Mathematics

PART - A

SINGLE ANSWER CORRECT:

1. If $f(x)=\begin{cases} x+\{x\}+x\sin\{x\} & for\ x\neq 0 \\ 0 & for\ x=0 \end{cases}$ where $\{x\}$ is fractional part of x

a) $f(x)$ is continuous and differentiable at $x=0$

b) $f(x)$ is continuous but not differentiable at $x=0$

c) $f(x)$ is neither continuous nor differentiable at $x=0$

d) differentiable everywhere

1.c

Concept code: M120408

Sol. $\lim\limits_{x\to 0+} f(x)=\lim\limits_{x\to 0+} x+x-[x]+x\sin(x-[x])$

$=0+0-0+0\sin(0-0)$

$=0$

$\lim\limits_{x\to 0-} f(x)=\lim\limits_{x\to 0-} x+x-[x]+x\sin(x-[x])$

$=0+0+1+0\sin(0+1)$

$=1$

2. Let $f(x)=x^3$, $g(x)=|x|$ then at $x=0$ the composite function

a) *gof* is differentiable but *fog* is not differentiable.

b) *fog* is differentiable but *gof* is not differentiable.

c) *gof* and *fog* both are derivable

d) neither *gof* nor *fog* is derivable

2.c

Concept code: M120407

Sol. $gof(x)=g(f(x))=g(x^3)=|x^3|$

$fog(x)=f(g(x))=f(|x|)=|x|^3$

$gof(x)=|x^3|=\begin{cases} x^3 & if\ x\geq 0 \\ -x^3 & if\ x<0 \end{cases}$ $\qquad fog(x)=\begin{cases} x^3\ if\ x\geq 0 \\ -x^3\ if\ x<0 \end{cases}$

$\lim\limits_{x\to 0}\dfrac{gof(x)-gof(0)}{x-0}$

$\lim\limits_{x\to 0}\dfrac{|x|^3}{x}=\lim\limits_{x\to 0}\dfrac{|x|}{x}x^2$

$= \lim_{x\to 0+} 1(0) = 0$

$\lim_{x\to 0-} -1(0) = 0$

3. $f(x) = \begin{cases} x\sin\frac{1}{x} & for\ -1 \le x \le 1\ and\ x \neq 0 \\ 0 & for\ x = 0 \end{cases}$

$g(x) = \begin{cases} x^2\sin\frac{1}{x} & for\ -1 \le x \le 1\ and\ x \neq 0 \\ 0 & for\ x = 0 \end{cases}$

$h(x) = |x|^3 \quad for\ -1 \le x \le 1$

Which of these functions are differentiable at $x = 0$

a) f *and* g b) f *and* h c) g *and* h d) none

3.c

Concept code: M120405

Sol. $f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{x\to 0} \frac{x\sin\frac{1}{x} - 0}{x} = \lim_{x\to 0} \sin\frac{1}{x}$ does not exist

$g'(0) = \lim_{x\to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0} \frac{x^2\sin\frac{1}{x}}{x}$

$= \lim_{x\to 0} x\sin\frac{1}{x} = 0$

$-1 \le \sin\frac{1}{x} \le 1$

$-x \le x\sin\frac{1}{x} \le x$

$\lim_{x\to 0} -x \le \lim_{x\to 0} x\sin\frac{1}{x} \le \lim_{x\to 0} x$

$0 \le \lim_{x\to 0} x\sin\frac{1}{x} \le 0$

$h'(0) = \lim_{x\to 0} \frac{h(x) - h(0)}{x - 0}$

$= \lim_{x\to 0} \frac{|x|^3}{x} = \lim_{x\to 0} \frac{|x|}{x} x^2 = 0$

4. The domain of derivative of $f(x)$

$= \begin{cases} Tan^{-1}x & if\ |x| \le 1 \\ \frac{1}{2}(|x| - 1) & if\ |x| > 1 \end{cases}$ is

a) $R - \{0\}$ b) $R - \{1\}$ c) $R - \{-1\}$ d) $R - \{-1, 1\}$

4.d

Concept code: M120407

Sol. $f(x) = \begin{cases} Tan^{-1}x & if\ -1 \le x \le 1 \\ \frac{1}{2}(x-1) & if\ x > 1 \\ \frac{1}{2}(-x-1) & if\ x < -1 \end{cases}$

$f(-1) = Tan^{-1}(-1) = \frac{-\pi}{4}$

$\lim\limits_{x \to -1-} \frac{1}{2}(-x-1) = 0$

$\lim\limits_{x \to -1+} Tan^{-1}(x) = -\frac{\pi}{4}$

5. Let $f: R \to R$, $f(1) = 3$, $f'(1) = 6$

$\lim\limits_{x \to 0} \left(\frac{f(1+x)}{f(1)} \right)^{1/x} =$

a) 1 b) $e^{\frac{1}{2}}$ c) e^2 d) e^3

5.c

Concept code: M120405

Sol. $\lim\limits_{x \to 0} \left(\frac{f(1+x)}{f(1)} \right)^{\frac{1}{x}} = e^{\lim\limits_{x \to 0} \frac{1}{x}\left(\frac{f(1+x)}{f(1)} - 1 \right)}$

$= e^{\lim\limits_{x \to 0} \left(\frac{f(1+x) - f(1)}{x\, f(1)} \right)}$

$= e^{\frac{1}{f(1)} \lim\limits_{x \to 0} \left(\frac{f(1+x) - f(1)}{x} \right)}$

$e^{\frac{1}{f(1)} \lim\limits_{x \to 0} \frac{f'(1+x)}{1}} = e^{\frac{f'(1)}{f(1)}} = e^{\frac{6}{3}} = e^2$

