IMPLEMENTING A NETLIST EQUIVALENCE CHECKER USING A SAT APPROACH

Manual

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1 Introduction

1.1 Aim

To formally check two given netlists for equivalence using a SAT-based approach. The method shall be implemented in a C++ or python program that reads in two netlist files in a specified format. If the netlists are functionally equivalent, the program shall display “OK”. If not, it shall print an input assignment which leads to different output values (counter example).

1.2 Previous knowledge

Basic familiarity with basic C++ syntax or python, including the usage of std::vector and std::map (accessing, adding, deleting and modifying items).

1.3 Task

Your task is to write a program that reads in two netlist files (combinational logic) and formally check these for equivalence. If equivalent, print Equivalent. If not equivalent, print a counter example.

The theory will be explained in section 2 in this manual, an example is provided in section 3.

To reduce workload, a C++ and python template file is provided. The template file will take care of reading the netlist files and creating data structures.

2 Theory

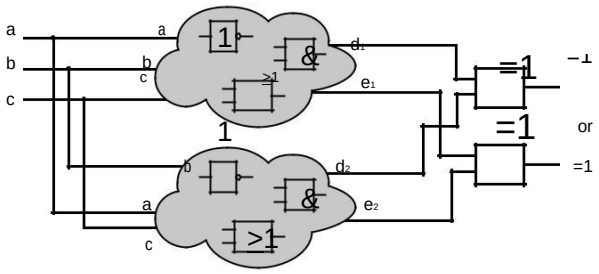


Figure 5: Sample miter circuit.

The idea behind this SAT-based method is to formally check if there is the possibility that at least one output of the two circuits has different values for the same input assignment.

Or - the other way round: To check for equivalence, you need to prove that all outputs have the same value for all possible input assignments. If this cannot be proven, the circuits are not equivalent.

To achieve this, the following steps are necessary:

1. Create an appropriate miter circuit,
2. Convert the miter circuit to a formula in CNF, and
3. Check the CNF for satisfiability.

If the circuits are not equivalent, print an according counter example (i. e. input assignment that leads to different outputs).

2.1 Miter circuit

First, we want to create an “imaginary” circuit that combines both netlists. Such a circuit is called miter. It has exactly one output. If it can be proven that this output is zero for all input assignments, the netlists are equivalent. Otherwise, the netlists are not equivalent.

A sample miter circuit is displayed in fig. 5. Let’s discuss the details of such a miter circuit.

To ensure that the input assignment is the same for both circuits, we can simply connect inputs that have the same name.

To check if a particular output port has the same value for both netlists, we connect the output ports by using an XOR gate. If the signals differ, the XOR output is 1, otherwise 0. We have to do this for each output.

In case of multiple circuit outputs, we have to find a way to combine all XOR outputs. We do this by ORing all XOR outputs.

Finally, assign signal names to all unnamed signals.

Now we have a so-called miter circuit. If we can prove that all miter outputs are 0 for all possible input cases, the two circuits are equivalent. If it is possible that at least one output becomes 1, the two circuits are not equivalent.

2.2 Conjunctive normal form

Now, we create a CNF from the circuit. We start with an empty CNF. For each gate, we add its characteristic function to the CNF. Additionally, we want to check if the circuits differ, therefore we set the output signal to 1. We do this by adding the according characteristic function of “one” to the CNF. Moreover, we have to connect the inputs of the netlists.

To successfully check netlists for equivalence, the CNF must contain the following:

Characteristic function for each gate in each netlist,

Connection of each input pair,

XOR for each output pair, and

a clause to combine all XOR outputs.

2.2.1 Characteristic functions

A characteristic function is a representation of a gate as boolean formula. It takes all variables (inputs and outputs) as function parameters. The function result shall be one if the parameters represent a valid variable assignment for that gate.

