Mathematics

PART - A

SINGLE ANSWER CORRECT:

1. Let $f(x)=\begin{cases}1+x, & x\in[0,2]\\ 3-x, & x\in(2,3]\end{cases}$. Suppose $g(x)=f(f(x))$ then

a) $g(x)$ is discontinuous only at $x=1$

b) $g(x)$ is discontinuous only at $x=2$

c) $g(x)$ is discontinuous at both $x=1$ and $x=2$

d) $g(x)$ is continuous everywhere

1. C

CONCEPT CODE: M120402

Sol: $f(2)=3$, $f(2^+)=1$ so $f(x)$ is Discontinuous at $x=2$.

Now $f(x)=2\Rightarrow 1+x=2\Rightarrow \boxed{x=1}\rightarrow$ Discontinuous.

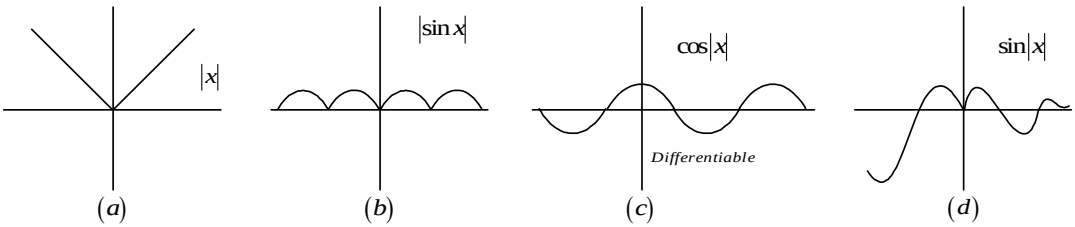
2. Which of the following function is differentiable at $x=0$?

a) $|x|$ b) $|\sin x|$ c) $\cos|x|$ d) $\sin|x|$

2. C

CONCEPT CODE: M120405

Sol:



3. Let $f(x)=(\sin\pi x)|x-1||x-2|$. Then

a) $f(x)$ is Not differentiable at $x=1$ b) $f(x)$ is Not differentiable at $x=2$

c) $f(x)$ is Not differentiable at $x=0,1,2$ d) $f(x)$ is Differentiable everywhere

3. D

CONCEPT CODE: M120405

Sol: Since at $x=1$ and 2 $\sin\pi x=0$

So Differentiable at all $x\in R$.

(Using Def. we can verify).

4. $f(x)=\begin{cases} x^2+ax+1, & x \text{ is Rational} \\ ax^2+2x+b, & x \text{ is Irrational} \end{cases}$. If $f(x)$ is continuous at $x=1$ and $x=2$. Then a and b equals

a) $a=\frac{1}{2}, b=0$ b) $a=0, b=\frac{1}{2}$ c) $a=b=\frac{1}{2}$ d) $a=\frac{1}{2}, b\in R$

4. A

CONCEPT CODE: M120405

Sol: $x^2+ax+1=ax^2+2x+b$

$\Rightarrow (a-1)x^2+(2-a)x+(b-1)=0$

$x=1 \quad x=2$ (are roots)

So, $\frac{b-1}{a-1}=2$ and $\frac{-(2-a)}{a-1}=3$

$\Rightarrow a=\frac{1}{2}, b=0$

5. If a function $f(x)$ is differentiable at $x=2$ then which is FALSE ?

a) $\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}$ must exist and is finite

b) $\lim\limits_{h\to 0}\frac{f(2+h)-f(2-h)}{h}$ must exist and is finite

c) $\lim\limits_{h\to 0} f(2+h)-f(2)=0$

d) $\lim\limits_{h\to 0} f(2+h)-f(2)\neq 0$

5. D

CONCEPT CODE: M120408

Sol: $f(x)$ must be continuous at $x=2$

So, $\lim\limits_{x\to 0} f(2+x)=\lim\limits_{x\to 0} f(2-x)=f(2)$

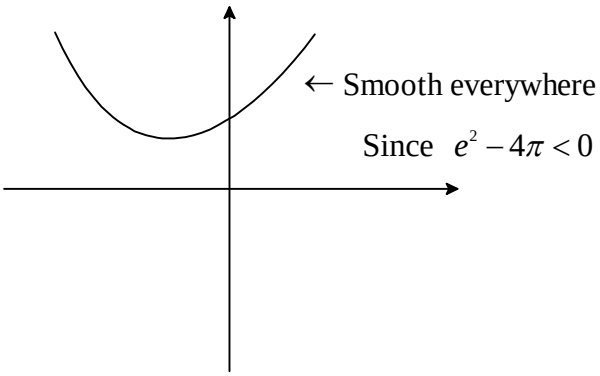
6. The number of points of Non-Differentiability of $f(x)=|x^2+ex+\pi|$ is

a) 4 b) 2 c) 1 d) 0

6. B

CONCEPT CODE: M120408

Sol:



7. If $f(x)=\begin{cases}\dfrac{x-|x|}{x} & ,\text{when } x<0\\ 5x^2+a & ,\text{when } 0\le x\le 1\\ b\left(\dfrac{x^2-1}{x^2-3x+2}\right) & ,\text{when } 1<x<3\\ -14 & ,\text{when } x\ge 3\end{cases}$ is a continuous function on IR, then $(a,b)=$

a) $\left(2,-\frac{7}{2}\right)$ b) $(2,-14)$ c) $\left(-\frac{7}{2},-14\right)$ d) $(2,7)$

7. A

CONCEPT CODE: M120402

Sol: $f(x)$ is continuous on $R\Rightarrow f(x)$ is continuous at 0, 3

$\Rightarrow \lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)=f(0)$ and $\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} f(x)=f(3)$

$\Rightarrow \lim\limits_{x\to 0^-}\dfrac{x-|x|}{x}=\lim\limits_{x\to 0^+}(5x^2+a)$ and $\lim\limits_{x\to 3^-}\dfrac{b(x^2-1)}{x^2-3x+2}=-14$

$\Rightarrow \lim\limits_{x\to 0^-}\dfrac{2x}{x}=a$ and $b\lim\limits_{x\to 3^-}\dfrac{x+1}{x-2}=-14\Rightarrow a=2,\ b=-7/2\Rightarrow (a,b)=(2,-7/2)$.

8. Let $f:IR\to IR$ be the function defined by $f(x)=\begin{cases}5 & ,\ \text{if } x\le 1\\ a+bx & ,\ \text{if } 1<x<3\\ b+5x & ,\ \text{if } 3\le x<5\\ 30 & ,\ \text{if } x\ge 5\end{cases}$ then f is

a) continuous if $a=5$ and $b=5$ b) continuous if $a=0$, $b=5$

c) continuous if $a=-5, b=10$ d) not continuous for any value of a and b

8. D

CONCEPT CODE: M120402

Sol: $\lim\limits_{x\to 3-} f(x)=\lim\limits_{x\to 3-}(a+bx)=a+3b,\ \lim\limits_{x\to 3+} f(x)=\lim\limits_{x\to 3+}(b+5x)=b+15$

$\lim\limits_{x\to 5-} f(x)=\lim\limits_{x\to 5-}(b+5x)=b+25,\ \lim\limits_{x\to 5+} f(x)=\lim\limits_{x\to 5+}(30)=30$

$\lim\limits_{x\to 1-} f(x)=5,\ \lim\limits_{x\to 1+} f(x)=\lim\limits_{x\to 1+} a+bx=a+b$

If f is continuous at 1 then $a+b=5$.

If f is continuous at 5 then $b+25=30\Rightarrow b=5$ and hence $a=0$

If f is continuous at 3 then $a+3b=b+15\Rightarrow a+2b=15$ which is not true.

$\therefore f$ is not continuous at any values of a and b.

