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(Q) Find the Truncation error and also check for consistency for the discretization of linear BVP by the central difference scheme.

[Ans] The TE is $a_i Y_{i-1} + b_i Y_i + c_i Y_{i+1} - d_i$ — (1)

where $a_i = \frac{1}{h^2} - \frac{A(x)}{2h}$, $b_i = \frac{-2}{h^2} + B(x_i)$

& $c_i = \frac{1}{h^2} + \frac{A(x)}{2h}$, $d_i = C(x_i)$

& Y_{i-1}, Y_i & Y_{i+1} are the exact solutions of

$$y'' + A(x)y' + B(x)y = C(x)$$

Now, using the Taylor's approx for Y_{i-1} & Y_{i+1},

we get $Y_{i-1} = Y(x_i - h) = Y(x_i) - hY'(x_i) + \frac{h^2}{2!}Y''(x_i)$

$$- \frac{h^3}{3!}Y'''(x_i) + O(h^4).$$

$$Y_{i-1} = Y_i - hY_i' + \frac{h^2}{2!}Y_i'' - \frac{h^3}{3!}Y_i''' + O(h^4).$$

& $Y_{i+1} = Y_i + hY_i' + \frac{h^2}{2!}Y_i'' + \frac{h^3}{3!}Y_i''' + O(h^4)$

Now substituting these in (1)

we get

$$T.E = a_i\left(Y_i - hY_i' + \frac{h^2}{2!}Y_i'' - \frac{h^3}{3!}Y_i''' + O(h^4)\right)$$

$$+ b_i Y_i + c_i\left(Y_i + hY_i' + \frac{h^2}{2!}Y_i'' + \frac{h^3}{3!}Y_i''' + O(h^4)\right)$$

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$$= (a_i + b_i + c_i)Y_i + (-a_i h + c_i h)Y_i' + \left(\frac{h^2}{2}\right)(a_i + c_i)Y_i''$$

$$+ \frac{h^3}{3!}(-a_i + c_i)Y_i''' + O(h^4)(a_i + c_i) - d_i$$

$$= \left(\frac{1}{h^2} - \frac{A(x)}{2h} + B(x) - \frac{2}{h^2} + \frac{1}{h^2} + \frac{A(x)}{2h}\right)Y_i$$

$$+ \left(-\frac{1}{h} + \frac{A(x)}{2} + \frac{1}{h} + \frac{A(x)}{2}\right)Y_i' + \frac{h^2}{2}\left(\frac{1}{h^2} + \frac{1}{h^2}\right)Y_i''$$

$$+ \frac{h^3}{3!}\left(-\frac{1}{h^2} + \frac{A(x)}{2h} + \frac{1}{h^2} + \frac{A(x)}{2h}\right)Y_i''' + O(h^4)\left(\frac{2}{h^2}\right) - C(x).$$

$$= B(x)Y_i + A(x)Y_i' + Y_i'' - C(x_i) + \frac{h^2}{3!}A(x)Y_i''' + O(h^2)$$

$\because$ Y_i is a solⁿ of $y'' + A(x)y + B(x)y = C(x)$

$\therefore$ this term becomes 0.

$TE = O(h^2)$.

So, the truncation error associated with the approximation is of the order of (h^2).

Also, as $h \to 0$

$TE \to 0$.

So, the numerical scheme is consistent with the ODE.

(1) $x^2y'' + xy' = 1$ $\qquad y(1) = 0, \quad y(1.4) = 0.0566$

$h = 0.1$

$x_0 = 1, \; x_1 = 1.1, \; x_2 = 1.2, \; x_3 = 1.3, \; x_4 = 1.4$

$y_0 = 0. \qquad y_4 = 0.0566.$

Unknowns → y_1, y_2, y_3

Now, let's discretize the ODE using B.C's.

$$x_i^2 y_i'' + x_i y_i' = 1$$

$$\Rightarrow x_i^2 \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \right) + x_i \left(\frac{y_{i+1} - y_{i-1}}{2h} \right) = 1.$$