Let’s use the AND gate as example. The AND function is defined as follows:

$$c = f_{\boldsymbol{AND}}(a, b) = a \wedge b$$

Now, we create a Karnaugh map which takes both the inputs and the output as parameters. Since the characteristic function shall result in 1 if its parameters represent a valid assignment, we mark the valid assignments with 1, the others with 0. These are the valid assignments:

a = 0; b = 0; c = 0

a = 0; b = 1; c = 0

a = 1; b = 0; c = 0

a = 1; b = 1; c = 1

And these are the invalid assignments:

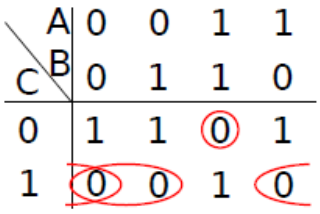
a = 0; b = 0; c = 1

a = 0; b = 1; c = 1

a = 1; b = 0; c = 1

a = 1; b = 1; c = 0

This is the resulting Karnaugh map:

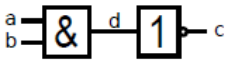


The characteristic function (in CNF) can be easily derived (as done in the exercises):

$$fc_{\text{AND}}(a,b,c) = (a \vee \overline{c})(b \vee \overline{c})(\overline{a} \vee \overline{b} \vee c)$$

2.2.2 Deriving a CNF from a netlist

To create a CNF from a netlist, we introduce names for all internal signals and concatenate the CNFs of all individual gates. Assume we have the following netlist:



Here, we introduced *d* as internal signal. The corresponding CNF is:

$$CNF_{netlist} = (a \vee \overline{d})(b \vee \overline{d})(\overline{a} \vee \overline{b} \vee d)(d \vee c)(\overline{d} \vee \overline{c})$$

... which is the result when we concatenate the characteristic functions of the AND and OR gate.

2.3 Solving the CNF

2.3.1 Setting a variable in a CNF

When we apply the Davis Putnam algorithm (next section), we have to be able to set a variable to a specific value and generate the simplified CNF afterwards.

Example task: Set c to 0 in the following CNF:

$$CNF = (a \vee b \vee \overline{c})(\overline{a} \vee \overline{b} \vee \overline{c})(\overline{a} \vee b \vee c)(a \vee \overline{b} \vee c)$$

When setting c to 0, we have to check every clause that contains either c or $\overline{c}$. All clauses that contain $\overline{c}$ can be removed from the CNF. Why? We know that $\overline{c}$ will be 1 when $c = 0$. Therefore, the whole clause will evaluate to 1, e.g., $(a \vee b \vee \overline{c}) = (a \vee b \vee 1) = 1$. Therefore, this clause can be removed.

In clauses containing c, we have to remove the literal c from the clause, since we know that c will evaluate to 0 when $c = 0$. E.g., $(\overline{a} \vee b \vee c) = (\overline{a} \vee b \vee 0) = (\overline{a} \vee b)$.

The resulting CNF for the example task is:

$$CNF_{c=0} = (\overline{a} \vee b)(a \vee \overline{b})$$

2.3.2 The Davis Putnam algorithm

The Davis Putnam algorithm takes a CNF as input and checks for satisfiability. This is the basic Davis Putnam algorithm:

```
def DP(cnf):
    # heuristics
    repeat
        apply pure literal rule to cnf
        apply unit clause rule to cnf
    until heuristics not applicable anymore

    # terminal conditions
    if cnf is empty:
        terminate algorithm, solution found
    else if cnf has empty clause:
        # no solution possible
        return
    else:
        # backtracking
        variable = choose variable from cnf
        cnf0 = set variable to zero in cnf
        DP(cnf0)
        cnf1 = set variable to one in cnf
        DP(cnf1)

        DP(cnf1)
```

The algorithm is implemented recursively. In each recursion step, the algorithm tries to simplify the CNF using the *pure literal* and *unit clause* heuristics. After that, terminal conditions are checked: An empty CNF means that a solution has been found, while a CNF with at least one empty clause means that no solution exists. If the terminal conditions do not apply, we have to backtrack (i. e. choose a variable and guess its assignment).

