State Population of Washington

The Data

This is the initial data since 1900 taken from Wolfram Alpha:

In[1]:=
```
StatePops = population of washington since 1900
```
Washington, United States (administrative division) [population]

Out[1]=
```
{{{1900, 1, 1, 0, 0, 0.}, 518 103},
 {{1910, 1, 1, 0, 0, 0.}, 1 141 990}, {{1920, 1, 1, 0, 0, 0.}, 1 356 621},
 {{1930, 1, 1, 0, 0, 0.}, 1 563 396}, {{1940, 1, 1, 0, 0, 0.}, 1 736 191},
 {{1950, 1, 1, 0, 0, 0.}, 2 378 963}, {{1960, 1, 1, 0, 0, 0.}, 2 853 214},
 {{1970, 1, 1, 0, 0, 0.}, 3 413 244}, {{1980, 1, 1, 0, 0, 0.}, 4 132 353},
 {{1990, 1, 1, 0, 0, 0.}, 4 866 692}, {{2000, 1, 1, 0, 0, 0.}, 5 911 122},
 {{2001, 1, 1, 0, 0, 0.}, 5 987 785}, {{2002, 1, 1, 0, 0, 0.}, 6 056 187},
 {{2003, 1, 1, 0, 0, 0.}, 6 113 262}, {{2004, 1, 1, 0, 0, 0.}, 6 184 289},
 {{2005, 1, 1, 0, 0, 0.}, 6 261 282}, {{2006, 1, 1, 0, 0, 0.}, 6 372 243},
 {{2007, 1, 1, 0, 0, 0.}, 6 464 979}, {{2008, 1, 1, 0, 0, 0.}, 6 566 073},
 {{2009, 1, 1, 0, 0, 0.}, 6 664 195}, {{2010, 1, 1, 0, 0, 0.}, 6 724 540},
 {{2011, 1, 1, 0, 0, 0.}, 6 822 112}, {{2012, 1, 1, 0, 0, 0.}, 6 896 325},
 {{2013, 1, 1, 0, 0, 0.}, 6 973 742}, {{2014, 1, 1, 0, 0, 0.}, 7 061 530}}
```

The next step is to only get the year and population together and disregard other data. To make the data easier to graph, the population has been scaled down by a factor of one million.

In[2]:=
```
StatePop = Table[{StatePops[[j]][[1]][[1]], StatePops[[j]][[2]] / 10.^6}, {j, 1, 25}]
```

Out[2]=
```
{{1900, 0.518103}, {1910, 1.14199}, {1920, 1.35662}, {1930, 1.5634}, {1940, 1.73619},
 {1950, 2.37896}, {1960, 2.85321}, {1970, 3.41324}, {1980, 4.13235}, {1990, 4.86669},
 {2000, 5.91112}, {2001, 5.98779}, {2002, 6.05619}, {2003, 6.11326}, {2004, 6.18429},
 {2005, 6.26128}, {2006, 6.37224}, {2007, 6.46498}, {2008, 6.56607}, {2009, 6.66419},
 {2010, 6.72454}, {2011, 6.82211}, {2012, 6.89633}, {2013, 6.97374}, {2014, 7.06153}}
```

Here is a neat table displaying the data:

In[3]:=
```
Text[Grid[Prepend[StatePop, {"Year", "Population (in millions)"}],
  Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]
```

