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Information and Communication Technologies Institute

# Wireless Sensor Networks
# Chapter 9: Localization & positioning

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Courtesy: Holger Karl, UPB

# Goals of this chapter

- Means for a node to determine its physical position (with respect to some coordinate system) or symbolic location
- Using the help of
  - Anchor nodes that know their position
  - Directly adjacent
  - Over multiple hops
- Using different means to determine distances/angles locally

# Overview

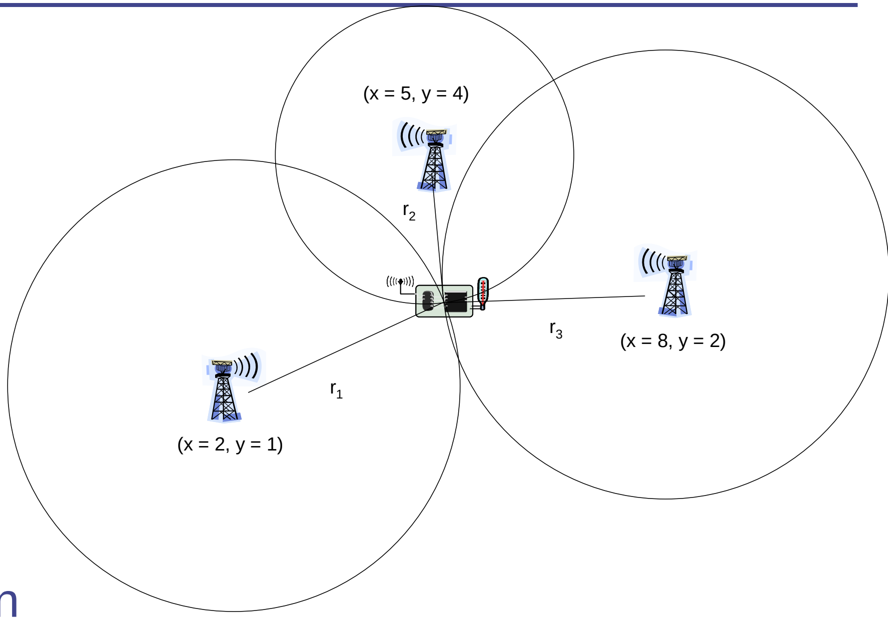
- ***Basic approaches***
- Trilateration
- Multihop schemes

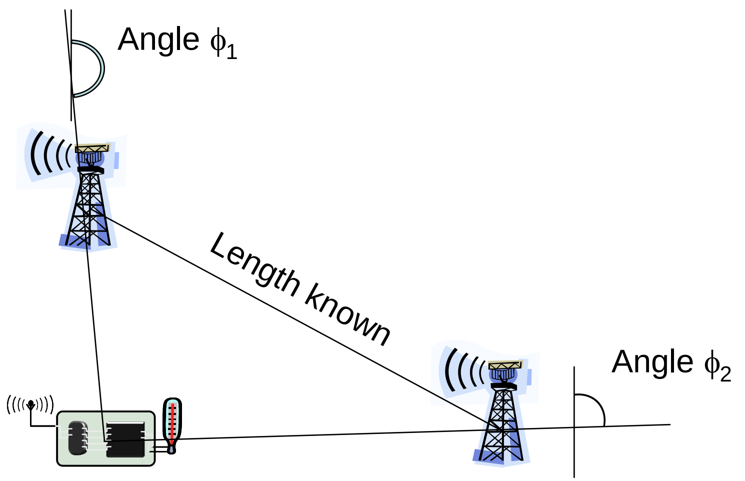
# Localization & positioning

- Determine ***physical position*** or ***logical location***
  - Coordinate system or symbolic reference
  - Absolute or relative coordinates
- Options
  - Centralized or distributed computation
  - Scale (indoors, outdoors, global, …)
  - Sources of information
- Metrics
  - Accuracy (how close is an estimated position to the real position?)
  - Precision (for repeated position determinations, how often is a given accuracy achieved?)
  - Costs, energy consumption, …

