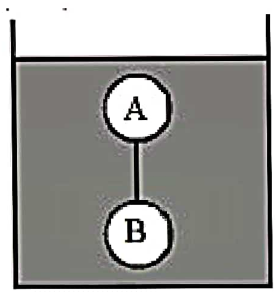
a) 8 b) 11 c) 34 d) 89

3. b

CONCEPT CODE : P111002

Sol. $\frac{v_i}{v} = \frac{d}{\rho}$

4. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



a) $d_A < d_F$ b) $d_B > d_F$ c) $d_A > d_F$ d) $d_A + d_B = 2d_F$

4. a, b, d

CONCEPT CODE : P111002

Sol. $F_{net} = \vec{F}_B + \vec{F}_g$ $F_B = V\rho g$

5. A log of wood of mass 120 kg floats in water. The weight that can be put on the raft to make it just sink, should be (density of wood = $600\ \text{kg/m}^3$)

a) 80 kg b) 50 kg c) 60 kg d) 30 kg

5. a

CONCEPT CODE : P111002

Sol. Volume of log of wood

$V = \frac{\text{mass}}{\text{density}} = \frac{120}{600} = 0.2\ m^3$

Let x weight that can be put on the log of wood.

So weight of the body $= (120 + x) \times 10\ N$

Weight of displaced liquid $V\sigma g = 0.2 \times 10^3 \times 10\ N$.

The body will just sink in liquid if the weight of the body will be equal to the weight of displaced liquid.

$(120 + x) \times 10 = 0.2 \times 10^3 \times 10 \Rightarrow 120 + x = 200$

$x = 80\ kg$.

6. A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be

a) 8 : 1 b) 4 : 1 c) 3 : 1 d) 2 : 1

6. b

<u>CONCEPT CODE</u> : P111002

Sol. $V = V_C + V_S$ $F_B = (V_C + V_S)\rho g$

7. A sample of metal weighs 210 gm in air, 180 gm in water and 120 gm in liquid. Then specific gravity of

a) Metal is 3 b) Metal is 7

c) Liquid is 3 d) Liquid is $\frac{1}{3}$

7. c

<u>CONCEPT CODE</u> : P111002

Sol. Density of metal $= \rho$, Density of liquid = s.

If V is the volume of sample then according to problem

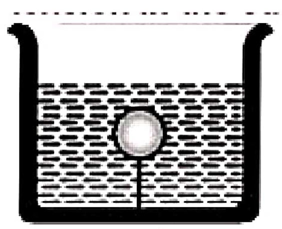
$210 = V\rho g$ ...... (i)

$180 = V(\rho - 1)g$ ...... (ii)

$120 = V(\rho - \sigma)g$ ...... (iii)

By solving (i), (ii) and (iii) we get $\rho = 7$ and $\sigma = 3$

8. A solid sphere of specific gravity $\eta < 1$, is suspended in a water tank by a string tied to its base as shown in figure. If the mass of the sphere is m then the tension in the string is given by



a) $\left(\frac{\eta - 1}{\eta}\right)mg$ b) ηmg c) $\frac{mg}{\eta - 1}$ d) $\left(\frac{1 - \eta}{\eta}\right)mg$

8. d

<u>CONCEPT CODE</u> : P111002

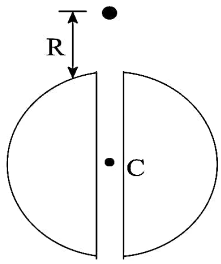
Sol. Tension in spring T = upthrust

Weight of sphere $= V\sigma g - V\rho g$

$= V\frac{1}{\eta}\rho g - V\rho g$ (As $\sigma = \eta\rho$)

$= \left(\frac{1}{\eta} - 1\right)V\rho g = \left(\frac{1}{\eta} - 1\right)mg$

9. A particle is dropped from a height equal to the radius of the earth above the tunnel dug through the earth as shown in the figure. (Radius of earth = R; mass of earth = M)



a) Particle will oscillate through the earth to a height R on both sides
b) Particle will execute simple harmonic motion
c) Motion of the particle is periodic
d) Particle passes the centre of earth with a speed $= \sqrt{\dfrac{2GM}{R}}$

