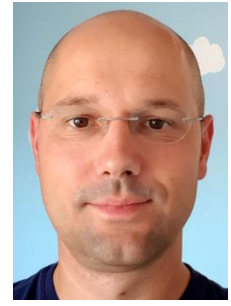# Fast Approximation for Calculating Deep Convection Cloud Top Heights from Satellite Brightness Temperature

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1. Why? (Motivation)

Deep convection cloud top height diagnosis is very important in aviation meteorology. One of the methods to estimate altitude of existing convective cloud tops is to compare infrared satellite brightness temperature (BT) with a calculated parcel curve temperature („BT-parcel" method). When we already have deep convective clouds we do not need the full vertical temperature profile (sounding). To calculate the cloud top pressure, which is directly related to altitude in the standard atmosphere, we only need the parcel curve (moist adiabat) and measured satellite brightness temperature. Moist adiabats are usually calculated iteratively from surface temperature and dewpoint, but this calculation can be computationally quite intensive. Inspired by previous work on non-iterative calculations of moist adiabats (*Bakhshaii and Stull 2013, Moisseeva and Stull 2017.*) **we wanted to test even simpler approximations, which are still accurate enough for estimating cloud top pressure level.**
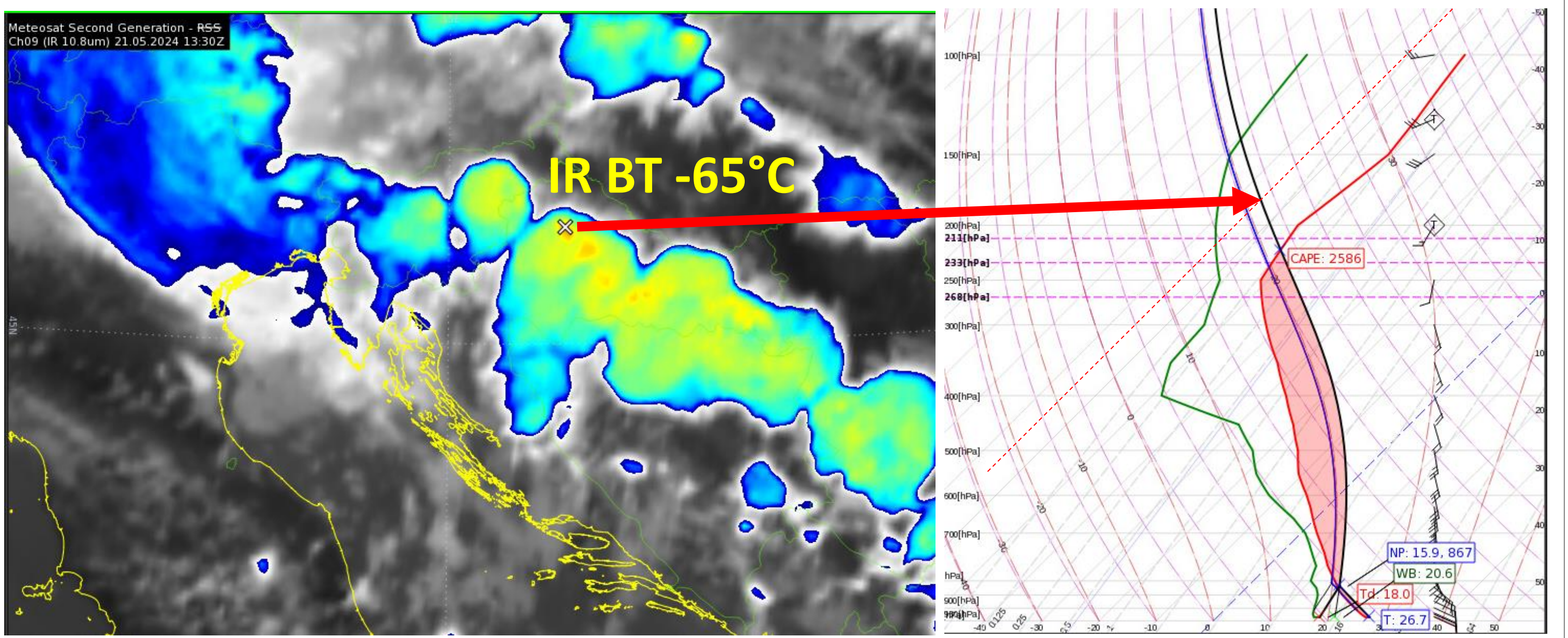

Fig 1. Manual method used in forecasting operations to estimate convective cloud top heights. Infrared brightness temperature is compared to parcel curve (from NWP or OBS) to get cloud top pressure which translates into pressure altitude (ICAO standard atmosphere) or flight level (FL).