2.3.3 Pure literal rule

The pure literal rule is a heuristic that can speed up algorithm runtime. Consider the following CNF:

$$sampleCNF1 = (a \vee b \vee \overline{c})(\overline{a} \vee \overline{b})(\overline{b} \vee \overline{c})$$

In this CNF, the literal c only exists in its negated form, $\overline{c}$. In this case we know that the only possible value of c is 0. The resulting CNF is:

$$sampleCNF1_{pure} = (a \vee b \vee 1)(\overline{a} \vee \overline{b})(\overline{b} \vee 1) = (\overline{a} \vee \overline{b})$$

2.3.4 Unit clause rule

The unit clause rule is another heuristic. Consider the following CNF:

$$sampleCNF2 = (a \vee b)(a \vee c)(\overline{a} \vee \overline{b})(\overline{a})$$

The last clause in this CNF consists of only one literal. Since we know that all clauses have to be fulfilled (including the last clause), we know that we have to set a to 0, otherwise the last clause would not be fulfilled. The resulting CNF is:

$$sampleCNF2_{unit} = (0 \vee b)(0 \vee c)(1 \vee \overline{b})(1) = (b)(c)$$

2.3.5 Terminal conditions

What does it mean when the algorithm terminates and a solution has been found? It means, there is a valid variable assignment for all variables (signals). Since the main output has been fixed to 1, we know that the variable assignment represents a case in which the circuits under test show different behaviour. Therefore, if a solution could be found, we know that the circuits are not equivalent, and we found a counter example.

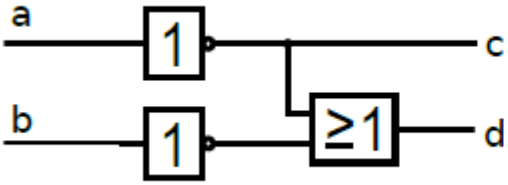
When no solution could be found, we know that there is no assignment that drives the global output to 1. This means that the circuits under test produce the same output for all possible input patterns. Therefore, the circuits are equivalent.

2.3.6 Backtracking

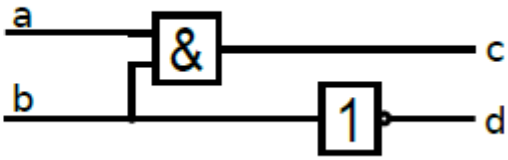
If we cannot apply any heuristic rule anymore, we have to start backtracking. That means that we choose one variable (Hint: for fastest algorithm execution, choose the rightmost variable in the CNF) and try both possible values for that variable – 0 and 1.

3 Example

Consider the following circuits

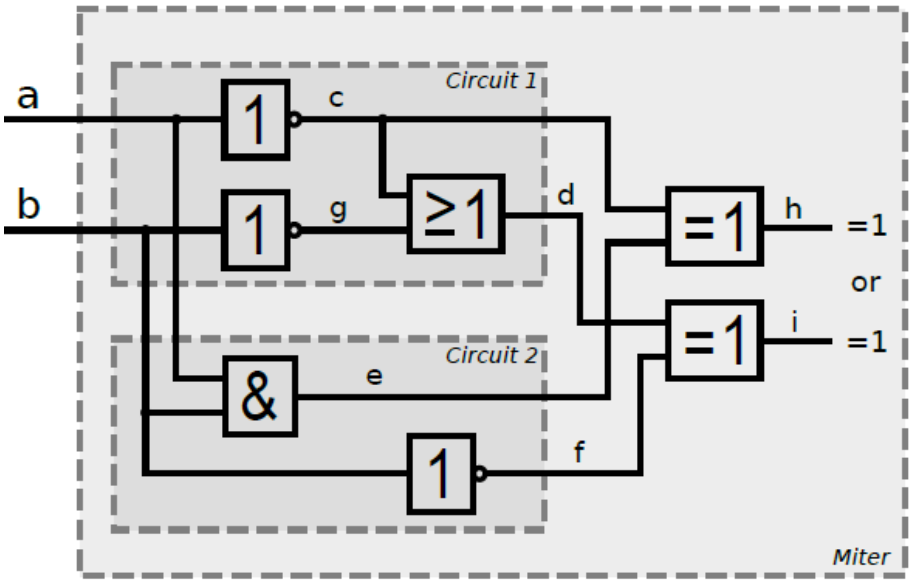


and



3.1 Creating the CNF

We add the miter circuit (connect equally-named inputs, combine outputs by XOR gates, com-bine XOR outputs by an OR gate, set OR output to “1”) and we assign signal names to all internal signals. Attention: Net numbers in the second netlist have to be modified. The result is one big circuit:



