

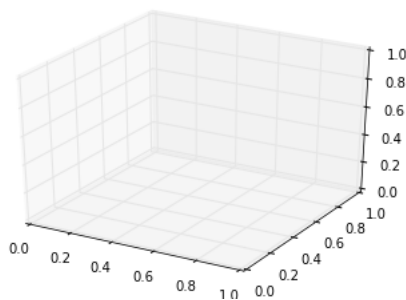
Three-Dimensional Plotting in Matplotlib

Creating an effective visualization requires leading the viewer in such a way that the figure conveys information. This narrative may be conveyed purely visually in some instances, with no further writing required, while in others, brief explanatory labels and indications are required. Axes labels and titles are likely the most fundamental annotations you will use, but they are certainly not the only ones. Let's have a look at some data and see if we can find a way to present it with intriguing visualizations and annotations. The first step is to load the necessary functions and prepare the notebook for plotting.

```
from mpl_toolkits import mplot3d
```

Once this submodule is imported, a three-dimensional axes can be created by passing the keyword `projection='3d'` to any of the normal axes creation routines:

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
fig = plt.figure()
ax = plt.axes(projection='3d')
```



With this three-dimensional axes enabled, we can now plot a variety of three-dimensional plot types. Three-dimensional plotting is one of the functionalities that benefits immensely from viewing figures interactively rather than statically in the notebook; recall that to use interactive figures, you can use `%matplotlib notebook` rather than `%matplotlib inline` when running this code.

Three-dimensional Points and Lines

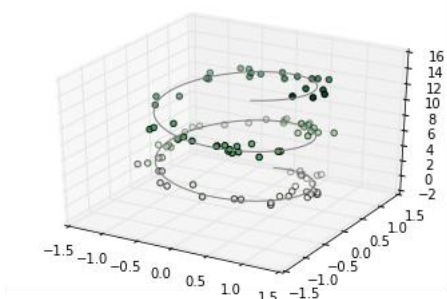
The most basic three-dimensional plot is a line or collection of scatter plot created from sets of (x, y, z) triples. In analogy with the more common two-dimensional plots discussed earlier, these can be created using the `ax.plot3D` and `ax.scatter3D` functions. The call signature for these is nearly identical to that of their two-dimensional counterparts, so you can refer to Simple Line Plots and Simple Scatter Plots for more information on controlling the output. Here we'll plot a trigonometric spiral, along with some points drawn randomly near the line:

```
ax = plt.axes(projection='3d')

# Data for a three-dimensional line
zline = np.linspace(0, 15, 1000)
xline = np.sin(zline)
yline = np.cos(zline)
```

```
ax.plot3D(xline, yline, zline, 'gray')

# Data for three-dimensional scattered points
zdata = 15 * np.random.random(100)
xdata = np.sin(zdata) + 0.1 * np.random.randn(100)
ydata = np.cos(zdata) + 0.1 * np.random.randn(100)
ax.scatter3D(xdata, ydata, zdata, c=zdata, cmap='Greens');
```



Notice that by default, the scatter points have their transparency adjusted to give a sense of depth on the page. While the three-dimensional effect is sometimes difficult to see within a static image, an interactive view can lead to some nice intuition about the layout of the points.

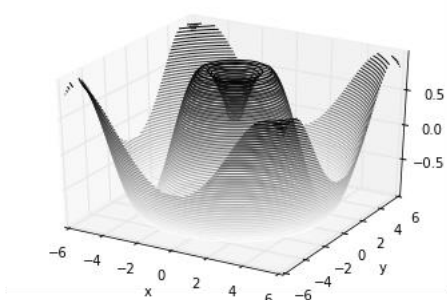
Three-dimensional Contour Plots

Analogous to the contour plots we explored in Density and Contour Plots, `mplot3d` contains tools to create three-dimensional relief plots using the same inputs. Like two-dimensional `ax.contour` plots, `ax.contour3D` requires all the input data to be in the form of two-dimensional regular grids, with the Z data evaluated at each point. Here we'll show a three-dimensional contour diagram of a three-dimensional sinusoidal function:

```
def f(x, y):
    return np.sin(np.sqrt(x ** 2 + y ** 2))

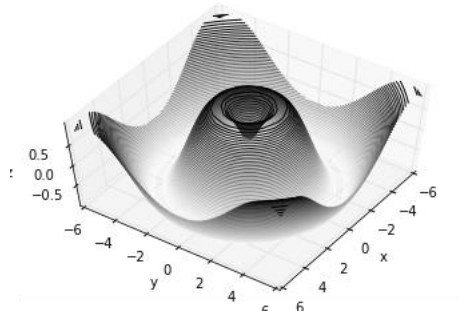
x = np.linspace(-6, 6, 30)
y = np.linspace(-6, 6, 30)

