Wasted Potential! Spring Energy Lab

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Section P

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Introduction

Introduction

Purpose

- The purpose of this lab is to design and perform an experiment which analyzes the conservation of energy in a spring-based system.

Researchable Question

- How does a change in the angle of inclination of a ramp affect the minimum height to which a frictionless cart starting at the top of the ramp travels?

Hypothesis

- If the angle of inclination is increased, then the minimum height to which the cart travels will decrease, where h_f is proportional to the square of the angle of inclination.

Methodology

Diagram

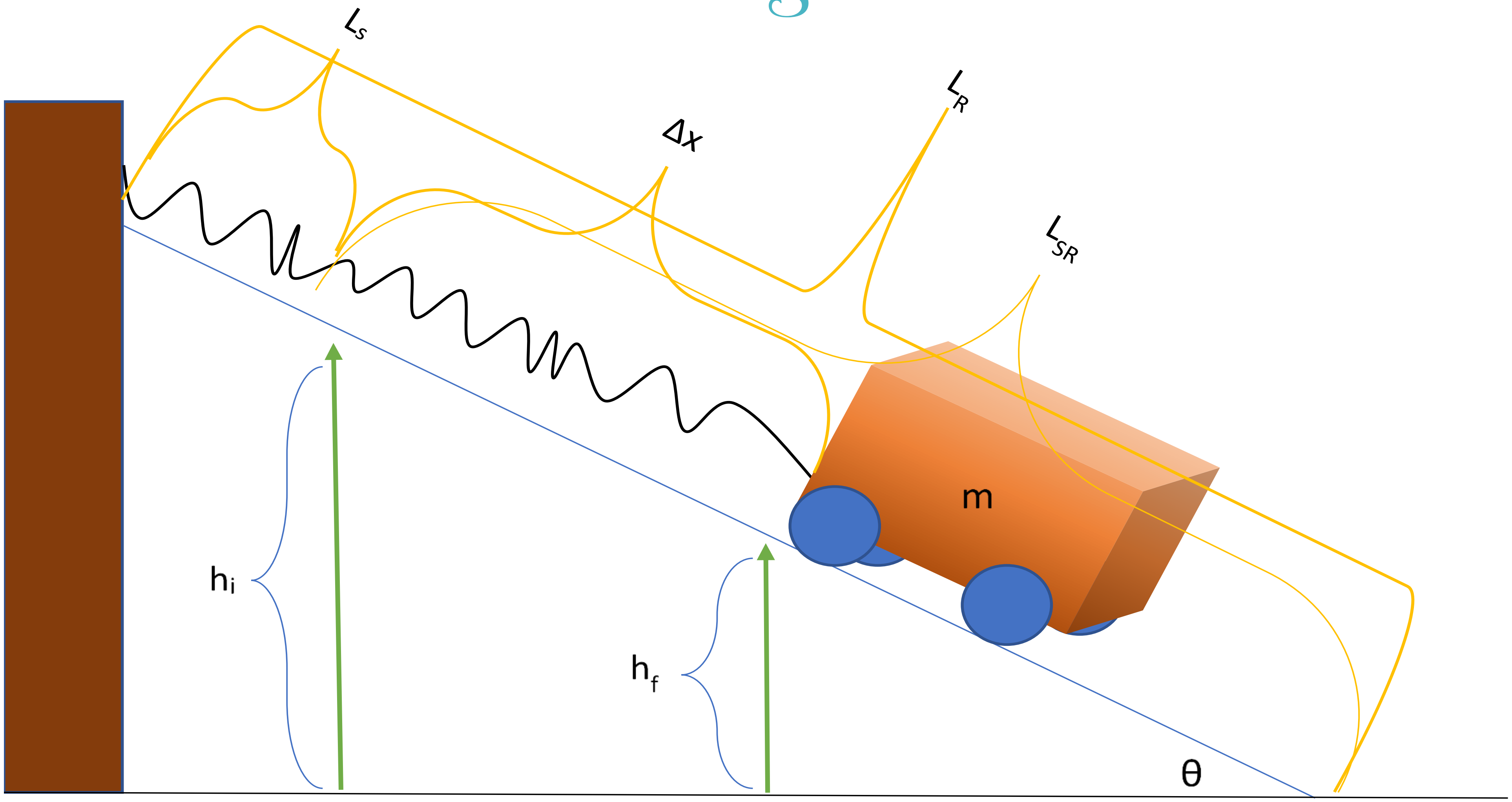

Figure 1: A diagram of the assembled system.

