CS711: Game Theory and Mechanism Design **Sep-Nov 2020**

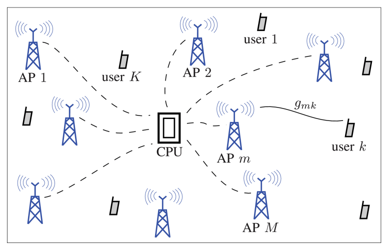
Project Report: PSP Auction with Weighted Bids for Resource Allocation

Chandra Shekhar Tiwari, Piyush Singh, Rangeet Bhattacharyya, Soumyadeep Datta, Vaibhav Verdhan

Abstract

We consider the downlink of a Cell Free Massive MIMO system, which is among the foremost candidates for Beyond-5G networks. We propose a progressive second price (PSP) based auction mechanism for dynamic allocation of spectrum among users by a telecom provider. We demonstrate its superiority in achieving overall social welfare as compared to currently used baselines in wireless communications.

1 Introduction



Massive multiple input - multiple output (MIMO), is a promising wireless access technology that can provide high throughput, reliability and energy efficiency with simple signal processing. Cell-Free Massive MIMO [1] is a distributed form of massive MIMO, where there are no cells or cell boundaries around various access points. It enhances user experience and improves the overall system performance by providing huge throughput and coverage probability to all users throughout the system.

Due to wireless signal decay over distance, beamforming the signals to the users has proven to be the next step in achieving efficient transmission. To achieve an efficient allocation-scheme, it is essential for the Access Point (or AP) to know the channel gain of its connections to each user. Each user device sends pilot signals to the APs, which then can estimate the channel gains. In almost all current work in network resource allocation, the reported channel gain is assumed to be truthful. However, a corrupt agent might, to maximize their own utility/valuation, modify their wireless receivers to falsely report their channel gains, which is one of the most important factors in resource allocation. This can deviate the allocation-scheme from the desired optimal outcome, potentially reducing the revenue for the service provider operating the APs. To tackle this problem, we design the strategy-proof mechanisms to ensure that each agent's dominant strategy is to truthfully reveal its channel gain regardless of other agents' strategies.

During high-network-congestion periods, in order to maximize user satisfaction, the service provider should be able to allot a higher bandwidth to users attempting to fulfill critical and time-sensitive tasks (for example: banking transactions, completion of online-tests and others), and a lower bandwidth to users demanding the same for leisure tasks (for example: movie playback, downloading of content for later consumption and others). We introduce mechanisms to enable the service provider to account for such differences in consumption types, and hence be able to prioritize more critical tasks.

1.1 Related work

Bandwidth can be looked at as a single-divisible resource, to be distributed among multiple users/agents. The problem of resource-allocation is a well-studied problem due to its application in various fields, communication networks, traffic networks, power-electricity systems. The resource needs to be allocated in a way to achieve a global objective i.e. maximization of a system-wide social welfare.

Our problem has been formulated as a auction-game, to enable a priority-based allocation of bandwidth. In [2], a dynamic-iterative-biding process was defined in the network-allocation setting for users, wherein at each step a reallocation of resource was done for the user whose bid was updated. The analysis provided showed existence of an ϵ-Nash equilibrium, i.e. after convergence no player can gain more than ϵ by updating their bid. Hence, by the introduction of a bidding fee, an equilibrium was guaranteed. The auction mechanism for resource allocation, was formulated for each user using a two-dimensional bid profile: $b_n = (\beta_n, d_n) \in [0, \infty) \times [0, \gamma]$ where β_n is the per-unit price user is willing to pay for a demand of d_n units of the resource.

In almost all the work that followed the above, a similar formulation of bid-profile was used for the users. Following the above-work, the progressive-second-price auction, its design and analysis was formally introduced in [3] by the same authors. PSP was designed to be economically efficient i.e. maximizing the total user value. Under elastic demand, the PSP-auction was shown to be incentive compatible with the price-bids being equal to users' marginal valuation of the resource. The PSP-auction setting provided quick-iterative-convergence in the order of $O(\frac{N}{\epsilon})$ guarantees along with the original ϵ-Nash equilibrium.

An improvement work on the original PSP-auction was proposed in [4] which did an in-depth analysis of tie-breaking in the allocation schemes for guaranteeing a complete utilization of the resource in high-demand, but equal-bid situations. The work additionally proved the original theoretical properties of the PSP-auction with the tie-breaking cases.

