Two-Way Independent ANOVA

ANDY FIELD

Aims

- Rationale of factorial ANOVA
- Partitioning variance
- Interaction effects
  - Interaction graphs
  - Interpretation

What is Two-Way Independent ANOVA?

- Two independent variables
  - Two-way = 2 Independent variables
  - Three-way = 3 Independent variables
- Different participants in *all* conditions
  - Independent = ‘different participants’
- Several independent variables is known as a factorial design.

Benefit of Factorial Designs

- We can look at how variables *interact*.
- Interactions
  - Show how the effects of one var might depend on the effects of another
  - Are often more interesting than main effects.
- Examples
  - Interaction between hangover and lecture topic on sleeping during lectures.
    - A hangover might have more effect on sleepiness during a stats lecture than during a clinical one.

An Example

- Field (2009): Testing the effects of alcohol and gender on ‘the beer-goggles effect’:
  - V 1 (**Alcohol**): none, 2 pints, 4 pints
  - V 2 (**Gender**): male, female
- Dependent variable (DV) was an objective measure of the attractiveness of the partner selected at the end of the evening.

Factorial anova as regression

- Subset the data
  - Gender: “Male” and “Female”
    - Dummy coding: X_1=1 for “female”, and 0 otw
  - For alcohol: keep the levels “None” and “4 pints”
    - Dummy coding: X_2=1 for “4 pints”, and 0 otw
  - We obtain 2 factors, each with two levels

$$\text{Attractiveness}_i=b_0+b_1*X_{1i}+b_2*aX_{2i}+b_3*X_{1i}*aX_{2i}+\varepsilon_i$$

$$\text{Attractiveness}_i=b_0+b_1*\text{genderF}_i+b_2*\text{alcohol}_i+b_3*\text{genderF}_i*\text{alcohol}_i+\varepsilon_i$$

Factorial anova as regression

$$\text{Attractiveness}_i = b_0 + b_1 * \text{genderF}_i + b_2 * \text{alcohol}_i + b_3 * \text{genderF}_i * \text{alcohol}_i + \varepsilon_i$$

E(A|M, none) = b0

E(A|F, none) = b0 + b1

E(A|M, 4pints) = b0 + b2

E(A|F, 4pints) = b0 + b1+ b2 + b3

Factorial anova as regression

E(A|M, none) = b0

E(A|F, none) = b0 + b1

E(A|M, 4pints) = b0 + b2

E(A|F, 4pints) = b0 + b1+ b2 + b3

b_0 is the mean attractiveness for men that drunk no alcohol (ref=men, no alcohol)

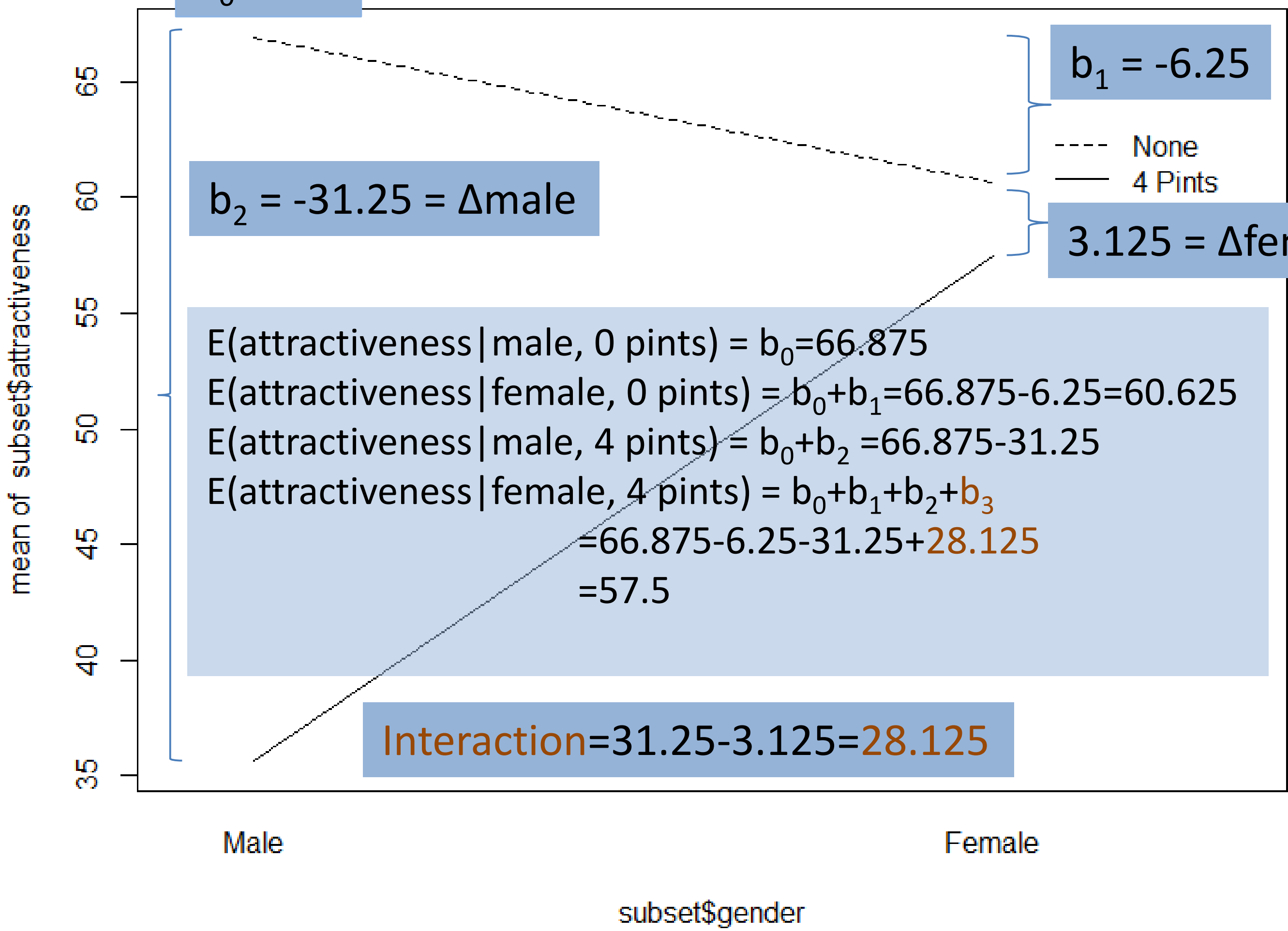
b_1 is the difference in mean attractiveness between women that drunk no alcohol and men that drunk no alcohol

