BOARD OF INTERMEDIATE EDUCATION (AP)

JUNIOR INTER MATHEMATICS – IA

MODEL PAPER

Time: 3 hours **Max. Marks: 75**

SECTION – A

I. **i) Very short answer type questions.**

ii) Answer ALL questions.

iii) Each question carries TWO marks. **10 × 2 = 20**

1. If f: R – {±1} ⟶ R is defined by $f(x) = \log\left|\frac{1+x}{1-x}\right|$ then show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

2. Find the domain of the real valued fuction $f(x) = \frac{1}{(x^2-1)(x+3)}$.

3. Define a symmetric matrix. Give one example of order 3×3.

4. Find the adjoint and the inverse of the matrix $\begin{pmatrix} 2 & -3 \\ 4 & 6 \end{pmatrix}$.

5. If the position vectors of the points A, B, C are $-2\bar{i} + \bar{j} - \bar{k}$; $-4\bar{i} + 2\bar{j} + 2\bar{k}$ and $6\bar{i} - 3\bar{j} - 13\bar{k}$ respectively and $\overline{AB} = \lambda\overline{AC}$. Then find λ.

6. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.

7. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$, $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$ then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.

8. If $\sin\theta = \frac{4}{5}$ and θ is not in the first quadrant then find the value of $\cos\theta$.

9. Prove that $\cos 48° \cos 12° = \frac{3+\sqrt{5}}{8}$.

10. For $x \in R$, show that $\cosh 2x = 2\cosh^2 x - 1$.

SECTION – B

II. **i) Short answer type questions.**

ii) Answer any FIVE questions.

iii) Each question carries FOUR marks. **5 × 4 = 20**

11. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ then show that $A^2 - 4A - 5I = 0$.

12. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are non coplaner find the points of intersection of the line passing through the points $2\bar{a} + 3\bar{b} - \bar{c}$; $3\bar{a} + 4\bar{b} - 2\bar{c}$; with the line joining the points $\bar{a} - 2\bar{b} + 3\bar{c}$, $\bar{a} - 6\bar{b} + 6\bar{c}$.

13. If $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$ then find the value of $(\bar{a} \times \bar{b})(\bar{b} \times \bar{c})$.

14. Prove that $\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right) = \frac{1}{16}$.

15. Solve the equation $4\cos^2\theta + \sqrt{3} = 2(\sqrt{3} + 1)\cos\theta$ and write the general solution.

16. Prove that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(2\tan^{-1}\frac{3}{4}\right)$.

17. In the triangle ABC, if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ then show that $\angle c = 60°$.

SECTION - C

III. i) Long answer type questions.

ii) Answer any FIVE questions.

iii) Each question carries SEVEN marks. **5 × 7 = 35**

18. Show that the function $f : Q \to Q$ defined by $f(x) = 5x + 4$ for all $x \in Q$ is a bijection and find f^{-1}.

19. Using mathematical induction prove that $1.2.3 + 2.3.4 + 3.4.5 +$....up to n terms $(n \in N)$

$= \frac{n(n+1)(n+2)(n+3)}{4}$.

20. With out expanding the determinant prove that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

21. Solve $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$ using Cramer's rule .

22. If $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$ verify that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$.

23. If $A + B + C = 2S$ then prove that $\cos(S - A) + \cos(S - B) + \cos C =$

$-1 + 4\cos\frac{S-A}{2}\cos\frac{S-B}{2}\cos\frac{C}{2}$.

24. In the triangle ABC, show that $\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$.

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