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Abstract. In this article we show the precise algorithm of functioning of quantum adder on the example of addition of two 2-bit numbers. It consists of the quantum Fourier transformer and conditional rotation gates that let us use the minimum number of qubits to get the addition realization of the sum. Despite the fact that the algorithm uses the minimum amount of operations this accelerates the process of the adder.

1. Introduction

Recently quantum technologies have developed headily. There are almost no fundamental problems left.

The benefit of quantum technologies nowadays is pretty obvious. Those things that some years ago were just theoretical now are experimentally approved and efficient. Although in modern science there is always a place for new ideas. One of those ideas is a quantum adder.

A quantum adder is a physical machine that can accept input states which represent a coherent superposition of many different possible inputs and subsequently evolve them into a corresponding superposition of outputs. Computation, i.e. a sequence of unitary transformations, affects simultaneously each element of the superposition, generating a massive parallel data processing albeit within one piece of quantum hardware [1]. This way quantum computers can efficiently solve some problems which are believed to be intractable on any classical computer [2,3].

First quantum addition algorithms mirrored their classical counterparts [4,5,6,] with necessary extensions for a reversible computation. Further quantum addition algorithms were based on the use of the complementary qubit transfer [7,8], but it still followed the classical model. The ideal addition algorithm for a quantum computer may not be similar to its classical counterpart, so that's why the quantum adder on the base of quantum Fourier transformation (QFT) was invented (Figure.1).

The horizontal lines are wires that carry the qubits. In the upper lines is encoded number A, presented in binary type. The number B is encoded in the lowest lines. Computing the Quantum Fourier Transformer may be done according to the following wire diagram (Figure.2). Each gate can be depicted as a matrix form. (Figure.3).
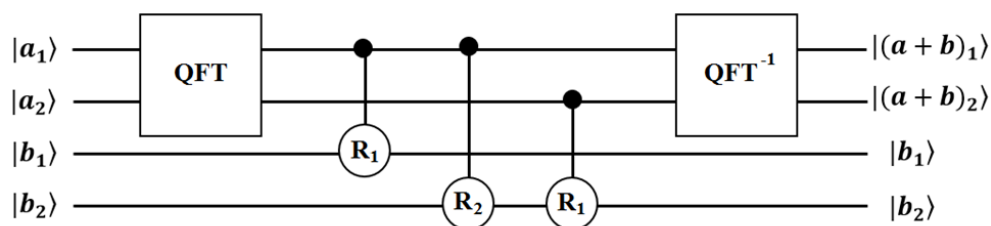
Figure 1. The quantum addition scheme based on the quantum Fourier transformer.
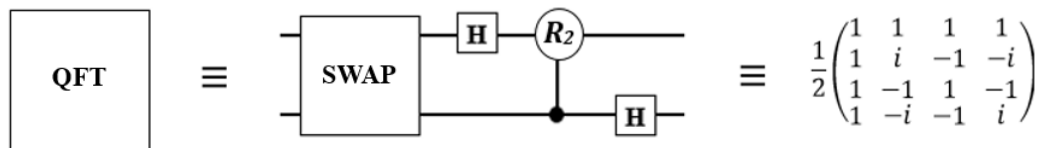
Figure 2. a) The Quantum Fourier Transformer scheme and it's mathematical form.

Figure 3. a) Swap gate. b) Hadamard transform. c) Conditional rotation.

2. Example «1+2»

For better understanding let's look upon the algorithm of addiction of two classical numbers 1 and 2. Thus, the number A is a 1, and the number B is a 2. Let's see what will happen when those numbers enter the quantum adder. Decimal number "1" in binary system is shown as "01". We have this number encoded by two qubits. In order to have quantum-mechanical characters for our qubits, we tensorially multiply them and get the next vector.

After this, our numbers enter QFT where they can be depicted as a matrix form. To know what will happen to our number afterwards, we multiply the matrix and the vector and get the following state. Then we take one qubit of "A" number and one qubit of "B" number. The result of their tensorial multiplication we apply to the conditional rotation gate.

We point out that all R1 can be done simultaneously as they include different qubits. This also applies to R2, etc. As a result, the number of operations is still the same, but the time necessary to the calculation declines in N times and can be taken as O (log2n).

But how can we distinguish the answer? In order to do it we use the reversal quantum Fourier transformation. Having multiplied the matrix to the obtained vector we will receive the final result.

You can see below the precise description of the following calculations.

2.1. Tensor product

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

2.2. Application of the QFT

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ i \\ -1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

2.3. Tensor product

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ i \end{pmatrix}$$

2.4. Application of the controlled phase modification operator (k=1)

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2.5. Tensor product

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix}$$

2.6. Application of the controlled phase modification operator (k=2)

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

2.7. Tensor product

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

2.8. Application of the controlled phase modification operator (k=1)

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

2.9. Tensor product

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix}$$

2.10. Application of the QFT[1]

$$\frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} 1 \\ -i \\ -1 \\ i \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle \otimes |1\rangle = 11_2 = 3_{10}$$

3. Conclusions

In this research we demonstrated the simplest scheme of computations on the quantum computer. The presented algorithm helps to understand in the best way how the quantum computations of classical numbers are made. This article has the educational character and aims to convince that the quantum informatics is a very appealing subject and it's not that hard to learn if you try to do those calculations by yourself with the help of the quantum adder. On the example of that research, we've shown you how quantum technologies can be applied for calculations of greater numbers and for solving more complicated tasks. On the base of that algorithm, we plan to elaborate the quantum multiplier, which will be especially useful in quantum cryptography.

4. Acknowledgments

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