Fluids

Fluid:

→ A Substance which can flow (liquid / gases) is known as fluid

→ Study of fluids at rest wrt container is called Fluid statics

Density:

→ Mass per unit volume is called density

$$\boxed{\rho = \frac{\text{mass}}{\text{Volume}}}$$

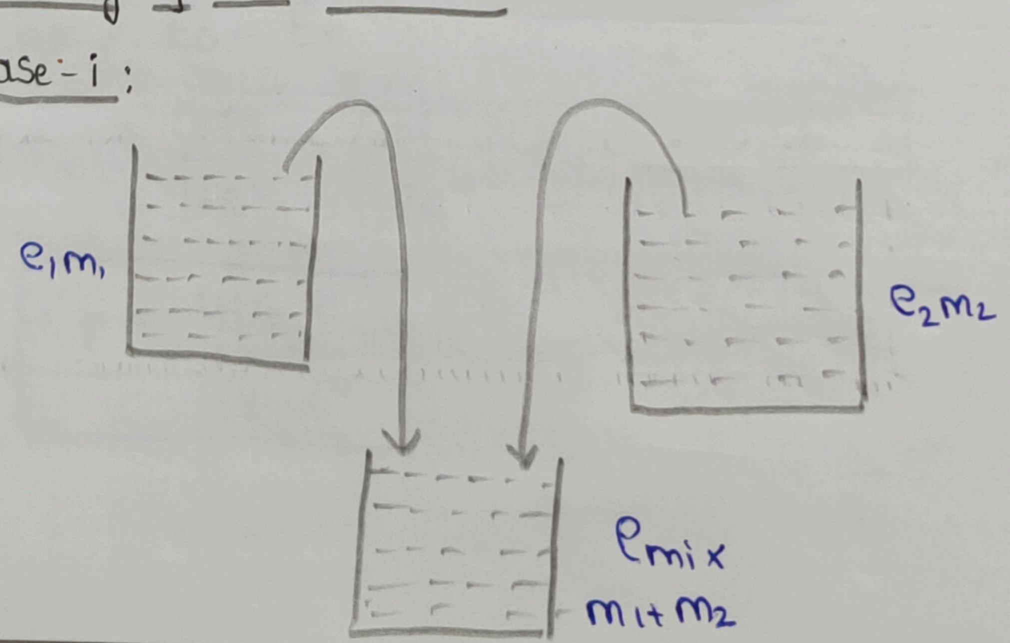
→ S.I unit of density is kg/m^3

⇒ $1\,g/cc = 1000\,kg/m^3$

→ $\rho_{water} = 1\,g/cc$

Density of the mixture:

Case - i :



$$\rho_{mix} = \frac{\text{Total mass}}{\text{Total volume}}$$

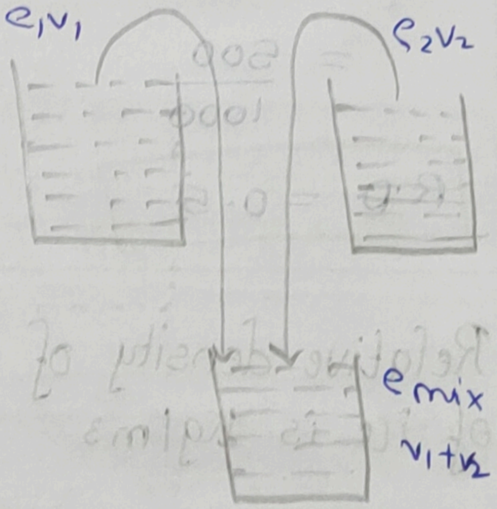
$$\Rightarrow \boxed{\rho_{mix} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}}$$

Note: If $m_1 = m_2 = m$, then.

$$\boxed{\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}}$$

Case-ii :

$$\boxed{\rho_{mix} = \frac{\rho_1 V_1 + \rho_2 V_2}{\rho_1 + \rho_2}}$$



Note: If $V_1 = V_2 = V$

$$\boxed{\rho_{mix} = \frac{\rho_1 + \rho_2}{2}}$$

Relative density:

→ The ratio of density of the substance and density of the water at 4°c is called Relative density of the substance.

$$\boxed{R.d = \frac{\text{Density of the substance}}{\text{Density of water at } 4°c}}$$

→ It's a dimensionless quantity.