6. If $f(x) = \cos \pi (|x| + [x])$ then $f(x)$ is/are (where $[\cdot]$ is GINF)

a) continuous everywhere b) continuous $x = 0$

c) differentiable in $(2,4)$ d) differentiable in $(0,1)$

6.d

Concept code: M120401

Sol. $f'\left(\frac{1}{2}\right)=\underset{x\to\frac{1}{2}}{lt}\frac{f(x)-f\left(\frac{1}{2}\right)}{x-\frac{1}{2}}=\underset{x\to\frac{1}{2}}{lt}\frac{\cos\left(\pi(x+0)-0\right)}{x}=0$

7. If $f(x)=|x+1|\left(|x|+|x-1|\right)$ then the number of points of the function is/are not differentiable at in $[-2,2]$

a) $x=-1$ b) $x=0$ c) $x=1$ d) all the above

7.d

Concept code: M120405

Sol. $f(x)=\begin{cases}2x^2+x-1, & -2\le x<-1\\ -2x^2-x+1 & -1\le x<0\\ x+1 & 0\le x<1\\ 2x^2+x-1 & 1\le x\le 2\end{cases}$

$f'(x)=\begin{cases}4x+1 & -2\le x<-1\\ -4x-1 & -1\le x<0\\ 1 & 0\le x<1\\ 4x+1 & 1\le x\le 2\end{cases}$

8. $f(x)=[x\sin\pi x]$ then f is $\left([\cdot]\ is\ GINF\right)$

a) continuous at $x=0$ b) continuous in $(-1,0)$

c) differentiable in $(-1,1)$ d) all the above

8.d

Concept code: M120405

Sol. $-1\le\sin\pi x\le 1$

$-x\le x\sin\pi x\le x$

$f(x)=value$

$f'(x)=0$

9. If x and y are the sides of two squares such that $y=x-x^2$. Then, the rate of change, if the area of second square with respect to the first square, when $x=2,$ is

a) 1 b) 4 c) 3 d) 6

9.c

Concept code: M120502

Sol. $\frac{d\left(y^2\right)}{d\left(x^2\right)}=\frac{2y\frac{dy}{dx}}{2x}=\frac{y}{x}\frac{dy}{dx}=\frac{\left(x-x^2\right)}{x}\frac{dy}{dx}$

$\frac{dy}{dx}=1-2x \qquad =(1-x)(1-2x)$

Where $x=2,\ \frac{d\left(y^2\right)}{d\left(x^2\right)}=(-1)(-3)=3$

10. Let f be a function which is continuous and differentiable for all real x, if $f(2)=-4$ and $f'(x)\geq 6$ for all $x\in[2,4]$, then

a) $f(4)\leq 8$ b) $f(4)\geq 8$ c) $f(4)\geq 12$ d) $f(4)\geq 20$

10.b

Concept code: M120607

Sol. $\frac{f(4)-f(2)}{4-2}=f'(c)$

$f(4)+4=2f'(c)\geq 2(6)=12$

$f(4)\geq 8$

11. A function $y=f(x)$ is given by $x=\frac{1}{1+t^2}$ and $y=\frac{1}{t\left(1+t^2\right)}$ for all $t>0$, then t is

a) increasing in $\left(0,\frac{3}{2}\right)$ and decreasing in $\left(\frac{3}{2},\infty\right)$

b) increasing in $(0,1)$

c) increasing in $(0,\infty)$

d) none

11.b

Concept code: M120604

Sol. $\frac{dx}{dt}=\frac{-2t}{\left(1+t^2\right)^2} \qquad \frac{dy}{dt}=\frac{-\left(1+3t^2\right)}{t^2\left(1+t^2\right)^2}$

$\frac{dy}{dx}=\frac{1+3t^2}{2+3}>0$ if $t>0$

$x=\frac{1}{1+t^2}\in(0,1)$

12. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence of same length x. The maximum area is enclosed by the park is

a) $\sqrt{\frac{x^3}{8}}$ b) $\frac{1}{2}x^2$ c) πx^2 d) $\frac{3}{2}x^2$

12.b

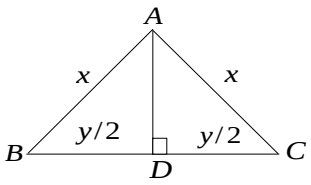
Concept code: M120606

Sol. Let AB = AC = x and BC = y

$$AD=\sqrt{x^2-\frac{y^2}{4}}=\frac{1}{2}\sqrt{4x^2-y^2}$$

$$\text{A = Area of } \Delta ABC=\frac{1}{2}(BC)(AD)=\frac{1}{4}y\sqrt{4x^2-y^2}$$

$$\frac{dA}{dy}=\frac{1}{4}\sqrt{4x^2-y^2}+\frac{y}{4}\frac{(-2y)}{2\sqrt{4x^2-y^2}}$$



$$\frac{dA}{dy}=0\Rightarrow y^2=2x^2\Rightarrow y=\sqrt{2}x$$

$$\frac{d^2A}{dy^2}<0 \text{ when } y=\sqrt{2}x$$

Maximum area $A=\frac{1}{4}\sqrt{2}x.\sqrt{2}x=\frac{1}{2}x^2$

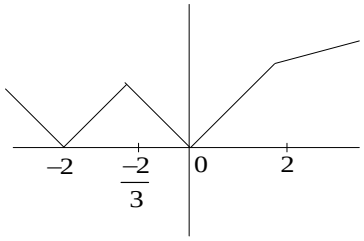
13. The function $f(x)=2|x|+|x-2|-||x+2|-2|x||$ has a local maximum at $x=$

a) -2 b) $\frac{-2}{3}$ c) 2 d) $\frac{2}{3}$

13. b

Concept code: M120605

Sol.



