Home work - 8

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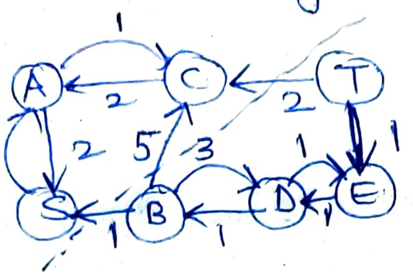
1.

a) Final residual graph



b) Max flow value = 3

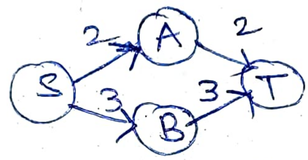
c) Min cut : $A^* = \{S, A, C\}$

$B^* = \{B, D, E, T\}$

2.

a) TRUE

Consider,

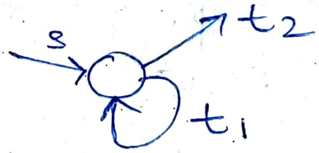


Number of disjoint sets are two

Max flow = 5 $\Rightarrow 2 \neq 5$

b) TRUE,

Consider



$s = t_1 + t_2$

$s + t_1 = t_2$

If the input flow is divided into two paths t_1 & t_2 (containing positive cycle flow)

Using conservation of flow

$s = t_1 + t_2$

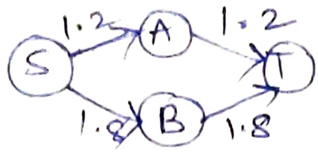
$s + t_1 = t_2$

$2s + t_1 = \cancel{t_1} + 2t_2$

$\Rightarrow s = t_2$

The flow going through cycle can be directly pushed through t_2

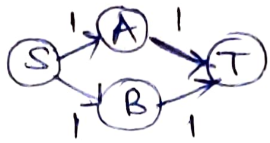
c) FALSE



Max flow $= 1.2 + 1.8$

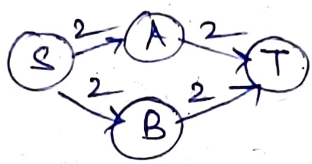
$= 3.0$ (integer)

d) FALSE



Max flow $= 2$

increasing every edge by 1



Max flow $= 4 > 3$ $(2+1)$

$= 2+2$

$\neq 2+1$

e) TRUE

The relative order of capacity of cuts will not change.

③ Assume max flow to be 'F'. Now we perform Ford-Fulkerson to get max flow. Compute residual graph for max flow. Compute min cut (A^*, B^*).

Now let 'F' be the number of edges going from A^* to B^* (Because each edge capacity is 1 and max flow is F, number of edges is also F)

If $F < K$, then removing k edges connected A^* & B^* will remove complete flow and make graph disconnected.

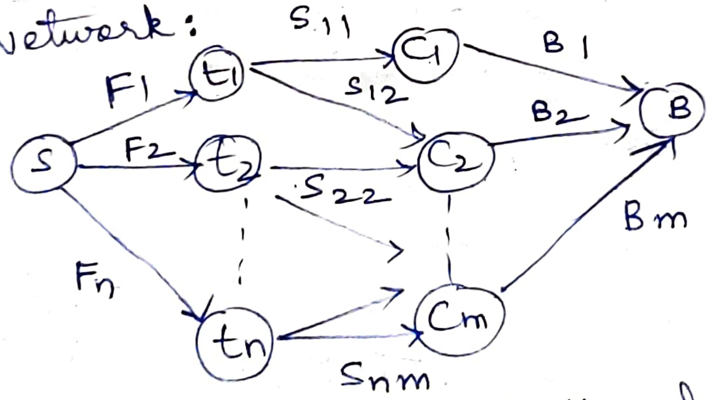
If F>K then flow will reduce by 'K'
Flow becomes F-K

Since the edge capacities are 1 the flow reduces by number of edges, but this cannot be guaranteed if edge capacities are greater than 1. We cannot guarantee max flow reduction.

Best way to remove edges (with capacity ≥1) is to remove top k edges with highest flows outgoing from source. This will guarantee maximum reduction.

