

# Quantum Mechanics

## from an Information Theory Perspective

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# Introduction

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  - unitary evolution - time reversible
  - wavefunction collapse - non reversible

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  - wavefunction collapse - non reversible
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- measurement  $\approx$  entanglement

# Outline

Motivation

Local Operators

Generalisation

Learning the Environment

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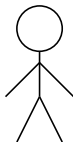
# Motivation

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- three participants: system, observer and environment (no-deletion [Arun Kumar Pati, 2000])
- system, observer and environment form a closed system
- system and observer entangled, environment absorbs observer state

# Motivation (A Schematic)



S

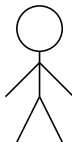


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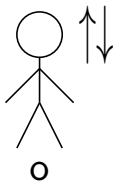


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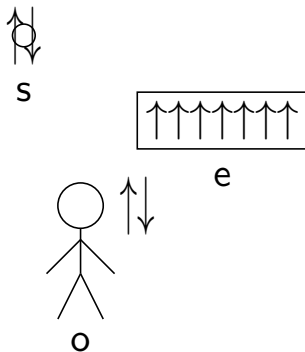


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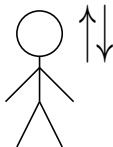
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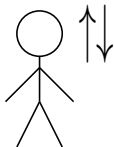
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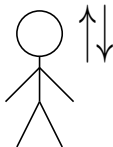


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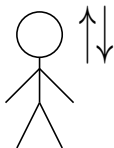


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$$|\psi\rangle_s, |\phi\rangle_o, |\chi\rangle_e \in \mathcal{H} = \text{span}\{|0\rangle, |1\rangle\}$$

# Motivation (The Measurement Procedure)

overall measurement procedure:

$$|\psi\rangle_s |\phi\rangle_o |\chi\rangle_e = (\psi_0 |0\rangle + \psi_1 |1\rangle)_s (\phi_0 |0\rangle + \phi_1 |1\rangle)_o |0\rangle_e$$

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$$|000\rangle_{soe} \rightarrow |000\rangle_{soe}$$

$$|010\rangle_{soe} \rightarrow |001\rangle_{soe}$$

$$|100\rangle_{soe} \rightarrow |110\rangle_{soe}$$

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# Motivation (Extension to Basis)

a natural unitary extension:

$$|001\rangle_{\text{soe}} \rightarrow |010\rangle_{\text{soe}}$$

$$|011\rangle_{\text{soe}} \rightarrow |011\rangle_{\text{soe}}$$

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$$|\psi\rangle_s |\phi\rangle_o |1\rangle_e \rightarrow (\psi_0 |01\rangle + \psi_1 |10\rangle)_{\text{so}} |\phi\rangle_e =: |\bar{\Psi}\rangle_{\text{so}} |\phi\rangle_e$$

# Motivation (Generic State)

generic environment state:

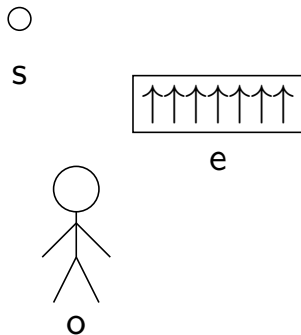
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# Motivation (Summarising)



