Calculating cloud top height from surface temperature, dewpoint and IR brightness temperature

1 Calculation procedure

1. From T, T_d [°C], and p [Pa] (pressure level of T and T_d) θ_e is calculated using Bolton's 1980 formulas:

$$T_L = \frac{1}{\frac{1}{T_D - 56} + \frac{ln(T_K/T_D)}{800}} + 56 \quad \text{(LCL temperature [K])}$$

$$\theta_{DL} = T_K \left(\frac{1000}{p - e_s}\right)^k \left(\frac{T_K}{T_L}\right)^{.28r_s}$$

$$\theta_e = \theta_{DL} \exp\left[\left(\frac{3036.}{T_L} - 1.78\right) * r_s(1 + .448r_s)\right]$$

where: $T_K = T + 273.15$K, $T_D = T_d + 273.15$K, $k = R_d/C_{pd}$ =0.28571,
$e_s(T_d) = 6.112e^{\frac{17.67T_d}{T_d+243.5}}$ (saturation vapor pressure)
$r_s = \frac{0.622e_s(T_d)}{(p-e_s(T_d))}$ (saturation mixing ratio)

2. From θ_e, θ_w is calculated using Davies Jones 2008 very accurate approximation:

$$\theta_w = \theta_e - 273.15K - exp(\frac{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}{1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4})$$

where $x = \theta_e/273.15$ K
when $\theta_e >= 173.15$ K
for $\theta_e < 173.15$ K $\quad \theta_w = \theta_e - 273.15K$

3. Moist adiabat polynomial coefficients depend on θ_w and are calculated as follows:

$$C_i(\theta_w) = \sum_{j=0}^{4} a_{ij}\theta_w^i$$

C_i	a_{i0}	a_{i1}	a_{i2}	a_{i3}	a_{i4}
C_0	9.981118e+02	-1.865352e+01	-7.228945e-02	-1.288899e-04	4.152094e-05
C_1	1.868462e+01	-1.584316e-01	-5.652978e-03	7.782649e-05	-1.697159e-07
C_2	2.347380e-01	2.461856e-03	-1.223192e-04	-1.929728e-07	1.532080e-08
C_3	2.285961e-03	5.360970e-05	-2.299950e-07	-6.829626e-08	9.717708e-10
C_4	1.275047e-05	2.984764e-07	1.986974e-08	-1.344314e-09	1.835750e-11
C_5	2.928147e-08	-7.653230e-11	1.876537e-10	-9.439792e-12	1.349002e-13

4. Moist adiabat approximation 5th degree polynomial $p(t, \theta_w)$:

$$p(t, \theta_w) = \sum_{i=0}^{5} C_i(\theta_w)t^i$$

Which gives us cloud top pressure level from IR BT.