FLUIDS

→ Fluid : A substance which can flow (liquid or gases) is called fluid

→ Study of fluids at rest with respect to container is called fluid statics

★ Density : Mass per unit volume is called density

$$\text{density } (\rho) = \frac{\text{Mass}}{\text{Volume}}$$

→ S.I unit of density is kg/m^3

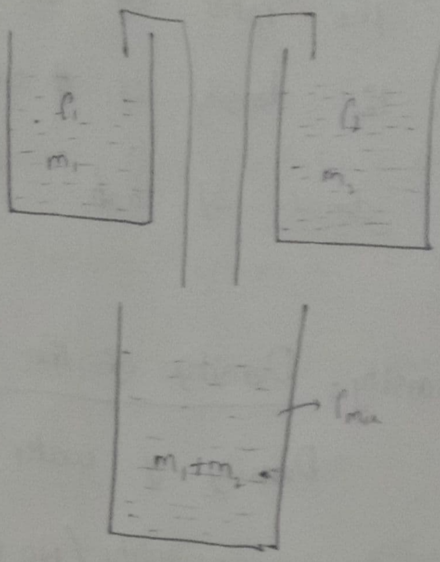
→ $1 g/cc = 1000 kg/m^3$

→ $\rho_{water} = 1 g/cc$

★ Density of a mixture :

Case (i) : If two containers A of density ρ_1 and mass m_1 and container B of density ρ_2 and mass m_2 are combined then the resultant density is given by

$$\rho_{mix} = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{\frac{\rho_1}{m_1} + \frac{\rho_2}{m_2}}$$

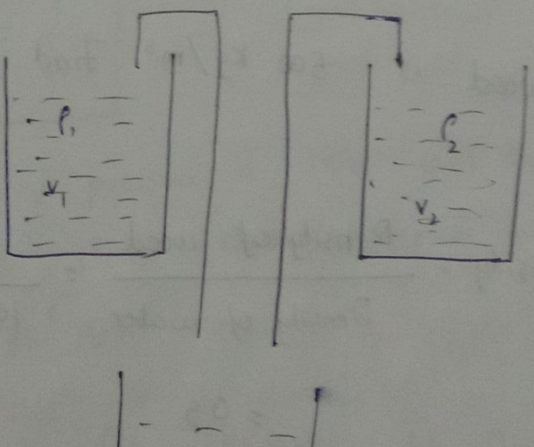


~~Case~~ If $m_1 = m_2 = m$

then $\rho_{mix} = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

Case ii) : If two containers A of density ρ_1 and volume V_1 and container B of density ρ_2 and ~~mass~~ volume V_2 are mixed, then the resultant density is given by.

$$\rho_{mix} = \frac{\text{Total mass}}{\text{Total volume}} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$



If $V_1 = V_2 = V$

$$\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$$

* Relative Density: The ratio of the density of the substance to the density of water at 4°c is called relative density of that substance.

$$\text{Relative density} = \frac{\text{Density of the substance}}{\text{Density of water at } 4°c}$$

→ It is a dimentionless quantity (No units)

→ Since, density of water is 1 g/cc. Relative density is numerically equal to the density of the substance in CGS units

Q) Relative density of the Mercury is 13.6. What is the density of mercury?

Sol: Since R.D = Density of Mercury.

∴ Density of Mercury = 13.6 g/cc.

Q) Density of wood is 500 kg/m³. Find Relative density of the wood.

Sol: $\text{Relative density} = \frac{\text{Density of wood}}{\text{Density of water}} = \frac{500 \text{ kg/m}^3}{1000 \text{ kg/m}^3}$

$= 0.5$

Q) Relative density of ice is 0.9. Find density of ice in kg/m^3!

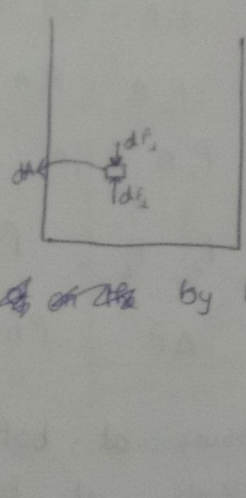
Sol: Relative density × Density of water = Density of ice

Density of ice = $0.9 \times 1000 \ kg/m^3$

Density of ice = $900 \ kg/m^3$

~~VARIA~~ PRESSURE IN A FLUID When fluid is at rest it exerts a force perpendicular to any surface with any contact with it such as the container wall (or) body immersed in a fluid

$$P = \frac{dF_\perp}{dA}$$



If pressure is uniform at all points at a finite plane surface of Area, then pressure on the surface ~~of the~~ by the fluid

$$\boxed{P = \frac{F_\perp}{A}}$$

→ S.I unit of pressure is N/m^2 (or) Pascal (Pa).

