Numbers & Indices

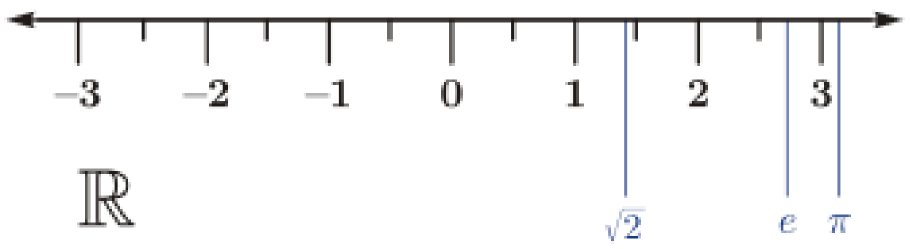
Outline

- Real Numbers
- Rational Numbers
- Irrational Numbers
- Integers
- Natural Numbers
- Prime Numbers
- Rules of Indices

Types of Numbers

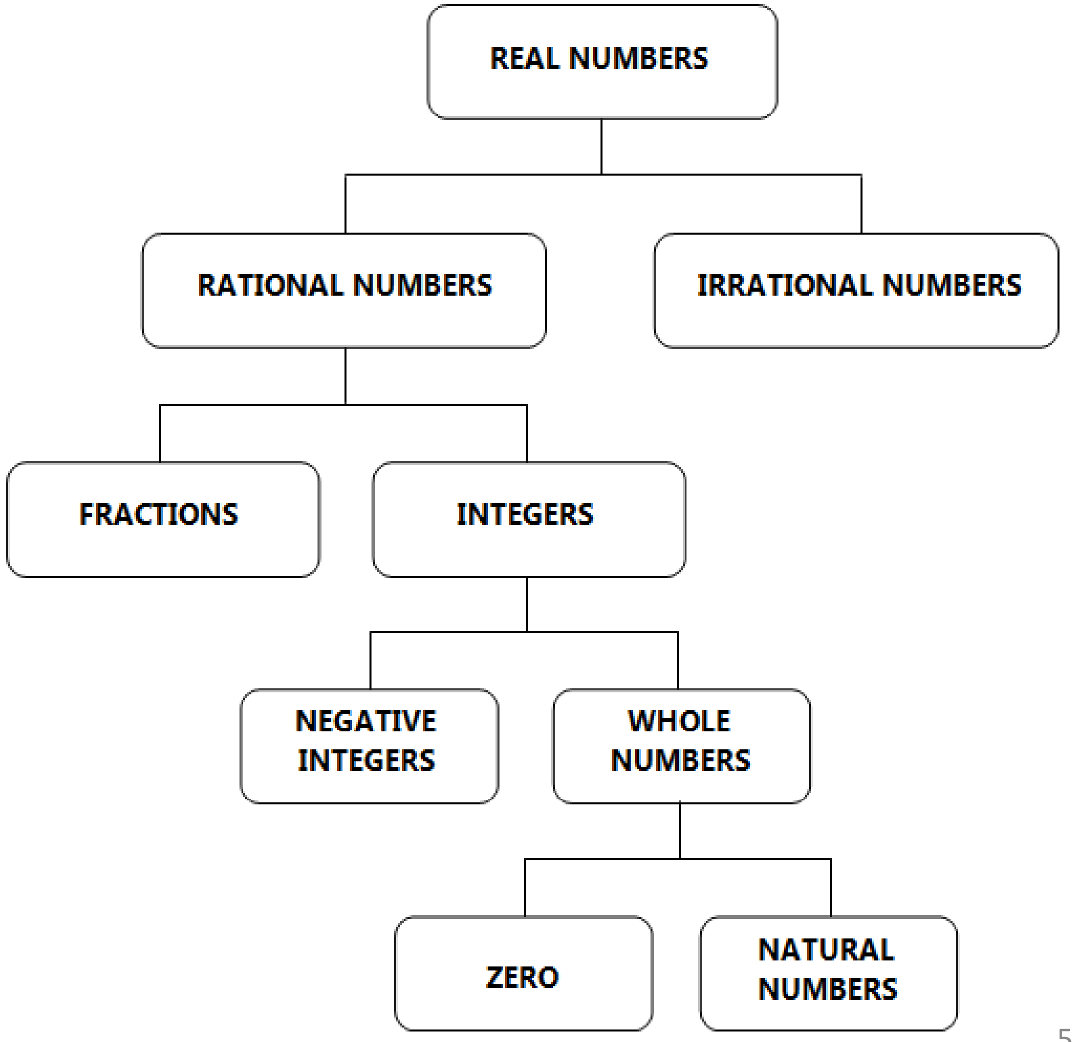
Real Numbers $(\mathbb{R})$

