Chapter SIX

Introduction to Continuous Probability Distributions

CHAPTER OUTCOMES

After studying the material in Chapter 6 you should be able to:

1. Convert a normal distribution to a standard normal distribution.
2. Determine probabilities using the normal distribution.
3. Calculate values of the random variable associated with specified probabilities from a normal distribution.
4. Calculate probabilities associated with a uniformly distributed random variable.
5. Determine probabilities for an exponential probability distribution.

PREPARING FOR CHAPTER SIX

- Review the methods for determining the probability for a discrete random variable in Chapter 5.
- Review the discussion of the mean and standard deviation in Sections 3.1 and 3.2.
- Review the concept of *z*-scores outlined in Section 3.3.

Why You Need To Know

As shown in Chapter 5, you will encounter many business situations where the variable of interest is discrete and probability distributions such as the binomial, Poisson, or the hypergeometric will be useful for helping analyze decision situations. However, you will also deal with applications where the variable of interest is *continuous* rather than discrete. For instance, Toyota managers are interested in a measure called cycle time, which is the time between cars coming off the assembly line. Their factory may be designed to produce a car every 55 seconds and the operations managers would be interested in determining the probability the actual time between cars will exceed 60 seconds. A pharmaceutical company may be interested in the probability that a new drug will reduce blood pressure by more than 20 points for patients. The Kellogg company is interested in the probability that cereal boxes labeled as containing 16 ounces will actually contain at least that much cereal.

In each of these examples, the value of the variable of interest is determined by measuring (measuring the time between cars, measuring the blood pressure reading, measuring the weight of cereal in a box). In every instance, the number of possible values for the variable is limited only by the capacity of the measuring device. The constraints imposed by the measuring devices produce a finite number of outcomes. In these and similar situations, a continuous probability distribution can be used to approximate the distribution of possible outcomes for the random variables. The approximation is appropriate when the number of possible outcomes is large. Chapter 6 introduces three specific continuous probability distributions of particular importance for decision making and the study of business statistics. The first of these, the *normal distribution*, is by far the most important because a great many applications involve random variables that possess the characteristics of the normal distribution. In addition, many of the topics in the remaining chapters of this textbook dealing with statistical estimation and hypothesis testing are based on the normal distribution.

In addition to the normal distribution, you will be introduced to the uniform distribution and the exponential distribution. Both are important continuous probability distributions and have many applications in business decision making. You need to have a firm understanding and working knowledge of all three continuous probability distributions introduced in this chapter.

6.1 The Normal Probability Distribution

Chapter 5 introduced three important discrete probability distributions: the binomial distribution, the Poisson distribution, and the hypergeometric distribution. For each distribution, the random variable of interest is discrete and its value is determined by *counting*. For instance, a market researcher may be interested in the probability that a new product will receive favorable reviews by at least three-fourths of those sampling the product. The researcher might sample 100 people and then count the number of people who provide favorable reviews. The random variable could be values such as 65, 66, 67, . . . , 75, 76, 77, etc. The number of possible positive reviews could never be a fraction such as 76.80 or 79.25. The possible outcomes are limited to integer values between 0 and 100 in this case.

In other instances, you will encounter applications in which the value of the random variable is determined by *measuring* rather than by counting. In these cases, the random variable is said to be approximately *continuous* and can take on any value along some defined continuum. For instance, a Pepsi-Cola can that is supposed to contain 12 ounces might actually contain any amount between 11.90 and 12.10 ounces, such as 11.9853 ounces. When the variable of interest is approximately continuous, the probability distribution associated with the random variable is called a *continuous probability distribution*.

One important difference between discrete and continuous probability distributions involves the calculation of probabilities associated with specific values of the random variable. For instance, in the market research example in which 100 people were surveyed, we could use the binomial distribution to find the probability of any specific number of positive reviews, such as $P(x = 75)$ or $P(x = 76)$. While these individual probabilities may be small values, they can be computed because the random variable is discrete. However, if the random variable is continuous as in the Pepsi-Cola example, there are an uncountably infinite number of possible outcomes for the random variable. Theoretically, the probability of any one of these individual outcomes is zero. That is, $P(x = 11.92) = 0$ or $P(x = 12.05) = 0$. Thus, when you are working with continuous distributions, you will find the probability for a range of possible values such as $P(x \leq 11.92)$ or $P(11.92 \leq x \leq 12.0)$. Likewise, you can conclude that

$$P(x \leq 11.92) = P(x < 11.92)$$

because we assume that $P(x = 11.92) = 0$.

There are many different continuous probability distributions, but the most important of these is the *normal distribution*.

The Normal Distribution

You will encounter many business situations in which the random variable of interest will be treated as a continuous variable. There are several continuous distributions that are frequently used to describe physical situations. The most useful continuous probability distribution is the **normal distribution**.[1] The reason is that the output from a great many processes (both man-made and natural) are normally distributed.

Normal Distribution

The normal distribution is a bell-shaped distribution with the following properties:

1. It is *unimodal*; that is, the normal distribution peaks at a single value.
2. It is *symmetrical*; this means that the two areas under the curve between the mean and any two points equidistant on either side of the mean are identical. One side of the distribution is the mirror image of the other side.
3. The mean, median, and mode are equal.
4. The normal approaches the horizontal axis on either side of the mean toward plus and minus infinity (∞). In more formal terms, the normal distribution is *asymptotic* to the x axis.
5. The amount of variation in the random variable determines the height and spread of the normal distribution.

Figure 6.1 illustrates a typical normal distribution and highlights the normal distribution's characteristics. All normal distributions have the same general shape as the one shown in Figure 6.1. However, they can differ in their mean value and their variation, depending on the situation being considered. The process being represented determines the scale of the horizontal axis. It may be pounds, inches, dollars, or any other attribute with a continuous measurement. Figure 6.2 shows several normal distributions with different centers and different spreads. Note that the total area (probability) under each normal curve equals 1.

The normal distribution is described by the rather-complicated-looking probability density function, shown in Equation 6.1.

Normal Distribution Density Function

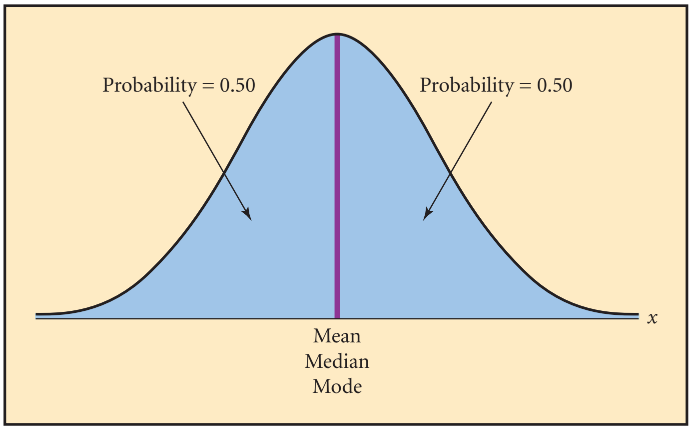
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{6.1}$$

where:

x = Any value of the continuous random variable
σ = Population standard deviation
π = 3.14159
e = Base of the natural log $\simeq$ 2.71828 . . .
μ = Population mean

FIGURE 6.1

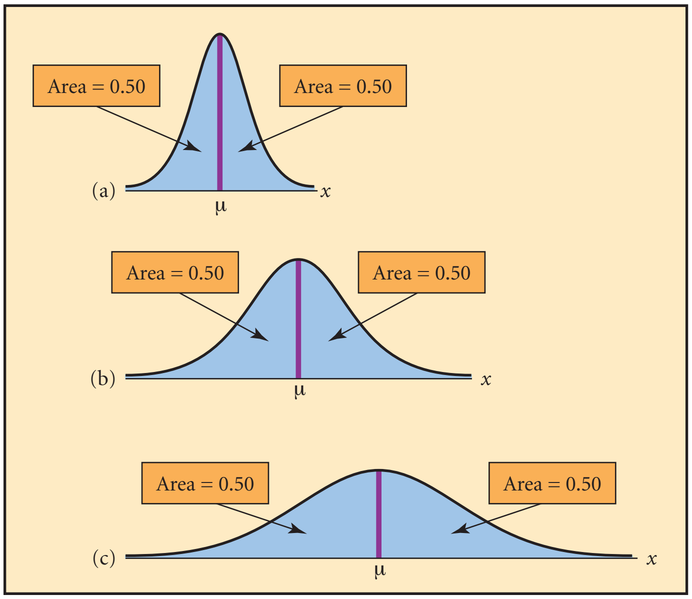
Characteristics of the Normal Distribution



[1] It is common to refer to the very large family of normal distributions as "*the* normal distribution." Keep in mind, however, that "the normal distribution" really is a very large family of distributions.

FIGURE 6.2

Difference Between Normal Distributions



To graph the normal distribution, we need to know the mean, μ, and the standard deviation, σ. Placing μ, σ, and a value of the variable, x, into the probability density function, we can calculate a height, $f(x)$, of the density function. If we try enough x-values, we will get a curve like those shown in Figures 6.1 and 6.2.

The area under the normal curve corresponds to probability. The probability, $P(x)$, is equal to 0 for any particular x. However, we can find the probability for a range of values between x_1 and x_2 by finding the area under the curve between these two values. A special normal distribution called the *standard normal distribution* is used to find areas (probabilities) for a normal distribution.

CHAPTER OUTCOME #1

The Standard Normal Distribution

The trick to finding probabilities for a normal distribution is to convert the normal distribution to a **standard normal distribution**.

Standard Normal Distribution
A normal distribution that has a mean = 0.0 and a standard deviation = 1.0. The horizontal axis is scaled in *z*-values that measure the number of standard deviations a point is from the mean. Values above the mean have positive *z*-values. Values below the mean have negative *z*-values.

To convert a normal distribution to a standard normal distribution, the values (x) of the random variable are standardized as outlined previously in Chapter 3. The conversion formula is shown as Equation 6.2.

Standardized Normal *z*-Value

$$z = \frac{x - \mu}{\sigma} \tag{6.2}$$

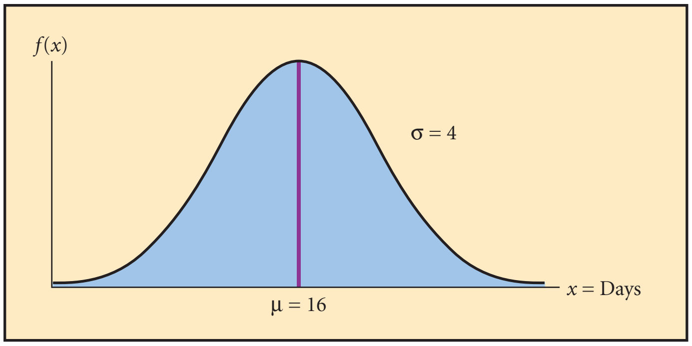
where:

z = Scaled value (the number of standard deviations a point x is from the mean)
x = Any point on the horizontal axis
μ = Mean of the normal distribution
σ = Standard deviation of the normal distribution

Equation 6.2 *rescales* any normal distribution axis from its true units (time, weight, dollars, barrels, and so forth) to the standard measure referred to as a *z-value*. Thus, any value of the normally distributed continuous random variable can be represented by a unique *z*-value.

FIGURE 6.3

Distribution of Days Homes Stay on the Market Until They Sell



Business Application

REAL ESTATE SALES A July 25, 2005, article in the *Washington Post* by Kirstin Downey and Sandra Fleishman entitled "D.C. Area Housing Market Cools Off" stated that the average time that a home remained on the market before selling in Fairfax County is 16 days. Suppose that further analysis performed by Metropolitan Regional Information Systems Inc., which runs the local multiple-listing service, shows the distribution of days that homes stay on the market before selling is approximated by a normal distribution with a standard deviation of 4 days. Figure 6.3 shows this normal distribution with $\mu = 16$ and $\sigma = 4$.

Three homes that were sold in Fairfax County were selected from the multiple-listing inventory. The days that these homes spent on the market were

Home 1: $x = 16$ days

Home 2: $x = 18.5$ days

Home 3: $x = 9$ days

Equation 6.2 is used to convert these values from a normally distributed population with $\mu = 16$ and $\sigma = 4$ to corresponding z-values in a standard normal distribution. For Home 1, we get

$$z = \frac{x - \mu}{\sigma} = \frac{16 - 16}{4} = 0$$

Note, Home 1 was on the market 16 days, which happens to be equal to the population mean. The standardized z-value corresponding to the population mean is zero. This indicates that the population mean is 0 standard deviations from itself.

