HW9

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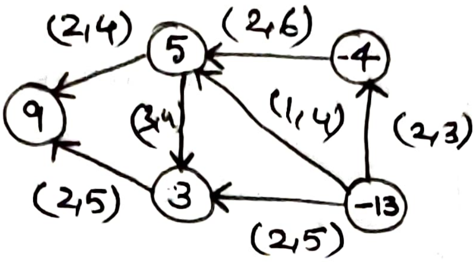
1.



ⓐ

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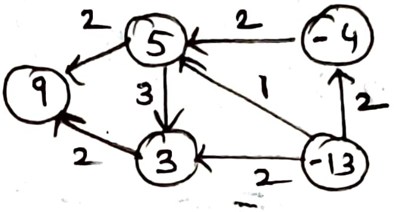
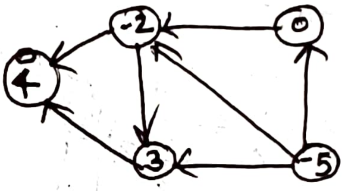


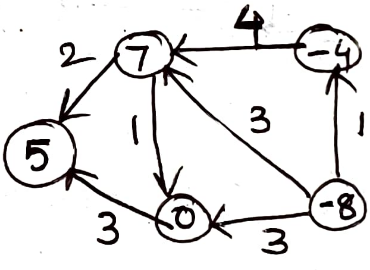
⇒ Assigning lower bounds

Lv



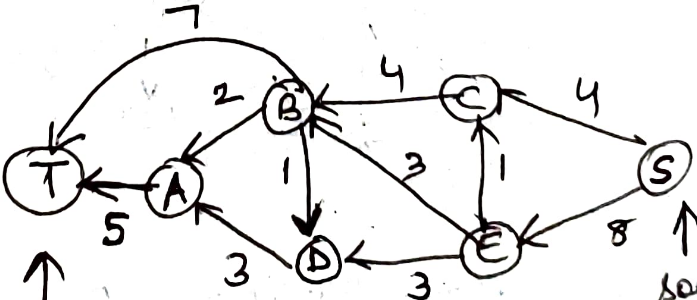
⇒ $L_v = f_{in} - f_{out}$ calculating L_v for all nodes

G'



⇒ ① upper bound − lower bound and ② demand value $D = L_v$

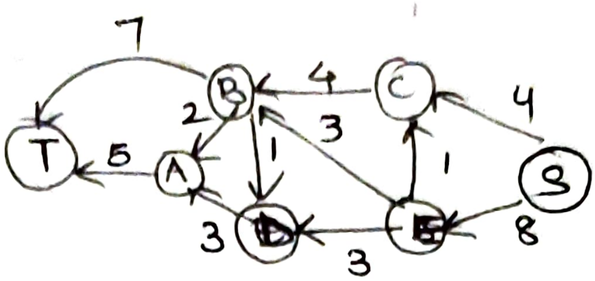
ⓑ



sink

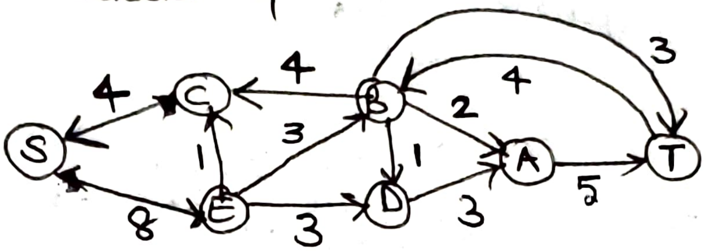
source

c)



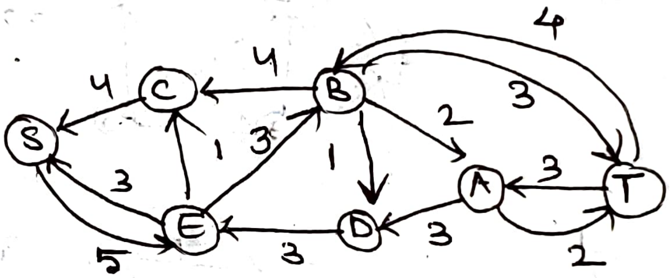
path: S → C → B → T   flow = 4

bottleneck = 4



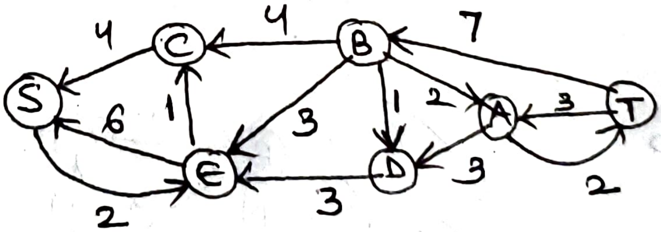
path: S → E → D → A → T

flow = 3



path: S → E → B → T

flow = 3.