9. If $f:R\to R$ is defined by $f(x)=\begin{cases}\dfrac{1+3x^2-\cos 2x}{x^2}, & for\quad x\neq 0\\ k, & for\quad x=0\end{cases}$ is continuous at $x=0$, then $k=$

a) 1 b) 5 c) 6 d) 0

9. B

CONCEPT CODE: M120402

Sol: $f(x)$ is continuous at 0

$\Rightarrow f(0)=\lim\limits_{x\to 0} f(x)\Rightarrow k=\lim\limits_{x\to 0}\dfrac{1+3x^2-\cos 2x}{x^2}=\lim\limits_{x\to 0}\dfrac{6x+2\sin 2x}{2x}=3+2=5.$

10. The value of $f(\pi)$ so that $f(x)=\dfrac{1-\cos 7(x-\pi)}{x-\pi}$ is continuous at the point $x=\pi$ is

a) 0 b) 1 c) π d) $\dfrac{\pi}{2}$

10. A

CONCEPT CODE: M120402

Sol: $f(\pi)=\lim\limits_{x\to\pi} f(x)=\lim\limits_{x\to\pi}\dfrac{1-\cos 7(x-\pi)}{x-\pi}=\lim\limits_{x\to\pi}7\sin(x-\pi)=7\sin 0=0.$

11. If $f:R\to R$ is defined by $f(x)=\begin{cases}\dfrac{\cos 3x-\cos x}{x^2} & \text{for}\quad x\neq 0\\ \lambda & \text{for}\quad x=0\end{cases}$ and if f is continuous at $x=0$, then $\lambda=$

a) -2 b) -4 c) -6 d) -8

11. B

CONCEPT CODE: M120402

Sol: f is continuous at $x=0\Rightarrow\lim\limits_{x\to 0} f(x)=f(0)$

$\Rightarrow\lambda=\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{-3\sin 3x+\sin x}{2x}=\lim\limits_{x\to 0}\dfrac{-9\cos 3x+\cos x}{2}=\dfrac{-9+1}{2}=-4.$

12. Consider $f(x)=\frac{x^2}{|x|}$, $x\neq 0$, $f(x)=0, x=0$

a) $f(x)$ is discontinuous at every $x\in R$ b) $f(x)$ is continuous at every $x\in R$

c) $f'(x)$ exists in $(-1, 1)$ d) $f'(x)$ exists in $(-2, 2)$

12. B

CONCEPT CODE: M120402

Sol: If $x>0$, $f(x)=x$, if $x<0$, $f(x)=-x$ is continuous everywhere.

13. The function $f(x)=(x^2-1)|x^2-3x+2|+\cos|x|$ is not differentiable at

a) -1 b) 0 c) 1 d) 2

13. D

CONCEPT CODE: M120408

Sol: $f(x)=(x+1)(x-1)|x-1||x-2|+\cos|x|$

14. Let $f(x)=\min\{x, x^2\}$, for every real x. Then

a) f is continuous for all x b) f is differentiable for all x

c) $f'(x)=0\ \forall\ x>1$ d) f is differentiable at 0, 1

14. A

CONCEPT CODE: M120408

Sol: $f(x)$ is not differentiable at $x=0$, $x=1$

$f'(x)=1$, for $x>1$.

15. If $f(x)=\frac{\sin(e^{x-2}-1)}{\ln(x-1)}$, $x\neq 2$ is continuous at $x=2$, then $f(2)=$

a) 0 b) 2 c) 1 d) -2

15. C

CONCEPT CODE: M120402

Sol: By LH rule, $f(2)=\lim\limits_{x\to 2}\frac{\cos(e^{x-2}-1)\times(e^{x-2})}{\left(\frac{1}{x-1}\right)}=1$

16. If p and q are number of points of discontinuous and non-differentiable respectively, of the function $f(x)=[2+3\sin x]$, where $x\in(-\pi,\ 2\pi)$, then the value of $p+q$ is (where $[x]$ is GIF)

a) 34 b) 32 c) 30 d) 28

16. C

CONCEPT CODE: M120408

Sol: $f(x)=2+[3\sin x]$

$\therefore$ f is discontinuous whenever $3\sin x$ is an integer.

i.e., $\sin x=-\frac{2}{3},\ -\frac{1}{3},\ 0,\ \frac{1}{3},\ \frac{2}{3},\ \frac{3}{3}$

If $x\in(-\pi,\ 2\pi)$ (If $[3\sin x]=-3$, it is continuous).

$\sin x=-\frac{2}{3}$ occurs 4 times

$=-\frac{1}{3}$ occurs 4 times

$=0$ occurs 2 times

$=\frac{1}{3}$ occurs 2 times

$=\frac{2}{3}$ occurs 2 times

$=\frac{3}{3}$ occurs 1 time

$\therefore P=15$ also $p=q$

17. If $f(x)=\begin{cases} e^x & \text{for } x<1 \\ a-bx & \text{for } x\geq 1 \end{cases}$ is differentiable for $x\in R$, then

a) $a=1,\ b=e-1$ b) $a=0,\ b=e$ c) $a=0,\ b=-e$ d) $a=e,\ b=1$

17. C

CONCEPT CODE: M120408

Sol: f is continuous at $x=1\Rightarrow e'=a-b$

f is differentiable at $x=1\Rightarrow e'=0-b\Rightarrow b=-e$

$\Rightarrow a=0$

18. Which of the following statements are TRUE about a function $f:R\to R$ (where $a\in R$)

a) f is continuous at $x=a\Rightarrow f$ is differentiable at $x=a$

b) f is differentiable at $x=a\Rightarrow f$ is continuous at $x=a$

c) f is not differentiable at $x=a\Rightarrow f$ is not continuous at $x=a$

d) f is not continuous at $x=a\Rightarrow f$ is differentiable at $x=a$

18. B

CONCEPT CODE: M120405

19. The number of non-differentiable points of $f(x)=|\sin 2x|$ where $x\in(0,2\pi)$ is
a) 0 b) 1 c) 3 d) 4

19. C

CONCEPT CODE: M120408

Sol: $\sin 2x=0\Rightarrow x=\frac{\pi}{2},\pi,\frac{3\pi}{2}$

20. $f(x)=x+|x|$. Then f is
a) Differentiable at all $x\in R$
b) Differentiable at all $x\in R$ except at one point
c) Differentiable at all $x\in R$ except at two points
d) Differentiable at all $x\in R$ except at three points

20. B

CONCEPT CODE: M120408

Sol: $f(x)=\begin{cases}0 & if\ x<0\\ 2x & if\ x\geq 0\end{cases}$

PART - D

NUMERICAL ANSWER TYPE:

1. If the function $f:IR\to IR$ defined by $f(x)=\begin{cases}a\left(\frac{1-\cos 2x}{x^2}\right) & ,\text{for}\quad x<0\\ b & ,\text{for}\quad x=0\\ \frac{\sqrt{x}}{\sqrt{4+\sqrt{x}}-2} & ,\text{for}\quad x>0\end{cases}$ is continuous at $x=0$, then $a+b=k$, then the value of $\frac{20k}{9}$ is

1. 13.33

CONCEPT CODE: M120405

Sol: $f(x)$ is continuous at $x=0\Rightarrow \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0^-}\frac{a(1-\cos 2x)}{x^2}=\lim_{x\to 0^+}\frac{\sqrt{x}}{\sqrt{4+\sqrt{x}}-2}=b\Rightarrow \lim_{x\to 0^-}\frac{2a\sin^2 x}{x^2}=\lim_{x\to 0^-}\frac{\sqrt{x}\left(\sqrt{4+\sqrt{x}}+2\right)}{\sqrt{x}}=b$

$\Rightarrow 2a=4=b\Rightarrow a=2,\ b=4\Rightarrow a+b=2+4=6=k$.

2. The number of points in the interval $(0, 2)$ at which $f(x)=|x-0.5|+|x-1|+\tan x$ is not differentiable is p then the value of $\frac{20p}{8}$ is

2. 7.5

CONCEPT CODE: M120408

Sol: The function $f(x)=|x-0.5|+|x-1|+\tan x$ is not differentiable in $(0,2)$ at $x=0.5,\ 1,\ \pi/2$. $\therefore\ p=3$

3. Define $f(x)=\begin{cases} x^2+bx+c & ,x<1 \\ x & ,x\geq 1 \end{cases}$. If $f(x)$ is differentiable at $x=1$, then $(b-c)=-p$, then the value of p^5 is

3. 32

CONCEPT CODE: M120405

Sol: $f'(1+)=f'(1-)$
$b=-1,\ c=1$
$-p=b-c=-2$.

