

GTC
Seminar 2

Lab work (programming)

Implement in a programming language of your choice (preferably C++) the following programs:

1. `void isPermutation(int n, int* p)` which takes as input arguments a positive integer n and a (pointer to) an array of n integers, and prints the message
 - "is a permutation" if $\langle p[0], \dots, p[n-1] \rangle$ is a permutation of the numbers $1, \dots, n$.
 - "not a permutation" otherwise.

Write a program that reads from console the values of $p[0], \dots, p[n-1]$ and then invokes `isPermutation(n,p)` to detect if what was read is a permutation or not.

2. `int factorial(int n)` which computes $n!$
3. `int rankPermutation(int n, int* p)` which takes as input arguments a positive integer n and a (pointer to) an array of n integers, and computes the rank of the permutation $\langle p[0], p[1], \dots, p[n-1] \rangle$ in lexicographic order. We assume implicitly that $\langle p[0], p[1], \dots, p[n-1] \rangle$ is a permutation of $1, 2, \dots, n$.
4. `permWithRank(int k, int n)` which prints the permutation with rank k of $\{1, 2, \dots, n\}$ in lexicographic order.
5. `void nextPermutation(int n, int* p)` which takes as input arguments a positive integer n and a (pointer to) an array of n integers, and displays the permutation next to $\langle p[0], p[1], \dots, p[n-1] \rangle$ in lexicographic order. We assume implicitly that $\langle p[0], p[1], \dots, p[n-1] \rangle$ is a permutation of $1, 2, \dots, n$.
6. (Optional) `void prevPermutation(int n, int* p)` which takes as input arguments a positive integer n and a (pointer to) an array of n integers, and displays the permutation next to $\langle p[0], p[1], \dots, p[n-1] \rangle$ in

lexicographic order. We assume implicitly that $\langle p[0], p[1], \dots, p[n-1] \rangle$ is a permutation of $1, 2, \dots, n$.

Homework

- (1) Suppose 50 socks lie in a drawer. Each one is either white or black, ankle-high or knee-high, and either has a hole or doesn't. 22 socks are white, four of these have a hole, and one of these four is knee-high. Ten white socks are knee-high, ten black socks are knee-high, and five knee-high socks have a hole. Exactly three ankle-high socks have a hole.
 - Use the principle of inclusion and exclusion to determine the number of black, ankle-high socks with no holes.
 - Draw a Venn diagram that shows the number of socks with each combination of characteristics.
- (2) How many positive integers between 50 and 100
 - (a) are divisible by 7? Which integers are these?
 - (b) are divisible by 11? Which integers are these?
 - (c) are divisible by both 7 and 11?
- (3) How many positive integers less than 1000
 - (a) are divisible by 7?
 - (b) are divisible by 7 but not by 11?
 - (c) are divisible by both 7 and 11?
 - (d) are divisible by either 7 or 11?
 - (e) are divisible by exactly one of 7 and 11?
 - (f) have distinct digits?
 - (g) have distinct digits and are even?
- (4) How many prime numbers smaller than 168 are there?
- (5) Consider the ordered set $A = \{a, b, c, d\}$ with $a < b < c < d$
 - (a) Enumerate the first seven 3-permutations of A in ordine lexicographic order.
 - (b) What is the rank of the 3-permutation $\langle c, b, d \rangle$ in lexicographic order?
 - (c) Care este 3-permutarea cu rangul 60 în ordine lexicografică?
- (6) Consider the ordered set $A = \{a, b, c, d\}$ with $a < b < c < d$.
 - (a) Enumerate the first 7 subsets of A in ascending order of the rank given by their binary representation.
 - (b) Which is the the subset with rank 14 of A ?
 - (c) What is the rank of the subset $\{a, c, d\}$?