→ Since density of water is 1g/cc; relative density is numerically equal to density of the substance in cgs units.

Ex: Relative density of mercury is 13.6 therefore density of mercury is 13.6 g/cc

(Q) Density of wood is 500kg/m³. Find relative density of the wood.

Sol

$$R.D = \frac{\rho_{wood}}{\rho_{water}}$$

$$= \frac{500}{1000}$$

$$\boxed{R.D = 0.5}$$

(Q) Relative density of ice is 0.9. Find density of ice is kg/m3

Sol

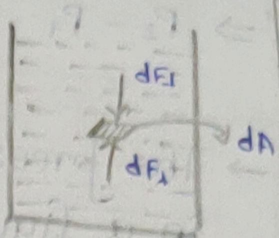
$$0.9 = \frac{\rho_{ice}}{1000}$$

$$\Rightarrow \boxed{\rho_{ice} = 900\ kg/m^3}$$

Pressure in a fluid:

→ When fluid is at rest, it exerts a force perpendicular to any surface in contact with it. Such as a container wall or body immersed in a fluid

$$\boxed{P = \frac{dF_\perp}{dA}}$$



If pressure is uniform at all points at a finite plane surface of area A then pressure on the surface of the fluid is given by:

$$\boxed{P = \frac{F_\perp}{A}}$$

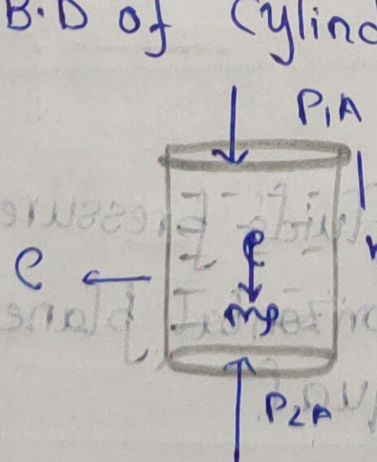
→ S.I unit of pressure is N/m^2 or pascal (Pa)

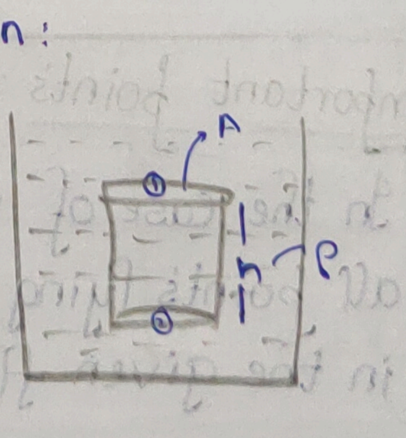
→ $P_{atm} = 1.01 \times 10^5\, Pa \simeq 10^5\, Pa$

Variation of pressure in a fluid due to depth:

Consider a cylindrical column (area A, height h) in a liquid of density ρ. The pressure difference b/w the bottom and top layer of the cylindrical column is given by

F.B.D of cylindrical column:





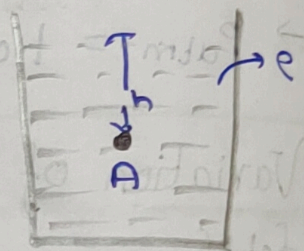
$P_1A + mg = P_2A$

$\Rightarrow P_1\cancel{A} + \rho \cancel{A} hg = P_2\cancel{A}$

$\Rightarrow \boxed{P_2 - P_1 = \rho g h}$

→ The pressure difference b/w two points in a liquid depends on density of the liquid, height difference b/w the two points and accelaration due to gravity.

→ From the above equation we can conclude that the pressure at a point inside the liquid at point A is given by is given by



$$\boxed{P_A = P_0 + h\rho g}$$

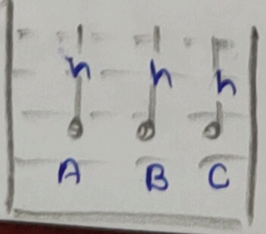
(or)

$$\boxed{P_A = P_{atm} + h\rho g}$$

P_0 - atmoshpheric pressure
P_A - pressure at point A
h - depth of poin A from the surface.
g - accelaration due to gravity.

