

Dual

	D_1	D_2	
S_1	x_{11}	x_{12}	a_1
S_2	x_{21}	x_{22}	a_2
	b_1	b_2	

$$\min: Z = C_{11}x_{11} + C_{12}x_{12} \\ + C_{21}x_{21} + C_{22}x_{22}$$

$$\text{s.t. } x_{11} + x_{12} = a_1$$

$$x_{21} + x_{22} = a_2$$

$$x_{11} + x_{21} = b_1$$

$$x_{12} + x_{22} = b_2$$

$$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$$

(1)

Dual :

$$\text{max: } Z' = a_1 u_1 + a_2 u_2 \\ + b_1 v_1 + b_2 v_2$$

$$\text{s.t. } u_1 + v_1 \leq C_{11}$$

$$u_1 + v_2 \leq C_{12}$$

$$u_2 + v_1 \leq C_{21}$$

$$u_2 + v_2 \leq C_{22}$$

u_1, u_2, v_1, v_2 are free variables.

(+ve, -ve, zero)

②

Primal-Dual Conditions:

$$(u_1 + v_1 - c_{11}) x_{11} = 0$$

$$(u_1 + v_2 - c_{12}) x_{12} = 0$$

$$(u_2 + v_1 - c_{21}) x_{21} = 0$$

$$(u_2 + v_2 - c_{22}) x_{22} = 0$$

$$(u_i + v_j - c_{ij}) x_{ij} = 0$$

\Rightarrow Either

$$u_i + v_j = c_{ij}$$

or

$$x_{ij} = 0$$

for all i and j

③

Example

		D_1	D_2	
		x_{11}	x_{12}	200 units
S_1		4	1	
	S_2	2	3	100 units
		150 units	150 units	

Balanced Transportation Problem

$$\text{min: } Z = 4x_{11} + x_{12}$$

$$0x_1 = \dots + 2x_{21} + 3x_{22}$$

$$0x_1 = \dots + x_{11} + x_{12} = 200$$

$$0x_1 = \dots + x_{21} + x_{22} = 200$$

$$x_{11} + x_{21} = 150$$

$$x_{12} + x_{22} = 150$$

$$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$$

(4)

	x_{11}	x_{12}	
u_1	4	1	200
u_2	2	3	100

$$150 \quad 150$$

min: $Z = 4x_{11} + x_{12} + 2x_{21} + 3x_{22}$

s.t. $x_{11} + x_{12} \leq 200$

$$5x_{11} + 15x_{12} + x_{21} + x_{22} = 100$$

$$x_{11} + x_{21} = 150$$

$$5x_{11} + 15x_{12} + x_{22} = 150$$

$$15x_{12} + x_{22} = 150$$

$$x_{11}, x_{12}, x_{21}, x_{22} \geq 0$$

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$$\begin{aligned} \text{Let } x_{11} &= x_1 \\ x_{12} &= x_2 \quad \rightarrow \text{homogeneous} \\ x_{21} &= x_3 \\ x_{22} &= x_4 \end{aligned}$$

Now

$$\min: Z = 4x_1 + x_2 + 2x_3 + 3x_4$$

$$\text{s.t. } \begin{aligned} x_1 + x_2 &\leq 200 \\ x_3 + x_4 &= 100 \end{aligned}$$

$$\begin{aligned} x_1 + x_3 &= 150 \\ x_2 + x_4 &= 150 \end{aligned}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$\text{Dual: } \max: Z' = 200u_1 + 100u_2$$

$$+ 150v_1 + 150v_2$$

$$\text{s.t. } u_1 + v_1 \leq 4, u_1 + v_2 \leq 1$$

$$u_2 + v_1 \leq 2, u_2 + v_2 \leq 3$$

u_1, u_2, v_1, v_2 are free.

⑥

Primal-Dual Conditions:

$$(u_1 + v_1 - c_{11}) x_{11} = 0$$

$$(u_1 + v_2 - c_{12}) x_{12} = 0$$

$$(u_2 + v_1 - c_{21}) x_{21} = 0$$

$$(u_2 + v_2 - c_{22}) x_{22} = 0$$

\Rightarrow Either $u_1 + v_1 = c_{11}$

or $u_2 + v_1 = c_{21}$

etc $x_{11} = 0$ etc

choose $x_{11} \neq 0$, then

$$u_1 + v_1 = c_{11} \text{ etc}$$

$$u_2 + v_1 = c_{21} \text{ etc}$$

$$x_{11} > 0 \text{ etc}$$

and $x_{11} \neq 0$ etc

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②

Phase - I Solution:

(NWCR)

	x_{11}	x_{12}	
u_1	150	50	200
u_2	100	100	100
	4	1	50%
	2	3	100%

$$x_{11} = 150, \quad x_{12} = 50$$

$$x_{22} = 100$$

$$Z = 150 \times 4 + 50 \times 1 + 100 \times 3 \\ = 950$$

From Dual $u_1 + x_{11} = 4, \quad u_1 + x_{12} = 1$

$$u_2 + x_{22} = 3$$

$$\Sigma P = P_1 + P_2$$

⑧

	U_1	U_2	
U_1	150	50	200
U_2	14	1	100
	100	12	100

$$Z = 950$$

$$U_1 + U_1 = 4, \quad U_1 + U_2 = 1$$

$$U_2 + U_2 = 2$$

since $x_{11} = 150, x_{12} = 50$

and $x_{22} = 100$

$$m+n-1 = 2+2-1 = 3$$

$$U_1 = 0, \quad U_1 = 4$$

$$U_2 = 1 \quad U_2 = 1$$

Check for optimality

For Cell (2,1)

$$U_2 + U_1 = 5 > 3 \times 1$$

For optimality $U_2 + U_1 \leq 3$

(9)

Present Solution is
not optimal.

Apply MODI Method:

(50)	4	+	(50)
(+)		-	(100)
3	2		

200

100

0 150 150
21 22

(50)	4	1
100	3	100

200

100

Check for optimality.

2's without lines

$$u_1 + u_2 = 4$$

$$u_1 + u_2 = 1$$

$$u_2 + u_1 = 3$$

$$u_1 = 0, \quad u_2 = 4$$

$$u_2 = -1, \quad u_1 = 1$$

$$u_2 + u_2 = 0 \leq 2$$

Present Solution is optimal

$$x_1 = 50, \quad x_2 = 150$$

$$x_{21} = 100$$

$$\begin{aligned} Z &= 50 \times 4 + 150 \times 1 + 100 \times 3 \\ &= 650 \end{aligned}$$

$$Z' = a_1 u_1 + a_2 u_2 + b_1 v_1 + b_2 v_2$$

$$\begin{aligned} &= 200 \times 0 + 100 \times (-1) + 150 \times 4 \\ &\quad + 150 \times 1 \end{aligned}$$

$$= 750 - 100 = 650$$

$$Z' = Z \Rightarrow \min: Z = \max: Z'$$

(11)

Least Cost Method:

		D_1	D_2	
		50	150	200 50
S_1	02	4	1	190 0
	01	100	2	
S_2	01	3	0	150
	S_1	150	8	150
		50		

$$x_{11} = 50, \quad x_{12} = 150$$

$$x_{21} = 100$$

$$Z = 200 + 150 + 300 \\ = 650$$

$u_1 = 0, \quad u_1 = 4$	
$u_2 = -1 \quad u_2 = 1$	

Present Solution is Optimal.

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