Budget Constraint

Venkata Khandrika

Outline

Topic

Subtopic

1. Budget Constraint
   - Assumptions
   - Budget Set
   - Example 1
   - Example 2

Assumptions

- Consider a simple 2-good economy model.
- Suppose that a consumer has income M to spend on goods 1 and 2.
- Let (x_1, x_2) be the amount that the consumer chooses of goods 1 and 2 respectively. This will be called the consumer's **consumption bundle**.
- Let p_1 and p_2 be the prices of goods 1 and 2 respectively.
- Then, our consumer can afford all bundles (x_1, x_2) such that:

$$p_1 x_1 + p_2 x_2 \leq M$$

- This is what we'll call the consumer's **budget constraint**.

Reality

One might believe that the 2-good economy model is unrealistic. In fact, it is.

However, suppose that we are interested in studying a consumer's demand for milk. We might let x_1 measure his or her consumption of milk in quarts per month.

We can then let x_2 stand for everything else the consumer might want to consume.

We say that good 2 represents a *composite good* that stands for everything else that the consumer might want to consume other than good 1.

When we adopt this interpretation, two goods are enough.

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Budget Set and Budget Line

Definition

The **budget set** is the set of consumption bundles which the consumer can afford:

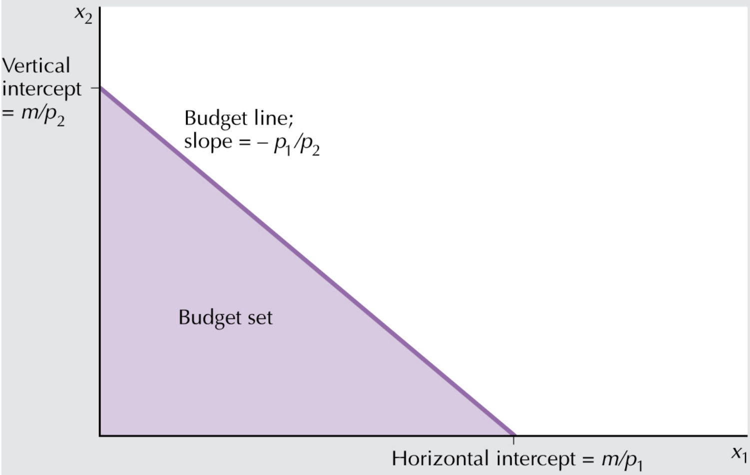
$$\{(x_1, x_2) : p_1 x_1 + p_2 x_2 \leq M, x_1 \geq 0, x_2 \geq 0\}$$

Definition

The set of consumption bundles that exhaust the income of the consumer form the **budget line**:

$$\{(x_1, x_2) : p_1 x_1 + p_2 x_2 = M, x_1 \geq 0, x_2 \geq 0\}$$

Budget Set Graphically



Slope and Intercepts of Budget Line

The budget line is:

$$p_1 x_1 + p_2 x_2 = M$$

In slope-intercept form with x_2 on the y-axis, the budget line is:

$$x_2 = \frac{M}{p_2} - \frac{p_1}{p_2} x_1 \quad (p_2 \neq 0)$$

Hence, the slope of the budget line is:

$$-\frac{p_1}{p_2}$$

Then, the intercepts are:

$$\text{y-intercept} = \frac{M}{p_2} \quad \text{and} \quad \text{x-intercept} = \frac{M}{p_1}$$

Budget Line and Opportunity Cost

Choosing between bundles on the budget line requires a tradeoff.

Economists sometimes say that the slope of the budget line measures the **opportunity cost** of consuming good 1. In order to consume more of good 1 you have to give up some consumption of good 2.

Giving up the opportunity to consume good 2 is the true economic cost of more good 1 consumption; and that cost is measured by the slope of the budget line.

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Example 1

- Suppose that I go to Starbucks with \$10 to spend on cookies and coffee.
- Assume that the price of cookies is \$1 and the price of coffee is \$2.
- Then, my budget constraint is:

$$x_1 + 2x_2 \leq 10$$

- I can, therefore, afford all consumption bundles that cost less than \$10. Put simply, I can buy any combination of cookies and coffee under \$10.
- I can afford all bundles that satisfy this inequality.

