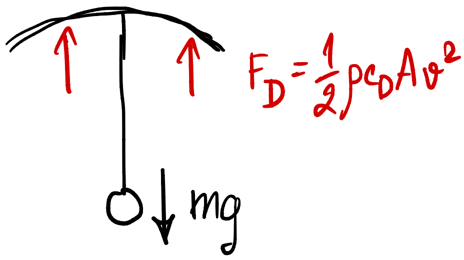
สัญลักษณ์ฟังก์ชัน เหมือน $f(x)$ ไม่ใช่ v คูณกับ t

ผลเฉลยสมการความเร็วจาก $v(t)$ ซึ่งมีแรงต้านอากาศ



จาก Newton's :
$$\Sigma \vec{F} = m\frac{d\vec{v}}{dt}$$

$$mg - \frac{1}{2}\rho C_D A v^2 = m\frac{dv}{dt}$$

ให้ $\alpha = \frac{1}{2}\rho C_D A$:
$$g - \frac{\alpha}{m}v^2 = \frac{dv}{dt}$$

$$\int_{t=0}^{t} 1\,dt = \Delta t = \int_{v(0)}^{v(t)} \frac{1}{g - \frac{\alpha}{m}v^2}\,dv$$

จัดรูปเป็น $\frac{m}{\alpha}\int_{v(0)}^{v(t)} \frac{1}{\frac{mg}{\alpha} - v^2}\,dv$

จากผลเฉลยของ
$$\int \frac{1}{a^2 - u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C = \frac{1}{a}\operatorname{arctanh}\left(\frac{u}{a}\right) + C = \frac{1}{a}\operatorname{arccoth}\left(\frac{u}{a}\right) + C$$

ใช้ผลเฉลยนี้เพราะ $\frac{u}{a} \in [-1,1] \rightarrow \frac{u^2}{a^2} \in [0,1]$

เมื่อคิด $u = v$ และ $a = \sqrt{\frac{mg}{\alpha}}$

หมายถึงว่า $0 \leq \frac{\alpha v^2}{mg} \leq 1$

พจน์แรงต้านอากาศ

พจน์น้ำหนัก

อธิบายเป็นภาษามนุษย์คือ แรงต้านอากาศน้อยกว่า/เท่ากับน้ำหนักเสมอ!

ได้ว่า $t = \Delta t = \sqrt{\frac{m}{\alpha g}}\operatorname{arctanh}\left(\sqrt{\frac{\alpha}{mg}}\,v\right)\Big|_{v=v(0)}^{v(t)}$

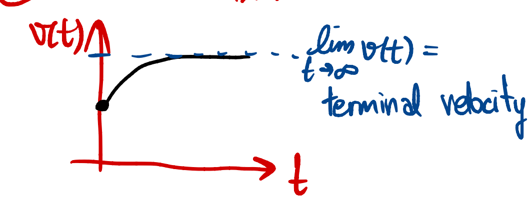
v ที่ $t=0$ หรือ "v เริ่มต้น"