$i = 1, 2, 3$

$$\Rightarrow x_1^2 \left(\frac{y_2 - 2y_1 + y_0}{h^2} \right) + x_1 \left(\frac{y_2 - y_0}{2h} \right) = 1. \quad \text{(i)}$$

$$x_2^2 \left(\frac{y_3 - 2y_2 + y_1}{h^2} \right) + x_2 \left(\frac{y_3 - y_1}{2h} \right) = 1. \quad \text{(ii)}$$

$$x_3^2 \left(\frac{y_4 - 2y_3 + y_2}{h^2} \right) + x_3 \left(\frac{y_4 - y_2}{2h} \right) = 1 \quad \text{(iii)}$$

$$\underbrace{\left(\frac{-2x_1^2}{h^2} \right)}_{b_1} y_1 + \underbrace{\left(\frac{x_1^2}{h^2} + \frac{x_1}{2h} \right)}_{c_1} y_2 = 1 - \underbrace{\left(\frac{x_1^2}{h^2} - \frac{x_1}{2h} \right)}_{a_1} y_0$$

$$\underbrace{\left(\frac{x_2^2}{h^2} - \frac{x_2}{2h} \right)}_{a_2} y_1 + \underbrace{\left(\frac{-2x_2^2}{h^2} \right)}_{b_2} y_2 + \underbrace{\left(\frac{x_2^2}{h^2} + \frac{x_2}{2h} \right)}_{c_2}$$

$$a_i = \frac{x_i^2}{h^2} - \frac{x_i}{2h}, \quad b_i = \frac{-2x_i^2}{h^2}, \quad c_i = \frac{x_i^2}{h^2} + \frac{x_i}{2h}$$

Now, we have,

$$b_1 y_1 + c_1 y_2 = 1 - a_1 y_0 \quad \text{(A)}$$
$$a_2 y_1 + b_2 y_2 + c_2 y_3 = 1 \quad \text{(B)}$$
$$a_3 y_2 + b_3 y_3 = 1 - a_3 y_4 \quad \text{(C)}$$

$Ax = d$

$$A = \begin{bmatrix} b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 \\ 0 & a_3 & b_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1-a_1 y_0 \\ 1 \\ 1-c_3 y_4 \end{bmatrix}$$

Now, we can write (A) (B) & (C) as

$$y_1 + \frac{c_1 y_2}{b_1} = \frac{(1-a_1 y_0)}{b_1}$$

So, $c_1' = \frac{c_1}{b_1}$ $\qquad d_1' = \frac{(1-a_1 y_0)}{b_1}$

Now, $a_2 y_1 + b_2 y_2 + c_2 y_3 = 1$.

$$a_2 y_1 + \frac{a_2 c_1 y_2}{b_1} = \frac{a_2}{b_1}$$

$$\left(b_2 - \frac{a_2 c_1}{b_1}\right) y_2 + c_2 y_3 = \left(1 - \frac{a_2}{b_1}\right)$$

$$y_2 + \frac{c_2 y_3}{(b_2 - a_2 c_1')} = \frac{(1 - a_2 d_1')}{(b_2 - a_2 c_1')}$$

$$c_2' = \frac{c_2}{b_2 - a_2 c_1'}, \quad d_2' = \frac{1 - a_2 d_1'}{b_2 - a_2 c_1'}$$

$$y_2 + c_2' y_3 = d_2'$$

Now, Now, $a_3 y_2 + b_3 y_3 = (1 - c_3 y_4)$

$$a_3 y_2 + a_3 c_2' y_3 = a_3 d_2'$$

$$y_3 (b_3 - a_3 c_2') = (1 - c_3 y_4) - a_3 d_2'$$

$$y_3 = \frac{\{(1 - c_3 y_4) - a_3 d_2'\}}{(b_3 - a_3 c_2')}$$

$\Rightarrow y_3 = 0.034437\ 45167$

Now, $y_2 = d_2' - c_2' y_3$

$= 0.0166557 4562$

Now,

$$y_1 = \frac{1}{b_1} - \frac{c_1 y_2}{b_1}$$

$= 4.574181079 \times 10^{-3}$

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0$

(2) $y'' = x + y$, $y(0) = 0$, $y(1) = 0$ $\quad h = 0.2$

$$y_i'' = x_i + y_i$$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) = x_i + y_i$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} - 1\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2}\right)}_{c_i} y_{i+1} = \underbrace{x_i}_{d_i}$$