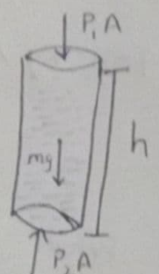
→ $P_{atm} = 1.01 \times 10^5$ Pa (atmospheric pressure)

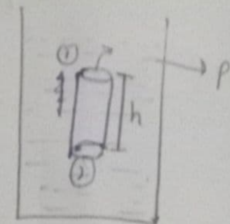
$P_{atm} \simeq 10^5$ Pa

VARIATION OF PRESSURE IN A FLUID DUE TO DEPTH

Consider a cylindrical column (Area - A, Height - H) in a liquid of density ρ, the pressure difference between bottom and top layer of the cylinderical column is given by

FBD of cylinderical column





$$P_1A + mg = P_2A$$

$$P_1A - P_2A = -mg$$

$$P_2A - P_1A = mg$$

$$P_2A - P_1A = \rho V g$$

$$P_2A \div P_1A = \rho A \times h g$$

$$P_2 - P_1 = \rho h g$$

$$\Delta P = \rho h g$$

P_2 - Pressure at bottom layer
P_1 - Pressure at top layer
h - height of cylinderical column
g - Acceleration due to gravity
ρ - Density of the liquid

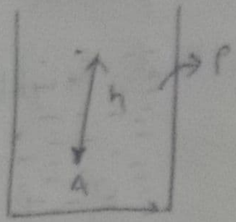
→ The pressure difference between two points in a liquid depends on ρ density of the liquid, height difference between two pants and acceleration due to gravity.

→ From the above equation, we can conclude that the pressure at a point inside the liquid at a point A inside the liquid is given by

$$P_A = P_0 + h\rho g$$

(or)

$$\Rightarrow P_A = P_{atm} + h\rho g$$



P_0 - Atmospheric pressure

ρ - density of the liquid

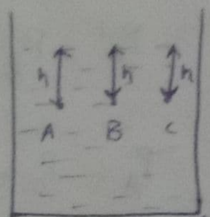
h - depth of point A from the surface of the liquid

g - Acceleration due to gravity

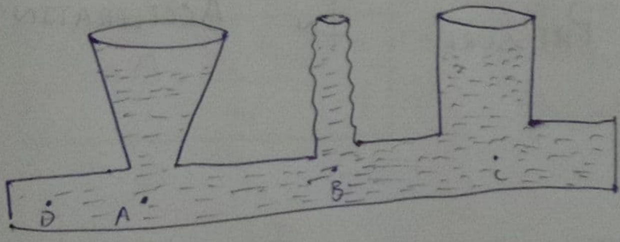
Important points:

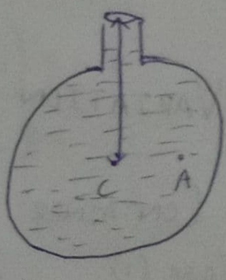
1) In the case of stationary fluids, pressure at all points lying on same horizontal plane in the given fluid are equal.

$$P_A = P_B = P_C$$



2) Hydrostatic Paradox: The pressure at a point inside the fluid does not depend on shape of the container or amount of fluid above that point



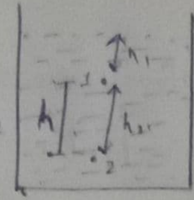


$$P_A = P_B = P_C = P_D$$

$$P_A = P_C$$

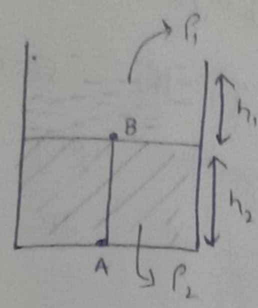
3) The pressure difference between two points depends on vertical height between two points

$$P_2 - P_1 = h\rho g$$



h - height difference between the two points.