Example 1

- Suppose that I spend all of my money on cookies (x_1). I can then buy 10 cookies.
- Suppose that I spend all of my money on coffee (x_2). I can then buy 5 cups of coffee.
- These correspond to the intercept bundles $(0, 5)$ and $(10, 0)$.
- Other affordable bundles on my budget line are:

$$(2, 4), (4, 3), (6, 2), (8, 1)$$

- In this case, the goods are not divisible; they are discrete. However, we will usually assume that goods are infinitely divisible for convenience.

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Example 2

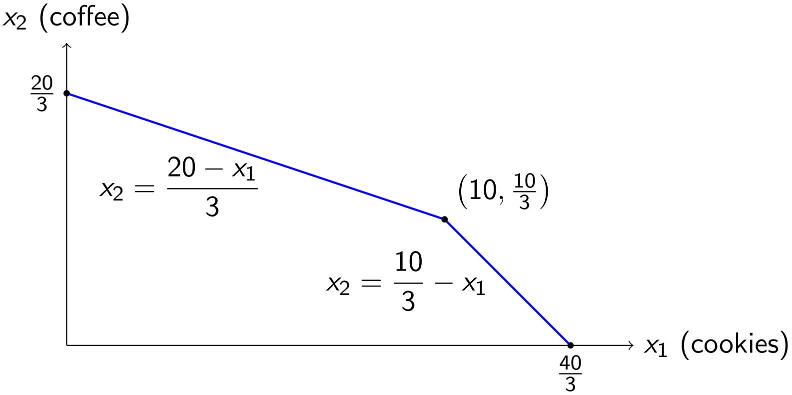
Suppose I have \$20. The price of coffee (p_2) at Starbucks is \$3, and the price of cookies (p_1) depends on whether I buy more or less than 10:

$$p_1 = \begin{cases} \$1 & \text{when} \quad x_1 \leq 10, \\ \$3 & \text{when} \quad x_1 > 10. \end{cases}$$

What will my budget constraint and budget line look like?

Example 2's Solution

$$\text{Budget Constraint} = \begin{cases} x_1 + 3x_2 \leq 20 \quad \text{when} \quad x_1 \leq 10, \\ 3x_1 + 3x_2 \leq 20 \quad \text{when} \quad x_1 > 10. \end{cases}$$



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2. Budget Changes
   - Change in Income
   - Change in Prices
   - Taxation
   - Subsidy

Increase in Income

Consider Example 1 again, wherein I go to Starbucks. Now, however, I go with \$20 instead of \$10. How does this impact the budget line?

The budget line in Example 1 is:

$$x_1 + 2x_2 = 10$$

In slope-intercept form, this looks like:

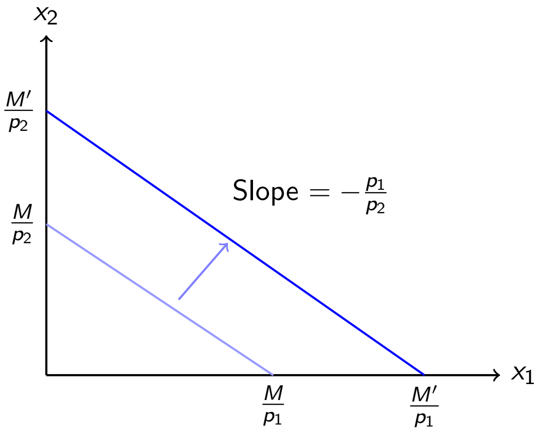
$$x_2 = 5 - \frac{1}{2}x_1$$

After the increase in income, the line becomes:

$$x_2 = 10 - \frac{1}{2}x_1$$

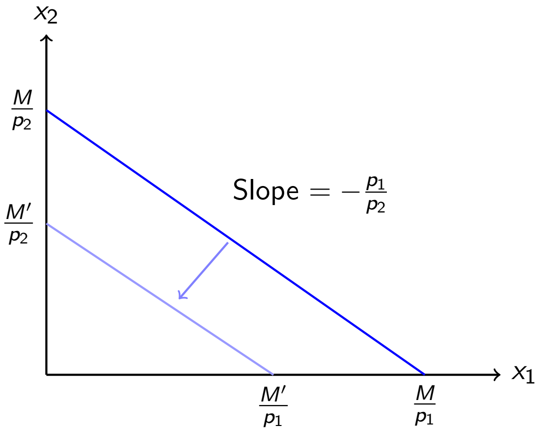
Increase in Income

The slope does not change, but the intercept changes.