# Main approaches (information sources)

- Proximity
  - Exploit finite range of wireless communication
  - E.g.: easy to determine location in a room with infrared room number announcements
- (Tri-/Multi-)***lateration*** and ***angulation***
  - Use distance or angle estimates, simple geometry to compute position estimates
- Scene analysis
  - Radio environment has characteristic "signatures"
  - Can be measured beforehand, stored, compared with current situation



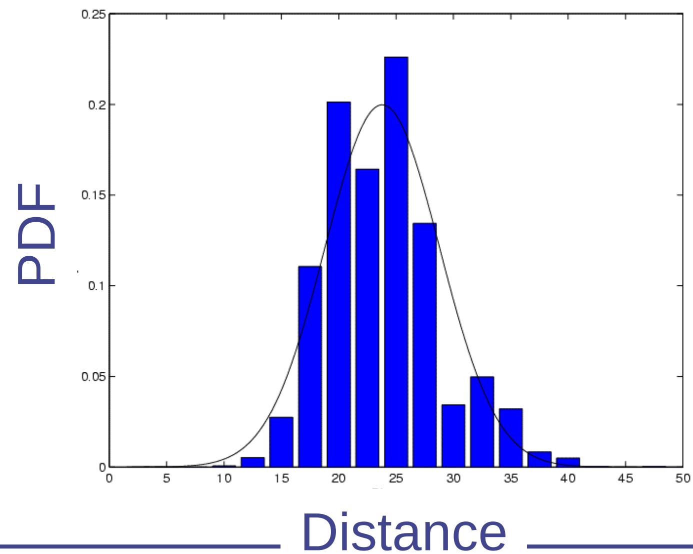



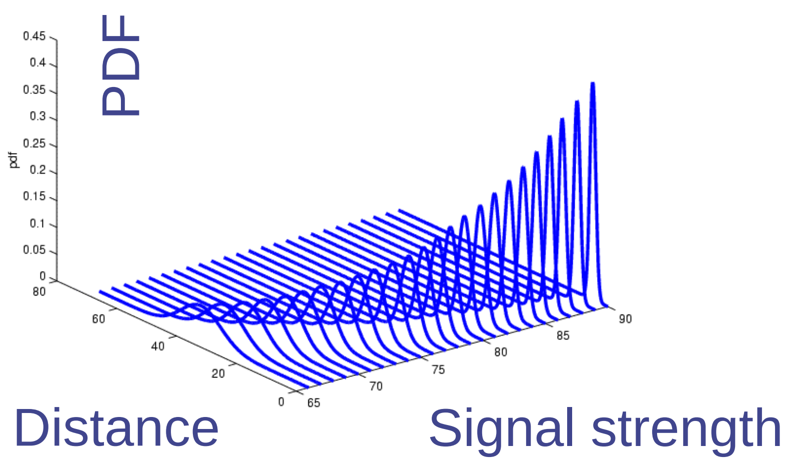
# Estimating distances – RSSI

- Received Signal Strength Indicator
  - Send out signal of known strength, use received signal strength and path loss coefficient to estimate distance

$$P_{\text{recv}} = c\frac{P_{\text{tx}}}{d^{\alpha}} \Leftrightarrow d = \sqrt[\alpha]{\frac{cP_{\text{tx}}}{P_{\text{recv}}}}$$