$$f(x)=-2x-4(x\le -2)$$

$$=2x+4\left(-2<x\le\frac{-2}{3}\right)$$

$$=-4x\left(\frac{-2}{3}<x\le 0\right)$$

$$=4x(0<x\le 2)$$

$$=2x+4(x>2)$$

14. If x is real, then the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

a) 1 b) $\frac{17}{7}$ c) $\frac{1}{4}$ d) 41

14.d

Concept code: M120606

Sol. Let $\frac{3x^2+9x+17}{3x^2+9x+7}=y$

$3x^2y+9xy+7y=3x^2+9x+17$

$3x^2(y-1)+9x(y-1)+7y-17=0$

x is real $\Rightarrow D\geq 0\Rightarrow 81(y-1)^2-4(3)(y-1)(7y-17)\geq 0$

$27(y^2-2y+1)-4(7y^2-24y+17)\geq 0$

$-y^2+42y-41\geq 0\Rightarrow y^2-42y+41\leq 0$

$(y-1)(y-41)\leq 0$

$1\leq y\leq 41$

15. The sides of a triangle are in the ratio $1:\sqrt{3}:2$ then the angles of the triangle are in the ratio

a) 1 : 3 : 5 b) 2 : 3 : 4 c) 3 : 2 : 1 d) 1 : 2 : 3

15.d

Concept code: M111303

Sol. Let $a=k,\ b=\sqrt{3}k,\ c=2k$

$\cos A=\frac{\sqrt{3}}{2}\Rightarrow A=\frac{\pi}{6},\ \cos B=\frac{1}{2}\Rightarrow B=\frac{\pi}{3}$

$C=\frac{\pi}{2}$

16. In ΔABC, if $b^2+c^2=2a^2$, then the value of $\frac{\cot A}{\cot B+\cot C}$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{5}{3}$

16.a

Concept code: M111303

Sol. $\frac{\cot A}{\cot B+\cot C}=\frac{b^2+c^2-a^2}{c^2+a^2-b^2+a^2+b^2-c^2}=\frac{a^2}{2a^2}$

17. If the angles of a triangle are in the ratio $4:1:1$, then the ratio of the longest side to the perimeter is

a) $\sqrt{3}:(2+\sqrt{3})$ b) $1:6$ c) $1:2+\sqrt{3}$ d) 2 : 3

17.a

Concept code: M111303

Sol. $4x+x+x=180^0 \Rightarrow x=30^0$

$A=120^0,\ B=30^0,\ C=30^0$

$$\frac{a}{a+b+c}=\frac{\sin A}{\sin A+\sin B+\sin C}=\frac{\sqrt{3}}{2+\sqrt{3}}$$

18. In a right angled isosceles triangle, the ratio of the circumradius and inradius is

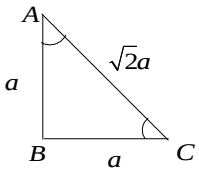
a) $2\left(\sqrt{2}+1\right):1$ b) $\left(\sqrt{2}+1\right):1$ c) $2:1$ d) $\sqrt{2}:1$

18.b

Concept code: M111507

Sol. $R=\frac{\sqrt{2}a}{2}=\frac{a}{\sqrt{2}}$

$$r=\frac{\Delta}{s}=\frac{\frac{1}{2}a^2}{\frac{1}{2}\left(a+a+\sqrt{2}a\right)}=\frac{a}{2+\sqrt{2}}$$



19. In ΔABC, medians AD and CE are drawn. If $AD=5$, $\angle DAC=\frac{\pi}{8}$ and $\angle ACE=\frac{\pi}{4}$ then area of ΔABC is equal to

a) $\frac{25}{9}$ b) $\frac{25}{3}$ c) $\frac{25}{18}$ d) $\frac{10}{3}$

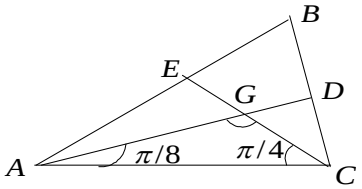
19.b

Concept code: M111501

Sol. $AG=\frac{2}{3}AD=\frac{10}{3}$

In ΔGAC,

$$\frac{GC}{\sin\frac{\pi}{8}}=\frac{GA}{\sin\frac{\pi}{4}}\Rightarrow GC=\frac{10}{3}\frac{\sin\frac{\pi}{8}}{\sin\frac{\pi}{4}}$$



Area of $\Delta AGC=\frac{1}{2}AG\cdot GC\sin\left(\frac{\pi}{2}+\frac{\pi}{8}\right)$

$$=\frac{1}{2}\cdot\frac{10}{3}\cdot\frac{10}{3}\frac{\sin\frac{\pi}{8}\cos\frac{\pi}{8}}{\sin\frac{\pi}{4}}=\frac{25}{9}$$

Area of $\Delta ABC=3\,Area\ of\ \Delta AGC=\frac{25}{3}$

20. In ΔABC, if $\cos A+\cos B+\cos C=\frac{7}{4}$, then $\frac{R}{r}$ is equal to

a) $\frac{3}{2}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

20.d

Concept code: M111506

Sol. $1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{7}{4}$

$4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{3}{4}$

$\frac{r}{R}=\frac{3}{4}$

PART - D

NUMERICAL ANSWER TYPE:

1. $f(x+y)=f(x)f(y)$ and $f(5)=2$, $f'(0)=3$ then $f'(5)$ is k then the value of $\frac{k}{5}$ is

1. 1.2

Concept code: M120408

Sol. $x=0,\ y=0\Rightarrow f(0)=(f(0))^2\Rightarrow f(0)=1$

$f'(5)=\lim\limits_{h\to0}\frac{f(h+5)-f(5)}{h}$

$=\lim\limits_{h\to0}\frac{f(h)f(5)-f(5)}{h}$

$=\lim\limits_{h\to0}f(5)\left(\frac{f(h)-1}{h}\right)$

$=f(5)\lim\limits_{h\to0}\left(\frac{f(h)-1}{h}\right)=f(5)\lim\limits_{h\to0}\frac{f'(h)}{1}$

$=f(5)f'(0)$

$=2(3)=6$

2. If the rate of change in the volume of sphere is equal to the rate of change of its radius, then the radius is $\frac{1}{k\sqrt{\pi}}$. The value of $\frac{k}{5}$ is