North points up

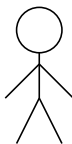
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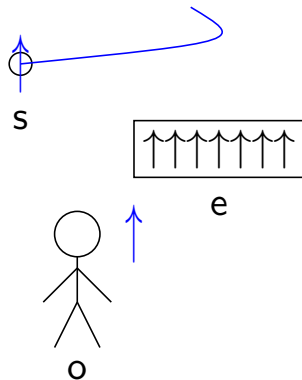
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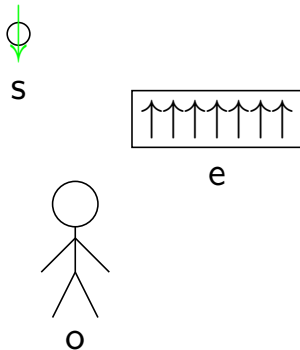
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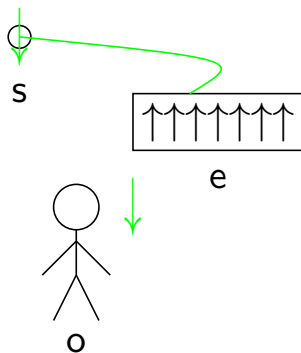
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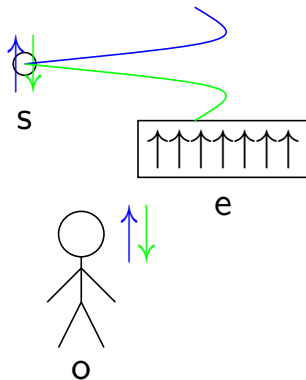
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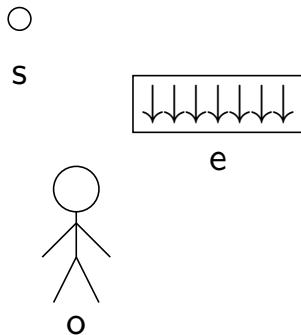


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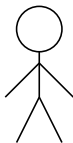
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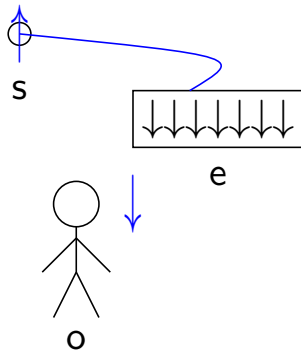
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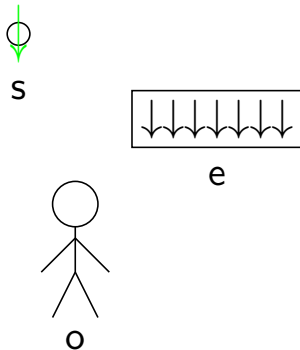
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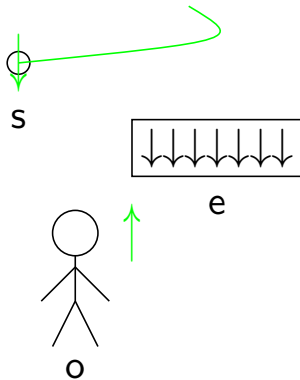
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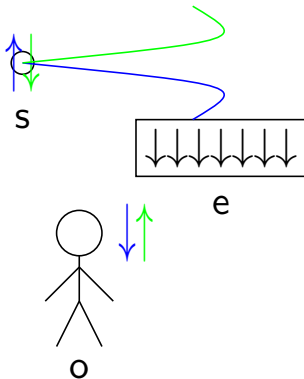
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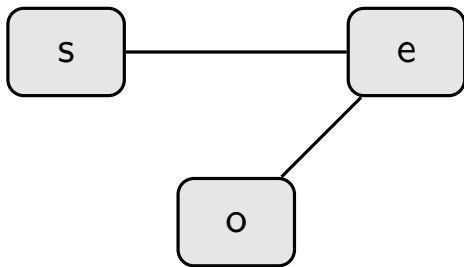


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# Local Operators



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# Local Operators (Imprint)

$\mathcal{I}_{a \rightarrow b}$ :

$$\mathcal{I}_{a \rightarrow b} |00\rangle_{ab} = |00\rangle_{ab}$$

$$\mathcal{I}_{a \rightarrow b} |01\rangle_{ab} = |01\rangle_{ab}$$

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$\equiv$  CNOT gate

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# Local Operators

	$\mathcal{I}_{s \rightarrow e}$		$\mathcal{S}_{o \leftrightarrow e}$	
$ 000\rangle_{\text{soe}}$	$\longrightarrow$	$ 000\rangle_{\text{soe}}$	$\longrightarrow$	$ 000\rangle_{\text{soe}}$
$ 010\rangle_{\text{soe}}$	$\longrightarrow$	$ 010\rangle_{\text{soe}}$	$\longrightarrow$	$ 001\rangle_{\text{soe}}$
$ 100\rangle_{\text{soe}}$	$\longrightarrow$	$ 101\rangle_{\text{soe}}$	$\longrightarrow$	$ 110\rangle_{\text{soe}}$
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$ 101\rangle_{\text{soe}}$	$\longrightarrow$	$ 100\rangle_{\text{soe}}$	$\longrightarrow$	$ 100\rangle_{\text{soe}}$
$ 111\rangle_{\text{soe}}$	$\longrightarrow$	$ 110\rangle_{\text{soe}}$	$\longrightarrow$	$ 101\rangle_{\text{soe}}$