(4.)

a). Network:



The above network is the design to solve the given problem.

→ Connecting a source node with all the tourists to with edge capacity as the amount of dollars they have.

→ Connecting each tourist with the currencies which they need with edge capacities as corresponding limit.

→ Connecting each currency nodes with bank with edge capacities as maximum exchange bank can perform.

If we perform ford-fulkerson on the given graph then we will get the amount of conversions and requests performed by bank.

b) The algorithm will provide solution if the max flow of the network is total amount of dollars ~~possed~~ possessed by tourists.

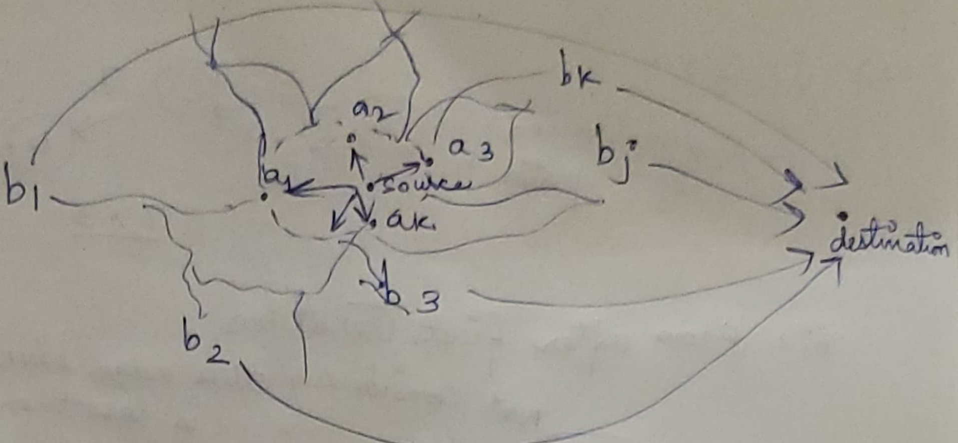
(i) Assuming solution exists need to prove max flow is total amount of dollars.

For solution to exist all the outgoing from 's' should have maxflow through them implying max flow is total amount of dollars.

(ii) Assume max flow is total amount of dollars then need to prove solution exists.

maxflow is only possible if all edges outgoing from source are flowing with max capacity ⇒ all the tourists were able to process their requests.

5.



Consider the following network.

Add two nodes "source" and "destination" such that all starting points are reached from "source" and all ending points are converging to "destination".

a) Compute ford-fulkerson from "source" to "destination" considering each edge (road) capacity as 1. Then in this way we will get all the paths from $a_i \to b_i$ without each edge not being shared.

This is because since each edge capacity is 1, it can only belong to one path.

b) In the above network, split each vertex into two nodes, let us say

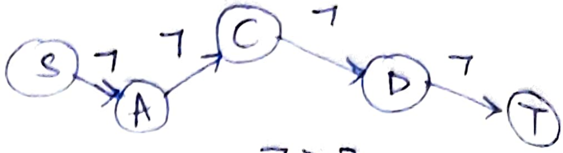
$(a_1) \to (a_1) \xrightarrow{1} (a_i')$

and add an edge with capacity 1. Now perform ford-fulkerson, this will give all paths from a_i to b_i and each path don't share an edge (or) vertex because if a vertex is considered implies $a_1 \to a_i'$ is fully saturated and vertex cannot be reused.

6.

a) (i) $\Delta = 4$

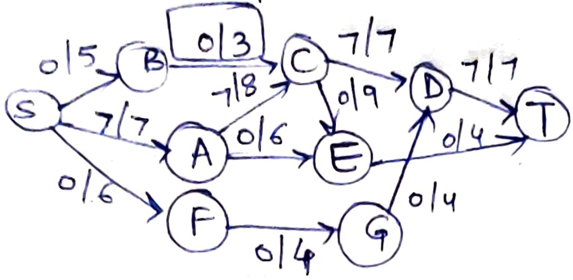
Augmentation path :



