SLAM Task

Geometric modelling

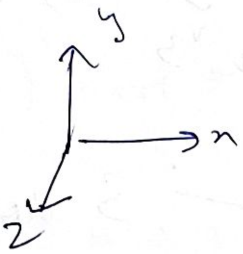
1) Boundary representation

2) Solid representation

2D linear transforms

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What happens to e^1 — What happens to e^2



$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \longrightarrow$ rotate by π

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \longrightarrow$ x shear

$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \longrightarrow$ y shear

For rotations

1) Preserve scale $\longrightarrow$ columns have length one

$\Rightarrow m_{11}^2 + m_{21}^2 = 1$

$m_{12}^2 + m_{22}^2 = 1$ — (2)

2) No shearing $\longrightarrow$ axis should be $\perp$.

$\boxed{m_{11} \cdot m_{12} + m_{21} m_{22} = 0}$ $\hookrightarrow$ (1)

3) No mirror images
(no orientation changing)

$\boxed{\det(M) = 1}$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

3 d Rotations

$$\begin{bmatrix} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$
$\downarrow \quad \downarrow \quad \downarrow$
$v_1 \quad v_2 \quad v_3$

1) $\|v_1\| = \|v_2\| = \|v_3\| = 1$ — 3

2) $v_1 v_2 = v_2 v_3 = v_3 v_1 = 0$ — 3

3) $\det\left(\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}\right) = 1$

— 3 dofs

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3 canonical rotations : Yaw, pitch, roll

1) Yaw. $R(\alpha) \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

No y disturbance.

→ xz plane

2) Pitch. $R(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix}$ No x disturbance yz plane

3) roll $R(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$ No z disturbance xy plane

$R = R(\alpha) R(\beta) R(\gamma)$

$\alpha \in [-\pi, \pi]$

$\beta \in [-\pi/2, \pi/2]$

$\gamma \in [-\pi, \pi]$

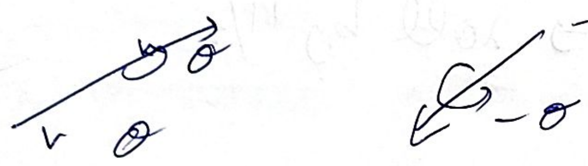
Problems   gimbal lock

Order matters (Non commutative)

· Kinematic singularities and non-uniform representation.

representation theorem

There exists an axis v ($||v||=1$) and θ for every rotation.



Exponential co-ordinates

$(v,\theta) \circ (v',\theta)$ $\Rightarrow$?

Open CV takes (v,θ) where $||v||=1$

$\rightarrow (v_1\theta, v_2\theta, v_3\theta)$ exp. co-ordinates

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Quaternions

$q=(a,b,c,d)$ $||q||=1 \Rightarrow a^2+b^2+c^2+d^2=1$

Set of all unit quaternions allowed (S_3) hypersphere.

$(x,y,z,w) = (b,c,d,a)$

$\Rightarrow a + b\hat{i} + c\hat{j} + d\hat{k}$.

$(v,\theta) \leftrightarrow (\cos\theta/2, v_1 \sin\theta/2, v_2 \sin\theta/2, v_3 \sin\theta/2)$

Useful examples

$(1,0,0,0) \rightarrow$ identity

$(0,1,0,0) \rightarrow$ pitch by π

$(0,0,1,0) \rightarrow$ yaw by π

$(0,0,0,1) \rightarrow$ roll by π

$(1/\sqrt{2}, 1/\sqrt{2}, 0, 0) \rightarrow$ pitch by $\pi/2$

$(1/\sqrt{2}, 0, 1/\sqrt{2}, 0) \rightarrow$ yaw by $\pi/2$

$(1/\sqrt{2}, 0, 0, 1/\sqrt{2}) \rightarrow$ roll by $\pi/2$

Inverses and multiple representations

$(a,b,c,d) \xleftrightarrow{\text{equivalent}} (-a,-b,-c,-d)$

$\updownarrow$ inverse

$(a,-b,-c,-d) \xleftrightarrow{\text{equivalent}} (-a,b,c,d)$

Multiplication

$q_1 \circ q_2$

$p = (b,c,d)$

$q_1 \circ q_2 = (a_1 a_2 - p_1 \cdot p_2, p_1 \times p_2 + a_1 p_2 + a_2 p_1)$

$p = (x, y, z, \cancel{d})$ where $(x,y,z) \in \mathbb{R}^3$

$p' = q \circ p \circ q^{-1}$

Homogeneous transforms

$$\begin{bmatrix}x'\\y'\\z'\end{bmatrix} = \underbrace{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}_{R}\begin{bmatrix}x\\y\\z\end{bmatrix} + \begin{bmatrix}x_T\\y_T\\z_T\end{bmatrix} \qquad Y = AX$$

Make a 4X4 algebraic equivalent.