  - Problem: Highly error-prone process – Shown: PDF for a fixed RSSI



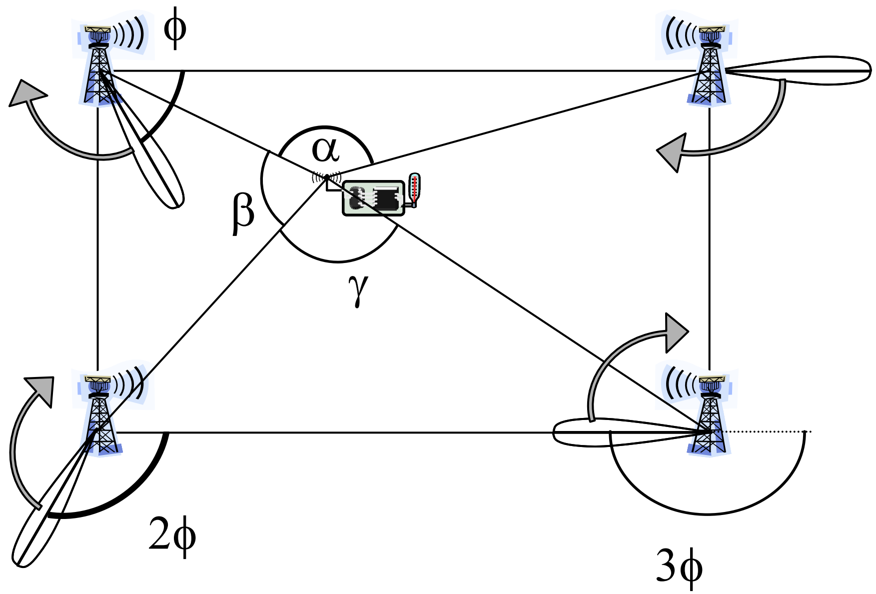
# Estimating distances – other means

- Time of arrival (ToA)
  - Use time of transmission, propagation speed, time of arrival to compute distance
  - Problem: Exact time synchronization

- Time Difference of Arrival (TDoA)
  - Use two different signals with different propagation speeds
  - Example: ultrasound and radio signal
    - Propagation time of radio negligible compared to ultrasound
  - Compute difference between arrival times to compute distance
  - Problem: Calibration, expensive/energy-intensive hardware

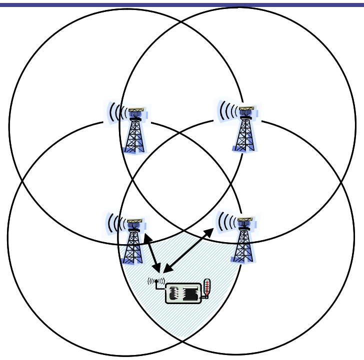
# Determining angles

- Directional antennas
  - On the node
  - Mechanically rotating or electrically “steerable”
  - On several access points
    - Rotating at different offsets
    - Time between beacons allows to compute angles

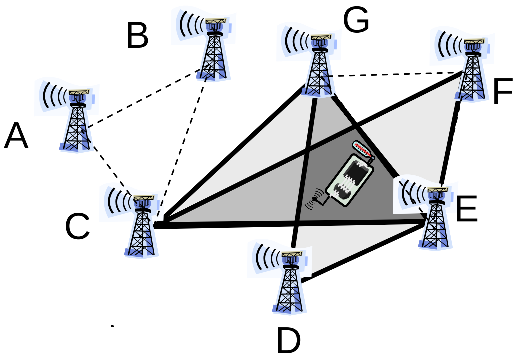


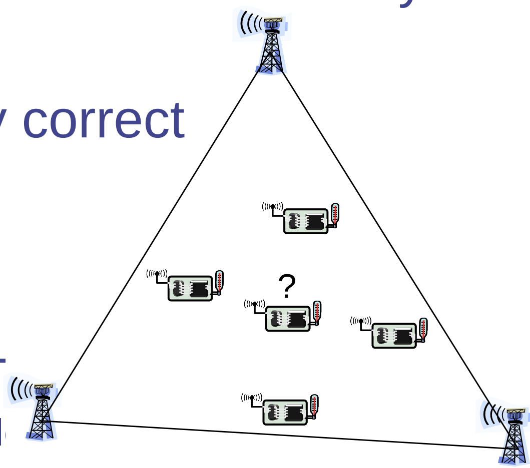
# Some range-free, single-hop localization techniques