Procedure

Procedure

- A 228-centimeter frictionless ramp was set up against a brick wall at the desired angle of inclination. The spring assembly was composed of two comparable springs, with dental floss attaching a varying number of coils at non-uniform intervals; these springs were attached together with the hooks at their ends. Vishnu then hooked one end of spring assembly onto the rear-end of a 500-gram frictionless cart and used his right thumb to hold the other end of the assembly where the center of the ramp was in contact with the wall, holding the spring parallel to the surface of the ramp. Vishnu then used his left hand to place the end of the cart 0.1925 m away from the upper end of the ramp, as this is the length where the spring was neither compressed nor stretched. Vishnu would then count down from three and on one, Vishnu would take his hand off the cart and let it travel down the ramp. When the cart reached its lowest point, Dan recorded the height of the ramp beneath the rear of the cart, measuring to the bottom edge of the ramp. Dan would then return the cart to Vishnu as Piyusha recorded the measured height. This process was repeated 10 times for each of the 6 angle settings.

Constants & Equations

Constants

Length of Ramp (L_R)	2.28 m
Length of Spring (L_s)	0.193 m
Length of Ramp from End of Spring (L_{SR})	2.09 m
Gravity (g)	9.80 m/s^2
Mass of Kart (m)	0.500 kg
Spring constant (k) (See Appendix A)	1.71 kg/s^2

Equations

$$\sum E_i = \sum E_f$$

$$TE_i = mgh_i$$

$$TE_f = PE_s + mgh_f$$

$$h_f[\theta] = h_i - \frac{2mg \sin \theta^2}{k}$$ **(See Appendix B)**

$$h_i = (L_R - L_S) \sin \theta$$

$$PE_s = 0.857 * \left(\frac{\Delta h}{\sin \theta}\right)^2 + 0.0161 * \frac{\Delta h}{\sin \theta}$$

(See Appendix C)

Results

Summary Data Table

Table 1

	θ (degrees)	h_i (m)	h_{favg} (m)	STDEV (m)	%RSD h_{favg}	h_{fT} (m)	| %err | of h_f	TE_i (J)	TE_F (J)	% TE change
IV1	4.024	0.146	0.125	0.002	1.24	0.118	5.80	0.718	0.687	4.22
IV2	7.054	0.256	0.182	0.002	1.16	0.170	6.98	1.26	1.20	4.77
IV3	8.576	0.311	0.195	0.001	0.650	0.184	5.68	1.53	1.47	3.91
IV4	10.10	0.366	0.209	0.003	1.59	0.190	10.0	1.79	1.70	5.44
IV5	13.18	0.476	0.195	0.002	1.09	0.179	9.18	2.33	2.24	4.11
IV6	14.74	0.531	0.184	0.003	1.83	0.161	14.1	2.60	2.48	4.85
				Avg	**1.26**	**Avg**	**8.62**		**Avg**	**4.55**

Note: This is a summary table consisting of all calculated values. Except for the IV (angle of inclination), all values are displayed with three significant figures.