$y_0 = 0$

$y_5 = 0$

$$\left.\begin{aligned} a_1 y_0 + b_1 y_1 + c_1 y_2 &= d_1 \\ a_2 y_1 + b_2 y_2 + c_2 y_3 &= d_2 \\ a_3 y_2 + b_3 y_3 + c_3 y_4 &= d_3 \\ a_4 y_3 + b_4 y_4 + c_4 y_5 &= d_4 \end{aligned}\right\} \text{Solve for } y_1, y_2, y_3 \,\&\, y_4$$

$$y_1 + c_1' y_2 = d_1'$$

$$c_1' = \frac{c_1}{b_1}$$

$$d_1' = \frac{(d_1 - a_1 y_0)}{b_1}$$

$$y_2 + c_2' y_3 = d_2',$$

$$y_3 + c_3' y_4 = d_3'$$

$$y_4 + c_4' y_5 = d_4'$$

$$y_4 = d_4' - c_4' y_5$$

$$c_i' = \frac{c_i}{b_i - a_i c_{i-1}'}, \quad d_i' = \frac{d_i - a_i d_{i-1}'}{b_i - a_i c_{i-1}'}$$

Solving this, we get

$~~y(0) = 0~~$ $y(0.2) = -0.02894$, $y(0.4) = -0.05031299$

$y(0.6) = -0.05880$ $\quad y(0.8) = -0.0541$

$$\Rightarrow y_3 = 0.034437\ 45167$$

Now, $y_2 = d_2' - c_2' y_3$

$$= 0.0166557\,4562$$

Now,

$$y_1 = \frac{1}{b_1} - \frac{c_1 y_2}{b_1}$$

$$= 4.574181079 \times 10^{-3}$$

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0$

(2) $y'' = x + y$, $y(0) = 0$, $y(1) = 0$ $h = 0.2$

$$y_i'' = x_i + y_i$$

$$\Rightarrow \frac{(y_{i+1} - 2y_i + y_{i-1})}{h^2} = x_i + y_i$$

$$\Rightarrow \cancel{\left(0 + \frac{1}{h^2}\right) y_{i+1}} \; \underbrace{\left(\frac{1}{h^2}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} - 1\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2}\right)}_{c_i} y_{i+1} = \underbrace{x_i}_{d_i}$$

$$\left.\begin{aligned} a_1 y_0 + b_1 y_1 + c_1 y_2 &= d_1 \\ a_2 y_1 + b_2 y_2 + c_2 y_3 &= d_2 \\ a_3 y_2 + b_3 y_3 + c_3 y_4 &= d_3 \\ a_4 y_3 + b_4 y_4 + c_4 y_5 &= d_4 \end{aligned}\right\} \text{Solve for } y_1, y_2, y_3 \text{ \& } y_4$$

$y_0 = 0$

$y_5 = 0$

$$y_1 + c_1' y_2 = d_1'$$

$$c_1' = \frac{c_1}{b_1}$$

$$d_1' = \frac{(d_1 - a_1 y_0)}{b_1}$$

$$y_2 + c_2' y_3 = d_2',$$

$$y_3 + c_3' y_4 = d_3'$$

$$y_4 + c_4' y_5 = d_4'$$

$$c_i' = \frac{c_i}{b_i - a_i c_{i-1}'}, \quad d_i' = \frac{d_i - a_i d_{i-1}'}{b_i - a_i c_{i-1}'}$$