b_2 is the difference in mean attractiveness between men that drunk 4 pints vs none

b_3 compares the difference between men and women in the no alcohol condition to the difference between men and women in the 4 pints condition

What Is an Interaction?

$$Attractiveness_i = b_0 + b_1 * genderF_i + b_2 * alcohol_i + b_3 * genderF_i * alcohol_i + \varepsilon_i$$



b_0=66.9

b_1 = -6.25

b_2 = -31.25 = Δmale

3.125 = Δfemale

E(attractiveness|male, 0 pints) = b_0=66.875
E(attractiveness|female, 0 pints) = b_0+b_1=66.875-6.25=60.625
E(attractiveness|male, 4 pints) = b_0+b_2 =66.875-31.25
E(attractiveness|female, 4 pints) = $b_0+b_1+b_2+b_3$
=66.875-6.25-31.25+28.125
=57.5

Interaction=31.25-3.125=28.125

Factorial anova as regression

- summary.lm(regr)

Call: aov(formula = attractiveness ~ gender * alcohol, data = subset)
Residuals: Min 1Q Median 3Q Max -16.875 -5.625 -0.625 5.156 19.375

Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept) b_0	66.875	3.055	21.890	< 2e-16 ***
Gender2 b_1	-6.250	4.320	-1.447	0.159
alcohol2 b_2	-31.250	4.320	-7.233	7.13e-08 ***
gender2:alcohol2 b_3	28.125	6.110	4.603	8.20e-05 ***

