

Homework 8

In the following, the common mathematical notation is used. Variables are denoted by lower case letters at the end of the alphabet: x, y, z, \dots . Usual symbols for function and predicate symbols will be used. used for this will be specified explicitly.

1. Which of the following expressions are predicate logic expressions and what kind of expressions they are (terms, formulae)? Identify the function and the predicate symbols in each of the proper¹ expressions. Which variables are free, which variables are bound?

(a) 4,

(b) $(8x - 5) + 7 \geq (3 - 5x \Leftrightarrow y > 8z)$,

(c) $\neg(x - y < x^2 + y\sqrt{z}) \wedge (\exists z((5 + 1) * y = 5\frac{x}{y^2}))$,

(d) $\forall x(\frac{x+1}{x^2+5} > \frac{x^3+5x+11}{1+\frac{x-8}{x^4-1}}))$,

(e) $\neg P(x, y) \Leftrightarrow (\forall x \exists y \forall z ((P(y, z) \vee Q(x, y, z)) \Rightarrow (R(x, z, y) \vee \neg P(x, z))))$.

2. Carry out the substitutions:

(a) $x_{\{x \leftarrow z+2\}}$,

(b) $(x + y)_{\{x \leftarrow z+2, y \leftarrow x+y\}}$,

(c) $(xy)_{\{x \leftarrow z+2\}}$,

(d) $(\neg P(x, y) \Leftrightarrow (\forall x \exists y \forall z ((P(y, z) \vee Q(x, y, z)) \Rightarrow (R(x, z, y) \vee \neg P(x, z))))_{\{x \leftarrow (y+z), y \leftarrow xy\}}$

3. Let

$$\theta = \{x \leftarrow x + 5, y \leftarrow 2x + 3, z \leftarrow y + u\},$$

$$\sigma = \{x \leftarrow 3x + 3, z \leftarrow u + v, v \leftarrow x + 2y\},$$

$$\lambda = \{y \leftarrow x + v, u \leftarrow 3y, v \leftarrow 4z\}.$$

Compute $\theta\sigma$, $\tau\lambda$, $\theta(\sigma\lambda)$, $(\theta\sigma)\lambda$.

4. Solve the following unification problems:

(a) $f(x, y) =^? f(h(a), x)$,

(b) $f(x, y) =^? f(h(x), x)$,

(c) $f(x, b) =^? f(h(y), z)$,

(d) $f(x, x) =^? f(h(y), y)$,

where f, h are function symbols (binary, unary respectively), a, b are constants, x, y, z are variables.

5. Consider the predicate logic language that contains the following symbols:

- function symbols \mathcal{F} : $+$ binary, $-$ unary, $*$ binary.

¹Expressions of predicate logic: terms or formulas.

- predicate symbols \mathcal{P} : $=, <, \leq$ all binary.
- constant symbols \mathcal{C} : $0, 1$.

Give interpretations of this language:

- in the universe of natural numbers,
- in the theory of univariate polynomials over reals,
- in the theory of sets,
- in the theory of strings.

For each interpretation, give a variable assignment and evaluate the meaning of the expressions under the interpretation and variable assignment for:

- $(x + (-y)) * z$,
- $(x * y + (-z)) \leq (-z + 1) * 0$,
- $(x * (y + z)) = (x * y) + (x * z)$.

6. Translate the following sentences into predicate logic syntax:

- “Some integer is larger than 23.”,
- “A positive number is not negative.”,
- “Adding two odd integers yields an even number.” (use only addition and multiplication - no division, mod, or predicates for characterizing odd or even numbers),
- “No integer is larger than all others.”,
- “Every integer is larger than one and smaller than another.”