

# Slutsky Equation

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# What is the Slutsky Equation About?

We want to consider how a consumer's choice of a good responds to changes in its price.

It is natural to think that when the price of a good rises the demand for it will fall. However, Giffen goods are an exception to this rule. Giffen goods are pretty peculiar and are primarily a theoretical curiosity, but they do exist in the real world.

For example, in the 19th century, potatoes were considered a Giffen good in Ireland during the Great Famine. As the price of potatoes rose, people could not afford to buy more expensive foods and ended up consuming more potatoes, which were a staple food.

What is going on here? How is it that changes in price can have these ambiguous effects on demand? Here, we'll try to sort out these effects.

# What is the Slutsky Equation About?

The Slutsky equation is a mathematical decomposition of the total effect on demand of a price change into two components:

- ① Substitution Effect
- ② Income Effect

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## 2 Substitution Effect

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# Substitution Effect

When the price of a good changes, there are two sorts of effects: the rate at which you can exchange one good for another changes, and the total purchasing power of your income is altered.

The first part — the change in demand due to the change in the rate of exchange between the two goods — is called the substitution effect.

In order to give a more precise definition we have to consider the effect in greater detail.

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# Pivot and Shift

Consider the following diagram:

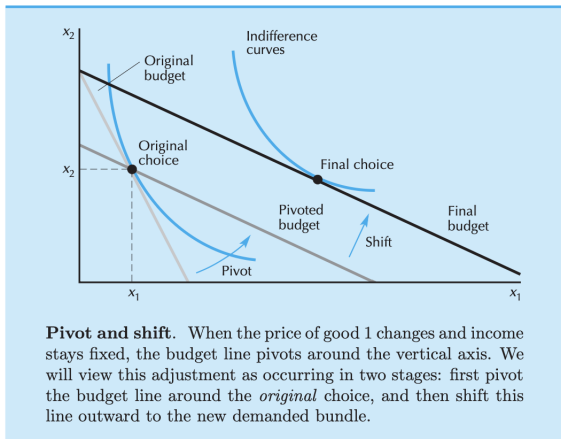


Figure 1: Pivot and Shift

# Pivot and Shift

Here we have a situation where the price of good 1 has declined. This means that the budget line rotates outwards about the vertical intercept  $\frac{m}{p_2}$  and becomes flatter.

We can break this movement of the budget line up into two steps: first *pivot* the budget line around the *original demanded bundle* and then *shift* the pivoted line out to the *new demanded bundle*.

This “pivot-shift” operation gives us a convenient way to decompose the change in demand into two pieces.

The first step — the pivot — is a movement where the slope of the budget line changes while its purchasing power stays constant, while the second step — the shift — is a movement where the slope stays constant and the purchasing power changes.

# Pivot and Shift

The purchasing power of the consumer has remained constant in the sense that the original bundle of goods is just affordable at the new pivoted line.

This necessitates the calculation of how much we have to change income by in order to keep the old bundle affordable at the new price.

Let  $m'$  be the new income associated with the pivoted budget line. Since  $(x_1, x_2)$  is affordable at both  $(p_1, p_2, m)$  and  $(p'_1, p_2, m')$ , we have:

$$m' = p'_1 x_1 + p_2 x_2$$

$$m = p_1 x_1 + p_2 x_2$$

Then,  $m' - m$  will be the change in income necessary to make the old bundle affordable at the new prices.

$$m' - m = x_1(p'_1 - p_1)$$

Letting  $\Delta p_1 = p'_1 - p_1$  represent the change in price of good 1, and  $\Delta m = m' - m$  represent the change in money income, we have:

$$\Delta m = x_1 \Delta p_1 \quad (1)$$

Note that the change in income and the change in price will always move in the same direction: if the price goes up, then we have to raise income to keep the same bundle affordable. This needs more elaboration.

# Pivot and Shift

If the price of good 1 decreases, as shown in Figure 1, then the consumer can afford to buy more of good 1 with the same amount of money.

However, this new budget line will not pass through the original bundle.

To make the new budget line pass through the original bundle, we need to reduce the consumer's income by an amount equal to  $x_1 \Delta p_1$ .

This is what we mean when we say that the change in income and the change in price will always move in the same direction. If the price goes up, then we have to raise income to keep the original bundle affordable. If the price goes down, then we have to lower income to keep the original bundle affordable.

The new income to keep the same purchasing power (original bundle) is given by:  $m' = m + \Delta m$  ( $\Delta m$  maybe positive or negative).

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# Substitution Effect

Although  $(x_1, x_2)$  is still affordable, it is generally not the optimal purchase at the pivoted budget line. In Figure 2 below, the optimal purchase is denoted by  $Y$  on the pivoted budget line.

