# SPX-VIX joint calibration: extending the quintic OU model

October 2023

## 1 Bibliography review

A quintic Ornstein-Uhlenbeck dynamic that jointly calibrates SPX and VIX smiles is introduced in [6, 5].

We wish to extend this model to capture more stylized facts (long memory, roughness, Zumbach effect).
Comparison to other models (path-dependency of volatility [4], rough volatility [3]).

### 1.1 Stylized facts

The equities spot and options market exhibit a lot of empirical properties. A good model should be able to capture and reproduce such stylized facts. Some statistical evidence highlighted in [2] and [7] gives a first set of stylized facts on asset returns:

- Absence of autocorrelation,
- Heavy tails: The tails of the return distributions exhibit power-law-like scaling behaviour. The exponents are consistent with the existence of variances, but the existence of higher moments is not guaranteed.
- Multi-timescale volatility clustering,
- Gain/loss asymettry,
- Leverage effect,
- etc.

- Gaussianity: At relatively high frequencies (less than 6 months) stock returns do not follow a Gaussian distribution. When looking at lower frequencies they appear closer to Gaussian, but the convergence is very slow.

On top of that, some other important effects were later discovered:

**Volatility persistence** analyzed in [1] that introduces a mean-reversion dynamic for stochastic volatility to address the long-memory behaviour.

**Rough volatility**

**Zumbach effect** fkjzenfgjaz

**VIX pricing**

## 1.2 Models

**Guyon**

**Bouchaud**

**Gatheral**

**Abi Jaber**

| | Black-Scholes | Heston | Gatheral | Quintic OU | LeBaron |
|---|---|---|---|---|---|
| Heavy tails | ✗ | ✓ | ✓ | ✓ | ✓ |
| Leverage effect | ✗ | ✗ | ✓ | ✗ | ✗ |
| Zumbach effect | ✗ | ✓ | ✗ | ✓ | ✗ |
| No Martingality | ✗ | ✗ | ✗ | ✗ | ✗ |
| Volatility persistence | ✗ | ✗ | ✗ | ✗ | ✗ |

The hunt for a perfect statistical model for financial markets is still going on.

# References


[1] Fabienne Comte and Eric Renault. Long memory in continuous-time stochastic volatility models. *Mathematical finance*, 8(4):291–323, 1998.

[2] Rama Cont. Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative finance*, 1(2):223, 2001.

[3] Jim Gatheral, Paul Jusselin, and Mathieu Rosenbaum. The quadratic rough heston model and the joint s&p 500/vix smile calibration problem. *arXiv preprint arXiv:2001.01789*, 2020.

[4] Julien Guyon and Jordan Lekeufack. Volatility is (mostly) path-dependent. *Quantitative Finance*, 23(9):1221–1258, 2023.

[5] Eduardo Abi Jaber, Camille Illand, et al. Joint spx-vix calibration with gaussian polynomial volatility models: deep pricing with quantization hints. *arXiv preprint arXiv:2212.08297*, 2022.

[6] Eduardo Abi Jaber, Camille Illand, et al. The quintic ornstein-uhlenbeck volatility model that jointly calibrates spx & vix smiles. *arXiv preprint arXiv:2212.10917*, 2022.

[7] Blake LeBaron. Stochastic volatility as a simple generator of apparent financial power laws and long memory. *Quantitative finance*, 1(6):621–631, 2001.