Choice

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Outline

1 Optimal Choice

2 Consumer Demand

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Topic

1. Optimal Choice
2. Consumer Demand
3. References

Subtopic

1. Optimal Choice
   - Optimal Bundle
   - Exceptions
   - Condition for Optimal Bundle

Optimal Bundle

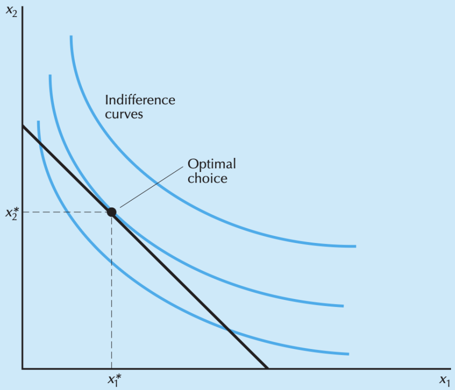
We will put together the budget set and the theory of preferences in order to examine the optimal choice of consumers.

The optimal bundle will be the one that lies on the budget line and is on the highest indifference curve.

Well-behaved preferences mean that we can restrict our attention to bundles of goods that lie on the budget line and not worry about those beneath the budget line.

Start at the right-hand corner of the budget line and move to the left. As we move along the budget line we note that we are moving to higher and higher indifference curves. We stop when we get to the highest indifference curve that just touches the budget line.

Optimal Bundle



Optimal choice. The optimal consumption position is where the indifference curve is tangent to the budget line.

Optimal Bundle

Note an important feature of this optimal bundle: at this choice, the indifference curve is tangent to the budget line.

If the indifference curve weren't tangent, it would cross the budget line, and if it crossed the budget line, there would be some nearby point on the budget line that lies above the indifference curve.

Does this tangency condition really have to hold at an optimal choice? Well, it doesn't hold in *all* cases. However, what is always true is that at the optimal point, the indifference curve can't cross the budget line.

When does "not crossing" imply tangent? Let's look at the exceptions.

Subtopic

1. Optimal Choice
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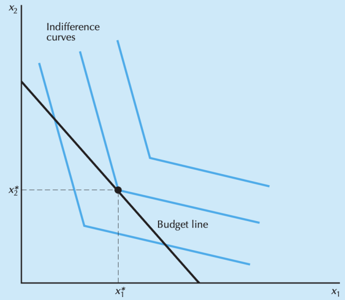
Exception 1: Kinky Tastes

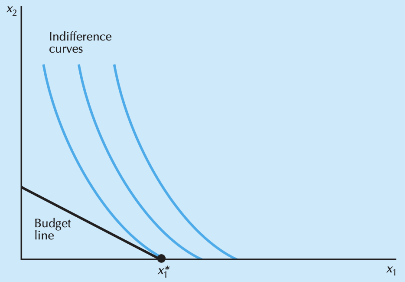
The indifference curve might have a kink at the optimal choice, and a tangent just isn't defined.



Kinky tastes. Here is an optimal consumption bundle where the indifference curve doesn't have a tangent.

Exception 2: Boundary Optimum

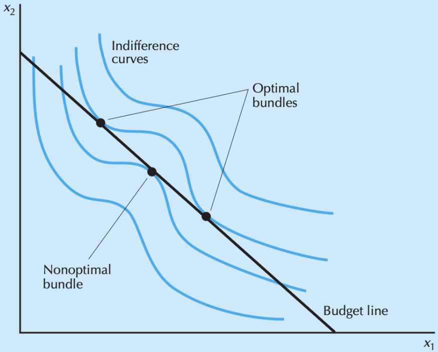
Suppose that the optimal point occurs where the consumption of some good is zero. This represents a boundary optimum.



Boundary optimum. The optimal consumption involves consuming zero units of good 2. The indifference curve is not tangent to the budget line.

Exception 3: More than One Tangency

Finally, it is possible that there are multiple tangencies.



More than one tangency. Here there are three tangencies, but only two optimal points, so the tangency condition is necessary but not sufficient.

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1. Optimal Choice
   - Optimal Bundle
   - Exceptions
   - Condition for Optimal Bundle

Condition for Optimal Bundle

At the optimal bundle, the indifference curve is tangent to the budget line. This means that the slope of the indifference curve must be equal to the slope of the budget line.

$$MRS = \frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

Suppose, for example, that the $MRS = -\frac{1}{2}$ and the price ratio is $\frac{1}{1}$. Economically speaking, this means that the consumer is willing to give up half a unit of good y to get one more unit of good x. Alternatively, the consumer is willing to give up one unit of good y to get two more units of good x. Meanwhile, the market is willing to exchange them on a one-to-one basis.