4) Pressure at a point in two immisible liquids



$$P_B = P_0 + h_1 \rho_1 g$$

$$P_A - P_B = h_2 \rho_2 g$$

$$P_A = P_B + h_2 \rho_2 g$$

$$P_A = P_0 + h_1 \rho_1 g + h_2 \rho_2 g$$

P_A - Pressure at point A

P_B - Pressure at point B.

VARIATION OF PRESSURE IN ACCELERATING CONTAINER

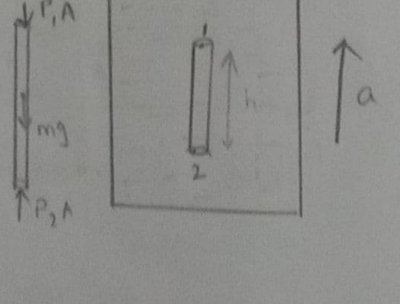
Case (i)

$$P_2 A - P_1 A - mg = ma$$

$$(P_2 - P_1) A = m(g + a)$$

$$(P_2 - P_1) A = \rho A h (g + a)$$

$$P_2 - P_1 = \rho h (g + a)$$

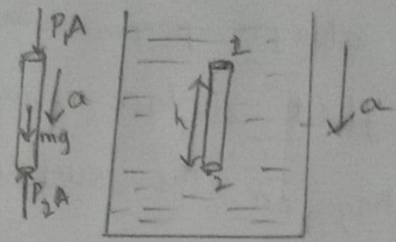


Case - 2

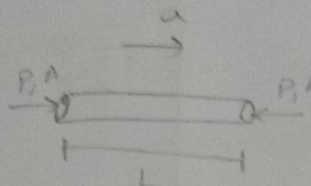
$$P_1 A + mg - P_2 A = mg$$

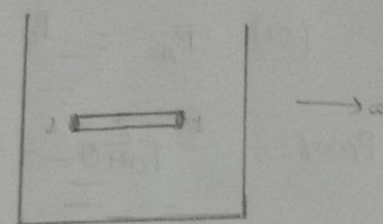
$$(P_2 - P_1) A = m(g - a)$$

$$P_2 - P_1 = h\rho(g - a)$$



Case -3



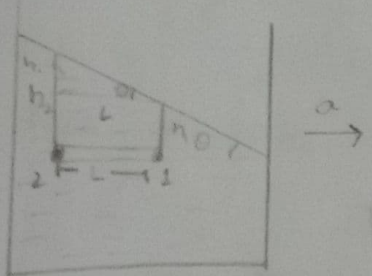


$$P_2A - P_1A = ma$$

$$P_2 A - P_1 A = \rho A L a$$

$$P_2 - P_1 = \rho L A$$

When the container moves horizontally, the water in the container makes an angle θ with the horizontal



Method - I

$$P_2 = P_0 + h_2 \rho g \quad -①$$

$$P_1 = P_0 + h_1 \rho g \quad -②$$

$$P_2 - P_1 = \rho L g \quad -③$$

① & ②

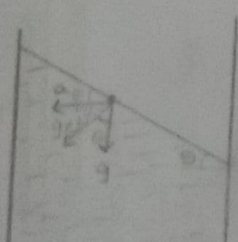
$$P_2 - P_1 = (h_2 - h_1) g \quad -④$$

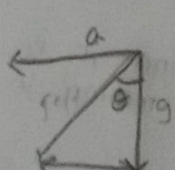
From ③ & ④

$$(h_2 - h_1) g = L \cancel{g} a$$

$$\boxed{\tan\theta = \frac{h_2 - h_1}{L} = \frac{a}{g}}$$

Method - II

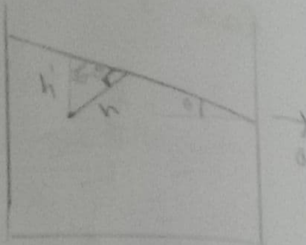




$$\tan\theta = \frac{a}{g}$$

NOTE: 1) When container accelerates, free surface of the liquid is always perpendicular to g_{eff} (or) when container accelerates a fluid surface always perpendicular to the net force acting on the surface molecule.

