HOMEWORK-11 USC id: 1231409457

(1) We will prove the given problem is NPC. Given a spanning tree of the graph, we can check the degree of each vertex is $\le k$ in polynomial time $\rightarrow$ efficient certification.

We will use Hamiltonian path to determine given problem is NP complete. Given an arbitrary graph, let H be the graph obtained by adding K-2 edges for each vertex as shown below

By the mentioned reduction the graph G will be having hamiltonian path if and only if there is spanning tree in H with degree $\le K$ at each vertex.

Suppose G has a Hamiltonian path, let T be spanning tree of H obtained by adding every fan edge in H to P. Every vertex V of H is either a leaf of T or a vertex of P. If $V \in P$, then $deg_P(V) \le 2$ and therefore $deg_H(V) = deg_P(V) + k - 2 \le k$

We can say that H is required spanning tree.

Suppose H has an required spanning tree. The leaves of T are precisely the vertices of H with degree 1, these are also precisely the vertices of H that are not vertices of G. Let P be the subtree of T obtained by deleting every leaf of T. Observe that P is a spanning tree of G, for every vertex $\in$ P, we have $deg_P(V) = deg_T(V) - (K-2) \le 2$. We can say that P is hamiltonian path in G.

(2) The given problem belongs to NP.

Efficient certification : Given a cycle, we can determine if the sum of weights of the edges = 0 in polynomial time.

We will use subset problem to determine given problem as NP- Complete.

Given an instance of subset sum with 'n' numbers construct an instance of subset of n numbers. Construct an instance of zero weight cycles as follows.

For each number a_i the graph contains two vertices u_i and v_i from each u_i, there is only one outgoing edge, which goes to v_i and has weight a_i. From each v_i, there are n outgoing edges which ~~go~~ go to each u_j and have weight 0. Any cycle in this graph have the form $u_1 - v_1 - u_2 - v_2 -- u_k - v_k$

The weight of a cycles is 0. iff the sum of all weights between each u_i and its corresponding v_i is 0. Iff the sum of all corresponding a_i is 0. Iff there is a subset with a sum of 0.

(3) Given problem belongs to NP.

Efficient certification: Given k set of clubs, we can determine by removing them and check if all people have atleast one club in which they belong. This can be done in polynomial time.

We will choose set cover ~~to~~ to determine it as NP complete. We translate inputs of set cover to inputs of redundant clubs, we need to specify how each redundant clubs input element is formed from the set cover instance. We use the set cover elements as our translated list of people, and make list of clubs, one for each member of the set cover family. The members of each club ~~are~~ just the elements of corresponding family. To ~~family~~ finish specifying the redundant clubs input, we need to say what k is, we let $k = f - k_{sc}$ where F is the number of families in set cover instance. This translation can clearly be done in polynomial time.

Finally we need to show that the translation preserves truth values. If we have a yes-instance of set cover consisting of k_{sc} subsets the other k subsets form a solution to the translated redundant clubs problem, because each person belongs to a club in the cover. Conversely if we have k redundant clubs, the remaining k_{sc} clubs form a cover. So the answer to the set cover instance is yes if and only if the answer to the translated redundant clubs instance is yes.

4) Given a graph $G(V,E)$ and certifier $S \subset V$, $|S| = |V|/2$, we can determine if no two nodes are adjacent in polynomial time $O(|S|^2) = O(|V|^2)$

Half is → NP.

We will use independent set to determine half is as NP-complete.

For a graph (G, V)

(i) if $K = |V|/2$, this reduces to half-is problem.

(ii) if $K < |V|/2$, then we need to add m new nodes such that $K + m = (|V| + m)/2$ is $m = |V| - 2K$.

modified set of V' has even number of nodes. Since the additional nodes are all disconnected from each other they form a subset of independent-set of size $\frac{|V'|}{2}$ if and if $G(V,E)$ has independent set of size K.

(ii) If $k > \frac{|V|}{2}$, then again $m = |V| - 2k$ new nodes to form the modified set of nodes V'. Connect these new nodes to all the other $|V| + m - 1$ nodes. Since these m new nodes are connected to every other none of them should belong to independent set.

Therefore the new graph $G'(V', E)$ has an independent set of size $|V'|/2$ if and only if $G(V, E)$ has independent set of size k

$\therefore$ Given problem is NP-complete.

(5) Given problem ~~is NP~~ belongs to NP.

Efficient certification: Given set of courses, we can determine if the course has any overlap in polynomial time, and also Check the count $\geq k$.

We will use Independent set to determine the problem as NP-Complete.

For a graph $G(V, E)$ and a number k, we construct the graph in such a way that for each vertex $v_i \in V$ construct a job b_i. For each edge $v_i v_j \in E$ we construct an interval, C_{ij} and let b_i, b_j require C_{ij}. Finally, the number k is copied from A to B. It is clear that this reduction can be performed in polynomial time.

Now, we can prove that G has independent set of size k iff there is a non conflicting schedule that complete k jobs.

If we assume that G contains an independent set $\{v_{i_1}, \ldots v_{i_k}\}$. Then we schedule corresponding k jobs. $b_{i_1}, b_{i_2}, \ldots b_{i_k}$. Since none of them overlap,

- On the other side, if there are not conflicting jobs of size k, then we can find k vertices which are independent.

$\therefore$ Given problem is NP-Complete.