Sample Means of an Exponential Function

Project by Vanshaj S.

Introduction

In this “study,” we’re going to take a look at a sample of exponential distributions and compare its mean and variance to the expected mean and variance, mathematically calculated.

Required Libraries

For ease (and beauty) of graphing, let’s use the ggplot2 library.

```
library(ggplot2)
```

Setup

To ensure reproducibility, let’s set a random seed. We’ll also define our variables for future use.

```
set.seed(8243)             # reproducibility

lambda <- 0.2              # as instructed
sample.size <- 40          # as instructed
sample.num <- 1000         # as instructed

binwidth <- 0.1            # a near-perfect binwidth :)
steelblue <- "steelblue"   # a great fill color
orange <- "#c56000"        # a great line color
```

Accumulation

Let’s begin by accumulating our samples. For this project, we take 1,000 samples of size 40 from an exponential distribution.

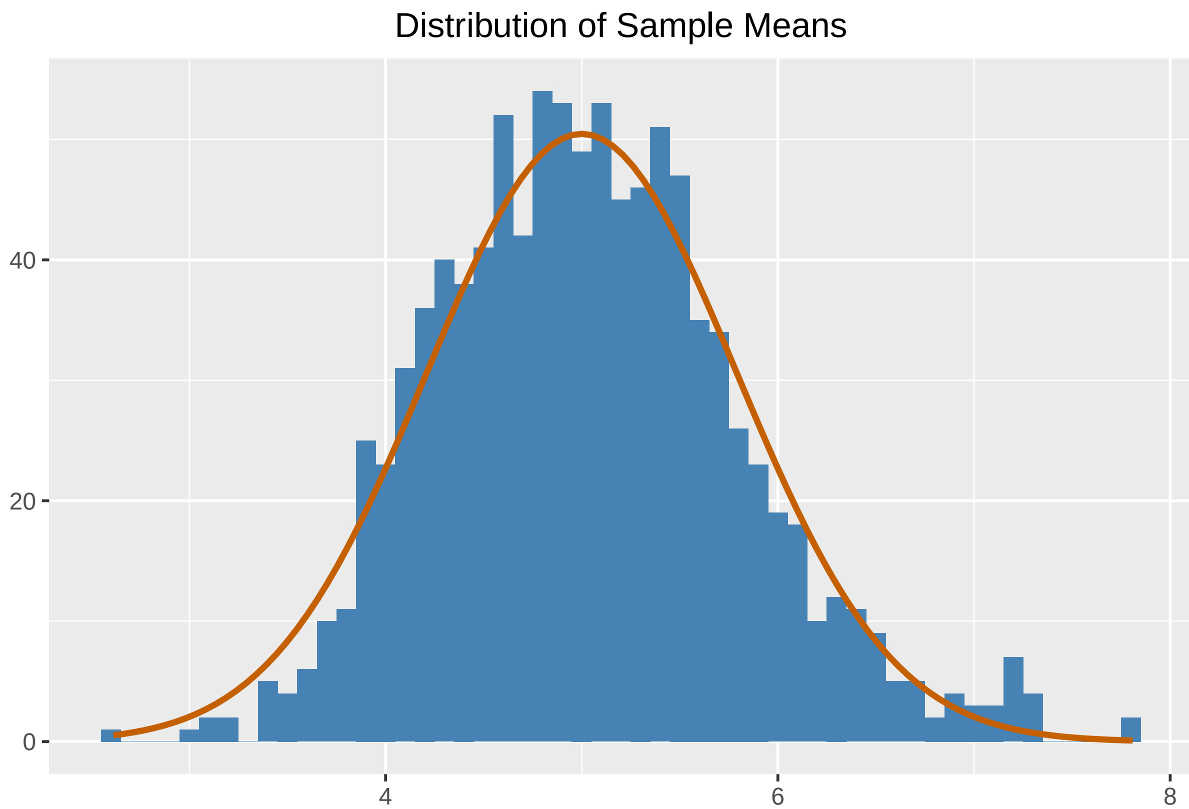
```
exp.mns <- NULL
for (i in 1 : sample.num) exp.mns = c(exp.mns, mean(rexp(sample.size, lambda)))
```

Display

Firstly, we’ll make a histogram of our sample means with a relevant normal density curve superimposed to show the decent normality of our data.

```
qplot(exp.mns, geom="histogram", binwidth = binwidth, fill = I(steelblue)) +
  stat_function(
    fun = function(x, mean, sd, n, bin){
      dnorm(x = x, mean = mean, sd = sd) * n * bin
    },
    args = c(mean = 1/lambda, sd = (1/lambda)/sqrt(sample.size),
             n = sample.num, bin = binwidth), col = I(orange), size = 1.1) +
```

```
labs(title = "Distribution of Sample Means", x = "") +
theme(plot.title = element_text(hjust = 0.5))
```



Comparisons

Let's take a look at the theoretical versus the sample means.

```
## Theoretical Mean
1/lambda

## Experimental Mean
mean(exp.mns)
```

Running this code gives us a theoretical mean of 5 and an experimental mean of 5.0365308. Pretty close.