$$\to y_4 = d_4' - c_4' y_5$$

Solving this, we get

$\cancel{y(0) = 0}$ $y(0.2) = -0.02859$, $y(0.4) = -0.05031299$

$y(0.6) = -0.05880$ $y(0.8) = -0.0441$

Q (3) $y'' - 2y = 0$, $y(0) = 1,\ y'(1) = 0.$

$h = 0.2$

$y_i'' - 2y_i = 0$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) - 2y_i = 0$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2}\right)}_{a_i} y_{i-1} - \underbrace{\left(\frac{2}{h^2} + 2\right)}_{b_i} y_i + \underbrace{\frac{1}{h^2}}_{c_i} y_{i+1} = \underbrace{0}_{d_i}$$

Now, $y_0 = 0$, $\quad y_n' = 0$

$$\Rightarrow \frac{(y_{n+1} - y_{n-1})}{2h} = 0 \Rightarrow y_{n+1} = y_{n-1}$$

Now, we have

$a_i y_{i-1} + b_i y_i + c_i y_{i+1} = 0 \qquad i = 1, 2, \dots, n$

(i) $a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1$ → we have to solve these (5) eq^{ns}

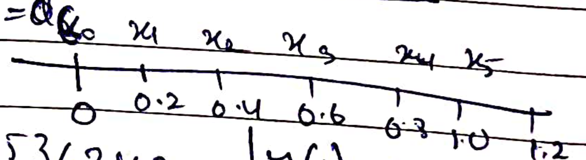
(ii) $- a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2$

(iii) $- a_3 y_2 + b_3 y_3 + c_3 y_4 = d_3$

(iv) $- a_4 y_3 + b_4 y_4 + c_4 y_5 = d_4$

(v) $- a_5 y_4 + b_5 y_5 + c_5 y_4 = d_5$ $\quad \boxed{\Rightarrow y_4 = y_6}$

$(a_5 + c_5) y_4 + b_5 y_5 + \ \ = d_5$



Solving these we get

$y(0.2) = 0.786509$ | $y(0.6) = 0.536243$ | $y(1) = 0.461007$

$y(0.4) = 0.635938$ | $y(0.8) = 0.479447$

x_{-1} x_0 x_1 x_2 x_3 x_4 x_5

-0.25 0 0.25 0.5 0.75 1 1.25

Q4

$$y'' - 2xy' - 2y = -4x, \qquad y(0) = y'(0).$$
$$2y(1) - y'(1) = 1.$$

$$y_i'' - 2x_i y_i' - 2y_i = -4x_i$$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) - 2x_i\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) - 2y_i = -4x_i$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2} + \frac{x_i}{h}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} - 2\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2} - \frac{x_i}{h}\right)}_{c_i} y_{i+1} = \underbrace{-4x_i}_{d_i}$$

So, we will have

$$a_0 y_{-1} + b_0 y_0 + c_0 y_1 = d_0$$
$$a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1$$
$$a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2 \quad (*)$$
$$a_3 y_2 + b_3 y_3 + c_3 y_4 = d_4$$
$$a_4 y_3 + b_4 y_4 + c_4 y_5 = d_5$$

Now, from the B.C. we have

$$y(0) = y'(0) \Rightarrow y_0 = y_0' \Rightarrow y_0 = \frac{y_1 - y_{-1}}{2h}$$
$$\Rightarrow 2h y_0 = y_1 - y_{-1}$$
$$y_{-1} = y_1 - 2h y_0$$

$$2y(1) - y'(1) = 1.$$
$$\Rightarrow 2y_4 - y_4' = 1$$
$$\Rightarrow 2y_4 - \left(\frac{y_5 - y_3}{2h}\right) = 1.$$
$$\Rightarrow 4h y_4 - y_5 + y_3 = 2h$$
$$\Rightarrow (4h y_4 + y_3 - 2h) = y_5$$