4. The number non-differentiable points of the function $f(x)=[\sin x]$ in the interval $(0,4\pi)$ is

4. 5

CONCEPT CODE: M120405

Sol: $x=\frac{\pi}{2},\pi,2\pi,\frac{5\pi}{2},3\pi$

5. Number of non-differential points of the function $f(x)=\left\{\frac{x}{2}\right\}$ in the interval $(0,100)$ is

5. 49

CONCEPT CODE: M120405

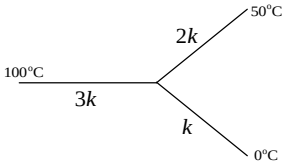
Sol: $\frac{x}{2}$ must be an integer $\Rightarrow x=2,4,6,....,98$

Physics

PART - A

SINGLE ANSWER CORRECT:

1. Three rods of identical dimensions have thermal conductivities 3k, 2k and k, their arrangement and temperatures are shown. Temperature at their junction is



a) $\frac{200}{3}\,^{o}C$ b) $\frac{200}{4}\,^{o}C$ c) $\frac{200}{5}\,^{o}C$ d) $\frac{200}{6}\,^{o}C$

1. a

CONCEPT CODE : P111207

Sol. $\frac{3kA(100-\theta)}{l} = \frac{2kA(\theta-50)}{l} + \frac{kA(\theta-0)}{l}$

$\Rightarrow \quad 6\theta = 400$

$\Rightarrow \quad \theta = \frac{200}{3}\,^{o}C$

2. A semicircular rod is joined at its ends to a rod of same material and same cross-sectional area. Their junctions are maintained at different temperatures. The ratio of heat current through then is

a) $1:\pi$ b) $\pi:1$ c) $2:\pi$ d) $\pi:2$

2. c

CONCEPT CODE : P111207

Sol. $i_1 : i_2 = \frac{kA\Delta\theta}{\pi r} : \frac{kA\Delta\theta}{2r}$

$= \frac{1}{\pi} : \frac{1}{2} = 2:\pi$

3. A rod of length 1 m, cross sectional are $4\,cm^2$ and thermal conductivity 42 J/ms ^{o}C is used to melt ice. One end of the rod is placed in boiling water and other in ice. The mass of ice melt in one day is

a) 216 g b) 432 g c) 648 g d) 864 g

3. b

CONCEPT CODE : P111208

Sol. $\frac{dQ}{dt} = \frac{kA\Delta\theta}{l} = \frac{Ldm}{dt}$

$\Rightarrow \quad dm = \frac{kA\Delta\theta}{lL}dt \Rightarrow m = \frac{kA\Delta\theta \cdot t}{lL}$

$\Rightarrow \quad m = \frac{42\times 4\times 10^{-4}\times 100\times 86400}{1\times 80\times 4.2} = 432\ g$

4. Two spheres of same material and radius 1 : 4 has surface temperatures in the ratio 2 : 1. The ratio of their power radiation is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 1 d) 4

4. c

CONCEPT CODE : P111208

Sol. $P_1 : P_2 = A_1T_1^4 : A_2T_2^4 = r_1^2T_1^4 : r_2^2T_2^4$

$= \left(r_1T_1^2\right)^2 : \left(r_2T_2^2\right)^2 = \left(1\times 2^2\right)^2 : \left(4:1^2\right) = 1:1$

5. A body cools from 50^oC to 40^oC in 5 min in a surrounding of 20^oC. Temperature of the body after another 5 min would be about

a) $\frac{100}{6}$ b) $\frac{100}{3}$ c) 200 d) $\frac{200}{3}$

5. b

CONCEPT CODE : P111208

Sol. $\frac{50^oC - 40^oC}{5\,\text{min}} = k\left(45^oC - 20^oC\right)$

$\Rightarrow \quad k = \frac{2}{25}/\,\text{min}$

$\frac{40^oC - \theta^oC}{5\,\text{min}} = \frac{2}{25\,\text{min}}\left[\frac{40^oC + \theta^oC}{2} - 20^oC\right]$

$\Rightarrow \quad 40 - \theta = \frac{2}{5}\times\frac{\theta}{2} \Rightarrow \theta = \frac{100^o}{3}C$

6. Two stars has maximum spectral emissive power at wavelengths $4000\,A^o$ and $5000\,A^o$. Ratio of their surface temperatures is

a) 4 : 5 b) 5 : 4 c) 16 : 25 d) 25 : 16

6. b

CONCEPT CODE : P111208

Sol. $T_1 : T_2 = \frac{1}{\lambda_1} : \frac{1}{\lambda_2} = \frac{1}{4} : \frac{1}{5}$

$= 5:4$

7. A metal sphere of radius 10 cm, emissivity 0.1 and temperature 127^oC is placed in a surrounding of temperature 27^oC. The initial rate of loss of heat by the sphere is

a) 3 W b) 7.5 W c) 12.5 W d) 19 W

7. c

CONCEPT CODE : P111208

Sol. $\frac{dQ}{dt} = e\sigma A\left(T^4 - T_0^4\right) = 4\pi r^2 e\sigma\left(T^4 - T_0^4\right)$

$= 4\times 3.14\times(0.1)^2\times 0.1\times 5.67\times 10^{-8}\left(400^4 - 300^4\right)$

$= 12.5\ \text{W}$

8. It takes 5 min cool a liquid from $70^{o}C$ to $60^{o}C$ in a surrounding of $30^{o}C$. The additional time taken to cool it to $50^{o}C$ is
a) 5 min b) 6 min c) 7 min d) 8 min

8. c

CONCEPT CODE : P111208

Sol. $$\frac{\frac{70-60}{5}}{\frac{60-50}{t}} = \frac{k(65-30)}{k(55-30)}$$

$$\Rightarrow \quad \frac{t}{5} = \frac{35}{25} = \frac{7}{5}$$

$$\Rightarrow \quad t = 7 \text{ min}$$

9. Temperature of a metal sphere is maintained at 500 K, in an evacuated chamber at 300 K, with the help of a power source of 210 W. When the metal sphere is completely blackened then it requires a power of 700 W to maintain same temperature. Emissivity of that metal is
a) 0.3 b) 0.4 c) 0.5 d) 0.6

9. a

CONCEPT CODE : P111208

Sol. $$P = e\sigma A\left(T^4 - T_0^4\right)$$

$$\Rightarrow \quad \frac{210}{700} = \frac{e\sigma A\left(T^4 - T_0^4\right)}{\sigma A\left(T^4 - T_0^4\right)}$$

$$\Rightarrow \quad e = \frac{3}{10} = 0.3$$

10. A body takes 30s to cool down from $60^{o}C$ to $55^{o}C$ when weather temperature is $45^{o}C$. Time taken by the body to cool down from $55^{o}C$ to $50^{o}C$ is about
a) 30 s b) 40 s c) 50 s d) 60 s

10. c

CONCEPT CODE : P111208

Sol. $$\frac{\Delta\theta}{\Delta t} = k\left(\theta_{av} - \theta_0\right)$$

$$\Rightarrow \quad \frac{\frac{50^{o}C}{30\,s}}{\frac{5^{o}C}{t}} = \frac{k(57.5-45)^{o}C}{k(52.5-45)^{o}C}$$

$$\Rightarrow \quad \frac{t}{30s} = \frac{12.5}{7.5} = \frac{5}{3} \Rightarrow t = 50s$$

11. Two cylindrical rods of same material has ratio of length 2 : 1 and ratio of diameters is 1 : 2. If temperature difference along their lengths is same, then the ratio of heat current through them is
a) 1 : 1 b) 1 : 2 c) 1 : 4 d) 1 : 8

11. d

CONCEPT CODE : P111207

Sol. $i=\frac{dQ}{dt}=\frac{kA\Delta\theta}{l}=\frac{k\pi d^2\Delta\theta}{l}$

$\Rightarrow \quad \frac{i_1}{i_2}=\left(\frac{d_1}{d_2}\right)^2\cdot\frac{l_2}{l_1}=\left(\frac{1}{2}\right)^2\cdot\frac{1}{2}=\frac{1}{8}$