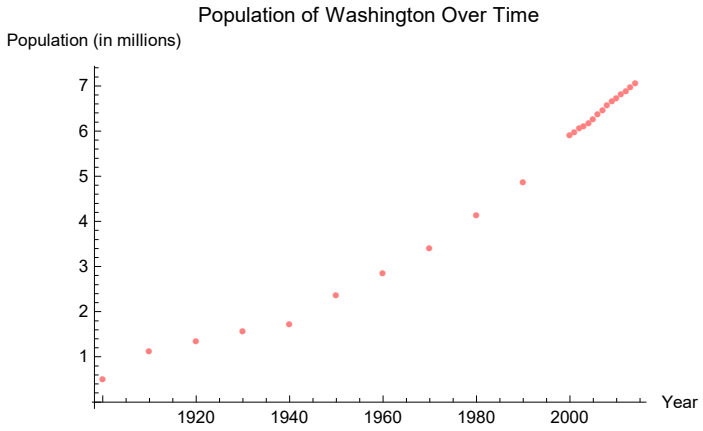
Out[3]=

Year	Population (in millions)
1900	0.518103
1910	1.14199
1920	1.35662
1930	1.5634
1940	1.73619
1950	2.37896
1960	2.85321
1970	3.41324
1980	4.13235
1990	4.86669
2000	5.91112
2001	5.98779
2002	6.05619
2003	6.11326
2004	6.18429
2005	6.26128
2006	6.37224
2007	6.46498
2008	6.56607
2009	6.66419
2010	6.72454
2011	6.82211
2012	6.89633
2013	6.97374
2014	7.06153

Here is the data plotted:

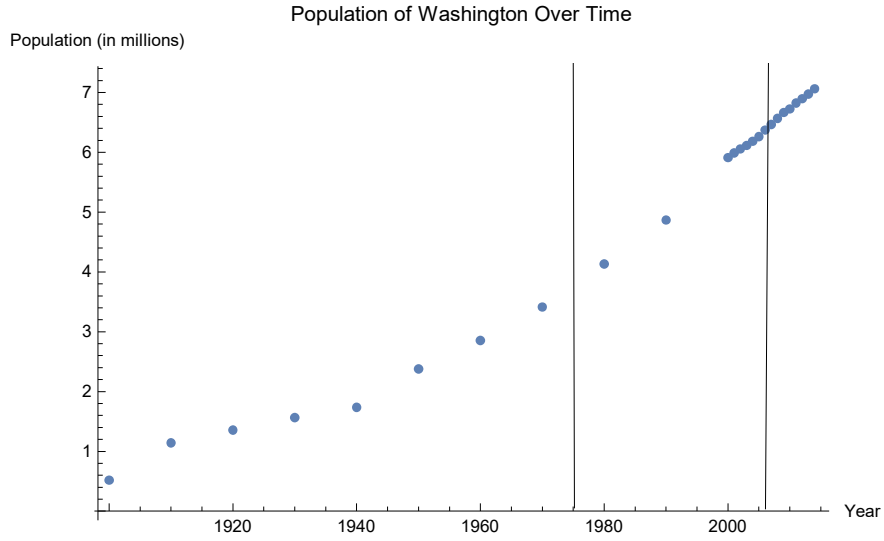
In[4]:=
```
PlotOfData = ListPlot[StatePop, PlotMarkers → {Automatic, 5},
  PlotStyle → Pink, AxesLabel → {"Year", "Population (in millions)"},
  PlotLabel → "Population of Washington Over Time"]
```

Out[4]=



Finding Medians

The next step is to split the data up into three sections. Since there are 25 points, the first and third sections will have 8 points while the middle section will have 9 points:



The next step is the find the median of the x values and median of the y values for each of the three sections, and then store them as a point.
First will be finding and storing median x-values of the points:

In[5]:=
```
firstPopX = Table[{StatePop[[j]][[1]]}, {j, 1, 8}];
secondPopX = Table[{StatePop[[j]][[1]]}, {j, 9, 17}];
thirdPopX = Table[{StatePop[[j]][[1]]}, {j, 18, 25}];
MidPointPopX = {Median[firstPopX], Median[secondPopX], Median[thirdPopX] * 1.}
```

Out[8]=
```
{{1935}, {2002}, {2010.5}}
```

Below is the finding and sorting of medians for the y-values of the three points:

In[9]:=
```
firstPopY = Table[{StatePop[[j]][[2]]}, {j, 1, 8}];
secondPopY = Table[{StatePop[[j]][[2]]}, {j, 9, 17}];
thirdPopY = Table[{StatePop[[j]][[2]]}, {j, 18, 25}];
MidPointPopY = {Median[firstPopY], Median[secondPopY], Median[thirdPopY]}
```

Out[12]=
```
{{1.64979}, {6.05619}, {6.77333}}
```

Now the corresponding x and y values of the medians will be transposed:

