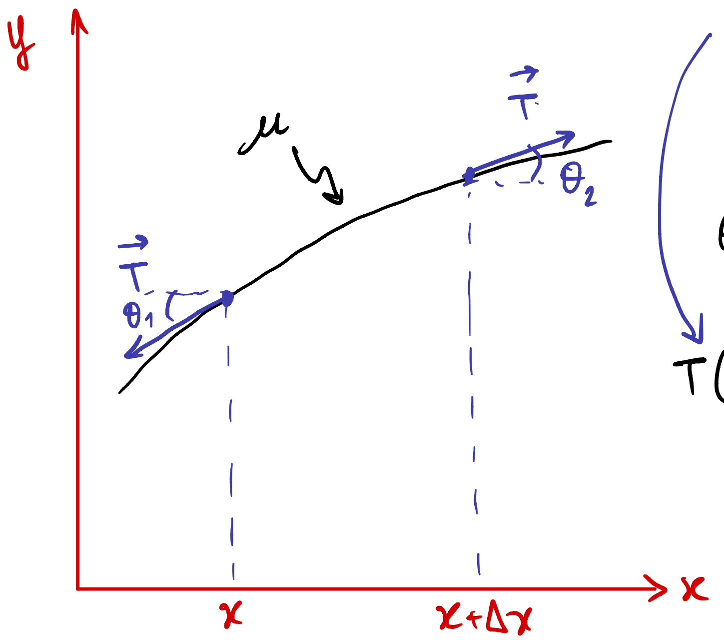
WAVE



$$y:\ T\sin\theta_2 - T\sin\theta_1 = m\ddot{y}$$

$$x:\ T\cos\theta_2 - T\cos\theta_1 = m\ddot{x}$$

$$\theta \approx 0:\quad \theta \approx \sin\theta \approx \tan\theta, \quad \cos\theta \approx 1$$

$$T(\tan\theta_2 - \tan\theta_1) = m\ddot{y}$$

$$T\left(\left.\frac{\partial y}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial y}{\partial x}\right|_{x}\right) = m\frac{\partial^2 y}{\partial t^2}$$

$$T\cancel{\Delta x}\frac{\partial^2 y}{\partial x^2} = \mu\cancel{\Delta x}\frac{\partial^2 y}{\partial t^2}$$

$$\rightarrow \frac{\partial^2 y}{\partial x^2} - \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = 0$$

$$\rightarrow \frac{\mu}{T} = \frac{1}{v^2}\ ;\ v = \sqrt{\frac{T}{\mu}}$$

Def.

$$\frac{dy}{dx} \equiv \lim_{x\to 0}\frac{y(x+\Delta x) - y(x)}{\Delta x}$$

$$\Delta x \cdot \frac{dy}{dx} = \lim_{x\to 0} y(x+\Delta x) - y(x)$$

$$y(x) = \frac{dy}{dx} \rightarrow \Delta x\frac{d^2 y}{dx^2} = \left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}$$

Sol'n

$$\boxed{y(x,t) = y(x \boxed{\pm} vt)}$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial y}{\partial z}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial^2 y}{\partial z^2}$$

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial t} = -v\cdot\frac{\partial y}{\partial z}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left(-v\frac{\partial y}{\partial z}\right) = -v\frac{\partial}{\partial z}\left(\frac{\partial y}{\partial t}\right)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial z^2}$$

$$\therefore\ \frac{1}{v^2}\cdot\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial z^2}$$

ELECTROMAGNETIC (*)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0(I + I_d) = \mu_0\left(I + \epsilon_0\frac{d\Phi_E}{dt}\right)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0\epsilon_0\frac{d\vec{E}}{dt}$$

ELECTROMAGNETIC WAVE

- **Plane wave:**

Assume $\vec{E} = \hat{k} E_z(y,t)$, $\vec{B} = \hat{i} B_x(y,t)$

and use def $\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_A \vec{B} \cdot d\vec{A} = -\frac{\partial \Phi_B}{\partial t}$,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \iint_A \vec{E} \cdot d\vec{A} = \mu_0 \epsilon_0 \frac{\partial \Phi_E}{\partial t}$$

$\rightarrow$ $\oint \vec{E} \cdot d\vec{A} = 0$ and $\oint \vec{B} \cdot d\vec{A} = 0$

$\rightarrow$ XZ-Plane : $\oint \vec{E} \cdot d\vec{s} = 0$ $\qquad$ $\oint \vec{B} \cdot d\vec{s} = 0$

YZ-Plane : $\boxed{\frac{\partial}{\partial y} E_z = -\frac{\partial}{\partial t} B_x}$ —(1) $\qquad$ $\oint \vec{B} \cdot d\vec{s} = 0$

XY-Plane : $\oint \vec{E} \cdot d\vec{s} = 0$ $\qquad$ $\boxed{\frac{\partial}{\partial y} B_x = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} E_z}$ —(2)

(1): $\frac{\partial^2 E_z}{\partial y^2} = -\frac{\partial}{\partial y}\left(\frac{\partial}{\partial t} B_x\right) = -\frac{\partial}{\partial t}\left(\frac{\partial}{\partial y} B_x\right)$

$$* \boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} *$$

$\boxed{\frac{\partial^2}{\partial y^2} E_z = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E_z}$: WAVE EQ.

