Correlation as a Resource

The role of redundant correlated records in the emergence of Objective Classical Reality

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Quantum25

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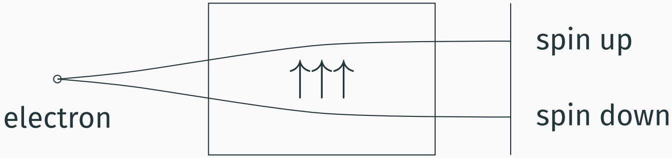
Motivation

Motivation: Stern Gerlach



Stern-Gerlach apparatus

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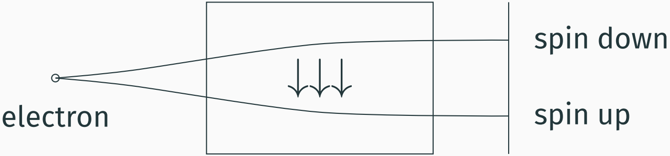

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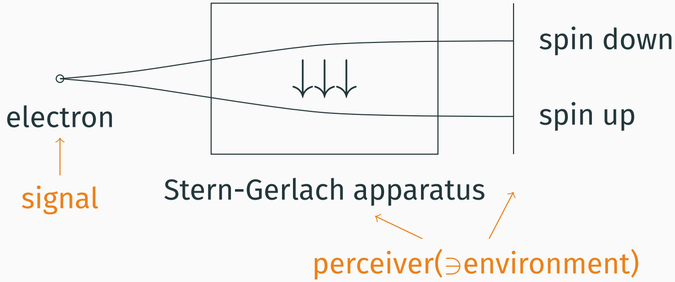

Definition (Objective Classical Reality[1])
The state of a signal s exists objectively if multiple observers (o_α) can probe the signal independently and without disturbing it.

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jointly called perceiver

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No-go Theorem: Simple Argument

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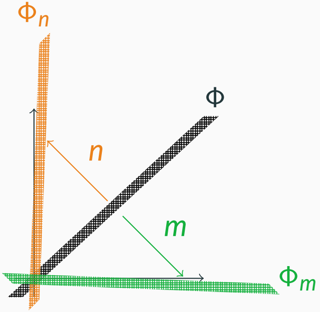

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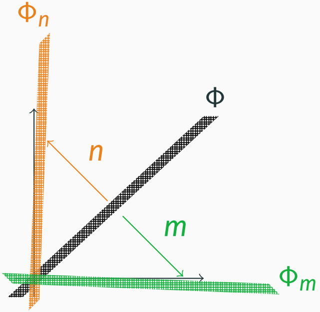

$\implies \Phi \subset \mathcal{H}_{\mathrm{p}}$

Qudit Measurement Procedure

Correlated Perceiver

signal: $\mathcal{H}_\mathsf{s} = \mathbb{C}^d$

perceiver: $\mathcal{H}_\mathsf{p} = (\mathbb{C}^d)^{\otimes \mathrm{N}}$

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Transferring Correlation (Imprint)

$$\begin{aligned}\mathcal{I}_{\mathrm{a}\to\mathrm{b}}|m\rangle_{\mathrm{a}}|n\rangle_{\mathrm{b}} &= |m\rangle_{\mathrm{a}}|n+m\rangle_{\mathrm{b}} \\ \mathcal{I}^{-1}_{\mathrm{a}\to\mathrm{b}}|m\rangle_{\mathrm{a}}|n\rangle_{\mathrm{b}} &= |m\rangle_{\mathrm{a}}|n-m\rangle_{\mathrm{b}}\end{aligned} \tag{4}$$

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repeating:

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\left(\sum \psi_m |m\rangle_{\mathrm{s}}|m\rangle_{\mathrm{p_1}} \dots |m\rangle_{\mathrm{p_K}}\right) \left(\sum \phi_l |l\rangle_{\mathrm{p_{K+1}}} \dots |l\rangle_{\mathrm{p_N}}\right) \tag{6}
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correlation is a resource!