2) When container accelerates along horizontal direction, the pressure at a point inside the fluid is given by



$$P_A = P_0 + h' \rho g$$

(or) $$P_A = P_0 + h \rho \sqrt{g^2 + a^2}$$

Proof:

$$\tan\theta = \frac{a}{g}$$

$$\cos\theta = \frac{g}{\sqrt{g^2+a^2}}$$

$$\sin(90-\theta) = \frac{h}{h'}$$

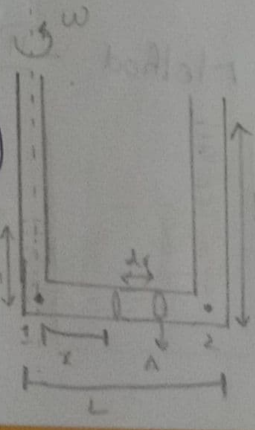
$$h' = \frac{h}{\cos\theta}$$

$$P_A = P_0 + \frac{h}{\cos\theta} \rho g$$

$$P_A = P_0 + \frac{h}{g}\sqrt{g^2+a^2}\, \rho g$$

$$P_A = P_0 + h \rho \sqrt{g^2+a^2}$$

Case (iv):



$$dP = \rho\, dx\, a$$

$$\int_{P_1}^{P_2} dP = \int_0^L \rho\, dx\, x\omega^2$$ (since it is moving in circular path)

$$P_2 - P_1 = \rho\omega^2 \left[\frac{x^2}{2}\right]_0^L$$

$$P_2 - P_1 = \frac{\rho\omega^2 l^2}{2} \quad -①$$

$$P_1 = P_0 + h_1 \rho g \quad -②$$

$$P_2 = P_0 + h_2 \rho g \quad -③$$

From ② and ③

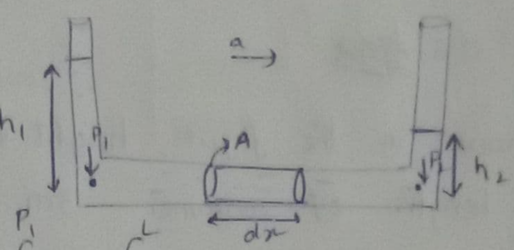
$$P_2 - P_1 = (h_2 - h_1)\rho g$$

① & ④

$$(h_2 - h_1)\rho g = \frac{\rho\omega^2 l^2}{2}$$

$$\boxed{h_2 - h_1 = \frac{\rho\omega^2 L^2}{2g}}$$

Q) Find the height difference of the liquid in the two limbs of U-tube when it accelerated horizontally



Sol: $\int_{P_2}^{P_1} dP = \int_0^L \rho\, dx\, a$

$$\Delta P = \rho a \left[x\right]_0^L$$

$$\Delta P = \rho L a \quad -①$$

$P_1 = \rho h_1 g$ -② $P_2 = \rho h_2 g$ -③

From ①, ②, ③

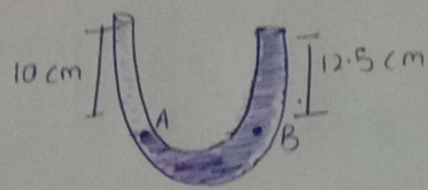
$$P_1 - P_2 = \rho L a$$

$$\rho g(h_1 - h_2) = \rho L a$$

$$h_1 - h_2 = \frac{La}{g}$$

Q) A U-tube contains water and methyl alcohol seperated by mercury The mercury columns in the two arms at the same level with 10cm of water in one arm as shown in figure Find relavtive density of methyl alcohol.

Sol.