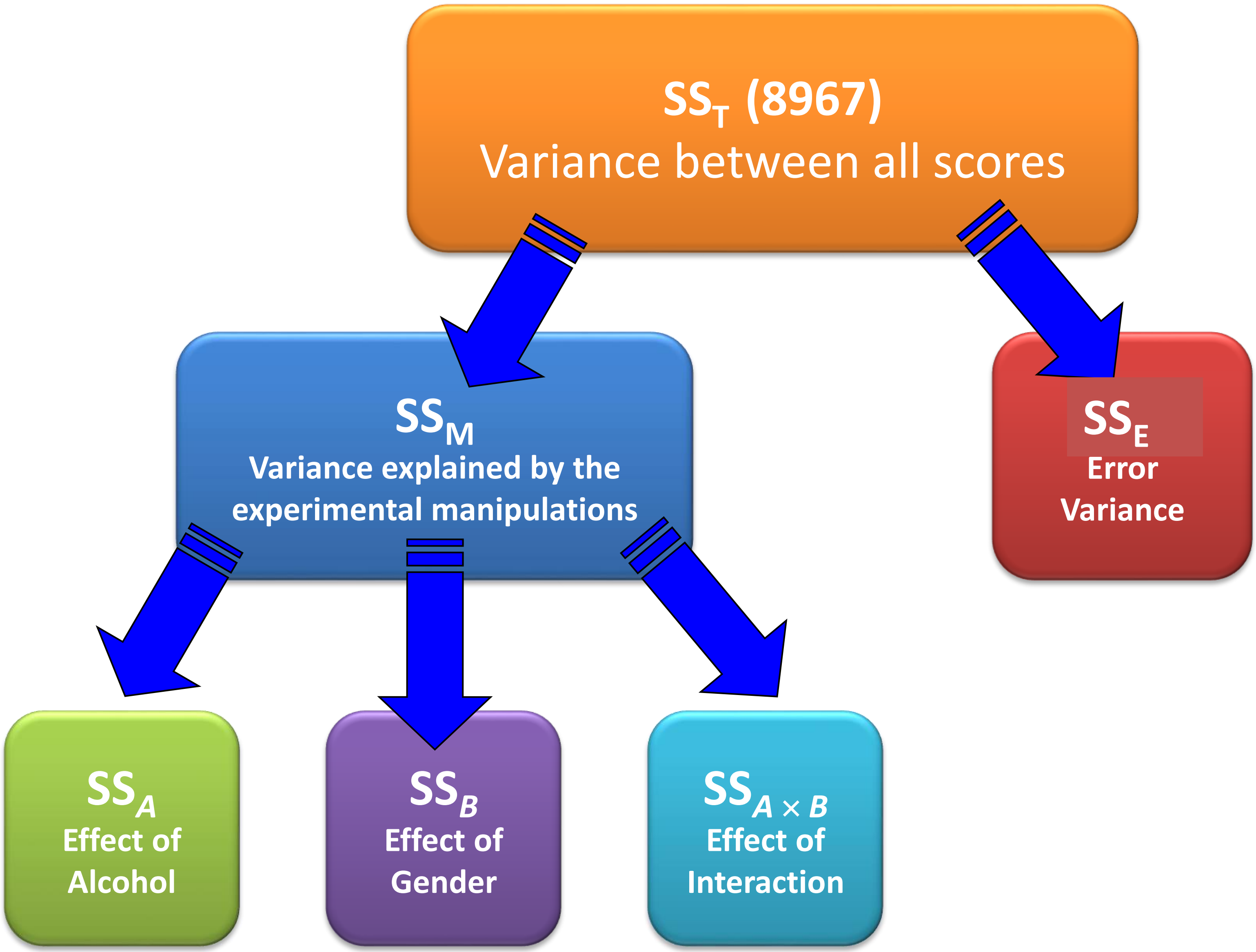
--- Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 8.641 on 28 degrees of freedom
Multiple R-squared: 0.6796, Adjusted R-squared: 0.6452
F-statistic: 19.79 on 3 and 28 DF, p-value: 4.367e-07

Variance partitioning balanced data

Table 12.1: Data for the beer-goggles effect

Alcohol	None j=1		2 Pints j=2		4 Pints j=3	
Gender	Female k=1	Male k=2	Female	Male	Female	Male
	65	50	70	45	55	30
	70	55	65	60	65	30
	60	80	60	85	70	30
R=8	60	65	70	65	55	55
	60	70	65	70	55	35
	55	75	60	70	60	20
	60	75	60	80	50	45
	55	65	50	60	50	40
Total	485	535	500	535	460	285
Mean $\bar{x}_{.jk}$	60.625	66.875	62.50	66.875	57.50	35.625
Variance s^2_{jk}	24.55	106.70	42.86	156.70	50.00	117.41

DISCOVERING STATISTICS USING R



SS_T (8967)
Variance between all scores
SS_M
Variance explained by the experimental manipulations
SS_E
Error Variance
SS_A
Effect of Alcohol
SS_B
Effect of Gender
$SS_{A \times B}$
Effect of Interaction

Step 1: Calculate SS_T

65	50	70	45	55	30
50	55	65	60	65	30
70	80	60	85	70	30
45	65	70	65	55	55
55	70	65	70	55	35
30	75	60	70	60	20
70	75	60	80	50	45
55	65	50	60	50	40

$\bar{x}_{\dots} =$ **Grand Mean = 58.33**

$$SS_T = s^2(N-1)$$
$$SS_T = 190.78(48-1)$$
$$SS_T = 8966.67$$

Step 2: Calculate SS_M

$$SS_M = \sum_{j=1}^{k} n_j(\bar{x}_{.j} - \bar{x}_{..})^2$$

$$SS_M = 8(60.625 - 58.33)^2 + 8(66.875 - 58.33)^2 + 8(62.5 - 58.33)^2$$
$$+8(66.875 - 58.33)^2 + 8(57.5 - 58.33)^2 + 8(35.625 - 58.33)^2$$
$$SS_M = 5479.167$$

Step 2a: Calculate SS_G

Female		
65	70	55
70	65	65
60	60	70
60	70	55
60	65	55
55	60	60
60	60	50
55	50	50

Mean Female = 60.21

$\bar{x}_{..k}$

Male		
50	45	30
55	60	30
80	85	30
65	65	55
70	70	35
75	70	20
75	80	45
65	60	40

Mean Male = 56.46

$$SS_{Gender} = \sum_{k=1}^{K} n_k(\bar{x}_{..k} - \bar{x}_{...})^2$$

$$SS_{Gender} = 24(60.21 - 58.33)^2 + 24(56.46 - 58.33)^2 = 168.75$$

Step 2b: Calculate SS_A

None	
65	50
70	55
60	80
60	65
60	70
55	75
60	75
55	65

Mean None = 63.75

$\bar{x}_{.j.}$

2 Pints	
70	45
65	60
60	85
70	65
65	70
60	70
60	80
50	60

Mean 2 Pints = 64.6875

4 Pints	
55	30
65	30
70	30
55	55
55	35
60	20
50	45
50	40

Mean 4 Pints = 46.5625

$$SS_{Alcohol} = \sum_{j=1}^{J} n_j(\bar{x}_{.j.} - \bar{x}_{...})^2$$

$$SS_{Alcohol} = 16(63.75 - 58.33)^2 + 16(64.6875 - 58.33)^2 + 16(46.5625 - 58.33)^2$$
$$SS_{Alcohol} = 3332.292$$

