VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY (VSSUT), ODISHA

Mid Semester Examination November - 2019

COURSE NAME: B.Tech SEMESTER: 1^{st}

BRANCH NAME: All Branches

SUBJECT NAME: Mathematics-I

FULL MARKS: **20** TIME: **2 Hours**

Answer **All** Questions.

The figures in the right hand margin indicate Marks. *Symbols carry usual meaning.*

Q1. Answer all Questions. **[1 × 5]**

a) Whether the vectors (4, -1, 3), (0, 8, 1), (1, 2, -5) and (2, 6, 1) are linearly dependent or linearly independent? - CO1

b) Suppose V is a vector space of all 2×2 symmetric matrices. Find the dimension of V. - CO1

c) If λ is an eigenvalue of a matrix A, then show that $k\lambda$ is an eigenvalue of the matrix kA, k is a scalar. - CO2

d) Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 0 & 6 \\ 3 & 4 & 2 \end{bmatrix}$. - CO2

e) Find the quadratic form of the matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 0 & 6 \\ 3 & 4 & 2 \end{bmatrix}$. - CO2

Q2. **[5]**

a) Determine a and b such that the system of linear equations: - CO1

$$x + 2y + z = 3, \qquad ay + 5z = 10, \qquad 2x + 7y + az = b$$

have no solution, unique solution and infinite solutions.

OR

b) (i) Show that all the vectors in R^3 satisfying - CO1

$$2x_1 + 3x_2 - x_3 = 0, x_1 - 4x_2 + x_3 = 0$$

is a subspace of R^3.

(ii) If A and B are square matrices of equal rank, then prove or disprove that rank(A^2)= rank(B^2).

Q3. **[5]**

a) Find all eigenvalues and eigenvectors of the matrix - CO2

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

OR

b) (i) Show that the eigenvalues of Hermitian matrices are real. - CO2

(ii) Show that eigenvalues of unitary matrix is of unit modulus.

Q4. **[5]**

a) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ using Gauss Jordan elimination method. - CO1

OR

b) Transform the quadratic form $Q = 3x_1^2 + 5x_2^2 + 5x_3^2 - 2x_1x_2 + 2x_2x_3 + 6x_1x_3$ into canonical form and express the new coordinates y in terms of x. - CO2