Geomagnetic Cutoff Rigidity in AMPS: Definitions, Penumbra, and Backtracing

AMPS Interface Documentation

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1 What is cutoff rigidity?

Charged particles moving in the geomagnetic field are filtered by their *magnetic rigidity*

$$R \equiv \frac{pc}{Ze}, \tag{1}$$

where p is particle momentum, Z is charge state, e is the elementary charge, and c is the speed of light. For a given location and arrival direction, there exists a threshold rigidity below which trajectories are typically *forbidden* (do not connect to interplanetary space) and above which they are *allowed*.

Why rigidity instead of energy? Magnetic deflection depends on momentum per unit charge. Energy-based thresholds depend on particle species through the $p(E)$ relation and Z.

2 Trajectory-based definition

AMPS evaluates cutoffs using *numerical trajectory tracing*:

- Choose an observation point $\mathbf{r}_0$ and an arrival direction $\hat{\mathbf{v}}_0$ (in GSM unless noted).
- For a trial rigidity R, construct the corresponding momentum and velocity for the chosen particle species.
- Integrate the Lorentz equation backward in time (equivalently forward for antiparticles):

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E}(\mathbf{r},t) + \mathbf{v} \times \mathbf{B}(\mathbf{r},t)\right). \tag{2}$$

- Apply stop conditions (magnetopause crossing, maximum time, or return to atmosphere/inner boundary).

A trajectory is labeled *allowed* if it reaches the upstream boundary (solar wind / magnetopause) without intersecting the inner loss boundary.

3 Penumbra and effective cutoff

Realistic geomagnetic fields produce a *penumbra*: alternating allowed/forbidden bands in rigidity. Following classic practice (e.g., Cooke et al., 1991; Smart & Shea, 2009), define:

- R_L: the *lower* cutoff (below which all trajectories are forbidden),
- R_U: the *upper* cutoff (above which all trajectories are allowed).

Between R_L and R_U the transmissivity is fractional. The *effective cutoff* is computed by summing allowed bands in the penumbra:

$$R_C = R_L + \sum_{k=1}^{N_{\text{allowed}}} \Delta R_k, \tag{3}$$

where ΔR_k are the rigidity widths of allowed intervals inside (R_L, R_U). In discrete scanning, this becomes a weighted count of allowed samples.

4 Vertical vs. directional cutoffs

A *vertical cutoff* uses $\hat{\mathbf{v}}_0$ aligned with the local zenith. For anisotropic SEP events and LEO particle telescopes, AMPS supports *directional cutoffs* using the instrument look direction. This matters because R_C can vary strongly with pitch/azimuth in the penumbra.

5 Implementation notes (AMPS interface conventions)

- Coordinates: the interface uses GSM for fields and particle state unless explicitly specified.
- Boundary surfaces: the magnetopause (e.g., Shue) is used as an "escape" boundary; crossing it ends the tracing.
- Time dependence: temporal modes (static, driven) are handled by updating $\mathbf{E}(t)$ and $\mathbf{B}(t)$ during integration.
- Output: AMPS reports R_L, R_U, and R_C when penumbra scanning is enabled.

6 References (selected)

- Cooke, D. J., et al. (1991). "On cosmic-ray cut-off terminology." *Nuovo Cimento C.*
- Smart, D. F., & Shea, M. A. (2009). "Fifty years of progress in geomagnetic cutoff rigidity determinations." *Adv. Space Res.*
- Kress, B. T., et al. (2015). "Modeling geomagnetic cutoffs for space weather applications." *J. Geophys. Res. Space Physics.*