$$P_A = P_B$$

$$P_0 + h_w \rho_w g = P_0 + h_a \rho_a g$$

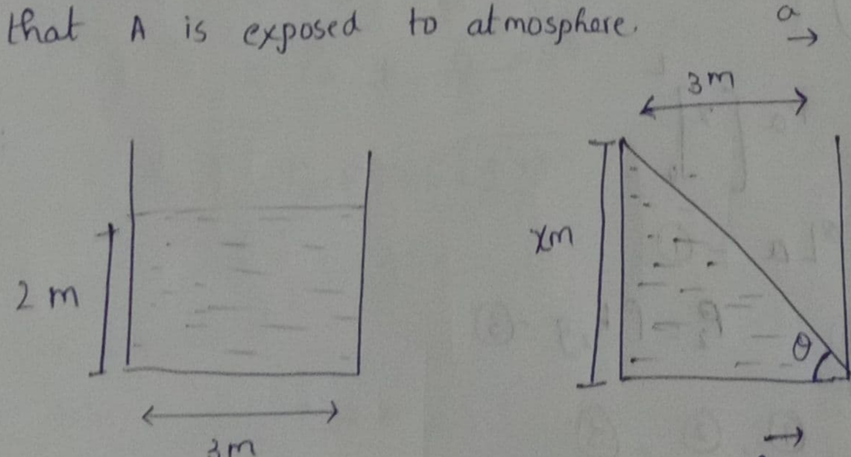
$$(10)\rho_w = (12.5)\rho_a$$

$$\frac{\rho_a}{\rho_w} = \frac{10}{12.5}$$

Relative density $= 0.8$

Q) In the container shown in the figure, height of the liquid in it is 2m and length of container 3m. With what minimum acceleration container should accelerate horizontally that A is exposed to atmosphere.

Sol



Areas are equal in both cases

$$2\times 3 = \frac{1}{2}\times 3\times x$$

$$x = 4\,m$$

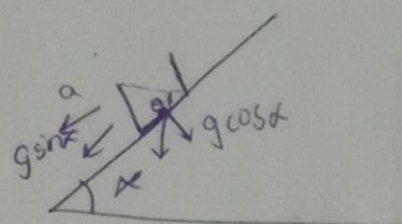
$$\tan\theta = \frac{a}{g}$$

$$\frac{4}{3} = \frac{a}{g}$$

$$a \Rightarrow \frac{4g}{3}$$

8) A fluid container containing water is acceleratiang with acceleration 'a'. The density of water in it is ρ. Then find angle of inclination of free space.

Sol:



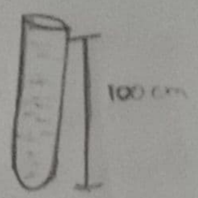
$$\tan\theta = \frac{a_{eff}}{g_{eff}}$$

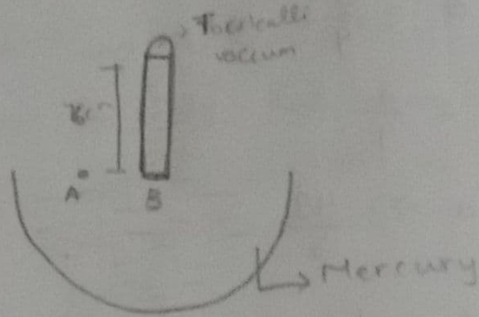
$$a_{eff} = a + g\sin\alpha$$

$$\tan\theta = \frac{a + g\sin\alpha}{g\cos\alpha}$$

PRESSURE MEASURING DEVICES (OR) PRESSURE GAUGES

Barometer: A device which measures atmospheric pressure is called Barometer





$P_A = P_B$

$P_A = P_{atm}$

$P_{atm} = h_m \rho_m g$

$P_{atm} = 76 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8$

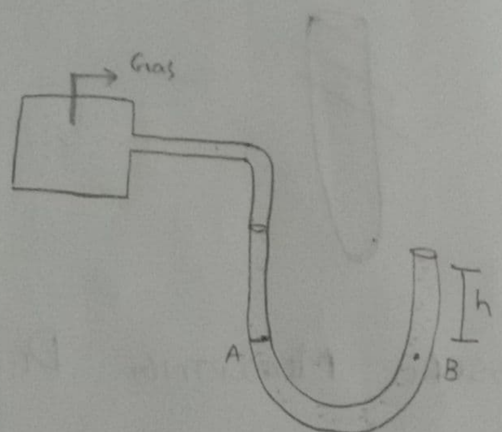
$P_{atm} = 1.01 \times 10^5$ Pa

$P_{atm} \simeq 10^5$ Pa

→ 1 atm = 1 bar = 10^5 Pa

→ Pressure due to 76cm height of mercury column $= 10^5$ Pa

Manometer:



$P_A = P_B$

$P_A = P_{Gas}$

$$\boxed{P_{gas} = P_0 + h\rho_{Hg} g}$$

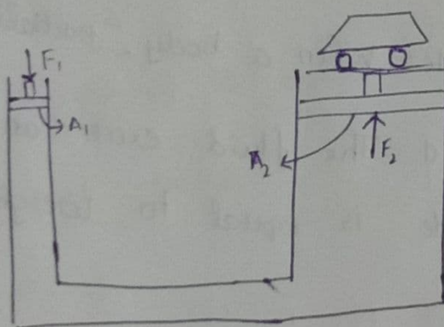
$h\rho_{Hg} g$ - Gauge Pressure

Gauge Pressure - Excess pressure ~~due~~ ~~to~~ than atmospheric pressure

• PASCAL'S LAW: It states that if the pressure in a fluid is increased at a point, the increase in the pressure would be transmitted to entire fluid without diminishing its value.

• Application OF PASCAL'S LAW:

• Hydraulic Machine: It's a force multiplying device



$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

$$\boxed{\begin{matrix} A_2 > A_1 \\ F_2 > F_1 \end{matrix}}$$

Force is increased

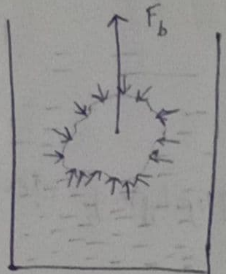
Eg: $F_1 = 100N$; $A_1 = 10\ cm^2$; $A_2 = 100 cm^2$

$F_2 = ?$

Sol $F_2 = F_1\left(\frac{A_2}{A_1}\right)$

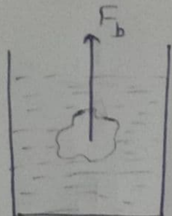
$F_2 = 100 \times \frac{100}{10} = 1000N$

Buoyant Force: The resultant of all the forces exerted by the fluid on the body is called Buoyant force



- Archamede's Principle: When a body is partially (or) completely immersed in a fluid. The fluid exerts an upward force whose magnitude is equal to weight of the fluid displaced.

F_b = Weight of the fluid displaced



$F_b = m_f g_{eff}$

***

$$\boxed{F_b = \rho_f V_i g}$$

V_i - Volume of the immersed part

ρ_f - Density of the fluid

g - Acceleration due to gravity

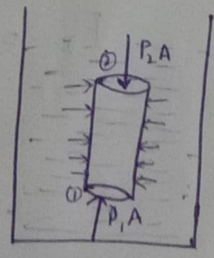
F_b - Buoyant force.

Proof: $F_{net} = F_b = P_2A - P_1A$

$F_b = (P_2 - P_1)A$

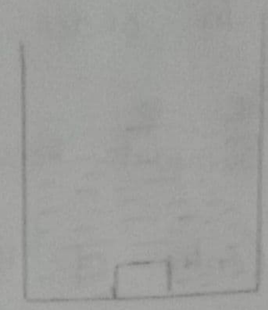
$F_b = \rho h g A$

$F_b = \rho(hA)g = \rho V g$



Case (i): $\rho_{body} > \rho_{water}$

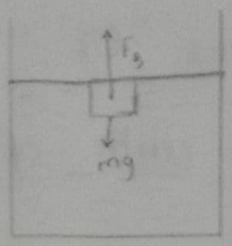
Body sinks



Case (ii): $\rho_{body} = \rho_{water}$

F_B = Weight of the body

Body just sinks



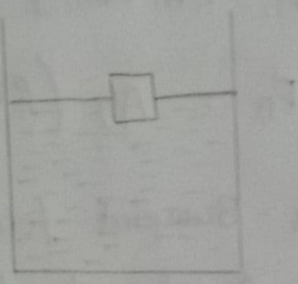
Case (iii) : $\rho_{body} < \rho_{fluid}$

Body floats

$$F_B = Mg$$

$$\rho_f V_i g = \rho_{body} V g$$

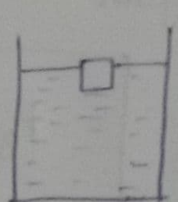
$$\frac{V_i}{V} = \frac{\rho_{body}}{\rho_{fluid}}$$



$\frac{V_i}{V}$ - fractional volume immersed in a fluid

Eg: $\rho_{ice} = 0.9$ g/cc

$\rho_w = 1$ g/cc. $\frac{V_i}{V} = ?$



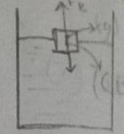
$$\frac{V_i}{V} = \frac{0.9}{1} = \frac{9}{10}.$$

* Important Points.