12. Two identical rods, when welded in series, carry 20 cal of heat at a particular temperature difference. If they are now welded in parallel, then the time taken to carry same amount of heat at same temperature difference is

a) 1 min b) 2 min c) 4 min d) 8 min

12. a

CONCEPT CODE : P111208

Sol. $i=\frac{dQ}{dt}=\frac{\Delta\theta}{R_{th}}$; in series $R_{th}=2R$ and in parallel, $R_{th}^1=\frac{R}{2}$

$\Rightarrow \quad t=\frac{Q\cdot R_{th}}{\Delta\theta} \Rightarrow \frac{t_2}{t_1}=\frac{R_{th}^1}{R_{th}}=\frac{\frac{R}{2}}{2R}=\frac{1}{4}$

$\Rightarrow \quad t_2=\frac{1}{4}t_1=1\,\text{min}$

13. A wall of two layers of same thickness has ratio of conductivity 2 : 1. At steady state, temperature difference across the wall is $36^\circ C$, then temperature difference across the layer of higher conductivity is

a) $6^\circ C$ b) $12^\circ C$ c) $18^\circ C$ d) $24^\circ C$

13. b

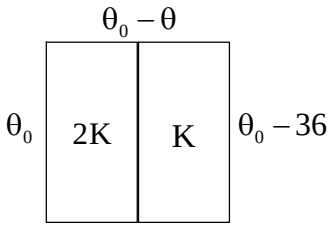
CONCEPT CODE : P111207

Sol. As the layers are in series,

$$\frac{2kA\left[\theta_0-(\theta_0-\theta)\right]}{l}=\frac{kA\left[(\theta_0-\theta)-(\theta_0-36)\right]}{l}$$

$\Rightarrow \quad 2\theta=36-\theta$

$\Rightarrow \quad \theta=\frac{36}{3}=12^\circ C$



14. A slab consists of two parallel layers of copper and brass of same thickness having ratio of thermal conductivities 4 : 1. If free face of copper and brass are kept at $0^\circ C$ and $100^\circ C$ respectively then temperature of the interface is

a) $20^\circ C$ b) $40^\circ C$ c) $60^\circ C$ d) $80^\circ C$

14. a

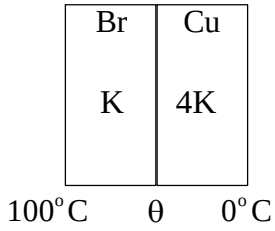
CONCEPT CODE : P111208

Sol.

$$\frac{kA(100-\theta)}{l} = \frac{4kA(\theta-0)}{l}$$

$$\Rightarrow \quad 100 = 5\theta$$

$$\Rightarrow \quad \theta = 20^{o}C$$



15. It takes 6 h to grow ice from 0 to 1 cm on a lake when atmospheric temperature is $-10^{o}C$. The time taken to grow thickness of ice from 1 cm to 2 cm is

a) 6 h b) 12 h c) 18 h d) 24 h

15. c

CONCEPT CODE : P111207

Sol. $t_1 : t_2 = y_2^2 - y_1^2 : y_3^2 - y_2^2$

$= 1^2 - 0^2 : 2^2 - 1^2 = 1:3$

$\Rightarrow \quad t_2 = 3t_1 = 18h$

16. The ratio of lengths of two rods is 1 : 2 and the ratio of coefficient of expansions is 2 : 3. The first rod is heated through $60^{o}C$. Find the temperature through which the second rod is to be heated so that its expansion is twice that of first is

a) $60^{o}C$ b) $40^{o}C$ c) $30^{o}C$ d) $10^{o}C$

16. b

CONCEPT CODE : P111201

Sol. $\ell_2^1 = 2\ell_1^1$

$\ell_2 = (1 + \alpha_2 \Delta t) = 2\ell_1(1 + \alpha_1 \Delta t)$

$3 \cdot \Delta t = 2.60$

$\Delta t = 40^{o}C$

17. A wire of length 60 cm is bent into a circle with a gap of 1 cm at its ends. On heating it by $100^{o}C$, the length of the gap increases to 1.02 cm. α of material of wire is

a) $2 \times 10^{-4}/^{o}C$ b) $4 \times 10^{-4}/^{o}C$ c) $6 \times 10^{-4}/^{o}C$ d) $1 \times 10^{-4}/^{o}C$

17. a

CONCEPT CODE : P111201

Sol. $\alpha = \frac{\Delta\ell}{\ell \Delta t} = \frac{0.02}{1 \times 10^{+2}}$

$\alpha = 2 \times 10^{-4}/^{o}C$

18. A metal metre scale gives correct measurement at $0^{o}C$. It is generally used at a temperature of $40^{o}C$. The correction to be made for every metre is ($\alpha = 10^{-6}/1^{o}C$)

a) 4×10^{-5} m b) 4×10^{-5} m to be added

c) 4×10^{-5} m must be deducted d) None of the above

18. b

CONCEPT CODE : P111201

Sol. $\Delta\ell = \ell\,\alpha\,\Delta t$

$= 1 \times 10^{-6} \times 40$

$= 4\times10^{-5}$ m.

19. A metal rod has a length of 1 m at $30^{\circ}C$. α of metal is $2.5\times10^{-5}/^{\circ}C$. The temperature at which it will be shortened by 1 mm is

a) $-30^{\circ}C$ b) $-40^{\circ}C$ c) $-10^{\circ}C$ d) $10^{\circ}C$

19. c

CONCEPT CODE : P111201

Sol. $\alpha = \dfrac{\Delta\ell}{\ell\Delta t}$

$2.5\times10^{-5} = \dfrac{-1\times10^{-3}}{1\times(t-30)}$

$t-30 = -40$

$t = -10^{\circ}C$

20. Density of a substance at $0^{\circ}C$ is 10.6 gm/c.c. and at $100^{\circ}C$ is 10 gm/c.c. coefficient of linear expansion of solid is

a) $0.0006/^{\circ}C$ b) $0.0004/^{\circ}C$ c) $0.0003/^{\circ}C$ d) $0.0002/^{\circ}C$

20. d

CONCEPT CODE : P111201

Sol. $\gamma = \dfrac{d_0 - d_t}{d_t(\Delta t)}$

$\gamma = \dfrac{10.6-10}{10\times100}$

$\gamma = 0.6\times10^{-3}\ /^{\circ}C$

$\gamma = 0.0006$

$\alpha = 0.0002\ /^{\circ}C$

PART - D

NUMERICAL ANSWER TYPE:

1. Three metal rods of same lengths and same area of cross section having conductivities 1, 2, 6 units are connected in series. Then their effective conductivity will be *x* units, the value of *x* is

1. 1.80

CONCEPT CODE : P111207

Sol. $\frac{3}{x}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}$

$\frac{3}{x}=\frac{1}{1}+\frac{1}{2}+\frac{1}{6}$ $\qquad x=\frac{18}{10}=1.8$

2. A body takes 8 minutes to cool from $90^{\circ}C$ to $80^{\circ}C$ in a surrounding of temperature $25^{\circ}C$. The time taken by it to cool from $80^{\circ}C$ to $70^{\circ}C$ in the same surroundings is in minutes

2. 9.60

CONCEPT CODE : P111208

Sol. $\frac{\theta_1-\theta_2}{t}=k\left(\frac{\theta_1+\theta_2}{2}-\theta_3\right)$

3. If the co-efficient of cubical expansion is x times co-efficient of superficial expansion then the value of x is

3. 1.50

CONCEPT CODE : P111201

Sol. Cubical expansion $=x$. Superficial expansion

$\gamma=x\cdot\beta$

$3\alpha=x\cdot 2\alpha \Rightarrow x=\frac{3}{2}=1.50$

4. Up on heating, the length of the side of a cube changes by 2%. The volume of cube changes by x %, the value of x is

4. 6.00

CONCEPT CODE : P111201

Sol. $\frac{\Delta V}{V}\times 100 = 3\cdot\frac{4\ell}{\ell}\times 100$

$= 3\times 2\% = 6\%$

5. In steady state condition, the temperatures at the two ends of a metal rod of length 25 cm are $100^{\circ}C$ and $0^{\circ}C$. Then temperature at a point 8 cm from the hot end is __ $^{\circ}C$

