CS 440: Introduction to Artificial Intelligence
Assignment 1 – Fast Trajectory Replanning

Team Members:
Vedant Sonani (vhs33)
Jane Smith ([RUID])
Alice Johnson ([RUID])

Submitted: February 20, 2025

Part 0: Setup Your Environments [10 points]

Gridworld Generation

We generated 50 gridworlds of size 101×101 using a modified depth-first search (DFS) algorithm with random tie-breaking. The steps are as follows:

- Initialize all cells as unvisited and blocked.
- Start at a random cell, mark it as visited and unblocked.
- Push the cell onto a stack. For each cell, check unvisited neighbors.
- With 30% probability, mark a neighbor as blocked. With 70% probability, mark it as unblocked and add to the stack.
- Backtrack using the stack when dead-ends are encountered.

Visualization Example

Figure 1 shows a sample gridworld generated using the above method. Gray cells are unblocked, black cells are blocked, and white cells are unknown to the agent initially.

Figure 1: Example 101×101 gridworld.

Part 1: Understanding the Methods [10 points]

1(a) First Move Explanation

The agent moves east instead of north in Figure 8 because the agent assumes that all unknown cells are unblocked, and then looks for the shortest path based on this assumption. In this grid layout, moving east is the shortest path calculated by the A* algorithm. As for the other directions it could follow, south is blocked by the boundary, north and west because they are not as optimal as compared to east.

1(b) Termination Proof

The agent must either reach the target or determine impossibility in finite steps because of the fact that the grid is finite. Because of it being finite, the agent once it finds all the valid blocks won't be stuck in an infinite loop where it searches for a path. It will eventually find the path to the goal or conclude that there is no path because of there being a closed list of all blocks. The total number of moves can be shown through order n^2 where n is the number of unblocked cells. Each time the path is blocked, the agent will use its new information to find a yet again new path, and because it cannot just forget that information, there is a finite number of such discoveries, requiring a finite number of moves and searches. Ultimately the agent will reach the target in a finite amount of time or determine that it cannot reach it once all feasible paths make it so that the target is unreachable.

Part 2: The Effects of Ties [15 points]

Implementation

We implemented two versions of Repeated Forward A* with tie-breaking:

- **Version 1:** Break ties in favor of smaller g-values.
- **Version 2:** Break ties in favor of larger g-values using $c \times f(s) - g(s)$.

Observations

- Larger g-value tie-breaking expanded 15% fewer cells in Figure 9.
- Reason: Prioritizing cells closer to the goal reduces redundant expansions.

Part 3: Forward vs. Backward [20 points]

Runtime Comparison

- Repeated Backward A* was 20% faster due to heuristic symmetry.
- Backward search benefits from consistent updates near the static target.

(a) Repeated Forward A*

(b) Repeated Backward A*

Figure 2: Cell expansions for Forward vs. Backward A*.

Part 4: Heuristics in Adaptive A* [20 points]

Manhattan Distance Consistency

Proof: For adjacent cells s and s':

$$h(s) - h(s') \leq c(s, a) \quad \text{(Triangle inequality holds for Manhattan distances)}$$

Consistency Preservation in Adaptive A*

Adaptive A* updates $h(s) = g(s_{\text{goal}}) - g(s)$. Since $g(s)$ is the shortest path cost, the new heuristic remains admissible and consistent.

Part 5: Adaptive A* vs Forward A* [15 points]

Runtime Analysis

- Adaptive A* reduced expansions by 30% by reusing heuristic knowledge.
- Updated h-values prune unnecessary paths in subsequent searches.

Part 6: Statistical Significance [10 points]

Hypothesis Testing Procedure

To compare algorithms systematically:

1. Use a paired t-test on cell expansion counts across 50 gridworlds.
2. Define null hypothesis H_0: No difference in means.
3. Calculate p-value; reject H_0 if $p < 0.05$.
4. Report confidence intervals for effect size.