2. 0.4

Concept code: M120502

Sol. $V=\frac{4}{3}\pi r^3 \qquad Given\ \frac{dV}{dt}=\frac{dr}{dt}$

$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}\Rightarrow 4\pi r^2=1$

$r=\frac{1}{2\sqrt{\pi}}$

3. The least positive integral value of "a", so that the line $(3-a)x+ay+a^2-1=0$ is normal to the curve $xy=1$ is

3. 4

Concept code: M120603

Sol. Slope = $\frac{-(3-a)}{a}=\frac{a-3}{a}$

$xy=1\Rightarrow y=\frac{1}{x}$

$\frac{dy}{dx}=\frac{-1}{x^2}=$ slope of the tangent

Slope of the normal = $x^2>0$

$\frac{a-3}{a}>0\Rightarrow a<0,\ a>3$

$\Rightarrow a=4$

4. In ΔABC, if $AB=x$, $BC=x+1$, $\angle C=\frac{\pi}{3}$ then the least positive integral value of x is

4. 7

Concept code: M111303

Sol. $x^2=(x+1)^2+b^2-2(x+1)b\cos\frac{\pi}{3}$

$x^2=x^2+2x+1+b^2-bx-b$

$b^2-b(x+1)+2x+1=0$

Since b is real, $D\geq 0$

$(x+1)^2-4(1)(2x+1)\geq 0$

$x^2-6x-3\geq 0\Rightarrow x\geq 3+\sqrt{12}$

5. In ΔABC, line joining the circumcentre and orthocenter is parallel to side AC, then the value of $\tan A\tan C$ is k then the value of $\frac{k}{6}$ is

5. 0.5

Concept code: M111507

Sol. $SD = 2R\cos B$

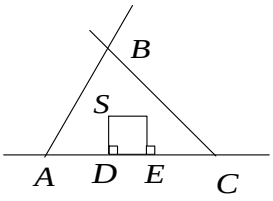
$OE = 2R\cos A\cos C$

$SD = OE \Rightarrow R\cos B = 2R\cos A\cos C$

$\cos\left(180^0 - \left(A+C\right)\right) = 2\cos A\cos C$

$-\cos\left(A+C\right) = \cos A\cos C$

$\sin A\sin C = 3\cos A\cos C$

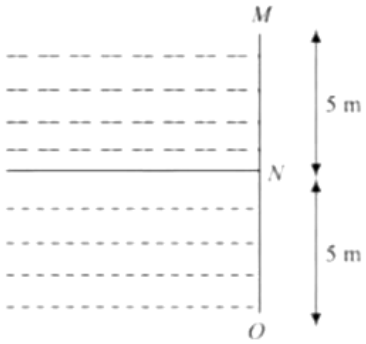


Physics

PART - A

SINGLE ANSWER CORRECT:

1. Two liquids of densities ρ_1 and ρ_2 $(\rho_2 = 2\rho_1)$ are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5m. The ratio of the magnitude of forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing) :



a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$

1. d

CONCEPT CODE : P111001

Sol. F = Pressure at centre of mass x-area.

2. Water flows into a large tank with flat bottom at the rate of $10^{-4}\,m^3\,s^{-1}$. Water is also leaking out of a hole of area $1\,cm^2$ at its bottom. If the height of the water in the tank remains steady, then this height is :

a) 5.1 cm b) 7 cm c) 4 cm d) 9 cm

2. a

CONCEPT CODE : P111005

Sol. $\theta = AV$

$V = \sqrt{2gn}$

3. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be

a) $\frac{1}{2}\rho v^2$ b) $\frac{3}{4}\rho v^2$ c) $\frac{1}{2}\rho v^2$ d) ρv^2

3. b

CONCEPT CODE : P111005

Sol. $F = \text{mass}\left(v_f - v_c\right)$

Mass $= \rho Av$

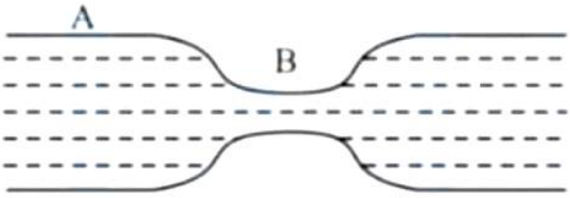
$F_2 = 25\%\ eAv\left(v-0\right)$

$F_2 = 25\%\ eAv\left(v-\left(-v\right)\right)$

$F_3 = 50\%\ eAv\left(v-v\right)$

$P = \dfrac{F_1 + F_2}{A}$

4. Water flows in a horizontal tube (see figure). The pressure of water changes by $700\,\text{Nm}^{-2}$ between A and B where the area of cross section are $40\,\text{cm}^2$ and $20\,\text{cm}^2$, respectively. Find the rate of flow of water through the tube. (density of water $= 1000\,\text{kgm}^{-2}$)



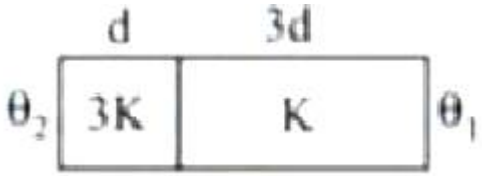
a) $3020\,\text{cm}^3/\text{s}$ b) $2720\,\text{cm}^3/\text{s}$ c) $2420\,\text{cm}^3/\text{s}$ d) $1810\,\text{cm}^3/\text{s}$

4. b

CONCEPT CODE : P111004

Sol. Apply equation of continuity
Apply Bernoulli's theorem
$\theta = AV$

5. Two materials having coefficients of thermal conductivity 3K and K and thickness d and $3d$, respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are θ_2 and θ_1 respectively, $(\theta_2 > \theta_1)$. The temperature at the interface is:



a) $\dfrac{\theta_1}{10} + \dfrac{9\theta_2}{10}$ b) $\dfrac{\theta_2 + \theta_1}{2}$ c) $\dfrac{\theta_1}{6} + \dfrac{5\theta_2}{6}$ d) $\dfrac{\theta_1}{3} + \dfrac{2\theta_2}{3}$