Graph of Data

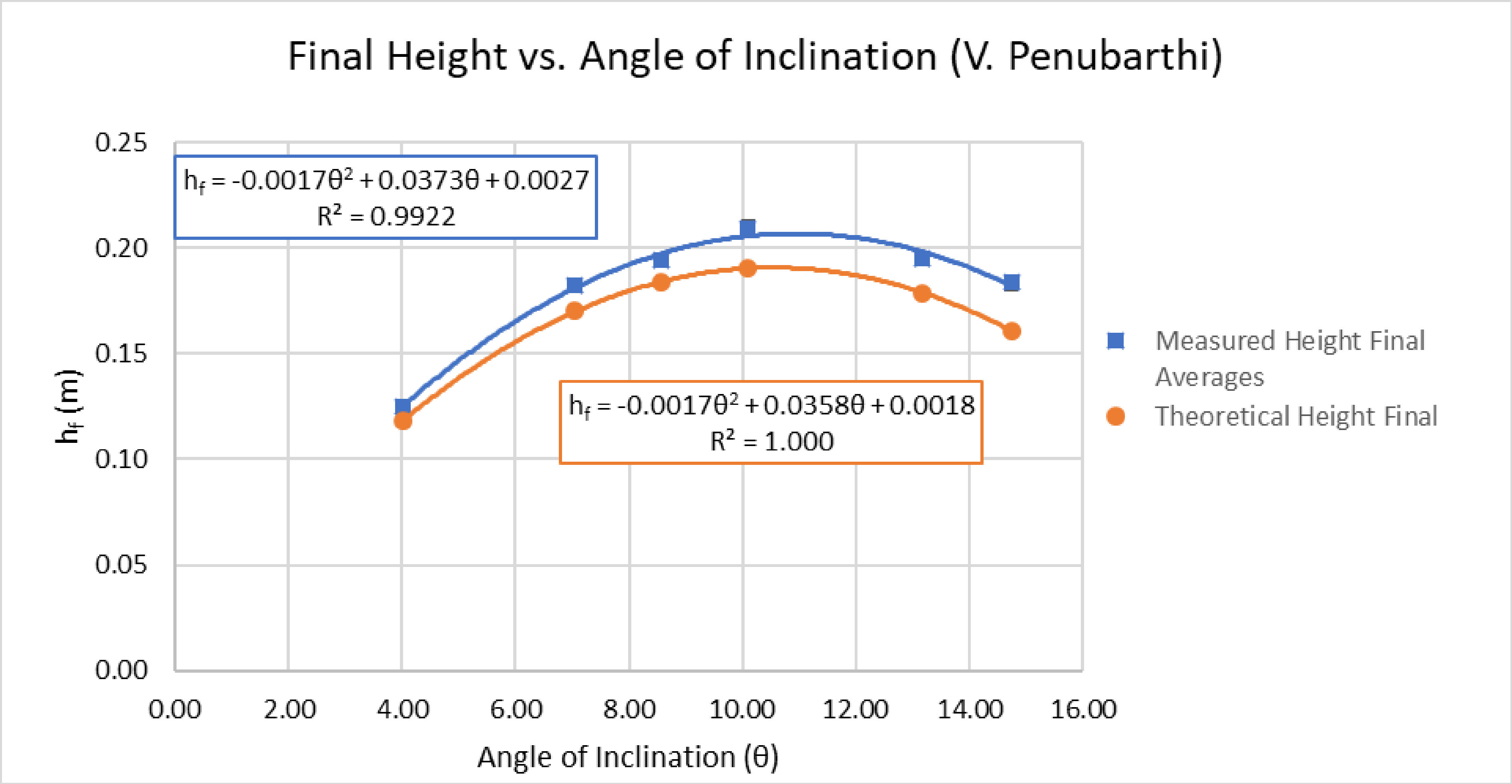

Figure 2: The error bars are present in the graph, but due the fact that they are small values, they cannot be seen.

Photographs

Figure 3: Dan measures final height of cart while Piyusha records the data.