$\frac{1}{v^2} = \mu_0 \epsilon_0 \rightarrow v = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 299,792,458 \ m \cdot s^{-1} \approx 3 \times 10^8 \ m \cdot s^{-1}$

Wave form : $F(y \pm vt)$: $\vec{E} = \hat{k} E_0 \sin(ky - \omega t)$, $\vec{B} = \hat{i} B_0 \sin(ky - \omega t)$

$\left.\begin{array}{l} \frac{\partial E_z}{\partial y} = k E_0 \cos(ky - \omega t) \\ \frac{\partial}{\partial t} B_x = -\omega B_0 \cos(ky - \omega t) \end{array}\right\}$ $k E_0 = \omega B_0$, $\boxed{E_0 = \frac{\omega}{k} B_0}$ $\rightarrow$ $\frac{\omega}{k} = \frac{k}{\omega} c^2$

$\therefore \boxed{c = \frac{\omega}{k} = \frac{E_0}{B_0}} = v$

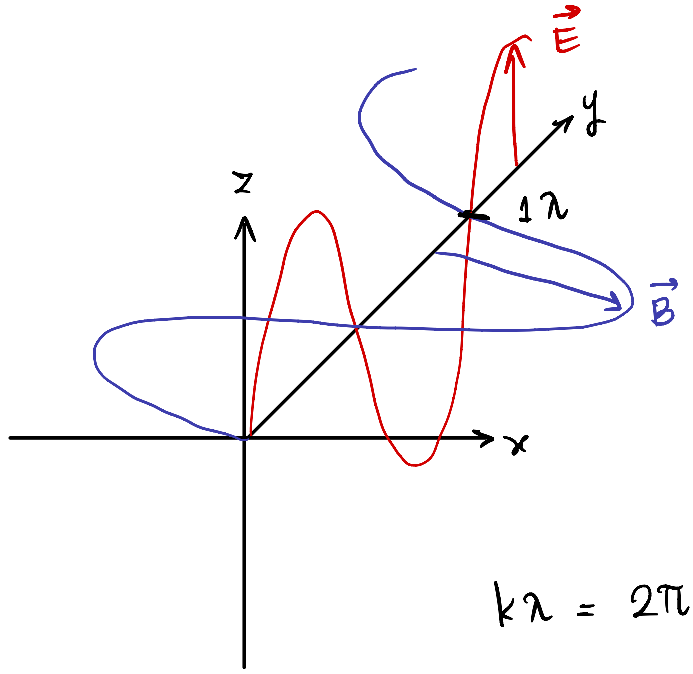
$\rightarrow E_0^2 = c^2 B_0^2$

$\left.\begin{array}{l} \frac{\partial}{\partial y} B_x = k B_0 \cos(ky - \omega t) \\ \frac{\partial}{\partial t} E_z = -\omega E_0 \cos(ky - \omega t) \end{array}\right\}$ $k B_0 = \mu_0 \epsilon_0 \omega E_0$

$E_0 = \frac{k}{\mu_0 \epsilon_0 \omega} B_0$

$\boxed{E_0 = \frac{k}{\omega} c^2 B_0}$

$\boxed{E_0^2 = \frac{1}{\mu_0 \epsilon_0} B_0^2}$



Propagation (y): $\vec{E}\times\vec{B}$

$$\begin{cases}\vec{E} = \hat{k}E_0\sin(ky-\omega t)\\ \vec{B} = \hat{i}B_0\sin(ky-\omega t)\end{cases}$$

$$\rightarrow \vec{E} = \hat{k}E_0\sin[\underbrace{k(y-\tfrac{\omega}{k}t)}_{y\pm vt\ \leftrightarrow\ y-ct}]$$

$$k\lambda = 2\pi \rightarrow \boxed{k = \frac{2\pi}{\lambda}} = \text{"Number of repetition" per length.}$$

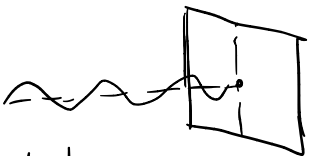
$$\omega T = 2\pi \rightarrow \boxed{\omega = \frac{2\pi}{T}} = \text{"Number of repitition" per time.}$$

$\epsilon_0 = 8.85\times10^{-12}$

$\mu_0 = 4\pi\times10^{-7}$

- Energy

$$\left.\boxed{\begin{aligned}U_E &= \frac{1}{2}\epsilon_0E^2\\ U_B &= \frac{1}{2\mu_0}B^2\end{aligned}}\right\}\ \text{Energy per unit volume}$$



Instantaneous Energy Density

$$\left.\begin{aligned}&\cdot\ \text{E-field}: \frac{1}{2}\epsilon_0E_0^2\sin^2(ky-\omega t)\\ &\cdot\ \text{B-field}: \frac{1}{2\mu_0}B_0^2\sin^2(ky-\omega t)\end{aligned}\right\}\ U_{tot} = \epsilon_0E_0^2\sin^2(ky-\omega t)$$

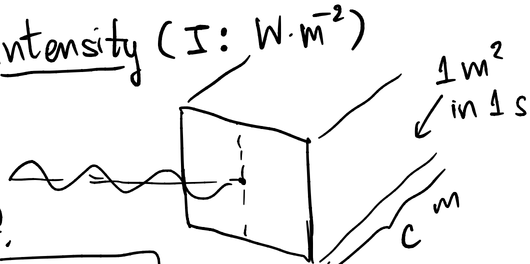
$$= \frac{1}{2\mu_0}\frac{E_0^2}{\cancel{c^2}_{\ \frac{1}{\mu_0\epsilon_0}}}\sin^2(ky-\omega t)$$

$$= \text{E-field}$$

$$\therefore \boxed{\langle U\rangle = \frac{1}{2}\epsilon_0E_0^2}$$

$$\langle U\rangle = \frac{1}{2\mu_0}B_0^2$$

- Intensity (I: $W\cdot m^{-2}$)



Def.

$$\boxed{I = \langle U\rangle\cdot c}\ \ast$$

$$S = I = \begin{cases}c\epsilon_0E_0^2\sin^2(ky-\omega t)\\ \dfrac{E_0B_0}{\mu_0}\sin^2(ky-\omega t)\\ \dfrac{c}{\mu_0}B_0^2\sin^2(ky-\omega t)\end{cases}$$

