Computational Physics - PH3264
Module 2 - Integration

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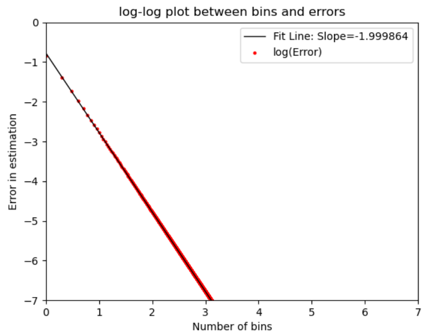
1. a. The error is measured as abs($\pi-$estimate), indicating that we our approximation does indeed approach π.

   b. The function's actual integrand is π. The following plot shows the errors plotted against the number of bins on a log-log scale. The slope of the line is -1.9998,



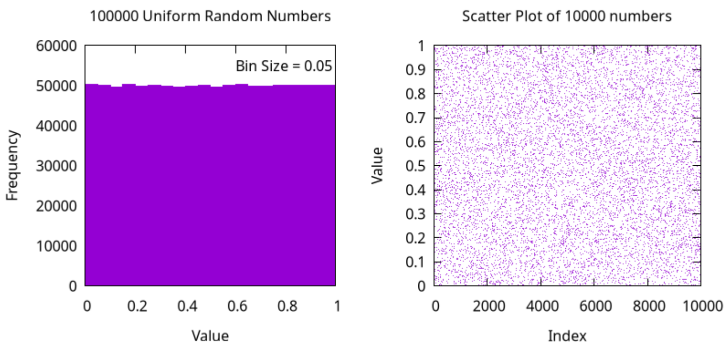
   which is close to what is predicted by theory (predicted slope: -2).

   c. Taking $sin(x)$ with 10,000 bins, the value estimated is 2 - which agrees with the theory and confirms that the integration scheme works.

   d. The answer obtained is 0.997300 which appears to match with gaussian integral tables.

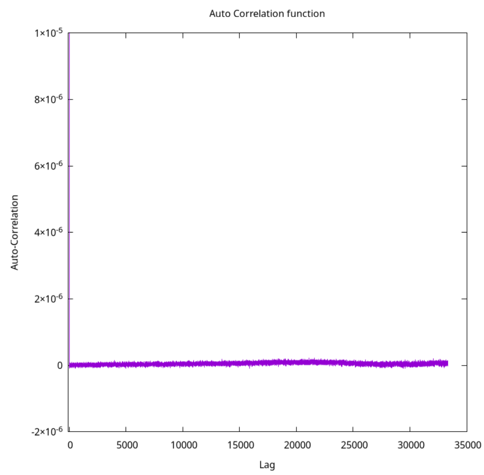
2. (a) The random numbers plotted as a histogram with bin size is shown in Figure 2b (left) - suggesting that the random numbers are uniform.

   (b) The first 10000 (out of 100,000) random numbers are plotted on a scatter plot in Figure 2b (right).



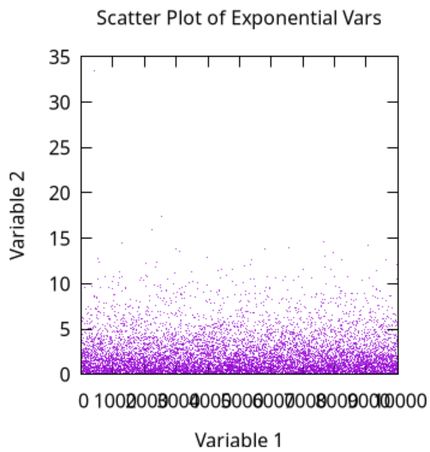
(c) The auto-correlation function plotted with increasing lag is shown in Figure 2c. The auto-correlation function has the following form:

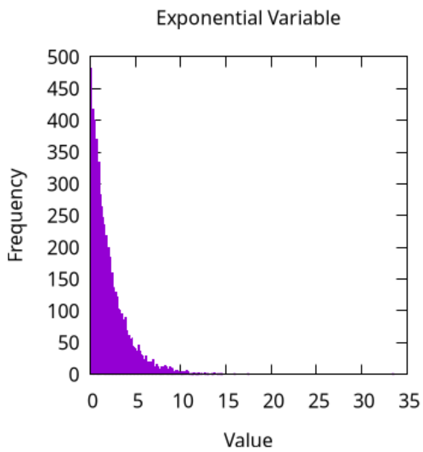
$$\hat{R(k)} = \frac{1}{(n-k)\sigma^2} \sum_{t=1}^{n-k} (X_t - \mu)(X_{t+k} - \mu)$$



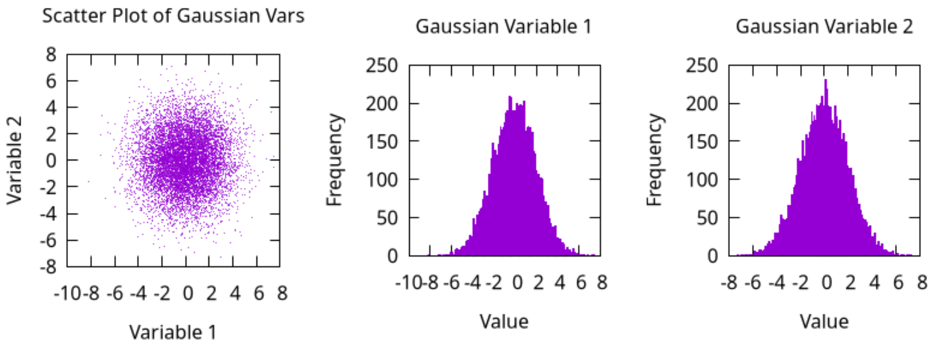
(d) The mean and standard deviation of this uniform distribution are 0.5001 and 0.28879.

3. (a) For generating a random variable that samples according to the distribution e^{-2x}, we scale uniformly generated random numbers by $-2 \cdot ln(1-x)$. The distribution generated by this random device is shown in Figure 3b.





(b) For generating random numbers with a gaussian distribution, we use the Box-Muller transform to convert a set of numbers in a 1x1 square to a set of exponentially distrubted numbers in the XY plane, such that any orthagonal axis is an independent variable.



4. (a) Brute Force Integration.

10	03.4258	4.706778
100	11.9365	6.961318
1000	10.8966	2.375228
10000	10.8879	0.742370
100000	10.8913	0.234514