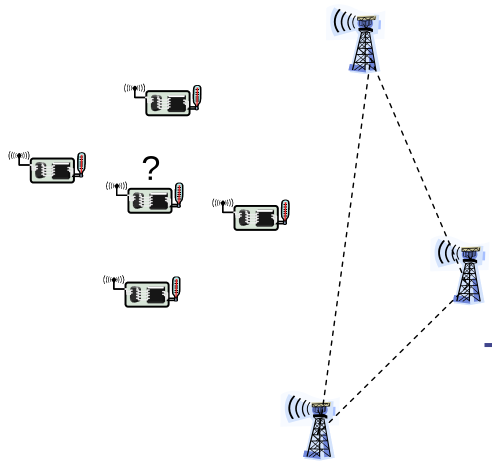
- ***Overlapping connectivity***: Position is estimated in the center of area where circles from which signal is heard/not heard overlap



- ***Approximate point in triangle***
  - Determine triangles of anchor nodes where node is inside, overlap them
  - Check whether inside a given triangle – move node or simulate movement by asking neighbors
  - Only approximately correct



# Overview

- Basic approaches
- ***Trilateration***
- Multihop schemes

# Trilateration

- Assuming distances to three points with known location are exactly given
- Solve system of equations (Pythagoras!)
  - $(x_i,y_i)$ : coordinates of ***anchor point*** i, $r_i$ distance to anchor i
  - $(x_u, y_u)$ : unknown coordinates of node

$$(x_i - x_u)^2 + (y_i - y_u)^2 = r_i^2 \text{ for } i = 1, \ldots, 3$$

  - Subtracting eq. 3 from 1 & 2:

$$(x_1 - x_u)^2 - (x_3 - x_u)^2 + (y_1 - y_u)^2 - (y_3 - y_u)^2 = r_1^2 - r_3^2$$
$$(x_2 - x_u)^2 - (x_2 - x_u)^2 + (y_2 - y_u)^2 - (y_2 - y_u)^2 = r_2^2 - r_3^2.$$

  - Rearranging terms gives a linear equation in $(x_u, y_u)$!

$$2(x_3 - x_1)x_u + 2(y_3 - y_1)y_u = (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2)$$
$$2(x_3 - x_2)x_u + 2(y_3 - y_2)y_u = (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)$$

# Trilateration as matrix equation

- Rewriting as a matrix equation:

$$2\begin{bmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\ (r_2^2 - r_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) \end{bmatrix}$$

- Example: $(x_1, y_1) = (2,1)$, $(x_2, y_2) = (5,4)$, $(x_3, y_3) = (8,2)$, $r_1 = 10^{0.5}$, $r_2 = 2$, $r_3 = 3$

$$2\begin{bmatrix} 6 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} 64 \\ 22 \end{bmatrix}$$

! $(x_u, y_u) = (5,2)$

# Trilateration with distance errors

- What if only distance estimation $r_i^0 = r_i + \varepsilon_i$ available?
- Use multiple anchors, overdetermined system of equations

$$2\begin{bmatrix} x_n - x_1 & y_n - y_1 \\ \vdots & \vdots \\ x_n - x_{n-1} & y_n - y_{n-1} \end{bmatrix} \begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} (r_1^2 - r_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) \\ \vdots \\ (r_{n-1}^2 - r_n^2) - (x_{n-1}^2 - x_n^2) - (y_{n-1}^2 - y_n^2) \end{bmatrix}$$

- Use $(x_u, y_u)$ that minimize mean square error, i.e, $||\mathbf{Ax} - \mathbf{b}||_2$