5. 68.00

CONCEPT CODE : P111207

Sol. $\frac{100-\theta}{8}=\frac{\theta}{17}$

Chemistry

PART - A

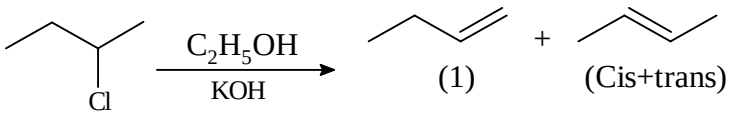
SINGLE ANSWER CORRECT:

1. How many different alkenes are formed when 2 – chlorobutane is treated with ethanolic solution of KOH ?

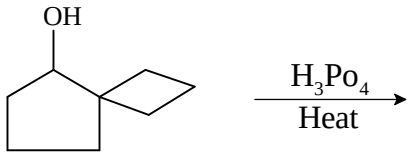
a) 1 b) 2 c) 3 d) 4

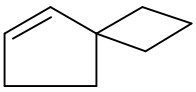
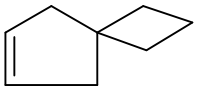
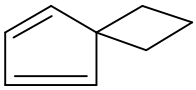
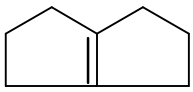
1. c

Concept code: C111705

Sol: 

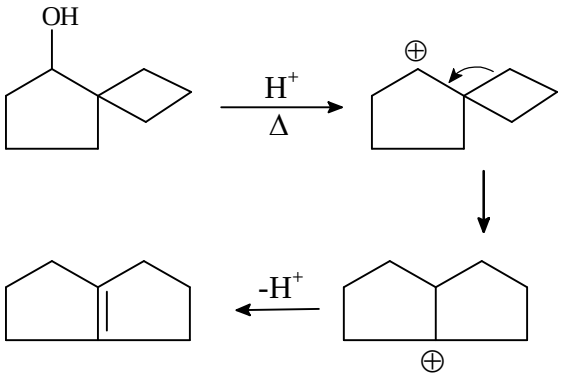
2. What is the major dehydration product for the following reaction :



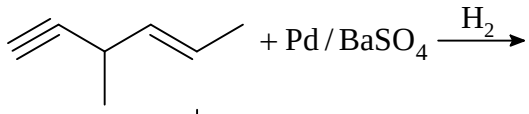
a)  b)  c)  d) 

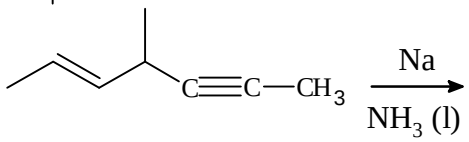
2. d

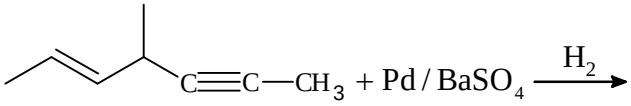
Concept code: C111705

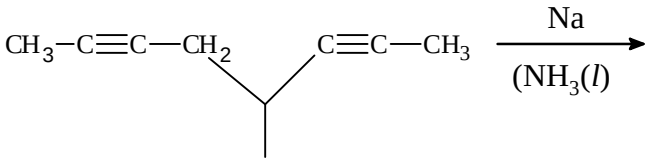
Sol: 

3. Which of the following compounds will lose optical activity after the reaction.

a) 

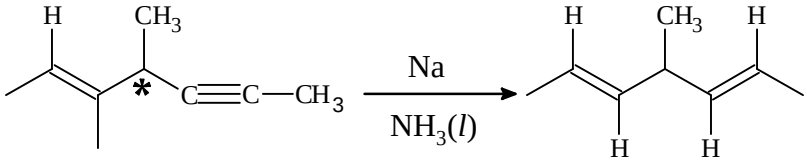
b) 

c) 

d) 

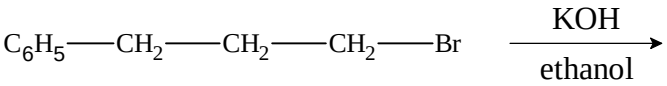
3. b

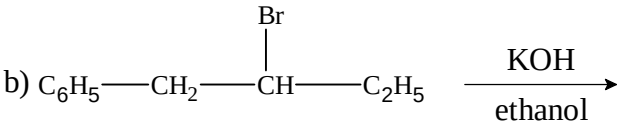
Concept code: C111705

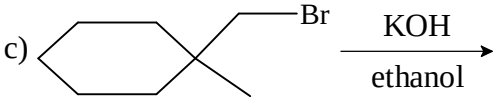
Sol: 

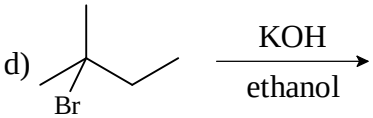
trans-trans optically inactive

4. In which of the following reactions only single isomer of alkene is formed ?

a) 

b) 

c) 

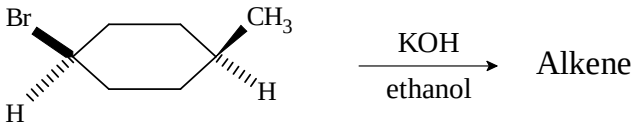
d) 

4. a

Concept code: C111705

Sol: It has only one type of $\beta-H$ and the product alkene does not have stereo isomers

5. Consider the following reaction

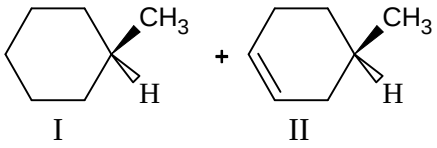


The correct statement concerning product of the above reaction is
a) only single alkene is formed
b) a pair of geometrical isomers are formed
c) a pair of enantiomers in equal amounts is formed
d) a pair of diastereomers in equal amount is formed

5. c

Concept code: C111705

Sol:



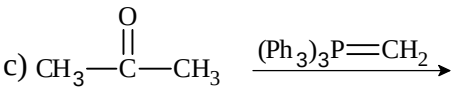
I II

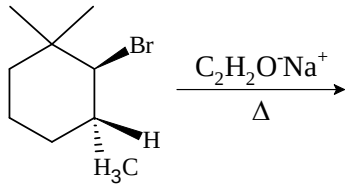
(I) and (II) are enantiomers formed in equal amounts

6. Which of the following reactions cannot produce an alkene

a) $CH_3—CH_2—OH \xrightarrow[\Delta]{\text{Conc. } H_2SO_4}$

b) $CH_3—CH_2—OH \xrightarrow[\Delta]{Al_2O_3}$

c) 

d) 

6. d

Concept code: C111705

Sol: Antielimination is favourable in E_2.

7. What is the major product of the reaction given below : ?

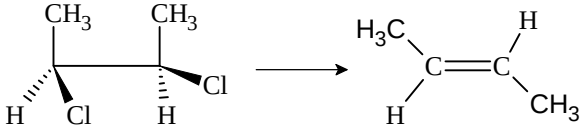
meso – 2, 3 – dichlorobutane $+NaI_{(aq)} \xrightarrow{\text{acetone}}$

a) cis – 2 – butene b) 1, 3 – butadiene c) trans – 2 – butene d) 1 – butene

7. c

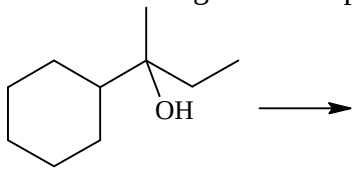
Concept code: C111705

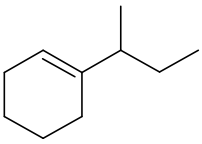
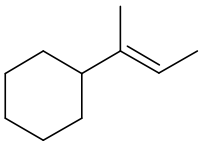
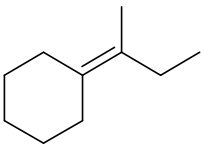
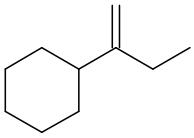
Sol: Anti dehalogenation.