เอาไปคำนวณหา max, average ได้

$$\left|\frac{\vec{E}\times\vec{B}}{\mu_0}\right| : I\propto\psi^2$$

$\vec{\nabla}\cdot\vec{S}+\vec{J}\cdot\vec{E} = -\dfrac{\partial U}{\partial t}$

Def. Poynting vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

$$|\vec{S}| = \frac{\text{Power}}{A}$$

$$= \frac{1}{A}\frac{d}{dt}(\vec{F}\cdot\vec{s})$$

$$= \frac{F}{A}\cdot v$$

$$= P\cdot c$$

$$P = \iint_A S\,dA$$

$$\boxed{P_{rad} = \frac{|\vec{S}|}{c}}$$

Def. Radiation pressure

$$I = S_{av} = \frac{E_0B_0}{2\mu}$$

$$= \frac{1}{2}c\epsilon_0E_0^2$$

Cont'd. Radiation Pressure

$$\frac{dP}{d\tau} = \frac{EB}{\mu_0 c^2} = \frac{|\vec{S}|}{c^2} \quad \text{for } d\tau = Ac\,dt$$

$$\rightarrow P_{rad} = \boxed{\frac{1}{A}\frac{dp}{dt} = \frac{|\vec{S}|}{c} = \frac{EB}{\mu_0 c}} \quad : \quad P_{rad} = \begin{cases} \frac{S}{c} & \text{if } 100\% \text{ absorbed} \\ \frac{2S}{c} & \text{if } 100\% \text{ reflected} \end{cases}$$

(momentum) $p = \begin{cases} U/c & \ldots \\ 2U/c & \ldots \end{cases}$

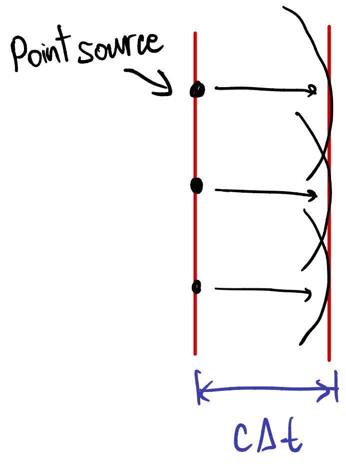
$$P \equiv \frac{\Delta E}{\Delta t} \quad [\text{power}]$$

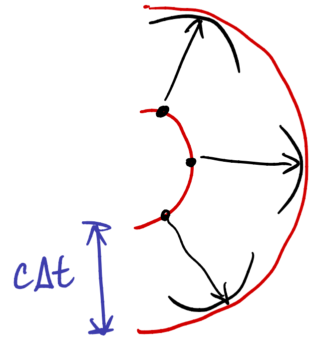
$$p = \frac{\Delta E}{c}$$

OPTICS

Interference

Ray-waves : Huygen's Principle





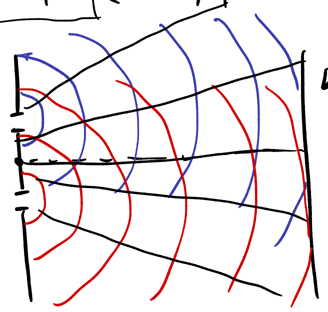
Note
Complex conjugate of z can be written as $\bar{z}$ or z^*

Re : $\dfrac{\partial^2 y}{\partial^2 x} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$, $y \rightarrow \psi$

Double slit source :
$S_1 : \psi_1$, $S_2 : \psi_2$
[Liner eq. system]

$$\begin{cases} C_1\left[\dfrac{\partial^2 \psi_1}{\partial x^2} - \dfrac{1}{v^2}\dfrac{\partial^2 \psi_1}{\partial t^2}\right] = 0 \\ C_2\left[\dfrac{\partial^2 \psi_2}{\partial x^2} - \dfrac{1}{v^2}\dfrac{\partial^2 \psi_2}{\partial t^2}\right] = 0 \end{cases} \rightarrow \left[\dfrac{\partial^2}{\partial x^2} - \dfrac{1}{v^2}\dfrac{\partial^2}{\partial t^2}\right](C_1\psi_1 + C_2\psi_2)$$

$\boxed{I \propto \psi^2}$ (Real, positive) $\rightarrow$ $I \propto \psi^*\psi$ (Quantum)



$$I_{\psi_1+\psi_2} \propto (\psi_1 + \psi_2)^2$$
$$\propto \underbrace{\psi_1^2 + \psi_2^2}_{I_1 + I_2} + \underbrace{2\psi_1\psi_2}_{\text{Can be either } +, -}$$

(Positive)

Superposition : $\psi_1 = A\sin\omega t$, $\psi_2 = A\sin(\omega t + \phi)$

$$\psi_t = \psi_1 + \psi_2$$
$$\psi_t = A[\sin\omega t + \sin(\omega t + \phi)]$$
$$\psi_t = 2A\sin\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)$$
$$\psi_t = \underbrace{2A\cos\left(\frac{\phi}{2}\right)}_{\text{Time-idp.}}\ \underbrace{\sin\left(\omega t + \frac{\phi}{2}\right)}_{\text{Time-dp.}} \leftarrow \text{Phase shift}$$

ω เดิม

$\phi = 0 :\ \psi_t = 2A\sin(\omega t)$

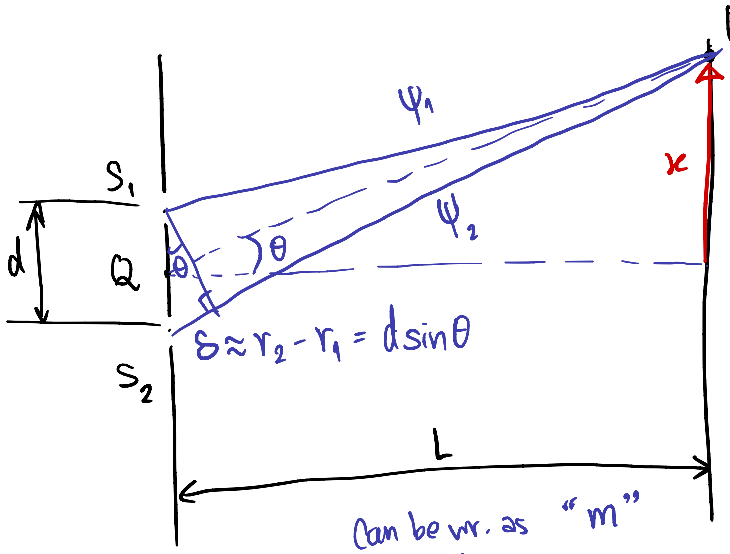
$\phi = \pi :\ \psi_t = 0$

$I = \psi_t^2 = 4A^2\cos^2\frac{\phi}{2}\sin^2\left(\omega t + \frac{\phi}{2}\right)$

$$\boxed{\langle I \rangle = I_{max}\cos^2\left(\frac{\phi}{2}\right)}$$

$\phi = \delta\left(\frac{2\pi}{\lambda}\right) = \dfrac{2\pi d\sin\theta}{\lambda}$