Thus, the consumer would certainly be willing to give up some of good x in order to purchase a little more of good y.

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Subtopic

2. Consumer Demand
    - Consumer Demand
    - Perfect Substitutes
    - Perfect Complements
    - General Strategy
    - Cobb-Douglas

Consumer Demand

The optimal choice of goods 1 and 2 at some set of prices and income is called the consumer's demanded bundle.

A demand function in terms of prices and income is called a **Marshallian Demand Function**. It is denoted as $x(p_x, p_y, m)$ and $y(p_x, p_y, m)$.

A demand function in terms of prices and utility is called a **Hicksian Demand Function**. It is denoted as $x(p_x, p_y, u)$ and $y(p_x, p_y, u)$.

Subtopic

2 Consumer Demand

- Consumer Demand
- Perfect Substitutes
- Perfect Complements
- General Strategy
- Cobb-Douglas

Perfect Substitutes

Suppose we have the general utility function $U(x, y) = ax + by$ for goods that are perfect substitutes. Furthermore, suppose that we have the general budget constraint $p_x x + p_y y = m$.

To determine the consumer's demand for goods x and y, we compare the MRS and price ratio.

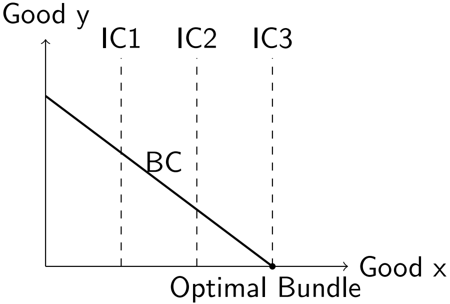
$$MRS = \frac{MU_x}{MU_y} = \frac{a}{b}$$

$$\text{Price ratio} = \frac{p_x}{p_y}$$

Perfect Substitutes

If $\frac{a}{b} > \frac{p_x}{p_y}$, then the consumer is unwilling to give up x for y. Why?

The slope of the budget line is flatter than the slope of the indifference curves, hence the higher indifference curves are to the right.

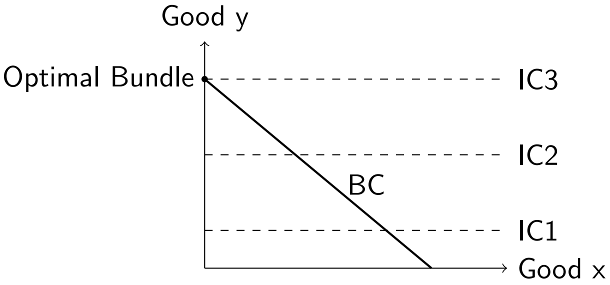


The demand for good x is $x = \frac{m}{p_x}$ and the demand for good y is $y = 0$.

Perfect Substitutes

If $\frac{a}{b} < \frac{p_x}{p_y}$, then the consumer is unwilling to give up y for x. Why?

The slope of the budget line is steeper than the slope of the indifference curves, hence the higher indifference curves are to the left.

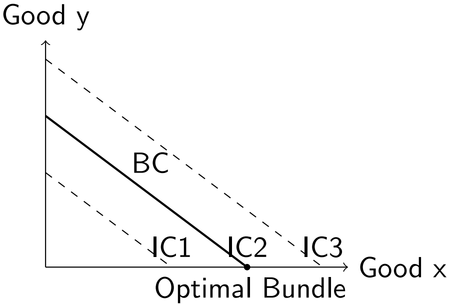


The demand for good x is $x = 0$ and the demand for good y is $y = \frac{m}{p_y}$.

Perfect Substitutes

If $\frac{a}{b} = \frac{p_x}{p_y}$, then the consumer is indifferent between x and y. Why?

The slope of the budget line is equal to the slope of the indifference curves, hence there is only one indifference curve and the budget constraint is coincident with it.



Perfect Substitutes

Thus, the Marshallian demand function for perfect substitutes is:

$$(x, y) = \begin{cases} \left(\frac{m}{p_x}, 0\right) & \text{if } \frac{a}{b} > \frac{p_x}{p_y} \\ \left(0, \frac{m}{p_y}\right) & \text{if } \frac{a}{b} < \frac{p_x}{p_y} \\ \text{Any point on the budget line} & \text{if } \frac{a}{b} = \frac{p_x}{p_y}. \end{cases}$$

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2. Consumer Demand
   - Consumer Demand
   - Perfect Substitutes
   - **Perfect Complements**
   - General Strategy
   - Cobb-Douglas

Perfect Complements

Suppose we have the general utility function $U(x, y) = \min\{ax, by\}$ for goods that are perfect complements. Furthermore, suppose that we have the general budget constraint $p_x x + p_y y = m$.