Important points:

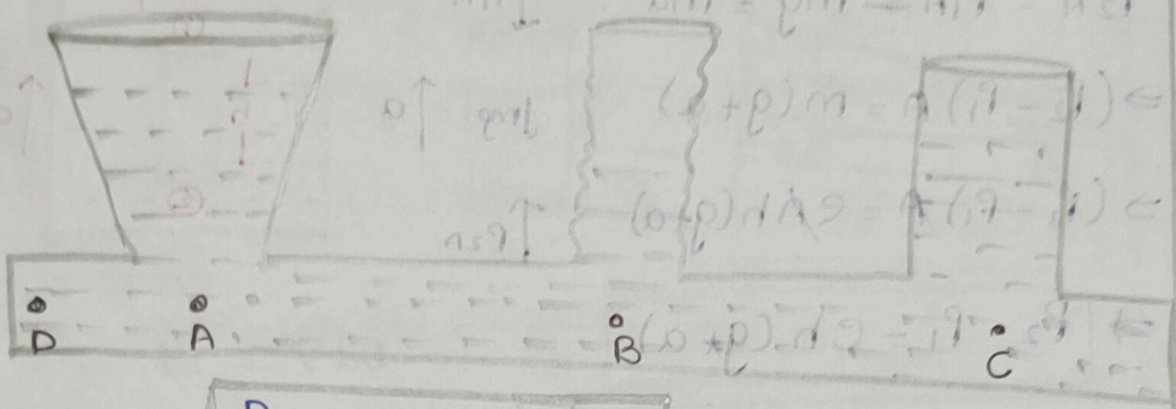
① → In the case of stationary fluids pressure at all points lying on same horizontal plane in the given fluid are equal.



$$P_A = P_B = P_C$$

② Hydro static paradox :

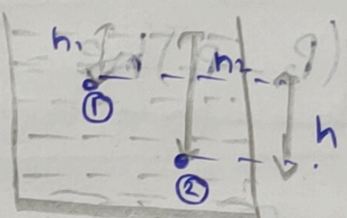
The pressure at a point in the fluid doesnot depend on shape of the container on amount of fluid aboave the point.



$$\boxed{P_A = P_B = P_C = P_D}$$

③ The pressure difference b/w two points depends on the height (vertical) b/w two points.

$$\boxed{P_2 - P_1 = \rho g h}$$

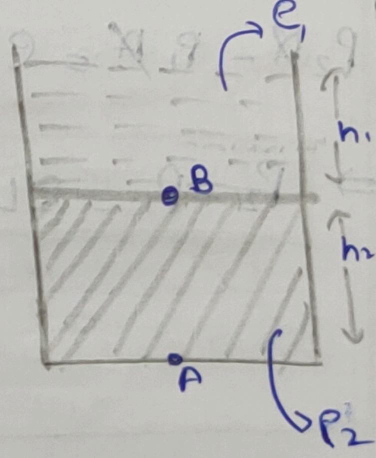


④ Pressure ~~due~~ at a point in two immesible liquids

$P_B = P_0 + h_1 \rho_1 g$

$P_A - P_B = h_2 \rho_2 g$

$\Rightarrow \boxed{P_A = P_0 + h_1 \rho_1 g + h_2 \rho_2 g}$



Variation of pressure in accelarating Container :