Back to Double Slit : Young's Experiment



$$\psi_1 = A\sin(kr_1 - \omega t)$$

$$\psi_2 = A\sin(kr_1 + k\delta - \omega t)$$

$$\psi_1 + \psi_2 = 2A\cos\left(\frac{k\delta}{2}\right)\sin\left(kr_1 - \omega t + \frac{k\delta}{2}\right)$$

$$\bar{I} = I_{max}\cos^2\left(\frac{k\delta}{2}\right)$$

Bright : $\cos^2\left(\frac{k\delta}{2}\right) = 1$

$k = \frac{2\pi}{\lambda}$: $\frac{k\delta}{2} = \pm(n \geq 0)\pi$

Dark : $k = \frac{2\pi}{\lambda}$: $\frac{k\delta}{2} = \pm(n > 0)\frac{\pi}{2}$

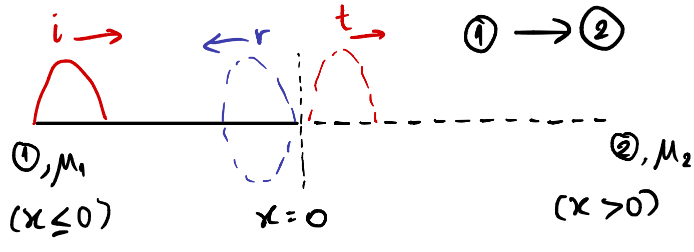
Can be wr. as "m"

$$\left.\boxed{\begin{array}{ll} \therefore \text{Max} : & \delta = (|n| \geq 0)\lambda \\ \text{Min} : & \delta = (|n| \geq 0 + \frac{1}{2})\lambda \end{array}}\right\} = d\sin\theta \approx \underbrace{d\tan\theta \approx \frac{dx}{L}}_{L >> d}$$

Index of Refraction : $n = \frac{c}{v} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$

↳ Snell's law : $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$

Reflection Phase Change (P.f.)



① → ②

(incident) $\psi_i = A\cos(k_1x - \omega t)$ } f_1
(reflection) $\psi_r = B\cos(-k_1x - \omega t)$
(transmit) $\psi_t = C\cos(k_2x - \omega t) \to f_2$

Boundary condition :

1. continuity : $f_1(x=0) = f_2(x=0)$

$$A\cos\omega t + B\cos\omega t = C\cos\omega t$$

$$A + B = C \quad \text{—(1)}$$

2. Smoothness : $\left.\frac{\partial f_1}{\partial x}\right|_0 = \left.\frac{\partial f_2}{\partial x}\right|_0$

$$-k_1A\sin(-\omega t) + k_1B\sin(-\omega t) = -k_2C\sin(-\omega t)$$

$$k_1(A - B) = k_2C$$

$$A - B = \frac{k_2}{k_1}C$$

$\frac{\omega}{k} = v$; $\frac{k_2}{k_1} = \frac{v_1}{v_2} \to A - B = \frac{v_1}{v_2}C$ —(2)

(1)+(2) : $2A = \left(1 + \frac{v_1}{v_2}\right)C$

$$\therefore C = \frac{2v_2}{v_1 + v_2}A$$

(1) $= \frac{v_2}{v_1}$(2) : $\frac{v_2}{v_1}(A - B) = A + B$

$$v_2A - v_2B = v_1A + v_1B$$

$$\therefore B = \frac{v_2 - v_1}{v_2 + v_1}A$$

$$\therefore \Psi_1 = A\cos(k_1x-\omega t) + \frac{v_2-v_1}{v_2+v_1}A\cos(-k_1x-\omega t) \quad ; \quad x \le 0$$

$$\Psi_2 = \frac{2v_2}{v_1+v_2}A\cos(k_1x-\omega t) \quad ; \quad x \ge 0$$

} Momentum & Energy Conservation

<u>Key</u>: Change $v \to n$ terms

<u>Reflected term</u> (B)

$n_2 > n_1$: $\frac{n_2}{n_1} = \frac{v_1}{v_2} > 1$ $(v_2 < v_1)$: $|\Psi_r|_{max} = -|\Psi_i|_{max} \to \cos(\xi + \pi)$

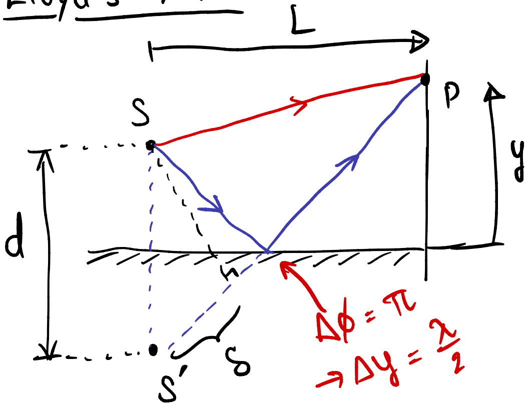
$\to$ Phase change $\Delta\phi = \pi$ ■

$n_1 > n_2$: $\frac{n_1}{n_2} = \frac{v_2}{v_1} > 1$ $(v_2 > v_1)$; $|\Psi_r|_{max} = |\Psi_i|_{max} \to \Delta\phi = 0$ ■

<u>Refracted term</u> (C)

