SAT Approach For 8-queens Problem

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Abstract

This project is to discover the solution for the 8-queens problem using SAT approach. The 8-queens problem, or general N-queens problem, is ideally suited to constraint programming, one of which is SAT (satisfiability) solving. The idea is that we encode the problem as a set of booleans, where each boolean represents one cell of the chessboard. If there exists a queen on a cell, the boolean value of that cell will be set to TRUE, and vice-versa.

1 Step-by-step Solver

a) Problem Formulation

- Input: 8x8 Chessboard, 8 queens.
- Output: Chessboard(s) with 8 queens placed on the in without attacking each other.
- Data structure: a 2D boolean array with 8 columns, 8 rows, 1 if there is a queen on that position of the board, 0 if otherwise.
- Initial states: No queen on the board
- Goal states: 8 queens on the board without attacking each other

b) Constraints Description With CNF Clauses

CNF clauses to describe restrictions required when placing a queen in the cell[3][3]

1. $-b[0][0]$
2. $-b[1][1]$
3. $-b[2][2]$
4. $-b[4][4]$
5. $-b[5][5]$
6. $-b[6][6]$
7. $-b[7][7]$
8. $-b[3][0]$
9. $-b[3][1]$
10. $-b[3][2]$
11. $-b[3][4]$
12. $-b[3][5]$
13. $-b[3][6]$
14. $-b[3][7]$
15. $-b[0][3]$
16. $-b[1][3]$
17. $-b[2][3]$

18. $-b[4][3]$
19. $-b[5][3]$
20. $-b[6][3]$
21. $-b[7][3]$
22. $-b[0][6]$
23. $-b[1][5]$
24. $-b[2][4]$
25. $-b[4][2]$
26. $-b[5][1]$
27. $-b[6][0]$
28. $b[3][3]$

Percentage completed: 100%

c) Find The Expected Set Of CNF Clauses

This function should return the set of CNF clauses that describe the whole chessboard when consecutively placing all the eight queens on the board.

Level 1: Each queen can only move on a single column (Percentage completed: 100%)
Level 2: The position of the queens is any cell on the chessboard (Percentage completed: 100%)
Percentage completed: 100%

d) Find A Set Of Satisfied Values

If the set of values that satisfy all the CNF clauses can be found, the exact positions of eight queens will show up. This program should generate CNFs clauses using Python3 and then find a set of satisfied values using the PySat library.

Percentage complete: 100%

e) Find A Suitable Assignment Of Variables

This function is implemented to find a suitable assignment of variables using A* algorithm, given that some queens are placed on the board in advance.We first pre-place all the remaining queens(the queens that is not given by the file) at the row 1 of each column(where no pre-placed queens). To do so, we iterate through cells in chess board and write a function to check whether queen can stand at the cell. This function mush comply with the rule that the queen at this cell are not attacked by others. We use the number of pairs of attacking queen as heuristic h(n) and the order of a board state pushed to priority queue as g(n) by adding 1 to current board state's order that needs to be expanded.

Percentage completed: 100%

f) Visualizing The Chessboard

A program is implemented to visualize the chessboard with the result found.
Percentage completed: 100%

2 Tasks Assigned

The project "SAT Approach For 8-queens Problem" was completed with the contribution of each member as follow:

- **a. Problem Formulation: All three members**
- **b. Constraints Description With CNF Clauses: All three members**
- **c. Find The Expected Set Of CNF Clauses: Vu Thien Hoang and Tran Vinh Long**
- **d. Find A Set Of Satisfied Values:Phan Dam Tung Lam**
- **e. Find A Suitable Assignment Of Variables: All three members**
- **f. Visualizing The Chessboard: Phan Dam Tung Lam**
- **Testing: Vu Thien Hoang(c, d), Phan Dam Tung Lam(e, f)**

3 Further insight

Link to c,d testing: https://youtu.be/$W_{6h}s6lybA$
Link to e,f testing: https://www.youtube.com/watch?v=K435T0TBgWY