Group Theory

Lecture 8

Defn A subgp N of G is called normal subgp of G $(N \trianglelefteq G)$ if $ghg^{-1} \in N$ $\forall h \in N$ and $\forall g \in G$

Question when a left is equal to a right coset in a group?

If the group is abelian then every left coset is equal to a right coset.

Let G be a gp and H be a subgp of G.

Let a left coset is equal to a right coset i.e $\underset{a = a \cdot 1}{a} \in aH = Hb$ for some $a, b \in G$. $\Rightarrow$ $a \in \overline{H\overset{= 1a}{a} \cap Hb}$ which implies that $Ha = Hb$ as they are not disjoint. $\Rightarrow aH = Ha$

$\Rightarrow aHa^{-1} = H$.

$\Rightarrow H$ is a normal subgp of G.

Question what about the converse?

The converse true.

Let H be a normal subgp.

WTS $aH = Ha$.

For all $h \in H$ we have $ah = (\underbrace{aha^{-1}}_{\in H})a$

$(\text{here } h' = aha^{-1}) \quad = h'a$

$\in Ha$.

Thus $aH = Ha$.

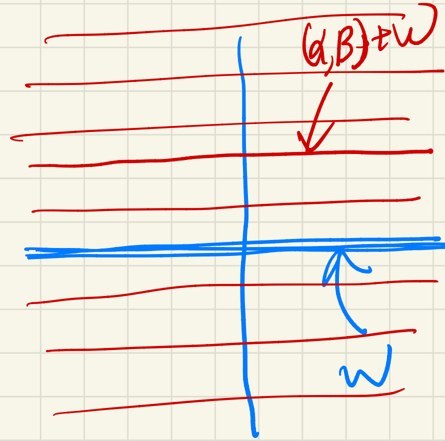
Propn A subgp H of G is normal iff every left coset is also a right coset.

Cor If $[G:H] = 2$ then $H \trianglelefteq G$.

Pf: For any $a \notin H$ $\quad$ $G = H \cup aH$

$= H \cup Ha$ $\Rightarrow$ $aH = Ha$, $H \trianglelefteq G$.

Linear algebra



Let $V = \mathbb{R}^2$

and $W = \{(x,0) \mid x \in \mathbb{R}\}$

$(\alpha,\beta) \in V$

$$(\alpha,\beta)+W = \{(\alpha,\beta)+(x,0) \mid x \in \mathbb{R}\}$$

$$= \{(\alpha+x,\beta) \mid x \in \mathbb{R}\}.$$

$$\boxed{V/W = \{(\alpha,\beta)+W \mid (\alpha,\beta) \in \mathbb{R}^2\}.}$$

↳ This is a vector space which is the quotient sp of W in V.

$\cong \mathbb{R}$.

$$T : V/W \longrightarrow \mathbb{R}.$$

$$T(\overline{x,y}) = T\big((x,y)+W\big) = y.$$

$$\ker T = W.$$

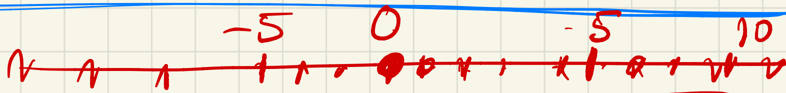
$$\boxed{x \sim y \quad \text{if} \quad x^{-1}y \in H}$$

$$(\alpha_1, \beta_1) \sim (\alpha_2, \beta_2)$$

$$\text{if} \quad (\alpha_1 - \alpha_2, \beta_1 - \beta_2) \in W,$$

$$\beta_1 = \beta_2.$$

$\mathbb{Z}$.



$a \sim b$ if $n \mid a-b$.

$$a - b = kn.$$

$$\Rightarrow a = b + kn.$$

<u>If $n = 5$.</u>

$$\bar{0} = 5k,$$

$$\bar{1} = 1 + 5k$$

$$\bar{2} = 2 + 5k.$$

$$\bar{3} = 3 + 5k$$

$$\bar{4} = 4 + 5k$$

$$\mathbb{Z}/5\mathbb{Z}$$

$$\left| G/H \right| = [G : H].$$

left cosets of H