5. a

CONCEPT CODE : P111207

Sol. $\dfrac{3KA\left(\theta_2 - \theta\right)}{d} = \dfrac{KA\left(\theta - \theta_1\right)}{3d}$

6. An ideal fluid flows (laminar flow) through pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is :

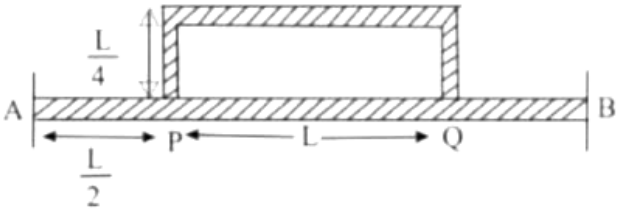
a) $\frac{9}{16}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{3}{4}$ d) $\frac{81}{256}$

6. a

CONCEPT CODE : P111004

Sol. $a_1v_1 = a_2v_2$

7. Temperature difference of $120^{\circ}C$ is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to :

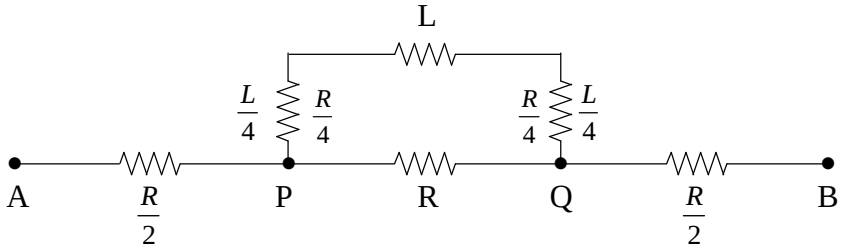


a) $45^{\circ}C$ b) $75^{\circ}C$ c) $60^{\circ}C$ d) $35^{\circ}C$

7. a

CONCEPT CODE : P111207

Sol.



$R = \frac{e\ell}{\pi}$

$\frac{\Delta T_{PQ}}{R_{eq}} = I = \frac{120 \times 5}{8R}$

$\Delta T_{PQ} = 45^{\circ}$

8. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acing on the submarine would be : (Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, $g = 10$ m/s^2)

a) $\frac{200}{3}\%$ b) $\frac{200}{5}\%$ c) $\frac{5}{200}\%$ d) $\frac{3}{200}\%$

8. a

CONCEPT CODE : P111001

Sol. $P = P_0 + ndg$

$$\frac{\Delta P}{P} \times 100 = \left(\frac{P_2 - P}{P}\right) \times 100$$

9. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is :

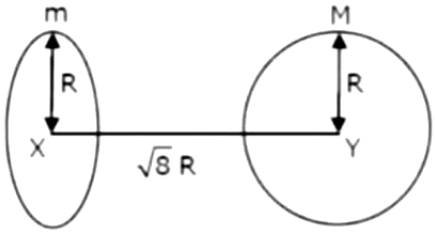
a) 4 b) 1 c) 2 d) 3

9. a

CONCEPT CODE : P111905

Sol. $v = \sqrt{\frac{2GM}{R}}$

10. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass m) and a sphere (mass M) where both have equal radius R.



a) $\frac{\sqrt{8}}{9} \cdot \frac{GmM}{R}$ b) $\frac{\sqrt{8}}{27} \cdot \frac{GmM}{R^2}$ c) $\frac{2\sqrt{2}}{3} \cdot \frac{GMm}{R^2}$ d) $\frac{1}{3\sqrt{8}} \cdot \frac{GMm}{R^2}$

10. b

CONCEPT CODE : P110901

Sol. $F = \frac{Gmx}{\left(R^2 + x^2\right)^{3/2}} \times M$

$x = \sqrt{8}R$

11. A fluid is flowing through a horizontal pipe of varying cross-section, with speed $v\,\text{ms}^{-1}$ at a point where the pressure is P Pascal. At another point where pressure is $\frac{P}{2}$ Pascal its speed is $v\,\text{ms}^{-1}$. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to

a) $\sqrt{\frac{P}{\rho} + v}$ b) $\sqrt{\frac{2P}{\rho} + v^2}$ c) $\sqrt{\frac{P}{2\rho} + v^2}$ d) $\sqrt{\frac{P}{\rho} + v^2}$

11. d

CONCEPT CODE : P111004

Sol. Apply Bernoulli's theorem.

12. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ \text{m/s}^2$ at the north pole and the radius of the earth = 6400 km)

a) 195.66 N b) 194.32 N c) 194.66 N d) 195.32 N

12. d

CONCEPT CODE : P110902

Sol. At equator

$W_e = W_p - W_R \times M$

$W_p = 196\ N = Mg$

$M = 19.6$

13. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass 2M. the gravitational field at distance $3a$ from the centre will be

a) $\frac{2GM}{9a^2}$ b) $\frac{GM}{9a^2}$ c) $\frac{GM}{3a^2}$ d) $\frac{2GM}{3a^2}$

13. c

CONCEPT CODE : P110901

Sol. $I = \frac{G(M+2M)}{(3R)^2}$

14. A test particle is moving in circular orbit in the gravitational field produced by a mass density $r(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T.

a) $\frac{T}{R}$ is a constant b) $\frac{T^2}{R^3}$ is a constant

c) $\frac{T}{R^2}$ is a constant d) TR is a constant

14. a

CONCEPT CODE : P110905

Sol. $\int_0^M dm = \int_0^R 4\pi r^2 e dr$

$m = 4\pi KR$

$T = \frac{2\pi R}{v}$

$$\frac{T}{R}=\frac{2\pi}{v}$$

15. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is

a) $\frac{K_1+K_2}{2}$ b) K_1+K_2 c) $\frac{2K_1+3K_2}{5}$ d) $\frac{K_1+3K_2}{4}$