For Home 2, we get

$$z = \frac{x - \mu}{\sigma} = \frac{18.5 - 16}{4} = 0.63$$

Thus, for this population, a home that stays on the market 18 days is 0.63 standard deviations higher than the mean. The standardized z-value for Home 3 is

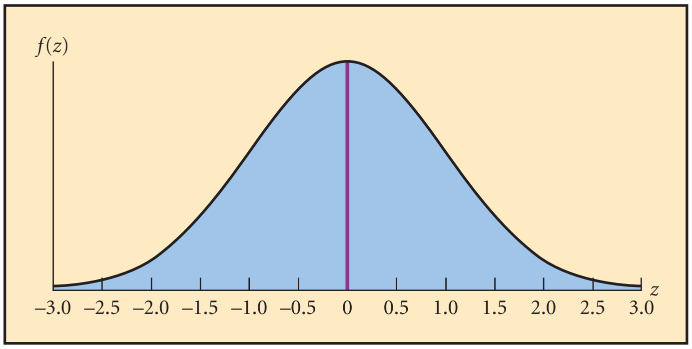
$$z = \frac{x - \mu}{\sigma} = \frac{9 - 16}{4} = -1.75$$

The z-value is -1.75. This means a home from this population that stays on the market for only 9 days has a value that is 1.75 standard deviations below the population mean. Note, a negative z-value always indicates the x-value is less than the mean, μ.

The z-value represents the number of standard deviations a point is above or below the population mean. Equation 6.2 can be used to convert any value of x from the population distribution to a corresponding z-value. If the population distribution is normally distributed as shown in Figure 6.3, then the distribution of z-values will also be normally distributed and is called the *standard normal distribution*. Figure 6.4 shows a standard normal distribution.

FIGURE 6.4

Standard Normal Distribution



You can convert the normal distribution to a standard normal distribution and use the standard normal table to find the desired probability. Example 6-1 shows the steps required to do this.

CHAPTER OUTCOME #2

Using the Standard Normal Table The *standard normal table* in Appendix D provides probabilities (or areas under the normal curve) for many different z-values. The standard normal table is constructed so that the probabilities provided represent the chance of a value falling between the z-value and the population mean.

The standard normal table is also reproduced in Table 6.1. This table provides probabilities for z-values between $z = 0.00$ and $z = 3.09$.

SUMMARY Using the Normal Distribution

If a continuous random variable is distributed as a normal distribution, the distribution is symmetrically distributed around the mean, or expected value, and is described by the mean and standard deviation. To find probabilities associated with a normally distributed random variable, use the following steps:

1. Determine the mean, μ, and the standard deviation, σ.
2. Define the event of interest, such as $P(x \geq x_1)$.
3. Convert the normal distribution to the standard normal distribution using Equation 6.2:

$$z = \frac{x - \mu}{\sigma}$$

4. Use the standard normal distribution tables to find the probability associated with the calculated z-value. The table gives the probability between the z-value and the mean.
5. Determine the desired probability using the knowledge that the probability of a value being on either side of the mean is 0.50 and the total probability under the normal distribution is 1.0.

EXAMPLE 6-1 Using The Standard Normal Table

TRY PROBLEM 6.1

Employee Commute Time After completing a study, a company in Kansas City concluded the time its employees spend commuting to work each day is normally distributed with a mean equal to 15 minutes and a standard deviation equal to 3.5 minutes. One employee has indicated that she commutes 22 minutes per day. To find the probability that an employee would commute 22 or more minutes per day, you can use the following steps:

Step 1 Determine the mean and standard deviation for the random variable.
The parameters of the probability distribution are

$$\mu = 15 \quad \text{and} \quad \sigma = 3.5$$

Step 2 Define the event of interest.
The employee has a commute time of 22 minutes. We wish to find

$$P(x \geq 22) = ?$$

Step 3 **Convert the random variable to a standardized value using Equation 6.2.**

$$z = \frac{x - \mu}{\sigma} = \frac{22 - 15}{3.5} = 2.00$$

Step 4 **Find the probability associated with the z-value in the standard normal distribution table (Appendix D).**

To find the probability associated with $z = 2.00$, [i.e., $P(0 \leq z \leq 2.00)$], do the following:

1. Go down the left-hand column of the table to $z = 2.0$.
2. Go across the top row of the table to 0.00 for the second decimal place in $z = 2.00$.
3. Find the value where the row and column intersect.

The value, 0.4772, is the probability that a value in a normal distribution will lie between the mean and 2.00 standard deviations above the mean.

Step 5 **Determine the probability for the event of interest.**

$$P(x \geq 22) = ?$$

We know that the area on each side of the mean under the normal distribution is equal to 0.50. In Step 4 we computed the probability associated with $z = 2.00$ to be 0.4772, which is the probability of a value falling between the mean and 2.00 standard deviations above the mean. Then, the probability we are looking for is

$$P(x \geq 22) = P(z \geq 2.00) = 0.5000 - 0.4772 = 0.0228$$

Business Application

REAL ESTATE SALES (*CONTINUED*) Earlier, we discussed the situation involving real estate sales in Fairfax County near Washington D.C. in which a report showed that the mean days a home stays on the market before it sells is 16 days. We assumed the distribution for days on the market before a home sells was normally distributed with $\mu = 16$ and $\sigma = 4$. A local D.C. television station interviewed an individual whose home had recently sold after 14 days on the market. Contrary to what the reporter had anticipated, this homeowner was mildly disappointed in how long her home took to sell. She said she thought it should have sold quicker given the fast-paced real estate market, but the reporter countered that he thought the probability was quite high that a home would require 14 or more days to sell. Specifically, we want to find

$$P(x \geq 14) = ?$$

This probability corresponds to the area under a normal distribution to the right of $x = 14$ days. This will be the sum of the area between $x = 14$ and $\mu = 16$ plus the area to the right of $\mu = 16$. Refer to Figure 6.5.

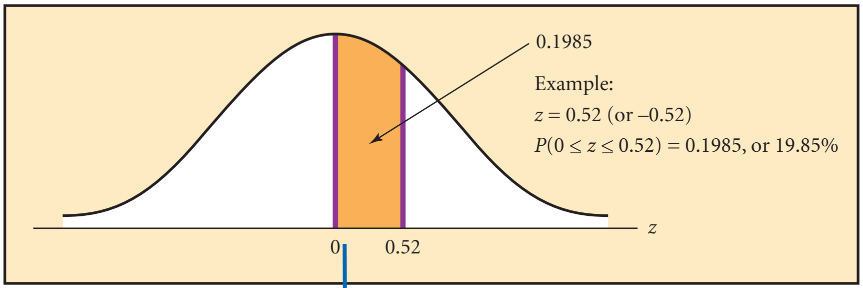
To find this probability, you first convert $x = 14$ days to its corresponding z-value. This is equivalent to determining the number of standard deviations $x = 14$ is from the population mean of $\mu = 16$. Equation 6.2 is used to do this as follows:

$$z = \frac{x - \mu}{\sigma} = \frac{14 - 16}{4} = -0.50$$

Because the normal distribution is symmetrical, even though the z-value is negative 0.50, we find the desired probability by going to the standard normal distribution table for a

TABLE 6.1 Standard Normal Distribution Table

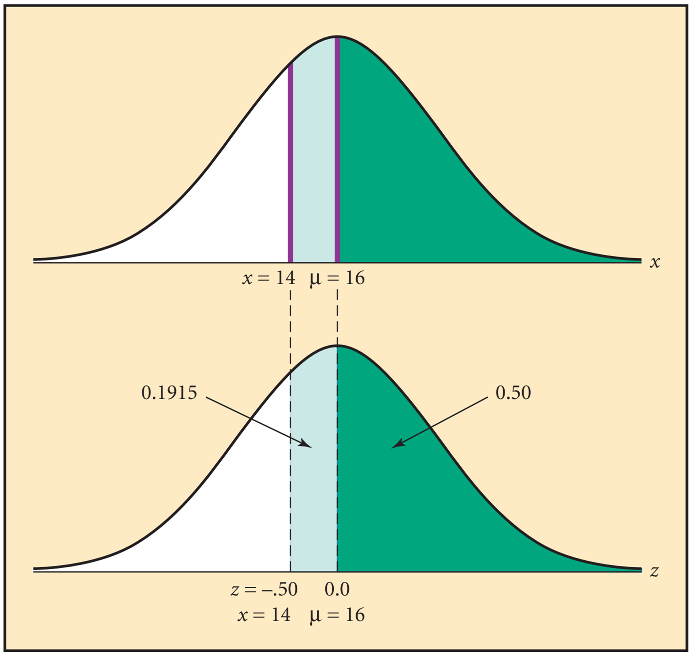
To illustrate: 19.85% of the area under a normal curve lies between the mean, μ, and a point 0.52 standard deviation units away.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846.	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

FIGURE 6.5

Probabilities from the Normal Curve for Fairfax Real Estate



positive $z = 0.50$. The probability in the table for $z = 0.50$ corresponds to the probability of a z-value occurring between $z = 0.50$ and $z = 0.0$. This is the same as the probability of a z-value falling between $z = -0.50$ and $z = 0.00$. Thus, from the standard normal table (Table 6.1 or Appendix D), we get

$$P(-0.50 \leq z \leq 0.00) = 0.1915$$

This is the area between $x = 14$ and $\mu = 16$ in Figure 6.5. We now add 0.1915 to 0.5000, which is the probability of a value exceeding $\mu = 16$. Therefore, the probability that a home will require 14 or more days to sell is

$$P(x \geq 14) = 0.1915 + 0.5000 = 0.6915$$

This is illustrated in Figure 6.5. Thus, there is nearly a 70% chance that a home will require at least 14 days to sell.

Business Application

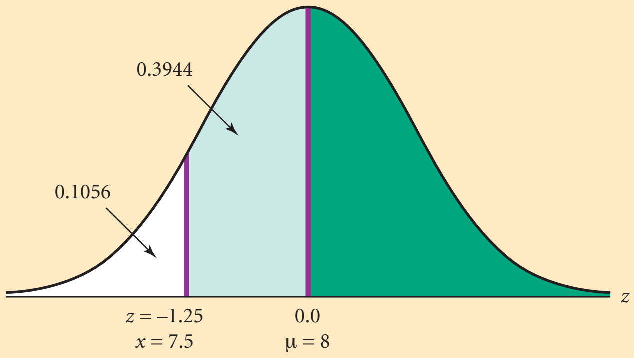
LONGLIFE BATTERY COMPANY Several states, including California, have passed legislation requiring automakers to sell a certain percentage of zero-emissions cars within their borders. One current alternative is battery-powered cars. The major problem with battery-operated cars is the limited time they can be driven before the batteries must be recharged. Longlife Battery, a start-up company, has developed a battery pack it claims will power a car at a sustained speed of 45 miles per hour for an average of 8 hours. But of course there will be variations: Some battery packs will last longer and some shorter than 8 hours. Current data indicate that the standard deviation of battery operation time before a charge is needed is 0.4 hours. Data show a normal distribution of uptime on these battery packs. Automakers are concerned that batteries may run short. For example, drivers might find an “8-hour” battery that lasts 7.5 hours or less unacceptable. What are the chances of this happening with the Longlife battery pack?

To calculate the probability the batteries will last 7.5 hours or less, find the appropriate area under the normal curve shown in Figure 6.6. There is approximately 1 chance in 10 that a battery will last 7.5 hours or less when the vehicle is driven at 45 miles per hour.

Suppose this level of reliability is unacceptable to the automakers. Instead of a 10% chance of an “8-hour” battery lasting 7.5 hours or less, the automakers will accept no more than a 2% chance. Longlife Battery asks the question, what would the mean uptime have to be to meet the 2% requirement?

FIGURE 6.6

Longlife Battery Company



$$z = \frac{x - \mu}{\sigma} = \frac{7.5 - 8}{0.4} = -1.25$$

From the normal table $P(-1.25 \le z \le 0) = 0.3944$

Then we find $P(x \le 7.5 \text{ hours}) = 0.5000 - 0.3944 = 0.1056$

Assuming that uptime is normally distributed, we can answer this question by using the standard normal distribution. However, instead of using the standard normal table to find a probability, we use it in reverse to find the z-value that corresponds to a known probability. Figure 6.7 shows the uptime distribution for the battery packs. Note, the 2% probability is shown in the left tail of the distribution. This is the allowable chance of a battery lasting 7.5 hours or less. We must solve for μ, the mean uptime that will meet this requirement.