1) → In order to be floating body in equilibrium

I. ~~the~~ Buoyant Force (F_B) = Weight of the body / object

II. Centre of Buoyancy and centre of gravity of the body should be on the same line (To maintain Rotational equilibrium $\tau_{net} = 0$)

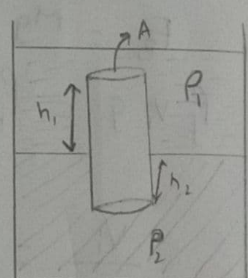


• CENTRE OF BUOYANCY (C.B): Centre of Buoyancy is the centre of gravity / centre of mass of the displaced liquid.

→ Body floating at the interface of two immiscible liquids

$$F_B = \rho_1 h_1 A g + \rho_2 h_2 A g$$

$$F_B = A g (\rho_1 h_1 + \rho_2 h_2)$$



F_B - Buoyant force
ρ_1 - Density of liquid 1
ρ_2 - Density of liquid 2

→ Experimental ~~calculation~~ way of Calculating ~~buoyancy~~ Buoyant Force:

$$\boxed{\begin{array}{l} F_B = W_{Air} - W_{liquid} \\ W_{liquid} = W_{air} - F_B \end{array}}$$

Relative density of the body $= \dfrac{W_1}{W_1 - W_2}$

R.D of body $= \dfrac{\rho_{body}}{\rho_w V g} = \dfrac{\rho_{body} V g}{\rho_w V \cdot g} = \dfrac{W_1}{W_1 - W_2}$

Relative density of the liquid $= \dfrac{W_1 - W_3}{W_1 - W_2}$

W_1 - Weight of the body in air.

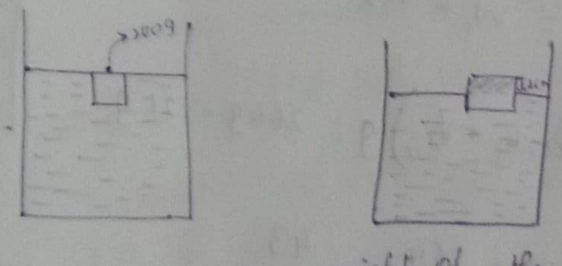
W_2 - Weight of the body in water

W_3 - Weight of the body in liquid

* The above two formulae are applicable only when the body is completly immersed in the liquid.

Q) When 200g ~~is~~ of mass is supported by a cubical block it just sinks in the water, when 200g of mass is removed from the cubical block. Cubical block rises by 2cm. neglect the size of 200g of mass. Find the side length of the cube.

Sol:



Due to 200g the ~~wat~~ extra weight of the water displaced $(a^2 \times 2)\,V$

$$200 \times g = (1) \times a^2 \times 2 \times g$$

$$a = 10cm$$

Q) An ornament weighing 36 g in air, weight only 34 g in water, assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it, specific gravity of Gold is 19.3 and that of copper is 8.9

Sol:

$$F_b = W_1 - W_2$$

$$(1)(V_c + V_G)\,g = (36-34)\,g$$

$$\left(\frac{m_c}{\rho_c} + \frac{m_G}{\rho_G}\right) = 2$$

$$\frac{x}{8.9} + \frac{36-x}{19.3} = 2$$

$$x = 2.2$$

Q) A piece of Copper having an internal cavity weight 264 grams in air and 221 g in water. Find the volume of the cavity. Density of the copper is 8.8 g/cc.

Sol:

$$F_B = W_1 - W_2$$

$$(1)(V_{cavity} + V_{Cu})\,g = 264\,g - 221\,g$$

$$V_{cavity} + \frac{264}{8.8} = 43$$

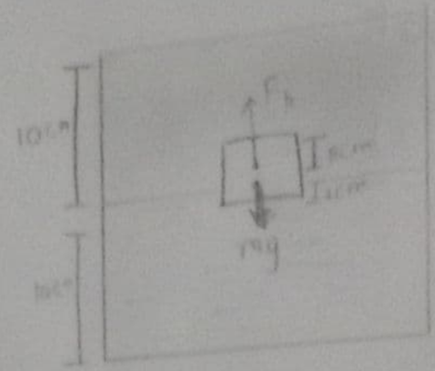
$$V_{cavity} = 43 - 30$$

$$V_{cavity} = 13 \text{ cc.}$$

Q) A cubical block of wood of side length 10cm floats at the interface ~~b~~ between an oil and water with its base 2cm below the interface. If the height of oil and water are 10cm each. and density of oil 0.8 g/cc

(a) Find mass of the block.

