Project 2

CEN 501-Systems

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Introduction

Blackman-Tukey PSD

The Blackman Tukey is one of the ways to construct an estimate of the power spectrum using a windowed fast Fourier transforms (FFT) of the autocorrelation function of the time series. It was developed by Blackman and Tukey and is based on the Wiener-Khinchin theorem, which states that if the Fourier transform of a series x(t) is X(f), and if the autocorrelation function of the series is R, then the Fourier transform of R yields PX(f)=|X(f)|2 or the power spectrum of x(t).

In the Blackman-Tukey approach $P_X(f)$ is estimated by

$$P_X(f) = |\sum_{k=0}^{M-1} w_k r_k e^{-2\pi i f k}|$$

Where, rk is the autocorrelation estimate at lag k, M is the maximum lag considered and window length, and wk is the windowing function.

For coding purpose the following steps have been used:

1. Creation of the test signals
2. Calculation of the autocorrelation function of the data
3. Applying a suitable window function, and finally
4. Computing a FFT of the data for obtaining the power density spectrum.

Test signals

Test signals (set1) were created by sampling 128 samples of a sinusoidal signal of different frequencies (10, 11, 25 Hz) at sampling frequency of 64 Hz.

Another set of test signals (set2) were created by adding a white Gaussian noise with SNR =0 dB.

Autocorrelation

An unbiased autocorrelation was carried out using:

$$Rk=\frac{1}{(N-l)}\sum_{n=0}^{N-1-|l|} x(n+|l|)x(n)$$

Where x is the data series, N is the length of x, and l is the lag.

The above equation has been implemented as the part of the blackman_tukey.m file.

Window Filter

For this Hamming window was used with zero-phase was used. The equation is as follows:

$$W(n)=0.54+0.46\cos\frac{2\pi n}{N-1}$$

The above equation has been implemented in both blackman_tukey.m as well as Welsh_period.m.

Fast Fourier Transform (FFT)

Wiener Khinchin theorem states that the autocorrelation function of a wide stationary random process has a spectral decomposition given by the power spectrum of that process. So, by taking FFT of the above autocorrelation we can obtain power spectral density of the function.

Welch's Periodogram PSD

Welch's periodogram is another method for estimation of power spectral density of the function. This method uses the concept of periodogram spectral estimates, which are result of converting a signal from the time domain to the frequency domain. It incorporates both approaches from Bartlett's method and periodogram and reduces noise in the estimated PSD in exchange for reduction in the frequency resolution.

The following algorithm was used for coding the estimation method.

1. Split the signal into L data segments of length M, overlapping by D points.
2. Then overlapped segments are then windowed. Each individual L overlapping segments are then multiplied with a window in the time domain. For this Hamming window was used.
3. Then the periodogram is calculated by computing the DFT and then its magnitude is squared.
4. Now the individual periodograms are then averaged, which reduces the variance of the individual power measurements.
5. Then FFT is computed on the resulted periodogram to obtain power spectral density.

   The algorithm is implemented in the welch_period.m.

Observations

By increasing lag we obtain more distinct peaks in the PSD in blackman-Tukey. Though we can say that by increasing the lag, the power in the PSD starts to converge on certain frequencies. Also the areas under the PSD starts to decrease means decrease in power with increase in the lag due to decrease in the autocorrelation.

Same is the case observed in Welch Periodogram PSD as we increase the shift S, we get better distinct peaks in PSD and it starts to converge onto certain frequencies.

Conclusion

Based on the above observation we can conclude that both blackman tukey and Welch's periodogram both uses different approaches to the same problem of estimation of power spectral density. In blackman tukey as the autocorrelation decreases or lag increases the area under the curve or PSD decreases. Though the peaks become distant but there is loss of avg. power.

Same can be concluded for Welch periodogram, as shift increases the PSD becomes more distinct but overall power decreases. This goes well with accordance to the theory too.

Result

Both blackman-tukey and Welch periodogram provide us with an estimate of PSD of a signal. They both reduce the variance compared to a pure periodogram but they take different approaches:

Blackman-tukey does it by smoothing the variations out of a single periodogram whereas Welch Periodogram does it by averaging away the variations over many computed periodograms.

Also Blackman tukey can only be used when all the data is obtained, therefore no computing can be done until all the data is obtained. Whereas in case of welch, DFTs can be started as soon as each block arrives. So welch could have real-time advantage.

But algorithmically, because of the no of computations needed, we can say that Welch is more computationally expensive than the blackman-tukey.

Bibliography

1. Wikipedia
2. Spectraworks

   http://www.spectraworks.com/Help/btffttheory.html