Spread of Staph Infections in Hospitals

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Abstract

Methicillin-resistant Staphylococcus aureus (MRSA), commonly referred to as staph, is a bacterium that causes serious infections in humans and is resistant to treatment with the widely used antibiotic methicillin. MRSA has traditionally been a problem inside hospitals, where elderly patients or patients with compromised immune systems could more easily contract the bacteria and develop bloodstream infections. Recently, a genetically different strain of MRSA has been found in the community at large. The new strain (CA-MRSA) is able to infect healthy and young people, which the traditional strain (HA-MRSA) rarely does. As CA-MRSA appears in the community, it is inevitably being spread into hospitals. In this paper we find out if CA-MRSA will overtake HA-MRSA in the hospital.

Introduction

Methicillin-resistant Staphylococcus aureus (MRSA) infection is caused by a type of staph bacteria that's become resistant to many of the antibiotics used to treat ordinary staph infections.

MRSA is spread by contact. So, you could get MRSA by touching another person who has it on the skin. Or you could get it by touching objects that have the bacteria on them. MRSA is carried by about 2% of the population (or 2 in 100 people), although most of them aren't infected.

Most MRSA infections occur in people who've been in hospitals or other health care settings, such as nursing homes and dialysis centers. When it occurs in these settings, it's known as health care-associated MRSA (HA-MRSA). HA-MRSA infections typically are associated with invasive procedures or devices, such as surgeries, intravenous tubing or artificial joints.

Another type of MRSA infection has occurred in the wider community among healthy people. This form, community-associated MRSA (CA-MRSA), often begins as a painful skin boil. It's spread by skin-to-skin contact. At-risk populations include groups such as high school wrestlers, child care workers and people who live in crowded conditions.

As CA-MRSA appears in the community, it is inevitably being spread into hospitals. Some studies suggest that CA-MRSA will overtake HA-MRSA in the hospital, which would increase the severity of the problem and likely cause more deaths per year.

Mathematical model

To predict whether or not CA-MRSA will overtake HA-MRSA, a compartmental model has been developed by mathematicians in collaboration with medical professionals. This model classifies all patients in the hospital into three groups:

$K(t)$ – patients colonized with HA-MRSA.

$C(t)$ – patients colonized with CA-MRSA.

$S(t)$ – patients not colonized with either strain.

The parameters of the model are:

β_C – the rate (per day) at which CA-MRSA is transmitted

β_H – the rate (per day) at which CA-MRSA is transmitted

δ_C – the rate (per day) at which patients with CA-MRSA exit the hospital by death or discharge

δ_H – the rate (per day) at which patients with HA-MRSA exit the hospital by death or discharge.

δ_S – the rate (per day) at which susceptible patients exit the hospital by death or discharge.

α_S – the rate (per day) at which patients with CA-MRSA successfully undergo decolonization measures.

α_H – the rate (per day) at which patients with HA-MRSA successfully undergo decolonization measures.

N – the total number of patients in the hospital.

Λ – the rate (per day) at which patients enter the hospital

Patients movements

Patients move between compartments as they become colonized or decolonized (figure 1). This type of model is typically known as an SIS (susceptible-infected-susceptible) model, in which patients who become colonized can become susceptible again and colonized again (there is no immunity).

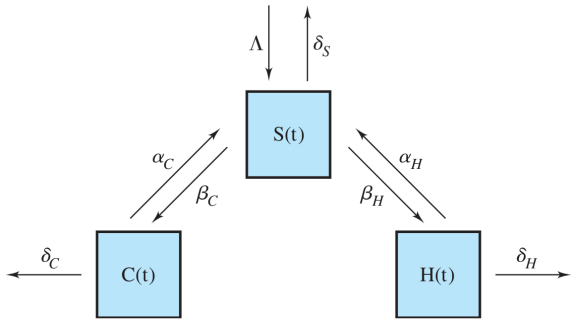

Figure 1: A diagram of how patients transit between the compartments

The transition between states is described by the following system of differential equations:

$$\frac{dS}{dt} = \underbrace{\Lambda}_{\text{entrance rate}} - \underbrace{\frac{\beta_H S(t)H(t)}{N}}_{\text{acquire HA-MRSA}} - \underbrace{\frac{\beta_H S(t)C(t)}{N}}_{\text{acquire CA-MRSA}} +$$

$$\underbrace{\alpha_H H(t)}_{\text{HA-MRSA decolonized}} + \underbrace{\alpha_C C(t)}_{\text{CA-MRSA decolonized}} - \underbrace{\delta_S S(t)}_{\text{exit hospital}} \quad (1)$$

$$\frac{dH}{dt} = \underbrace{\frac{\beta_H S(t)H(t)}{N}}_{\text{from S}} - \underbrace{\alpha_H H(t)}_{\text{decolonized}} - \underbrace{\delta_H H(t)}_{\text{exit hospital}} \quad (2)$$

$$\frac{dC}{dt} = \underbrace{\frac{\beta_H S(t)C(t)}{N}}_{\text{from S}} - \underbrace{\alpha_C C(t)}_{\text{decolonized}} - \underbrace{\delta_C C(t)}_{\text{exit hospital}} \quad (3)$$