Step 2c: Calculate SS_{A*G}

$$SS_{A*G} = SS_M - SS_{Alcohol} - SS_{Gender}$$
$$SS_{A*G} = 5479.167 - 168.75 - 3332.292$$
$$SS_{A*G} = 1978.125$$

Note: True in balanced designs

Step 3: Calculate SS_E

$$SS_E = \sum_{j=1}^{J} \sum_{k=1}^{K} s_{jk}(n_{jk} - 1)$$

$$\begin{aligned}
SS_E &= s_{11}^2(n_{11} - 1) + s_{12}^2(n_{12} - 1) \\
&+ s_{21}^2(n_{21} - 1) + s_{22}^2(n_{22} - 1) \\
&+ s_{31}^2(n_{31} - 1) + s_{32}^2(n_{32} - 1) \\
SS_E &= (24.55 * 7) + (106.7 * 7) \\
&+ (42.86 * 7) + (156.7 * 7) \\
&+ (50 * 7) + (117.41 * 7) \\
&= 3487.52
\end{aligned}$$

Two-way anova table

Source of variation	Degrees of freedom	Sum of squares	Mean square	F-ratio
Factor A	J-1	SS_A	$MS_A=SS_A/(J-1)$	$F_A=MS_A/MS_E$
Factor G	K-1	SS_G	$MS_G=SS_G/(K-1)$	$F_G=MS_G/MS_E$
Interaction	(J-1)(K-1)	SS_{AG}	$MS_{AG}=SS_{AG}/(J-1)(K-1)$	$F_{AG}=MS_{AG}/MS_E$
Error	JK(R-1)=N-JK	SS_E	$MS_E=SS_E/JK(R-1)$	

Fitting a Factorial ANOVA Model

```
> gogglesModel <- lm(attractiveness ~ gender + alcohol + gender:alcohol, data = gogglesData)
```

Or:

```
> gogglesModel <- lm(attractiveness ~ alcohol*gender, data = gogglesData)
```

Sums of squares

- When data is unbalanced, there are different ways to calculate the sums of squares. Assume the model A + B + A*B
  - ***Type I SS***: Tests for the presence of an effect given that the previous one stated is already in the model
    - SS(A): reduction in residual SS attributable to A
    - SS(B|A): reduction in residual SS attributable to B when A is already in the model
    - SS(A*B|A,B): reduction in residual SS attributable to A*B when A and B are already in the model
  - ***Type II SS***: Tests for the presence of an effect, given that the others not containing this term are already in the model
    - SS(A|B), SS(B|A), SS(A*B|A,B)
  - ***Type III SS***: Tests for the presence of an effect, given that the others are in the model
    - SS(A|B, A*B), SS(B|A, A*B), SS(A*B|A, B)
- Do not interpret a main effect if interactions are present (generally speaking, if a significant interaction is present, the main effects should not be further analysed).
- When data is balanced, types I, II and III all give the same results.

Type III SS in R

- If we want to look at the Type III sums of squares for the model, we need to also execute this command after we have created the model:

```
> Anova(gogglesModel, type="III")
```