Numerical Results

Correlation Proxy

pairwise correlations : level of agreement of measurement outcomes

$$ \tag{7} $$

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compare initial C_p with final C_s!

Simulation Procedure

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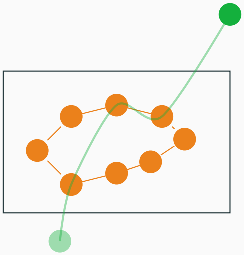
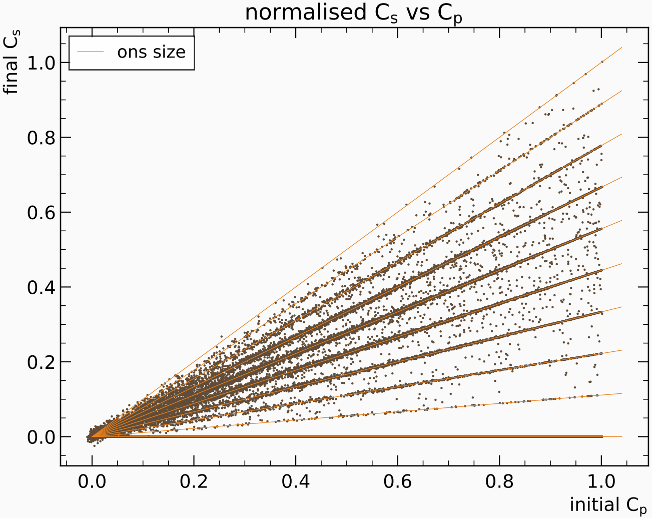
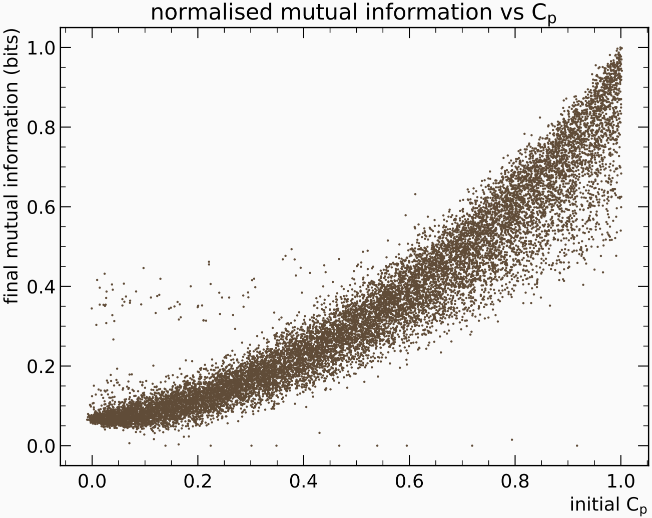


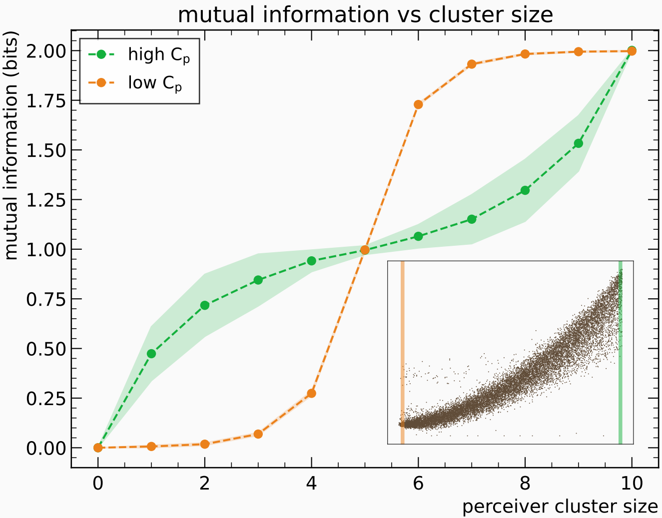
Simulation Results: C_s vs C_p



Simulation Results: mutual information vs $-C_p$



Simulation Results: mutual information plateau?



Summary

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Thanks! Any Questions?