1. Go to the body of the standard normal table, where the probabilities are located, and find the probability as close to 0.48 as possible. This is 0.4798.
2. Determine the z-value associated with 0.4798. This is $z = 2.05$. Because we are below the mean, the z is negative. Thus, $z = -2.05$.
3. The formula for z is

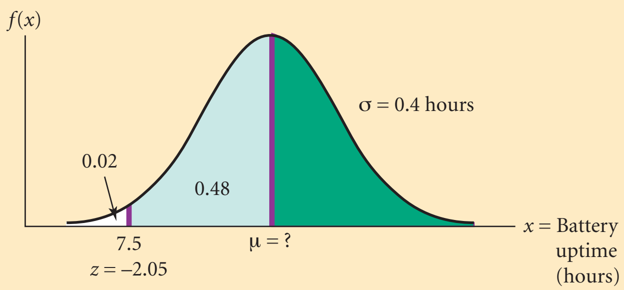
$$z = \frac{x - \mu}{\sigma}$$

4. Substituting the known values, we get

$$-2.05 = \frac{7.5 - \mu}{0.4}$$

FIGURE 6.7

Longlife Battery Company, Solving for the Mean



Solve for μ:

$$z = \frac{x - \mu}{\sigma}$$

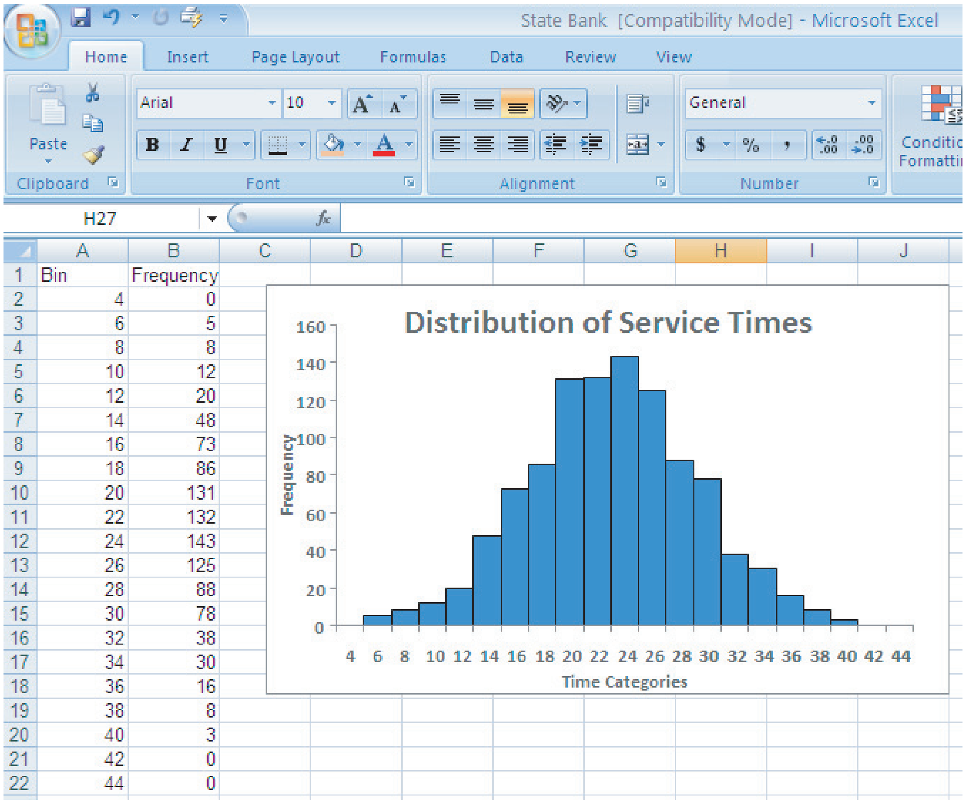
$$-2.05 = \frac{7.5 - \mu}{0.4}$$

$$\mu = 7.5 + 2.05(0.4)$$

$$\mu = 8.32$$

FIGURE 6.8

Excel 2007 Output for State Bank and Trust Service Times



Bin	Frequency
4	0
6	5
8	8
10	12
12	20
14	48
16	73
18	86
20	131
22	132
24	143
26	125
28	88
30	78
32	38
34	30
36	16
38	8
40	3
42	0
44	0

Excel 2007 Instructions:

1. Open file: State Bank.xls.
2. Create bins (upper limit of each class).
3. Select **Data > Data Analysis**.
4. Select **Histogram**.
5. Define data and bin ranges.
6. Check **Chart Output**.
7. Define Output Location.

Minitab Instructions (for similar results):

1. Open file: State Bank. MTW.
2. Choose **Graph > Histogram**.
3. Click **Simple**.
4. Click **OK**.
5. In **Graph Variables**, enter data column **Service Time**.
6. Click **OK**.

5. Solve for μ:

$$\mu = 7.5 + 2.05(0.4) = 8.32 \text{ hours}$$

Longlife Battery will need to increase the mean life of the battery pack to 8.32 hours to meet the automakers' requirement that no more than 2% of the batteries fail in 7.5 hours or less.

Excel and Minitab Tutorial

CHAPTER OUTCOME #3

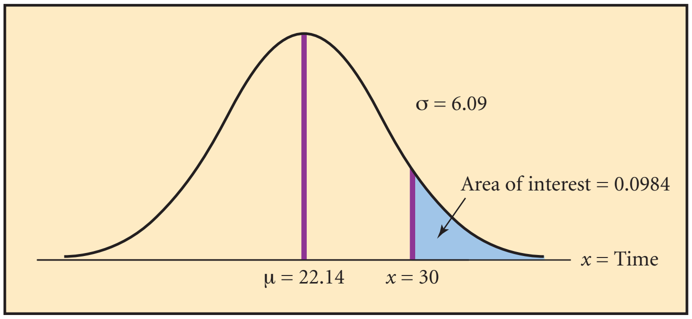
STATE BANK AND TRUST The director of operations for the State Bank and Trust recently performed a study of the time bank customers spent from the time they walk into the bank until they complete their banking. The data file, **State Bank**, contains the data for a sample of 1,045 customers randomly observed over a four-week period. The customers in the survey were limited to those who were there for "basic bank business," such as making a deposit or a withdrawal, or cashing a check. The histogram in Figure 6.8 shows that the times appear to be distributed quite closely to a normal distribution.[2]

The mean service for the 1,045 customers was 22.14 minutes, with a standard deviation equal to 6.09 minutes. On the basis of these data, the manager assumes that the service times are normally distributed with $\mu = 22.14$ and $\sigma = 6.09$. Given these assumptions, the

[2] A statistical technique known as the chi-square goodness-of-fit test, introduced in Chapter 13, can be used to determine statistically whether the data follow a normal distribution.

FIGURE 6.9

Normal Distribution for the State Bank and Trust Example



manager is considering providing a gift certificate to a local restaurant to any customer who is required to spend more than 30 minutes in the service process for basic bank business. Before doing this, she is interested in the probability of having to pay off on this offer.

Figure 6.9 shows the theoretical distribution, with the area of interest identified. The manager is interested in finding

$$P(x > 30 \text{ minutes})$$

This can be done manually or with Excel or Minitab. Figure 6.10A and Figure 6.10B show the output. The cumulative probability is

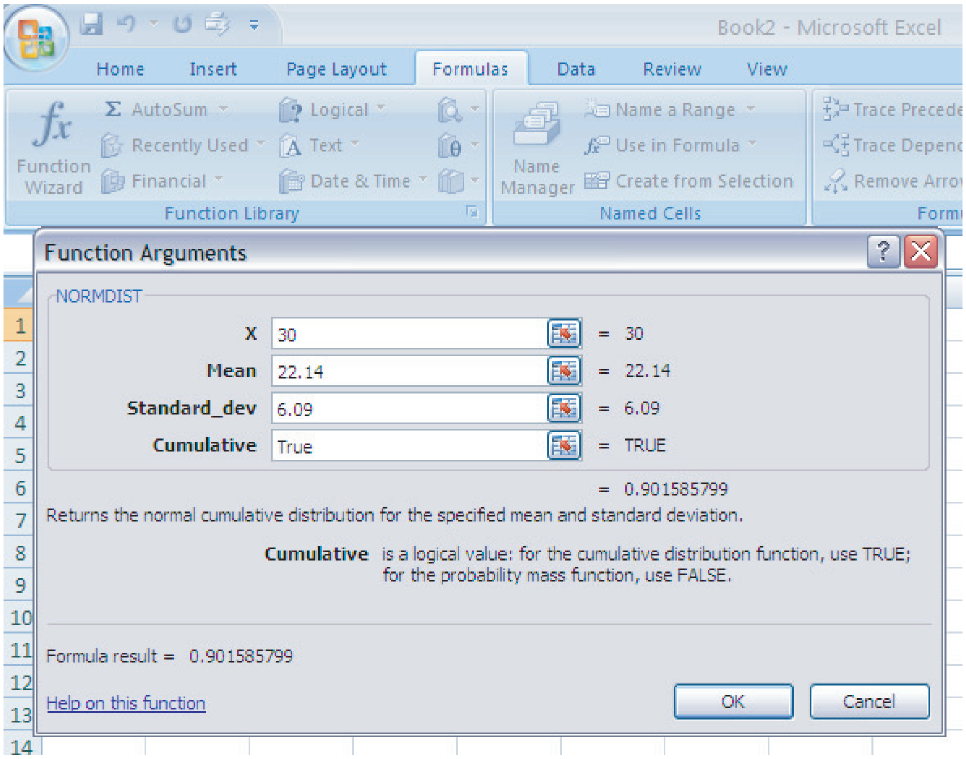
$$P(x \leq 30) = 0.9016$$

Then to find the probability of interest, we subtract this value from 1.0, giving

$$P(x > 30 \text{ minutes}) = 1.0 - 0.9016 = 0.0984$$

FIGURE 6.10A

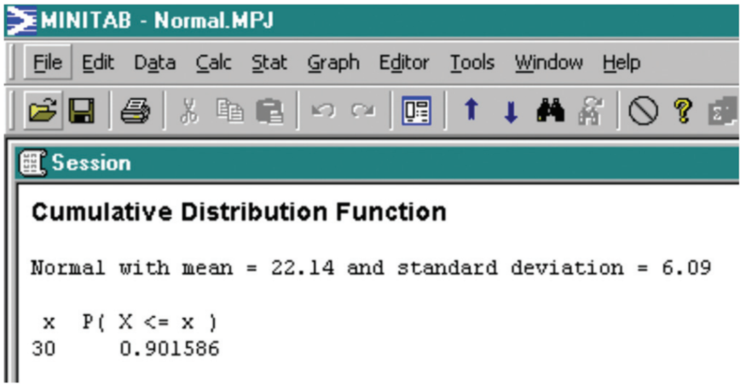
Excel 2007 Output for State Bank and Trust



Excel 2007 Instructions:
1. Open a blank worksheet.
2. Select **Formulas**.
3. Click on f_x **(Function Wizard)**.
4. Select the **Statistical** category.
5. Select the **NORMDIST** function.
6. Fill in the requested information in the template.
7. **True** indicates cumulative probabilities.
8. Click **OK**.

FIGURE 6.10B

Minitab Output for State Bank and Trust



Minitab Instructions:

1. Choose **Calc > Probability Distribution > Normal**.
2. Choose **Cumulative probability**.
3. In **Mean**, enter μ.
4. In **Standard deviation**, enter σ.
5. In **Input constant**, *x*.
6. Click **OK**.

Thus, there are just under 10 chances in 100 that the bank would have to give out a gift certificate. Suppose the manager believes this policy is too liberal. She wants to set the time limit so that the chance of giving out the gift is only 5%. You can use the standard normal table, the **Probability Distribution** command in Minitab, or the **NORMSINV** function in Excel to find the new limit.[3] To use the table, we first consider that the manager wants a 5% area in the upper tail of the normal distribution. This will leave

$$0.50 - 0.05 = 0.45$$

between the new time limit and the mean. Now go to the body of the standard normal table, where the probabilities are, and locate the value as close to 0.45 as possible (0.4495 or 0.4505). Next, determine the z-value that corresponds to this probability. Because 0.45 lies midway between 0.4495 and 0.4505, we interpolate halfway between $z = 1.64$ and $z = 1.65$ to get

$$z = 1.645$$

Now, we know

$$z = \frac{x - \mu}{\sigma}$$

We then substitute the known values and solve for x:

$$1.645 = \frac{x - 22.14}{6.09}$$
$$x = 22.14 + 1.645(6.09)$$
$$x = 32.158 \text{ minutes}$$

Therefore, any customer required to wait more than 32.158 (or 32) minutes will receive the gift. This should result in about 5% of the customers getting the restaurant certificate. Obviously, the bank will work to reduce the average service time or standard deviation so even fewer customers will have to be in the bank for more than 32 minutes.

[3] The function is =NORMSINV(.95) in Excel. This will return the z-value corresponding to the area to left of the upper tail equaling .05.

EXAMPLE 6-2 Using the Normal Distribution

TRY PROBLEM 6.15

Premier Technologies Premier Technologies has a contract to assemble components for radar systems to be used by the U.S. military. The time required to complete one part of the assembly is thought to be normally distributed, with a mean equal to 30 hours and a standard deviation equal to 4.7 hours. In order to keep the assembly flow moving on schedule, this assembly step needs to be completed in 26 to 35 hours. To determine the probability of this happening, use the following steps:

Step 1 Determine the mean, μ, and the standard deviation, σ.

The mean assembly time for this step in the process is thought to be 30 hours, and the standard deviation is thought to be 4.7 hours.

Step 2 Define the event of interest.

We are interested in determining the following:

$$P(26 \le x \le 35) = ?$$

Step 3 Convert values of the specified normal distribution to corresponding values of the standard normal distribution using Equation 6.2:

$$z = \frac{x - \mu}{\sigma}$$

We need to find the z-value corresponding to $x = 26$ and to $x = 35$.

$$z = \frac{x - \mu}{\sigma} = \frac{26 - 30}{4.7} = -0.85 \quad \text{and} \quad z = \frac{35 - 30}{4.7} = 1.06$$

Step 4 Use the standard normal table to find the probabilities associated with each z-value.