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generalised imprint:

$$\mathcal{I}_{a \rightarrow b} |i\rangle_a |j\rangle_b = |i\rangle_a |j+i\rangle_b$$

'+' defined appropriately

## Generalisation (The Procedure)

$$|\zeta\rangle_{\text{soe}} = |\psi\rangle_s |\phi\rangle_o |\chi\rangle_e = \left( \sum_i \psi_i |i\rangle_s \right) \left( \sum_j \phi_j |j\rangle_o \right) \left( \sum_k \chi_k |k\rangle_e \right)$$

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# Learning the Environment

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starting with "classical" environment:

$$|\chi\rangle_e = \sum_k \chi_k |k\rangle_{e_1} |k\rangle_{e_2} |k\rangle_{e_3} \dots |k\rangle_{e_N}$$

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$|i\rangle$

$|\phi\rangle$

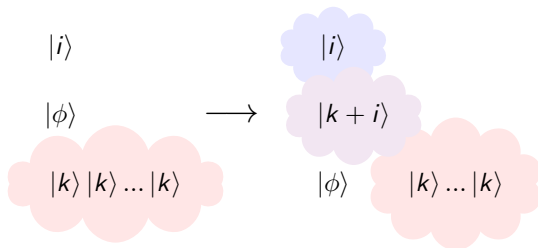
$|k\rangle |k\rangle \dots |k\rangle$

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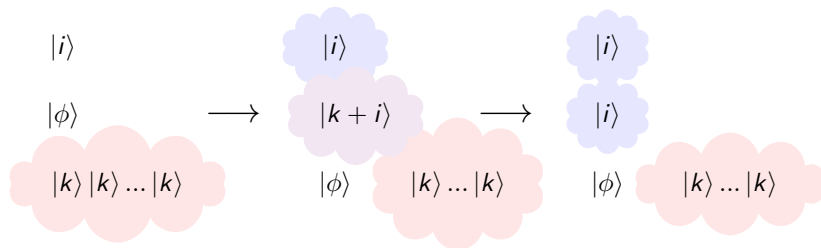


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# Learning the Environment (The Procedure)

$$\mathcal{S}_{o \leftrightarrow e_1} \circ \mathcal{I}_{s \rightarrow e_1}$$

$$s : |i\rangle_s$$

$$o : |\phi\rangle_o$$

$$e : |k\rangle_{e_1} |k\rangle_{e_2} \dots |k\rangle_{e_N}$$

$\hookrightarrow$

$$s : |i\rangle_s$$

$$o : |k + i\rangle_o$$

$$e : |\phi\rangle_{e_1} |k\rangle_{e_2} \dots |k\rangle_{e_N}$$

# Learning the Environment (The Procedure)

$$\mathcal{I}_{e_2 \rightarrow o}^{-1}$$

$\hookrightarrow$

$=$

$$s : |i\rangle_s$$

$$o : |k + i\rangle_o$$

$$e : |\phi\rangle_{e_1} |k\rangle_{e_2} \dots |k\rangle_{e_N}$$

$$s : |i\rangle_s$$

$$o : |k + i - k\rangle_o$$

$$e : |\phi\rangle_{e_1} |k\rangle_{e_2} \dots |k\rangle_{e_N}$$

$$s : |i\rangle_s$$

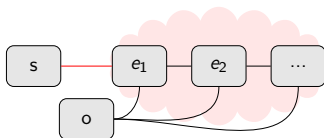
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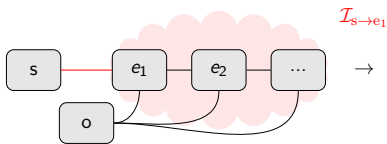
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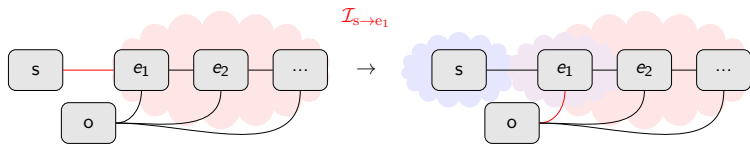
process of entangling system and observer to achieve measurement