Trans-2-butene

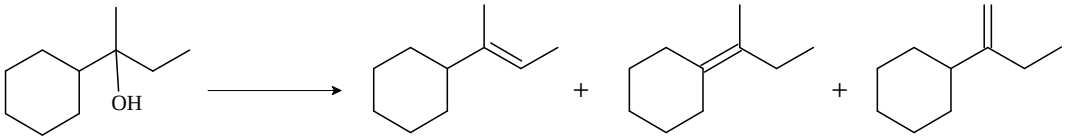
8. Which of the following is not the product of dehydration of



a)  b)  c)  d) 

8. a

Concept code: C111705

Sol: 

Possible products.

9. In the reaction below X is

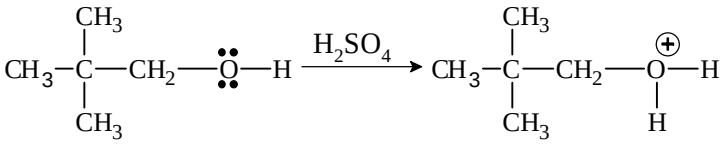
Neopentyl alcohol $\xrightarrow{H_2SO_4}$ X

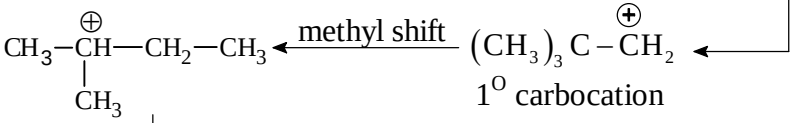
a) 2 – methyl pentane
b) 2 – methyl pent – 2 – ene
c) 2 – methylbut – 2 – ene
d) Neopentane

9. c

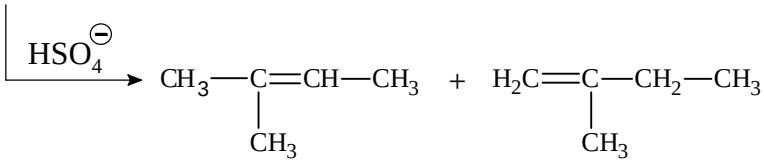
Concept code: C111705

Sol:





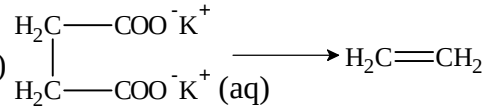
3O carbocation

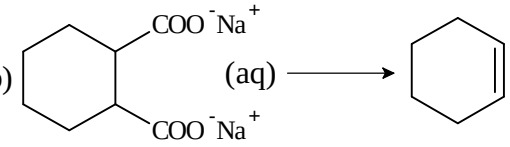


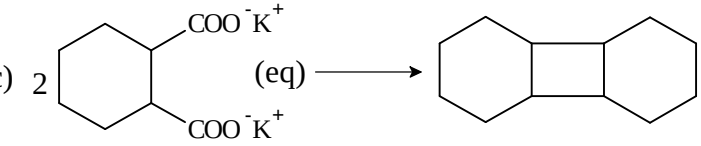
(major) (minor)

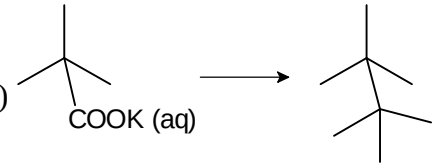
2 - methyl But-2-ene

10. Which of the following reactions is incorrect for Kolbe's electrolysis ?

a) 

b) 

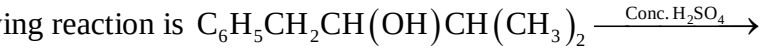
c) 

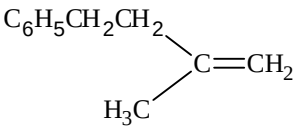
d) 

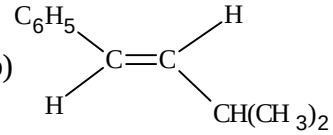
10. d

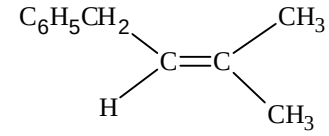
Concept code: C111705

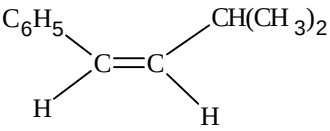
Sol: Sterically crowded free radical is formed.

11. The main product of the following reaction is 

a) 

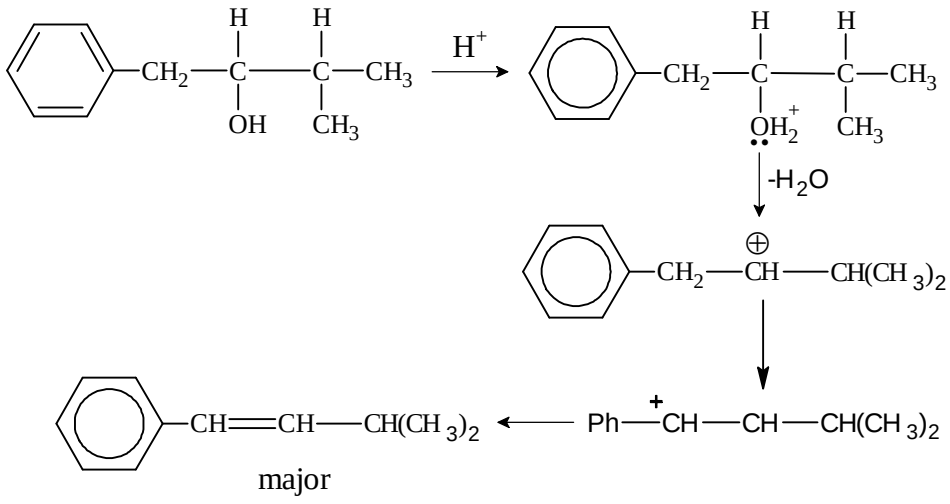
b) 

c) 

d) 

11. b

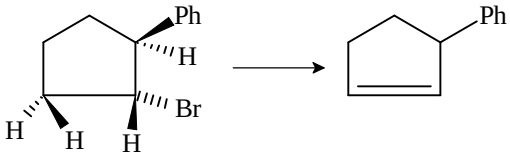
Concept code: C111705

Sol: 

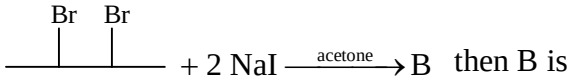
12. Trans 2 – Phenyl – 1 – bromo cyclopentane on reaction with alcoholic KOH produces.
a) 4 – phyenyl cyclopentene b) 2 – phenyl cyclopentene
c) 1 – phenyl cyclopentene d) 3 – phenyl cyclopentene

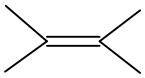
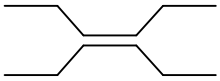
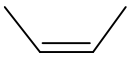
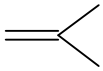
12. d

Concept code: C111705

Sol: 

3 – phenyl cyclopentene

13. 

then B is
a)  b)  c)  d) 

13. c

Concept code: C111705

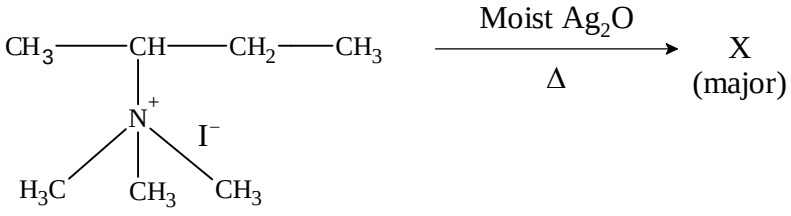
Sol: Debromination

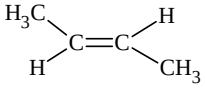
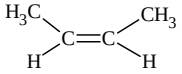
14. Which of the following statements is incorrect ?
a) $3^{o}\,R-X > 2^{o}\,R-X > 1^{\circ}(R-X)$ $[E_1$ (or) $E_2]$
b) rate $\propto [R-X]$ (Unimolecular elimination)
c) rate $\propto [R-X][OH^-]$ (Bimolecular elimination)
d) $I^- < Br^- < Cl^-$ (Leaving ability)

