6/5/04 This is the fourth draft of the chapter on Expected Value in Hold'em Brain by King Yao. Please email feedback, suggestions, comments, opinions, questions to KingYao@HoldemBrain.com or you could use the Feedback Form to email me at the bottom of the page

Hold'em Brain: Expected Value

You are in a restaurant looking at a menu. You see two entrees that you like equally, but one is cheaper than the other. You decide to order the cheaper one because you will be just as happy with it. You have just made a decision based on the comparison of the expected value of the two entrees.

You are driving on a highway during rush hour. Your lane seems to be going slower than the lane to your left. The first chance you get, you switch over to the left lane so you can get home faster. You have just made a decision based on the comparison of the expected value of the two lanes.

You are playing poker. The pot is very big, but your hand is mediocre. On the last round of betting, you say "ah, what the heck, I'm going to call, the pot is just too big." You have just made a decision based on the perceived expected value using information about the size of the pot and the strength of your hand.

Expected value is a concept that everybody uses in their daily lives, although they may not realize it. Whenever we have a choice, we use expected value to guide us on our decision. Sometimes the value of the choices are not purely monetary as it could be based on happiness, a term that academics like to call utility. Usually there is no need to use a formula to calculate the expected value of a decision, but there are some cases where the use of calculating expected value will show us something that is counterintuitive or simply show us why a certain idea is correct or incorrect. It can also help us to pinpoint what factors we need to consider when we are playing poker.

Expected value (EV) is a term used to describe the value of an event over the course of all possibilities. It is an easy way to describe situations that can have many different results, and shows the average result over all the probabilities. A simple example involves a basketball player at the free throw line. If the basketball player has made 750 free throws out of 1000 free throw attempts, you could estimate that he has a 75% chance of making a free throw attempt. Then you can say the EV of the number of points that he will score on one free throw attempt is 0.75. He will either make the free throw and score one point or miss the free throw and not score a point, but on average, he is expected to score 0.75 points with one free throw. The concept of EV is used throughout this book to demonstrate the values of certain poker plays and ideas. This section shows how EV can be calculated and demonstrates how it can be used, in preparation for its usage throughout this book.

The way to calculate the EV of an event is to take all possible events and assign a probability and a result to them. The sum of the probabilities will equal 100%, and the sum of each individual result multiplied by its probability will equal the EV. If the EV of the event is a positive number, we can say the event has a positive expectation or positive value. If the EV of the even is a negative number, we can say the event has a negative expectation or negative value.

Here's an example with a roll of a single die.
A game is played where a fair die is rolled. This means each of the six numbers on the die have an equal chance of coming up. If the die comes up 1 or 2, the result is -$2, if the die comes up 3 or 4, the result is +$3, if it comes up 5 or 6, the result is -$1. Here is a table with the probabilities of each roll and the results.

Result of Die	**Probability**	**Result in $**	**Prob. x Result**
1	1/6	- $2	- $0.333
2	1/6	- $2	- $0.333
3	1/6	+ $3	+$0.500
4	1/6	+ $3	+$0.500
5	1/6	- $1	- $0.167
6	1/6	- $1	- $0.167
Total	**6/6**	**N/A**	**$0.000**

The column that is labeled Probability x Result is the multiplication of the Probability and the Result column. If we add up all the numbers in that column, we would get the EV of a roll. In this case, it adds up to $0. This means that on average, the person who plays this game has no expectations as to winning or losing from a dollars perspective. Since they lose when the die comes up 1, 2, 5 or 6, they will lose more often than they win. However when they do win, they will win more money during those times, enough to compensate for the greater frequency of losses, since each loss is a smaller amount. The numbers add up in this case so that the player does not expect to profit or lose, however, he will win or lose a certain amount on any given roll.