- In Mathematics a real number is a value that represents a quantity along a continuous line.
- The Symbol of the set of real numbers is denoted by: $\mathbb{R}$
- Positive or negative, large or small, whole numbers or decimal numbers are all Real Numbers.



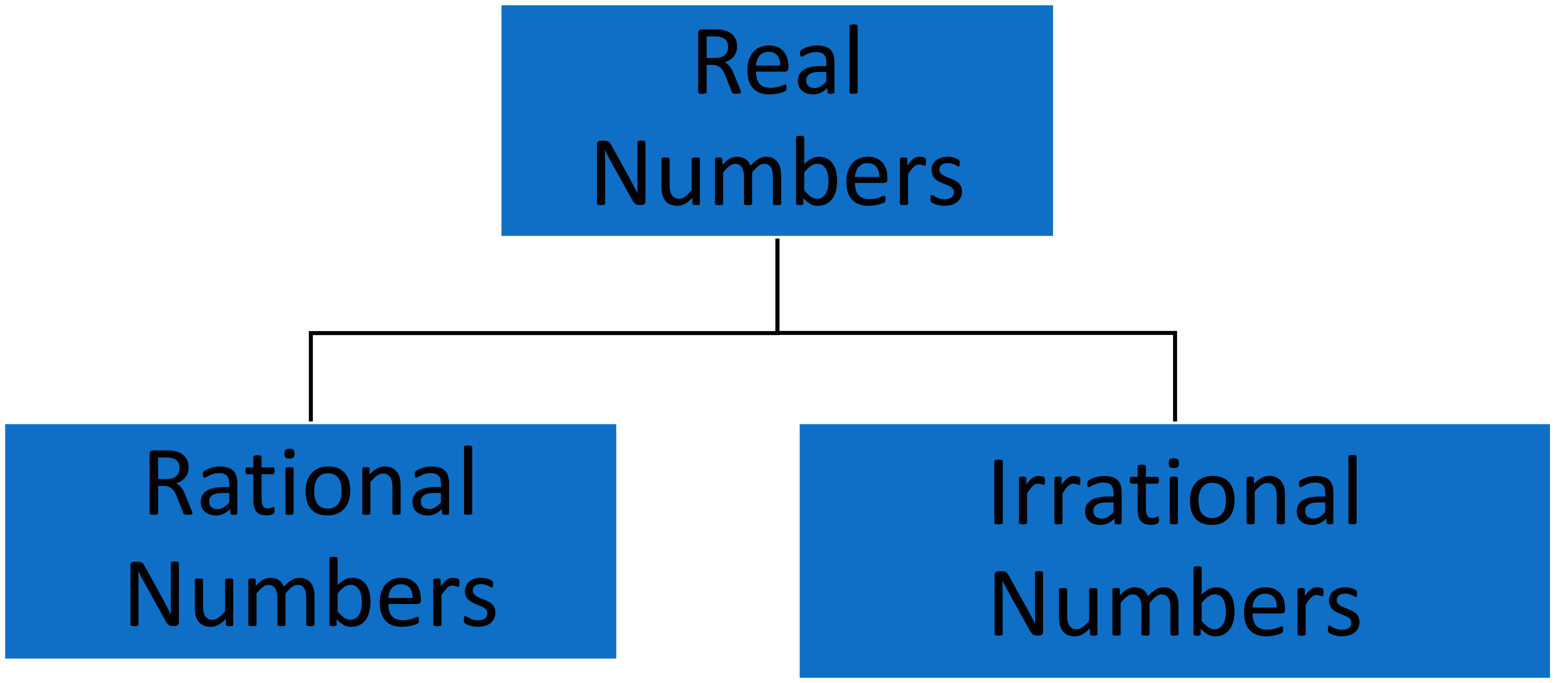
Types of Real Numbers ($\mathbb{R}$)