$n_2 > n_1, n_1 > n_2$: $\Psi_{r'}$ ทิศเดียวกันกับ Ψ_i

<u>Lloyd's Mirror</u>



Total Path Diff : $\delta + \frac{\lambda}{2}$

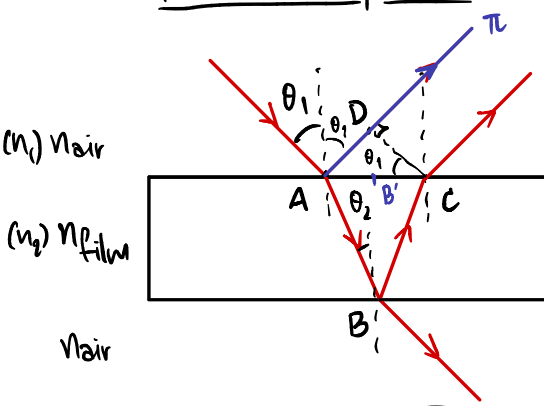
<u>Max</u> : $\delta + \frac{\lambda}{2} = (n \ge 0)\lambda \to (n \ge 0 - \frac{1}{2})\lambda$

<u>Min</u> : $\delta + \frac{\lambda}{2} = (n \ge 0 + \frac{1}{2})\lambda$

$$\boxed{\begin{aligned} \text{Max}: \ \delta &= (n \ge 0 + \tfrac{1}{2})\lambda \\ \text{Min}: \ \delta &= (n \ge 0)\lambda \end{aligned}}$$

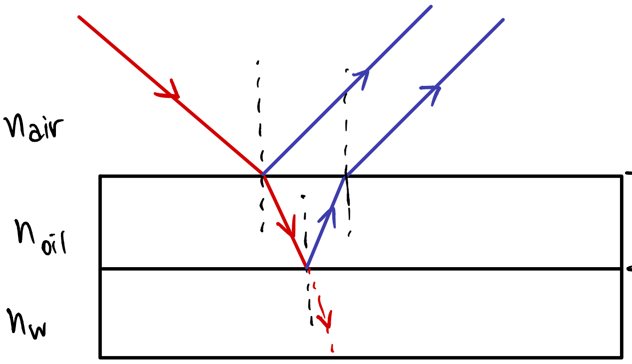
" Equivalent to double slit with $\Delta\phi = \pi$ (Max ⟲ Min) "

<u>Thin Film Experiment</u>



$AB\cos\theta_2 = d$
$AB = BC$

$AD = AC\sin\theta_1$
$AD = 2AB'\sin\theta_1$
$AD = 2AB\sin\theta_1\sin\theta_2$
$\therefore AD = 2d\sin\theta_1\tan\theta_2$



$$OPD = n_2(\overline{AB}+\overline{BC}) - n_1(\overline{AD})$$

$$OPD = n_2\left(\frac{2d}{\cos\theta_2}\right) - \underbrace{2n_1 d\sin\theta_1\tan\theta_2}_{n_2 d\sin\theta_2}$$

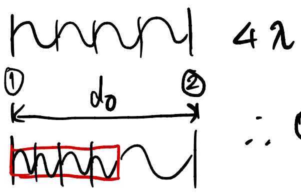
$$OPD = 2n_2 d\cos\theta_2$$

$$OPD_{tot} = 2n_2 d\cos\theta_2 + \frac{\lambda}{2} = (n \ge 0)\lambda$$

<u>Max</u> $\to 2n_2 d\cos\theta_2 = (n \ge 0 + \frac{1}{2})\lambda$	$\theta_2 \to 0 : \cos\theta_2 \to 1$
<u>Min</u> $\to 2n_2 d\cos\theta_2 = (n \ge 0)\lambda$	

<u>Optical Path</u> :

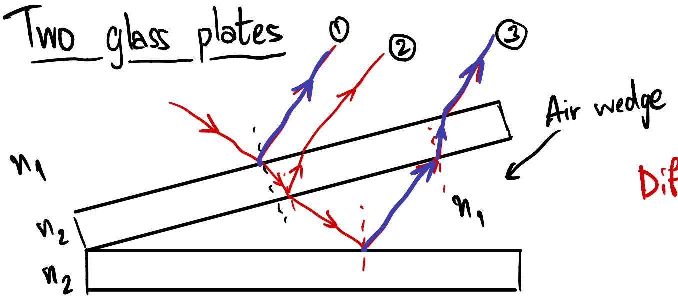
$$n = \frac{c}{v} = \frac{d_0/t}{d/t} \to d_0 = nd \leftarrow OPL$$



$$\therefore OPD = n_2 d_2 - n_1 d_1$$

Cont'd. Def. $\lambda_n = \frac{\lambda}{n} \rightarrow 2d = (n \geq 0 + \frac{1}{2})\lambda_n$, $2d = (n \geq 0)\lambda_n$

Two glass plates



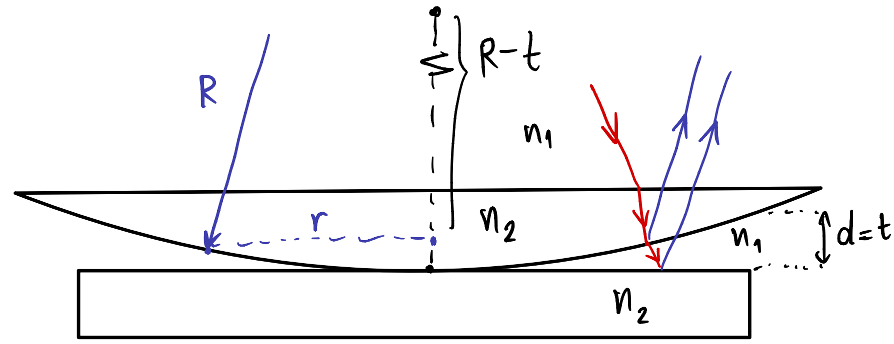
Max : $2d = (m + \frac{1}{2})\frac{\lambda}{n_1}$

Min : $2d = m \cdot \frac{\lambda}{n_1}$

Difficult to see interference ①—②

- Properties of material :
  For thick film : reflected element is less than thin
- Properties of light source (max & min) → Av.