Then, we start with an empty CNF. We append the clauses of the characteristic function for each gate and also assign the static 1 to the output:

$$
\begin{aligned}
CNF_{initial} = (& \\
& \text{\# NOT gates} \\
& (a \vee c)(\overline{a} \vee \overline{c}) \\
& (b \vee g)(\overline{b} \vee \overline{g}) \\
& (b \vee f)(\overline{b} \vee \overline{f}) \\
& \\
& \text{\# AND gates} \\
& (a \vee \overline{e})(b \vee \overline{e})(\overline{a} \vee \overline{b} \vee e) \\
& \\
& \text{\# OR gates} \\
& (\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \\
& \\
& \text{\# XOR gates (miter)} \\
& (c \vee e \vee \overline{h})(\overline{c} \vee \overline{e} \vee \overline{h})(\overline{c} \vee e \vee h)(c \vee \overline{e} \vee h) \\
& (d \vee f \vee \overline{i})(\overline{d} \vee \overline{f} \vee \overline{i})(\overline{d} \vee f \vee i)(d \vee \overline{f} \vee i) \\
& \\
& \text{\# Final ONE assignment} \\
& (h \vee i) \\
)&
\end{aligned}
$$

3.2 Solving the CNF

We call the Davis Putnam algorithm with the initial CNF:

$$DP\ (CNF_{initial})$$

Step 1

No heuristics are applicable to the resulting CNF and this CNF is not a trivial case, therefore we have to start backtracking. We choose the rightmost variable in the CNF, i. We create two new CNFs, one by setting i to 0, and one by setting i to 1:

$$\begin{aligned} CNF_{j=1,i=0} = (& \\ & (a \vee c)(\overline{a} \vee \overline{c}) \\ & (b \vee g)(\overline{b} \vee \overline{g}) \\ & (b \vee f)(\overline{b} \vee \overline{f}) \\ & (a \vee \overline{e})(b \vee \overline{e})(\overline{a} \vee \overline{b} \vee e) \\ & (\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \\ & (c \vee e \vee \overline{h})(\overline{c} \vee \overline{e} \vee \overline{h})(\overline{c} \vee e \vee h)(c \vee \overline{e} \vee h) \\ & (\overline{d} \vee f)(d \vee \overline{f}) \\ & (h) \\ &) \end{aligned}$$

$$\begin{aligned} CNF_{j=1,i=1} = (& \\ & (a \vee c)(\overline{a} \vee \overline{c}) \\ & (b \vee g)(\overline{b} \vee \overline{g}) \\ & (b \vee f)(\overline{b} \vee \overline{f}) \\ & (a \vee \overline{e})(b \vee \overline{e})(\overline{a} \vee \overline{b} \vee e) \\ & (\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \end{aligned}$$

$$
\begin{aligned}
&(c \vee e \vee \overline{h})(\overline{c} \vee \overline{e} \vee \overline{h})(\overline{c} \vee e \vee h)(c \vee \overline{e} \vee h \\
&(d \vee f)(\overline{d} \vee \overline{f}) \\
)&
\end{aligned}
$$

We recurse on both CNFs.

Step 2

This is the first recursion step that was invoked in step 1 above. We apply
$CNF_{j=1,i=0}$. We find the unit clause (h), therefore, we set h to 1 and get:

$$
\begin{aligned}
CNF_{j=1,i=0,h=1} = (& \\
&(a \vee c)(\overline{a} \vee \overline{c}) \\
&(b \vee g)(\overline{b} \vee \overline{g}) \\
&(b \vee f)(\overline{b} \vee \overline{f}) \\
&(a \vee \overline{e})(b \vee \overline{e})(\overline{a} \vee \overline{b} \vee e) \\
&(\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \\
&(c \vee e)(\overline{c} \vee \overline{e}) \\
&(\overline{d} \vee f)(d \vee \overline{f}) \\
&)
\end{aligned}
$$