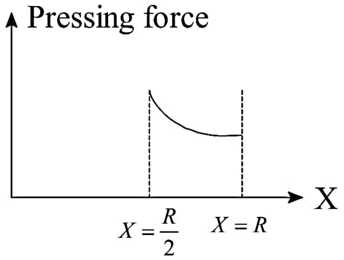
9. a, c, d

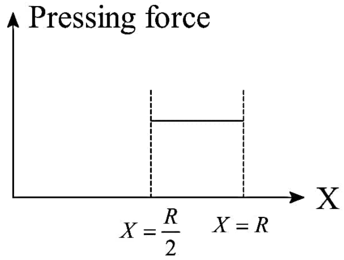
<u>CONCEPT CODE</u> : P110902

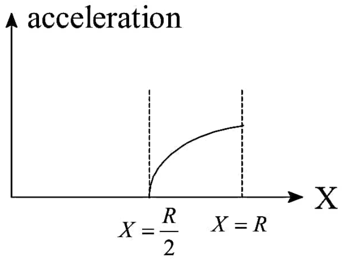
Sol. From COE (A) is correct.
The force outside the earth varies as inverse square of the distance
$\therefore$ Motion is not simple harmonic. However, from symmetry of motion, the motion will be periodic.
$\therefore$ (C) is correct.
From COE

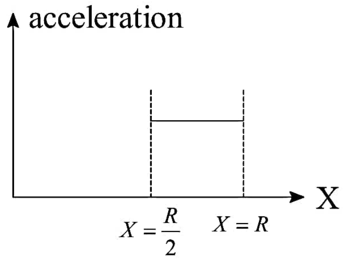
$$\frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{3GMm}{2R}\right).$$

10. A tunnel is dug along a chord of the earth at a perpendicular distance $\frac{R}{2}$ from the earth's centre. The wall of the funnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The passing force by the particle on the wall and the acceleration of the particle varies with × (distance of the particle from the centre) according to :

a)


b)


c)


d)


10. b, c

<u>CONCEPT CODE</u> : P110902

Sol. Net force towards centre of earth $= mg' = \dfrac{mgx}{R}$

Normal force $N = mg'\sin\theta$

Thus pressing face $N = \dfrac{mgx}{R}\dfrac{R}{2x}$

$N = \dfrac{mg}{2}$ constant and independent of X

Tangential force $F = ma = mg'\cos\theta$

$$a = g'\cos\theta = \frac{gx}{R}\frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$$

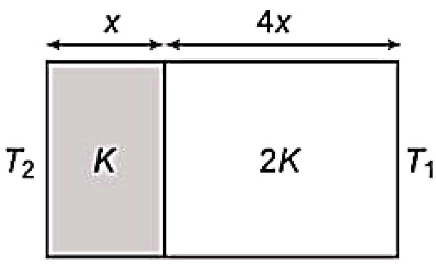
$$a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

Curve is parabolic and at $x = \frac{R}{2}$, $a = 0$.

PART - C

INTEGER ANSWER TYPE:

1. The temperature of the two outer surfaces of a composite slab consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and $4x$ respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab in steady state is $\frac{AK(T_2 - T_1)}{nx}$. The value of n is



1. 3

CONCEPT CODE : P111207

Sol. $R_{net} = R_1 + R_2 = \left(\frac{X}{KA}\right) + \frac{4X}{2KA} = \frac{3X}{KA}$

Now, $H = \frac{dQ}{dt} = \frac{TD}{R_{net}}$

$$= \frac{(T_2 - T_1)}{(3X/KA)} = \left[\frac{KA(T_2 - T_1)}{x}\right]\left(\frac{1}{3}\right)$$

2. A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of 2 g of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is $\frac{1}{4}$ that of first, the rate at which ice melts in $\frac{g}{s}$ will be

2. 4

CONCEPT CODE : P111207

Sol. $R = \frac{l}{KA}$

l is halved, A is four times and K is $\frac{1}{4}$ times.

$\therefore$ R will become half. Hence, heat current will become two times. Therefore, rate of melting of ice will also become two times or 4 g/s.

3. The earth is assumed to be a sphere of radius R. A platform is arranged at a height 3R from the surface of the earth. The escape velocity of a body from this platform is $\frac{v_e}{f}$, where v_e is its escape velocity from the surface of the earth. Find the value of f.

3. 2

CONCEPT CODE : P110904

Sol. $v_e = \sqrt{\frac{2GM}{R}}$

$v_e' = \sqrt{\frac{2GM}{4R}} = \frac{1}{2}\sqrt{\frac{2GM}{R}} = \frac{v_e}{f}$

$f = 2$

4. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}$th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is ______ . (Ignore energy loss due to atmosphere)

