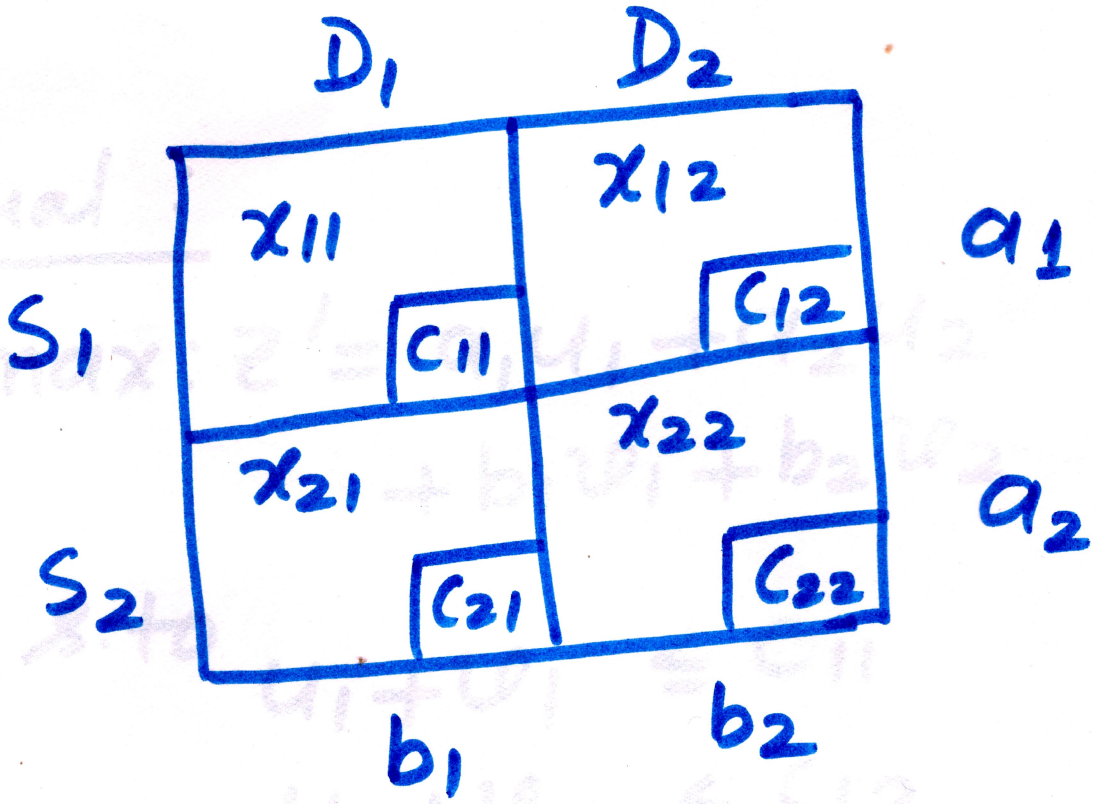


$$\min: Z = c_{11}x_{11} + c_{12}x_{12} + c_{21}x_{21} + c_{22}x_{22}$$

$$\text{s.to} \quad x_{11} + x_{12} = a_1$$

$$x_{21} + x_{22} = a_2$$

$$x_{11} + x_{21} = b_1$$

$$x_{12} + x_{22} = b_2$$

$$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$$

Dual :

$$\max: z' = a_1u_1 + a_2u_2 + b_1v_1 + b_2v_2$$

s.to

$$u_1 + v_1 \leq c_{11}$$
$$u_1 + v_2 \leq c_{12}$$
$$u_2 + v_1 \leq c_{21}$$
$$u_2 + v_2 \leq c_{22}$$

u_1, u_2, v_1, v_2 are free variables.
(+ve, -ve, zero)