Max flow = 4+3+3 = 10

Demand = 5+7 = (−4−8) = 12

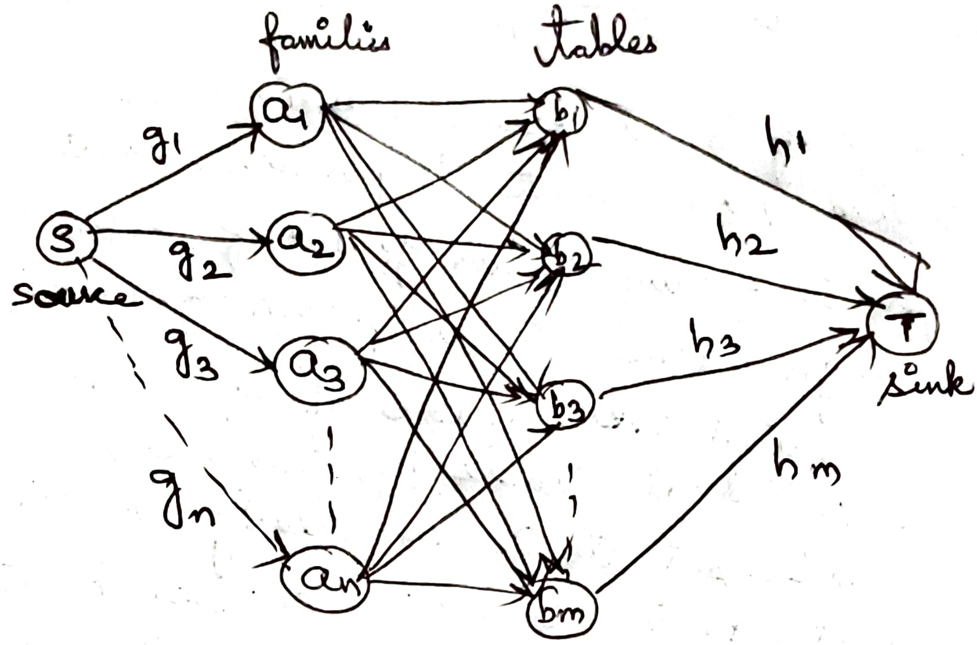
As max flow < Demand ⇒ there is no feasible circulation

2. Given pile of boxes which needs to be placed inside one another to reduce the number of visible boxes. Each box is of rectangular parallelopiped shape with side lengths of $\{i_1, i_2, i_3\}$ for a box i, we can reduce this problem to a circulation flow problem with lower bounds, where a unit of flow represents boxes that are placed inside one visible box whose dimensions are larger than other boxes which are feasible to place one box inside another box.

Each box can either be visible or can be empty, which represents with a lower bound of 0 & capacity 1 with connecting source to each box. Similarly the lower bound is 0 and capacity is 1 for connecting each box which is empty to sink (T). the remaining edges (P_i, q_i) for box i represents a box, we represent each box as an edge with lower & capacity values being 1 & connecting to other boxes (q_i, P_j) be the box j & lower bound can be 0 and capacity of 1. The demand here is $-k$ and demand with k and sink node which represents boxes that are filled & boxes that are empty respectively. The claim is that there is a nesting of k visible boxes iff there is a feasible circulation of demand k in G.

Suppose there is an arrangement of k boxes which are visible in a nested order. The nested boxes arrangement defines a path from source to sink being $(S, P_1, q_1, P_2, q_2, \ldots t)$ This implies we have k paths from s to t & corresponding circulation with lower bounds is formed. The next part of proof is that we have a feasible circulation and as edges are unit capacity & we want to consider boxes that are nested & we make use of node disjoint technique to get boxes ~~assigned~~ arranged in nested order to form k feasible visible boxes. Here we can make use of scaled version or Edmond Karp algorithm to compute feasible circulation in polynomial time. So the overall complexity is polynomial time.

3.



We can solve the problem using max-flow technique. We can assume that $a_1, a_2 -- a_n$ are n families to be seated across $b_1, b_2 -- b_m$ tables with each table having $h_1, h_2, -- h_m$ seats. ~~and connecting~~ We connect tables to sink (T) with capacities of $h_1, h_2, -- h_m$ and Connecting each person from family a_i to a table b_j with no other person from a_i is seated over b_j & capacity is 1 as one person can sit at one table. Also Connecting the source node to n families $a_1, a_2, -- a_n$ with capacities $g_1, g_2, - g_n$ as each family a_i has g_i no. of people.

Now we find the max flow & if max flow is Σg_i, then there is a feasible solution possible with no 2 members of same family seated over same table otherwise No.