{car}

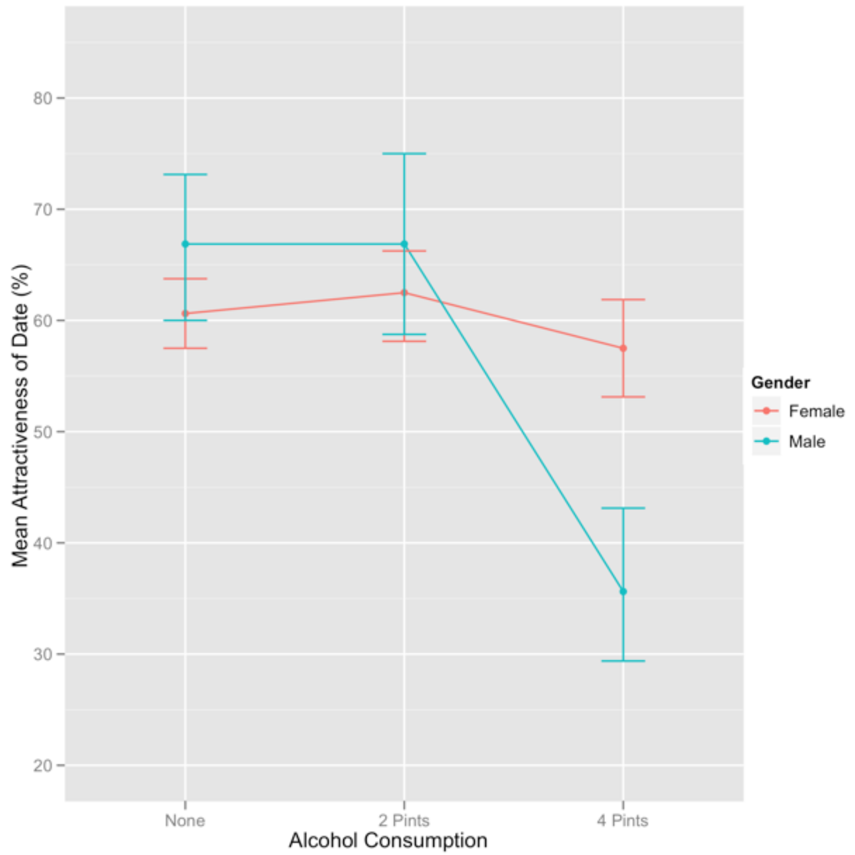
Interpreting Factorial ANOVA

```
Anova Table (Type III tests)

Response: attractiveness
                 Sum Sq Df   F value    Pr(>F)
(Intercept)      163333  1 1967.0251 < 2.2e-16 ***
gender              169  1    2.0323    0.1614
alcohol            3332  2   20.0654 7.649e-07 ***
gender:alcohol     1978  2   11.9113 7.987e-05 ***
Residuals          3488 42
```

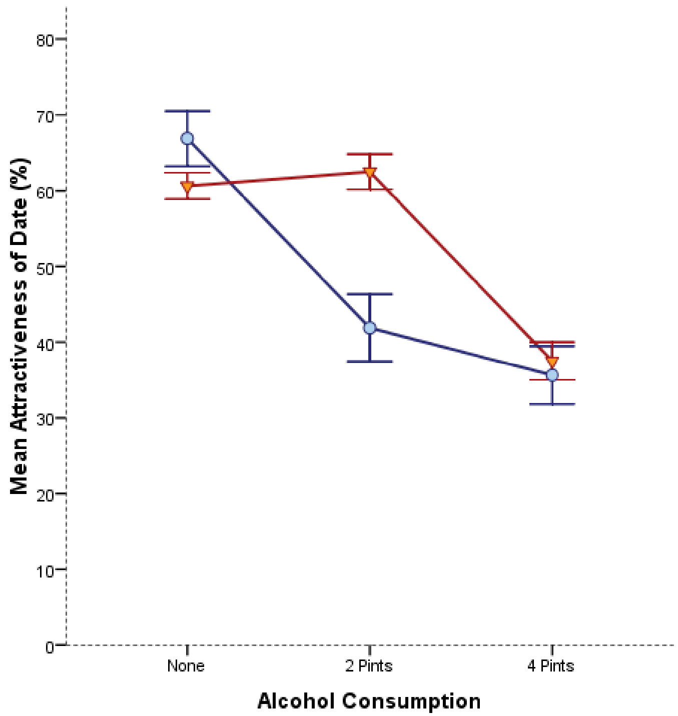
Output 12.4

Interpretation: Interaction

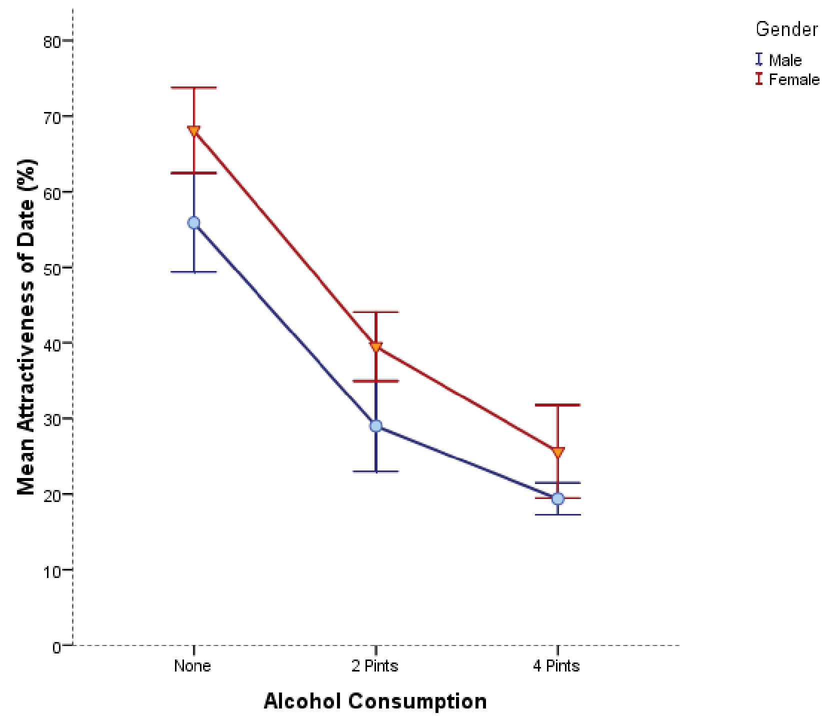


There was a significant interaction between the amount of alcohol consumed and the gender of the person selecting a mate, on the attractiveness of the partner selected (p=8e-5).