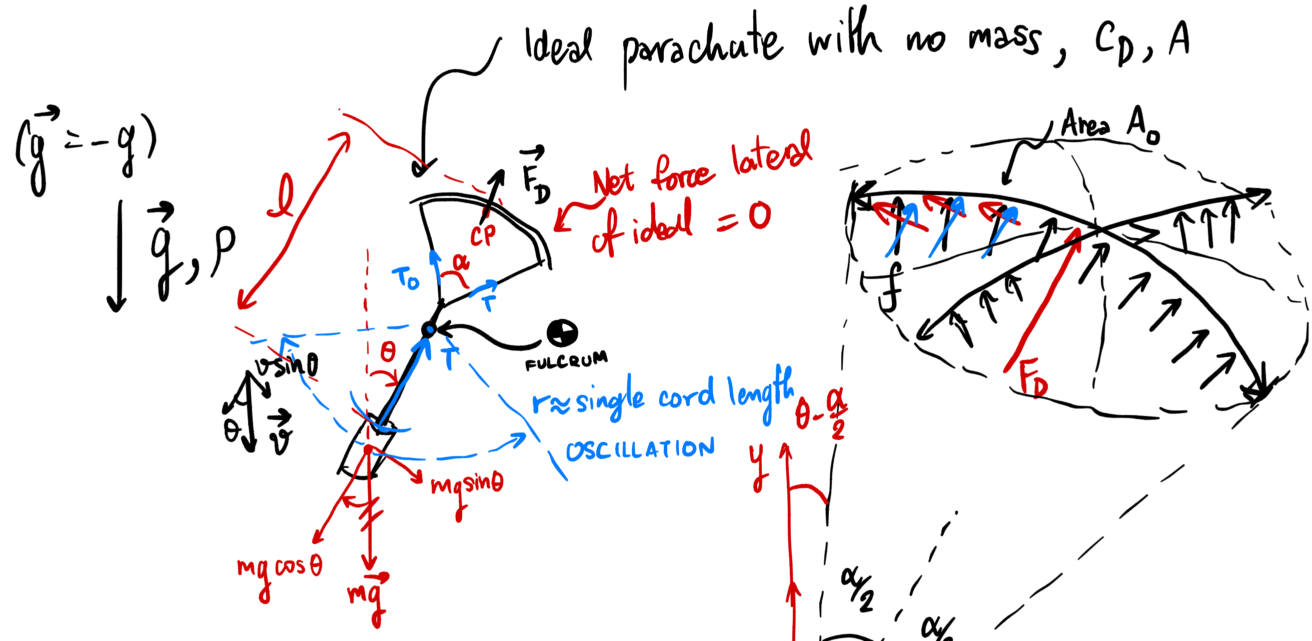
$$\sqrt{\frac{\alpha g}{m}}\Delta t = \operatorname{arctanh}\left(\sqrt{\frac{\alpha}{mg}}\,v(t)\right) - \operatorname{arctanh}\left(\sqrt{\frac{\alpha}{mg}}\,v(0)\right)$$

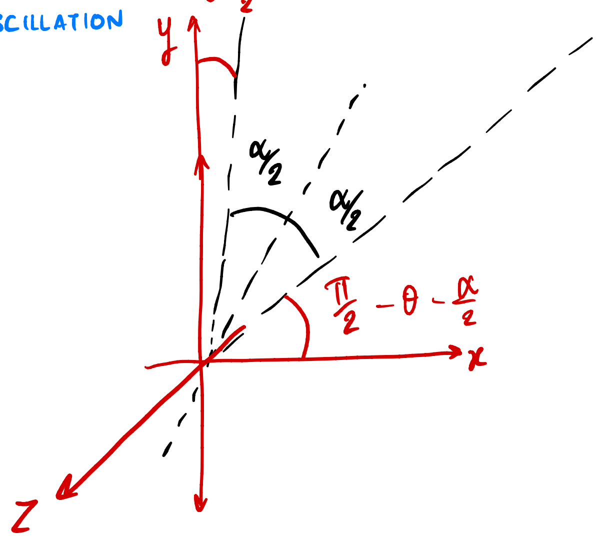
จัดรูป :
$$v(t) = \sqrt{\frac{mg}{\alpha}}\tanh\left(\operatorname{arctanh}\left(\sqrt{\frac{\alpha}{mg}}\,v(0)\right) + \sqrt{\frac{\alpha g}{m}}\,t\right) \quad ; \alpha = \frac{1}{2}\rho C_D A$$

$$\therefore v(t) = v_0 \pm \sqrt{\frac{mg}{\alpha}}\tanh\left(\sqrt{\frac{\alpha g}{m}}\,t\right)$$

นำไปพล็อตกราฟ $v(t) - t$ ได้







$$\vec F_{D(x,y)} = \oint_L \vec f \, d\theta_{yx}$$

$$\boxed{\vec F_{D(x,y,z)} = \oiint_A \vec f \, d\theta_{yx} \, d\theta_{yz}}$$

with integration boundary of $\left[\theta - \frac{\alpha}{2}, \frac{\pi}{2} - \theta - \frac{\alpha}{2}\right]$

while f is composite force of aerodynamic drag $F.D. = \frac{1}{2}\rho c_D A v^2$

$$\therefore f(v) = \frac{1}{2}\rho c_D A_{xz} v^2 = \alpha_{xz} v^2 .$$

Newton's 2nd:

$$\Sigma \vec F = m \frac{d\vec v}{dt}$$

$$\vec F_D + m\vec g = m \frac{d\vec v}{dt}$$

$$\vec F_{D(x,y,z)}(\vec v) + m\vec g = m \frac{d\vec v}{dt}$$

$$\int_0^t \frac{1}{m} \, dt = \int_{v_0}^{v} \frac{1}{\vec F_{D(x,y,z)}(\vec v) + m\vec g} \, d\vec v$$

