Homework 2

Due Week 5

1. For the expressions below that are propositional formulas, choose 3 different interpretations and compute their truth value under those interpretations, according to the definition given in the lecture:

   (a) $(((P \to Q) \vee S) \leftrightarrow T)$,
   (b) $((P \to (Q \wedge (S \to T))))$,
   (c) $(\neg(B(\neg Q)) \wedge R)$.

2. For the following propositional formulae, choose a truth valuation, and compute the truth value under the chosen valuation:

   (a) $((P \to Q) \wedge ((\neg Q) \wedge P))$
   (b) $((P \to Q) \to (Q \to P))$
   (c) $((\neg(P \vee Q)) \wedge (\neg Q))$

3. For the propositional translation of the text below, choose your own interpretation, then compute the truth value under that interpretation:

   "If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent."

4. Consider the formulas given in Exercise 2. Construct the truth tables corresponding to each formula. For each, say whether they are valid, invalid, satisfiable, unsatisfiable.

5. Consider the propositional translation of the Superman story at Exercise 3. Does Superman exist? I.e. is "Superman exists" a logical consequence of the story? Or is "Superman does not exist" a logical consequence of the story?

6. **(Optional, but strongly encouraged)** The construction of the tree representation of a proposition (abstract syntax) is basically a manipulation of language objects (formula trees) for the purpose of solving a problem (whether an expression is a proposition). Therefore it looks like it may fit in our definition of reasoning (it is abstract). Show that this process is also verifiable. That is, describe the steps for the construction of the formula trees as a set of rules.