There is no more unit clause or pure literal in $CNF_{j=1,i=0,h=1}$, and this CNF i
Therefore, we take the rightmost variable f and set it once to 0 and once to 1

$$
\begin{aligned}
CNF_{j=1,i=0,h=1,f=0} = (& \\
& (a \vee c)(\overline{a} \vee \overline{c}) \\
& (b \vee g)(\overline{b} \vee \overline{g}) \\
& (b) \\
& (a \vee \overline{e})(b \vee \overline{e})(\overline{a} \vee \overline{b} \vee e) \\
& (\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \\
& (c \vee e)(\overline{c} \vee \overline{e}) \\
& (\overline{d})
\end{aligned}
$$

$$
\begin{aligned}
CNF_{j=1,i=0,h=1,f=1} = (& \\
& (a \vee c)(\overline{a} \vee \overline{c}) \\
& (b \vee g)(\overline{b} \vee \overline{g}) \\
& (\overline{b}) \\
& (a \vee \overline{e})(b \vee \overline{e})(\overline{a} \vee \overline{b} \vee e)
\end{aligned}
$$

$$
\begin{aligned}
& (\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \\
& (c \vee e)(\overline{c} \vee \overline{e}) \\
& (d) \\
&)
\end{aligned}
$$

We recurse on both CNFs.

Step 3

This is the first recursion step that was invoked in step 2 above. We appl
$CNF_{j=1,i=0,h=1,f=0}$. The unit clause rule can be applied several times:

$$
\begin{aligned}
CNF_{j=1,i=0,h=1,f=0,b=1} = (& \\
& (a \vee c)(\overline{a} \vee \overline{c}) \\
& (\overline{g}) \\
& (a \vee \overline{e})(\overline{a} \vee e) \\
& (\overline{c} \vee d)(\overline{g} \vee d)(c \vee g \vee \overline{d}) \\
& (c \vee e)(\overline{c} \vee \overline{e}) \\
& (\overline{d}) \\
&)
\end{aligned}
$$

$$CNF_{j=1,i=0,h=1,f=0,b=1,g=0} = ($$
$$(a \vee c)(\overline{a} \vee \overline{c})$$
$$(a \vee \overline{e})(\overline{a} \vee e)$$
$$(\overline{c} \vee d)(c \vee \overline{d})$$
$$(c \vee e)(\overline{c} \vee \overline{e})$$
$$(\overline{d})$$
$$)$$

$$CNF_{j=1,i=0,h=1,f=0,b=1,g=0,d=0} = ($$
$$(a \vee c)(\overline{a} \vee \overline{c})$$
$$(a \vee \overline{e})(\overline{a} \vee e)$$
$$(\overline{c})$$
$$(c \vee e)(\overline{c} \vee \overline{e})$$
$$)$$

$$CNF_{j=1,i=0,h=1,f=0,b=1,g=0,d=0,c=0} = ($$
$$(a)$$
$$(a \vee \overline{e})(\overline{a} \vee e)$$
$$(e)$$

$$)$$

$$CNF_{j=1,i=0,h=1,f=0,b=1,g=0,d=0,c=0,a=1} = ($$
$$(e)$$
$$(e)$$
$$)$$

$$CNF_{j=1,i=0,h=1,f=0,b=1,g=0,d=0,c=0,a=1,e=1} = ()$$

At this point, no more heuristic rules are applicable. But the current CNF is empty (it has no clauses), and therefore, the algorithm terminates and a solution has been found.

What does that mean? We have an assignment which fulfills the CNF. That means that we have an assignment that will drive the output of the miter circuit to 1. This in turn means that we have found an assignment that shows that both circuits under test are not equivalent – the assignment provides a counter example.

The actual counter example can be derived from the assignments that have been made. But only the inputs and outputs are interesting for us. This is the counter example:

Inputs:
a: 1
b: 1

Outputs of first circuit:
c: 0
d: 0

Outputs of second circuit:
c: 1
d: 0

4 Task

The task is to write a program that implements the flow introduced above. C++ or python is preferred, and a C++, python template file is provided which contains a function that reads in the netlist files and creates according data structures.