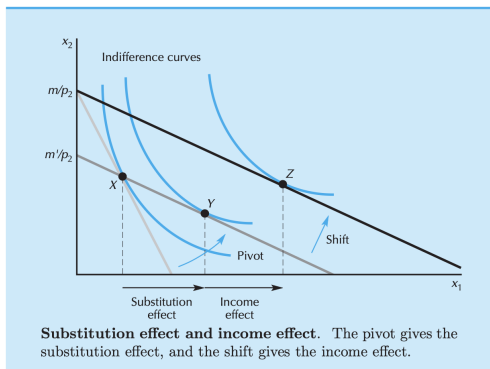


Figure 2

# Substitution Effect

This bundle of goods is the optimal bundle of goods when we change the price and then adjust dollar income so as to keep the old bundle of goods just affordable.

The movement from X to Y is known as the **substitution effect**.

More precisely, the substitution effect,  $\Delta x_1^s$ , is the change in the demand for good 1 when the price of good 1 changes to  $p'_1$  and money income changes to  $m'$ :

$$\Delta x_1^s = x_1(p'_1, m') - x_1(p_1, m)$$

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# Substitution Effect

The precise definition of the substitution effect is “the change in the demand for good 1 when the price of good 1 changes to  $p'_1$  and income changes to  $m'$ .”

In essence, the substitution effect is the change in demand of good 1 when its price changes from  $p_1$  to  $p'_1$ , keeping purchasing power (real income) constant. Then, there are two possible scenarios:

- 1 **Price of good 1 increases** ( $p'_1 > p_1$ ): If a good's price increases, then consumers substitute away from the good because it is relatively more expensive compared to other goods. This means that the demand falls  $\Rightarrow$  SE is  $-ve$ .
- 2 **Price of good 1 decreases** ( $p'_1 < p_1$ ): If a good's price decreases, then consumers substitute towards the good because it is relatively cheaper compared to other goods. This means that the demand rises  $\Rightarrow$  SE is  $+ve$ .

# Substitution Effect Simplified

Substitution effect always moves opposite the price change.

- 1 If price  $\uparrow$ , then SE is  $-ve$ .
- 2 If price  $\downarrow$ , then SE is  $+ve$ .

This is ALWAYS true.

## 2 Substitution Effect

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# Question

Suppose that John has the following demand function for milk:

$$x_1 = 10 + \frac{M}{10p_1}.$$

His income is \$120 and the initial price of milk is \$3. Find the substitution effect when the price falls to \$2.

# Solution

Originally his income is \$120 and the price of milk is \$3. Thus his demand for milk will be:  $x_1(p_1, m) = 10 + [120/(10 \times 3)] = 14$  units of milk.

In order to calculate the substitution effect, we must first calculate how much income would have to change in order to make the original bundle just affordable when the price of milk is \$2. We know that:

$$\Delta m = x_1 \Delta p_1 = 14 \times (2 - 3) = -14.$$

Therefore, the new income to keep the same purchasing power is given by:  $m' = m + \Delta m = 120 - 14 = 106$ .

At this level of income and the new price of milk, John's demand for milk will be:  $x_1(p'_1, m') = 10 + [106/(10 \times 2)] = 10 + 5.3 = 15.3$  units of milk.

Therefore, the substitution effect is:

$$\Delta x_1^s = x_1(p'_1, m') - x_1(p_1, m) = 15.3 - 14 = 1.3$$

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## 3 Income Effect

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When the price of a good changes, there are two sorts of effects: the rate at which you can exchange one good for another changes, and the total purchasing power of your income is altered.

The second part — the change in demand due to the change in purchasing power — is called the income effect.

The second step — the shift — is a movement where the slope stays constant and the purchasing power changes.

Thus the second stage of the price adjustment is called the **income effect**.

We simply change the consumer's income from  $m'$  to  $m$ , keeping the prices constant at  $(p'_1, p_2)$ , and undoing the step in the substitution effect.

More precisely, the income effect,  $\Delta x_1^n$ , is the change in the demand for good 1 when we change income from  $m'$  to  $m$ , holding the price of good 1 fixed at  $p'_1$ :

$$\Delta x_1^n = x_1(p'_1, m) - x_1(p'_1, m')$$

# Income Effect

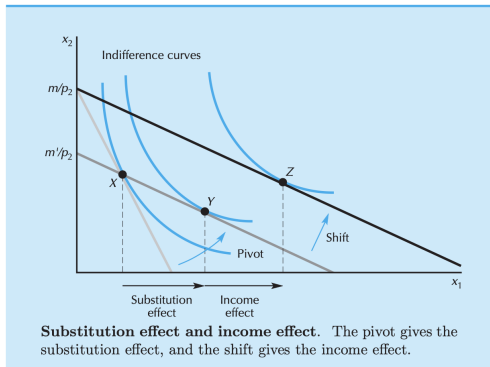


Figure 2

The movement from  $Y$  to  $Z$  is the **income effect**.

## 3 Income Effect

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This part of the change in demand happens because a price change affects your purchasing power (real income).