(b) Find the ~~hazard~~ gauge pressure at the base, (or) bottom of the cubical block



Sol: (a) Since the block is floating

$$F_b = Mg$$

$$F_{B(oil)} + F_{B(water)} = Mg$$

$$0.8\,(10^2)(8)\,g + (1)(10^2)(2)\,g = Mg$$

$$M = 640 + 200$$

$$M = 840\,g$$

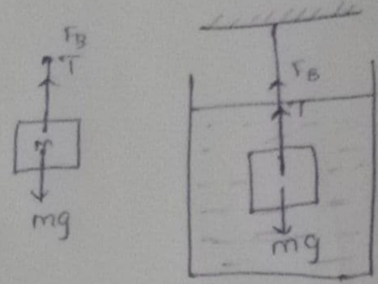
(b) $P_A = P_{oil} + P_{water}$

$$P_A = 10\times10^{-2}\times0.8\times10^3\times10 + 2\times10^{-2}\times10^3\times10$$

$$P_A = 1000\ Pa$$

$/cm^3$ floats

Q) A block of mass m is suspended by a massless inextensible string, the block is submerged completly in a liquid of density 'σ', find Tension in the string given that, density of the block is 'ρ', 'g' is acceleration due to gravity. $(\rho > \sigma)$



Sol:

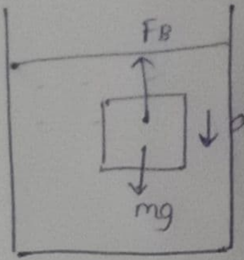
$$F_B + T = mg$$

$$T = mg - F_B$$

$$T = mg - \sigma \frac{m}{\rho} g$$

$$T = mg\left(1 - \frac{\sigma}{\rho}\right)$$

Q) In the above question, if the string is cut, with what accleration the block sinks?



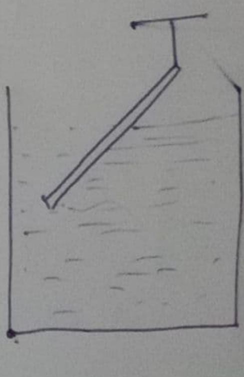
Sol:

$$mg - F_B = ma$$

$$mg - \sigma \frac{m}{\rho} g = ma$$

$$a = g\left(1 - \frac{\sigma}{\rho}\right)$$

Q) A uniform rod of length 2L floats partly immersed in water being supported by a string fast end to one of its end. The specific gravity of the rod is 0.75g/cc The length of the rod that extends out of the water is.



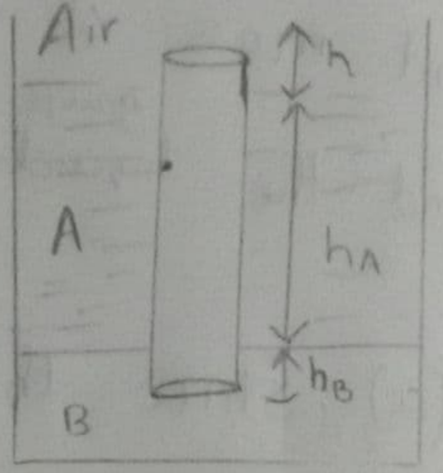
HW

Q) A uniform solid cylinder of density $0.8\ g/cm^3$ floats at equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of A and B are $0.7\ g/cc$ and $1.2\ g/cc$ respectively. The height of liquid A is h_A is $1.2\ cm$, the length of the part of the cylinder with liquid B is $0.8\ cm$. Find

(a) Total force exerted by liquid A on the cylinder

(b) Find ~~th~~ value of h.

(C) The cylinder is dipressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder, just after released?



Sol:

(a)