Primal - Dual Conditions:

$$(u_1 + v_1 - c_{11})\, x_{11} = 0$$
$$(u_1 + v_2 - c_{12})\, x_{12} = 0$$
$$(u_2 + v_1 - c_{21})\, x_{21} = 0$$
$$(u_2 + v_2 - c_{22})\, x_{22} = 0$$

$$(u_i + v_j - c_{ij})\, x_{ij} = 0$$

$\Rightarrow$ Either

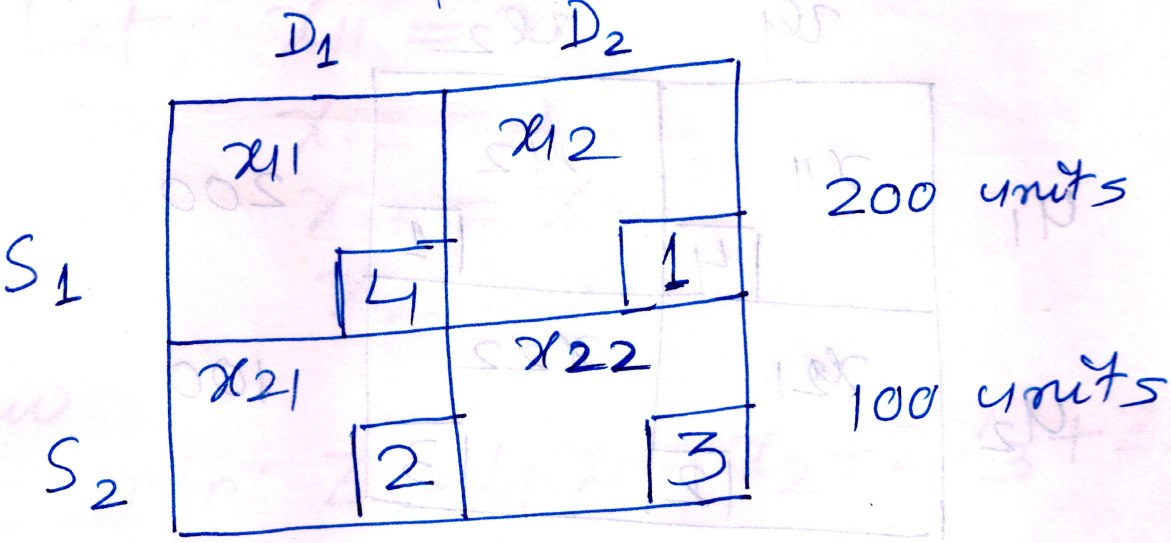
$$u_i + v_j = c_{ij}$$

or

$$x_{ij} = 0$$

for all i and j

Example



Balanced Transportation Problem

$$\min: Z = 4x_{11} + x_{12} + 2x_{21} + 3x_{22}$$

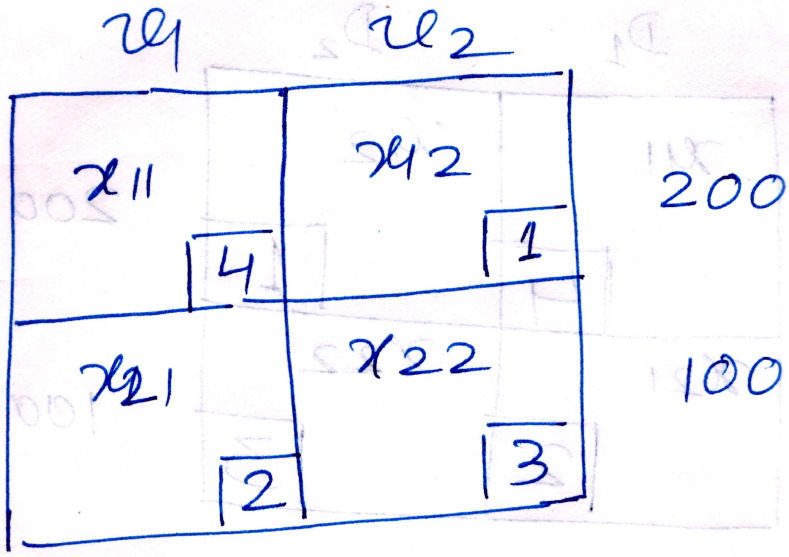
s.to $x_{11} + x_{12} = 200$

$x_{21} + x_{22} = 200$

$x_{11} + x_{21} = 150$

$x_{12} + x_{22} = 150$

$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$



$$\min: Z = 4x_{11} + x_{12} + 2x_{21} + 3x_{22}$$

s.to $x_{11} + x_{12} \leq 200$

$x_{21} + x_{22} = 100$

$x_{11} + x_{21} = 150$

$x_{12} + x_{22} = 150$

$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$

Let $x_{11} = x_1$

$x_{12} = x_2$

$x_{21} = x_3$

$x_{22} = x_4$

Now

$$\min: Z = 4x_1 + x_2 + 2x_3 + 3x_4$$

s.to $x_1 + x_2 \leq 200$

$x_3 + x_4 = 100$

$x_1 + x_3 = 150$

$x_2 + x_4 = 150$

$x_1, x_2, x_3, x_4 \geq 0$

Dual:

$$\max: Z' = 200u_1 + 100u_2 + 150v_1 + 150v_2$$

s.to $u_1 + v_1 \leq 4, \quad u_1 + v_2 \leq 1$

$u_2 + v_1 \leq 2, \quad u_2 + v_2 \leq 3$

u_1, u_2, v_1, v_2 are free.

Primal - Dual Conditions:

$$(u_1 + v_1 - c_{11})\, x_{11} = 0$$
$$(u_1 + v_2 - c_{12})\, x_{12} = 0$$
$$(u_2 + v_1 - c_{21})\, x_{21} = 0$$
$$(u_2 + v_2 - c_{22})\, x_{22} = 0$$

$\Rightarrow$ Either

$$u_1 + v_1 = c_{11}$$

or

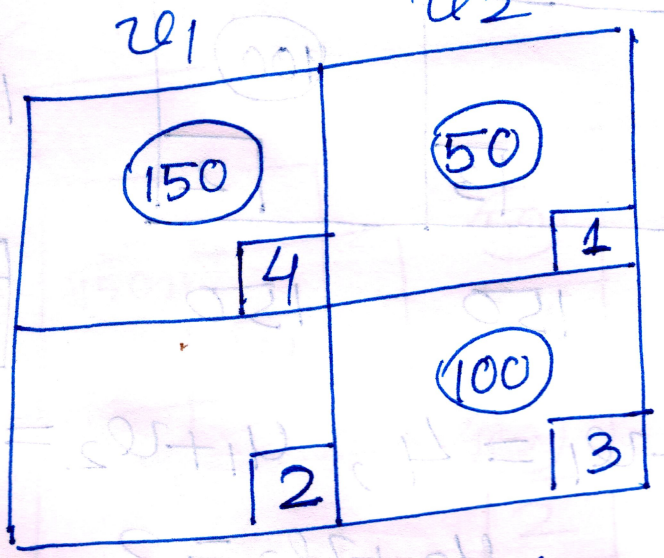
$$x_{11} = 0 \quad \text{etc}$$

If $x_{11} \neq 0$, then

$$u_1 + v_1 = c_{11} \quad \text{etc}$$

Phase – I Solution:

(NWCR)



$x_{11} = 150$, $x_{12} = 50$

$x_{22} = 100$

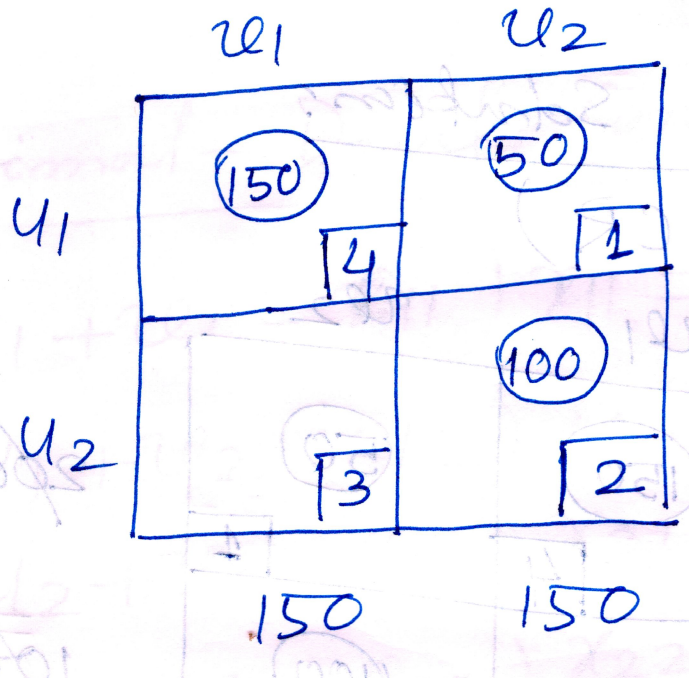
$Z = 150 \times 4 + 50 \times 1 + 100 \times 3$