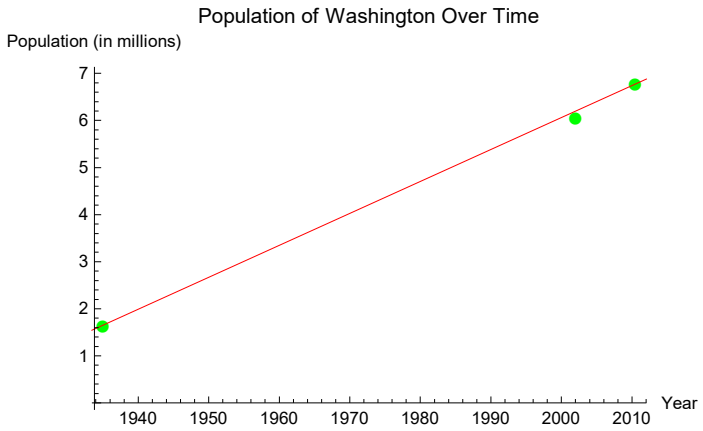
In[13]:=
```
TableOfMedians = Transpose[{MidPointPopX, MidPointPopY}];
```

Initial Phase of Median-Median Line

Now a line will be made that goes through median 1 and 3 and it will be displayed along with median 2:

In[14]:=
```
PlotOfMedians =
  ListPlot[{{1935, 1.6497935}, {2002, 6.0561869999999995}, {2010.5, 6.773325999999999}},
   PlotMarkers → {Automatic, 10}, PlotStyle → Green,
   AxesLabel → {"Year", "Population (in millions)"},
   PlotLabel → "Population of Washington Over Time"];
Medians1 = {1935, 1.6497935};
Medians3 = {2010.5, 6.773325999999999};
LineMedians1and3 = Graphics[{RGBColor[1, 0, 0], InfiniteLine[{Medians1, Medians3}]}];
Show[PlotOfMedians, LineMedians1and3, AxesLabel → {"Year", "Population (in millions)"},
 PlotLabel → "Population of Washington Over Time"]
```

Out[18]=



Finding the Median-Median Line

To complete the Median-Median Line, this line must be shifted down 1/3 of the way to the second point.

To do this, the slope of LineMedians1and3 will be found and fitted to Median 2 and a linear equation will be found. Then the line will be moved down 1/3 of the distance, which will be the “b” value if the linear equation is in the form of y=mx+b.

In[19]:=
```
SlopeOfLine = (6.773325999999999 - 1.6497935) / (2010.5 - 1935);
```

Solving for b by plugging point 1 in:

In[20]:=
```
bVal = Solve[1.6497935 == (SlopeOfLine * 1935) + b, b];
```

So the b-value is -129.662.

Solving for b of parallel line through Point 2 by plugging point 2 in:

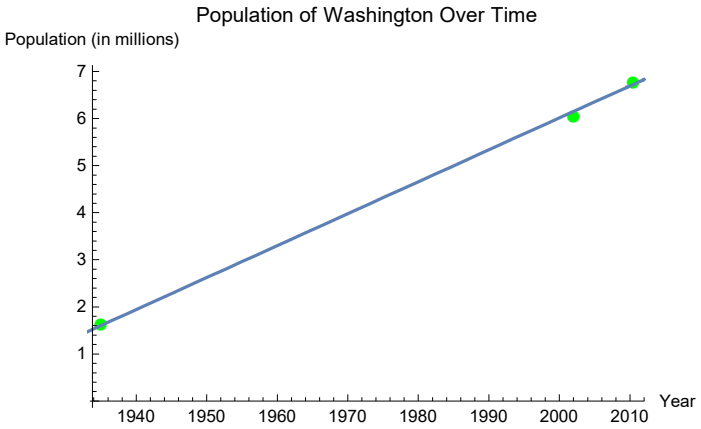
In[21]:=
```
bVal2 = Solve[6.0561869999999995 == (SlopeOfLine * 2002) + c, c];
```

The b-value of this line is -129.802.