- There are many types of Real Numbers
- Here are some of them
    - Natural numbers
    - Whole numbers
    - Integers
    - Fractions
    - Rational numbers
    - Irrational numbers



Real Numbers

There are two kinds of real numbers:

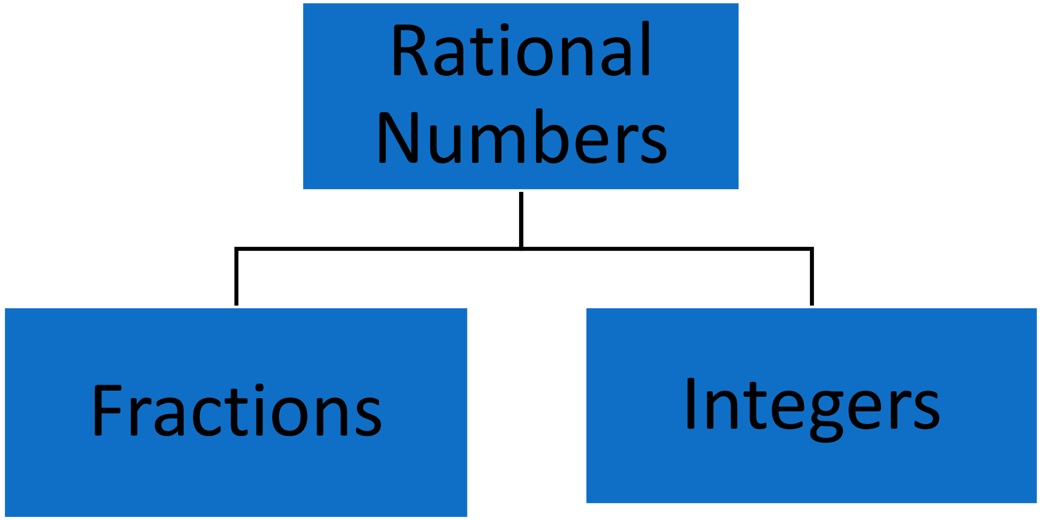


Rational Numbers ($\mathbb{Q}$)

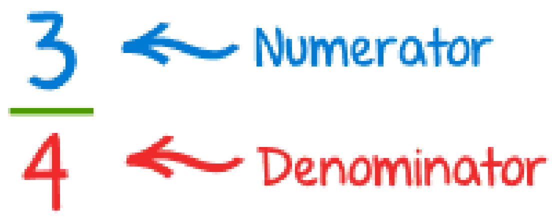
- In mathematics, a rational number is any number that can be expressed as the quotient or fraction p/q of two integers, p and q, with the denominator q not equal to zero.

E.g. $\frac{2}{3}, \frac{-1}{3}, \frac{1}{2}, 5$

Note: 5 is also a rational number since $5 = \frac{5}{1}$



Rational Numbers-Fractions



- We call the top number the **Numerator.**
- We call the bottom number the **Denominator**.