Decrease in Income

Consider Example 1 again, wherein I go to Starbucks. Now, however, I go with \$5 instead of \$10. How does this impact the budget line?



Subtopic

2. Budget Changes
   - Change in Income
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Increase in Price of x_1

Consider Example 1 again, wherein I go to Starbucks. Now, however, the cookies cost \$2 instead of \$1. How does this impact the budget line?

Initially the budget line was:

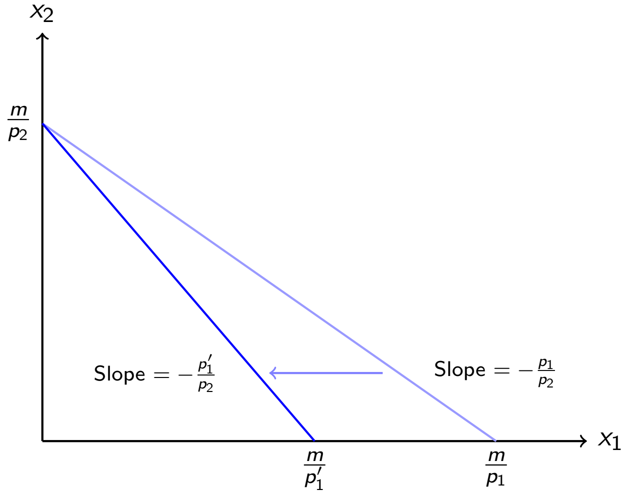
$$x_1 + 2x_2 = 10 \quad \Rightarrow \quad x_2 = 5 - \frac{1}{2}x_1$$

The budget line now becomes:

$$2x_1 + 2x_2 = 10 \quad \Rightarrow \quad x_2 = 5 - x_1$$

The absolute value of the slope has increased. This means that the line gets steeper.

Increase in Price of x_1



Increase in Price of x_2

Consider Example 1 again, wherein I go to Starbucks. Now, however, the coffee cost \$4 instead of \$2. How does this impact the budget line?

Initially the budget line was:

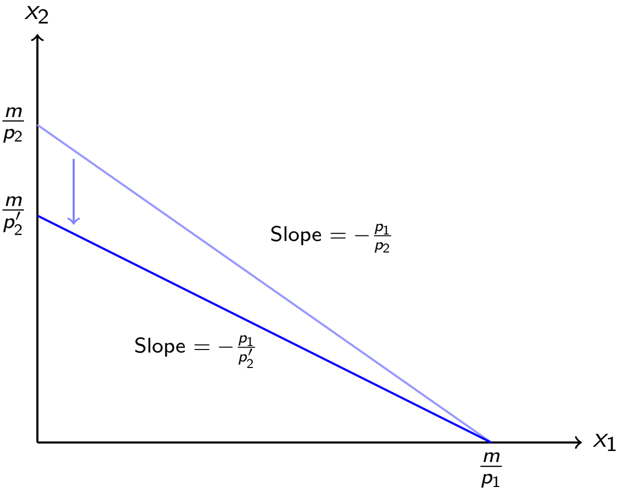
$$x_1 + 2x_2 = 10 \quad \Rightarrow \quad x_2 = 5 - \frac{1}{2}x_1$$

The budget line now becomes:

$$x_1 + 4x_2 = 10 \quad \Rightarrow \quad x_2 = \frac{10}{4} - \frac{1}{4}x_1$$

The absolute value of the slope has decreased. Therefore, the curve becomes flatter.

Increase in Price of x_2

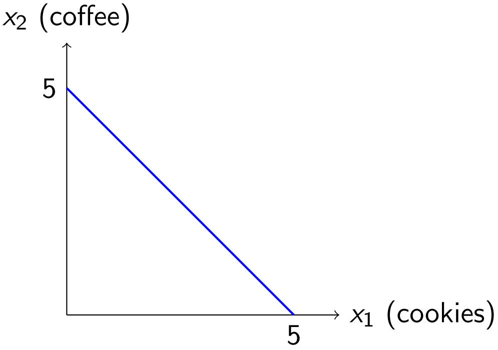


Decrease in Prices of x_1 and x_2

Consider Example 1 again, but now with initial prices $p_1 = \$2$ and $p_2 = \$2$.

