

# Calculating cloud top height from surface temperature, dewpoint and IR brightness temperature

## 1 Calculation procedure

1. From  $T$ ,  $T_d$  [°C], and  $p$  [Pa] (pressure level of  $T$  and  $T_d$ )  $\theta_e$  is calculated using Bolton's 1980 formulas:

$$T_L = \frac{1}{\frac{1}{T_D - 56} + \frac{\ln(T_K/T_D)}{800}} + 56 \quad (\text{LCL temperature [K]})$$

$$\theta_{DL} = T_K \left( \frac{1000}{p - e_s} \right)^k \left( \frac{T_K}{T_L} \right)^{.28r_s}$$

$$\theta_e = \theta_{DL} \exp \left[ \left( \frac{3036}{T_L} - 1.78 \right) * r_s (1 + .448r_s) \right]$$

where:  $T_K = T + 273.15\text{K}$ ,  $T_D = T_d + 273.15\text{K}$ ,  $k = R_d/C_{pd} = 0.28571$ ,

$e_s(T_d) = 6.112e^{\frac{17.67T_d}{T_d + 243.5}}$  (saturation vapor pressure)

$r_s = \frac{0.622e_s(T_d)}{(p - e_s(T_d))}$  (saturation mixing ratio)

2. From  $\theta_e$ ,  $\theta_w$  is calculated using Davies Jones 2008 very accurate approximation:

$$\theta_w = \theta_e - 273.15\text{K} - \exp\left(\frac{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}{1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4}\right)$$

where  $x = \theta_e/273.15\text{K}$

when  $\theta_e \geq 173.15\text{K}$

for  $\theta_e < 173.15\text{K}$   $\theta_w = \theta_e - 273.15\text{K}$

3. Moist adiabat polynomial coefficients depend on  $\theta_w$  and are calculated as follows:

$$C_i(\theta_w) = \sum_{j=0}^4 a_{ij}\theta_w^j$$

$C_i$	$a_{i0}$	$a_{i1}$	$a_{i2}$	$a_{i3}$	$a_{i4}$
$C_0$	9.981118e+02	-1.865352e+01	-7.228945e-02	-1.288899e-04	4.152094e-05
$C_1$	1.868462e+01	-1.584316e-01	-5.652978e-03	7.782649e-05	-1.697159e-07
$C_2$	2.347380e-01	2.461856e-03	-1.223192e-04	-1.929728e-07	1.532080e-08
$C_3$	2.285961e-03	5.360970e-05	-2.299950e-07	-6.829626e-08	9.717708e-10
$C_4$	1.275047e-05	2.984764e-07	1.986974e-08	-1.344314e-09	1.835750e-11
$C_5$	2.928147e-08	-7.653230e-11	1.876537e-10	-9.439792e-12	1.349002e-13

4. Moist adiabat approximation 5th degree polynomial  $p(t, \theta_w)$ :

$$p(t, \theta_w) = \sum_{i=0}^5 C_i(\theta_w)t^i$$

Which gives us cloud top pressure level from IR BT.