Rational Numbers-Integers ($\mathbb{Z}$)

- Set of numbers that consists of positive and negative whole numbers including zero is called Integers.
- Integers are denoted by ($\mathbb{Z}$)
- We can write them as $\mathbb{Z}$ **= {..., -3, -2, -1, 0, 1, 2, 3, ...}**
- Examples of integers: -16, -3, 0, 1, 198

Irrational Numbers

- If a number cannot be expressed as the quotient or fraction p/q of two integers, p and q, with the denominator q not equal to zero, then, the number is called an irrational number.

E.g. $\sqrt{2}$, π , $\frac{\sqrt{3}}{3}$ are irrational numbers

Different Types of Decimal Numbers

Terminating Decimals	Non terminating - Recurring decimals	Non terminating-Non recurring decimal
• 3/4 = 0.75 • Rational Number	• 2/3 = 0.666... • Rational Number	• $\sqrt{2}$ = 1.414... • Irrational Number

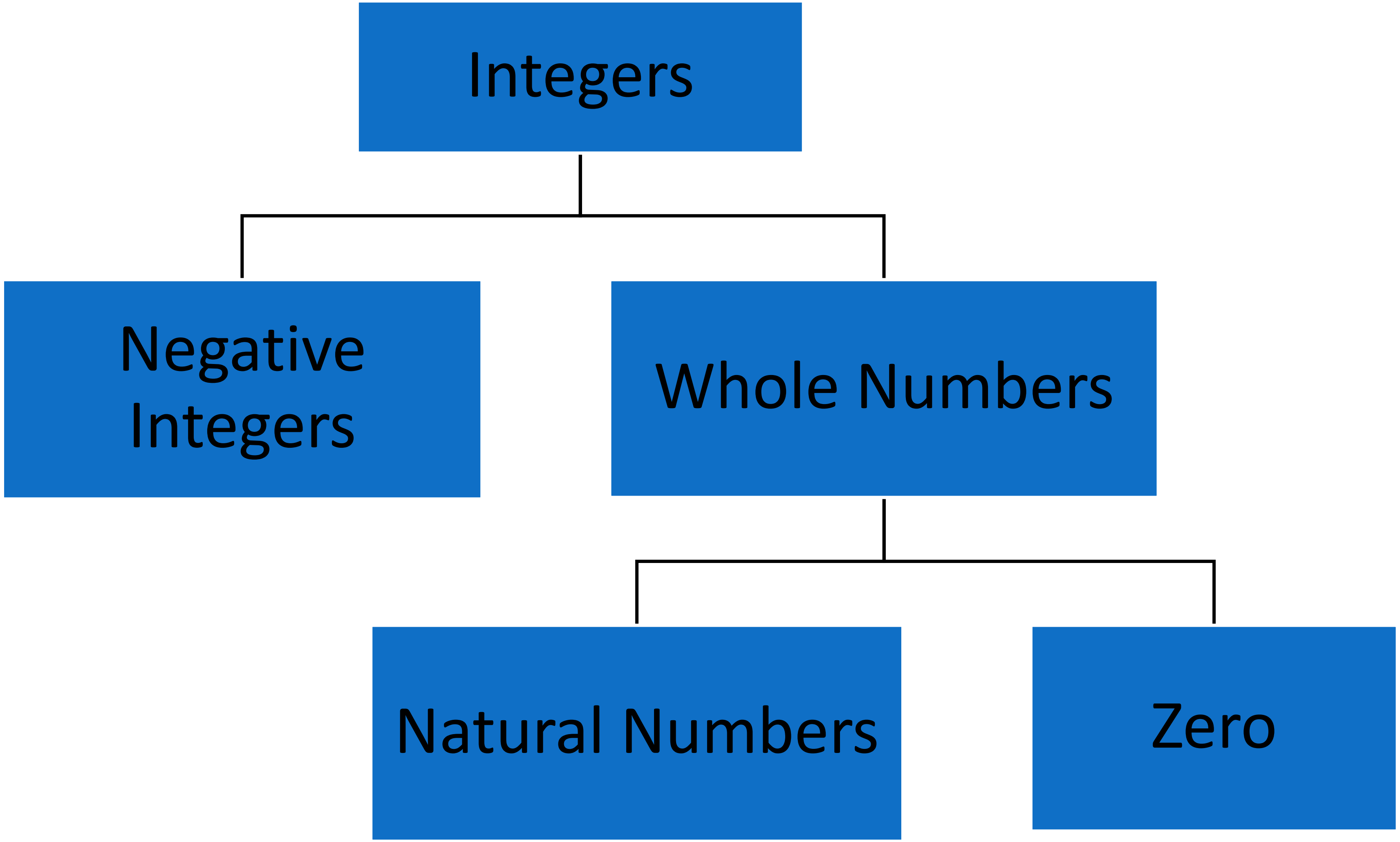
Questions

Q1) Identify which of the following are rational / irrational numbers.

1. 2
2. 5
3. 0.333…
4. 0.666
5. 0.125
6. $\sqrt{5}$
7. $2\sqrt{3}$
8. $\frac{\sqrt{64}}{\sqrt{169}}$

