

Infinite Series Test

Test	Series	Conv. if	Div. if	Note
Test for Divergence	$\sum_{n \geq 1} a_n$	-	$\lim_{n \rightarrow \infty} a_n \neq 0$	Inconclusive if the limit equals zero.
Geometric Series Test (GST)	$\sum_{n \geq 1} ar^{n-1}$	$ r < 1$	$ r \geq 1$	Geometric Series if there is a common ratio $S_n = \frac{a}{1-r}$.
P-Series Test	$\sum_{n \geq 1} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	Harmonic series ($p=1$) are useful for comparison tests.
Integral Test (IT)	$\sum_{n \geq 1} a_n$ $a_n = f(x)$	$\int_1^{\infty} f(x) dx$ conv.	$\int_1^{\infty} f(x) dx$ div.	$f(x)$ must be - continuous, - positive, - decreasing.
Direct Comparison Test (DCT)	$\sum_{n \geq 1} a_n$	$0 \leq a_n \leq b_n$, and $\sum b_n$ conv.	$0 \leq b_n \leq a_n$, and $\sum b_n$ div.	- Use larger conv. series to show conv. - Use larger div. series to show div.
Limit Comparison Test (LCT)	$\sum_{n \geq 1} a_n$	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} \geq 0$, and $\sum b_n$ conv.	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} > 0$, $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ div.	-
Alternating Series Test (AST)	$\sum_{n \geq 1} b_n$, $b_n = (-1)^{n+1} \cdot a_n \geq 0$	$b_{n+1} \leq b_n$, $\lim_{n \rightarrow \infty} b_n = 0$	$\lim_{n \rightarrow \infty} b_n \neq 0$	Must prove that limit equals zero. Must prove that b_n is decreasing for $n \geq 1$
Absolute Convergence Test (ACT)	$\sum_{n \geq 1} a_n$	$\sum_{n \geq 1} a_n $ conv.	-	Inconclusive if absolute series div.
Ratio Test	$\sum_{n \geq 1} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1$ $\sum a_n$ also abs. conv.	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1$	Inconclusive if $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = 1$. Useful with term $n!$ or n^{th} power.
Root Test	$\sum_{n \geq 1} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } < 1$ $\sum a_n$ also abs. conv.	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } > 1$	Inconclusive if $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = 1$. Useful with term n^{th} power.

The List

$$\lim_{n \rightarrow \infty} \ln n \ll n^p \ll b^n \ll n! \ll n^n ; p > 0, b > 1$$

Flowchart for most Series

Test for Divergence \rightarrow Root/Ratio Test \rightarrow AST \rightarrow DCT/LCT