X, Y = np.meshgrid(x, y)
Z = f(X, Y)
fig = plt.figure()
ax = plt.axes(projection='3d')
ax.contour3D(X, Y, Z, 50, cmap='binary')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z');
```



Sometimes the default viewing angle is not optimal, in which case we can use the `view_init` method to set the elevation and azimuthal angles. In the following example, we'll use an elevation of 60 degrees (that is, 60 degrees above the x-y plane) and an azimuth of 35 degrees (that is, rotated 35 degrees counter-clockwise about the z-axis):

```
ax.view_init(60, 35)
fig
```

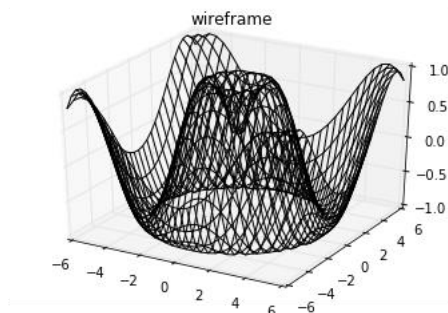


Again, note that this type of rotation can be accomplished interactively by clicking and dragging when using one of Matplotlib's interactive backends.

Wireframes and Surface Plots

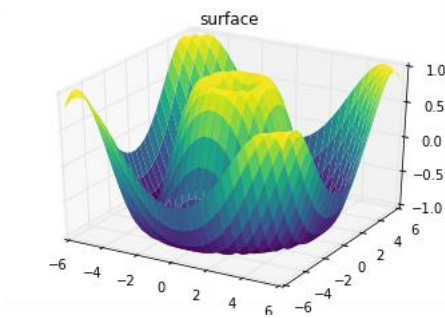
Two other types of three-dimensional plots that work on gridded data are wireframes and surface plots. These take a grid of values and project it onto the specified three-dimensional surface, and can make the resulting three-dimensional forms quite easy to visualize. Here's an example of using a wireframe:

```
fig = plt.figure()
ax = plt.axes(projection='3d')
ax.plot_wireframe(X, Y, Z, color='black')
ax.set_title('wireframe');
```



A surface plot is like a wireframe plot, but each face of the wireframe is a filled polygon. Adding a colormap to the filled polygons can aid perception of the topology of the surface being visualized:

```
ax = plt.axes(projection='3d')
ax.plot_surface(X, Y, Z, rstride=1, cstride=1,
               cmap='viridis', edgecolor='none')
ax.set_title('surface');
```

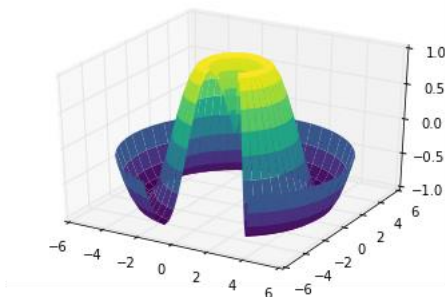


Note that though the grid of values for a surface plot needs to be two-dimensional, it need not be rectilinear. Here is an example of creating a partial polar grid, which when used with the `surface3D` plot can give us a slice into the function we're visualizing:

```
r = np.linspace(0, 6, 20)
theta = np.linspace(-0.9 * np.pi, 0.8 * np.pi, 40)
r, theta = np.meshgrid(r, theta)

X = r * np.sin(theta)
Y = r * np.cos(theta)
Z = f(X, Y)

ax = plt.axes(projection='3d')
ax.plot_surface(X, Y, Z, rstride=1, cstride=1,
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```



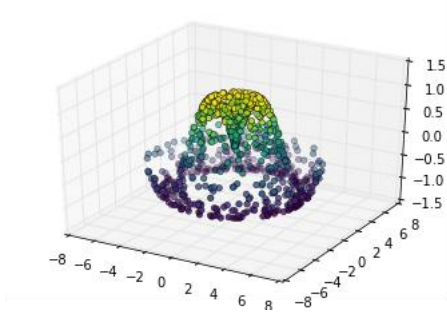
Surface Triangulations

For some applications, the evenly sampled grids required by the above routines is overly restrictive and inconvenient. In these situations, the triangulation-based plots can be very useful. What if rather than an even draw from a Cartesian or a polar grid, we instead have a set of random draws?

```
theta = 2 * np.pi * np.random.random(1000)
r = 6 * np.random.random(1000)
x = np.ravel(r * np.sin(theta))
y = np.ravel(r * np.cos(theta))
z = f(x, y)
```