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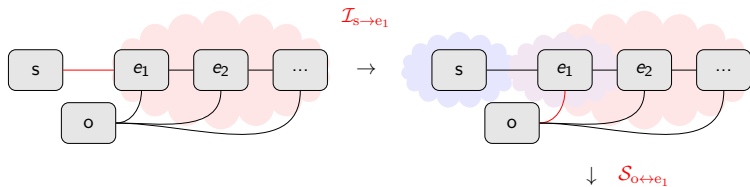
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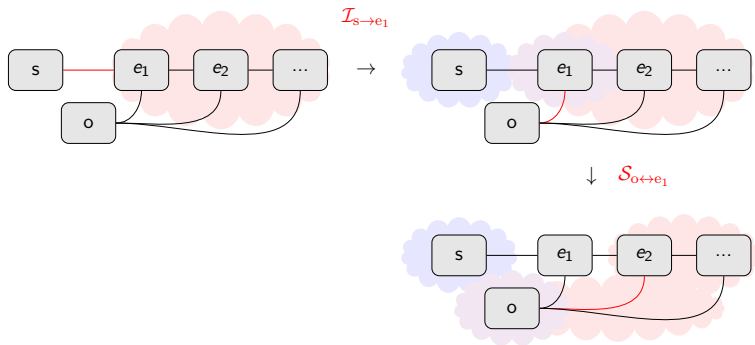
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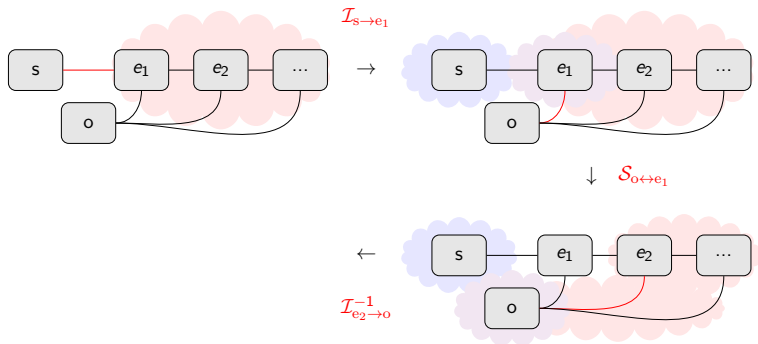
process of entangling system and observer to achieve measurement

# Summary



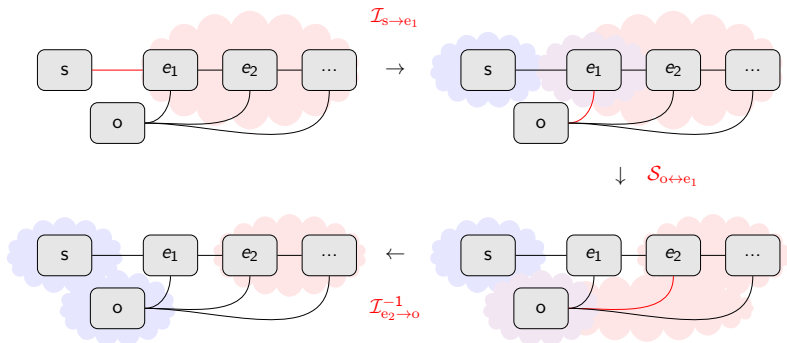
process of entangling system and observer to achieve measurement

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# Summary

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- perfect entanglement between system and observer




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- classical environment required for entanglement

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- perfect entanglement between system and observer
- classical environment required for entanglement
- wavefunction also contains counterfactual branches!

# Bibliography

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# Thanks

Firstly, thanks to Torsten Enßlin for the all the guidance.

And thanks for listening!