In [5] [6], multi-bid auction was introduced as extension to PSP-auction. Original PSP proposition required rounds of network communication with the users (for updation of the bids) to achieving the equilibrium, however, the multi-bid auction proposed a setting wherein a collection of bids (representative of the users' private valuation) were simultaneously submitted in-order to reduce the associated network overhead. The proposed-setting was shown to follow the the incentive compatibility and efficiency properties as in the original PSP-auction while being more efficient in terms of time and network-resource overhead. In [7], a quantized version of PSP (Q-PSP) was introduced, with simultaneous updating of players' bids, which was shown to have a fast and limited-steps convergence to the quantized Nash equilibrium.

The state-of-the-art divisible resource allocations propose the use of decentralized auction mechanisms as in [8] and [9]. The decentralized dynamic process is assisted with a similar set of bid profile, and at each iteration the single user updates its best bid under a constrained set of demand. Such auction mechanisms further reduce the network overhead communication with the central authority and provide (also derive) the incentive compatibility and similar set of efficiencies as the PSP-auction did.

From all the recent works in the field, it was observed that most have been a modification and extension on the original-PSP-auction. Hence, for proposing the truthful channel-gain and the service provider weighted allocation modification, we focus on the basic-PSP-auction itself. Provided the guarantees as the original-PSP-auction, these modifications can be easily introduced to the art-mechanisms as described above.

Prioritization-weight of network-usage type has been analysed in [10], wherein network usage has been classified into different classes and weights can assigned based upon the bandwidth, latency and cost requirements. In the analysis that follows, we use a price-bid-based valuation for the users as simplification, however, a much richer representation as described in [11] can be used to determine what the users value the most. For example, for a user at 8% (lower battery level), a lower-energy-consumption would be of a much higher value than a higher-data-rate.

1.2 Brief overview of the report

We motivate the problem and review existing literature in Section 1. We introduce the problem setup for our PSP mechanism design in Section 2. We state and prove our main results and theoretical guarantees for in Section 3. In Section 4, we introduce two baseline approaches, inspired by wireless communication literature, and through simulation results, we compare their performance with our proposed PSP mechanism. We conclude this report in Section 5.

2 Formal problem setup

There are K users and M access points in the system. Let mk be the channel between the kth user and mth access point. We denote the actual and estimated fading loss (channel gain) for the channel between the k^{th} user and m^{th} access point with β_{mk} and γ_{mk} respectively. Using equalizing power control coefficients, we can calculate the achievable spectral efficiency (bitrate/Hz) for the k^{th} user as [1]

$$\mathrm{SP}_k = \tau_f \log_2 \left(1 + \frac{N_t \rho (\sum_{m=1}^{M} \sqrt{\gamma_{mk}})^2}{\rho \sum_{m=1}^{M} \beta_{mk} + K} \right). \tag{1}$$

where, τ_f and ρ are pilot estimation overhead and max. downlink transmit signal-to-noise-ratio (SNR), resp.

Now, we define the type of the k^{th} user, θ_k, by a $3-$tuple $\theta_k \triangleq (\bar{R}_k, p_k, \gamma_{mk})$, where

- $\bar{R} \in [0, B\ \mathrm{SP}_k]$, is the *data rate requested* by the k^{th} user,
- $p_k \in [0, \infty)$, is the *price (per bit)* which the k^{th} user is willing to pay, and
- $\gamma_{mk} \in [0, \infty)$, is the *reported channel gain* between the k^{th} user and the m^{th} access point (AP).

Here, assume the k^{th} user is assigned with bandwidth share $b_k \in [0, B]$. Therefore, the bitrate of the k^{th} user can be given by;

$$R_k = b_k \mathrm{SP}_k, \tag{2}$$

Using the bitrate, we define the value of the k^{th} user as;

$$v_k = p_k R_k = p_k b_k \mathrm{SP}_k, \tag{3}$$

Now, we assume that the ISP assigns a weight, $w_k \in \tilde{\boldsymbol{W}}$, to the k^{th} user. Where $\tilde{\boldsymbol{W}} = \{\tilde{w}_1, \ldots, \tilde{w}_N\}$ is the set of weights assigned to each user based on prioritization mechanisms [10]. For instance, time-sensitive network utilization (banking operations, online-education/test submissions) to be allotted a higher priority weight. Here we assume that a higher weight w_k implies more priority of the user.