Putting these in the eqn (*)

$$a_0(y_1 - 2h y_0) + b_0 y_0 + c_0 y_1 = d_0$$
$$\underbrace{(b_0 - 2a_0 h)}_{b_0} y_0 + \underbrace{(c_0 + a_0)}_{c_0} y_1 = d_0 \quad \text{(I)}$$
$$a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1 \quad \text{(II)}$$
$$a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2 \quad \text{(III)}$$
$$a_3 y_2 + b_3 y_3 + c_3 y_4 = d_3 \quad \text{(IV)}$$
$$a_4 y_3 + b_4 y_4 + c_4(4h y_4 + y_3 - 2h) = d_4$$

$$\Rightarrow \underbrace{(a_4 + c_4)}_{a_4} y_3 + \underbrace{(b_4 + c_4 \cdot 4h)}_{b_4} y_4 = \underbrace{d_4 + 2hc_4}_{d_4} \quad \text{(V)}$$

i	a_i	b_i	c_i	$b_0 - 2a_0 h = -42$
0	16	-34	16	$c_0 + a_0 = 32$
1	17	-34	15	$a_4 + c_4 = 32$
2	18	-34	14	$b_4 + c_4 \times 4h = -22$
3	19	-34	13	$d_4 + 2hc_4 = 2$
4	20	-34	12	

Now, solving these eqn using thomas meth

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$$y_0 + \frac{1c_0}{1b_0} y_1 = \frac{d_0}{1b_0}$$

$$\frac{1c_0}{1b_0} \rightarrow c_0' \qquad \frac{d_0}{1b_0} \rightarrow d_0'$$

$$y_0 + c_0' y_1 = d_0'$$

$$y_1 + c_1' y_2 = d_1'$$

$$y_2 + c_2' y_3 = d_2'$$

$$y_3 + c_3' y_4 = d_3'$$

$$y_4 + c_4' y_5 = d_4'$$

Solving these, $c_4' = 0$

we get,

$$y(0) = 0.731802$$

$$y(0.25) = 0.96049$$

$$y(0.5) = 1.28107$$

$$y(0.75) = 1.73339$$

$$y(1) = 2.43039$$

$x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5$

Q5) A $\; y'' + \overset{B}{2x}\, y' + \overset{C}{2} y = \overset{D}{4}x, \qquad y(0)=1, \qquad y(0.5) = 1.279$

$y_0 = 1 \quad y_5 = 1.279$

$y_i'' + 2x_i y_i' + 2y_i = 4x_i$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) + 2x_i\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) + 2y_i = 4x_i$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2} - \frac{2x_i}{2h}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} + 2\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2} + \frac{2x_i}{2h}\right)}_{c_i} y_{i+1} = \underbrace{4x_i}_{d_i}$$

So, we have

$$\left.\begin{aligned} a_1 y_0 + b_1 y_1 + c_1 y_2 &= d_1 \\ a_2 y_1 + b_2 y_2 + c_2 y_3 &= d_2 \\ a_3 y_2 + b_3 y_3 + c_3 y_4 &= d_3 \\ a_4 y_3 + b_4 y_4 + c_4 y_5 &= d_4 \end{aligned}\right\} \text{ we have to solve for } y_1, y_2, y_3 \,\&\, y_4$$

Now solving them using thomas method

$$\begin{aligned} y_1 + c_1' y_2 &= d_1' \\ y_2 + c_2' y_3 &= d_2' \\ y_3 + c_3' y_4 &= d_4' \\ y_4 + c_4' y_5 &= d_5' \end{aligned}$$

we get, $y(0.1) = 1.09029$

$y(0.2) = 1.16117$

$y(0.3) = 1.21434$

$y(0.4) = 1.25249$

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Q6 $y'' - 2y = 0$, $y_0 = 1$, $y'(1) = 0$ using 2nd order backword diff. for $y'(1) = 0$, $h = 0.2$

Ans $y_i'' - 2y_i = 0$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) - 2y_i = 0$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} - 2\right)}_{b_i} y_i + \underbrace{\frac{1}{h^2}}_{c_i} y_{i+1} = \underbrace{0}_{d_i}$$