# Minimize mean square error

- Look at square of the of Euclidean norm expression (note that $\|\mathbf{v}\|_2^2 = \mathbf{v}^{\mathrm{T}}\mathbf{v}$ for all vectors v)

$$\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 = (\mathbf{A}\mathbf{x} - \mathbf{b})^{\mathrm{T}}(\mathbf{A}\mathbf{x} - \mathbf{b}) = \mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} - 2\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b} + \mathbf{b}^{\mathrm{T}}\mathbf{b}$$

- Look at derivative with respect to x, set it equal to 0:

$$2\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} - 2\mathbf{A}^{\mathrm{T}}\mathbf{b} = 0 \Leftrightarrow \mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

  - ***Normal equation***
  - Has unique solution (if A has full rank), which gives desired minimal mean square error

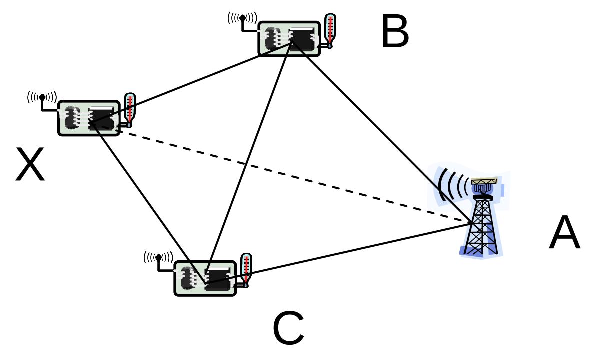
- Essentially similar for angulation as well

# Overview

- Basic approaches
- Trilateration
- ***Multihop schemes***

# Multihop range estimation

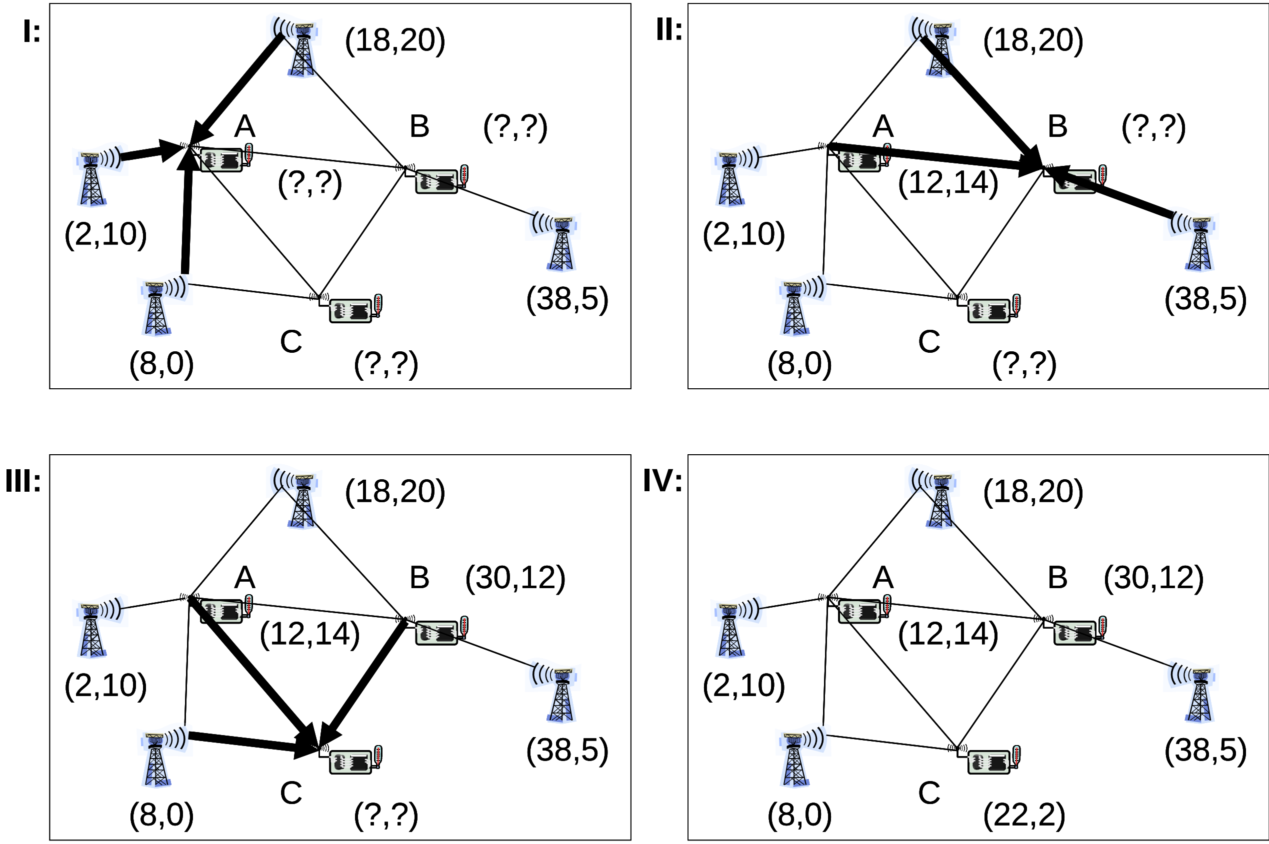
- How to estimate range to a node to which no direct radio communication exists?
  - No RSSI, TDoA, …
  - But: Multihop communication is possible