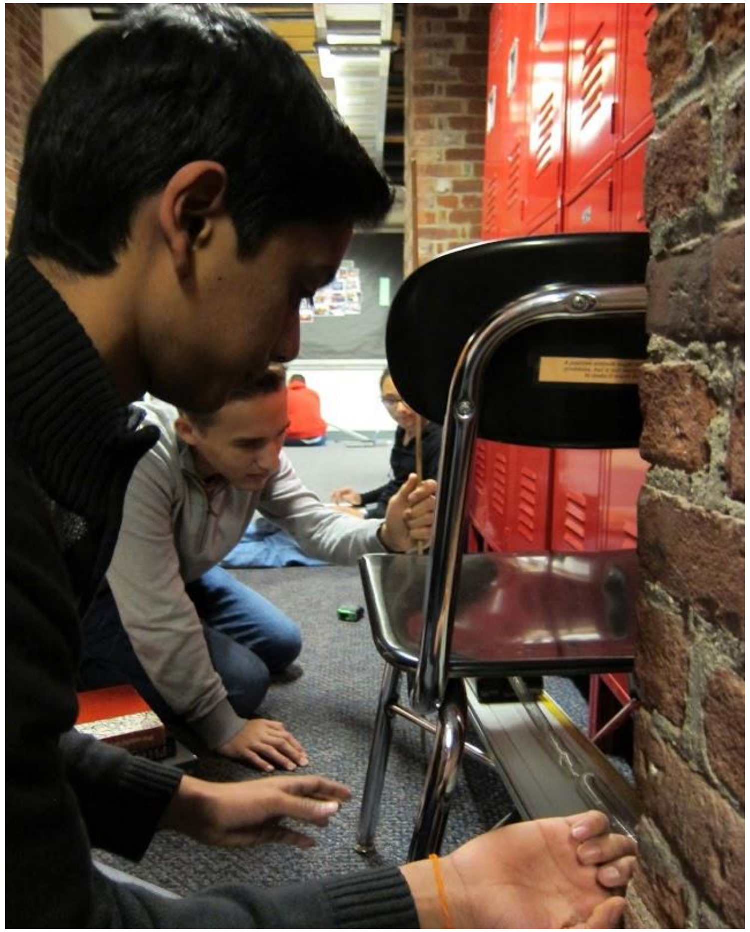

Figure 4: Vishnu secures spring while cart rolls down the ramp.

Analysis

Discussion

Discussion

- A primary objective was to find the theoretical final heights of the cart based on the angle of inclination. This was done by manipulating an energy equation to simplify it down to an equation which only contained the independent variable, the dependent variable, and the spring constant (k) was found by taking the derivative of the line of best fit in the Pre-k Force vs. Displacement Graph. This was then utilized in the equation to find theoretical final height. Calculating the initial energy of the system required the potential energy received from the height of the cart. Calculating the final energy of the system required both the potential energy received from the height of the cart and the potential energy of the spring, which was found by integrating the Pre-k Force vs. Displacement Graph.

Statistical Analysis

Statistical Analysis

- Overall, the first graph showed a quadratic relationship between the angle of inclination and the final height of the cart; the maximum final height recorded was 0.209 m at an angle of 10.1 degrees. The average %RSD of 1.26 indicates that the data showed a high precision. Furthermore, the average percent error of 8.62% illustrates that this data has moderate accuracy. A comparison between the graphs of the measured model and the theoretical model (derived in Appendix B), also suggests that the measured values were accurate as they are close to the expected values. Moreover, the R2 value of 0.992 suggests that the mathematical model is a strong fit, as 99.2% of the data can be accounted by the model. A general trend is that there was energy loss in all the settings, with 4.55% the average percentage of energy lost.

Sources of Error

Sources of Error

- The differences between the theoretical values and measured values can be explained through possible sources of error. One possible source of error is the change in k value, which was calculated to be 1.714 kg/s^2 before testing (See Appendix A), and was calculated to be 1.700 kg/s^2 (See Appendix E) after all testing was completed. Although there is a slight change in the k-value, the difference is miniscule enough that the impact on the data would be minimum. Another possible source of error is human error in measurement. It is possible that Vishnu may have been off placing the cart 0.1925 m away from upper end of the ramp, resulting in a spring which was either slightly compressed or slightly stretched before the cart was released. A third possible source of error is human reaction time. It is possible that when Dan was measuring the final height of the ramp, he was not able react quick enough to mark the lowest spot the cart reached before it began its ascent back up the ramp, resulting in final heights which are greater than what is expected.

Conclusion

Conclusion

Summary of Outcomes

- It is determined that as the angle of inclination increases, the final height of the cart on the ramp increases and then decreases, following a parabolic path. The final height is proportional to $h_i - \frac{2mg\sin\theta^2}{k}$ (See Appendix B). The data supports the hypothesis, as the final height of the cart on the ramp is proportional to the square of the angle of inclination.

Future Extensions

- Future extensions of this experiment include running more trials per angle of inclination and testing more angles to obtain a larger sample size. Another future extension could be to elaborate on the trend of energy loss observed and run an experiment with the purpose of discovering the contributing factors of energy loss. To maximize accuracy, a digital measuring device could be implemented to measure initial starting position and minimum height.

Appendices

Appendix A

Finding k

- k was found by pulling the spring to various lengths and recording the force of the spring
- The equation for F_s comes from the linear line of best fit for the 12 points