For $z = -0.85$, the probability is 0.3023.
For $z = 1.06$, the probability is 0.3554.

Step 5 Determine the desired probability for the event of interest.

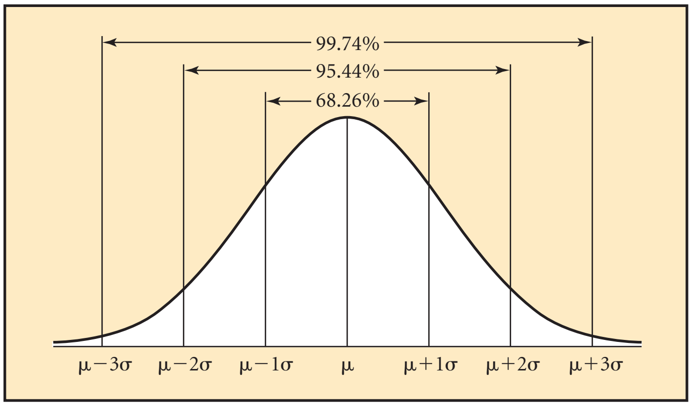
$$P(26 \le x \le 35) = 0.3023 + 0.3554 = 0.6577$$

Thus, there is a 0.6577 chance that this step in the assembly process will stay on schedule.

Approximate Areas Under the Normal Curve In Chapter 3 we introduced the Empirical Rule for probabilities with bell-shaped distributions. For the normal distribution we can make this rule more precise. Knowing the area under the normal curve between $\pm 1\sigma$, $\pm 2\sigma$, and $\pm 3\sigma$ provides a useful benchmark for estimating probabilities and checking reasonableness of results. Figure 6.11 shows these benchmark areas for any normal distribution.

FIGURE 6.11

Approximate Areas Under the Normal Curve



6-1: Exercises

Skill Development

6-1. A population is normally distributed with $\mu = 100$ and $\sigma = 20$.

a. Find the probability that a value randomly selected from this population will have a value greater than 130.

b. Find the probability that a value randomly selected from this population will have a value less than 90.

c. Find the probability that a value randomly selected from this population will have a value between 90 and 130.

6-2. For a standardized normal distribution, calculate the following probabilities:

a. $P(0.00 < z \leq 2.33)$

b. $P(-1.00 < z \leq 1.00)$

c. $P(1.78 < z < 2.34)$

6-3. For a normally distributed population with $\mu = 200$ and $\sigma = 20$, determine the standardized z-value for each of the following:

a. $x = 225$

b. $x = 190$

c. $x = 240$

6-4. For a standardized normal distribution, calculate the following probabilities:

a. $P(z < 1.5)$

b. $P(z \geq 0.85)$

c. $P(-1.28 < z < 1.75)$

6-5. A random variable is known to be normally distributed with the following parameters:

$$\mu = 5.5 \quad \text{and} \quad \sigma = .50$$

a. Determine the value of x such that the probability of a value from this distribution exceeding x is at most 0.10.

b. Referring to your answer in part a, what must the population mean be changed to if the probability of exceeding the value of x found in part a is reduced from 0.10 to 0.05?

6-6. For a standardized normal distribution, determine a value, say z_0, so that

a. $P(0 < z < z_0) = 0.4772$

b. $P(-z_0 \leq z < 0) = 0.45$

c. $P(-z_0 \leq z \leq z_0) = 0.95$

d. $P(z > z_0) = 0.025$

e. $P(z \leq z_0) = 0.01$

6-7. Assume that a random variable is normally distributed with a mean of 1,500 and a variance of 324.

a. What is the probability that a randomly selected value will be greater than 1,550?

b. What is the probability that a randomly selected value will be less than 1,485?

c. What is the probability that a randomly selected value will be either less than 1,475 or greater than 1,535?

6-8. A randomly selected value from a normal distribution is found to be 2.1 standard deviations above its mean.

a. What is the probability that a randomly selected value from the distribution will be greater than 2.1 σ_s above the mean?

b. What is the probability that a randomly selected value from the distribution will be less than 2.1 σ from the mean?

6-9. A random variable is normally distributed with a mean of 60 and a standard deviation of 9.

a. What is the probability that a randomly selected value from the distribution will be less than 46.5?

b. What is the probability that a randomly selected value from the distribution will be greater than 78?

c. What is the probability that a randomly selected value will be between 51 and 73.5?

6-10. A random variable is normally distributed with a mean of 25 and a standard deviation of 5. If an observation is randomly selected from the distribution:

a. What value will be exceeded 10% of the time?
b. What value will be exceeded 85% of the time?
c. Determine two values of which the smallest has 25% of the values below it and the largest has 25% of the values above it.
d. What value will 15% of the observations be below?

6-11. Consider a random variable, z, that has a standardized normal distribution. Determine the following probabilities:

a. $P(0 < z < 1.96)$
b. $P(z > 1.645)$
c. $P(1.28 < z \leq 2.33)$
d. $P(-2 \leq z \leq 3)$
e. $P(z > -1)$

6-12. For the following normal distributions with parameters as specified, calculate the required probabilities:

a. $\mu = 5, \sigma = 2$; calculate $P(0 < x < 8)$.
b. $\mu = 5, \sigma = 4$; calculate $P(0 < x < 8)$.
c. $\mu = 3, \sigma = 2$; calculate $P(0 < x < 8)$.
d. $\mu = 4, \sigma = 3$; calculate $P(x > 1)$.
e. $\mu = 0, \sigma = 3$; calculate $P(x > 1)$.

6-13. A random variable, x, has a normal distribution with $\mu = 13.6$ and $\sigma = 2.90$. Determine a value, x_0, so that

a. $P(x > x_0) = 0.05$.
b. $P(x \leq x_0) = 0.975$.
c. $P(\mu - x_0 \leq x \leq \mu + x_0) = 0.95$.

Business Applications

6-14. The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal.

a. Compute the probability in any year that more than 5,000 acres will be burned.
b. Determine the probability in any year that fewer then 4,000 acres will be burned.
c. What is the probability that between 2,500 and 4,200 acres will be burned?
d. In those years when more than 5,500 acres are burned, help is needed from eastern-region fire teams. Determine the probability help will be needed in any year.

6-15. In the National Weekly Edition of the *Washington Post* (December 19–25, 2005), firstStreet, Inc. advertised an atomic digital watch from LaCrosse Technology. It is radio-controlled and maintains its accuracy by reading a radio signal from a WWVB radio signal from Colorado. It neither loses nor gains a second in 20 million years. It is powered by a 3-volt lithium battery expected to last three years. Suppose the life of the battery has a standard deviation of 0.3 years and is normally distributed.

a. Determine the probability that the watch's battery will last longer than $3\frac{1}{2}$ years.
b. Calculate the probability that the watch's battery will last more than 2.75 years.
c. Compute the length-of-life value for which 10% of the watch's batteries last longer.

6-16. A global financial institution transfers a large data file every evening from offices around the world to its New York City headquarters. Once the file is received, it must be cleaned and partitioned before being stored in the company's data warehouse. Each file is the same size and the time required to transfer, clean, and partition a file is normally distributed, with a mean of 1.5 hours and a standard deviation of 15 minutes.

a. If one file is selected at random, what is the probability that it will take longer than 1 hour and 55 minutes to transfer, clean, and partition the file?
b. If a manager must be present until 85% of the files are transferred, cleaned, and partitioned, how long will the manager need to be there?
c. What percentage of the data files will take between 63 minutes and 110 minutes to be transferred, cleaned, and partitioned?

6-17. Bowser Bites Industries (BBI) sells large bags of dog food to warehouse clubs. BBI uses an automatic filling process to fill the bags. Weights of the filled bags are approximately normally distributed with a mean of 50 kilograms and a standard deviation of 1.25 kilograms.

a. What is the probability that a filled bag will weigh less than 49.5 kilograms?
b. What is the probability that a randomly sampled filled bag will weigh between 48.5 and 51 kilograms?
c. What is the minimum weight a bag of dog food could be and remain in the top 15% of all bags filled?
d. BBI is unable to adjust the mean of the filling process. However, it is able to adjust the standard deviation of the filling process. What would the standard deviation need to be so that no more than 2% of all filled bags weigh more than 52 kilograms?

6-18. T & S Industries manufactures a wash-down motor that is used in the food processing industry. The motor is marketed with a warranty that guarantees it will be replaced free of charge if it fails within the first 13,000 hours of operation. On the average, T & S wash-down motors operate for 15,000 hours with a standard deviation of 1,250 hours before failing. The number of operating hours before failure is approximately normally distributed.

a. What is the probability that a wash-down motor will have to be replaced free of charge?
b. What percentage of T & S wash-down motors can be expected to operate for more than 17,500 hours?
c. If T & S wants to design a wash-down motor so that no more than 1% are replaced free of charge, what would the average hours of operation before failure have to be if the standard deviation remains at 1,250 hours?

6-19. An Internet retailer stocks a popular electronic toy at a central warehouse that supplies the eastern United States. Every week the retailer makes a decision about how many units of the toy to stock. Suppose that weekly demand for the toy is approximately normally distributed with a mean of 2,500 units and a standard deviation of 300 units.

a. If the retailer wants to limit the probability of being out of stock of the electronic toy to no more than 2.5% in a week, how many units should the central warehouse stock?
b. If the retailer has 2,750 units on hand at the start of the week, what is the probability that weekly demand will be greater than inventory?
c. If the standard deviation of weekly demand for the toy increases from 300 units to 500 units, how many more toys would have to be stocked to ensure that the probability of weekly demand exceeding inventory is no more than 2.5%?

6-20. A private equity firm is evaluating two alternative investments. Although the returns are random, each investment's return can be described using a normal distribution. The first investment has a mean return of $2,000,000 with a standard deviation of $125,000. The second investment has a mean return of $2,275,000 with a standard deviation of $500,000.

a. How likely is it that the first investment will return $1,900,000 or less?
b. How likely is it that the second investment will return $1,900,000 or less?
c. If the firm would like to limit the probability of a return being less than $1,750,000, which investment should it make?

6-21. A-1 Plumbing and Repair provides customers with firm quotes for a plumbing repair job before actually starting the job. In order to be able to do this, A-1 has been very careful to maintain time records over the years. For example, it has determined that the time it takes to remove a broken sink disposal and install a new unit is normally distributed with a mean equal to 47 minutes and a standard deviation equal to 12 minutes. The company bills at $75.00 for the first 30 minutes and $2.00 per minute for anything beyond 30 minutes.

Suppose the going rate for this procedure by other plumbing shops in the area is $85.00, not including the cost of the new equipment. If A-1 bids the disposal job at $85, on what percentage of such jobs will the actual time required exceed the time for which it will be getting paid?

6-22. J.J. Kettering & Associates is a financial planning group in Fresno, California. The company specializes in doing financial planning for schoolteachers in the Fresno area. As such, it administers a 403(b) tax shelter annuity program in which public schoolteachers can participate. The teachers can contribute up to $20,000 per year on a pretax basis to the 403(b) account. Very few teachers have incomes sufficient to allow them to make the maximum contribution. The lead analyst at J.J. Kettering & Associates has recently analyzed the company's 403(b) clients and determined that the annual contribution is approximately normally distributed with a mean equal to $6,400. Further, he has determined that the probability a customer will contribute over $13,000 is 0.025. Based on this information, what is the standard deviation of contributions to the 403(b) program?

6-23. A senior loan officer for Wells Fargo Bank has recently studied the bank's real estate loan portfolio and found that the distribution of loan balances is approximately normally distributed with a mean of $155,600 and a standard deviation equal to $33,050. As part of an internal audit, bank auditors recently randomly selected 100 real estate loans from the portfolio of all loans and found that 80 of these loans had balances below $170,000. The senior loan officer is concerned that the sample selected by the auditors is not representative of the overall portfolio. In particular, he is interested in knowing the expected proportion of loans in the portfolio that would have balances below $170,000. You are asked to conduct an appropriate analysis and write a short report to the senior loan officers with your conclusion about the sample.

6-24. MP-3 players, and most notably the Apple iPod, have become an industry standard for people who want to have access to their favorite music and videos in a portable environment. The iPod can store massive numbers of songs and videos with its 60-GB hard drive. Although owners of the iPod have the potential to store lots of data, a recent study showed that the actual disk storage being used is normally distributed with a mean equal to 1.95 GB and a standard deviation of 0.48 GB. Suppose a competitor to Apple is thinking of entering the market with a low-cost iPod clone that has only 1.0 GB of storage. The marketing slogan will be "Why Pay for Storage Capacity that You Don't Need?"

Based on the data from the study of iPod owners, what percentage of owners, based on their current usage, would have enough capacity with the new 1-GB player?