1. $9\frac{5}{4}$
2. $\sqrt{53}$
3. $\frac{2\pi}{7}$
4. $\frac{\sqrt{5}}{5}$
5. -1.508
6. $\sqrt{36}$
7. 10
8. 0.3434…

Integers


Integers
Negative Integers
Whole Numbers
Natural Numbers
Zero

Natural Numbers and Whole Numbers

- Positive integers **excluding zero** are called “Natural Numbers”.
- Includes:
  - the counting numbers **{1, 2, 3, 4 ...}**

- Positive integers **including zero** are called “Whole Numbers”.
- Includes:
  - numbers **{0, 1, 2, 3, 4 ...}**

Prime Numbers

- If a natural number r excluding 1 is divisible by r alone, then r is known as a "**Prime Number**".
  - Example **{ 2, 3, 5, 7, 11, 13, 17 ...}**,

- Numbers which are not prime are called "**Composite Numbers**".

- 0,1 are neither prime nor composite.

Questions

Q2) Find all the prime numbers less than 50.

Q3) Apart from 2 are there any even-numbers that could be treated as Prime Numbers?

Complex Numbers ($\mathbb{C}$)

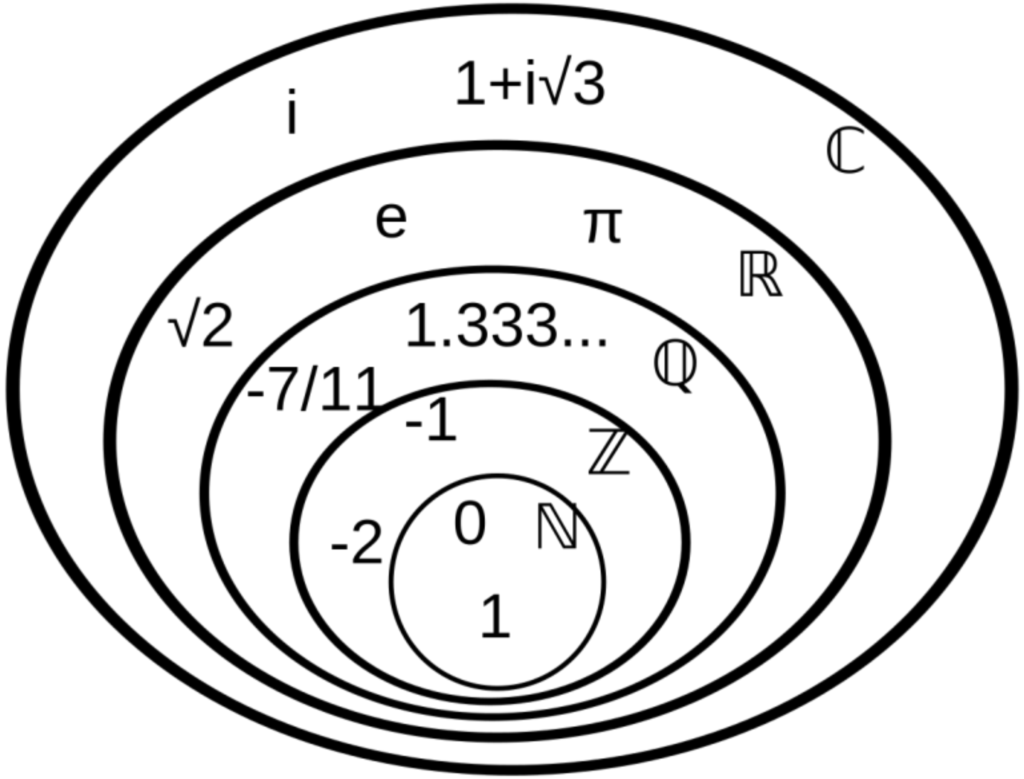
- A number of the form $a + ib$ where a and b are real numbers.