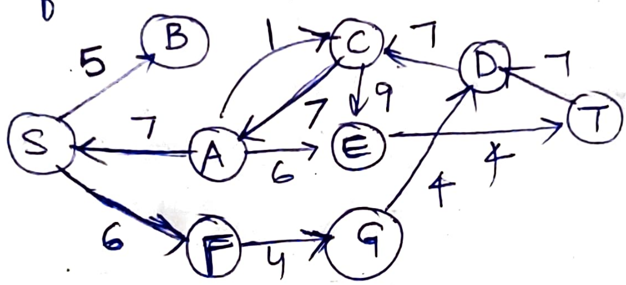
$7 > 3$

(ii) Flow after first iteration :

Not considering this edge since capacity is less than 4

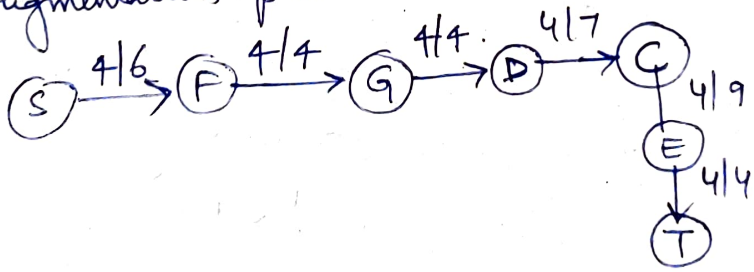


(iii) $G_f(\Delta)$

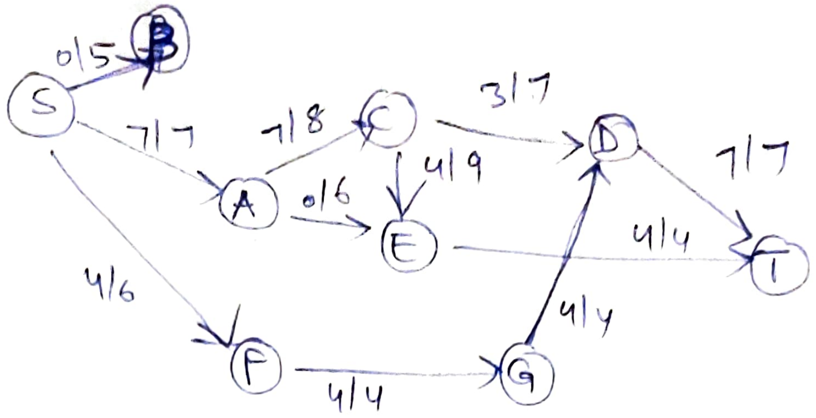


b) (i) $\Delta = 4$

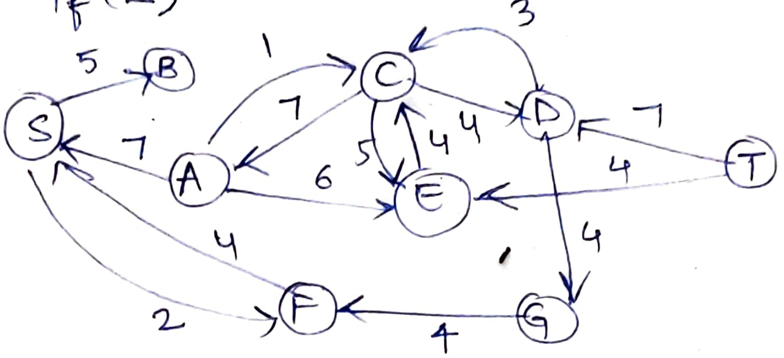
Augmentation path



(ii) Flow after second iteration.



(iii) $G_f(\Delta)$



c) YES.

Choice of paths affect the no. of iterations

In the above case there is no path from s–t after 2 iterations in $\Delta = 4$ scaling phase.

In $\Delta = 2$ & $\Delta = 1$ there will be no iterations because we have reached max-flow and there will be no path $s \to t$.

However if we consider paths $S \to A \to E \to E$ & $S \to F \to G \to D \to T$ in $\Delta = 4$ scaling phase then we need to consider path $S \to B \to C \to D \to T$ in $\Delta = 2$ scaling phase.

$\therefore$ Selection of augmentation paths change the no. of iterations in each Δ scaling phase.