If we assume that the hospital is always full, we can conserve the system by letting $\Lambda = \delta_S S(t) + \delta_H h(t) + \delta_C C(t)$. In this case $S(t) + C(t) + H(t) = N$ for all t (assuming you stat with population of size N). Now we can see that S is determined by the equation $S(t) = N - H(t) - C(t)$. Under the assumption that the hospital always has patience, we can replace S in (2) we obtain:

$$\frac{dH}{dt} = (\beta_H/N)(N - C - H)H - (\beta_H + \alpha_H)H \quad (4)$$

If we do the same thing for (3) we obtain:

$$\frac{dC}{dt} = (\beta_C/N)(N - C - H)C - (\beta_C + \alpha_C)C \quad (5)$$

Explore Model

Parameter values obtained from the Beth Israel Deaconess Medical Center are given in table 1. We can plug these values in our model and see what we can find out.

Parameter	Symbol	Baseline Value
Total nutients	N	400
Length of stay		
Susceptible	$1/\beta_S$	5 days
Colonized CA-MRSA	$1/\beta_C$	7 days
Colonized HA-MRSA	$1/\beta_H$	5 days
Transmission rate per susceptible patient to		
Colonized CA-MRSA per colonized CA-MRSA	β_C	0.45 per day
Colonized HA-MRSA per colonized HA-MRSA	β_H	0.4 per day
Decolonization rate per colonized patient per day per length of stay		
CA-MRSA	α_C	0.1 per day
HA-MRSA	α_H	0.1 per day

Table 1: Parameter values obtained from the Beth Israel Deaconess Medical Center

Using these values we'll get a system of equations and we'll find out the critical points:

$$\begin{cases} \frac{0.4H(400 - H - C)}{400} - (\frac{1}{5} + 0.1)H = 0 \\ \frac{0.45y(400 - H - C)}{400} - (\frac{1}{7} + 0.1)C = 0 \end{cases}$$

If we get rid of the brackets and do some simplification, we'll get:

$$\begin{cases} \frac{160H - 0.4CH - 0.4C^2}{400} - 0.3H = 0 \\ \frac{180C - 0.45CH - 0.45C^2}{400} - 0.242857C = 0 \end{cases}$$

$$\begin{cases} \frac{160H}{400} - \frac{0.4H^2H}{400} - \frac{0.4HC}{400} - 0.3H = 0 \\ \frac{180C}{400} - \frac{0.45CH}{400} - \frac{0.45C^2}{400} - 0.242857C = 0 \end{cases}$$

$$\begin{cases} -0.001HC - 0.001H^2 + 0.1H = 0 \\ -0.001125HC - 0.001125C^2 - 0.207143C = 0 \end{cases}$$

We can assume that H and C are positive numbers and obtain:

$$\begin{cases} H(H + C - 100) = 0 \\ C(H + C - 184.127) = 0 \end{cases}$$

Using this, we found three critical points

$$\begin{cases} H = 0 & \text{C = 0} \\ H = 100 & \text{C = 0} \\ H = 0 & \text{C = 184.127} \end{cases}$$

Interpreting results

Now that we found the solution of our system, we can sketch the direction field for it.

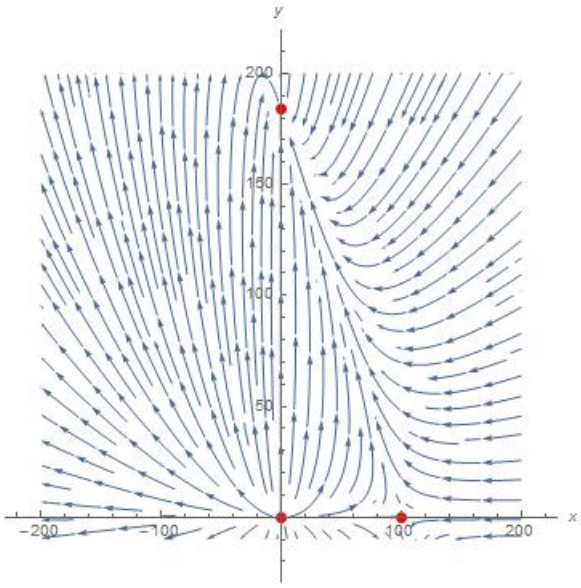

Figure 2: Direction field for the equation system

We can see that we have an asymptotically stable point $\{H = 0, C = 184.127\}$. Based on the direction field, we can observe that this node it's also a spiral on.

The other two nodes, $\{H = 0, C = 0\}$ and $\{H = 100, C = 0\}$ seems to be unstable points since all the arrows are getting away from them.

Given enough time, any point from this plane will converge to $\{H = 0, C = 184.127\}$, meaning that, using this model, CA-MRSA will overtake HA-MRSA.

References

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