

**Computational Physics - PH3264**  
Module 2 - Integration

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1. The total magnetic moment of the system is -8000.
2. The total energy of the system is -3000.

**Some notes on profiling and performace bottlenecks:**

In a Monte Carlo step, the code uses three random numbers to pick a random spin to flip. This computation can be reduced to just one random number. (This is possible thanks to a bijection between the sets  $\{0, 1, 2, 3 \dots L^3 - 1\}$  and  $\{(0, 0, 0), (0, 0, 1) \dots (L - 1, L - 1, L - 1)\}$ .) The conversion function is given below:

$$i\_to\_index(int\ i) = \{i/(L*L), (i/L)\%L, i\%L\}$$

Profiling with **gprof** (with 50,000 iterations) if three random numbers are generated, the program spends 46.03% of its 2.548 s runtime generating the three numbers. When reduced to one, the program spends only 14.17% of its 1.877 s runtime generating the index, providing a significant performance boost. Additionally, this index conversion is carried out only once per lattice and stored in an array, looked up when required, saving computation time over long cycles.

3. The magnetization per spin of the system fluctuates around 0 - as expected at a higher temperature.

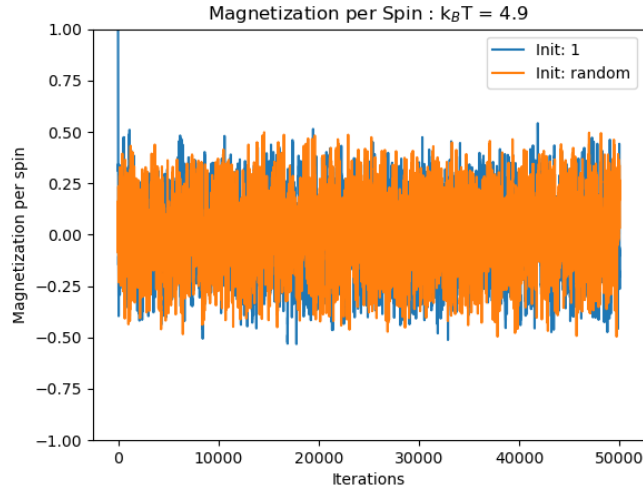


Figure 1: Magnetization per spin at  $k_B T=4.9$

4. The instantaneous energy per spin fluctuates around 2.0 at  $k_B T=3.9$

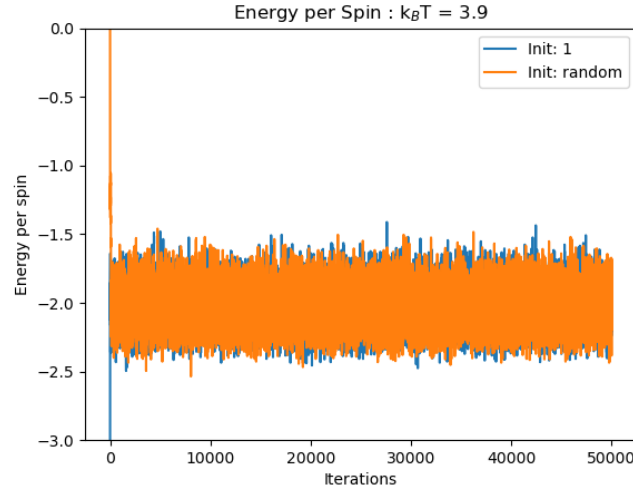


Figure 2: Energy per spin at  $k_B T=3.9$

5. The energy and magnetisation per spin fluctuate about -1.84 and  $\pm 0.73$  respectively.

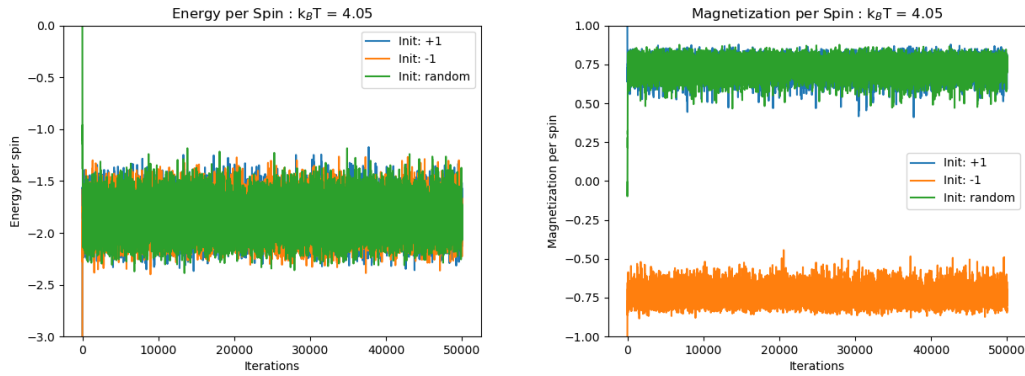


Figure 3: Energy and Magnetization per spin at  $k_B T=4.05$

6. The graphs for Energy and Magnetization at various box sizes have been shown in Figure 4.

The simulation was run as instructed and the following graphs were produced as a result. The magnetisation per spin of the lattice(s) at various temperatures has been plotted in Figure 5. Likewise, the energy per spin can be seen in Figure 6. The plots for  $\chi$  and  $C_v$  are in 7 and 8 respectively.