2. How did we do it? (Method)

1. We calculated true moist adiabats using the iterative formula (MetPy Python package) for a range of wet bulb potential temperatures (θ_w) from 0 to 40°C in 500 steps, representing different combinations of surface temperature (T) and dewpoint (T_d).
2. For each θ_w value moist adiabat was fitted with 5th-degree polynomial (6 coefficients C_i).
3. As moist adiabat shape changes with θ_w, this C_i dependency on θ_w was also modelled with polynomials. We tried different degrees of polynomials for $C_i(\theta_w)$ and found that 4th degree had the smallest absolute errors in the given θ_w range (Fig 3.).
4. As a result, we got the total of 6 x 5 coefficients (6 C_i coefficients from which pressure is calculated and 5 coefficients for each $C_i(\theta_w)$ calculation) shown in Table 1. Wet bulb potential temperature is calculated from input T, T_d, and p (pressure level of T and T_d) using Bolton's 1980 equations for equivalent potential temperature (θ_e) and Davies-Jones 2008 approximation of θ_w from θ_e. **With input values of T, T_d, p, and BT, cloud top pressure can be calculated in a few simple steps** (see procedure on the right). Compared to e.g. Mioisseva and Stull 2017 who had a total of 200 coefficients this is a significant simplification.

Table 1. All polynomial coefficients needed to calculate pressure at some temperature (BT) .

C_i	a_{i0}	a_{i1}	a_{i2}	a_{i3}	a_{i4}
C_0	9.981118e+02	-1.865352e+01	-7.228945e-02	-1.288899e-04	4.152094e-05
C_1	1.868462e+01	-1.584316e-01	-5.652978e-03	7.782649e-05	-1.697159e-07
C_2	2.347380e-01	2.461856e-03	-1.223192e-04	-1.929728e-07	1.532080e-08
C_3	2.285961e-03	5.360970e-05	-2.299950e-07	-6.829626e-08	9.717708e-10
C_4	1.275047e-05	2.984764e-07	1.986974e-08	-1.344314e-09	1.835750e-11
C_5	2.928147e-08	-7.653230e-11	1.876537e-10	-9.439792e-12	1.349002e-13

T, T_d [°C], p [hPa] => θ_e (Bolton's 1980 formulas), θ_e => θ_w (Davies-Jones 2008 formula). Six C_i coefficients are calculated using values from the Table 1. and θ_w[°C] as follows:

$$C_i(\theta_w) = \sum_{j=0}^{4} a_{ij}\,\theta_w^j \qquad\Longrightarrow\qquad p(t) = \sum_{i=0}^{5} C_i(\theta_w)\,t^i$$