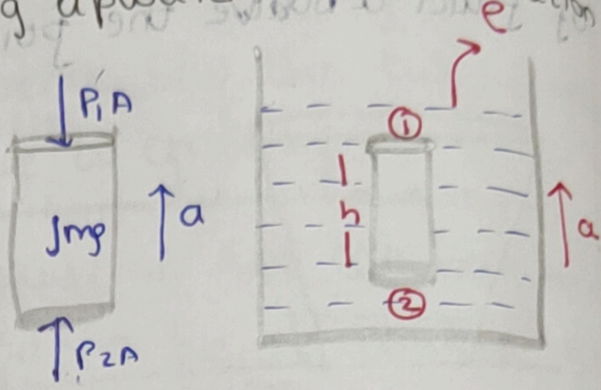
Case-I: Container moving upward with accelaration

$$P_2A - P_1A - mg = ma$$

$$\Rightarrow (P_2 - P_1)A = m(g+a)$$

$$\Rightarrow (P_2 - P_1)A = \rho A h(g+a)$$

$$\Rightarrow \boxed{P_2 - P_1 = \rho h(g+a)}$$

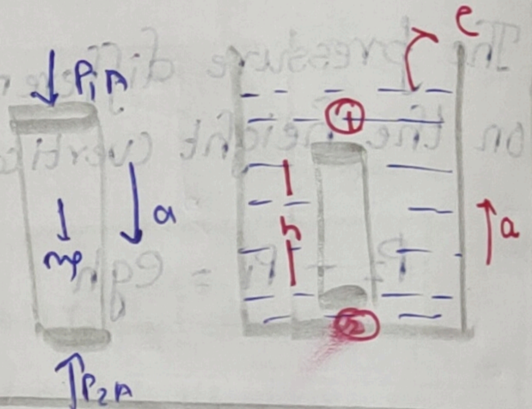


Case-II: Container accelarating downward

$$P_1A + mg - P_2A = ma$$

$$\Rightarrow (P_2 - P_1)A = m(g-a)$$

$$\Rightarrow \boxed{(P_2 - P_1)\cancel{h\rho} = h\rho(g-a)}$$

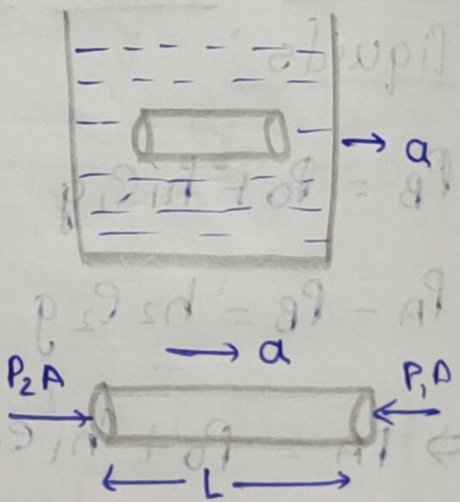


Case-III: Container accelarating horizontally.

$$P_2A - P_1A = ma$$

$$P_2\cancel{A} - P_1\cancel{A} = \rho\cancel{A}La$$

$$\Rightarrow \boxed{P_2 - P_1 = \rho La}$$



Case-IV :

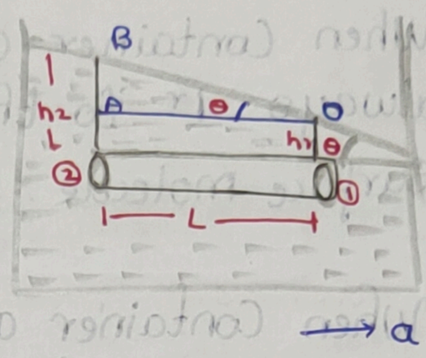
In case of container accelarating horizontally the pressure due to fluid on points lying on same horizontal surface is different.

Method-1:

$P_2 = P_0 + h_2 \rho g \longrightarrow ①$

$P_1 = P_0 + h_1 \rho g \longrightarrow ②$

$P_2 - P_1 = \rho g (h_2 - h_1) \longrightarrow ③$



① and ②

$P_2 - P_1 = \rho L a \longrightarrow ④$

From ③ and ④

$\cancel{\rho} g (h_2 - h_1) = \cancel{\rho} L a$

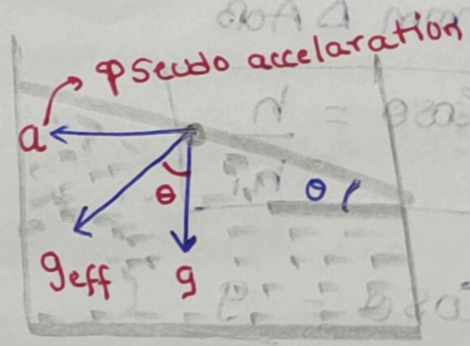
$$\boxed{\frac{a}{g} = \frac{h_2 - h_1}{L}}$$