The optimal bundle will be at the corner of the L-shaped indifference curve, where $ax = by$. This implies that $y = \frac{a}{b}x$.

Substituting this into the budget constraint gives us:

$$p_x x + p_y \left(\frac{a}{b}x\right) = m$$

$$x = \frac{m}{p_x + \frac{a}{b}p_y}$$

$$y = \frac{a}{b}x = \frac{a}{b} \cdot \frac{m}{p_x + \frac{a}{b}p_y}$$

Perfect Complements

Thus, the Marshallian demand function for perfect complements is:

$$(x, y) = \left(\frac{bm}{bp_x + ap_y}, \frac{am}{bp_x + ap_y} \right)$$

Subtopic

2. Consumer Demand
   - Consumer Demand
   - Perfect Substitutes
   - Perfect Complements
   - General Strategy
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General Strategy for Optimal Bundle

(1) Find the MRS by taking the ratio of the marginal utilities $\left(\frac{MU_x}{MU_y}\right)$

(2) Find the price ratio by taking the ratio of the prices $\left(\frac{p_x}{p_y}\right)$

(3) Obtain the tangency condition by setting MRS equal to price ratio. This will provide a relationship between x and y.

(4) Plug it into the budget constraint to get the demand functions

Subtopic

2. Consumer Demand
    - Consumer Demand
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Consumer Demand for Cobb-Douglas Utility

Given the Cobb-Douglas utility function $U(x, y) = x^\alpha y^\beta$ and prices P_x and P_y, we can find the consumer's demand for goods x and y.

$$\frac{\partial U}{\partial x} = \alpha x^{\alpha-1} y^\beta \quad \text{and} \quad \frac{\partial U}{\partial y} = \beta x^\alpha y^{\beta-1}.$$

$$\text{MRS} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{\alpha x^{\alpha-1} y^\beta}{\beta x^\alpha y^{\beta-1}} = \frac{\alpha y}{\beta x}.$$

$$\text{Price Ratio} = \frac{P_x}{P_y}.$$

Consumer Demand for Cobb-Douglas Utility

$$\text{Tangency condition} \;\Rightarrow\; \text{MRS} = \text{price ratio}$$

$$\frac{\alpha y}{\beta x} = \frac{P_x}{P_y} \;\Rightarrow\; P_x = \frac{\alpha y}{\beta x} P_y.$$

$$\text{Budget constraint:} \quad P_x x + P_y y = M.$$

$$x\left(\frac{\alpha y}{\beta x} P_y\right) + yP_y = M \;\Rightarrow\; \frac{\alpha}{\beta} yP_y + yP_y = M.$$

$$\Rightarrow\; yP_y\left(\frac{\alpha}{\beta} + 1\right) = M \;\Rightarrow\; yP_y\left(\frac{\alpha + \beta}{\beta}\right) = M.$$

Consumer Demand for Cobb-Douglas Utility

$$\Rightarrow \; yP_y = \left(\frac{\beta}{\alpha+\beta}\right) M \quad \text{(share of income spent on } y\text{)}$$

$$\Rightarrow \; y^* = \left(\frac{\beta}{\alpha+\beta}\right) \frac{M}{P_y} \quad \text{(Marshallian demand).}$$

Consumer Demand for Cobb-Douglas Utility

Similarly, $P_y = \frac{\beta x}{\alpha y} P_x$.

$$xP_x + y\left(\frac{\beta x}{\alpha y} P_x\right) = M \;\Rightarrow\; xP_x + \frac{\beta}{\alpha} xP_x = M.$$

$$\Rightarrow\; xP_x\left(\frac{\beta}{\alpha} + 1\right) = M \;\Rightarrow\; xP_x\left(\frac{\alpha+\beta}{\alpha}\right) = M.$$

$$xP_x = \left(\frac{\alpha}{\alpha+\beta}\right) M \quad \text{(share of income spent on } x\text{)}$$

$$x^* = \left(\frac{\alpha}{\alpha+\beta}\right) \frac{M}{P_x} \quad \text{(Marshallian demand).}$$

Consumer Demand for Cobb-Douglas Utility

Marshallian Demand Functions for Cobb-Douglas Utility:

$$(x^*, y^*) = \left(\frac{\alpha}{\alpha + \beta} \cdot \frac{M}{P_x}, \frac{\beta}{\alpha + \beta} \cdot \frac{M}{P_y} \right).$$

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References

Varian, H. R. (2014). Intermediate Microeconomics: A Modern Approach (9th ed.). W. W. Norton & Company.