HW-12

USC ID: 1231409457

Name: SRI MANVITH VADDEBOYINA

① We are using 2-approximation of vertex cover problem here for MIN-2SAT problem

Let the c be the set of clauses of original SAT problem. We construct a graph $G'(V', E')$ where V' are nodes & clause c. We connect two nodes v & u if that vertex's clause has a negation of variable of other vertex's clause variable. This makes one of clauses to be true when vertex cover is constructed.

Now, we find the minimum no. of true clauses using the vertex cover's 2-approximation & we get a set V' which are set to true & remaining $V-V'$ are set to false. Thus we have the 2-approximation to the given MIN-2SAT problem.

② We have to minimise the summation of flows on edges where min-cut is formed on the min s-t cut of network. Let the edge e is on min-cut & e_x denote the vertex x on side of s & e_y denote vertex y on side of t $\Rightarrow$ $e_{(x,y)}$ is the edge on min-cut & we set $e_x = 1$ as it is on s-side & $e_y = 0$ as it is on t-side $e_{(x,y)} \in \{0,1\} \forall (x,y) \in E$ & $e_x \in \{0,1\} \forall x \in V$ & $x \neq s$ & $x \neq t$

And $e_y - e_x + e_{(x,y)} \geq 0 \ \forall (x,y) \in E$. Objective function is to minimize $\sum_{(x,y) \in E} c(x,y) \cdot e_{(x,y)}$.

(3) The objective function here is to maximise total student happiness is:

maximise $\left(\sum_{i=1}^{16} \alpha_i (D_i - S_i) \right)$,

where

$S_i \rightarrow$ actual no. of students assigned to section i

$\alpha_i \rightarrow$ parameter reflecting how well the A.C system works for the room used for section i.

$D_i \rightarrow$ capacity of section i.

In total there are 16 sections & total students.

So, the constraints are : $\sum_{i=1}^{16} S_i = 720$

$0 < i \leq 16$

$0 \leq S_i \leq D_i$ (constraint)

$\forall i \; ; \; 0 < i \leq 16.$

(4) a) The variables includes, $r_i \rightarrow$ the radar power of i^{th} radar and $1 \leq i \leq n$ is the no. of available radars / space stations

b) We need to minimize $\left(\sum_{i=1}^{n} r_i \right)$ is the objective function

$r_i \rightarrow$ radar power of i^{th} station.

c) The constraints are $r_i + r_j \geq \text{distance}(i, j)$

i.e the distance between two space stations i & j. And the radius of space stations should be atleast distance (i, j) (a certain distance such that any spaceship travelling should be in radar of either i or j station) The no. of constraints are $\frac{n(n-1)}{2}$ as there are n stations & for every pair of stations the above constraint must be applied:

$1 \leq i \leq n \; , \; 1 \leq j \leq n \; \& \; i \neq j.$