Homework 5

1. Prove that the following are complete sets of boolean operators:

   (a) $\{\neg, \wedge\}$;

   (b) $\{\neg, \vee\}$;

   (c) $\{\rightarrow, \perp\}$;

   (d) $\{|\}$;

   (e) $\{\triangledown\}$;

   where, for all formulae F, G:

$$F|G = \neg(F \wedge G),$$
$$F \triangledown G = \neg(F \vee G).$$

2. Let $F_1, \ldots, F_n, G$ be propositional formulae.
   Show that

$$F_1, \ldots, F_n \models G$$
$$\text{iff}$$
$$(F_1 \wedge \ldots \wedge F_n \wedge \neg G) \text{ is unsatisfiable.}$$

3. Let F and G be propositional formulae. Show that $F \sim G$ iff $F \leftrightarrow G$ is valid.

4. Use the resolution to decide:

   "If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent."

   Does Superman exist?(based on the text above)

   Hint: is "Superman exists" a logical consequence of the text?

   How does it compare to the solution by truth tables?

5. For the following clause sets:

   (a) $\{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{B, \neg C\}, \{\neg B\}\}$,

   (b) $\{\{A, \neg B\}, \{A, C\}, \{\neg B, C\}, \{\neg A, B\}, \{B, \neg C\}, \{\neg A, \neg C\}\}$.

   - give the corresponding formula(e),
   - apply the resolution: are the corresponding formulae satisfiable, not satisfiable? If yes, give a satisfying truth valuation.

6. Decide, using resolution, whether the clause set containing the following clauses:

$$\begin{array}{ll} (1) & \{P, Q, \neg R\}, \\ (2) & \{\neg P, R\}, \\ (3) & \{P, \neg Q, S\}, \\ (4) & \{\neg P, \neg Q, \neg R\}, \\ (5) & \{P, \neg S\}. \end{array}$$

   is satisfiable or not. If yes, construct a satisfying truth valuation.

7. Establish the validity of the following formula, using resolution:

$$\begin{pmatrix} (P_1 \to (P_2 \vee P_3)) \wedge (\neg P_1 \to (P_3 \vee P_4)) \\ \wedge \\ (P_3 \to (\neg P_6)) \wedge (\neg P_3 \to (P_4 \to P_1)) \\ \wedge \\ (\neg (P_2 \wedge P_5)) \wedge (P_2 \to P_5) \end{pmatrix} \to \neg (P_3 \to P_6).$$