$= 950$

From Dual

$u_1 + v_1 = 4$, $u_1 + v_2 = 1$

$u_2 + v_2 = 3$



$Z = 950$

$u_1 + v_1 = 4$, $u_1 + v_2 = 1$

$u_2 + v_2 = 2$

since $x_{11} = 150$, $x_{12} = 50$

and $x_{22} = 100$

$m + n - 1 = 2 + 2 - 1 = 3$

$u_1 = 0$, $v_1 = 4$

$u_2 = 1$ $v_2 = 1$

Check for optimality

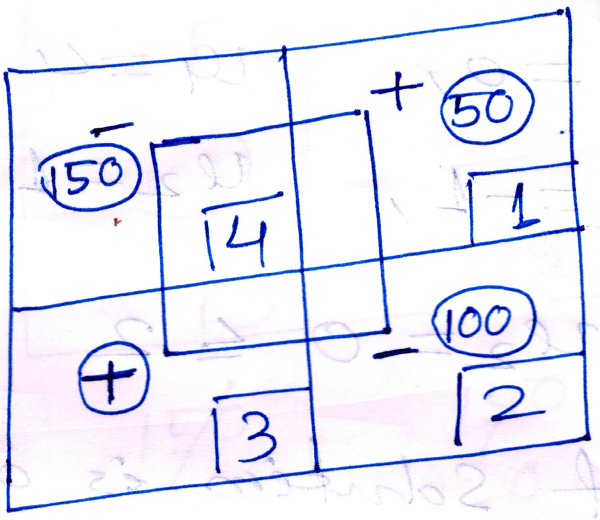
For cell (2, 1)

$u_2 + v_1 = 5 > 3$ ✗

For optimality $u_2 + v_1 \leq 3$

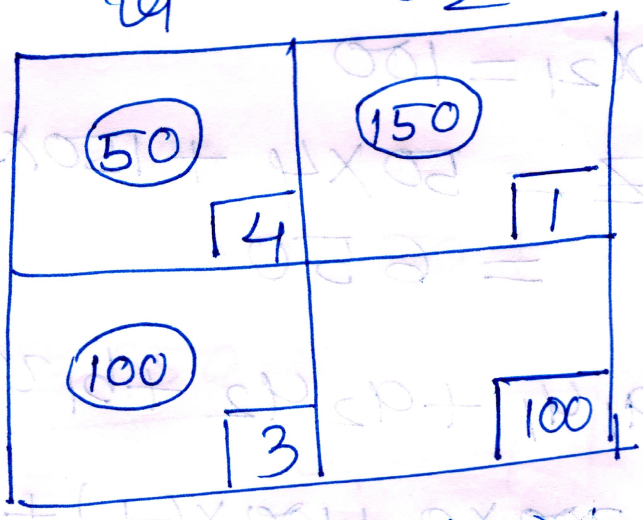
Present Solution is not optimal.

Apply MODI Method:



150 150

v_1 v_2



u_1

u_2

150 150

Check for optimality.

$u_1 + v_1 = 4$

$u_1 + v_2 = 1$

$u_2 + v_1 = 3$

$u_1 = 0, \quad v_1 = 4$

$u_2 = -1, \quad v_2 = 1$

$u_2 + v_2 = -1 \leq 2$

Present Solution is optimal

$x_{11} = 50, \quad x_{12} = 150$

$x_{21} = 100$

$Z = 50 \times 4 + 150 \times 1 + 100 \times 3$

$= 650$

$Z' = a_1 u_1 + a_2 u_2 + b_1 v_1 + b_2 v_2$

$= 200 \times 0 + 100 \times (-1) + 150 \times 4 + 150 \times 1$

$= 750 - 100 = 650$

$Z' = Z \Rightarrow \min: Z = \max: Z'$

Least cost Method:

	D_1	D_2	
S_1	(50) 4	(150) 1	200 150
S_2	(100) 3	2	100 0
	150 50	150 0	

$x_{11} = 50, \quad x_{12} = 150$

$x_{21} = 100$

$Z = 200 + 150 + 300$

$= 650$

$u_1 = 0, \quad v_1 = 4$

$u_2 = -1 \quad v_2 = 1$

Present Solution is Optimal.