Instead of writing out this whole table, we can write it in one algebraic equation. Here is the equation:

EV of rolling one die = 1/6 x (-$2) + 1/6 x (-$2) + 1/6 x (+$3) + 1/6 x (+$3) + 1/6 x (-$1) +1/6 x (-$1) = $0.00

Throughout this book, the EV equations will be put into a box. This will make it easier for those that would prefer to only look at the Result. Meanwhile, the Computation will be shown as well. For example, the above equation would look like this:

Action	**Computation**	**Result**
EV of rolling one die	1/6 x (-$2) + 1/6 x (-$2) + 1/6 x (+$3) + 1/6 x	$0.00

	(+$3) + 1/6 x (-$1) +1/6 x (-$1)	

Remember that in algebra, the order of operations is to first do everything within parenthesis (in this case there is none, but I wanted to mention it for future equations), then we multiply and divide, then we add or subtract. In the EV of rolling one die formula, we would multiply 1/6 x (-$2) to get -$0.333, and do the same for each term before adding all the terms together.

Now imagine the distribution was different. Instead of the payoff table that is shown above, change it so if the roll is a 1, the player gets $100, and with any other number the player will lose $1. It should be clear that this is a great game for the player, provided there is no cheating going on. Instead of writing out each term, we can simplify it to this shorter equation:

Action	**Computation**	**Result**
EV of new game	(1/6 x $100) + (5/6 x $-1)	$15.83

In the computation, we do not need to write out each of the rolls from 2 thru 6, because we know they all have the same result, losing $1. The solution for that equation is $15.83. The player expects to make $15.83 on average per roll of the die, although he will lose more often than win. On average, the player will lose 5 out of every 6 rolls, but when he does win, the winning amount overwhelms the losing amount so much that the player has a positive expectation of $15.83 average per roll.

Here's an example in Hold'em
You are playing $10-$20 Hold'em and the pot is currently $80 after the Turn card.. You have an open-ended straight draw and you are 100% sure your opponent has a hand that you will not beat unless you make a straight. But if you do hit your straight, you will win the hand. You believe there is a 17% chance that you will make your straight and a 83% chance that you will not. (In the chapters on Outs and Pot Odds, I will go into further detail on how to estimate your chances of winning and losing.)

Your opponent bets $20 and you must decide to call or fold. You only have $20 left in your stack, and if you call, you cannot lose more or win more on the River as you are considered all-in. If you call and win, you will win $100. If you call and lose, you will lose $20. You have to figure out if calling has a positive expectation.

Action	**Computation**	**Result**
EV of 10/20 problem	(17% x $100) + (83% x -20)	+$0.40

So you expect to make $0.40 by calling, which means it is better to call than fold. Sometimes you will win $100, more often you will lose $20. However on average, you expect to make $0.40. Calculations like this are difficult for most players to do in their heads while at the poker table. In the chapter on Pot Odds, a simpler way to make the determination of calling or folding is shown.

It is practical and much easier to implement, and yet it will be consistent with the EV equations. It is still useful to understand and apply the EV equations when studying the game and thinking about certain situations when not at the poker table. That is its purpose in this book, using it to study the game as opposed to using the equations directly at the table. There are simpler ways to make those calculations and not give up any accuracy.

In poker, whether they know it or not, players are always trying to put themselves into situations where they have positive EV. Good players are able to distinguish between situations that have positive EV and negative EV. When they have positive EV, they will decide to get involved in the hand. When they do not have positive EV, they will get out of the hand. Meanwhile, bad players are not able to distinguish between positive and negative EV. Thus they will often get involved in hands that have negative EV. Sometimes they will get out of hands that have positive EV. The crux of this book is to help the reader identify the difference between positive EV situations and negative EV situations. Once the positive EV situations are identified, the goal is identify the best play that will maximize the EV.

Expected Value Quiz and Answer

1. You have a fair coin, one that is expected to come up heads 50% of the time and tails the other 50% of the time. You are told that you will win $5 if you flip two tails in row, but if either of the flips comes up head, you will lose $2. What is the EV of this game?