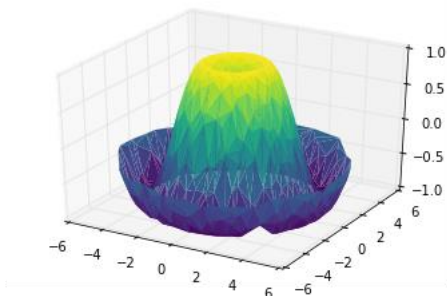
We could create a scatter plot of the points to get an idea of the surface we're sampling from:

```
ax = plt.axes(projection='3d')
ax.scatter(x, y, z, c=z, cmap='viridis', linewidth=0.5);
```



This leaves a lot to be desired. The function that will help us in this case is `ax.plot_trisurf`, which creates a surface by first finding a set of triangles formed between adjacent points (remember that `x`, `y`, and `z` here are one-dimensional arrays):

```
ax = plt.axes(projection='3d')
ax.plot_trisurf(x, y, z,
                cmap='viridis', edgecolor='none');
```



The result is certainly not as clean as when it is plotted with a grid, but the flexibility of such a triangulation allows for some really interesting three-dimensional plots. For example, it is actually possible to plot a three-dimensional Möbius strip using this, as we'll see next.

Example: Visualizing a Möbius strip

A Möbius strip is similar to a strip of paper glued into a loop with a half-twist. Topologically, it's quite interesting because despite appearances it has only a single side! Here we will visualize such an object using Matplotlib's three-dimensional tools. The key to creating the Möbius strip is to think about its parametrization: it's a two-dimensional strip, so we need two intrinsic dimensions. Let's call them θ

, which ranges from 0 to 2π around the loop, and w

which ranges from -1 to 1 across the width of the strip:

```
theta = np.linspace(0, 2 * np.pi, 30)
w = np.linspace(-0.25, 0.25, 8)
w, theta = np.meshgrid(w, theta)
```

Now from this parametrization, we must determine the (x, y, z) positions of the embedded strip.

Thinking about it, we might realize that there are two rotations happening: one is the position of the loop about its center (what we've called θ), while the other is the twisting of the strip about its axis (we'll call this ϕ). For a Möbius strip, we must have the strip makes half a twist during a full loop, or $\Delta\phi = \Delta\theta/2$

.

```
phi = 0.5 * theta
```

Now we use our recollection of trigonometry to derive the three-dimensional embedding. We'll define r , the distance of each point from the center, and use this to find the embedded (x,y,z)

coordinates:

```
# radius in x-y plane
r = 1 + w * np.cos(phi)

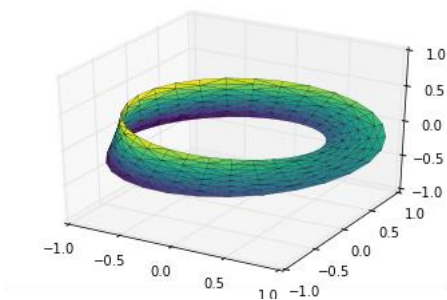
x = np.ravel(r * np.cos(theta))
y = np.ravel(r * np.sin(theta))
z = np.ravel(w * np.sin(phi))
```

Finally, to plot the object, we must make sure the triangulation is correct. The best way to do this is to define the triangulation *within the underlying parametrization*, and then let Matplotlib project this triangulation into the three-dimensional space of the Möbius strip. This can be accomplished as follows:

```
# triangulate in the underlying parametrization
from matplotlib.tri import Triangulation
tri = Triangulation(np.ravel(w), np.ravel(theta))

ax = plt.axes(projection='3d')
ax.plot_trisurf(x, y, z, triangles=tri.triangles,
               cmap='viridis', linewidths=0.2);

ax.set_xlim(-1, 1); ax.set_ylim(-1, 1); ax.set_zlim(-1, 1);
```



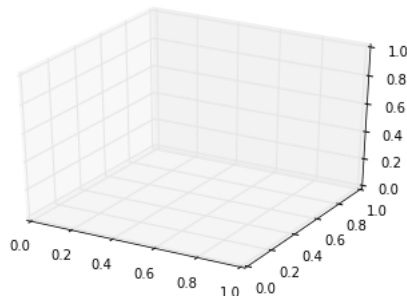
Combining all of these techniques, it is possible to create and display a wide variety of three-dimensional objects and patterns in Matplotlib.