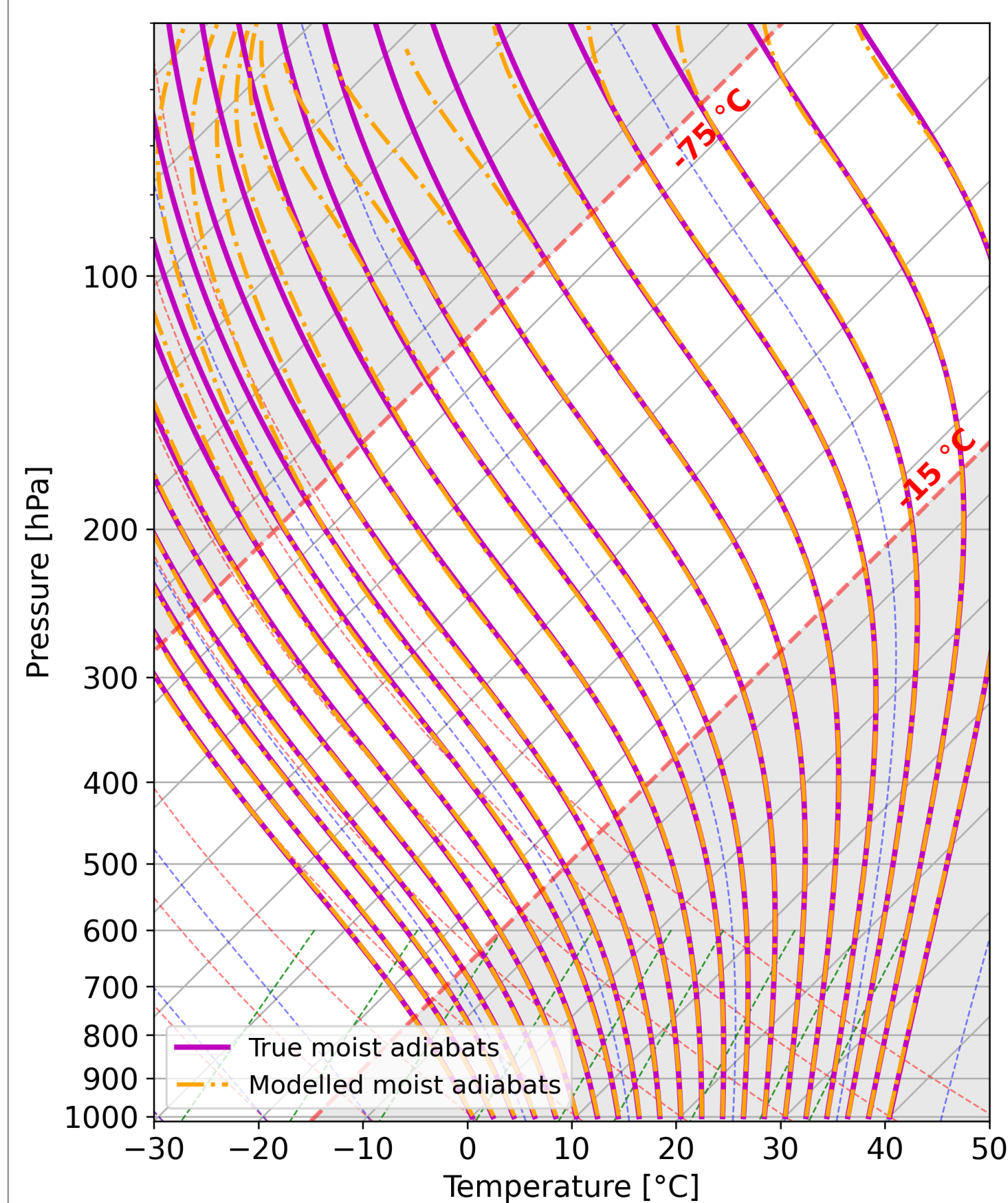## 3. What did we get? (Approximation test results)