The budget line is:

$$2x_1 + 2x_2 = 10$$

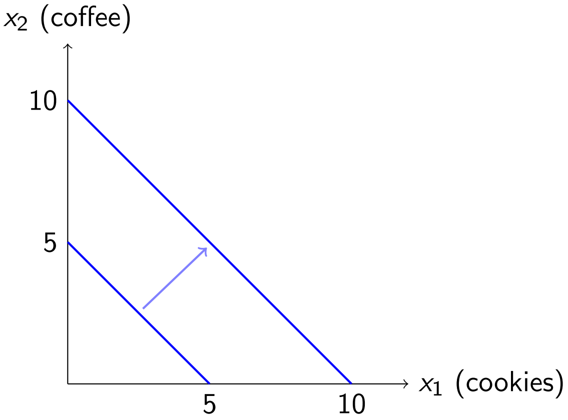


Decrease in Prices of x_1 and x_2

Now, consider a decrease in both prices from \$2 to \$1.

The budget line, now, is:

$$x_1 + x_2 = 10$$



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2 Budget Changes

- Change in Income
- Change in Prices
- Taxation
- Subsidy

Specific Tax

If the government imposes a *specific tax*, this means that the consumer has to pay a certain amount to the government for each unit of a good they purchase.

For example, in India, an excise duty of INR 19.90 (as of 10th April 2025) is paid per litre of petrol.

How does a specific tax affect the budget line of a consumer?

From a consumer's perspective, a tax is just like a higher price. Thus, a quantity tax of $\$t$ per unit of good 1 changes the price of good 1 from p_1 to $p_1 + t$. Therefore, the budget line gets steeper.

Ad-Valorem Tax

A government can also impose a *ad-valorem tax*, this means that the consumer has to pay a percentage amount of the retail value of a good as tax. Consequently, the amount of tax increases as the price of the good or service increases.

For example, most states in the U.S.A. have sales taxes. If the sales tax is 6 percent, then a good that is priced at \$1 will actually sell for \$1.06.

How does an ad-valorem tax affect the budget line of a consumer?

The consumer has to pay p_1 to the supplier and $\tau p1$ to the government for each unit of the good so the total cost of the good to the consumer is $(1 + \tau)p1$. Therefore, the budget line gets steeper.

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2 Budget Changes

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Specific Subsidy

If, for example, the consumption of milk were subsidized, the government would pay some amount of money to each consumer of milk depending on the amount that consumer purchased (in reality, the price just decreases).

If the subsidy is $\$s$ per unit of consumption of good 1, then from the viewpoint of the consumer, the price of good 1 would be $\$p_1 - \$s$.

This would, therefore, make the budget line flatter.

Ad-valorem Subsidy

If the government gives you back \$1 for every \$2 you donate to charity, then your donations to charity are being subsidized at a rate of 50 percent.

In general, if the price of good 1 is p_1 and good 1 is subject to an ad-valorem subsidy at rate σ, then the actual price of good 1 facing the consumer is $(1 - \sigma)p1$.

This would, therefore, make the budget line flatter.

Topic

Summary

- The budget set consists of all bundles of goods that the consumer can afford at given prices and income. We will typically assume that there are only two goods, but this assumption is more general than it seems.
- The budget line is written as $p_1x_1 + p_2x_2 = M$. It has a slope of $-\frac{p_1}{p_2}$, a vertical intercept of $\frac{m}{p_2}$, and a horizontal intercept of $\frac{m}{p_1}$.
- Increasing income shifts the budget line outward, and decreasing income shift it inward.
- Increasing the price of good 1 makes the budget line steeper, and decreasing the price of good 1 makes the budget line flatter.
- Increasing the price of good 2 makes the budget line flatter, and decreasing the price of good 2 makes the budget line steeper.
- Taxes and subsidies change the slope of the budget line by changing the prices paid by the consumer.