From ΔOAB

$\tan\theta = \dfrac{h_2 - h_1}{L}$

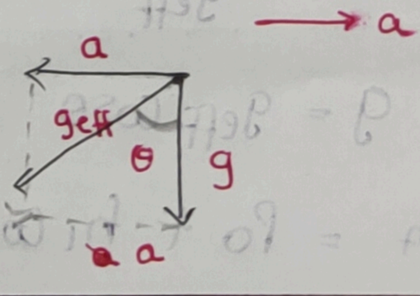
$$\Rightarrow \boxed{\frac{a}{g} = \tan\theta}$$

Method-2 :



$g_{eff} = \sqrt{g^2 + a^2}$

$$\Rightarrow \boxed{\tan\theta = \frac{a}{g}}$$



Note:

① When Container accelarates free surface of the liquid always $\perp^r$ to g_{eff}

(or)

When Container accelarates a fluid surface always $\perp^r$ to the net force acting on the surface molecule.

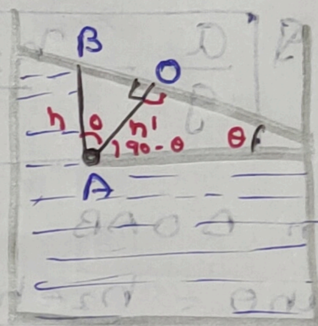
② When Container accelarates along horizontal direction the pressure at a point inside the fluid is given by

$P_A = P_0 + h\rho g_{eff}$

$$\boxed{P_A = P_0 + h\rho\sqrt{g^2+a^2}}$$

(or)

$$\boxed{P_A = P_0 + h'\rho g}$$

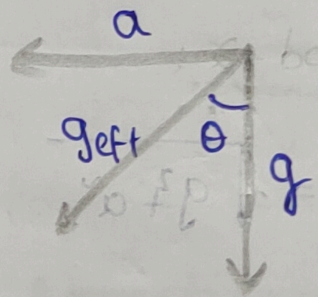


Proof:

$P_A = P_0 + h\rho\sqrt{g^2+a^2}$

From ΔAOB

$$\boxed{\cos\theta = \frac{h}{h_1}}$$



$\cos\theta = \frac{g}{g_{eff}}$

$\Rightarrow g = g_{eff}\cos\theta$ and $h_1\cos\theta = h$

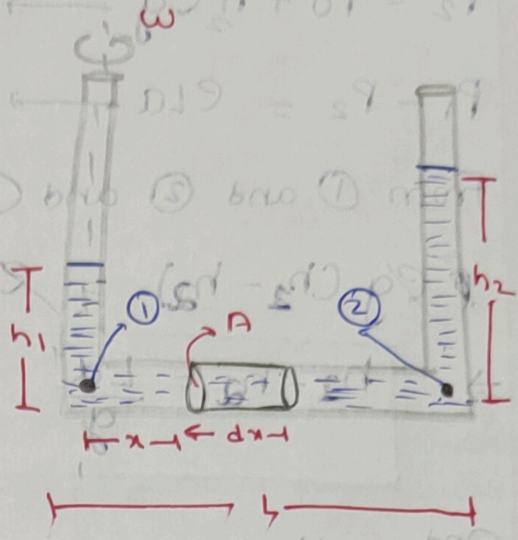
$P_A = P_0 + h_1\cos\theta\,\rho\, g_{eff}$

$$\boxed{P_A = P_0 + h_1 \rho g}$$

Case-V :