6-25. According to *Business Week* (Pallavi Gogoi, "Pregnant with Possibility," December 26, 2005), Maternity Chic, a purveyor of designer maternity wear, sells dresses and pants priced around $150 each for an average total sale of $1,200. The total sale has a normal distribution with a standard deviation of $350.

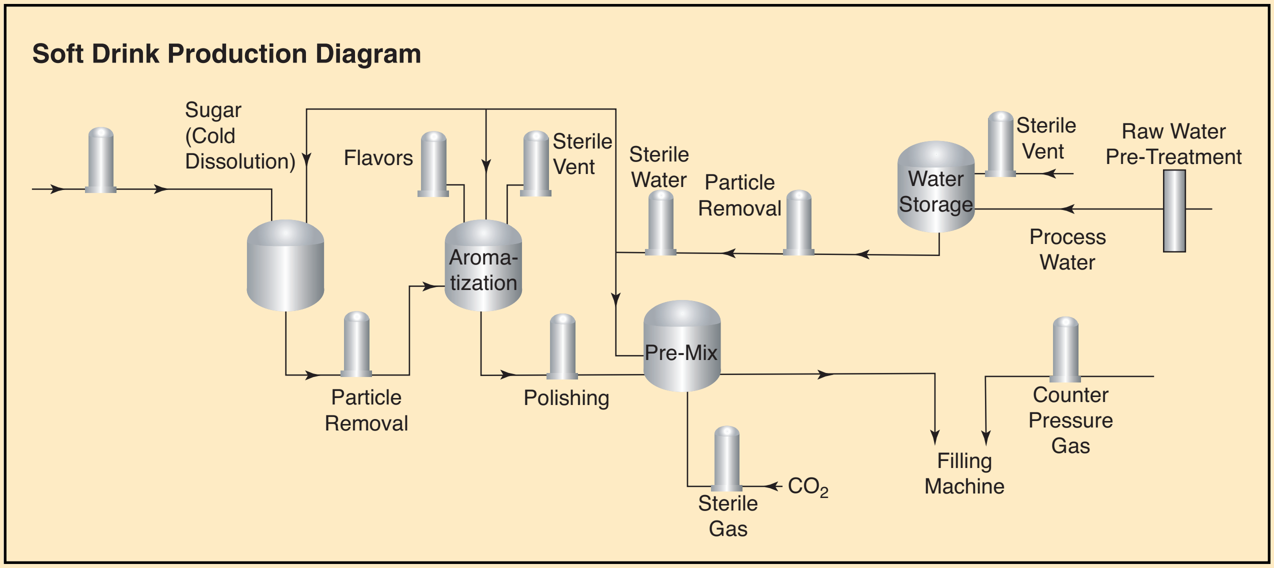
a. Calculate the probability that a randomly selected customer will have a total sale of more than $1,500.
b. Compute the probability that the total sale will be within 2 standard deviations of the mean total sales.
c. Determine the median total sale.

6-26. *USA Today* reported (Sandra Block, "Costs rising, so tame credit card bills now," October 18, 2005) that the average credit card debt per U.S. household was $9,312 in 2004. Assume that the distribution of credit card debt per household has a normal distribution with a standard deviation of $3,000.

a. Determine the percentage of households that have a credit card debt of more than $15,000.
b. One household has a credit card debt that is at the 95th percentile. Determine its credit card debt.
c. If four households were selected at random, determine the probability that at least half of them would have credit card debt of more than $15,000.

6-27. The Aberdeen Coca-Cola Bottling located in Aberdeen, North Carolina, is the bottler and distributor for Coca-Cola products in the Aberdeen area. The company's product line includes 12-ounce cans of Coke products. The cans are filled by an automated filling process that can be adjusted to any mean fill volume and will fill cans according to a normal distribution. However, not all cans will contain the same volume due to variation in the filling process. Historical records show that regardless of what the mean is set at, the standard deviation in fill will be 0.035 ounces. Operations managers at the plant know that if they put too much Coke in a can, the company loses money. If too little is put in the can, customers are short-changed and the North Carolina Department of Weights and Measures may fine the company. The following diagram shows how soft drinks are made:

a. Suppose the industry standards for fill volume call for each 12-ounce can to contain between 11.98 and 12.02 ounces. Assuming that the Aberdeen manager sets the mean fill at 12 ounces, what is the probability that a can will contain a volume of Coke product that falls in the desired range?
b. Assume that the Aberdeen manager is focused on an upcoming audit by the North Carolina Department of Weights and Measures. She knows that their process is to select one Coke can at random and if it contains less than 11.97 ounces, the company will be reprimanded and potentially fined. Assuming that the manager wants at most a 5% chance of this happening, at



Source: Pall Corporation, January 2006.

what level should she set the mean fill level? Comment on the ramifications of this step, assuming that company fills tens of thousands of cans each week.

6-28. Georgia-Pacific is a major forest products company in the United States. In addition to timberlands, the company owns and operates numerous manufacturing plants that make lumber and paper products. At one of their plywood plants, the operations manager has been struggling to make sure that the plywood thickness meets quality standards. Specifically, all sheets of their $\frac{3}{4}$-inch plywood must fall within the range 0.747 to 0.753 inches in thickness. Studies have shown that the current process produces plywood that has thicknesses that are normally distributed with a mean of 0.751 inches and a standard deviation equal to 0.004 inches.

a. Use either Excel or Minitab to determine the proportion of plywood sheets that will meet quality specifications (0.747 to 0.753), given how the current process is performing.
b. Referring to part a, suppose the manager is unhappy with the proportion of product that is meeting specifications. Assuming that he can get the mean adjusted to 0.75 inches, what must the standard deviation be if he is going to have 98% of his product meet specifications?

Computer Database Exercises

6-29. An article in *USA Today* (Julie Appleby, "AARP: Drug prices zoom past inflation," November 2, 2005) indicated that the wholesale prices for the 200 brand-name drugs most commonly used by Americans over age 50 rose an average of 6.1% in the 12 months ended June, 2005. The report listed the 10 drugs with the largest increases in the first 6 months of 2005. Atrovent, a treatment for lung conditions such as emphysema, had the largest increase. The price per day, on average, rose from \$2.12 to \$2.51. The file entitled **Drug$** contains data similar to that obtained in AARP's research.

a. Produce a relative frequency histogram for these data. Does it seem plausible the data were sampled from a population that was normally distributed?
b. Compute the mean and standard deviation for the sample data in the file **Drug$**.
c. Assuming the sample came from a normally distributed population and the sample standard deviation is a good approximation for the population standard deviation, determine the probability that a randomly chosen transaction would yield a price of \$2.12 or smaller even though the mean population was still \$2.51.

6-30. *USA Today*'s annual survey of public flagship universities (Arienne Thompson and Breanne Gilpatrick, "Double-digit hikes are down," October 5, 2005) indicates that the median increase in in-state tuition was 7% for the 2005–2006 academic year. A file entitled **Tuition** contains the percentage change for the 67 flagship universities.

a. Produce a relative frequency histogram for these data. Does it seem plausible that the data is from a population which has a normal distribution?
b. Suppose the decimal point of the three largest numbers had inadvertently been moved one place to the right in the data. Move the decimal point one place to the left and reconstruct the relative frequency histogram. Now does it seem plausible that the data have an approximate normal distribution?
c. Use the normal distribution of part b to approximate the proportion of universities that raised their in-state tuition more than 10%. Use the appropriate parameters obtained from this population.
d. Use the normal distribution of part b to approximate the fifth percentile for the percent of tuition increase.

6-31. The PricewaterhouseCoopers Saratoga release, 2005/2006 Human Capital Index Report, indicated that the average cost for an American company to fill a job vacancy during that time period was \$3,270. Sample data similar to that used in the study is in a file entitled **Hired**.

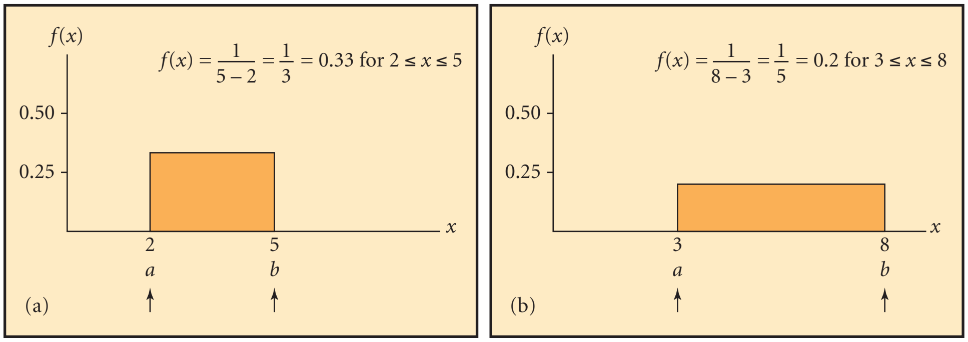
a. Produce a relative frequency histogram for these data. Does it seem plausible the data were sampled from a normally distributed population?
b. Calculate the mean and standard deviation of the cost of filling a job vacancy.
c. Determine the probability that the cost of filling a job vacancy would be between \$2,000 and \$3,000.
d. Given that the cost of filling a job vacancy was between \$2,000 and \$3,000, determine the probability that it would be more than \$2,500.

6.2 Other Continuous Probability Distributions

The normal distribution is the most frequently used continuous probability distribution in statistics. However, there are other continuous distributions that apply to business decision making. This section introduces two of these: the uniform distribution and the exponential distribution.

FIGURE 6.12

Uniform Distributions



CHAPTER OUTCOME #4

Uniform Probability Distribution

The *uniform distribution* is sometimes referred to as the *distribution of little information*, because the probability over any interval of the continuous random variable is the same as for any other interval of the same width.

Equation 6.3 defines the *continuous uniform density function*.

Continuous Uniform Density Function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(6.3)}$$

where:

$f(x)$ = Value of the density function at any x-value
a = Lower limit of the interval from a to b
b = Upper limit of the interval from a to b

Figure 6.12 illustrates two examples of uniform probability distributions with different a to b intervals. Note the height of the probability density function is the same for all values of x between a and b for a given distribution. The graph of the uniform distribution is a rectangle.

EXAMPLE 6-3 Using the Uniform Distribution

TRY PROBLEM 6.32

Stern Manufacturing Company The Stern Manufacturing Company makes seat-belt buckles for all types of vehicles. The inventory level for the spring mechanism used in producing the buckles is only enough to continue production for two more hours. The purchasing clerk estimates that the springs will be delivered one to four hours from the time they are ordered. Because the dispatcher offers no other information about the pending delivery schedule, the time it will take to replenish the inventory is said to be *uniformly distributed* over the interval of one to four hours. We are interested in the probability that the company will run out of parts due to the shipment taking more than two hours. The probability can be determined using the following steps:

Step 1 **Define the density function.**

The height of the probability rectangle, $f(x)$, for the delivery-time interval of one to four hours is determined using Equation 6.3, as follows:

$$f(x) = \frac{1}{b-a}$$

$$f(x) = \frac{1}{4-1} = \frac{1}{3} = 0.33$$

Step 2 **Define the event of interest.**
The production scheduler is specifically concerned that shipment will take longer than two hours to arrive. This event of interest is $x > 2.0$.

Step 3 **Calculate the required probability.**
We determine the probability as follows:

$$\begin{aligned}P(x > 2.0) &= 1 - P(x \leq 2.0)\\ &= 1 - f(x)(2.0 - 1.0)\\ &= 1 - 0.33(1.0)\\ &= 1 - 0.33\\ &= 0.67\end{aligned}$$

Thus, there is a 67% chance that production will be delayed because the shipment is more than two hours late.

Like the normal distribution, the uniform distribution can be further described by specifying the mean and the standard deviation. These values are computed using Equations 6.4 and 6.5.

Mean and Standard Deviation of a Uniform Distribution

Mean (Expected Value):

$$E(x) = \mu = \frac{a+b}{2} \quad \textbf{(6.4)}$$

Standard Deviation:

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} \quad \textbf{(6.5)}$$

where:

a = Lower limit of the interval from a to b
b = Upper limit of the interval from a to b

EXAMPLE 6-4 The Mean and Standard Deviation of a Uniform Distribution

TRY PROBLEM 6.33

Austrian Airlines The service manager for Austrian Airlines is uncertain about the time needed for the ground crew to turn an airplane around from the time it lands until it is ready to take off. He has been given information from the operations supervisor indicating that the times seem to range between 15 and 45 minutes. Without any further information, the service manager will apply a uniform distribution to the turnaround.

Based on this, he can determine the mean and standard deviation for the airplane turn-around times using the following steps:

Step 1 **Define the density function.**
Equation 6.3 can be used to define the distribution:

$$f(x) = \frac{1}{b-a} = \frac{1}{45-15} = \frac{1}{30} = 0.0333$$

Step 2 **Compute the mean of the probability distribution using Equation 6.4.**

$$\mu = \frac{a+b}{2} = \frac{15+45}{2} = 30$$

Thus, the mean turnaround time is 30 minutes.