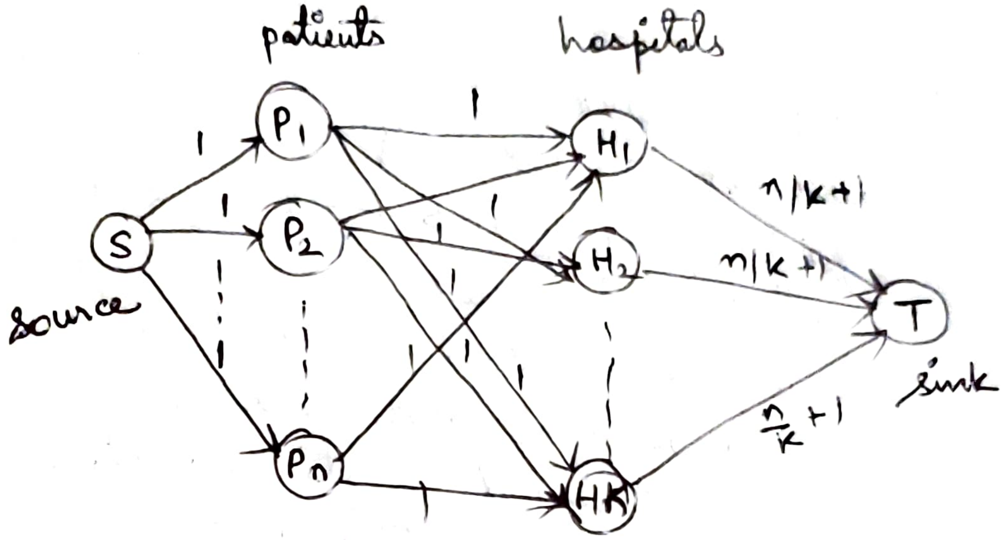
The claim here is feasible solution is possible if and only if max flow across the network is $\sum_{i=1}^{n} g_i$

The proof is, at first assuming there is a valid seating arrangement is possible. As the flow for a family a_i has g_i no. of people & we assign value 1 if a person from family a_i is seated over table b_j. If no person is seated at table b_j, the edge capacity can be set to 0. So, clearly we can notice that g_i is flowing from source to a_i & each person is seated over different tables & here flow is conserved and this arrangement is possible for all families with no person left without a seat at a table. So, the maximum flow here is $\sum_{i=1}^{n} g_i$.

Conversely if we are given that maximum flow is $\sum_{i=1}^{n} g_n$ then, as capacities are integers. Consider a family a_i where g_i members are at the party & we assign each person to a different table where that table doesn't have a family member from a_i & there is a seat available at that table b_j. We assign or create an arrangement of seating by making the edge capacities altering between 0 and 1. So flow of an edge is 1 if a person is seated at a table. Thus as all the edges from the family node a_i are saturated that implies that the every member of family a_i is seated at different tables. Also the seats filled will be atmost b_j at table b_j, thus we have a valid seating with the given constraints.

4.

a)



Here, we can find a feasible solution using max flow method. Assuming S as source and T as sink and given there are 'n' patients and 'k' hospitals and each hospital is assigned to a hospital if patient's location from that hospital is 30 minutes journey and the hospital has less than $n/k+1$ no of patients. So edge connecting patient to hospital varies from 0 ~~or~~ 1 with 0 being not assigned & 1 being assigned to that hospital. Also each patient is connected to source node with each edge capacity being 1. So, based on this network max flow is performed and if max flow becomes 'n' then all the patients are assigned to hospital thus giving us the feasible solution.

b) To prove, we can claim that a feasible or balanced allocation of patients is possible if and only if the max flow is 'n'.

To prove this, at first assuming there is balanced allocation of patients to the hospital then as each patient is assigned a nearest hospital such that the n patients are admitted at the hospital with the total flow coming from s being n and each patient is assigned a hospital & with each hospital having less than n/k+1 no. of patients assigned, then that patient is mapped to that hospital & at max n/k patients are assigned to a hospital & that amount of flow reaching sink node & thus all the n patients flow is being mapped successfully as feasible solution is given, then the maximum flow is n for this network.

This maximum flow can be found using ford-fulkerson algorithm. Conversely, given the maximum flow of value 'n', that means each patient is assigned a hospital nearest to his location. As max flow is n that implies all edges from source to patients are saturated & each patient is allocated to the hospital with capacity of each patient being saturated & each hospital is admitted less than

$n/k + 1$ patients, that implies there is valid assignment of patients to the hospitals with given constraints.

c) we make use of ford-fulkerson algorithm and time complexity of this algorithm is $O(Cm)$ where C is max flow & here $C = n$ (max flow) & m is no. of edges and $m = n + k + n * k$ because there are n patients connected from source, k hospitals connected to target & in between n patients are being mapped to k hospitals ~~so~~ $n * k$.

$$T.C = O(n(n+k+nk)) = \underline{\underline{O(n^2 k)}}$$