Now, we define our social welfare function (which we want to maximize) as the weighted sum (over K users) of the valuations, as defined in (3), as:

$$f(\boldsymbol{b}|\boldsymbol{\theta}) = \sum_{k=1}^{K} w_k p_k R_k = \sum_{k=1}^{K} w_k p_k b_k \mathrm{SP}_k \tag{4}$$

3 Main results

The allocation rule devised to satisfy the mechanism design problem as described above is:

$$a_i(s) = \min \left\{ \frac{R_i}{\mathrm{SP}_i}, \bar{B}(p_i, s_{-i}) \right\} \tag{5}$$

$$c_i(s) = \frac{1}{\lambda} \sum_{j \neq i} p_j\ [a_j(0, s_{-i}) - a_j(s_i, s_{-i})],\ \text{where} \tag{6}$$

$$\bar{B}(y, s_{-i}) = \max \left\{ 0,\ B - \sum_{w_k p_k \mathrm{SP}_k \geq y, k \neq i} \frac{R_k}{\mathrm{SP}_k} \right\}. \tag{7}$$

Here, λ is a constant factor set by the telecom operator to account for the unit mismatch in user bids (in Rs/Mbps) and bandwidth allocation (in MHz). It is explained in detail later in Appendix B.

Computational complexity: The rule is computationally simple- $O(I^2)$ where I is the number of players and can be efficiently used in real time dynamic network-allocation. [3]

Due to the above allocation-scheme being derived from the progressive second price auction, and the payments charged being a modification of the Vickrey Clarke Groves mechanism, the following results follow:

Proposition 1 *The allocation-scheme described in (5) maximizes the social welfare in (4).*

Proposition 2 *(ϵ-Nash equilibrium) Under the above assumptions, there exists an ϵ-Nash equilibrium.*

Moreover, the procedure to to achieve the given Nash equilibrium is efficient in the following way (expressed using the notations used above in the results that follow from [3].

Proposition 3 *(Efficiency) Let s^* be an ϵ-Nash equilibrium and $a^* = a(s^*)$.
Let $A = \{a'(s^*) = (a'_1(R_1, p_1), ..., a'_I(R_I, p_I)) : \forall, a'_i(R_i, p_i) \leq B_i\}$ and $a'_i(R_i) = B_i$ if $\sum_i \frac{R_i}{SP_i} \leq B$ and $\sum_i a'_i(R_i, p_i) = B$ if $\sum_i \frac{R_i}{SP_i} \geq B$. Efficiency proposition follows using the above-given changed notation in Proposition 3 from [12].*

Proposition 4 *(Incentive compatibility) Each user k truthfully reveals its private valuation for the price-per-bit-data-rate i.e. p_k is reported truthfully by each user.*

Proof: In (6) the change introduced from the payment vector described in eq. (2) [12], is the constant-factor λ. The given factor is derived as described in Appendix B. Due to no impact being created by a proportional reduction or increase in the user payments, provided the constrained selection of parameter, the incentive compatibility proof as described in Proposition 1 from [12] follows. ■

Proposition 5 *For network bandwidth allocation, the payment described in (6) ensures that the truthful reporting of $\beta_{mk} = \gamma_{mk}$ is the dominant strategy for each user under the given allocation in (5).*

Proof: Appendix A. ■

Remark 1: Adopting the price in 6, each user maximizes its own utility when it reports the true valuation of the channel gain to the service provider. By receiving the truthful reported channel gains, the service provider allocates the bandwidth to 5 and the social welfare as described in 4 is thus maximized.

Remark 2: We observe that a user with reduced channel gain would require a higher-bandwidth for achieving a particular data-rate as compared to a user with higher channel gain, hence, the cost of such user to the system would be higher, which would imply an increased cost for the same data-rate.

4 Experiments/Simulations

We consider $M = 64$ access points with $N_t = 2$ antennas each and a maximum downlink transmit SNR $\rho = 0$ dB. We vary the number of players, n, from 30 to 90 in intervals of 10, for the simulations. Along with our proposed PSP mechanism, we consider two other simple approaches:

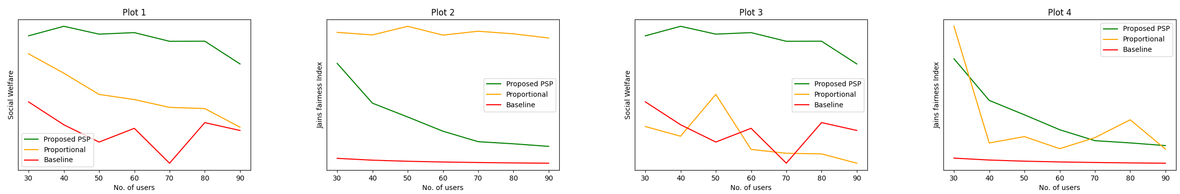
- *Proportional*: Allot bandwidth in proportion to the required valuations, $v_k^{\text{req}} = p_k \bar{R}_k$.