Step 3 **Compute the standard deviation using Equation 6.5.**

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(45-15)^2}{12}} = \sqrt{75} = 8.66$$

The standard deviation is 8.66 minutes.

CHAPTER OUTCOME #5

The Exponential Probability Distribution

Another continuous probability distribution that is frequently used in business situations is the *exponential distribution*. The exponential distribution is used to measure the time that elapses between two occurrences of an event, such as the time between "hits" on an Internet home page. The exponential distribution might also be used to describe the time between arrivals of customers at a bank drive-in teller window or the time between failures of an electronic component. Equation 6.6 shows the probability density function for the exponential distribution.

Exponential Density Function

A continuous random variable that is exponentially distributed has the probability density function given by

$$f(x) = \lambda e^{-\lambda x}, \qquad x \geq 0 \qquad \textbf{(6.6)}$$

where:

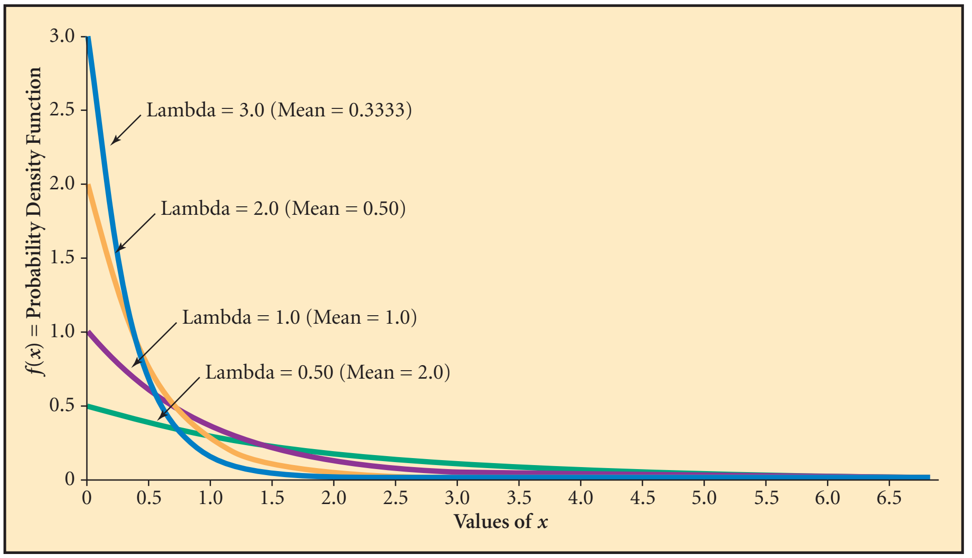
$e = 2.71828\ldots$
$1/\lambda =$ the mean time between events ($\lambda > 0$)

Note, the parameter that defines the exponential distribution is λ (lambda). You should recall from Chapter 5 that λ is the mean value for the Poisson distribution. If the number of occurrences per time period is known to be Poisson distributed with a mean of λ, then the time between occurrences will be exponentially distributed with a mean time of $1/\lambda$.

If we select a value for λ, we can graph the exponential distribution by substituting λ and different values for x into Equation 6.6. For instance, Figure 6.13 shows exponential density functions for $\lambda = 0.5$, $\lambda = 1.0$, $\lambda = 2.0$, and $\lambda = 3.0$. Note in Figure 6.13 that for any exponential density function, $f(x)$, $f(0) = \lambda$, as x increases, $f(x)$ approaches zero. It can

FIGURE 6.13

Exponential Distributions



also be shown that *the standard deviation of any exponential distribution is equal to the mean, 1/λ.*

As with any continuous probability distribution, the probability that a value will fall within an interval is the area under the graph between the two points defining the interval. Equation 6.7 is used to find the probability that a value will be equal to or less than a particular value for an exponential distribution.

Exponential Probability

$$P(0 \le x \le a) = 1 - e^{-\lambda a} \quad \textbf{(6.7)}$$

where:

a = the value of interest
$1/\lambda$ = mean
e = natural number = 2.71828

Appendix E contains a table of $e^{-\lambda a}$ values for different values of λa. You can use this table and Equation 6.7 to find the probabilities when the λa of interest is contained in the table. You can also use Minitab or Excel to find exponential probabilities, as the following application illustrates.

Business Application

Excel and Minitab Tutorial

HAINES INTERNET SERVICES The Haines Internet Services Company has determined that the number of customers who attempt to connect to the Internet per hour is Poisson distributed with $\lambda = 30$ per hour. The time between connect requests is exponentially distributed with a mean time between calls of 2.0 minutes, computed as follows:

$$\lambda = 30 \text{ per } 60 \text{ minutes} = 0.50 \text{ per minute}$$

The mean time between calls, then, is

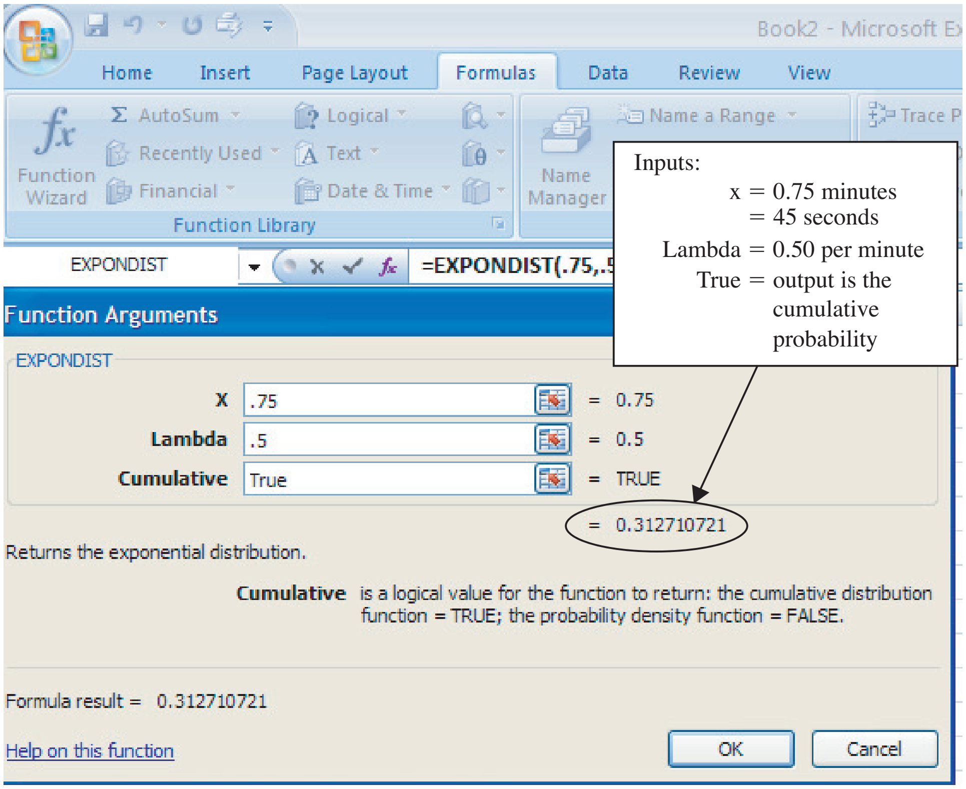
$$1/\lambda = \frac{1}{0.50} = 2.0 \text{ minutes}$$

Because of the system that Haines uses, if customer requests are too close together—45 seconds (0.75 minutes) or less—some customers fail to connect. The managers at Haines are analyzing whether they should purchase new equipment that will eliminate this problem. They need to know the probability that a customer will fail to connect. Thus, they want

$$P(x \le 0.75 \text{ minutes}) = ?$$

FIGURE 6.14A

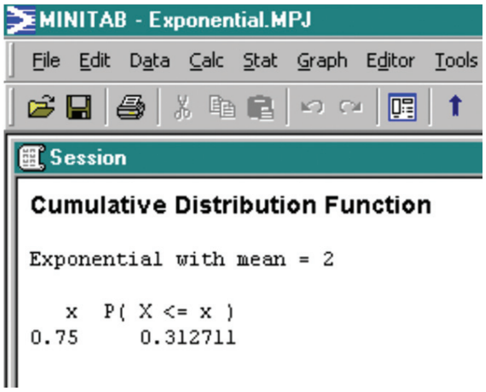
Excel 2007 Exponential Probability Output for Haines Internet Services



Excel 2007 Instructions:
1. On the **Data** tab, click on **Function Wizard**, f_x
2. Select **Statistical** category.
3. Select **EXPONDIST** function.
4. Supply x and lambda.
5. Set **Cumulative = TRUE** for cumulative probability.

FIGURE 6.14B

Minitab Exponential Probability Output for Haines Internet Services



Minitab Instructions:
1. Choose **Calc > Probability Distribution > Exponential**.
2. Choose **Cumulative probability**.
3. In **Scale**, enter μ.
4. In **Input constant**, enter value for x.
5. Click **OK**.

To find this probability using a calculator, we need to first determine λa. In this example, $\lambda = 0.50$ and $a = 0.75$. Then,

$$\lambda a = (0.50)(0.75) = 0.3750$$

We find that the desired probability is

$$\begin{aligned} 1 - e^{-\lambda a} &= 1 - e^{-0.3750} \\ &= 0.3127 \end{aligned}$$

The managers can also use the **EXPONDIST** function in Excel or the **Probability Distribution** command in Minitab to compute the precise value for the desired probability.[4] Figure 6.14A and Figure 6.14B show that the chance of failing to connect is 0.3127. This means that nearly one-third of the customers will experience a problem with the current system.

[4] The Excel EXPONDIST function requires that λ be inputted rather than $1/\lambda$.

6-2: Exercises

Skill Development

6-32. A continuous random variable is uniformly distributed between 100 and 150.

a. What is the probability a randomly selected value will be greater than 135?

b. What is the probability a randomly selected value will be less than 115?

c. What is the probability a randomly selected value will be between 115 and 135?

6-33. Suppose a random variable, x, has a uniform distribution with $a = 5$ and $b = 9$.

a. Calculate $P(5.5 \le x \le 8)$.

b. Determine $P(x > 7)$.

c. Compute the mean, μ, and standard deviation, σ, of this random variable.

d. Determine the probability that x is in the interval $(\mu \pm 2\sigma)$.

6-34. Let x be an exponential random variable with $\lambda = 0.5$. Calculate the following probabilities:

a. $P(x < 5)$

b. $P(x > 6)$

c. $P(5 \le x \le 6)$

d. $P(x \ge 2)$

e. the probability that x is at most 6

6-35. The time between telephone calls to a cable television payment processing center follows an exponential distribution with a mean of 1.5 minutes.

a. What is the probability that the time between the next two calls will be 45 seconds or less?

b. What is the probability that the time between the next two calls will be greater than 112.5 seconds?

6-36. The useful life of an electrical component is exponentially distributed with a mean of 2,500 hours.

a. What is the probability the circuit will last more than 3,000 hours?

b. What is the probability the circuit will last between 2,500 and 2,750 hours?

c. What is the probability the circuit will fail within the first 2,000 hours?

6-37. Determine the following:

a. the probability that a uniform random variable whose range is between 10 and 30 assumes a value in the interval (10 to 20) or (15 to 25).

b. the quartiles for a uniform random variable whose range is from 4 to 20.

c. the mean time between events for an exponential random variable that has a median equal to 10.

d. the 90th percentile for an exponential random variable that has the mean time between events equal to 0.4.

Business Applications

6-38. When only the value-added time is considered, the time it takes to build a laser printer is thought to be uniformly distributed between 8 and 15 hours.

a. What are the chances that it will take more than 10 value-added hours to build a printer?

b. How likely is it that a printer will require less than 9 value-added hours?

c. Suppose a single customer orders two printers. Determine the probability that the first and second printer each will require less than 9 value-added hours to complete.

6-39. The time to failure for a power supply unit used in a particular brand of personal computer is thought to be exponentially distributed with a mean of 4,000 hours as per the contract between the vendor and the PC maker. The PC manufacturer has just had a warranty return from a customer who had the power supply fail after 2,100 hours of use.

a. What is the probability that the power supply would fail at 2,100 hours or less? Based on this probability, do you feel the PC maker has a right to require that the power supply maker refund the money on this unit?

b. Assuming that the PC maker has sold 100,000 computers with this power supply, approximately how many should be returned due to failure at 2,100 hours or less?

6-40. *USA Today* reported (Dennis Cauchon and Julie Appleby, "Hospitals go where the money is," January 3, 2006) that the average patient cost per stay in American hospitals was $8,166. Assume that this cost is exponentially distributed.

a. Determine the probability that a randomly selected patient's stay in an American hospital will cost more than $10,000.

b. Calculate the probability that a randomly selected patient's stay in an American hospital will cost less than $5,000.

c. Compute the probability that a randomly selected patient's stay in an American hospital will cost between $8,000 and $12,000.