Matplotlib was initially designed with only two-dimensional plotting in mind. Around the time of the 1.0 release, some three-dimensional plotting utilities were built on top of Matplotlib's two-dimensional display, and the result is a convenient (if somewhat limited) set of tools for three-dimensional data visualization. three-dimensional plots are enabled by importing the `mplot3d` toolkit, included with the main Matplotlib installation:

```
from mpl_toolkits import mplot3d
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```
%matplotlib inline
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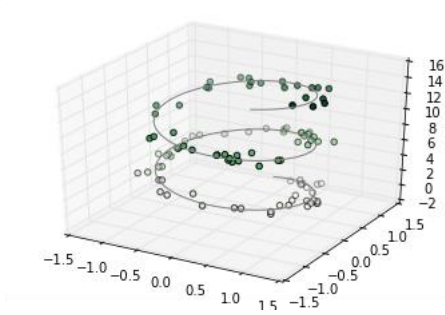
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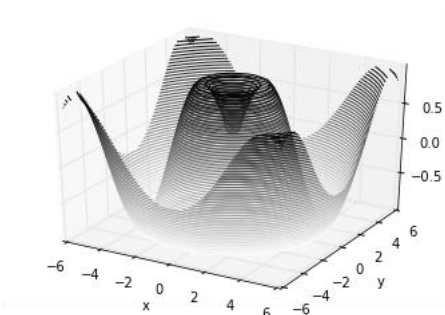
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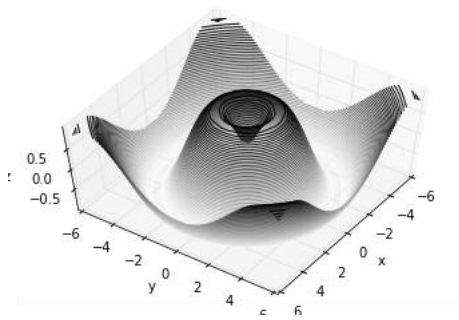
x = np.linspace(-6, 6, 30)
y = np.linspace(-6, 6, 30)

X, Y = np.meshgrid(x, y)
Z = f(X, Y)
fig = plt.figure()
ax = plt.axes(projection='3d')
ax.contour3D(X, Y, Z, 50, cmap='binary')
ax.set_xlabel('x')
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ax.set_zlabel('z');
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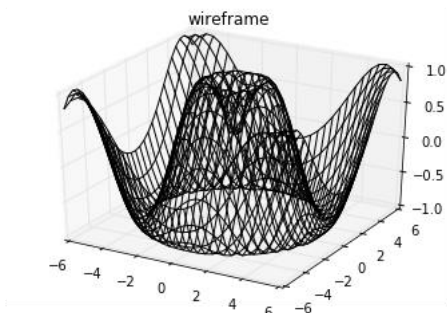



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Wireframes and Surface Plots

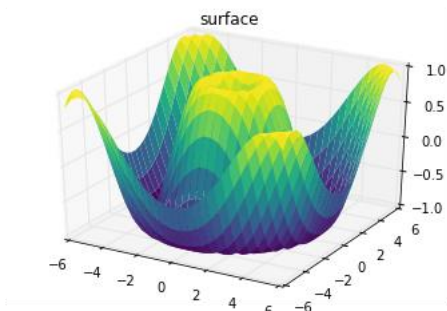
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ax.set_title('wireframe');
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A surface plot is like a wireframe plot, but each face of the wireframe is a filled polygon. Adding a colormap to the filled polygons can aid perception of the topology of the surface being visualized:

```
ax = plt.axes(projection='3d')
ax.plot_surface(X, Y, Z, rstride=1, cstride=1,
               cmap='viridis', edgecolor='none')
ax.set_title('surface');
```

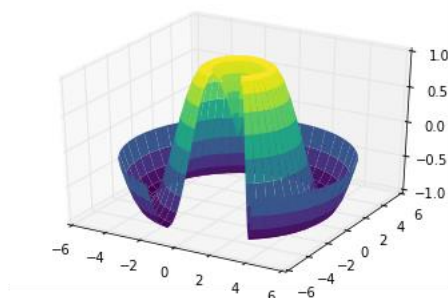


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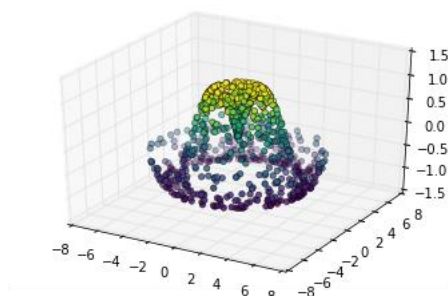
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y = np.ravel(r * np.cos(theta))
z = f(x, y)
```

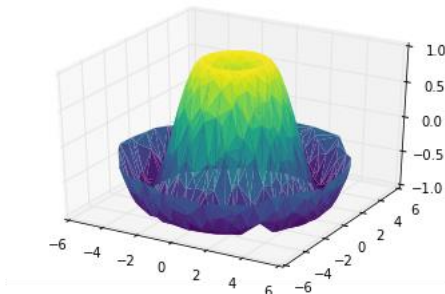
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This leaves a lot to be desired. The function that will help us in this case is `ax.plot_trisurf`, which creates a surface by first finding a set of triangles formed between adjacent points (remember that `x`, `y`, and `z` here are one-dimensional arrays):

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