$$b_k = \frac{p_k \bar{R}_k}{\sum_{i=1}^{n} p_i \bar{R}_i} B, \tag{8}$$

- *Baseline*: Allot entire bandwidth to the user j^* which maximizes a sigmoidal utility function [13].

$$j^* = \arg\max_j \frac{\sigma_j e^{-\sigma_j(B\mathrm{SP}_j - \bar{R}_j))}}{\left(1 + e^{-\sigma_j(B\mathrm{SP}_j - \bar{R}_j)}\right)^2} \mathrm{SP}_j, \text{ where } \sigma_j = \frac{\log\left(\frac{1-\delta}{\delta}\right)}{\rho_{\mathrm{th}} \bar{R}_j}. \tag{9}$$

Here, we take $\delta = 0.01$, $\rho_{\mathrm{th}} = 0.5$ [13].



(a) Social Welfare: Non-strategized

(b) Fairness: Non-strategized

(c) Social Welfare: Strategized

(d) Fairness: Strategized

Figure 1: Comparative performance of different bandwidth allocation schemes

In Fig. 1a, we plot the social welfare, as obtained in (4), versus the number of players, n. We observe that the proposed PSP mechanism achieves a considerably higher social welfare than the proportional allocation, as well as the baseline scheme. This shows how our proposed mechanism is suitably designed and overcomes the deficiencies of the commonly used mechanisms in wireless communications literature.

In Fig. 1b, we compare the bandwidth allocation schemes using the Jain's fairness index [14], defined as:

$$\mathcal{J}(x_1, \ldots, x_n) = \frac{\left(\sum_{i=1}^n x_i\right)^2}{n \sum_{i=1}^n x_i^2}, \tag{10}$$

where x_i denotes the data rate of the ith user. We observe that the proportional allocation scheme is the most fair of the three, while the baseline allocation is least fair. The proposed PSP scheme achieves a good balance in terms of fairness while also being the best at maximizing the social welfare.

In Fig. 1c and 1d, we show the superiority of our proposed mechanism in terms of guaranteeing non-user-strategizable channel gain reporting. For Fig. 1c, we use the same simulation setup as in Fig. 1a, but using pre-defined user strategies, the channel gain for some users is misreported, hence, we can observe that the increased difference in performance. Similarly, for Fig. 1d, we use the same simulation setup as in Fig. 1b, and hence assume strategic users. Clearly, when assumed that the users can misreport their channel gains for a proportional allocation, we can observe that our proposed algorithm performs better in terms of the fairness index in most iterations.

5 Summary and Discussions

We propose a mechanism design of an end-user compatible auction setting with priority-weighted allocation for social-welfare maximization and truthful channel gain reporting. We compare with baseline schemes showing superior performance in achieving social welfare. Possible extensions of our work include

- Improved models of user valuation for different social welfare objectives, such as energy efficiency.
- Allocation from multiple divisible goods (chunks of spectrum)
- Applying these modifications to more SoTA PSP approaches.

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Appendices

A Proof of Proposition 5

Assume the data-rate received by the k^{th} user under the above-described allocation scheme is given by $R_k(\alpha_k, \theta_{-k})$ when the k^{th} user reports its type information (manipulated) as α_k, while other users report truthful type information θ_{-k}.
Since the users are assumed to be able to manipulate their reported private type information, the k^{th} user would report its type information such that:

$$\max_{\alpha_k} U_k = \max_{\alpha_k} \{R_k(\alpha_k, \theta_{-k}) - c_k\}$$

From Proposition 4, we know that the price-bid type information is ensured to be truthful, due to its implications in the ascending sort as utilized for the allocation vector. Hence, only the reported channel gain and reported data-rate requirement can be strategized.

We consider different cases for possible types of user allocations from the allocation-scheme.

Say the set of users who are allotted a non-zero bandwidth is given by $Y \subseteq K$, which will be a subset of the total users available K. Denote user $y_{(N)} = \min_{p_k} Y$, i.e. $y_{(N)}$ denotes the last user who was allotted the bandwidth according to the given allocation-scheme.