The program shall implement the creation of the miter and the SAT solver. If the circuits are equivalent, the program shall output Equivalent. If the circuits are not equivalent, the program shall print a counter example by displaying all input signal names and an according variable assignment.

The output should look similar to these sample (reference) outputs:

```
#  ./sat netlists/xor2.net netlists/xor2.net
Equivalent!
```

```
#  ./sat netlists/xor2.net netlists/xor2_nand.net
Equivalent!
```

```
#  ./sat netlists/xor2.net netlists/xor2_nand_wrong.net
Not equivalent! Counter example:
Inputs:
a: 1
b: 1

Outputs netlist 1: f: 0


Outputs netlist 2: f: 1
```

4.1 Input format

Note: If you use the provided C++ template, you can skip this section.

This is a sample input netlist file:

```
10
b a
f
2 b
1 a
10 f

and     1 2 3
inv     3 4
```

```
10 and 1 4 5
11 inv 5 6
12 and 2 4 7
13 inv 7 8
14 and 6 8 9
15 inv 9 10
```

Line 1: Number of nets (here: 10).

Line 2: Names of all input nets.

Line 3: Names of all output nets.

Line 4-6: Mapping from net numbers to port names (for all inputs and outputs).

Line 7: Empty line.

Line 8 to end of file: Gate instantiations in the form: <Gate type> <input nets> <output nets>. Depending on the gate type, there number of parameters may vary. E. g. and has three parameters, while not has only two parameters.

4.2 Gate types

There are four gate types which are allowed in a netlist:

Gate type	Input ports	Output ports
and	2	1
or	2	1
inv	1	1
xor	2	1

According constants are predefined in the C++ template.

4.3 C++ template

If you use the provided C++ template, call the compiled program with two netlist files as pa-rameters. The template provides a function to read in the netlist files. Following variables are predefined:

int netCount1, netCount2: Number of nets in netlist 1 / 2.

std::vector<std::string> inputs1, inputs2: List of input names in netlist 1 / 2.

std::vector<std::string> outputs1, outputs2: List of output names in netlist 1 / 2.

std::map<std::string, int> map1, map2: Mapping from input/output names to net numbers in netlist 1 / 2.

GateList gates1, gates2: List of gates in netlist 1 / 2.
The type GateList is defined as std::vector<Gate>, where Gate is a struct with

members type and nets. Member type will be one of AND, OR, INV or XOR. nets is the list of the actual nets (net numbers) which are connected to this gate.

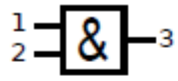
Refer to the comments in the C++ template file for additional details and usage information.

Hints

When creating the CNF, use positive net numbers for positive literals and negative net numbers for negative literals.

Since the netlists are described by net numbers (and not letters), you can create a CNF as list of list of numbers, e. g., std::vector< std::vector<int> >. Use positive numbers for positive literals/variables and negative numbers for negative literals/variables.

Consider this netlist:



The corresponding CNF could be represented as:

$$\{\{1, -3\}, \{2, -3\}, \{-1, -2, 3\}\}$$

Do not use net numbers twice.

Example: If you read in two netlists, the first netlist might use nets from 1 to 10, and the second netlist uses nets from 1 to 20. Note that both netlists will be instantiated in parallel in the miter. Therefore, you have to use different net numbers for the second netlist, e. g. by adding the number of nets of the first netlist to the nets of the second netlist. This would result in nets from 1 to 10 for the first netlist, and nets from 11 to 30 for the second netlist.

Do not reuse previously used net numbers for the miter circuit.

When you add the additional miter circuitry, you will need some more nets. Make sure that their net numbers are not already used by the first or the second netlist. E. g., use numbers that are greater than the sum of nets in the first and second netlist.

Track the current variable assignment in the Davis Putnam algorithm.

In the algorithm depicted on page 10, variable assignments are not tracked. You have to find a way to track variable assignments. Otherwise, you are not able to output a counter example.