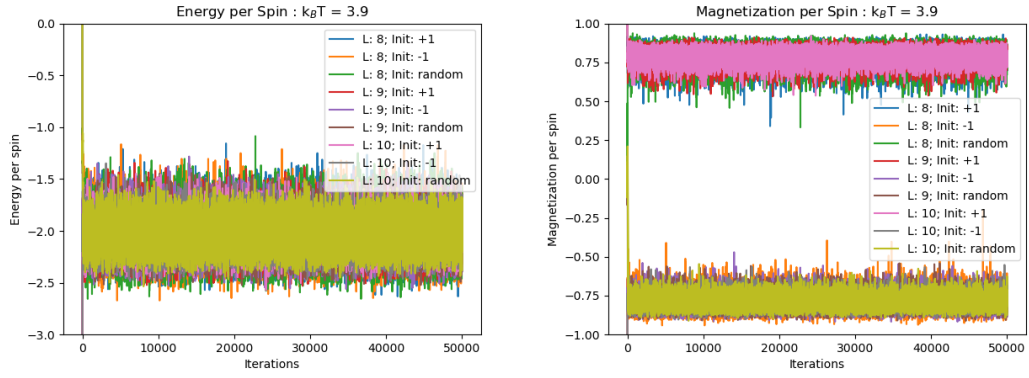


Figure 4: Energy and Magnetization per spin at  $k_B T=3.9$

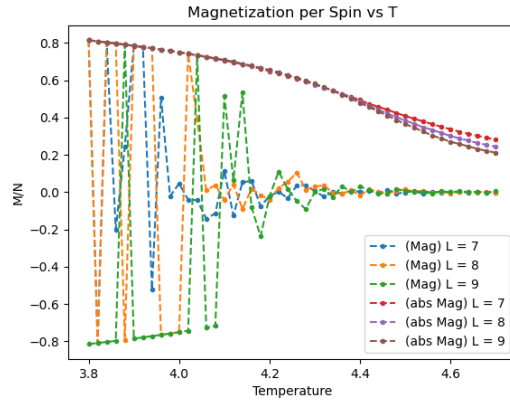


Figure 5: Magnetization per spin vs Temperature

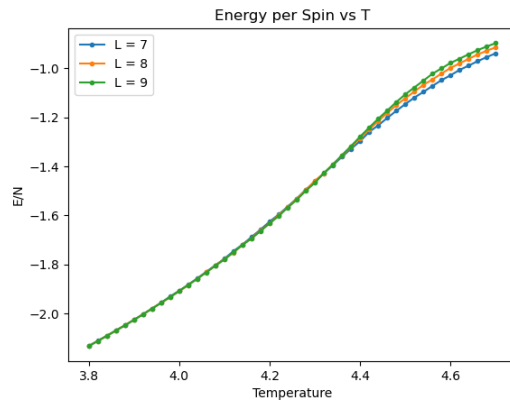


Figure 6: Energy per spin vs Temperature

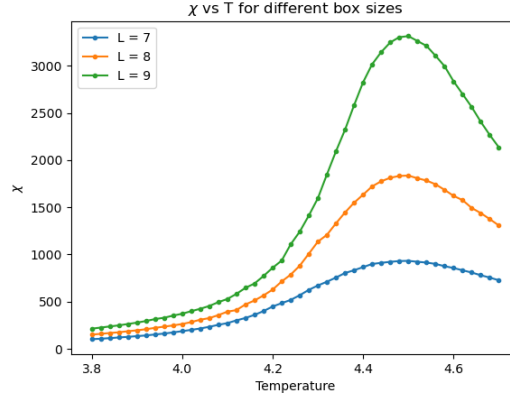


Figure 7:  $\chi$  vs Temperature

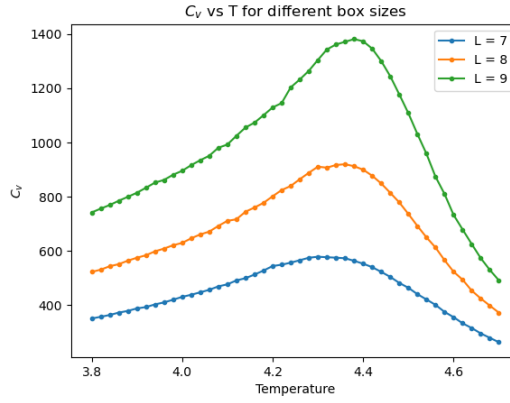


Figure 8:  $C_v$  vs Temperature

7. The values of  $\chi$  at  $T=4.5$  are 932.1, 1835.9 and 3313.7 (in simulation units) for lattice sizes of 7, 8 and 9 respectively.