- Idea 1: Count number of hops, assume length of one hop is known (***DV-Hop***)
  - Start by counting hops between anchors, divide known distance
- Idea 2: If range estimates between neighbors exist, use them to improve total length of route estimation in previous method (***DV-Distance***)

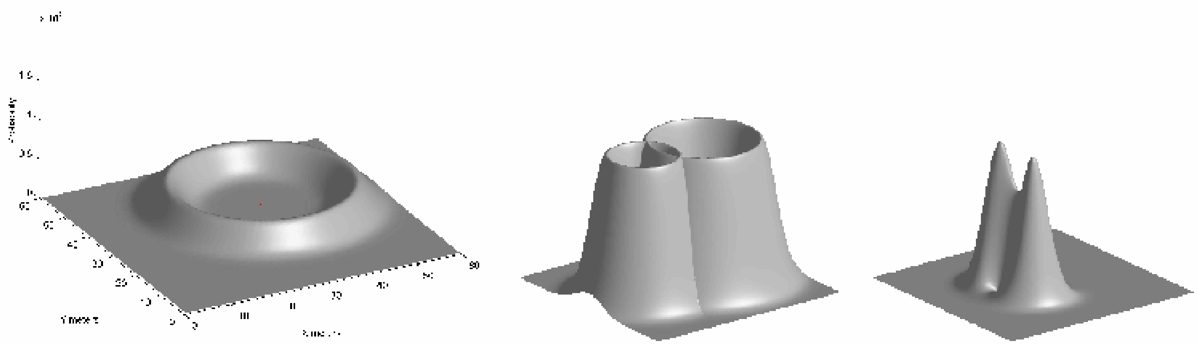
# Iterative multilateration

- Assume some nodes can hear at least three anchors (to perform triangulation), but not all
- Idea: let more and more nodes compute position estimates, spread position knowledge in the network
  - Problem: Errors accumulate



# Probabilistic position description

- Similar idea to previous one, but accept problem that position of nodes is only probabilistically known
  - Represent this probability explicitly, use it to compute probabilities for further nodes



(a) Probability density function of a node positions after receiving a distance estimate from one anchor

(b) Probability density functions of two distance measurements from two independent anchors

(c) Probability density function of a node after intersecting two anchor's distance measurements

# Conclusions

- Determining location or position is a vitally important function in WSN, but fraught with many errors and shortcomings
  - Range estimates often not sufficiently accurate
  - Many anchors are needed for acceptable results
  - Anchors might need external position sources (GPS)
  - Multilateration problematic (convergence, accuracy)