15. d

CONCEPT CODE : P111207

Sol. $K_{eq}=\frac{K_1A_1+K_2A_2}{A_1+A_2}$

16. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

a) $\frac{E}{64}$ b) $\frac{E}{32}$ c) $\frac{E}{4}$ d) $\frac{E}{16}$

16. d

CONCEPT CODE : P110903

Sol. Minimum energy + Potential energy = 0

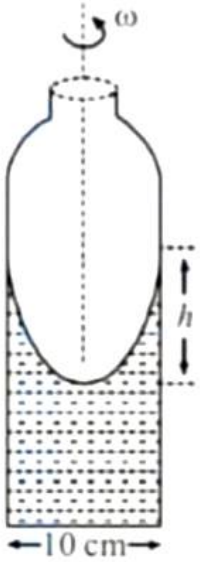
E + U = 0

$M_e=64\,m$ moon

$R_e=4R_m$

$E=\frac{GMm}{R}$

17. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rads^{-1}. The difference in the height *h* (in cm) of liquid at the centre of vessel and at the side will be :



a) $\frac{2\omega^2}{25g}$ b) $\frac{5\omega^2}{2g}$ c) $\frac{25\omega^2}{2g}$ d) $\frac{2\omega^2}{5g}$

17. c

CONCEPT CODE : P111001

Sol. $P_1 - P_2 = \frac{1}{2}ew^2\left(x_1^2 - x_2^2\right)$

18. The mass density of a planet of radius R varies with the distance *r* from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at :

a) $r = \sqrt{\frac{3}{4}}R$ b) $r = R$ c) $r = \frac{1}{\sqrt{3}}R$ d) $r = \sqrt{\frac{5}{9}}R$

18. d

CONCEPT CODE : P110901

Sol. $\int dm = \int e_0\left(1 - \frac{r^2}{R^2}\right)4\pi r d\sigma$

$E = \frac{GM}{r^2}$ and $\frac{dE}{dr} = 0$

19. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :
a) 0.5 b) 0.8 c) 0.6 d) 0.7

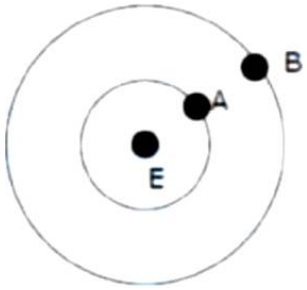
19. c

CONCEPT CODE : P111002

Sol. In equilibrium

$$v_e e_b g = \frac{v_b}{2}\rho_0 g + \frac{v_b}{2}\rho_w g$$

20. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$



[Given : radius of earth = 6400 km, mass of earth $= 6\times10^{24}$ kg]
a) 4.24×10^2 s b) 3.33×10^2 s
c) 1.33×10^3 s d) 4.24×10^3 s

20. c

CONCEPT CODE : P110905

Sol. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

PART - D

NUMERICAL ANSWER TYPE:

1. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(\text{beaker})} = 6\times10^{-6}\ ^\circ\text{C}^{-1}$ and $\gamma_{(\text{mercury})} = 1.5\times10^{-4}\ ^\circ\text{C}^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____.

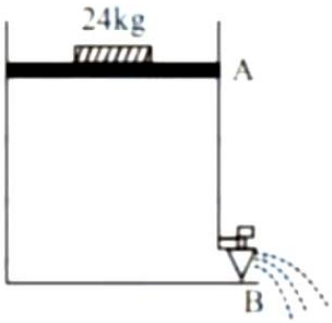
1. 20

CONCEPT CODE : P111201

Sol. Expansion of the liquid = expansion of solid

$$v_1 y_1 = v_2 y_2$$

2. Consider a water tank as shown in the figure. It's cross-sectional area is $0.4\,m^2$. The tank has an opening B near the bottom whose cross-section area is $1\,cm^2$. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is $v\,ms^{-1}$. The value of v, to the nearest integer, is ____ [Take $g = 10\,m/s^2$]



2. 3.00

CONCEPT CODE : P111004

Sol. Bernoulli's theorem

$$\rho_0 + \frac{mg}{A} + egH = \rho_0 + \frac{1}{2}ev^2$$

3. A hydraulic press can life 100 kg when a mass m is placed on the smaller piston. It can lift X kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass m on the smaller piston, then $\frac{X}{1000}$,

3. 25.6

CONCEPT CODE : P111001

Sol. Using Pascal law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

4. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes $0.01\,cm^3$ of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area $4\,cm^2$ by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}\,cm$. Then the thickness of oleic acid layer will be $x \times 10^{-14}\,m$. Where x is _____.

(Given : density of water is 1000 kg m^{-3} and $g = 9.8\,ms^{-2}$)

4. 25

CONCEPT CODE : P111007

Sol. $4t_T = 100 \times \frac{4}{3}\pi r^3$

$t_0 = 0.01\,t_T$

5. A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at $0^{o}C$ such that its height above the water surface is 20 cm. When the temperature of water is increased to $4^{o}C$, the height of the cylinder above the water surface becomes 21 cm. The density of water at $T = 4^{o}C$, relative to the density at $T = 0^{o}C$ is close to :

5. 1.01

CONCEPT CODE : P111002

Sol. When cylinder is floating in water at $0^{o}C$

Net thrust $= A(n_2 - n_1) \times e\, g$

When cylinder is floating at $4^{o}C$

Net thrust $= A(n_2 - n_1) \times e\, g$

Chemistry

PART - A

SINGLE ANSWER CORRECT:

1. Which of the following statements is not correct ?
a) The elements of Group 1 can be distinguished on the basis of flame photometry
b) The metals of Group 1 are strong reducing agents
c) The abundance of potassium in the earth's crust is more than that of sodium
d) The ionization energies, melting and boiling points of Group 1 decrease on moving down the group

1. C

Concept code: C110801

Sol: The abundance of sodium is more than that of potassium.