Fig 2. Visual comparison of true and modelled moist adiabats in the usual range of convective cloud top temperatures. Modelled moist adiabats are approximated using 5th degree polynomial, with each coefficient C_i dependency on θ_w approximated with 4th degree polynomial.
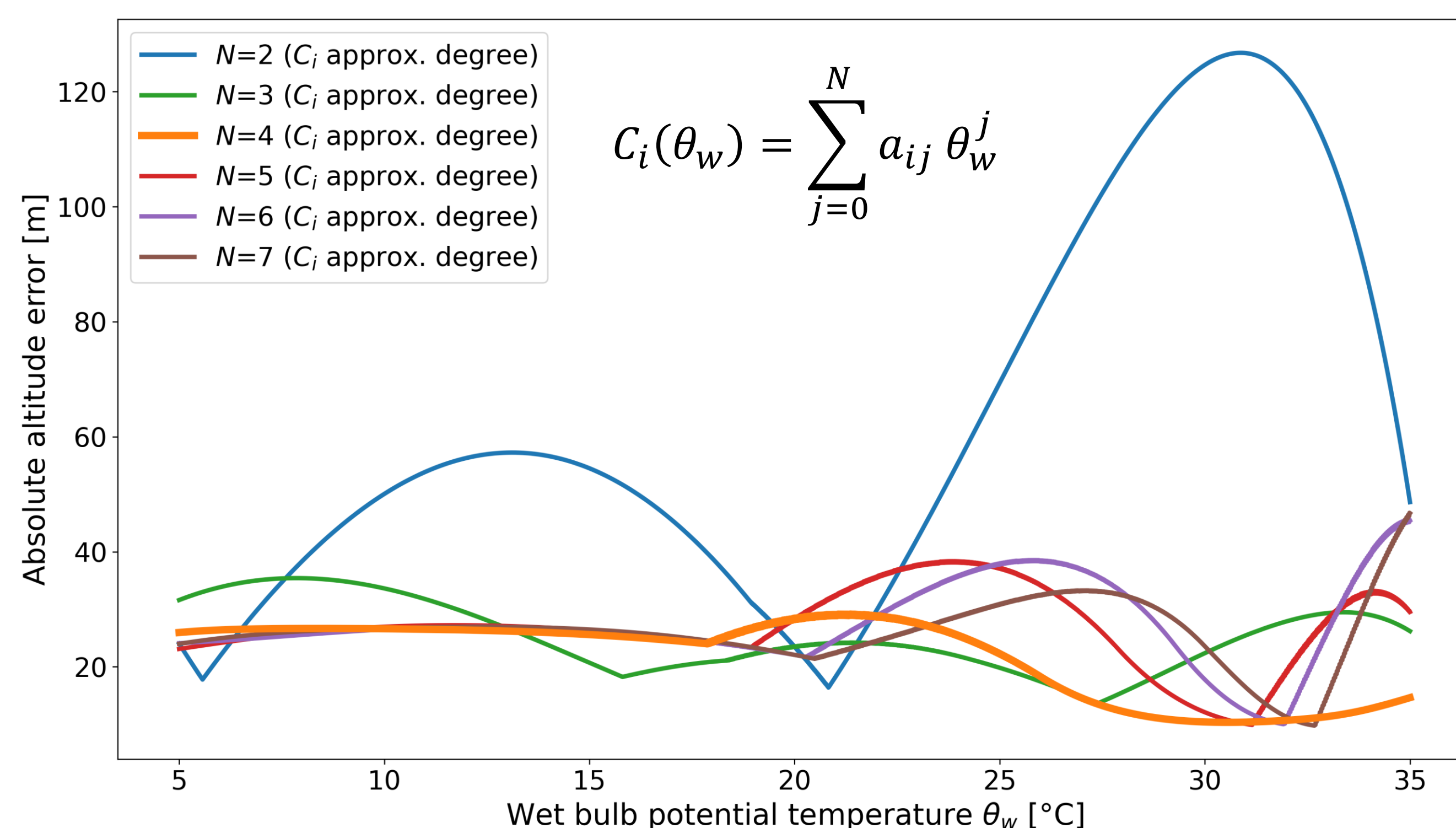

$$C_i(\theta_w) = \sum_{j=0}^{N} a_{ij}\,\theta_w^j$$

Fig 3. Absolute altitude errors for different coefficient $C_i(\theta_w)$ approximations.

Table 2. Error metrics for different degrees (N) of $C_i(\theta_w)$ polynomial approximation.