8. The value for  $C_v$  at peak position for lattice of length 8 is 919.8 (also in simulation units).
9. The value for  $C_v$  at peak position for lattice of length 9 is 1381.7.
10. The magnetization per spin at  $T = 3.8$  for a lattice of length 7 is  $\pm 0.814$ .
11. The Binder Cumulant plot is shown in Figure 9. The point where the curves intersect is  $T \approx 4.5$ .
12. The number of particles jumping from  $E_{10}$  to  $E_5$  is  $10 \cdot e^{\Delta E \beta}$  where  $\Delta E = E_5 - E_{10}$  and  $\beta = k_B T$ .

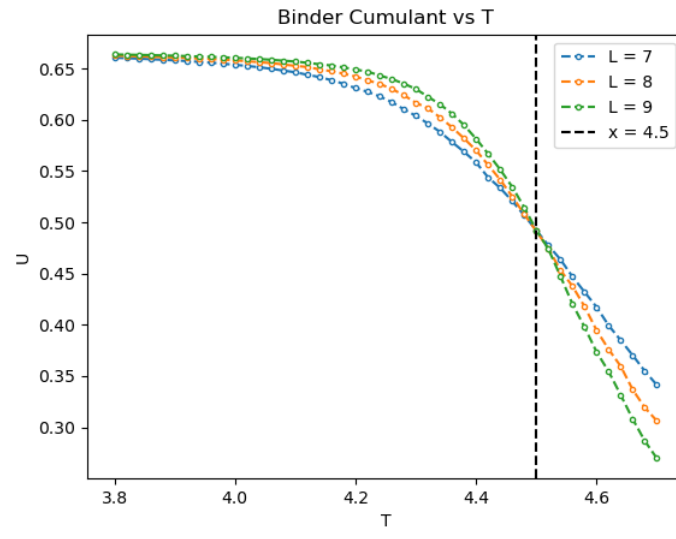


Figure 9: Binder Cumulant for different lattice sizes