6-41. Suppose you are traveling on business to a foreign country for the first time. You do not have a bus schedule or a watch with you. However, you have been told that buses stop in front of your hotel every 20 minutes throughout the day. If you show up at the bus stop at a random moment during the day, what is the probability that

a. you will have to wait for more than 10 minutes?

b. you will only have to wait for 6 minutes or less?

c. you will have to wait between 8 and 15 minutes?

6-42. A delicatessen located in the heart of the business district of a large city serves a variety of customers. The delicatessen is open 24 hours a day every day of the week. In an effort to speed up take-out orders, the deli accepts orders by fax. If, on the average, 20 orders are received by fax every two hours throughout the day, find the

a. probability that a faxed order will arrive within the next 9 minutes.
b. probability that the time between two faxed orders will be between and 3 and 6 minutes.
c. probability that 12 or more minutes will elapse between faxed orders.

6-43. During the busiest time of the day customers arrive at the Daily Grind Coffee House on the average of 15 customers per 20-minute period.

a. What is the probability that a customer will arrive within the next 3 minutes?
b. What is the probability that the time between the arrivals of customers is 12 minutes or more?
c. What is the probability that the next customer will arrive between 4 and 6 minutes?

6-44. The time required to prepare a dry cappuccino using whole milk at the Daily Grind Coffee House is uniformly distributed between 25 and 35 seconds. Assuming a customer has just ordered a whole-milk dry cappuccino,

a. What is the probability that the preparation time will be more than 29 seconds?
b. What is the probability that the preparation time will be between 28 and 33 seconds?
c. What percentage of whole-milk dry cappuccinos will be prepared within 31 seconds?
d. What is the standard deviation of preparation times for a dry cappuccino using whole milk at the Daily Grind Coffee House?

6-45. American Airlines states that the flight between Fort Lauderdale, Florida, and Los Angeles takes 5 hours and 37 minutes. Assume that the actual flight times are uniformly distributed between 5 hours and 20 minutes and 5 hours and 50 minutes.

a. Determine the probability that the flight will be more than 10 minutes late.
b. Calculate the probability that the flight will be more than 5 minutes early.
c. Compute the average flight time between these two cities.
d. Determine the variance in the flight times between these two cities.

6-46. A corrugated container company is testing whether a computer decision model will improve the uptime of its box production line. Currently, knives used in the production process are checked manually and replaced when the operator believes the knives are dull. Knives are expensive, so operators are encouraged not to change the knives early. Unfortunately, if knives are left running for too long, the cuts are not made properly, which can jam the machines and require that the entire process be shut down for unscheduled maintenance. Shutting down the entire line is costly in terms of lost production and repair work, so the company would like to reduce the number of shutdowns that occur daily. Currently, the company experiences an average of 0.75 knife-related shutdowns per shift, exponentially distributed. In testing, the computer decision model reduced the frequency of knife-related shutdowns to an average of 0.20 per shift, exponentially distributed. The decision model is expensive but the company will install it if it can help achieve the target of four consecutive shifts without a knife-related shutdown.

a. Under the current system, what is the probability that the plant would run four or more consecutive shifts without a knife-related shutdown?
b. Using the computer decision model, what is the probability that the plant could run four or more consecutive shifts without a knife-related shutdown? Has the decision model helped the company achieve its goal?
c. What would be the maximum average number of shutdowns allowed per day such that the probability of experiencing four or more consecutive shifts without a knife-related shutdown is greater than or equal to 0.70?

6-47. The average amount spent on electronics each year in U.S. households is $1,250 according to an article in *USA Today* (Michelle Kessler, "Gadget makers make mad dash to market," January 4, 2006). Assume that the amount spent on electronics each year has an exponential distribution.

a. Calculate the probability that a randomly chosen U.S. household would spend more than $5,000 on electronics.
b. Compute the probability that a randomly chosen U.S. household would spend more than the average amount spent by U.S. households.
c. Determine the probability that a randomly chosen U.S. household would spend more than 1 standard deviation below the average amount spent by U.S. households.

Computer Database Exercises

6-48. Rolls-Royce PLC provides forecasts for the business jet market and covers the regional and major aircraft markets. In a recent release, Rolls-Royce indicated that in both North America and Europe the number of delayed departures has declined since a peak in 1999/2000. This is partly due to a reduction in the number of flights at major airports and the younger aircraft fleets, but also results from improvements in Air Traffic Management (ATM) capacity, especially in Europe. Comparing January–April 2003 with the same period in 2001 (for similar traffic levels), the average en route delay per flight was reduced by 65%, from 2.2 minutes to 0.7 minutes. The file entitled **Delays** contains a possible sample of the en route delay times in minutes for 200 flights.

a. Produce a relative frequency histogram for this data. Does it seem plausible the data come from a population that has an exponential distribution?
b. Calculate the mean and standard deviation of the en route delays.
c. Determine the probability that this exponential random variable will be smaller than its mean.
d. Determine the median time in minutes for the en route delays assuming they have an exponential distribution with a mean equal to that obtained in part b.
e. Using only the information obtained in parts c and d, describe the shape of this distribution. Does this agree with the findings in part a?

6-49. The city of San Luis Obispo, California, Transit Program provides daily fixed-route transit service to the general public within the city limits and to Cal Poly State University's staff and students. The most heavily traveled route schedules a city bus to arrive at Cal Poly at 8:54 A.M. The file entitled **Late** lists plausible differences between the actual and scheduled time of arrival rounded to the nearest minute for this route.

a. Produce a relative frequency histogram for this data. Does it seem plausible the data came from a population that has a uniform distribution?
b. Provide the density for this uniform distribution.
c. Classes start 10 minutes after the hour and classes are a 5-minute walk from the drop-off point. Determine the probability that a randomly chosen bus on this route would cause the students on board to be late for class. Assume the differences form a continuous uniform distribution with a range the same as the sample.
d. Determine the median difference between the actual and scheduled arrival times.

6-50. *USA Today* has reported (Kathy Chu, "Banks rake in more in fees but don't share," December 1, 2005) that as of October, 2005 the average fee charged by banks to process a ATM transaction was \$2.91. The file entitled **ATM Fees** contains a list of ATM fees that might be required by banks.

a. Produce a relative frequency histogram for this data. Does it seem plausible the data came from a population that has an exponential distribution?
b. Calculate the mean and standard deviation of the ATM fees.
c. Assume that the distribution of ATM fees is exponentially distributed with the same mean as that of the sample. Determine the probability that a randomly chosen bank's ATM fee would be greater than \$3.00.

Summary and Conclusions

Chapter 6 has extended the discussion of probability distributions by introducing three continuous probability distributions: the normal, uniform, and exponential distributions. Although all three distributions have applications in business decision making, the normal distribution is by far the most important. As you will see in subsequent chapters, the normal distribution is used in many ways in both hypothesis testing and estimation.

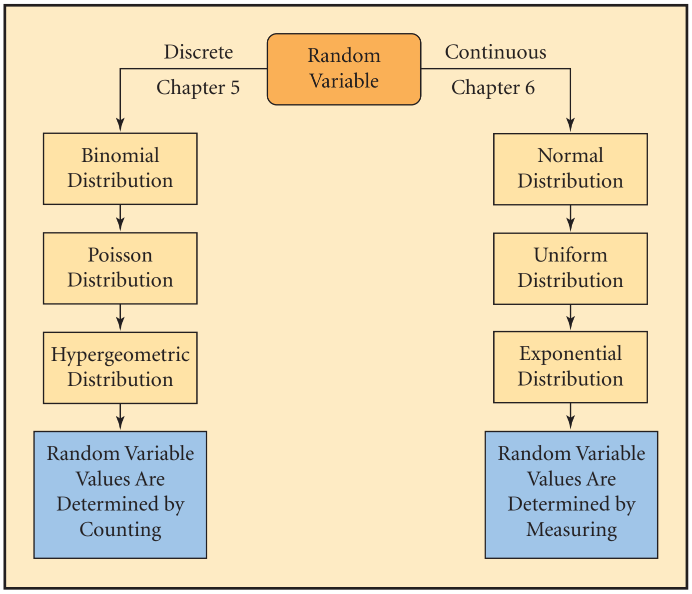
In Chapter 6, we showed that the normal distribution, with its special properties, is used extensively in statistical decision making. You will encounter the normal distribution numerous times as you study the material in subsequent chapters. An important point is that any normal distribution can be transformed to the corresponding standard normal probability distribution. You can then use the standard normal distribution to find probabilities associated with the original normal distribution. We also illustrated how Excel and Minitab can also be used to find probabilities associated with normal and exponential distributions.

The continuous distributions introduced in this chapter (normal, uniform, and exponential) combined with the discrete distributions (binomial, Poisson, and hypergeometric) introduced in Chapter 5 are the distributions that you will very frequently encounter in business decision-making situations. Figure 6.15 summarizes these discrete and continuous probability distributions.

Subsequent chapters will introduce other continuous probability distributions. Among these will be the *t-distribution*, the *chi-square distribution*, and the *F-distribution*. These additional distributions also play important roles in statistical decision making. The basic concept that the area under a continuous curve is equivalent to the probability is true for all continuous distributions.

FIGURE 6.15

Probability Distribution Summary



Equations

Normal Distribution Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad \textbf{(6.1)}$$

Standardized Normal z-Value

$$z = \frac{x-\mu}{\sigma} \quad \textbf{(6.2)}$$

Continuous Uniform Density Function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(6.3)}$$

Mean of the Uniform Distribution

$$E(x) = \mu = \frac{a+b}{2} \quad \textbf{(6.4)}$$

Standard Deviation of the Uniform Distribution

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} \quad \textbf{(6.5)}$$

Exponential Density Function

$$f(x) = \lambda e^{-\lambda x}, \qquad x \ge 0 \quad \textbf{(6.6)}$$

Exponential Probability

$$P(0 \le x \le a) = 1 - e^{-\lambda a} \quad \textbf{(6.7)}$$

Key Terms

Normal distribution 253
Standard normal distribution 254

Chapter Exercises

Conceptual Questions

6-51. The probability that a value from a normally distributed random variable will exceed the mean is 0.50. The same is true for the uniform distribution. Why is this not necessarily true for the exponential distribution? Discuss and show examples to illustrate your point.

6-52. Suppose you tell one of your fellow students that when working with a continuous distribution, it does not make sense to try to compute the probability of any specific value since it will be zero. She says that this can't be true, since when the experiment is performed some value must occur so the probability can't be zero. Your task is to respond to her statement and in doing so explain why it is appropriate to find the probability for specific ranges of values for a continuous distribution.

6-53. Discuss the difference between discrete and continuous probability distributions. Discuss two situations where a variable of interest may be considered either continuous or discrete.

6-54. Recall the Empirical Rule from Chapter 3. It stated that if the data distribution is bell-shaped, then the interval $\mu \pm \sigma$ contains approximately 68% of the values, $\mu \pm 2\sigma$ contains approximately 95%, and $\mu \pm 3\sigma$ contains virtually all of the data values. The "bell-shaped" distribution referenced is the normal distribution.

a. Verify that a standard normal distribution contains approximately 68% of the values in the interval $\mu \pm \sigma$.
b. Verify that a standard normal distribution contains approximately 95% of the values in the interval $\mu \pm 2\sigma$.
c. Verify that a standard normal distribution contains virtually all of the data in the interval $\mu \pm 3\sigma$.

6-55. The exponential distribution has a characteristic that is called the "memoryless" property. This means $P(X > x) = P(X > x + x_0 | X \geq x_0)$. To illustrate this, consider the calls coming into 911. Suppose that the distribution of the time between occurrences has an exponential distribution with a mean of one half hour (= 0.5).

a. Calculate the probability that no calls come in during the first hour.
b. Now suppose that you are monitoring the call frequency and you note that a call does not come in during the first two hours, determine the probability that no calls will come in during the next hour.

6-56. Revisit problem 6.55, but here, let's examine whether it would matter when you started monitoring the 911 calls if the time between occurrences had a uniform distribution with a mean of 2 and a range of 4.

a. Calculate the probability that no call comes in during the first hour.
b. Now suppose that you are monitoring the call frequency and you note that no call comes in during the first two hours. Determine the probability that no calls will arrive during the next hour.

Business Applications

6-57. Two automatic dispensing machines are being considered for use in a fast-food chain. The first dispenses an amount of liquid that has a normal distribution with a mean of 11.9 ounces and a standard deviation of 0.07 ounces. The second dispenses an amount of liquid that has a normal distribution with a mean of 12.0 ounces and a standard deviation of 0.05 ounces. Acceptable amounts of dispensed liquid are between 11.9 and 12.0 ounces. Calculate the relevant probabilities and determine which machine should be selected.

6-58. An online article (http://beauty.about.com) by Julyne Derrick, "Shelf Lives: How Long Can you Keep Makeup," suggests that eye shadow and eyeliner each have a shelf life of up to three years. Suppose the shelf lives of these two products are exponentially distributed with an average shelf life of one year.

a. Calculate the probability that the shelf life of eye shadow will be longer than three years.
b. Determine the probability that at least one of these products will have a shelf life of more than three years.
c. Determine the probability that a purchased eyeliner that is useful after one year will be useful after three years.