Answer

To get two tails in a row = 50% x 50% = 25%

This means you will not get two tails in a row 75% of the time (100% - 25%)

So the EV equation is:

EV = 25% x $5 + 75% x -$2 = $1.25 - $1.50 = -$0.25

Action	**Computation**	**Result**
EV for Quiz 1	(25% x $5) + (75% x -$2)	-$0.25

On average, you will lose playing this game, to the tune of -$0.25 per game.

2. You roll a fair die and you receive the equivalent dollar amount as your roll. If you did not have to pay anything to roll the die, what is the EV of your roll?

Answer

Action	**Computation**	**Result**
EV for Quiz 2	(1/6 x $1) + (1/6 x $2) + (1/6 x $3) + (1/6 x $4) + (1/6 x $5) + (1/6 x $6)	+$3.50

3. You are the owner and the coach of a NBA team and you only care about winning this current game. Your team is down by 2 points and there is only 5 seconds to go in the game. You know that if the game goes to overtime that you have a 55% chance of winning since the your team is

slightly better than the other team (you really feel this way, its not your ego talking). You know your team can get either a two point shot off or a three point shot off, but your team will need to take about 4 seconds to get a shot off, so you are assuming there is no chance you can get an offensive rebound and get a second chance. You expect your team has a 30% chance of making a three point shot (which would win the game if they made it) and a 51% chance of making a two point shot (which would tie the game and send it to overtime if they made it). You have a choice of drawing up one play, do you go for a three point shot or a two point shot?

Answer
To solve this problem, you want to compare the EV of taking a two point shot and taking a three point shot. As for the value of the results, assign winning the game as worth 1 and losing the game as worth 0, since you have no ulterior motive other than winning the game.

EV of going for Three = (Probability you make the Three x 1) + (Probability you do not make the Three x 0)

If you decide to go for a three point shot, there is no need for an overtime. Your team will either win the game or lose the game on the last shot. However, if you decide to go for a two point shot, you will go to overtime if you do make it since the game will be tied. This means you will need to multiply the probability of making the two point shot by the probability of winning the game in overtime to find out the total probability that your team will win the game.

EV of going for Two = (Probability you make the Two x Probability you win in OT x 1) + (Probability you make the Two x Probability you lose in OT x 0) + (Probability you do not make the Two x 0)

Action	**Computation**	**Result**
EV of going for Three	(30% x 1) + (70% x 0)	+0.30
EV of going for Two	(51% x 55% x 1) + (51% x 45% x 0) + (49% x 0)	+0.28

The EV analysis says you should go for three because the EV of going for three is 0.30 which is higher than the EV of going for two which is only 0.28. Notice that in reality, if you are a basketball head coach (and not concurrently the owner), and you feel you will get ridiculed by the media and fans if you choose to go for the three and miss, you may actually decide to go for two even though the EV of going for two is lower than the EV of going for three. This is because the value of the results may be different for you personally, as you may get fired if it looks like you made a controversial decision and it does not work. Instead of using 0 as the value of the result when you go for three and lose, you may decide that the value is actually negative, such as -1, since if the three point shot is missed, you may get fired and lose your job.

4. You are playing 10-20 Hold'em. On the Turn you have four cards to a flush draw. Here are the facts:

a. You know with certainty that if you do not make your flush draw you will lose the hand.

b. You know with certainty that if you do make the flush draw that you will win.
c. There are 46 unknown cards left in the deck (you have two in your hand and you can see four on the board). 9 of those cards will give you the flush, the other 37 will not.
d. After your opponent makes a bet, the pot contains $100.
e. It is $20 to you and coincidentally, that is all you have left. This means there will be no more betting on the River as you will be all-in if you call.

Should you call or should you fold?

Answer
There are 46 cards that are unknown left in the deck. 9 of these cards will give you a flush, the other 37 cards will not. This means you have a 9/46 chance of winning and a 37/46 chance of losing. If you win, you win $100, if you lose, you lose $20. Now we have everything we need to know to set up the EV equation.

Action	**Computation**	**Result**
EV for Quiz 4	(9/46 x $100) + (37/46 x -$20)	+$3.48

This shows there is a positive EV in calling.