Homework 1

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Contents

1 Exercise 1

Using the definition of well formed propositional formulae (wffs), decide which of the following are propositional formulae:

1.1 Point a

$$(((P \to Q) \lor S) \leftrightarrow T)$$

$$(\qquad\qquad\qquad\qquad\qquad\qquad \leftrightarrow \qquad\qquad) \tag{1}$$

$$(\qquad\qquad\qquad\qquad \lor \qquad\qquad) \qquad\quad T \tag{2}$$

$$(\quad \to \quad) \qquad\quad S \tag{3}$$

$$P \qquad\quad Q \tag{4}$$

After we split the formula in different layers we can easily identify the atoms and other formula. We can ses this spiting as an AST with parenthesis.

On layer 4 we can see that P and Q are atoms, therefore P and Q are formulae.

Those are used in layer 3 in order to form the formula $(P \to Q)$. We also observe that S it's an atom, therefore it is a formula.

In layer 2, $(P \to Q) \lor S)$ is a formula because $(P \to Q)$ and S are formulae. We also observe that T it's a formula, because it's an atom.

Finally, on layer 1, it can be seen that $(((P \to Q) \lor S) \leftrightarrow T)$ it's a well-formed formula because $((P \to Q) \lor S)$ and T are formulae.

1.2 Point b

$$((P \leftarrow (Q \land (S \leftarrow T))))$$

$$(\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad) \tag{5}$$

$$(\qquad \to \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad) \tag{6}$$

$$P \qquad (\qquad\qquad \land \qquad\qquad\qquad) \tag{7}$$

$$Q \qquad (\quad \leftrightarrow \quad) \tag{8}$$

$$S \qquad T \tag{9}$$

First we split the formula in different layers so we can see the formulae and atoms.

On the last layer (9) we have S and T which are atoms and based on the definition, they are formulae.

Going up to 8, we can see that Q is an atom and based on definition, will be a formula. Also, still based on definition, $(S \leftrightarrow T)$ is a formula because S and T are formulae.

On layer 7 we observe that P is a formula because it's an atom. Also, based on definition, $(Q \land (S \leftrightarrow T))$ is a formula, because Q and $(S \leftrightarrow T)$ are formulae.

In 6, because P and $(Q \wedge (S \leftrightarrow T))$ are formuale, then $(P \rightarrow (Q \wedge (S \leftrightarrow T)))$ is a formula.

Finally, on layer 5 we have an extra set of parenthesis and based on the definition, even if $(P \rightarrow (Q \wedge (S \leftrightarrow T)))$ is a formula, $((P \rightarrow (Q \wedge (S \leftrightarrow T))))$ isn't a valid one.

1.3 Point c

$$(\neg(B(\neg Q)) \wedge R)$$

$$
\begin{aligned}
&(\qquad\qquad\qquad\qquad\qquad \wedge \qquad\quad) && (10)\\
&\quad \neg(\qquad\qquad\qquad\quad) \qquad R && (11)\\
&\qquad\quad B \qquad\quad) && (12)\\
&\qquad\qquad (\neg \quad\;\;) && (13)\\
&\qquad\qquad\quad Q && (14)
\end{aligned}
$$

First we split the formula in different layers so we can see the formulae and atoms.

At the bottom (14), we can see that Q is a formula because it is an atom.

Going up, at layer 13, based on definition, $(\neg Q)$ is a formula, because Q is a formula.

On layer 12 we found B a formula, but using the downstream formula $(\neg Q)$, $B(\neg Q)$ can't be a formula, therefor, even if the upstream layers could form formulae, $(\neg(B(\neg Q)) \wedge R)$ it's not a well-formed formula.

2 Exercise 2

In practice, parentheses can be dropped, if there are no ambiguities. More- over, a precedence for propositional connectives is defined: $\leftrightarrow$, $\rightarrow$, $\vee$, $\wedge$, $\neg$ ($\neg$ binds the strongest). For the following, decide which are wffs (in the relaxed sense). For those that are wffs, place the parentheses in the appropriately, such that the formula is a wff in the strong sense, then give the formula tree (the abstract syntax):

2.1 Point a

$$P \wedge Q \rightarrow R \neg B \vee G$$

This formula it's not a wff, because $R\neg B \vee G$ can't form a wff, no matter how will place the parentheses. $\neg$ it's an unary logic operator and will bind to B. $\vee$ is a binary operator and will form a wff with $\neg B$ and G $(\neg B \vee G)$. Using the definition of a wff, $R(\neg B \vee G)$ can't be a wff because will need a binary logic operator in order to connect R and $\neg B \vee G$.

2.2 Point b

$$P \to \neg\neg\neg\neg\neg B \leftrightarrow Q \land S$$

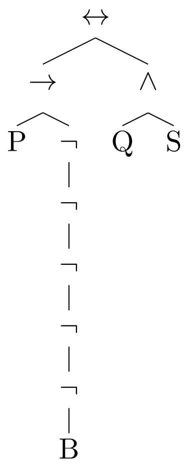
We can make a small observation. For one binary logical operator, will need two operands. For two binary operators we need three operands. Given P a wff, and l the numbers of binary logical operators from P, the numbers of atoms in P, a will be $a = l + 1$.

$\neg\neg\neg\neg\neg B$ can be reduced to $\neg B$ and now the formula will be $P \to \neg B \leftrightarrow Q \land S$.

This is a wff, because the number of binary logical operators is equal to the number of atoms, minus one, and we can find a form of parentheses placement which will satisfy the definition of a wff, following the precedence rule.

$$((P \to (\neg(\neg(\neg(\neg(\neg B)))))) \leftrightarrow (Q \land S))$$

Now we can draw the AST:



3 Exercise 3

Translate the following text into propositional formulae: “If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent.”

$$\begin{aligned}
\text{Superman is able} &\implies A \\
\text{Superman is willing} &\implies W \\
\text{Superman prevents evil} &\implies P \\
\text{Superman not is able} &\implies \neg A \\
\text{Superman is impotent} &\implies I \\
\text{Superman is unwilling} &\implies \neg W \\
\text{Superman exists} &\implies E
\end{aligned}$$

$$\begin{aligned}
(A \wedge W) \rightarrow P \\
\neg A \rightarrow I \\
\neg W \rightarrow M \\
\neg P \\
E \rightarrow \neg M \wedge \neg I
\end{aligned}$$