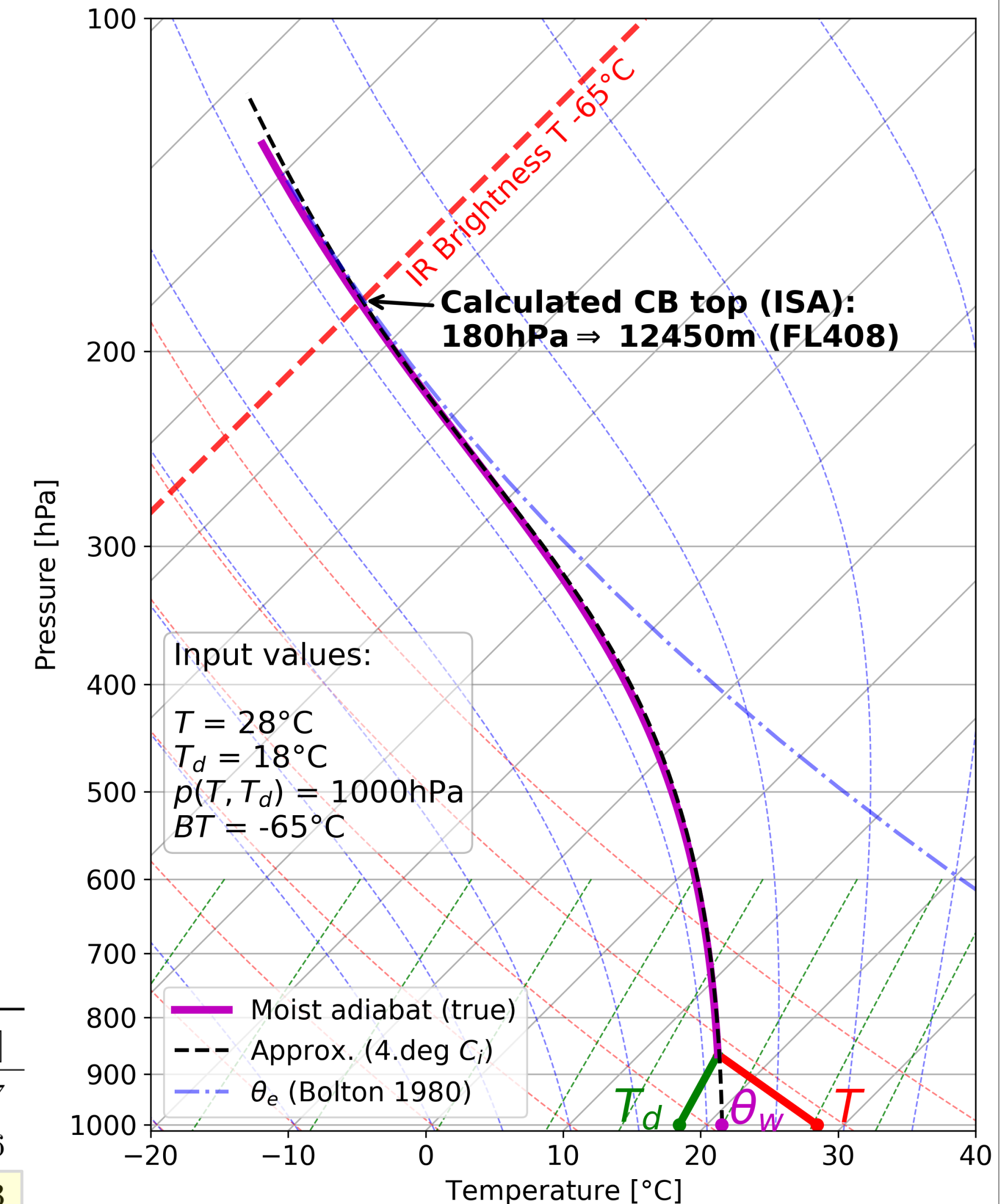| Degree (N)	RMSE min [m]	RMSE max [m]	MAX ABS ERR min [m]	MAX ABS ERR max [m]
2	0.429428	3.25341	16.7	126.7
3	0.204931	0.782863	13.7	33.6
4	0.132028	0.759327	10.3	28.3
5	0.132075	0.765255	9.5	37.4
6	0.11539	0.772867	9.7	37.7
7	0.114245	0.774773	9.6	33.2

Fig 4. Visualized example of cumulonimbus (CB) top height calculation.

4. Discussion

This work is novel in the way that shape dependency of moist adiabats on θ_w was modelled through coefficient dependency on θ_w. It should also be noted that the „BT-parcel" method for estimating convective cloud top heights is not without its flaws. It relies on the inherent limitations of parcel theory (neglecting dynamical effects and entrainment), infrared satellite resolution (brightness temperature accuracy), and, most importantly, the choice and accuracy of starting temperature and dewpoint (e.g., elevated convection). As a result, we anticipate errors of up to 500 meters when using this method. Therefore, the moist adiabat approximation with an altitude error around 30 meters is quite acceptable for this purpose. In operational practice, this method remains as one of the best options, especially over areas without radar coverage.

5. Future work

- **Our moist adiabat approximation can also be used for more precise calculations with higher degree polynomials**
- **Next step is implementation of this calculation in Visual Weather software (IBL), in order to automate CB top labels on IR satellite image**
- Improvement of the whole „BT-parcel" method can also be made by smart choice of T and T_d by e.g. search for the most unstable parcel (highest θ_w) in the convective area.

References:
Bakhshaii, Atoossa, and Roland Stull. "Saturated Pseudoadiabats—A Noniterative Approximation." Journal of Applied Meteorology and Climatology 52, no. 1 (2013): 5–15.
Bolton, David. "The Computation of Equivalent Potential Temperature." *Monthly Weather Review* 108, no. 7 (July 1, 1980): 1046–53. https://doi.org/10.1175/1520-0493(1980)108<1046:TCOEPT>2.0.CO;2.
Davies-Jones, Robert. "An Efficient and Accurate Method for Computing the Wet-Bulb Temperature along Pseudoadiabats." *Monthly Weather Review* 136, no. 7 (2008): 2764–85. https://doi.org/10.1175/2007MWR2224.1.
Moisseeva, Nadya, and Roland Stull. "Technical Note: A Noniterative Approach to Modelling Moist Thermodynamics." Atmospheric Chemistry and Physics 17, no. 24 (December 19, 2017): 15037–43.