4. 2

CONCEPT CODE : P110904

Sol. $gh = \frac{1}{4}g$

$\frac{GM}{r^2} = \frac{1}{h}\frac{GM}{R^2}$

$r = 2R$

$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{2R}$

$v = \sqrt{\frac{GM}{R}}$

$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2}\sqrt{\frac{GM}{R}}$

$N = 2$

5. Two satellites A and B have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, $\frac{T_A}{T_B}$ is

5. 1

CONCEPT CODE : P110905

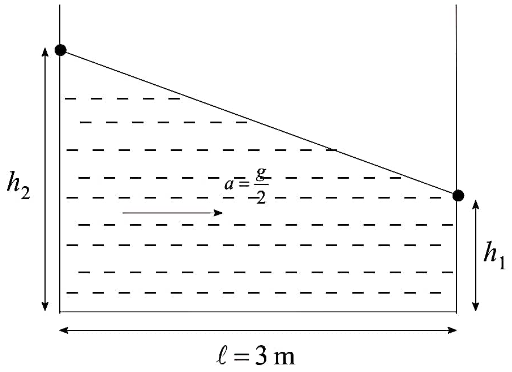
Sol. $V_0 = \sqrt{\frac{GM}{r}}$

$K_A = \frac{1}{2}m\left(\frac{GM}{R}\right)$

$K_B = \frac{1}{2}2m\left(\frac{GM}{2R}\right) = K_A$

$\frac{K_A}{K_B} = \frac{1}{1}$

6. A container felled with water is accelerated horizontally at an acceleration $a = \frac{g}{2}$. (g is magnitude of acceleration due to gravity. If $h_1 = 1\,\text{m}$ then the value of h_2 is meters is



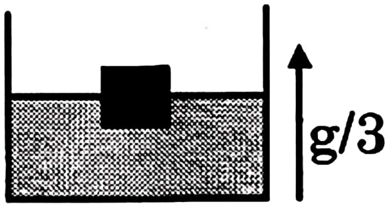
6. 3

CONCEPT CODE : P111001

Sol. $\tan\theta = \frac{a}{g} = \frac{h_2 - h_1}{\ell} = \frac{1}{2}$

$h_2 = 3\,\text{m}$

7. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with a net acceleration of $\frac{g}{3}$. The fraction of volume immersed in the liquid is $\frac{1}{n}$, the value of n is



7. 2

CONCEPT CODE : P111002

Sol. $\frac{V_{\text{immesed}}}{V_{\text{body}}} = \frac{d}{p}$

8. A ball whose density is $0.4 \times 10^3\,\text{kg/m}^3$ falls into water from a height of 9 cm. To what depth does the ball sink. (in cm)

8. 6

CONCEPT CODE : P111002

Sol. The velocity of ball before entering the water surface

$v = \sqrt{2gh} = \sqrt{2g \times 9}$

When ball enters into water, due to upthrust of water the velocity of ball decreases (or retarded).

The retardation, $a = \frac{\text{apparent weight}}{\text{mass of ball}}$

$= \frac{V(\rho - \sigma)g}{V_\rho}$

$= \left(\frac{\rho - \sigma}{\rho}\right)g = \left(\frac{0.4 - 1}{0.4}\right) \times g$

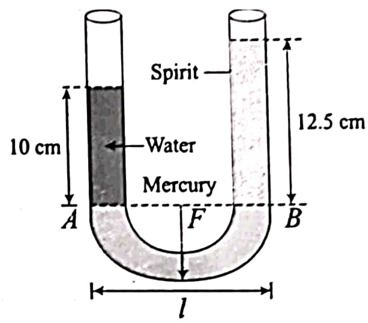
$= -\frac{3}{2}g$.

If h be the depth upto which ball sink, then,

$$0 - v^2 = 2 \times \left(-\frac{3}{2} g \right) \times h$$

$$2g \times 9 = 3gh \qquad h = 6\ cm$$

9. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in the same level with 10 cm of water in one arm and 12.5 cm of spirit in the other. The specific gravity of spirit is $\frac{x}{5}$, the value of x is



9. 4

<u>CONCEPT CODE</u> : P111001

Sol. $P_A = P_B$

$$10 \times \rho_W \times g = 12.5 \times \rho_S \times g$$

$$\frac{\rho_S}{\rho_W} = \frac{10}{12.5} = \frac{4}{5}$$

10. Two solids A and B float in water. It is observed that A floats with $\frac{1}{2}$ of its body immersed in water and B floats with $\frac{1}{4}$ of its volume above the water level. The ratio of the density of A to that of B is $2:n$, the value of n is

10. 3

<u>CONCEPT CODE</u> : P111002

Sol. Upthrust = weight of body

For A : $\frac{V_A}{2} \times \rho_w \times g = V_A \times \rho_A \times g = \rho_A$

For B : $\frac{3}{4} V_B \times \rho_w \times g = V_B \times \rho_B \times g = \rho_B$

(Since $\frac{1}{4}$ of volume of B is above the water surface)

$$\frac{\rho_A}{\rho_B} = \frac{\rho_w / 2}{3/4 \rho_w} = \frac{2}{3}.$$