Case-I: User $k \in Y - \{y_{(N)}\}$. We observe that the allotted bandwidth to such user will be $\frac{R_k}{SP_k}$. Where SP_k denotes the actual spectral efficiency for this user. Observe that the allotted bandwidth a_k is directly proportional to the required data-rate, and inversely proportional to the reported spectral efficiency. *[Case-Ia]* Now say this user over-reports its channel-gain, i.e. reports a better spectral efficiency than actual and reports the actual required data-rate, then due to the inverse relationship, we observe that the bandwidth allotted will be less than what the user needs. The payment for this user would reduce proportionally, however, using the basic principles of economics, the user valued the lost bandwidth more than the benefit achieved due to the reduced payment, hence the user would have a reduced overall utility. *[Case-Ib]* Now say this user under-reports its channel-gain while reporting the actual bandwidth required, in this case, a higher bandwidth would be allotted, however due to the ratio-ed relationship between spectral efficiency and required data-rate, this is equivalent to demanding a higher-data-rate. Hence, the player would need to pay a higher amount to occupy the excess bandwidth, but would not increase the goods-valuation, thereby, reducing the overall utility for this user. Additionally, the higher bandwidth can easily be acquired by simply requesting a higher data-rate, hence, the user doesn't gain by misreporting the channel-gain. From the ratio-ed relationship, all combined manipulations to data-rate and channel-gain can be simplified to the above two sub-cases described. Hence, the truthful channel-gain reporting is a (weakly) dominant strategy for this user.

Case-II: User $k \in y_{(N)}$. *[Case-II a]* If the user under-reports the channel gain, a higher bandwidth would be requested, however due to unavailability of the bandwidth, the same bandwidth as in case of truthful reporting would be allotted, therefore, the data-rate achieved remains the same, and so does the payment made. If the user over-reports the channel gain, we consider two cases, *[Case-II b1]* $\bar{B}(p_i, s_{-i}) < \frac{R_k}{SP_\alpha}$ where SP_α is the derived spectral efficiency from the misreported-channel-gain. In this case, as before, due to a reduced availability of the bandwidth, the same bandwidth would be allotted as before, and the payment remains the same too. *[Case-II b2]* $\bar{B}(p_i, s_{-i}) > \frac{R_k}{SP_\alpha}$ where SP_α, in this case under the revised allocation-scheme, the user $\notin y_{(N)}$, but $\in Y - y_{(N)}$. Hence, a lower bandwidth would be allotted, which would result in a lower data-rate being achieved, and from the basic principles of pricing, as in Case-I, the overall utility of the user would decrease. Similar to Case-I, the above two cases cover all possible manipulations using the combination of data-rate and channel-gain. Hence, the truthful channel-gain reporting is a (weakly) dominant strategy for this user.

Case-III: User $k \in K - Y$. It implies that: $\bar{B}(p_i, s_{-i}) = B$ i.e. when sorted ascendingly by the price-bids, the entire bandwidth is exhausted by the other users who had a higher price-bid.

According to the payment-scheme, it can be observed that this user doesn't cause any loss of bandwidth to any other user since the bandwidth allotted is zero. Hence, $c_k = 0$. The inherent property of the allocation-scheme to allot the users of higher-price-bid first has caused a zero allocation to this user, and since we have already established that the price-bid is ensured truthful for every user, this user can't gain by manipulating the channel-gain or the required data-rate.

Hence, in all the above explained cases the truthful channel-gain reporting is the (weakly) dominant strategy.

B Parameter λ in VCG payments

The parameter λ occurs in lieu of the spectral efficiency component when converting the VCG payment, which is derived proportional to the bandwidth allotted, into a data-rate based payment.

In real-time, this parameter can be easily computer and set by the service provider, and can be controlled similar to surge-pricing or increased when the overall system efficiency is low.

The constraints set on this parameter are as follows:

$$c_i(s) = \frac{1}{\lambda} \sum_{j \neq i} p_j \, [a_j(0, s_{-i}) - a_j(s_i, s_{-i})] \;\; \leq \;\; p_i a_i SP_i \;\; \forall \; i \in K \tag{11}$$

$$\implies \lambda \geq \;\; \frac{1}{p_i a_i SP_i} \sum_{j \neq i} p_j \, [a_j(0, s_{-i}) - a_j(s_i, s_{-i})] \;\; \forall \; i \in K \tag{12}$$

Properties of λ:

- Increases with the increase in average spectral efficiency, i.e. if the system is efficient in signal transmission, payments charged should be lower.
- Decreases with the increase in bandwidth demand, i.e. during peak data-requirement, a higher payment should be charged from the users.