- $i^2 = -1$ where i $= \sqrt{-1}$.

- A complex number has two part:
  - Real part: a
  - Imaginary part: b

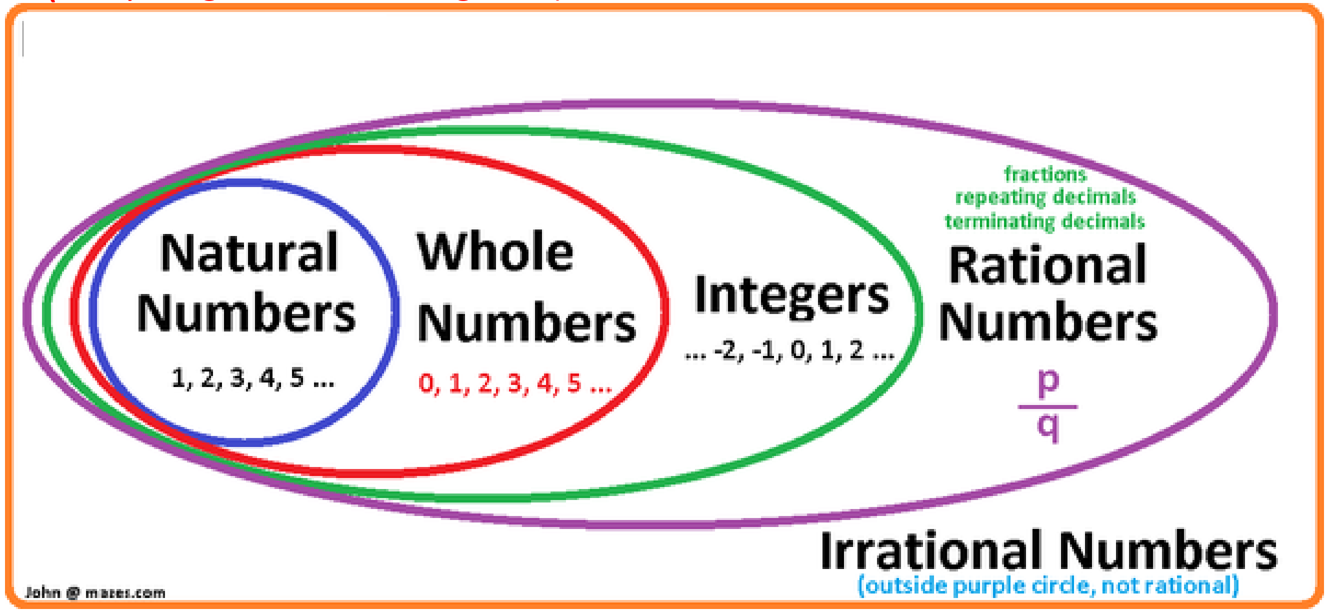
Example: $2 + 3i,\ i,\ -3i$ …

Different Number Sets



Real Numbers

(Everything inside the orange box)



Rules of Indices

Rules of Indices

- Indices are a useful way of more simply expressing large numbers.
- To manipulate expressions, we can consider using the following Laws of Indices.