14. D

Concept code: C111705

Sol: Leaving ability $I^- > Br^- > Cl^-$

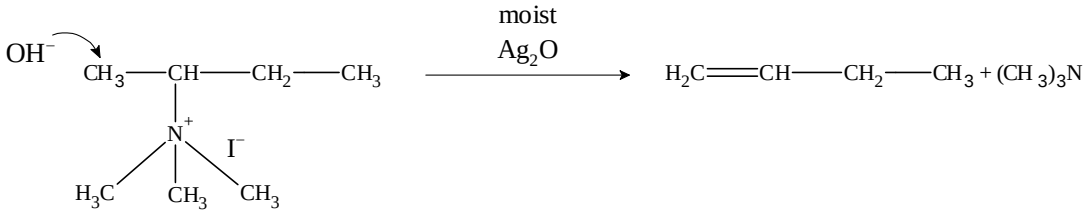
15. 

a) $H_2C{=}CH_2$ b)  c)  d) $CH_2=CH-CH_2-CH_3$

15. d

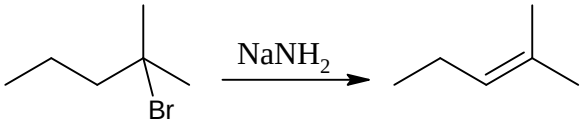
Concept code: C111705

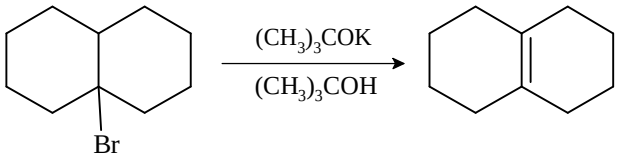
Sol:

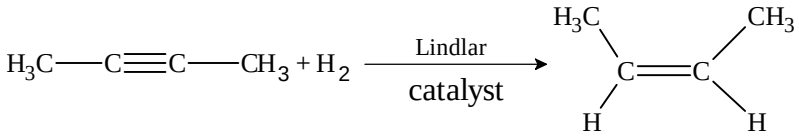


16. Which of the following reaction is incorrect ?

a) $CH_3 - CHBr_2 + 2Zn + Br_2CH - CH_3 \rightarrow CH_3 - CH = CH - CH_3 + 2ZnBr_2$

b) 

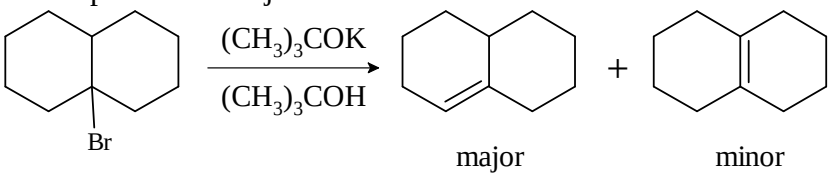
c) 

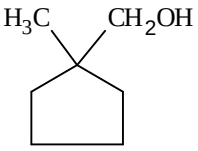
d) 

16. c

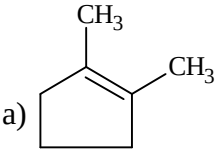
Concept code: C111705

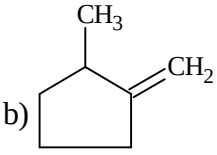
Sol: Hoffman product is major.

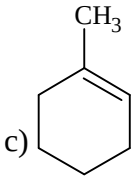


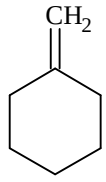
17. The major product obtained in acid catalyzed dehydration 

is

a) 

b) 

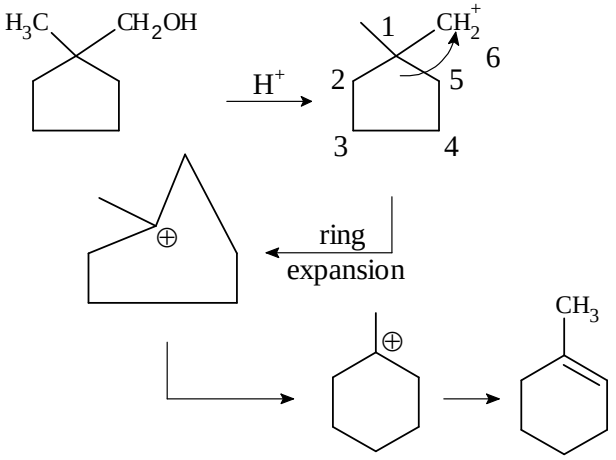
c) 

d) 

17. c

Concept code: C111705

Sol:



18. Which of the following reaction cannot produce propene as one of the important organic product ?

a) $H_3C-\overset{O}{\overset{\|}{C}}-CH_3 + Zn(Hg) \xrightarrow{HCl}$

b) $H_3C-CH_2-CH_2-\overset{\oplus}{N}(CH_3)_3Br^- \xrightarrow[\Delta]{AgOH}$

c) $H_3C-\underset{\underset{COOK}{|}}{CH}-CH_2-COOK(aq) \xrightarrow{Electrolysis}$

d) $H_3C-CHO + (C_6H_5)_3P=CH_2 \xrightarrow{\Delta}$

18. a

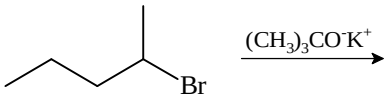
Concept code: C111705

Sol: $H_3C-\overset{O}{\overset{\|}{C}}-CH_3 + Zn(Hg) \xrightarrow{HCl} H_3C-CH_2-CH_3$

19. Which of the following reaction produce saytzeff product as major ?

a) $H_3C-\underset{\underset{\overset{+}{S}(CH_3)(CH_3)}{|}}{CH}-CH-CH_3 \xrightarrow{OH^-}$

b) $H_3C-\underset{\underset{CH_3}{|}}{\overset{CH_3}{\overset{|}{C}}}-CH_2-\underset{Br}{\underset{|}{CH}}-CH_3 \xrightarrow[\Delta]{Alc\ KOH}$

c)  $\xrightarrow{(CH_3)_3CO^-K^+}$

d) $H_3C-CH_2-CH_2-Br \xrightarrow{C_2H_5O^-Na^+}$

19. d

Concept code: C111705

Sol: $CH_3-CH_2-CH_2-Br \xrightarrow{C_2H_5O^-Na^+} CH_3-CH=CH_2$

20. Correct statement for E_2 reaction is

a) It is two step process
b) Rearrangement is possible
c) Weak base is favourable
d) It is one step concerted process

20. D

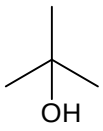
Concept code: C111705

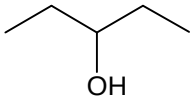
Sol: It is one step concerted process

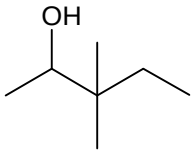
PART - D

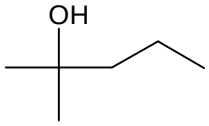
NUMERICAL ANSWER TYPE:

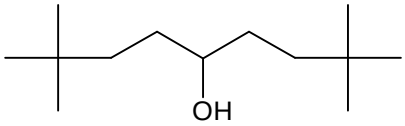
1. How many of the following alcohols can undergo dehydration faster than C_2H_5OH .

(a) 

(b) 

(c) 

(d) 

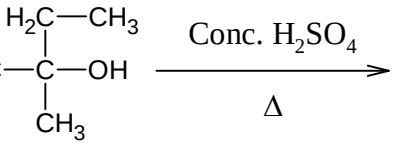
(e) 

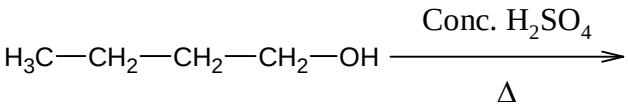
1. 5.00

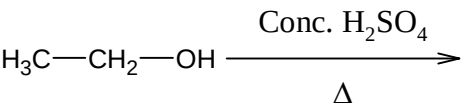
Concept code: C111705

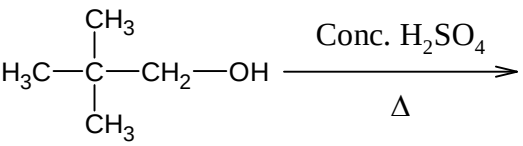
Sol: All react faster.

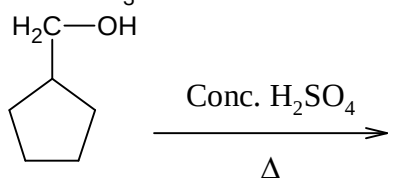
2. How many of the following reactions are produce alkene ?