Let's look at the variances as well.

```
## Theoretical Variance
((1/lambda)^2)/sample.size

## Experimental Variance
var(exp.mns)
```

Running this code gives us a theoretical variance of 0.625 and an experimental variance of 0.600561. Again, pretty close.

Check for Normality

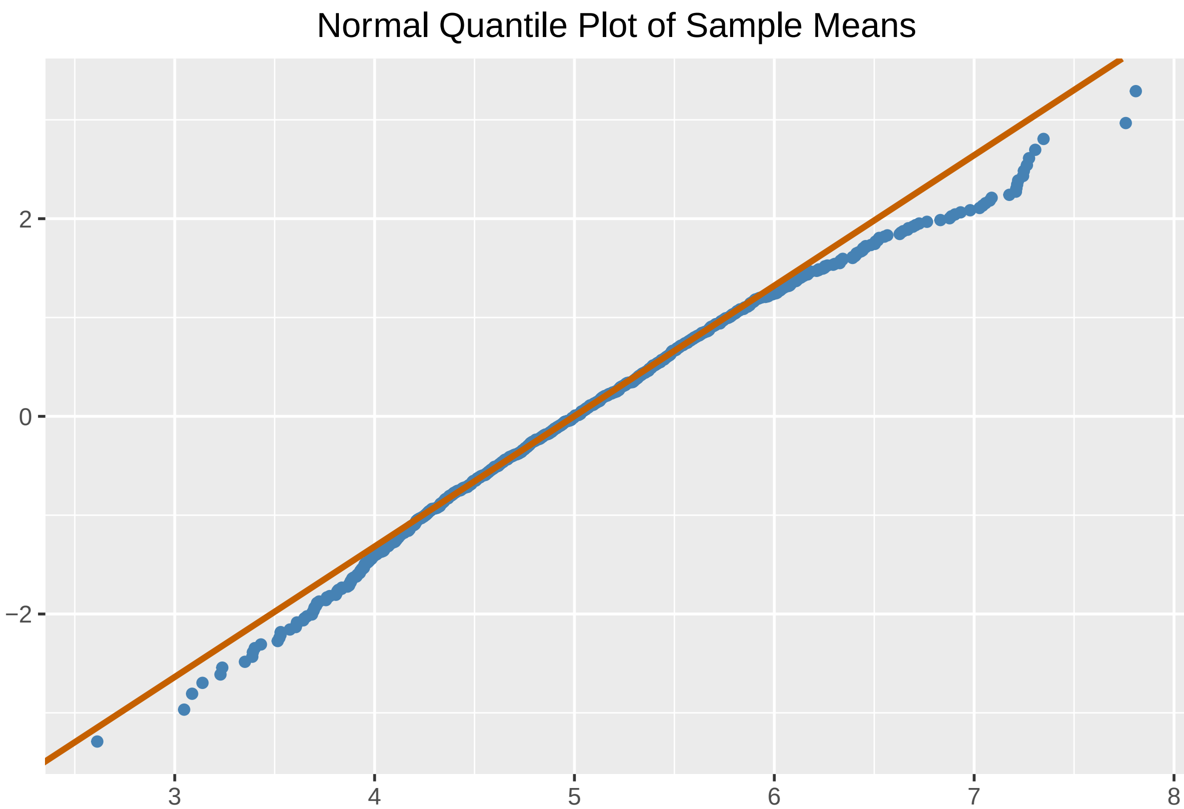
If you weren't hold from the histogram, let's make a normal quantile plot of the data. Essentially, the normal quantile plot's linearity is what proves the normality of the data – the more linear the plot, the more normal the data.

```
x <- qnorm(c(0.25, 0.75))
y <- quantile(exp.mns, c(0.25, 0.75))

slope <- diff(y)/diff(x)
int <- y[1L] - (slope * x[1L])

d <- data.frame(exp.mns)

ggplot(d, aes(sample = exp.mns, col = I(steelblue))) + geom_qq() +
  geom_abline(slope = slope, intercept = int, col = I(orange), size = 1.1) +
  labs(title = "Normal Quantile Plot of Sample Means", x = "", y = "") +
  theme(plot.title = element_text(hjust = 0.5)) + coord_flip()
```



The graph here looks *pretty* linear, as seen by its proximity to the approximation line (were the data perfectly normal, all data was reside on said line). We can safely conclude that the data here is normal.