Newton's Rings



Radii of dark rings

$$2d = m\frac{\lambda}{n_1}$$

$$r^2 + (R-t)^2 = R^2$$

$$r^2 = 2Rt - t^2$$

$R >> t$: $r^2 = 2Rt$

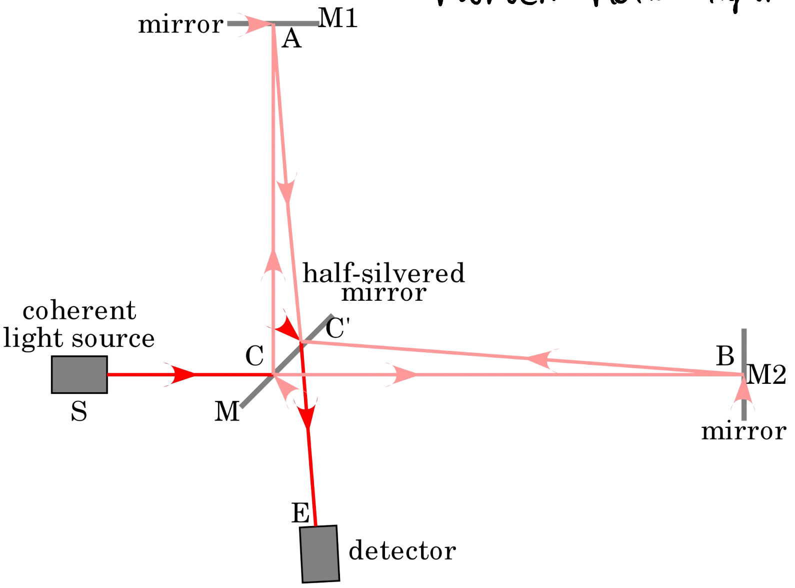
$$\therefore d = t = \frac{r^2}{2R}$$

$$\frac{r^2}{R} = m\frac{\lambda}{n_1}$$

$$\rightarrow r = \sqrt{m\frac{\lambda}{n_1}R}$$

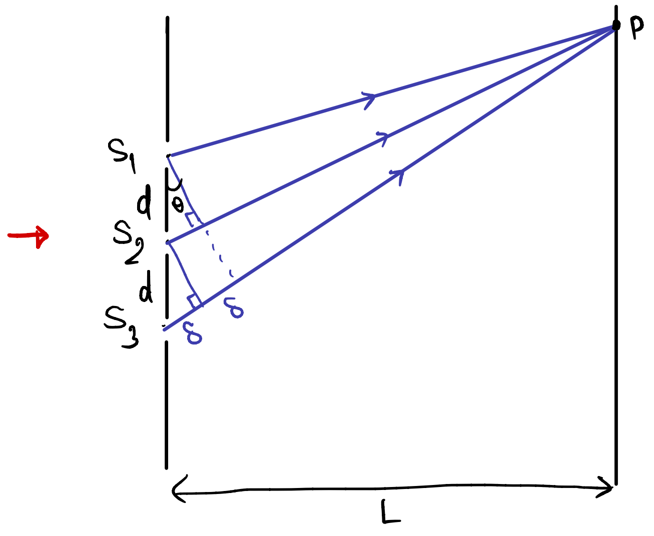
(radii of m^{th} dark ring)

Michelson interferometer : Measure wavelength of monochromatic light



Diffraction

3-slit :



* δ corr. ϕ

$E_1 = E_0 \sin(\omega t)$
$E_2 = E_0 \sin(\omega t + \phi)$
$E_3 = E_0 \sin(\omega t + 2\phi)$

$E_{tot} = E_1 + E_2 + E_3$, $I \propto E_{tot}^2$

$$E_1 + E_2 + E_3 = \underbrace{2E_0 \sin(\omega t + \phi)\cos(\phi)}_{E_1 + E_3} + \underbrace{E_0 \sin(\omega t + \phi)}_{E_2}$$

$$E_{tot} = E_0 \sin(\omega t + \phi)\left[2\cos\phi + 1\right]$$

$$\boxed{\begin{aligned} &\delta = d\sin\theta \\ &\lambda \leftrightarrow 2\pi \\ &\delta \leftrightarrow \frac{2\pi\delta}{\lambda} = \frac{2\pi d\sin\theta}{\lambda} = \phi \end{aligned}}$$

$$\langle I \rangle \propto \frac{E_0^2}{2}\left(1 + 2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right)^2 \rightarrow I_{max} = \frac{E_0^2}{2} \cdot 9$$

max $\cos(\dots) = 1$

$$\frac{I}{I_{max}} = \frac{(\dots)}{9}$$

$$\boxed{I = \frac{I_0}{9}\left[1 + 2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2}$$

Max: $\dfrac{\cancel{2\pi} d\sin\theta}{\lambda} = m'(\cancel{2\pi})$

$d\sin\theta = m\lambda$: $\boxed{\delta = m\lambda}$

$\hookrightarrow \Delta\theta = \dfrac{\lambda}{d}$

2^{nd} Max: $\cos\left(\dfrac{2\pi d\sin\theta}{\lambda}\right) = -1$

$\dfrac{2\cancel{\pi} d\sin\theta}{\lambda} = (2m+1)\cancel{\pi}$

$d\sin\theta = \dfrac{1}{2}(2m+1)\lambda$

: $\boxed{\delta = (m + \tfrac{1}{2})\lambda}$

$\hookrightarrow \Delta\theta = \dfrac{\lambda}{d}$

Min: $1 + 2\cos\left(\dfrac{2\pi d\sin\theta}{\lambda}\right) = 0$

$\cos\left(\dfrac{2\pi d\sin\theta}{\lambda}\right) = -\dfrac{1}{2}$

$\dfrac{2\pi d\sin\theta}{\lambda} = \dfrac{2\pi}{3}$

$d\sin\theta = m\dfrac{\lambda}{3}$: $\boxed{\delta = \dfrac{1}{3}(m+1)\lambda}$

$\hookrightarrow \Delta\theta = \dfrac{\lambda}{3d}$

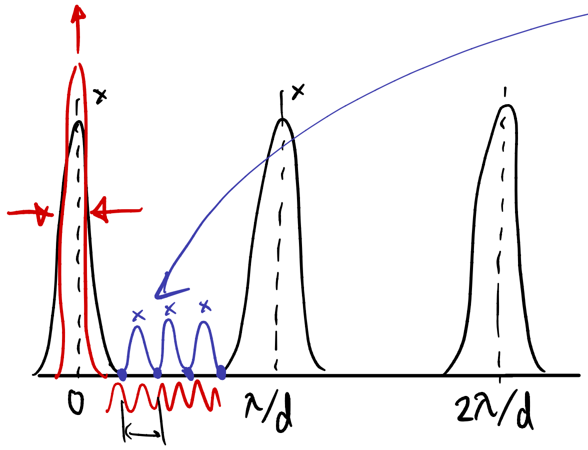
Mixed wavelength

$I = \dfrac{I_0}{9}\left(1 + 2\cos\left(\dfrac{2\pi d\sin\theta}{\lambda}\right)\right)^2$

$I = I(\lambda)$

* 3 Slit -1 = 2 darks
↓
N Slit -1 = $N-1$ Min.
= $N-2$ 2^{nd} Max.