2. Which of the following is not the minerals of potassium ?
a) Chile saltpeter b) Carnallite c) Kainite d) Potassium feldspar

2. A

Concept code: C110801

Sol: Chile saltpeter is $NaNO_3$.

3. Which of the following statements is not correct ?
a) Alkali metals cannot be obtained by the chemical reduction of their compounds
b) Alkali metals are usually obtained by the electrolytic reduction
c) Electrolytic reduction of sodium ions is done in a Down's cell to get sodium in a large scale
d) Electrolytic reduction of potassium ions can be done in a Downs cell to get potassium in a large scale

3. D

Concept code: C110807

Sol: Downs cell is used for the production of sodium.

4. Which of the following statements is not correct ?
a) Sodium and potassium are soft and silvery-white metals
b) Sodium and potassium in air get tarnished due to the formation of a layer of oxide or carbonate
c) Burning of sodium and potassium in dry oxygen produces peroxides
d) Sodium and potassium are kept under kerosene to avoid the contact with air and moisture

4. C

Concept code: C110803

Sol: Sodium produces monoxide $\left(Na_2O\right)$ which, in turn, produces peroxide $\left(Na_2O_2\right)$ whereas potassium produces superoxide $\left(KO_2\right)$.

5. Which of the following chemical equations is not actually observed ?

a) $4Na + \underset{(dry)}{O_2} \rightarrow 2Na_2O \xrightarrow[dry]{O_2} 2Na_2O_2$

b) $4K + O_2 \rightarrow 2K_2O \xrightarrow[dry]{O_2\ excess} 2K_2O_2$

c) $2M + 2H_2O \rightarrow 2MOH + H_2$; where M is Na or K

d) $2M + 2NH_3 \rightarrow 2MNH_2 + H_2$; where M is Na or K

5. B

Concept code: C110803

Sol: $K + O_2 \rightarrow KO_2$

6. Which of the following peroxide is white in colour ?

a) Li_2O_2 b) Cs_2O_2 c) K_2O_2 d) Rb_2O_2

6. A

Concept code: C110803

Sol: Li_2O_2 is colourless, rest of peroxide are coloured.

7. In the reaction $Na_2S + O_2 + H_2O \rightarrow A + NaOH$ the product A is

a) Na_2SO_3 b) $Na_2S_2O_3$ c) $Na_2S_4O_6$ d) Na_2SO_4

7. B

Concept code: C110806

Sol: The reaction is $2Na_2S + 2O_2 + H_2O \rightarrow Na_2S_2O_3 + 2NaOH$

8. The correct order of density of alkali metal follows the order

a) $Li < Na < K$ b) $Li < K < Na$ c) $Na < Li < K$ d) $Na < K < Li$

8. B

Concept code: C110801

Sol: The densities are $Li\left(0.54\,g\ cm^{-3}\right)$, $Na\left(0.97\,g\ cm^{-3}\right)$ and $K\left(0.86\,g\ cm^{-3}\right)$

9. Which of the following ions is expected to have maximum hydration energy ?

a) Be^{2+} b) Mg^{2+} c) Ca^{2+} d) Sr^{2+}

9. A

Concept code: C110901

Sol: Because of the small size of Be^{2+} ions, it will have more hydration energy

10. Which of the following ions is expected to have minimum standard reduction electrode potential ?
a) Be^{2+} b) Mg^{2+} c) Ca^{2+} d) Sr^{2+}

10. D

Concept code: C110903

Sol: Standard reduction electrode potential becomes more negative as we travel down the Group 2

11. Which of the following atoms will have maximum oxidizing ability ?
a) *Be* b) *Mg* c) *Ca* d) *Sr*

11. D

Concept code: C110903

Sol: Smaller the reduction potential of the ion, more easily it can be produced from the atom. Hence, larger will be the oxidizing ability of the atom.

12 Which of the following statements is not correct ?
a) The lattice energy of salts of alkaline earth metals decreases with increase in ionic sizes from Be^{2+} to Ba^{2+}
b) The hydration energies of ions of alkaline earth metals decrease regularly from Be^{2+} to Ba^{2+}
c) The salts of alkaline earth metals are less hydrated than those of alkali metals
d) Carbonates of alkaline earth metals are insoluble in water

12. C

Concept code: C110902

Sol: The salts of alkaline earth metals are more hydrated because of +2 charge of the ions.

13. Which of the following statements is not true ?
a) Boron is a nonmetal while all other elements of Group 13 are metals
b) Boron trichloride acts as a Lewis acid
c) Boron does not form the hydrated B^{3+} ion
d) The stability of the +1 oxidation state decreases down the group whereas that of +3 increases

13. D

Concept code: C111502

Sol: The stability of the +1 oxidation state increases down the group. This is because of the participation of only *p* electrons. The two of the n^{th} shell *s*-electrons $\left(ns^2\right)$ do not take part in the chemical reaction (inert pair effect).

14. Very pure hydrogen (99.9 %) can be made by which of the following processes ?
a) Reaction of methane with steam
b) Mixing natural hydrocarbons of high molar mass
c) Electrolysis of water
d) Reaction of salt like hydrides with water

14. C

Concept code: C111001

Sol: Pure hydrogen can be obtained by the electrolysis of water which is made conductor by addition of a little acid or an alkali.

15. Which of the following statements is not correct ?
a) *ortho*- and *para*-dihydrogen are similar in chemical properties
b) At room temperature, ordinary hydrogen is a mixture of 75% *ortho*- dihydrogen and 25% *para*-dihydrogen
c) At lower temperature, proportion of *ortho*-dihydrogen increases whereas that of *para*-dihydrogen decreases
d) It is not possible to isolate pure *ortho*-dihydrogen

15. C

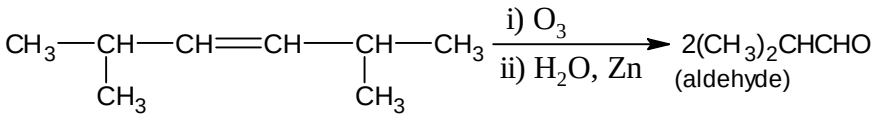
Concept code: C111001

Sol: The ratio para/ortho dihydrogen increases with decrease in temperature and at room temperature this ratio is about 0.25.