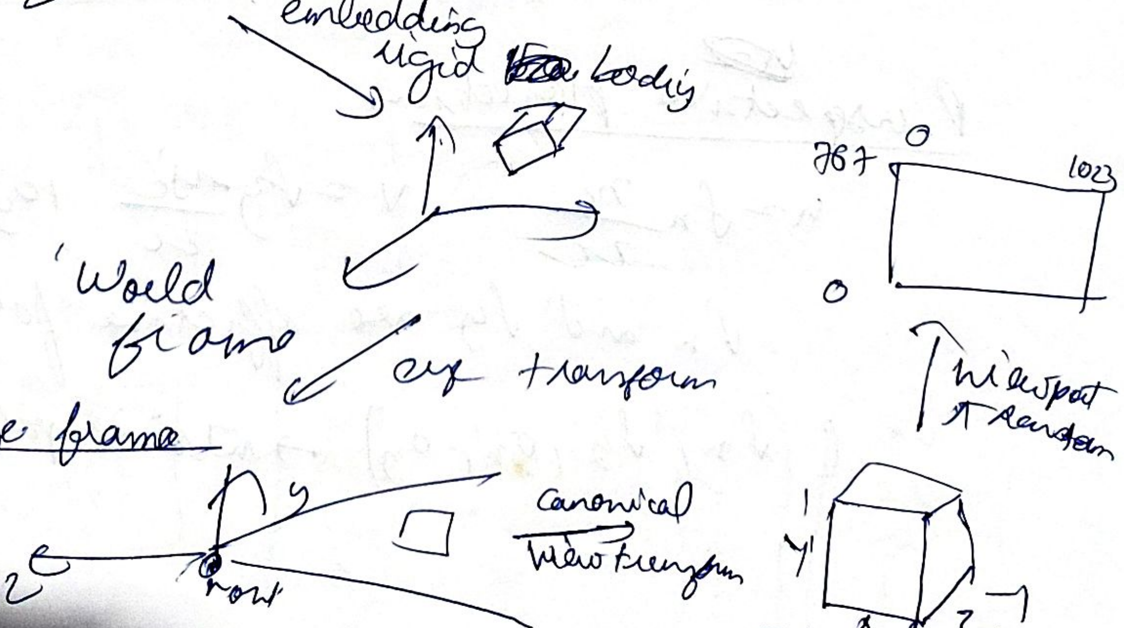
$$\begin{bmatrix}x'\\y'\\z'\\1\end{bmatrix} = \underbrace{\begin{bmatrix} & R & & x_t \\ & & & y_t \\ & & & z_t \\ 0 & 0 & 0 & 1\end{bmatrix}}_{T}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

First apply ~~rotatuition~~ rotation then translate.

What is T^{-1}

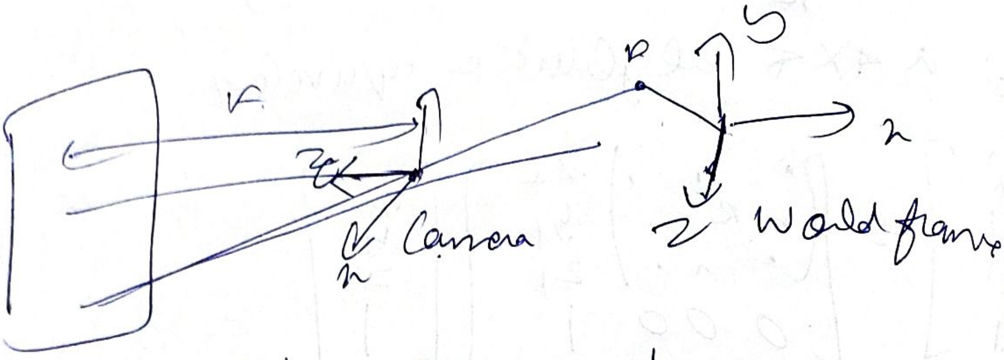
$$T^{-1} = \begin{bmatrix} & R^T & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -x_t \\ 0 & 1 & 0 & -y_t \\ 0 & 0 & 1 & -z_t \\ 0 & 0 & 0 & 1\end{bmatrix}$$

Body frame



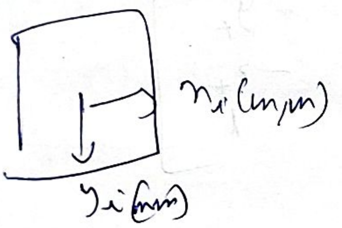
External parameters — position / orientation

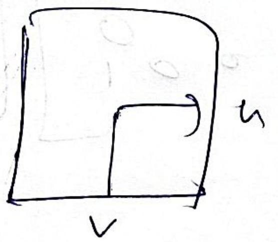
Internal parameters - like focal length



$$\frac{x_i}{f} = \frac{x_c}{z_c} \quad \text{and} \quad \frac{y_i}{f} = \frac{y_c}{z_c}$$

$$\Rightarrow x_i = f\frac{x_c}{z_c} \quad y_i = f\frac{y_c}{z_c}$$





$$u = m_x x_i = \frac{m_x f x_c}{z_c} + o_x$$

$$v = m_y y_i = \frac{m_y f y_c}{z_c} + o_y$$

Principle point (~~xx~~ centre

Perspective projections

$$u = f_x \frac{x_c}{z_c} \qquad v = f_y \frac{y_c}{z_c} + o_y$$

f_x and f_y are effective focal lengths

$(f_x, f_y, o_x, o_y) \rightarrow$ intrinsic parameters

$$\tilde{u} = w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \tilde{u}$$

$$\tilde{x} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{w} \end{bmatrix} = \tilde{x}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \begin{bmatrix} f_x x_c + z_c o_x \\ f_y y_c + z_c o_y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

$\downarrow$

Intrinsic matrix.

$$K = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \text{Calibration matrix.}$$

Extrinsic Parameters

position c_w and orientation R

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$x_c = R(x_w - c_w) = R x_w + t$$

$$\tilde{x}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{22} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$\hat{v}$ = Must Ment ने w → $P \propto w$