- 1 If price of good 1 increases  $\rightarrow$  your purchasing power falls.
- 2 If price of good 1 decreases  $\rightarrow$  your purchasing power rises.

Real income (purchasing power) changes when price changes. How this change in real income affects demand depends on whether the good is normal or inferior.

## 3 Income Effect

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# Normal Good – Price Increase

$$\text{Price} \uparrow \Rightarrow \text{Real Income} \downarrow$$

For a normal good, when real income falls, its demand falls.

$$\therefore \text{Demand} \downarrow$$

In the case of a normal good, an increase in price causes a fall in demand.  
This means that IE is –ve.

# Normal Good – Price Decrease

$$\text{Price} \downarrow \Rightarrow \text{Real Income} \uparrow$$

For a normal good, when real income increases, its demand increases.

$$\therefore \text{Demand} \uparrow$$

In the case of a normal good, a decrease in price causes a rise in demand.  
This means that IE is +ve.

# Inferior Good – Price Increase

$$\text{Price} \uparrow \Rightarrow \text{Real Income} \downarrow$$

For an inferior good, when real income falls, its demand increases.

$$\therefore \text{Demand} \uparrow$$

In the case of an inferior good, an increase in price causes an increase in demand.

This means that IE is +ve.

# Inferior Good – Price Decrease

$$\text{Price} \downarrow \Rightarrow \text{Real Income} \uparrow$$

For an inferior good, when real income increases, its demand decreases.

$$\therefore \text{Demand} \downarrow$$

In the case of an inferior good, a decrease in price causes a decrease in demand.

This means that IE is –ve.

## 3 Income Effect

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# Question

Suppose that John has the following demand function for milk:

$$x_1 = 10 + \frac{M}{10p_1}.$$

His income is \$120 and the initial price of milk is \$3. Find the income effect when the price falls to \$2.

# Solution

Firstly, we know that:

$$\begin{aligned}\text{Total Change in Demand} &= \text{Substitution Effect} + \text{Income Effect} \\ &= \Delta x_1^s + \Delta x_1^n\end{aligned}$$

We also know that  $\Delta x_1^s = 1.3$  units of milk.

The total change in demand when the price falls to \$2 is:

$$x_1(2, 120) - x_1(3, 120) = \left(10 + \frac{120}{10 \cdot 2}\right) - \left(10 + \frac{120}{10 \cdot 3}\right) = 2$$

Therefore, the income effect is:

$$\Delta x_1^n = 2 - 1.3 = 0.7$$

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## 4 Total Change in Demand

- Slutsky Identity
- Total Change in Demand

# Slutsky Identity

The total change in demand of good 1,  $\Delta x_1$ , is the change in demand due to a change in its price, holding income constant:

$$\Delta x_1 = x_1(p', m) - x_1(p, m)$$

We have seen above how this change can be broken up into two changes: the substitution effect and the income effect.

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^i$$

$$x_1(p', m) - x_1(p, m) = [x_1(p', m') - x_1(p, m)] + [x_1(p', m) - x_1(p', m')]$$

This equation is called the **Slutsky identity**.

## 4 Total Change in Demand

- Slutsky Identity
- Total Change in Demand

# Total Change in Demand

Here, there are four cases:

- (1) Normal Good – Price Increases
- (2) Normal Good – Price Decreases
- (3) Inferior Good – Price Increases
- (4) Inferior Good – Price Decreases

For these, we must analyse the impact of the substitution effect and income effect.

# Total Change in Demand – Normal Good

## (1) Normal Good – Price Increases

If price  $\uparrow$ , then SE is  $-ve$ .

Price  $\uparrow \Rightarrow$  Real income  $\downarrow$ . For a normal good, IE is  $-ve$ .

Total Change in Demand is  $-ve$ .

## (2) Normal Good – Price Decreases

If price  $\downarrow$ , then SE is  $+ve$ .

Price  $\downarrow \Rightarrow$  Real income  $\uparrow$ . For a normal good, IE is  $+ve$ .

Total Change in Demand is  $+ve$ .

# Total Change in Demand – Inferior Good

If the IE is stronger than the SE for an inferior good, we get a Giffen good.

## (1) Inferior Good – Price Increases

If price  $\uparrow$ , then SE is  $-ve$ .

Price  $\uparrow \Rightarrow$  Real income  $\downarrow$ . For an inferior good, IE is  $+ve$ .

Total Change in Demand is ambiguous. ( $\downarrow$  usually)

## (2) Inferior Good – Price Decreases

If price  $\downarrow$ , then SE is  $+ve$ .

Price  $\downarrow \Rightarrow$  Real income  $\uparrow$ . For an inferior good, IE is  $-ve$ .

Total Change in Demand is ambiguous. ( $\uparrow$  usually)

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Varian, H. R. (2014). Intermediate Microeconomics: A Modern Approach (9th ed.). W. W. Norton & Company.