Is There Likely to Be a Significant Interaction Effect?

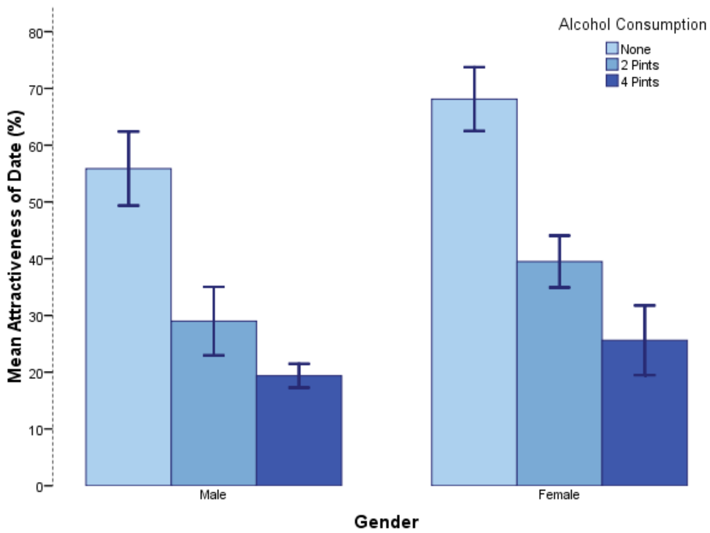


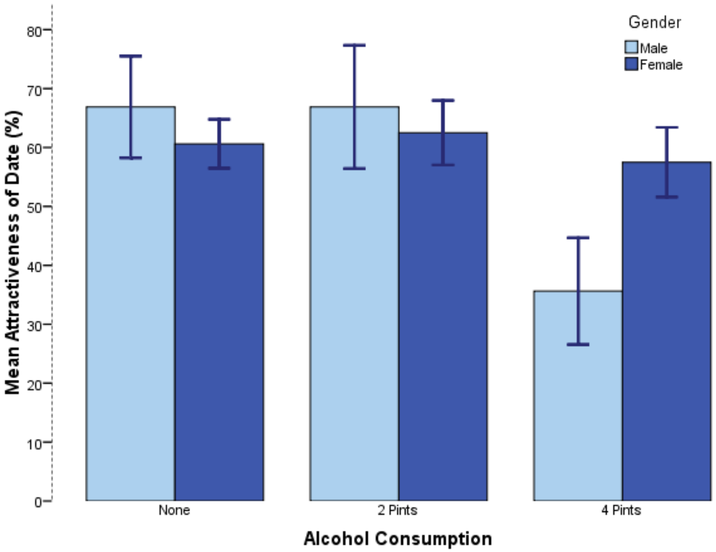
Yes



No

Is There Likely to Be a Significant Interaction Effect?





No

Yes

Post-hoc tests

- Cf Demo

Workflow two-way anova

1. Summary statistics

```
> ddply(df, .(factor1, factor2), summarise,
    Nobs = sum(!is.na(contvar)),
    Nmiss = sum(is.na(contvar)),
    mean = mean(contvar, na.rm=TRUE),
    sd = sd(contvar, na.rm=TRUE),
    se   = sd/sqrt(Nobs),
    t = qt(0.975, Nobs-1),
    lower = mean - t*se,
    upper = mean + t*se)
```

Workflow two-way anova

2. Graphs:

- bar chart

> plot <- ggplot(**df**, aes(**factor1**, **contvar**))

> plot +

stat_summary(fun.data = mean_cl_normal, geom = "errorbar", position=position_dodge(width=0.90), width = 0.2) +

stat_summary(fun.y = mean, geom = "bar", position="dodge") +

facet_wrap(~**factor2**) +

labs(x = “**factor1labe**l", y = “**contvarlabel**")

Workflow two-way anova

- Boxplots

```
> plot <- ggplot(df, aes(factor1, contvar))
> plot + geom_boxplot() +
  facet_wrap(~factor2) +
  labs(x = " factor1label ", y = " contvarlabel )
```

DISCOVERING STATISTICS USING R

Workflow two-way anova

- interaction plot

```
> summ <- ddply(...)
> ggplot(summ, aes(x=factor1, y=mean, group=factor2,
color=factor2)) +
  geom_errorbar(aes(ymin=lower, ymax=upper), width=.1,
              position=position_dodge(0.05)) +
  geom_line() +
  geom_point() +
  labs(x = “factor1label", y = “contvarlabel") +
  scale_color_brewer(palette="Paired") + theme_minimal()
```