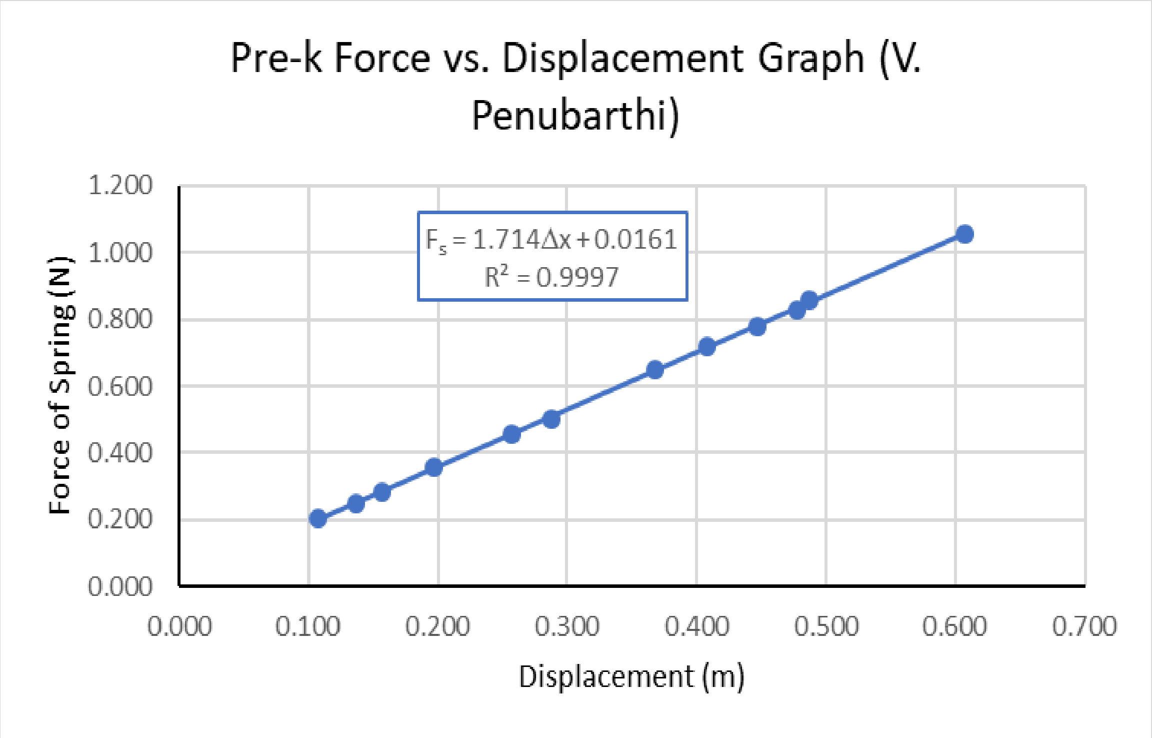

Figure 5: Pre-experimental force versus displacement data recorded.

$$F_s = 1.714\Delta x + 0.0161$$

$$\frac{d}{d\Delta x}F_s = \frac{d}{d\Delta x}(1.714\Delta x + 0.0161)$$

$$\underline{k = 1.714\ {}^{kg}/_{s^2}}$$

Table 2

Pre-k	
Stretch Length (m)	Force (N)
0.108	0.203
0.138	0.250
0.158	0.282
0.198	0.360
0.258	0.459
0.288	0.500
0.368	0.650
0.408	0.721
0.448	0.777
0.478	0.830
0.488	0.857
0.608	1.06

Appendix B

Finding h_{ftheo}

$$\sin\theta = \frac{\Delta h}{\Delta x}$$
$$\Delta x \sin\theta = \Delta h$$
$$\underline{\Delta x = \Delta h / sin\theta}$$

$$PE_{gi} = PE_{gf} + PE_s$$
$$mgh_i = mgh_f + \frac{1}{2}k(\Delta x)^2$$
$$mg(h_i - h_f) = \frac{1}{2}k(\Delta x)^2$$
$$mg(\Delta h) = \frac{1}{2}k(\Delta x)^2$$
$$mg(\Delta h) = \frac{1}{2}k(\frac{\Delta h}{\sin\theta})^2$$
$$\frac{2mg\Delta h}{k} = (\frac{\Delta h}{\sin\theta})^2$$
$$\frac{2mg\Delta h}{k} = \frac{\Delta h^2}{\sin\theta^2}$$

$$\frac{2mg\Delta h \sin\theta^2}{k} = \Delta h^2$$
$$\frac{2mg\sin\theta^2}{k} = \Delta h$$
$$\frac{2mg\sin\theta^2}{k} = h_i - h_f$$
$$\underline{h_f = h_i - 2mg\sin\theta^2 / k}$$