$dP = \rho\, dx\, a$

$$\int_{P_1}^{P_2} dP = \int_0^L \rho\, dx\, x\omega^2$$



$$\Rightarrow P_2 - P_1 = \rho\omega^2 \left[\frac{x^2}{2}\right]_0^L$$

$$\Rightarrow P_2 - P_1 = \frac{\rho\omega^2 L^2}{2} \longrightarrow (4)$$

$P_1 = P_0 + h_1 \rho g \longrightarrow (1)$

$P_2 = P_0 + h_2 \rho g \longrightarrow (2)$

From (1) and (2)

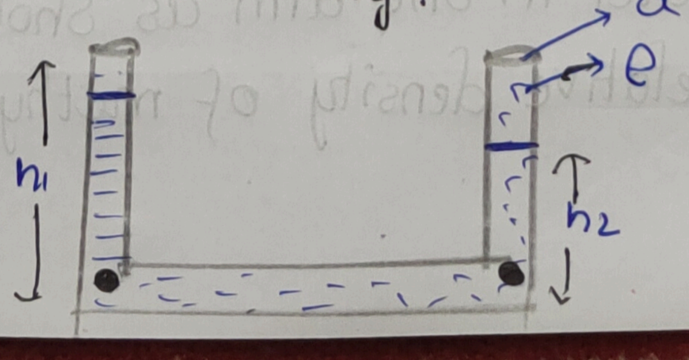
$P_2 - P_1 = \rho g (h_2 - h_1) \longrightarrow (3)$

From (3) and (4)

$$\Rightarrow \cancel{\rho} g (h_2 - h_1) = \frac{\cancel{\rho}\omega^2 L^2}{2}$$

$$\Rightarrow \boxed{h_2 - h_1 = \frac{\omega^2 L^2}{2g}}$$

(Q) Find the height difference of the liquid in the two limbs of U-tube when it is accelarated horizontally.



Sol $P_1 = P_0 + h_1 \rho g \longrightarrow$ (1)

$P_2 = P_0 + h_2 \rho g \longrightarrow$ (2)

$P_1 - P_2 = \rho L a \longrightarrow$ (3)

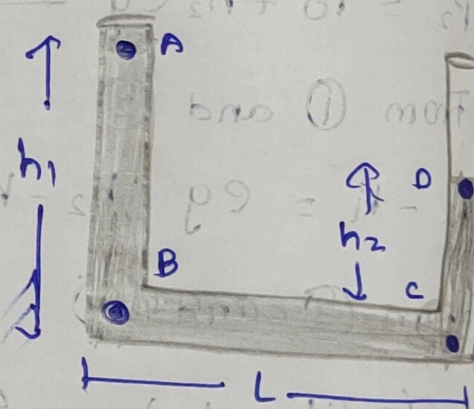
From (1) and (2) and (3)

$\cancel{\rho} g (h_1 - h_2) = \cancel{\rho} L a$

$$\Rightarrow \boxed{h_1 - h_2 = \frac{La}{g}}$$

IInd method;

Write the pressure equation from A to B and C to D



$P_A + h_1 \rho g - \rho L a - \rho h_2 g = P_D$

$P_A = P_D = P_0$

$\Rightarrow \cancel{P_0} + h_1 \rho g - \rho L a - h_2 \rho g = \cancel{P_0}$

$\Rightarrow h_1 - h_2 = \frac{La}{g}$

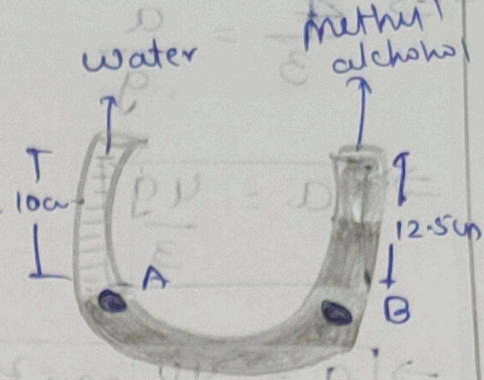
(Q) A utube contains water and methyl alcohol separated by mecury. The mecury columns in the two arms at the same level with 10cm of water in one arm as shown in fig. Find relative density of methyl alcohol.