$$\frac{t}{m} = \int_{v_0}^{v} \left\{ mg - \alpha v^2 \right\}^{-1} d\vec v$$

$$\frac{t}{m} = \frac{1}{\sqrt{\alpha m g}} \operatorname{arctanh}\left(\sqrt{\frac{\alpha}{mg}}\, v\right)\Bigg|_{v_0}^{v}$$

$$2t\sqrt{\frac{\alpha g}{m}} = \ln\left[\frac{1+\sqrt{\frac{\alpha}{mg}}\,v}{1-\sqrt{\frac{\alpha}{mg}}\,v}\right] - \ln\left[\frac{1+\sqrt{\frac{\alpha}{mg}}\,v_0}{1-\sqrt{\frac{\alpha}{mg}}\,v_0}\right]$$

$$2t\sqrt{\frac{\alpha g}{m}} = \ln\left[\frac{(1+kv)}{(1-kv)}\frac{(A)}{(B)}\right]$$

$$(1+kv)A = Be^{2t\sqrt{\frac{\alpha g}{m}}}(1-kv)$$

$$A + Akv = Be^{\{\}} - Be^{\{\}}kv$$

$$v(Ak + Be^{\{\}}k) = Be^{\{\}} - A$$

$$v = \frac{Be^{\{\}} - A}{k(Be^{\{\}} + A)}$$

<u>DESCENT</u> : $\therefore$ $$\boxed{v(t) = \sqrt{\frac{mg}{\alpha}}\,\frac{\left(1+\sqrt{\frac{\alpha}{mg}}\,v_0\right)e^{2\sqrt{\frac{\alpha g}{m}}t} - \left(1-\sqrt{\frac{\alpha}{mg}}\,v_0\right)}{\left\{\left(1+\sqrt{\frac{\alpha}{mg}}\,v_0\right)e^{2\sqrt{\frac{\alpha g}{m}}t} + \left(1-\sqrt{\frac{\alpha}{mg}}\,v_0\right)\right\}}}$$

$v \rightarrow v\cos\theta$

$v_0 \rightarrow v_0\cos\theta$

$\alpha \rightarrow \alpha_{xz}$

Oscillation

$$\Sigma F = m\frac{d^2x}{dt^2} \quad ; \quad \theta = \frac{x}{l}$$

$$\cancel{m}g\sin\theta = -\cancel{m}r\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{r}\sin\theta = 0$$

*Given that descending mass with parachute has approximately inertial frame of reference as the velocity is slowly increasing.

In oscillation, realistic parachute calc. method is required.

The F_D in lateral component must be included:

$$mg\sin\theta + F_D + mr\frac{d^2\theta}{dt^2} = 0 \quad ; \quad F_D = \overbrace{\frac{1}{2}\rho C_D A_{xy}}^{\alpha_{xy}} v^2\sin^2\theta$$

$$mr\frac{d^2\theta}{dt^2} + mg\sin\theta + \alpha_{xy}v^2\sin^2\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{r}\sin\theta + \frac{\alpha v^2}{mr}\sin^2\theta = 0$$

Thus, the Period of oscillation will follow small-angle approximation if $\theta \ll 1$ and $\sin\theta > \sin^2\theta$. The $\frac{\alpha v^2 \sin^2\theta}{mr}$ term will be negligible:

$$T = 2\pi\sqrt{\frac{r}{g}}\sum_{n=0}^{\infty}\left(\left(\frac{(2n)!}{(2^n n!)^2}\right)^2 \cdot \sin^{2n}\frac{\theta}{2}\right)$$

$$T \approx 2\pi\sqrt{\frac{r}{g}} \quad \text{(small-angle approximation)}$$

But the term v^2 can be formulated as $\left(\frac{dx}{dt}\right)^2 = \left(r\frac{d\theta}{dt}\right)^2$

$$mr\frac{d^2\theta}{dt^2} + mg\sin\theta + \alpha\cdot\left((l-r)\frac{d\theta}{dt}\right)^2\sin^2\theta = 0$$

$$\therefore \boxed{\frac{d^2\theta}{dt^2} + \frac{g}{r}\sin\theta + \alpha(l-r)^2\left(\frac{d\theta}{dt}\right)^2\sin^2\theta = 0}$$