ANDY FIELD

Workflow two-way anova

3. Statistical model
4. Testing the assumptions (cfr regression)
5. Post-hoc tests provided that a significant interaction was found

Correcting Data Problems

- Log transformation:

  df$logvar <- log(df$var)

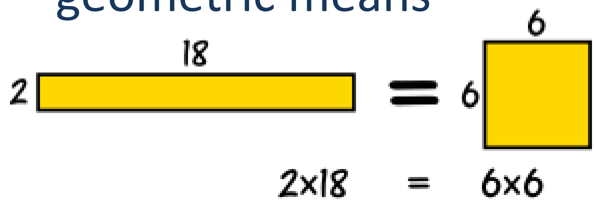
  df$logvar <- log(df$var + 1)

- Square root transformation:
  - df$sqrtvar <- sqrt(df$var)
- Reciprocal transformation:

  df$recvar <- 1/(df$var + 1)

To Transform … Or Not

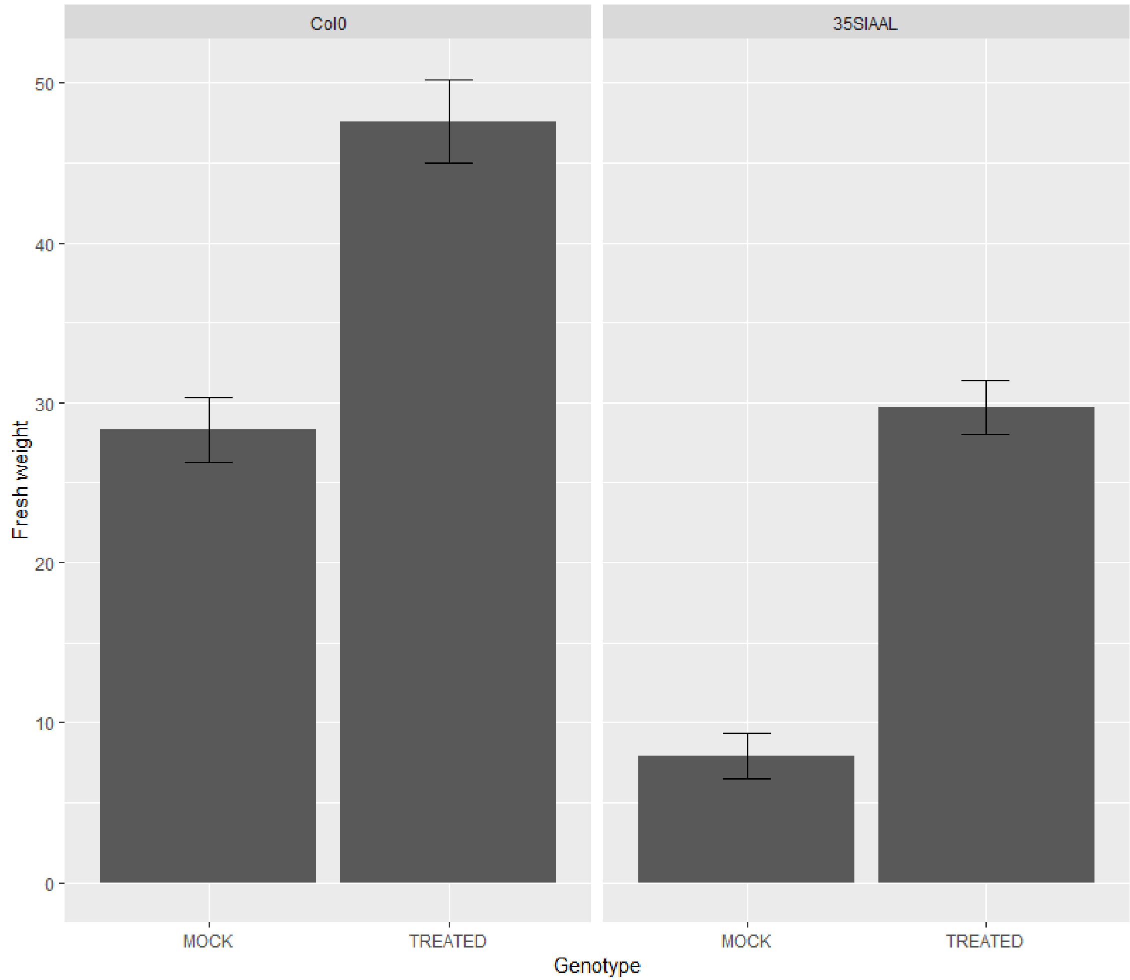
- Transforming the data helps as often as it hinders the accuracy of *F* (Games & Lucas, 1966).
- Games (1984):
  - The central limit theorem: sampling distribution of the mean will be normal in samples > 30 anyway.
  - Transforming the data changes the hypothesis being tested
    - E.g. when using a log transformation and comparing means, you change from comparing arithmetic means to comparing geometric means



$$\sqrt[n]{\prod_{i=1}^{n} x_i} = exp\left(\frac{\sum_{i=1}^{n} log_e(x_i)}{n}\right)$$

  - In small samples it is tricky to determine normality one way or another.
  - The consequences for the statistical model of applying the ‘wrong’ transformation could be worse than the consequences of analysing the untransformed scores.

