Twenty Four

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1 Introduction

Twenty Four, an innocent looking number that holds the secret to the universe. The idea was inspired during a week long seminar at the Indian Statistical Institute, Bengaluru sometime during 2012. Details are hazy but the director of the institute (it's all supposition at this stage) came up to a bunch of us students hanging around and asked us if we could come up with ways of combine three "*n's*" to form a 24. Much of the article would be blatant copying of ideas that the fellows around me came up with but towards the end there is an attempt at a sort of generalisation.

2 The Easy Cases

The challenge is to use three instances of a number, and no other numbers, and combine them using common (easily understood) mathematical operators to form twenty four. I believe it is best illustrated through example.

Eight Eight is almost trivial.

$$8 + 8 + 8 = 24$$

Four Four can be done in a few ways. Four is really the key to the problem so the reader is implored to pay attention!

$$(4 + 4 - 4)! = 24$$

$$\left(\frac{4 \times 4}{4}\right)! = 24$$

$$\left(4^{\frac{4}{4}}\right)! = 24$$

Three and Five Three and Five are easily obtained from Four.

$$\left(3 + \frac{3}{3}\right)! = 24, and,$$

$$\left(5 - \frac{5}{5}\right)! = 24$$

Nine Nine requires a little bit of extra thought beyond Three.

$$\left(\sqrt{9} + \frac{9}{9}\right)! = 24$$

Six

$$\left(6 - log_{\sqrt{6}}6\right)! = 24$$

Two and Seven and... Once the idea of logarithms sets in, it becomes quite easy to generalise.

$$log_{\sqrt{2}}\left(2 \times 2\right)! = 24$$

$$log_{\sqrt{7}}\left(7 \times 7\right)! = 24$$

But this would work for almost any positive real number! In fact this idea can be extended to negative numbers as well!

$$log_{\sqrt{|r|}}\left(|r| \times |r|\right)! = 24, r \in R, |r| \neq 0, 1$$

But we could keep going further! This would even work for most complex numbers.

$$log_{\sqrt{\|z\|}}\left(\|z\| \times \|z\|\right)! = 24, z \in C, \|z\| \neq 0, 1$$

Of course, the difficulty arises with zero and one, and a discussion of this is what would occupy the next section.

3 The Harder Cases

A solution for one seems so close at hand but yet so far. The reason, of course, is that one is special. It is very hard to start with one and end up with a non-special number. It is, in fact, even harder with zero. The solution given by the director is, in the author's opinion, quite brilliant and innovative. But most people faced with this solution seem to be quite offended. And therefore, an attempt will be made to come up with a slightly less elegant, slightly less offensive solution.

Director's Solution

$$\left(\sum (1+1)+1\right)! = 24$$

$$\left(\sum (0!+0!)+0!\right)! = 24$$

Is this not brilliant! Surprisingly, most people consider this cheating. The reader shall be a judge of whether or not this constitutes cheating, however, the article shall proceed to demonstrate the other trick.

Author's Solution

$$\left(\frac{1}{\int_{\arccos 1}^{\arctan 1} cos(x)sin(x)dx}\right)! = 24$$

$$\left(\frac{0!}{\int_{\arcsin 0}^{\arctan 0!} cos(x)sin(x)dx}\right)! = 24$$

The Entire Solution

$$log_{\sqrt{\|z\|}}(\|z\| \times \|z\|)! = 24, z \in C, \|z\| \neq 0, 1$$

$$\left(\frac{\|z\|}{\int_{\arccos \|z\|}^{\arctan \|z\|} cos(x)sin(x)dx}\right)! = 24, \|z\| = 1$$

$$\left(\frac{\|z\|!}{\int_{\arcsin \|z\|}^{\arctan \|z\|!} cos(x)sin(x)dx}\right)! = 24, \|z\| = 0$$

In fact this idea can easily be generalised to any normed vector space. This seems to be quite general!

4 Conclusion

Why twenty four? What is it that makes twenty four so special? It may seems like a philosophically void exercise to go through all this trouble to show something specific to twenty four. The author has been baffled by this question for a while now and has finally reached enlightenment. All one has to do, is reverse the digits to realise its cosmic significance!