Find difference and divide by 3 and move median line down by this amount and display on graph with summary points:

In[22]:=
```
ShiftDown = ((-129.662 + 129.802) / 3);
newB = -129.662 - ShiftDown;
MedianmedianL = Plot[(SlopeOfLine * x) + newB, {x, 1900, 2012}];
Show[PlotOfMedians, MedianmedianL, AxesLabel → {"Year", "Population (in millions)"},
 PlotLabel → "Population of Washington Over Time"]
```

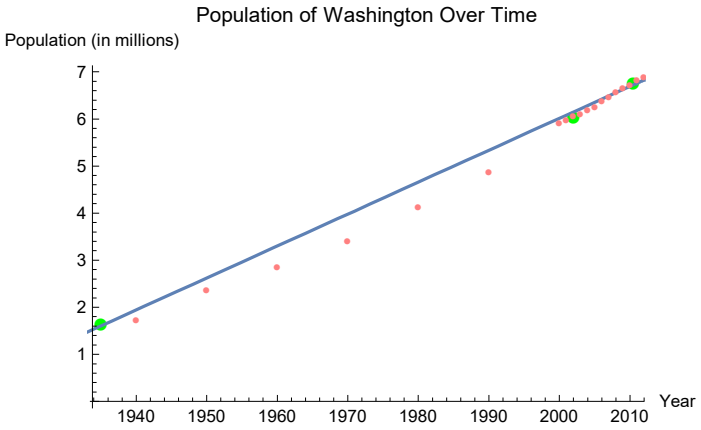
Out[25]=



Here is a view of the line with all the data and summary points:

In[26]:=
```
Show[PlotOfMedians, MedianmedianL, PlotOfData,
 AxesLabel → {"Year", "Population (in millions)"},
 PlotLabel → "Population of Washington Over Time"]
```

Out[26]=

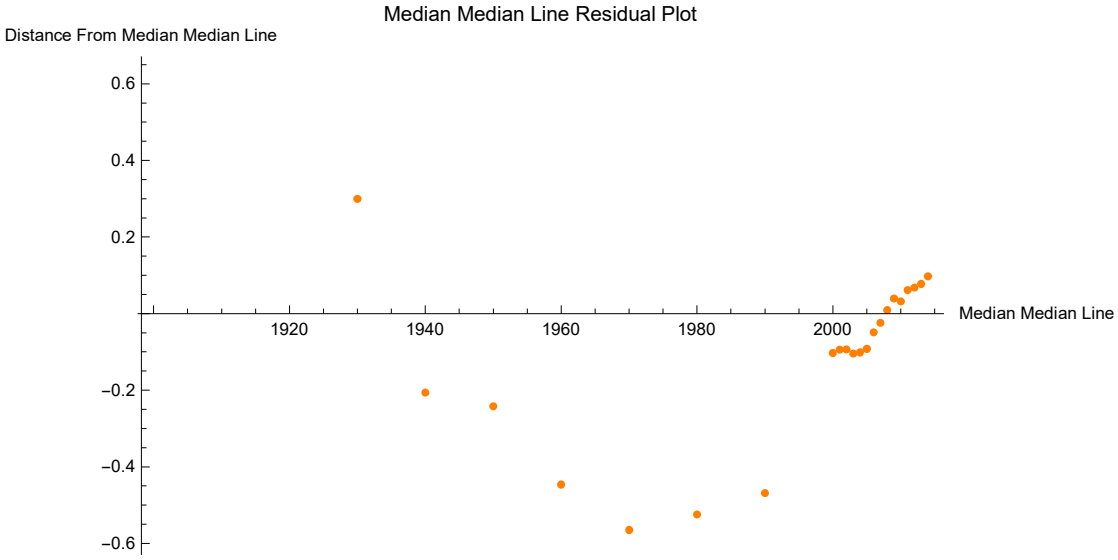


Residuals for Median-Median Line

To plot residuals the residuals must first be calculated and can then be displayed:

In[27]:=
```
StatePopX = Table[StatePops[[j]][[1]][[1]], {j, 1, 25}];
StatePopY = Table[StatePops[[j]][[2]] / 10.^6, {j, 1, 25}];
MedianMedianvals = (StatePopX * SlopeOfLine) + newB;
MedianMedianresids = StatePopY - MedianMedianvals;
ResidTable = Table[{StatePops[[j]][[1]][[1]], MedianMedianresids[[j]]}, {j, 1, 25}];
MedianMedianPlot = ListPlot[ResidTable,
  AxesLabel → {"Median Median Line", "Distance From Median Median Line"},
  PlotLabel → "Median Median Line Residual Plot", PlotStyle → {PointSize → 0.01, Orange}]
```

Out[32]=



Sum of residuals:

In[33]:=
```
SumOfResids = Total[MedianMedianresids]
```

Out[33]=
```
0.867523
```