16. The reductive ozonlysis of 2, 5-dimethylhex-3-ene produces
a) two molecules of aldehydes
b) two molecules of ketones
c) one molecule of aldehyde and one molecule of ketone
d) neither aldehyde nor ketone

16. A

Concept code: C111708

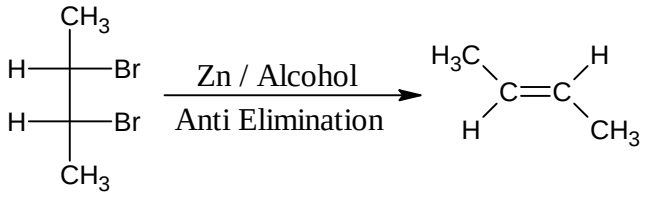
Sol: 

17. During debromination of *meso*-dibromobutane, the major compound formed is
a) *n*-butane b) 1-butene c) *cis*-2-butene d) *trans*-2-butene

17. D

Concept code: C111705

Sol:



18. Reaction of one molecule of *HBr* with one molecule of 1, 3-butadiene at $40°C$ gives predominantly.
a) 3-bromobutene under thermodynamically controlled conditions
b) 1-bromo-2-butene under kinetically controlled conditions
c) 3-bromobutene under kinetically controlled conditions
d) 1-bromo-2-butene under thermodynamically controlled conditions

18. D

Concept code: C111707

Sol: At $40°C$, thermodynamically controlled, 1, 4 – addition product predominates.

$$CH_2{=}CH{-}CH{=}CH_2 \xrightarrow[40°C]{HBr} \underset{\text{1-Bromo-2-butene}}{CH_3CH{=}CHCH_2Br}$$

At low temperature $(0°C)$, kinetically controlled 1, 2-addition product predominates.

19. Which of the following statements is not correct ?
a) The addition of acetylene to lithium amide dissolved in ether produces NH_3 and $HC \equiv \overline{C}Li^+$
b) If water can generate $HC \equiv CH$ from $HC \equiv CLi$ then water must be a stronger acid than acetylene
c) Acetylide ion (which has a pair of electrons in *sp* orbital of carbon) is more stronger base than ethyl carbanion, $C_2H_5^-$.
d) Lindlar catalyst is $Pb/BaSO_4$

19. C

Concept code: C111712

Sol: Acetylide is a weaker base than ethyl carbanion . A pair of electrons is more tightly bound in acetylene than in ethyl carbanion.

20. Which of the following statements is correct ?
a) The polymerization of ethyne gives Benzene in presence of red hot metal
b) The oxidative ozonlysis of propyne produces acetic acid only
c) The IUPAC name of $HC \equiv C - CH_2 = CH_2$ is 4-pentyne-1-ene
d) The number of isomers of alkyne C_6H_{10} is six

20. A

Concept code: C111712

Sol: B) $CH_3C \equiv CH \xrightarrow{O_3} CH_3COOH + HCOOH$

C) The correct name is 1-penten-4-yne. $C=C$ has priority over $C \equiv C$ and gets the smaller number

D) There are sever isomers.

1-hexyne, 2-hexyne, 3-hexyne, 4-methyl-1-pentyne, 4-methyl-2-pentyne, 3-methyl-1-pentyne,3, 3-dimethyl-1-1-butyne

PART - D

NUMERICAL ANSWER TYPE:

1. The ratio of number of water molecules present in Epson salt and Glauber salt as water of crystalisation is

1. 0.70

Concept code: C110801

Sol: Epson salt is $MgSO_4.7H_2O$

Glauber salt is $CaSO_4.10H_2O$

2. The mass of oxygen liberated by decomposition of 200mL of H_2O_2 solution with a strength of $34\%\left(\frac{w}{v}\right)$.

2. 32.00

Concept code: C111004

Sol: Volume strength = Percentage strength $\times \frac{56}{17} = 34 \times \frac{56}{17} = 112$

Volume of O_2 at S.T.P = Volume of H_2O_2 $\times$ Volume strength

$= 200 \times 112 = 22400\,mL$

Wt of 22,400 mL of O_2 at S.T.P = 32g.

3. Washing soda has *x* number of water molecules as water of crystallisation and it has *y* number of water molecules in it's formula when exposed to atmosphere then find $\frac{x-y}{x}$ value.

3. 0.90

Concept code: C110808

Sol: $\underset{\text{washing soda}}{Na_2CO_3.10H_2O} \longrightarrow Na_2CO_3.H_2O + 9H_2O$

$x = 60; \quad y = 1$

$\frac{x-y}{x} = \frac{10-1}{10} = \frac{9}{10} = 0.90$

4. The degree of hardness of sample of water containing 24mg of $MgSO_4$ per kg of water is _____ ppm.

4. 20.00

Concept code: C111003

Sol: Degree of Hardness $= \frac{wt\ of\ MgSO_4}{GMW\ of\ MgSO_4} \times \frac{10^8}{Wt\ of\ water(gm)}$

$= \frac{24\times10^{-3}}{120} \times \frac{10^8}{10^3} = \frac{2400}{120} = 20\ ppm$

5. Among B, Al, Ga, In and Tl . x elements have high I.E than Al and y elements have less atomic radius than Al. Then find $\frac{x}{y}$ is

5. 1.50

Concept code: C111501

Sol: Order $I.E_1 = \underbrace{B > Tl > Ga}_{high\ I.E} > Al > In$

$x = 3$

order of atomic radius – $\underbrace{B < Ga}_{less\ radius} < Al < In < Tl$

$y = 2$

$\therefore \frac{x}{y} = \frac{3}{2} = 1.5$