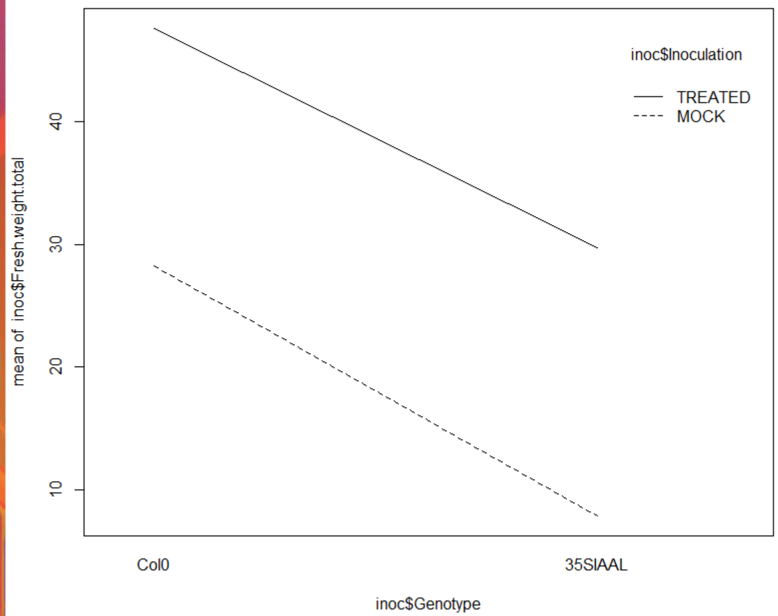
Effect log transformation



Effect log transformation

Original scale

H0: Col T – Col M = 35S T – 35S M



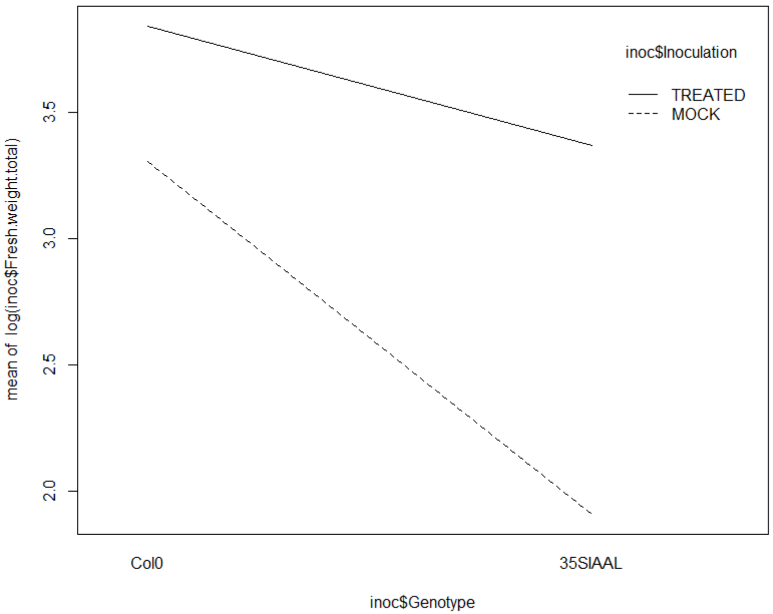
Log scale

H0: log(Col T) – log(Col M)

= log(35S T) – log(35S M)

After backtransformation:

Col T/Col M = 35S T/35S M



DEMO TWO-WAY ANOVA

Open the program Ch12_glm2.R

Exercises

- Task 1
  - The data “**weightgain**” arise from an experiment to study the gain in weight of rats fed on four different diets, distinguished by amount of protein (low and high) and by source of protein (beef and cereal). The data is available in the library HSAUR3. What is the effect of protein level and source of protein on weightgain? Check the assumptions by investigating the residuals.

Exercises

- Task 2
  - An experiment is designed to investigate change in systolic blood pressure after administering one of four different drugs to patients with one of three different blood diseases. These data (**drugs2.txt**) are from Afifi and Azen (1972). Investigate the effect of both factors on the outcome. Explore the residuals.

Exercises

- Task 3
  - Differences in food consumption when rancid lard was substituted for fresh lard was examined in male and female rats aged 30 to 34 days. Food eaten (in grams) during 73 days was recorded. Examine the effects of the factors on food consumption. Explore the residuals. Data taken from Biometry (Sokal and Rohlf, 1998) and entered as:
    - consumption <- c(709,679,699,592,538,476,508,505,539, 657,594,677)
    - gender <- gl(2, 6, labels = c("M", "F"))
    - fat <- gl(n=2,k=3,length=12,labels=c("Fresh","Rancid"))
    - rats = data.frame(consumption, gender, fat)