Appendix C

Finding PE_s

$$PE_s = \int_0^{\frac{\Delta h}{\sin\theta}} (1.714x + 0.0161)dx$$

$$PE_s = \left(1.714 * \frac{1}{2} * (\frac{\Delta h}{\sin\theta})^2\right) - (0.0161(\frac{\Delta h}{\sin\theta}))$$

$$PE_s = 0.857(\frac{\Delta h}{\sin\theta})^2 - 0.0161(\frac{\Delta h}{\sin\theta}))$$

Appendix D

Full Data Table

Table 3

	θ	h_i	h_{f1}	h_{f2}	h_{f3}	h_{f4}	h_{f5}	h_{f6}	h_{f7}	h_{f8}	h_{f9}	h_{f10}	h_{favg}	STDEV	%RSD	h_{fT}	l %err l	TE_i	TE_f	% TE
	(degrees)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	h_{favg}	(m)	of h_f	(J)	(J)	change
IV1	4.024	0.146	0.123	0.124	0.124	0.127	0.126	0.126	0.128	0.125	0.125	0.124	0.125	0.002	1.24	0.118	5.80	0.718	0.687	4.22
IV2	7.054	0.256	0.179	0.182	0.183	0.184	0.184	0.180	0.184	0.181	0.179	0.184	0.182	0.002	1.16	0.170	6.98	1.26	1.20	4.77
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IV4	10.10	0.366	0.205	0.204	0.216	0.209	0.210	0.211	0.208	0.210	0.209	0.211	0.209	0.003	1.59	0.190	10.0	1.79	1.70	5.44
IV5	13.18	0.476	0.198	0.196	0.191	0.196	0.194	0.197	0.195	0.197	0.194	0.193	0.195	0.002	1.09	0.179	9.18	2.33	2.24	4.11
IV6	14.74	0.531	0.184	0.185	0.184	0.186	0.183	0.184	0.187	0.178	0.188	0.178	0.184	0.003	1.83	0.161	14.1	2.60	2.48	4.85
														Avg	**1.26**	Avg	**8.62**		Avg	**4.55**

Note: The trials were recorded to the third significant figure, so the calculations have been kept at three significant figures as well

Appendix E

Post-k Data Table and Graph

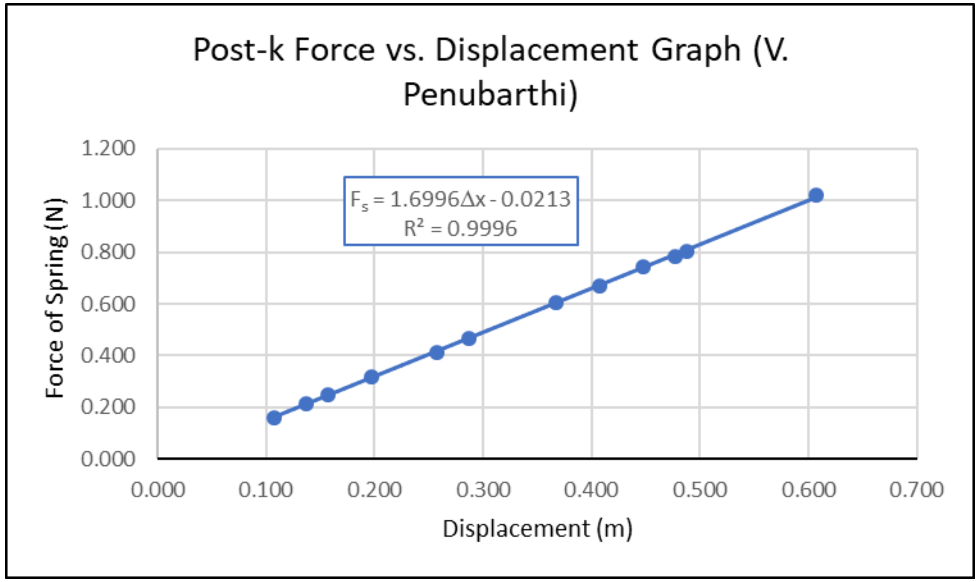

Figure 6: Post-experimental data used to find the spring constant after all trials had been completed.

Table 4

Post-k	
Stretch Length (m)	Force (N)
0.108	0.158
0.138	0.215
0.158	0.250
0.198	0.320
0.258	0.410
0.288	0.466
0.368	0.605
0.408	0.670
0.448	0.745
0.478	0.782
0.488	0.801
0.608	1.02