Now, $y_n' = 0$

$$\Rightarrow \frac{3y_n - 4y_{n-1} + y_{n-2}}{2h} = 0$$

$$\Rightarrow 3y_n = 4y_{n-1} - y_{n-2} \quad \text{— (I)}$$

& we have, $\Rightarrow 3y_5 = 4y_4 - y_3$

$$a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1 \quad \text{— (II)}$$
$$a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2 \quad \text{— (III)}$$
$$a_3 y_2 + b_3 y_3 + c_3 y_4 = d_3 \quad \text{— (IV)}$$
$$a_4 y_3 + b_4 y_4 + c_4 y_5 = d_4 \quad \text{— (V)}$$

Using (I) in (V), we get,

$$a_4 y_3 + b_4 y_4 + c_4\left(\frac{4y_4 - y_3}{3}\right) = d_4.$$

$$\Rightarrow \left(a_4 - \frac{c_4}{3}\right) y_3 + \left(b_4 + \frac{4c_4}{3}\right) y_4 = d_4$$

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Let's solve (II)-(V) using thomas algo.

$y_1 + c_1' y_2 = d_1'$ $\qquad c_1' = \frac{c_1}{b_1}$, $\quad d_1' = \frac{d_1 - a_1 y_0}{b_1}$

$y_2 + c_2' y_3 = d_2'$

$y_3 + c_3' y_4 = d_3'$

$y_4 + 0 = d_4'$

$y_1 = y(0.2) = \cancel{0.71186}$ 0.7861564

$y_2 = y(0.4) = \cancel{0.48068}$ 0.635205

$y_3 = y(0.6) = \cancel{0.281959}$ 0.5350706

$y_4 = y(0.8) = \cancel{0.118227}$ 0.4777416

$y_5 = y(1) = \underline{\underline{0.4586319}}$.

(A) $y'' + A(x) y' + B(x) y = C(x)$

$$\left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) + A(x)\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) + B(x) y_i = C(x)$$

$$\underbrace{\left(\frac{1}{h^2} - \frac{A(x)}{2h}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} + B(x)\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2} + \frac{A(x)}{2h}\right)}_{c_i} y_{i+1} = \underbrace{C(x)}_{d_i}$$

$a_i = \frac{1}{h^2} - \frac{A(x)}{2h}$ $\qquad d_i = C(x)$

$b_i = \frac{-2}{h^2} + B(x)$

$c_i = \frac{1}{h^2} + \frac{A(x)}{2h}$

(A) Show that the control vol. method reduces to central diff. scheme if p, q & r are const.

For the control vol. method,

$$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + q(x)y = r(x).$$

$$\Rightarrow \quad p(x)y'' + p'(x)y' + q(x)y = r(x)$$

If $p(x)$, $q(x)$ & $r(x)$ are continuous, then,

$$p_{i+1/2}\left(\frac{y_{i+1}-y_i}{h}\right) - p_{i-1/2}\left(\frac{y_i - y_{i-1}}{h}\right) + y_i\left[\frac{q_i h + q_i h}{2}\right]$$

$$= r_{i-}\,\delta x_{i-1} + r_{i+}\,\delta x_{i+}$$

$$\Rightarrow \left(\frac{p_i + p_{i+1}}{2}\right)\left(\frac{y_{i+1}-y_i}{h}\right) - \left(\frac{p_i - p_{i-1}}{2}\right)^2\left(\frac{y_i - y_{i-1}}{2}\right) + y_i q_i h$$

$$= r_i h$$

$$\Rightarrow \frac{p_i}{2}\left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h}\right) + \frac{p_{i+1}}{2}\left(\frac{y_{i+1}-y_i}{h}\right) + \frac{p_{i-1}}{2}\left(\frac{y_i - y_i}{2}\right)$$

$$+ y_i q_i h = r_i h.$$

$$\Rightarrow \quad p_i y_i'' + p_i y_i' + y_i q_i = r_i$$

using the central diff. app. $\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} = y_i''$

thus, it reduces to CDA.