GTC

# Seminar 2

## Lab work (programming)

Implement in a programming language of your choice (preferably `C++`) the following programs:

1. `void isPermutation(int n, int* p)` which takes as input arguments a positive integer $n$ and a (pointer to) an array of $n$ integers, and prints the message

   - `"is a permutation"` if $\langle p[0], \ldots, p[n-1] \rangle$ is a permutation of the numbers $1, \ldots, n$.
   - `"not a permutation"` otherwise.

   Write a program that reads from console the values of $p[0], \ldots, p[n-1]$ and then invokes `isPermutation(n,p)` to detect if what was read is a permutation or not.

2. `int factorial(int n)` which computes $n!$

3. `int rankPermutation(int n, int* p)` which takes as input arguments a positive integer $n$ and a (pointer to) an array of $n$ integers, and computes the rank of the permutation $\langle p[0], p[1], \ldots, p[n-1] \rangle$ in lexicographic order. We assume implicitly that $\langle p[0], p[1], \ldots, p[n-1] \rangle$ is a permutation of 1,2,...,$n$.

4. `permWithRank(int k, int n)` which prints the permutation with rank $k$ of $\{1, 2, \ldots, n\}$ in lexicographic order.

5. `void nextPermutation(int n, int* p)` which takes as input arguments a positive integer $n$ and a (pointer to) an array of $n$ integers, and displays the permutation next to $\langle p[0], p[1], \ldots, p[n-1] \rangle$ in lexicographic order. We assume implicitly that $\langle p[0], p[1], \ldots, p[n-1] \rangle$ is a permutation of 1,2,...,$n$.

6. (Optional) `void prevPermutation(int n, int* p)` which takes as input arguments a positive integer $n$ and a (pointer to) an array of $n$ integers, and displays the permutation next to $\langle p[0], p[1], \ldots, p[n-1] \rangle$ in

lexicographic order. We assume implicitly that $\langle p[0], p[1], \ldots, p[n-1] \rangle$ is a permutation of 1,2,...,$n$.

## Homework

(1) Suppose 50 socks lie in a drawer. Each one is either white or black, ankle-high or knee-high, and either has a hole or doesn't. 22 socks are white, four of these have a hole, and one of these four is knee-high. Ten white socks are knee-high, ten black socks are knee-high, and five knee-high socks have a hole. Exactly three ankle-high socks have a hole.

- Use the principle of inclusion and exclusion to determine the number of black, ankle-high socks with no holes.
- Draw a Venn diagram that shows the number of socks with each combination of characteristics.

(2) How many positive integers between 50 and 100

(a) are divisible by 7? Which integers are these?
(b) are divisible by 11? Which integers are these?
(c) are divisible by both 7 and 11?

(3) How many positive integers less than 1000

(a) are divisible by 7?
(b) are divisible by 7 but not by 11?
(c) are divisible by both 7 and 11?
(d) are divisible by either 7 or 11?
(e) are divisible by exactly one of 7 and 11?
(f) have distinct digits?
(g) have distinct digits and are even?

(4) How many prime numbers smaller than 168 are there?

(5) Consider the ordered set $A = \{a, b, c, d\}$ with $a < b < c < d$

(a) Enumerate the first seven 3-permutations of $A$ in ordine lexicographic order.
(b) What is the rank of the 3-permutation $\langle c, b, d \rangle$ in lexicographic order?
(c) What is the 3-permutation with rank 60 in lexicographic order?

(6) Consider the ordered set $A = \{a, b, c, d\}$ with $a < b < c < d$.

(a) Enumerate the first 7 subsets of $A$ in ascending order of the rank given by their binary representation.
(b) Which is the the subset with rank 14 of $A$?
(c) What is the rank of the subset $\{a, c, d\}$?