Rule 1	$a^m \times a^n = a^{(m+n)}$
Rule 2	$(a^m)^n = a^{m \times n}$
Rule 3	$a^m \div a^n = a^{(m-n)}$
Rule 4	$a^m = \frac{1}{a^{-m}}$ and $a^{-m} = \frac{1}{a^m}$
Rule 5	$a^0 = 1$
Rule 6	$a^{\frac{1}{m}} = \sqrt[m]{a}$ and $a^{m/n} = \sqrt[n]{a^m}$
Rule 7	$(ab)^m = a^m \times b^m$
Rule 8	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Examples

1) Rule 1: $2^3 \times 2^2 = 2^{(3+2)} = 2^5 = 32$
2) Rule 2: $(2^3)^2 = 2^{(6)} = 64$
3) Rule 3: $2^3 \div 2^2 = 2^{(3-2)} = 2$
4) Rule 4: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $2^3 = \frac{1}{2^{-3}}$
5) Rule 6: $2^{\frac{1}{3}} = \sqrt[3]{2}$ and $2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4} = 4^{\frac{1}{3}}$
6) Rule 7: $(2 \times 3)^2 = 2^2 \times 3^2 = 4 \times 9 = 36$
7) Rule 8: $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

Answer the following questions

Q1) Find the simplified form of the following. Each expression should have positive exponents.

i. $(p^5)^4$
ii. $(p^4)^5$
iii. $(p^{-5})^4$
iv. $y^3(y^{-5})^2$
v. $(4m)^3$
vi. $\frac{1}{y^{-4}}$
vii. $(2x^3y^{-3})^{-2}$

Questions

Q2) Simplify each expression.

i. $3^2(3x)^3$

ii. $(4.1)^5(4.1)^{-5}$

iii. $(b^5)^3b^2$

iv. $(-5x)^2+5x^2$

v. $(-2a^2b)^3(ab)^3$

vi. $(2x^{-3})^2(0.2x)^2$

vii. $4\mathrm{x}y^2 0^4(-y)^{-3}$

viii. $(10^3)^4(4.3 \times 10^{-8})$

ix. $(3^7)^2(3^{-4})^3$

x. $(2xy^4)(xy)^6$

Questions

Q3) Complete each equation.

i. $(b^2)^{\square} = b^8$

ii. $\left(m^{\square}\right)^3 = m^{-12}$

iii. $\left(x^{\square}\right)^7 = x^6$

iv. $(n^9)^{\square} = 1$

v. $(y^{-4})^{\square} = y^{12}$

vi. $7(c^1)^{\square} = 7c^8$

vii. $\left(5x^{\square}\right)^2 = 25x^{-4}$

viii. $\left(3x^3y^{\square}\right)^3 = 27x^9$

ix. $(m^2n^3)^{\square} = \frac{1}{m^6n^9}$

x. $\frac{a^{\square}b^2}{a^2b^{\square}} = \frac{a}{b^2}$

Questions

Q4) Solve each equation. Use the fact that if a^x=a^y then $x = y$.

i. $5^x = 25^x$

ii. $3^x = 27^4$

iii. $8^2 = 2^x$

iv. $4^x = 2^6$

v. $3^{2x} = 9^4$

vi. $2^x = \frac{1}{32}$

vii. $(27)^{2/3} = 3^x$

Questions

Q5) What is the simplified form of each expression?

i. $\frac{y^5}{y^4}$

ii. $\frac{d^3}{d^9}$

iii. $\frac{k^6 j^2}{k j^5}$

iv. $\frac{a^{-3} b^7}{a^5 b^2}$

v. $\frac{x^4 y^{-1} z^8}{z x^4 y^5}$

Thank You !