N-slit (grating) : many N



N-1 Min. = N-2 2^{nd} Max

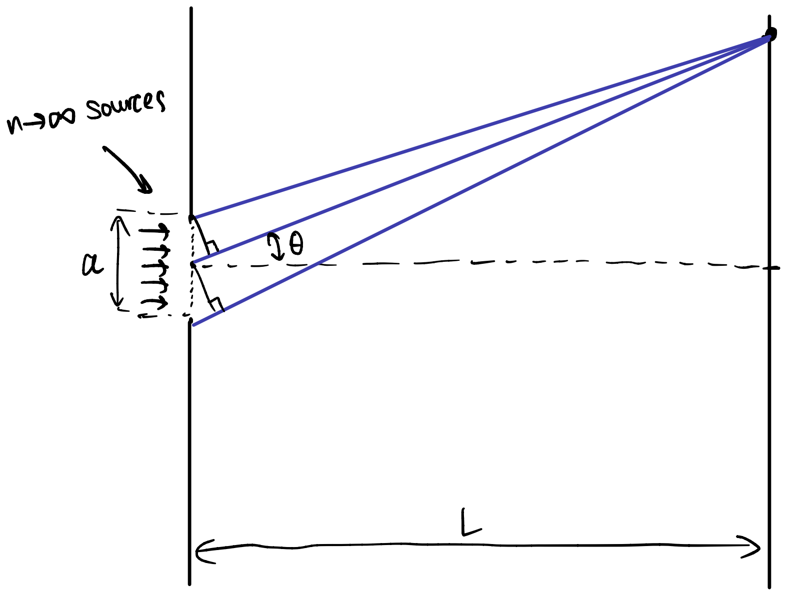
$\therefore I \propto N^2$

$$\Delta\theta = \frac{\lambda}{Nd}$$

N=1 : Diffraction

- * Fraunhofer : $L \gg a$
- · Fresnel : $L \approx a$

Single slit



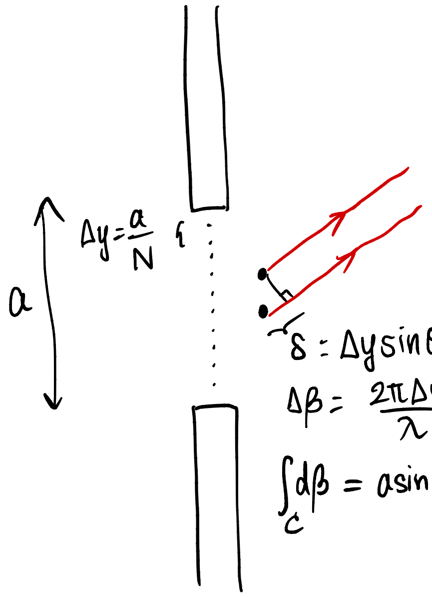
Min.

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$\rightarrow a\sin\theta = (n>0)\lambda$

$\rightarrow a\sin\theta = n\lambda$

Intensity of Single slit



$\delta = \Delta y \sin\theta$

$\Delta\beta = \frac{2\pi \Delta y}{\lambda}\sin\theta$

$\int_C d\beta = a\sin\theta\left(\frac{2\pi}{\lambda}\right)$

$E_1 = E_0 \sin(\omega t)$
$E_2 = E_0 \sin(\omega t + \Delta\beta)$
$E_3 = E_0 \sin(\omega t + 2\Delta\beta)$
$\vdots$
$E_N = E_0 \sin(\omega t + (N-1)\Delta\beta)$

Phase Shift between points: (1, N)

$$\beta = N\Delta\beta = N \cdot \frac{2\pi}{\lambda} \cdot \frac{a}{N}\sin\theta = \frac{2\pi\, a\sin\theta}{\lambda}$$

Sol 1: Trig.: $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$

$$\cos\left(\omega t - \frac{\Delta\beta}{2}\right) - \cos\left(\omega t + \frac{\Delta\beta}{2}\right) = 2\sin\omega t \sin\left(\frac{\Delta\beta}{2}\right)$$

$$\cos\Big(\underbrace{\omega t + \frac{\Delta\beta}{2}}_{\omega t + \Delta\beta - \frac{\Delta\beta}{2}}\Big) - \cos\Big(\underbrace{\omega t + \frac{3\Delta\beta}{2}}_{\omega t + \Delta\beta + \frac{\Delta\beta}{2}}\Big) = 2\sin(\omega t + \Delta\beta)\sin\left(\frac{\Delta\beta}{2}\right)$$

$$\cos\left(\omega t + \frac{3\Delta\beta}{2}\right) - \cos\left(\omega t + \frac{5\Delta\beta}{2}\right) = 2\sin(\omega t + 2\Delta\beta)\sin\left(\frac{\Delta\beta}{2}\right)$$

$$\rightarrow \cos\left(\omega t - \frac{\Delta\beta}{2}\right) - \cos\left(\omega t + \left(N - \tfrac{1}{2}\right)\Delta\beta\right) = 2\sin\left(\frac{\Delta\beta}{2}\right) E_0 \sum_{i=1}^{N} \sin(\omega t + (i-1)\Delta\beta)$$

$$\rightarrow \cancel{-}2\sin\left[\omega t + \frac{N-1}{2}\Delta\beta\right]\sin\Big(\cancel{-}\underbrace{N\Delta\beta}_{\frac{2\pi a\sin\theta}{\lambda}}\Big) = \frac{E_0}{2\sin\left(\frac{\Delta\beta}{2}\right)}\left(\Sigma \ldots\right)$$