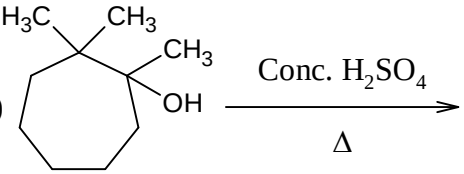
1) 

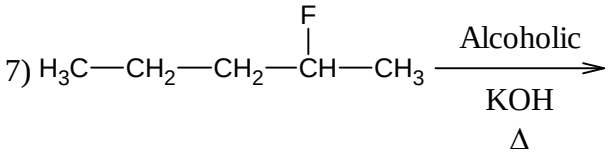
2) 

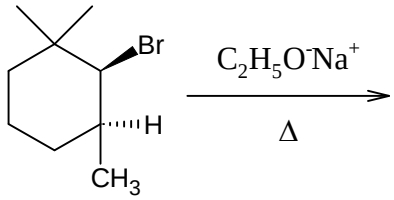
3) 

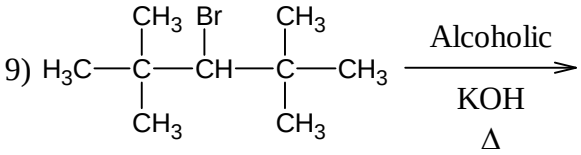
4) 

5) 

6) 

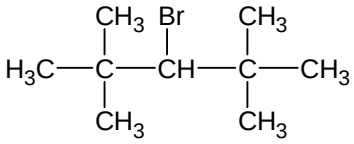
7) 

8) 

9) 

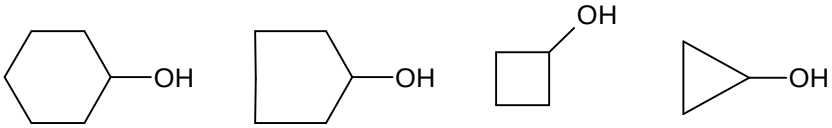
2. 8.00

Concept code: C111705

Sol: 

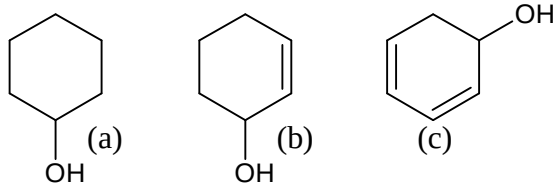
NO – β – Hydrogen

3. How many of the following statements is/are correct ?

a) 

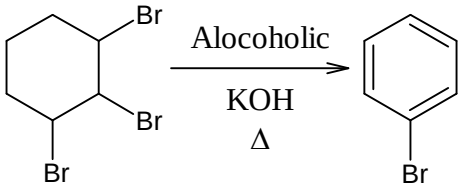
(P) (Q) (R) (S)

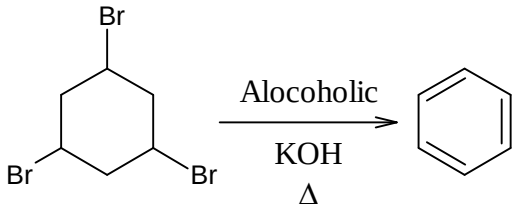
P > Q > R > S [Dehydration rate]

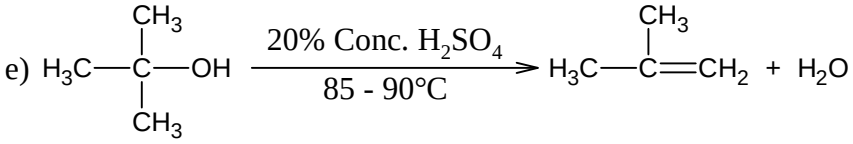
b) 

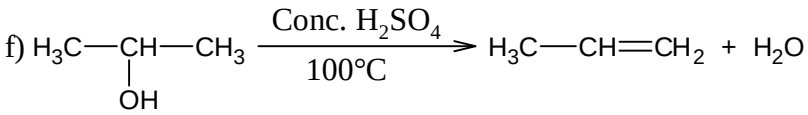
(a) (b) (c)

c < b < a

c) 

d) 

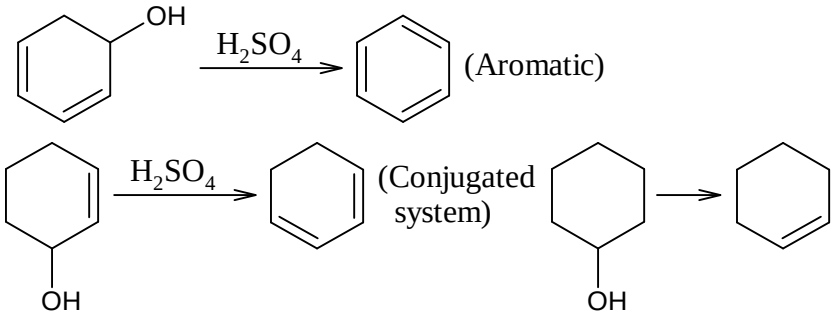
e) 

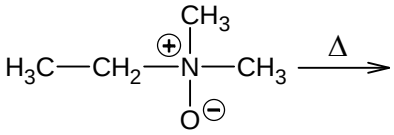
f) 

3. 4.00

Concept code: C111705

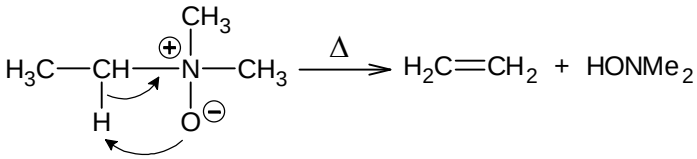
Sol: a, d, e, f are correct.
c, b are incorrect



4.  X is alkene. Find $Y=\frac{M.W\ of\ X}{7}+11.28$.

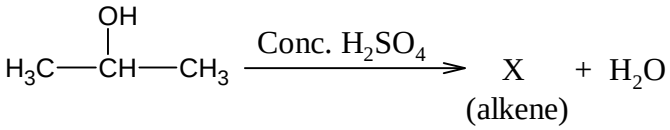
4. 15.28

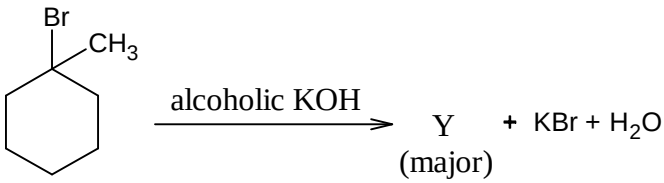
Concept code: C111705

Sol: 

$M.W=28$

$Y=\frac{28}{7}+11.28=15.28$

5. 

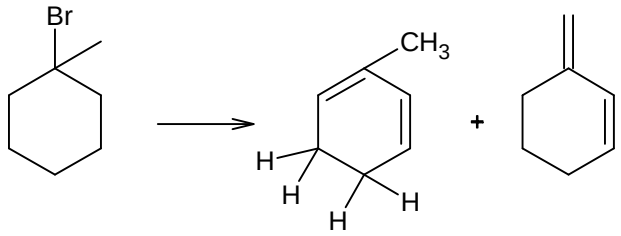


No. of αH hydrogen in X (major) + No. of α hydrogen in Y (major) + 0.58

5. 10.58

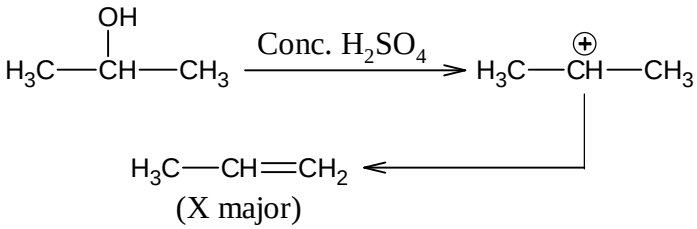
Concept code: C111705

Sol:



Y
(major)

$7\ \alpha\ H$



(X major)

$3\ \alpha\ H$

$0.58 + X(\alpha H) + Y(\alpha H) = 0.58 + 7 + 3 = 10.58$