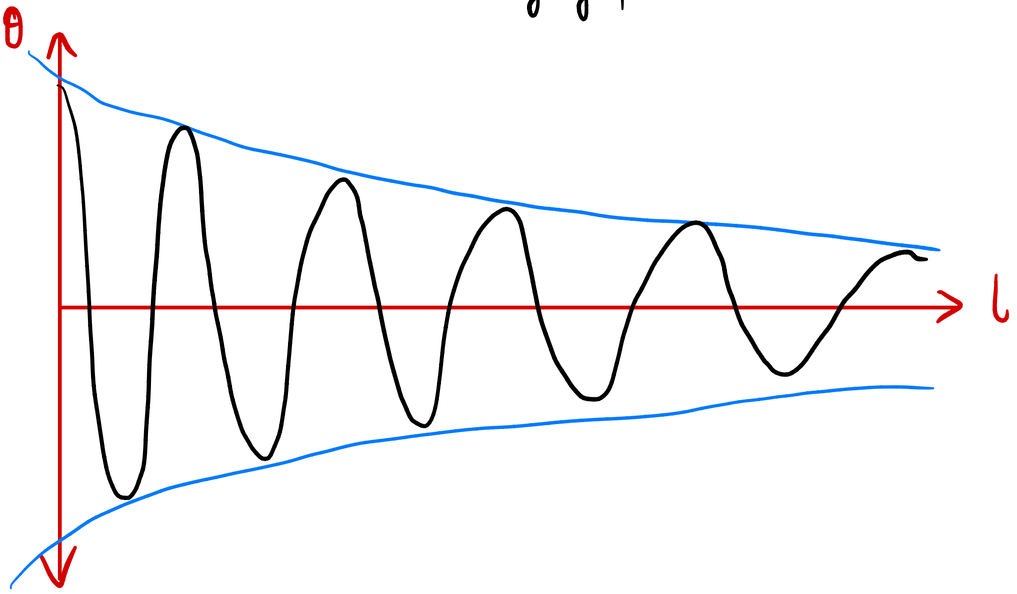
$\alpha \to \alpha_{xy}$

OSCILLATION

This equation describes the motion of oscillation of a descending mass object (payload) attached to parachute.

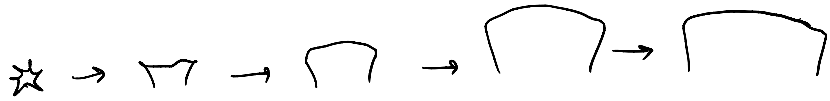
The first and second term represents oscillation with full amplitude $\theta_0 / r(1-\cos\theta_0)$, $\omega^2 = \frac{g}{r}$, and period $2\pi\sqrt{\frac{r}{g}}$.

The third term describes decaying pattern of the oscillation.

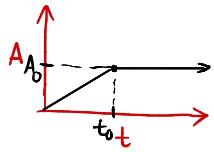


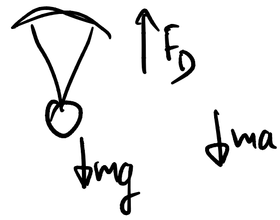
Which is similar to damped oscillation with solution of $\theta(t) = Ae^{-\gamma t}\cos(\omega t)$.

PARACHUTE OPENING : Linear Approximation



① $A = \begin{cases} A_0\left(\frac{t}{t_0}\right) & 0 \le t \le t_0 \\ A_0 & t > t_0 \end{cases}$





② $A = A_0\left(\frac{1-e^{-kt}}{1+e^{-kt}}\right)$

③ $A = A_0 \tanh(kt)$

④ $A = A_0(1-e^{-kt})$

$$mg - \frac{1}{2}\rho C_D \underline{A(t)}\,\underline{v}^2 = m\frac{\cancel{dv}}{\cancel{dt}}$$

สมมติ $A(t) = A_0 t$

$$\rightarrow \quad mg - ktv^2 = m\frac{dv}{dt}$$

$$g - \frac{k}{m}\cdot t\cdot v^2 = \frac{dv}{dt}$$

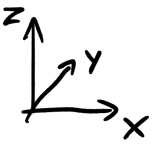
$$\frac{g}{t} - \frac{k}{m}v^2 = \frac{dv}{dt}$$

$$u - v = \frac{dv}{du}$$

Double Pendulum with 2 Parachute

Consider a 3D system :

* m_1 and m_2 is approx. point mass
* using coordinates



and ZYX vectors (Tait-Bryan) $V(\Psi, \theta, \phi)$