6-59. One of the products of Pittsburg Plate Glass Industries (PPG) is laminated safety glass. It is made up of two pieces of glass 0.125 inch thick, with a thin layer of vinyl sandwiched between them. The average thickness of the laminated safety glass is 0.25 inch. The thickness of the glass does not vary from the mean by more than 0.10 inch. Assume the thickness of the glass has a uniform distribution.

a. Provide the density for this uniform distribution.
b. If the glass has a thickness that is more than 0.05 inch below the mean, it must be discarded for safety considerations. Determine the probability that a randomly selected automobile glass is discarded due to safety considerations.
c. If the glass is more than 0.075 above the mean, it will create installation problems and must be discarded. Calculate the probability that a randomly selected automobile glass will be rejected due to installation concerns.
d. Given that a randomly selected automobile glass is not rejected for safety considerations, determine the probability that it will be rejected for installation concerns.

6-60. The Sea Pines Golf Course is preparing for a major LPGA golf tournament. Since parking near the course is extremely limited (room for only 500 cars), the course officials have contracted with the local community to provide parking and a bus shuttle service. Sunday, the final day of the tournament, will have the largest crowd and the officials estimate there will be between 8,000 and 12,000 cars needing parking spaces, but think no value is more likely than another. The tournament committee is discussing how many parking spots to contract from the city. If they want to limit the chance of not having enough provided parking to 10%, how many spaces do they need from the city on Sunday?

6-61. The manager for Select-a-Seat, a company that sells tickets to athletic games, concerts, and other events, has determined that the number of people arriving at the Broadway location on a typical day is Poisson distributed with a mean of 12 per hour. It takes approximately four minutes to process a ticket request. Thus, if customers arrive in intervals that are less than four minutes, they will have to wait. Assuming that a customer has just arrived and the ticket agent is starting to serve that customer, what is the probability that the next customer who arrives will have to wait in line?

6-62. The personnel manager for a large company is interested in the distribution of sick-leave hours for employees of her company. A recent study revealed the distribution to be approximately normal, with a mean of 58 hours per year and a standard deviation of 14 hours.

An office manager in one division has reason to believe that during the past year, two of his employees have taken excessive sick leave relative to everyone else. The first employee used 74 hours of sick leave, and the second used 90 hours. What would you conclude about the office manager's claim and why?

6-63. The Bryce Brothers Lumber Company is considering buying a machine that planes lumber to the correct thickness. The machine is advertised to produce "6-inch lumber" having a thickness that is normally distributed, with a mean of 6 inches and a standard deviation of 0.1 inch.

a. If building standards in the industry require a 99% chance of a board being between 5.85 and 6.15 inches, should Bryce Brothers purchase this machine? Why or why not?
b. To what level would the company that manufactures the machine have to reduce the standard deviation for the machine to conform to industry standards?

6-64. A small private ambulance service in Oklahoma has determined that the time between emergency calls is exponentially distributed with a mean of 41 minutes. When a unit goes on call, it is out of service for 60 minutes. If a unit is busy when an emergency call is received, the call is immediately routed to another service. The company is considering buying a second ambulance. However, before doing so, the owners are interested in determining the probability that a call will come in before the ambulance is back in service. Without knowing the costs involved in this situation, does this probability tend to support the need for a second ambulance? Discuss.

6-65. The St. Maries plywood plant is part of the Potlatch Corporation's Northwest Division. The plywood superintendent organized a study of the tree diameters that are being shipped to the mill. After collecting a large amount of data on diameters, he concluded that the distribution is approximately normally distributed with a mean of 14.25 inches and a standard deviation of 2.92 inches. Because of the way plywood is made, there is a certain amount of waste on each log because the peeling process leaves a core that is approximately 3 inches thick. For this reason, he feels that any log less than 10 inches in diameter is not profitable for making plywood.

a. Based on the data he has collected, what is the probability that a log will be unprofitable?
b. An alternative is to peel the log and then sell the core as "peeler logs." These peeler logs are sold as fence posts and for various landscape projects. There is not as much profit in these peeler logs, however. The superintendent has determined that he can make a profit if the peeler log's diameter is not more than 32% of the diameter of the log. Using this additional information, calculate the proportion of logs that will be unprofitable.

Computer Database Exercises

6-66. Continental Fan Manufacturing Inc. was established in 1986 as a manufacturer and distributor of quality ventilation equipment. Continental Fan's products include the AXC range hood exhaust fans. The file entitled **Fan Life** contains the length of life of 125 randomly chosen AXC fans that were used in an accelerated life-testing experiment.

a. Produce a relative frequency histogram for the data. Does it seem plausible the data came from a population that has an exponential distribution?
b. Calculate the mean and standard deviation of the fans' length of life.
c. Calculate the median length of life of the fans.
d. Determine the probability that a randomly chosen fan will have a life of more than 25,000 hours.

6-67. Team Marketing Report (TMR) produces the Fan Cost Index™ (FCI) survey, now in its 13th year, that tracks the cost of attendance for a family of four at NFL football games. The FCI includes four average-price tickets, four small soft drinks, two small beers, four hot dogs, two game programs, parking, and two adult-size caps. The league's average FCI in 2005 was $329.82. The file entitled **NFL Price** is a sample of 175 randomly chosen fans' FCIs.

a. Produce a relative frequency histogram for these data. Does it seem plausible the data were sampled from a population that was normally distributed?

b. Calculate the mean and standard deviation of the league's FCI.

c. Calculate the 90th percentile of the leagues fans' FCI.

d. The San Francisco 49ers had an FCI of $347. Determine the percentile of the FCI of a randomly chosen family whose FCI is the same as that of the 49ers average FCI.

6-68. Championship Billiards, owned by D & R Industries, Lincolnwood, Illinois, provides some of the finest billiard fabrics, cushion rubber, and component parts in the industry. It sells billiard cloth in bolts and half-bolts. A half-bolt of billiard cloth has an average length of 35 yards with widths of either 62 or 66 inches. The file entitled **Half Bolts** contains the lengths of 120 randomly selected half-bolts.

a. Produce a relative frequency histogram for this data. Does it seem plausible the data came from a population that has a uniform distribution?

b. Provide the density, $f(x)$, for this uniform distribution.

c. A billiard retailer, Sticks & Stones Billiard Supply, is going to recover the pool tables in the local college pool hall, which has eight tables. It takes approximately 3.8 yards per table. If Championship ships a randomly chosen half-bolt, determine the probability that it will contain enough cloth to recover the eight tables.

6-69. The Hydronics Company is in the business of developing health supplements. Recently, the company's R&D department came up with two weight-loss plans that included products produced by Hydronics. To determine whether these products are effective, the company has conducted a test. A total of 300 people who were 30 pounds or more overweight were recruited to participate in the study. Of these, 100 people were given a placebo supplement, 100 people were given plan 1, and 100 people were given plan 2. As might be expected, some people dropped out of the study before the four-week study period was completed. The weight loss (or gain) for each individual is listed in the data file called **Hydronics**. Note, positive values indicate that the individual actually gained weight during the study period.

a. Develop a frequency histogram for the weight loss (or gain) for those people on plan 1. Does it appear from this graph that weight loss is approximately normally distributed?

b. Referring to part a, assuming that a normal distribution does apply, compute the mean and standard deviation weight loss for the plan 1 subjects.

c. Referring to parts a and b, assume that the weight-change distribution for plan 1 users is normally distributed and that the sample mean and standard deviation are used to directly represent the population mean and standard deviation. Then, what is the probability that a plan 1 user will lose over 12 pounds in a four-week period?

d. Referring to your answer in part c, would it be appropriate for the company to claim that plan 1 users can expect to lose as much as 12 pounds in four weeks? Discuss.

6-70. The Future-Vision Cable TV Company recently surveyed its customers. A total of 548 responses were received. Among other things, the respondents were asked to indicate their household income. The data from the survey are found in a file named **Future-Vision**.

a. Develop a frequency histogram for the income variable. Does it appear from the graph that income is approximately normally distributed? Discuss.

b. Compute the mean and standard deviation for the income variable.

c. Referring to parts a and b and assuming that income is normally distributed and the sample mean and standard deviation are good substitutes for the population values, what is the probability that a Future-Vision customer will have an income exceeding $40,000?

d. Suppose that Future-Vision managers are thinking about offering a monthly discount to customers who have a household income below a certain level. If the management wants to grant discounts to no more than 7% of the customers, what income level should be used for the cutoff?

6-71. The Cozine Corporation runs the landfill operation outside Little Rock, Arkansas. Each day, each of the company's trucks makes several trips from the city to the landfill. On each entry the truck is weighed. The data file **Cozine** contains a sample of 200 truck weights. Determine the mean and standard deviation for the garbage truck weights. Assuming that these sample values are representative of the population of all Cozine garbage trucks, and assuming that the distribution is normally distributed,

a. determine the probability that a truck will arrive at the landfill weighing in excess of 46,000 pounds.

b. compare the probability in a to the proportion of trucks in the sample that weighed over 46,000 pounds. What does this imply to you?

c. Suppose the managers are concerned that trucks are returning to the landfill before they are fully loaded. If they have set a minimum weight of 38,000 pounds before the truck returns to the landfill, what is the probability that a truck will fail to meet the minimum standard?

CASE 6.1

State Entitlement Programs

Franklin Joiner, director of health, education and welfare, had just left a meeting with the state's newly elected governor and several of the other recently appointed department heads. One of the governor's campaign promises was to try to halt the rising cost of a certain state entitlement program. In several speeches, the governor indicated the state of Idaho should allocate funds only to those individuals ranked in the bottom 10% of the state's income distribution. Now the governor wants to know how much one could earn before being disqualified from the program and he also wants to know the range of incomes for the middle 95% of the state's income distribution.

Frank had mentioned in the meeting that he thought incomes in the state could be approximated by a normal distribution and that mean per capita income was about $33,000 with a standard deviation of nearly $9,000. The Governor was expecting a memo in his office by 3:00 P.M. that afternoon with answers to his questions.

CASE 6.2

Credit Data, Inc.

Credit Data, Inc. has been monitoring the amount of time its bill collectors spend on calls that produce contacts with consumers. Management is interested in the distribution of time a collector spends on each call in which they initiate contact, inform a consumer about an outstanding debt, discuss a payment plan, and receive payments by phone. Credit Data is mostly interested in how quickly a collector can initiate and end a conversation to move on to the next call. For employees of Credit Data, time is money in the sense that one account may require one call and 2 minutes to collect, while another account may take five calls and 10 minutes per call to collect. The company has discovered that the time that collectors spend talking to consumers about accounts is approximated by a normal distribution with a mean of 8 minutes and a standard deviation of 2.5 minutes. The managers believe that the mean is too high and should be reduced by more efficient phone call methods. Specifically, they wish to have no more than 10% of all calls require more than 10.5 minutes.

Required Tasks:

1. Assuming that training can affect the average time but not the standard deviation, the managers are interested in knowing to what level the mean call time needs to be reduced in order to meet the 10% requirement.
2. Assuming that the standard deviation can be affected by training but the mean time will remain at 8 minutes, to what level must the standard deviation be reduced in order to meet the 10% requirement?
3. If nothing is done, what percent of all calls can be expected to require more than 10.5 minutes?

CASE 6.3

American Oil Company

Chad Williams, field geologist for the American Oil Company, settled into his first-class seat on the Sun-Air flight between Los Angeles and Oakland, California. Earlier that afternoon, he had attended a meeting with the design engineering group at the Los Angeles New Product Division. He was now on his way to the home office in Oakland. He was looking forward to the one-hour flight because it would give him a chance to reflect on a problem that surfaced during the meeting. It would also give him a chance to think about the exciting opportunities that lay ahead in Australia.

Chad works with a small group of highly trained people at American Oil who literally walk the earth looking for new sources of oil. They make use of the latest in electronic equipment to take a wide range of measurements from many thousands of feet below the earth's surface. It is one of these electronic machines that is the source of Chad's current problem. Engineers in Los Angeles have designed a sophisticated enhancement that will greatly improve the equipment's ability to detect oil. The enhancement requires 800 capacitors, which must operate within ± 0.50 microns from the specified standard of 12 microns.

The problem is that the supplier can provide capacitors that operate according to a normal distribution, with a mean of 12 microns and a standard deviation of 1 micron. Thus, Chad knows that not all capacitors will meet the specifications required by the new piece of exploration equipment. This will mean that in order to have at least 800 usable capacitors, American Oil will have to order more than 800 from the supplier. However, these items are very expensive, so he wants to order as few as possible to meet their needs. At the meeting, the group agreed that they wanted a 98% chance that any order of capacitors would contain the sufficient number of usable items. If the project is to remain on schedule, Chad must place the order by tomorrow. He wants the new equipment ready to go by the time he leaves for an exploration trip in Australia. As he reclined in his seat, sipping a cool lemonade, he wondered whether a basic statistical technique could be used to help determine how many capacitors to order.

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