Homework 3

1. Let F, G, H are propositional formulae. Show that the following hold:

   - Reduction Laws:

   $$\begin{array}{ll} \text{(a)} & (F \leftrightarrow G) \sim (F \to G) \wedge (G \to F), \\ \text{(b)} & (F \to G) \sim (\neg F \vee G). \end{array}$$

   - Commutative Laws:

   $$\begin{array}{ll} \text{(a)} & F \vee G \sim G \vee F, \\ \text{(b)} & F \wedge G \sim G \wedge F, \\ \\ \text{(c)} & F \leftrightarrow G \sim G \leftrightarrow F. \end{array}$$

   - Associative Laws:

   $$\begin{array}{ll} \text{(a)} & (F \vee G) \vee H \sim F \vee (G \vee H), \\ \text{(b)} & (F \wedge G) \wedge H \sim F \wedge (G \wedge H), \\ \text{(c)} & (F \leftrightarrow G) \leftrightarrow H \sim F \leftrightarrow (G \leftrightarrow H). \end{array}$$

   - Distributive Laws:

   $$\begin{array}{ll} (a) & F \vee (G \wedge H) \sim (F \vee G) \wedge (F \vee H), \\ (b) & F \wedge (G \vee H) \sim (F \wedge G) \vee (F \wedge H), \\ \\ (c) & (F \vee G) \to H \sim (F \to H) \wedge (G \to H), \\ (d) & (F \wedge G) \to H \sim (F \to H) \vee (G \to H), \\ (e) & F \to (G \vee H) \sim (F \to G) \vee (F \to H), \\ (f) & F \to (G \wedge H) \sim (F \to G) \wedge (F \to H), \\ (g) & (F \wedge G) \to H \sim F \to (G \to H). \end{array}$$

   - Laws of “True” and “False”:

   $$\begin{array}{ll} (a) & \neg\top \sim \bot, \\ (b) & \neg\bot \sim \top, \\ (c) & F \vee \bot \sim F, \\ (d) & F \wedge \top \sim F, \\ (e) & F \vee \top \sim \top, \\ (f) & F \wedge \bot \sim \bot, \\ (g) & \bot \to F \sim \top, \\ (h) & F \to \top \sim \top. \end{array}$$

   - Idempocy rules:

   $$\begin{array}{ll} (a) & F \wedge F \sim F, \\ (b) & F \vee F \sim F. \end{array}$$

   - Absorbtion Laws:

   $$\begin{array}{ll} (a) & F \vee (F \wedge G) \sim F, \\ (b) & F \wedge (F \vee G) \sim F. \end{array}$$

- "Annihilation" Laws:

$$\begin{array}{ll} (a) & F \vee \neg F \sim \top, \text{ ("tertium non datur")} \\ (b) & F \wedge \neg F \sim \bot, \\ & \\ (c) & F \to F \sim \top. \end{array}$$

- Negation Laws:

$$\begin{array}{ll} (a) & \neg(\neg F) \sim F, \text{ ("double negation")} \\ (b) & \neg(F \vee G) \sim \neg F \wedge \neg G, \text{ ("De Morgan")} \\ (c) & \neg(F \wedge G) \sim \neg F \vee \neg G, \text{ ("De Morgan")} \\ & \\ (d) & \neg(F \to G) \sim F \wedge (\neg G), \\ (e) & \neg(F \leftrightarrow G) \sim F \leftrightarrow (\neg G). \end{array}$$

- Other Laws:

$$\begin{array}{ll} (a) & F \to G \sim F \leftrightarrow (F \wedge G), \\ (b) & F \to G \sim G \leftrightarrow (F \vee G). \end{array}$$

2. Design a digital circuit that implements the *minority* function. The minority function has 3 inputs and the output is $\mathbb{F}$ if most of the inputs are $\mathbb{T}$, and $\mathbb{T}$ otherwise.

3. Design digital circuits that implements one bit addition (that is two circuits, each with 3 inputs – the two things to be added, the two bits to be added, and the carry in – and the outputs are the sum, and the carry out respectively). For this, use $\mathbb{F}$ for 0 and $\mathbb{T}$ for 1.