Uber Problem - Hamster Huey and Calculus Cam

Vishnu S. Penubarthi Section P September 26, 2017

The Problem (Calculus): One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Solving for the total time that the rocket is in the air

Givens:

Initial velocity: $v_i = 0$ m/s
Initial position: $x_i = 0$ m
Net acceleration as engine burns: $a_y[t] = -1.2t^2 + 17$ m/s^2
Engine burn time: $t_E = 4.2$ s
Parachute deployment height: 109 m below max height
Speed of rocket with parachute: $v_p[t] = -17(1-e^{-t/11})$ m/s

Strategy:

1. Calculate the final y-position, velocity, and time when the engine stops burning.
2. Find the maximum height which the rocket reaches.
3. Find the height of the rocket at parachute deployment, and the amount of time it took after the engine stopped burning to reach this point.
4. Find the amount of time it takes after parachute deployment for the rocket to reach the ground.
5. Find the total amount of time it takes for the rocket to reach the ground after launch.

Solution:

Part 1:

The first step is to integrate $a_y[t]$ to get the equation for velocity and position, so that final velocity and position can be calculated.

$a_y[\mathrm{t}] = \mathrm{A}t^2 + \mathrm{B}t + \mathrm{C}$

$a_y[t] = -1.2t^2 + 17$

$v_y[t] = \int -1.2t^2 + 17\, dt$

$v_y[t] = -1.2 \times \frac{t^3}{3} + 17t + C$

$v_y[t] = -0.4t^3 + 17t + C$

Given that $v_y[0] = 0$

$v_y[0] = -0.4 \times 0^3 + 17 \times 0 + C$

$\underline{0 = C}$

$\underline{v_y[t] = -0.4t^3 + 17t}$

Final velocity after engine stops burning:

$v_y[4.2] = -0.4 \times 4.2^3 + 17 \times 4.2$

$v_y[4.2] = -29.635 + 71.400$

$\underline{v_y[4.2] = 41.765 \text{ m/s}}$

Integrate velocity equation to get y-position equation:

$v_y[t] = -0.4t^3 + 17t$

$y_y[t] = \int -0.4t^3 + 17t\, dt$

$y_y[t] = -0.4 \times \frac{t^4}{4} + \frac{17t^2}{2} + C$

$y_y[t] = -0.1t^4 + \frac{17t^2}{2} + C$

Given that $y_y[0] = 0$

$y_y[0] = -0.1 \times 0^4 + \frac{17 \times 0^2}{2} + C$

$\underline{0 = C}$

$\underline{y_y[t] = -0.1t^4 + 17t^2/2}$

Final y-position after engine stops burning:

$y_y[4.2] = -0.1 \times 4.2^4 + \frac{17 \times 4.2^2}{2}$

$y_y[4.2] = -31.12 + 149.94$

$\underline{y_y[4.2] = 118.82\, m}$

Part 2:

Amount of time until rocket reaches max height from the moment it stops burning:

EQ2: $v_f = a\Delta t + v_i$

$v_{maxheight} = -9.8\Delta t + 41.765$

$0 = -9.8\Delta t + 41.765$

$-41.765 = -9.8\Delta t$

$\frac{-41.765}{-9.8} = \Delta t$

$\underline{\Delta t = 4.26s}$

Find the maximum height the rocket reaches:

EQ3: $y_f = \frac{1}{2}a\Delta t^2 + v_i\Delta t + y_i$

$y_{max} = \frac{1}{2} \times -9.8 \times 4.26^2 + 41.765 \times 4.26 + 118.82$

$y_{max} = -88.92 + 177.90 + 118.82$

$\underline{y_{max} = 207.82\, m}$

Height at which rocket stops free-falling:

$\Delta y = y_{max} - y_{parachute}$

$109 = 207.82 - y_{parachute}$

$-98.82 = -y_{parachute}$
$\underline{y_{parachute} = 98.82}$
Time from end of engine burning to end of free-fall:
EQ3: $y_{parachute} = \frac{1}{2}a\Delta t^2 + v_i\Delta t + y_y$
$98.82 = \frac{1}{2} \times -9.8\Delta t^2 + 41.765\Delta t + 118.82$
$0 = -4.9\Delta t^2 + 41.765\Delta t + 20$
$t = \cancel{-0.45s}, \underline{\ t = 8.98s}$
Part 4:
Integrate velocity parachute equation for position equation:
$v_p[t] = v_T(1 - e^{-\frac{t}{D}})$
$v_p[t] = -17(1 - e^{-\frac{t}{11}})$
$y_p[t] = \int -17 \times (1 - e^{-t/11})\, dt$
$y_p[t] = \int -17 + 17e^{-t/11}\, dt$
$y_p[t] = -17t + 17 \times -11e^{-\frac{t}{11}} + C$
$y_p[t] = -17t - 187e^{-\frac{t}{11}} + C$
$y_p[0] = -17 \times 0 - 187e^{-\frac{0}{11}} + C$
$98.82 = -187 + C$
$\underline{285.82 = C}$
$\underline{y_p[t] = -17t - 187e^{-\frac{t}{11}} + 285.82}$
Find amount of time it takes rocket to reach ground after parachute deploys:
$y_p[t] = -17t - 187e^{-\frac{t}{11}} + 285.82$
$0 = -17t - 187e^{-\frac{t}{11}} + 285.82$
$t = \cancel{-9.661s}, \underline{\ t = 13.625s}$
Part 5:
Total time:
$t_{total} = t_E + t_{parachute} + t_{ground}$
$t_{total} = 4.2 + 8.98 + 13.625$
$\boxed{t_{total} = 26.81s}$

Stage 1: Until engine stops burning
Stage 2: Until parachute deploys
Stage 3: Until rocket reaches ground

Time	Acceleration	Velocity	Position	Stage
(s)	m/s^2	m/s	m	
0.00	17.00	0.0	0.00	1
1.00	15.80	16.6	8.40	
2.00	12.20	30.8	32.40	
3.00	6.20	40.2	68.40	
4.00	-2.20	42.4	110.40	
4.20	-4.17	41.8	118.82	
4.21	-9.80	41.667	119.24	2
5.00	-9.80	33.925	149.10	
6.00	-9.80	24.125	178.12	
7.00	-9.80	14.325	197.35	
8.00	-9.80	4.525	206.77	
9.00	-9.80	-5.275	206.40	
10.00	-9.80	-15.075	196.22	
11.00	-9.80	-24.875	176.25	
12.00	-9.80	-34.675	146.47	
13.00	-9.80	-44.475	106.90	
13.17	-9.80	-46.141	99.19	
13.18	-1.55	0.00	98.82	3
14.00	-1.43	-1.22	98.31	
15.00	-1.31	-2.59	96.40	
16.00	-1.20	-3.84	93.17	
17.00	-1.09	-4.99	88.74	
18.00	-1.00	-6.03	83.23	
19.00	-0.91	-6.98	76.71	
20.00	-0.83	-7.85	69.28	
21.00	-0.76	-8.65	61.03	
22.00	-0.69	-9.38	52.01	
23.00	-0.63	-10.04	42.30	
24.00	-0.58	-10.64	31.95	
25.00	-0.53	-11.20	21.03	
26.00	-0.48	-11.70	9.58	
26.81	-0.45	-12.07	0.01