Sol $P_A = P_B$

$\cancel{P_0} + h_w \rho_w g = \cancel{P_0} + h_a \rho_a g$

$\Rightarrow 10(1) = 12.5\, \rho_a$

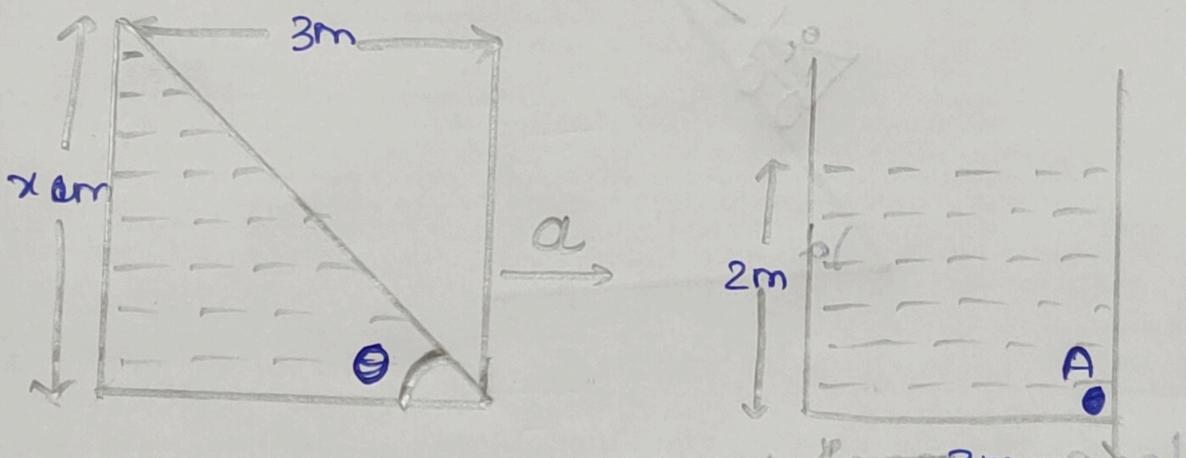
$\Rightarrow \rho_a = 0.8 g/c.c$



Relative density (R.d) = 0.8

(Q) The container shown in the figure, height of the liquid in it is 2m, and length of container 3m, with what minimum accelaration container should accelarate horizontally that A is exposed to atmo-sphere.

Sol



Area's are equal in both cases

Hence

$3 \times 2 = \frac{1}{2} \times x \times 3$

$\Rightarrow \boxed{x = 4m}$

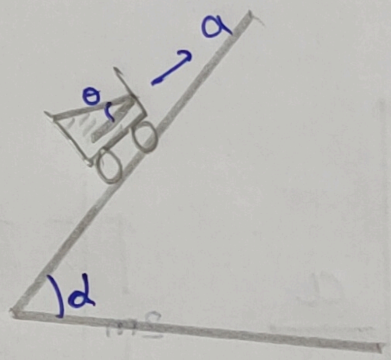
$\tan\theta = \frac{a}{g}$

$\Rightarrow \frac{\cancel{4}}{3} = \frac{a}{g}$

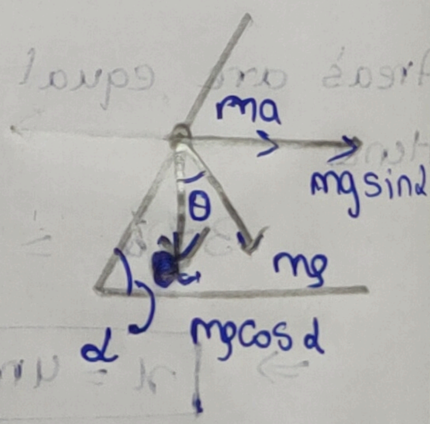
$\Rightarrow \boxed{a = \frac{4g}{3}}$

$\Rightarrow \boxed{a = \frac{40}{3}\, ms^{-2}}$

(Q) A fluid container containing a is accelarating ~~a afor~~ with accelaration a along the liquid of density ρ as shown in the figure. Find the angle of inclination of free surface.



$\tan\theta = \frac{mg\sin\alpha + ma}{mg\cos\alpha}$



$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{a + g\sin\alpha}{g\cos\alpha}\right)}$