Research paper template

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Abstract

Research paper template, with content taken from different university and personal projects.

Contents

1 My research interests

1.1 Stochastic differential equations

Simulation schemes for path-dependent options[1]

We are interested in a path-dependent payoff on an asset with diffusion $(X_t)_{0\leq t\leq T}$. In particular, we kill the diffusion when it leaves the open set D.
With $\tau := \inf\{t > 0 : X_t \notin D\}$, this leads to the computation of $\mathbb{E}\left[\mathbb{1}_{T<\tau} f(X_T)\right]$.

Proposition 1.0.1. *Price of a double no-touch option with lower and upper barriers S_{min}, S_{max}:*

$$V(S,t) = 2\pi\left(\frac{S}{S_{min}}\right)^{\alpha} e^{\beta(T-t)} \sum_{n=1}^{\infty} n\left[\frac{(1-(-1)^m e^{-\alpha H})}{\alpha^2 H^2 + n^2\pi^2}\right] \exp\left(-\tfrac{1}{2}\sigma^2\frac{n^2\pi^2}{H^2}t\right)\sin\left(\frac{n\pi}{H}\ln\tfrac{S}{S_{min}}\right).$$

with $H = \ln(S_{max}/S_{min})$, $\alpha = -\tilde{r}/\sigma^2$ and $\beta = -\tilde{r}/2\sigma^2 - r$, $\tilde{r} = r - \frac{1}{2}\sigma^2$.

Proof. See A □

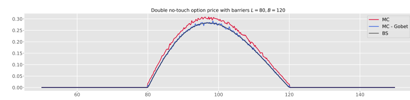

Figure 1: Evolution of the double no-touch option that pays \$1 against S_0

Quadratic Volterra Heston stochastic volatility model

A microstructure-based model for spot and instantaneous variance process with several kernels to plug in that generalized [GJR20]:

$$\begin{cases} dS_t = S_t\sqrt{V_t}dW_t, \\ V_t = a(Z_t - b)^2 + c, \\ Z_t = \xi_0(t) + \displaystyle\int_0^t K(t-s)\sqrt{V_s}dB_s, \end{cases}$$

where $\langle dW, dB\rangle_t = \rho dt$.

[1] Emmanuel Gobet. "Weak approximation of killed diffusion using Euler schemes". In: *Stochastic processes and their applications* 87.2 (2000), pp. 167–197

Model name	$K(t)$	Domain of H	Semi-mart.	Markovian
rough	$\eta t^{H-1/2}$	$(0, 1/2]$	✗	✗
path-dependent	$\eta(t+\varepsilon)^{H-1/2}$	$(-\infty, 1/2]$	✓	✗
one-factor	$\eta\varepsilon^{H-1/2}e^{-(1/2-H)\varepsilon^{-1}t}$	$(-\infty, 1/2]$	✓	✓
two-factor	$\eta_1\varepsilon^{H_1-1/2}e^{-(1/2-H_1)\varepsilon^{-1}t} + \eta_2\varepsilon^{H_2-1/2}e^{-(1/2-H_2)\varepsilon^{-1}t}$	$(-\infty, 1/2]$	✓	✓

Table 1: Different kernels used through the paper, table and names from [AL24].

1.2 Deep learning

Generating synthetic data[2]

Monte Carlo risk engines based on historical events may lead to an overfitting in the backtesting process. Being able to generate unseen yet but coherent market scenarios can be crucial from a risk-management perspective. In mathematical terms, generating realistic market scenarios is "sampling from the joint distribution of risk factors". A simple **parametric** approach to this problem would be calibrating a normal law to the distribution of log-returns and sampling from it. This empirical distribution exhibiting – among other particularities – skewness and kurtosis [Con01], it is often a poor approximation of reality.
Alternative **non-parametric** approaches can be found in Generative Adversarial Network (GAN) [Goo+14], where the target distribution is directly learned from the data without the need for assumptions about the form of it. With this framework though, we are already entering the realm of neural networks.

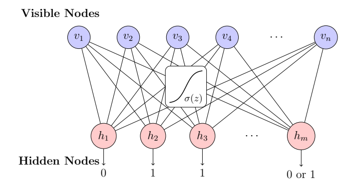

Figure 2: Bernoulli Restricted Boltzmann Machine (RBM) bipartite graph

[2] Alexei Kondratyev and Christian Schwarz. "The market generator". In: *Available at SSRN 3384948* (2019)

References

[Gob00] Emmanuel Gobet. "Weak approximation of killed diffusion using Euler schemes". In: *Stochastic processes and their applications* 87.2 (2000), pp. 167–197.

[Con01] Rama Cont. "Empirical properties of asset returns: stylized facts and statistical issues". In: *Quantitative finance* 1.2 (2001), p. 223.

[Goo+14] Ian Goodfellow et al. "Generative adversarial nets". In: *Advances in neural information processing systems* 27 (2014).

[KS19] Alexei Kondratyev and Christian Schwarz. "The market generator". In: *Available at SSRN 3384948* (2019).

[GJR20] Jim Gatheral, Paul Jusselin, and Mathieu Rosenbaum. "The quadratic rough Heston model and the joint S&P 500/VIX smile calibration problem". In: *arXiv preprint arXiv:2001.01789* (2020).

[AL24] Eduardo Abi Jaber and Shaun Xiaoyuan Li. "Volatility models in practice: Rough, Path-dependent or Markovian?" In: *Path-Dependent or Markovian* (2024).

Appendices

A Double no-touch option derivation

Proof. We write the Black-Scholes equation under it's heat equation version, then using Fourier series as an *ansatz* and the boundary conditions we identify the coefficients and plug everything back in to have the option value. □