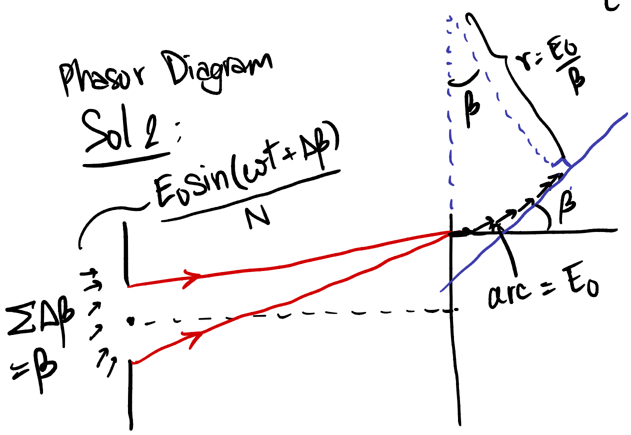
$$\boxed{E = E_0\left(\frac{\sin\frac{\beta}{2}}{\beta/2}\right)} \; ; \; \boxed{\beta = a\sin\theta\left(\frac{2\pi}{\lambda}\right)}$$

and $I \propto E^2$:

$$\therefore \boxed{I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2}$$

Phasor Diagram

Sol 2:



Min (Dark): $\frac{\beta}{2} = n\pi$

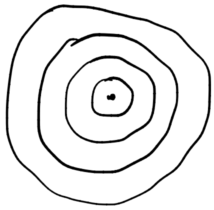
$a\sin\theta \cdot \frac{\pi}{\lambda} = n\pi$

$\therefore \boxed{a\sin\theta = n\lambda}$

Sol 3: Imaginary

$$\Sigma\sin(\ldots) = \mathrm{Im}\left(e^{i\theta} + e^{i2\theta} + e^{i3\theta} + \ldots\right)$$

Circular Aperture



ลักษณะ:วง "Airy's disc"

$n = 1$: min $\quad a\sin\theta = \lambda[1.22]$

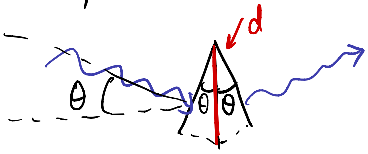
- Resolution of 2 sources

  "Rayleigh criterion" เริ่มแยกออก เมื่อ $D \geq \text{Dist}(\text{Central max, First min})$

  $$\theta_{m=1} = 1.22\frac{\lambda}{a}$$

Xray

- Crystal Diffraction

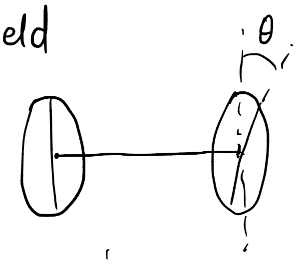


Bragg

$$\boxed{2d\sin\theta = n\lambda}$$

Polarization

Direction & Magnitude of oscillating E-field

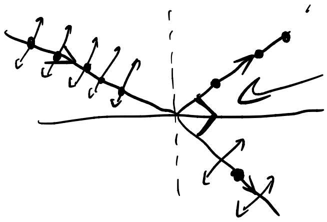


- Selective Absorption

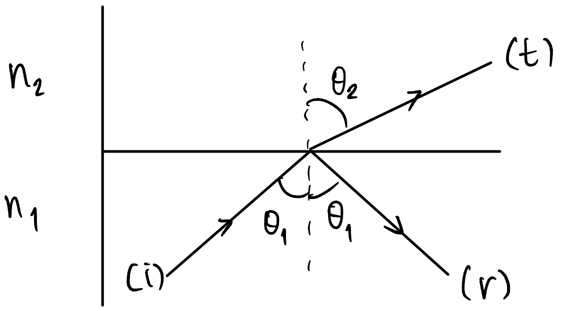
  Malus's law : $I = I_0\cos^2\theta$

- By reflection

  Brewster's angle : $\tan\theta_i = \frac{n_2}{n_1}$



Fresnel's equations



$$E_r = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}E_i$$

$$E_t = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2}E_i$$

SPECIAL THEORY OF RELATIVITY

ELECTROMAGNETIC (✱)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt}\right)$$

Introduction

Aether (Maxwell) เป็น medium แห่ง → Michelson-Morley exp.

$$\Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1}$$

$$\Delta t_1 = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1/2}$$

$$\rightarrow \Delta t = \Delta t_2 - \Delta t_1 = \frac{2L}{c}\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

binomial expansion : $(1+x)^n = 1 + nx + \dots$; $x \ll 1$ $(v \ll c)$

$$\Delta t = \frac{2L}{c}\left(1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\Delta t = \frac{Lv^2}{c^3}$$; จาก Aether wind → b

หมุนกระจก 90° : Arm 2 → Arm 1 ; Arm 1 → Arm 2 : Shifted by $2\Delta t$; $\delta = 2c\Delta t \quad \frac{2Lv^2}{c^2}$

$$n = \frac{\delta}{\lambda} = \frac{2Lv^2}{\lambda c^2}$$ ⟶ No shift found. with precision 0.018

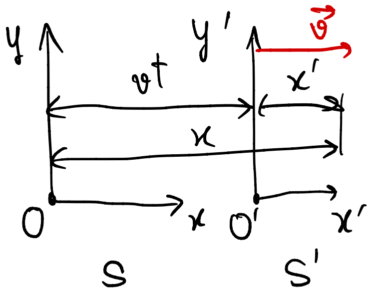
explain

Fitzgerald & Lorentz assumption : object contracts by $\sqrt{1 - \frac{v^2}{c^2}}$ factor

→ ∴ No Aether

Two postulates: 1. Inertial frame
2. Constant speed of light

Lorentz transformation



S: $x = ct$, S': $x' = ct'$

(1) $x' = [x - vt]\gamma$
(2) $x = [x' + vt']\gamma$

$t \neq t'$
each observer observes different length

(1)×(2): $xx' = \gamma^2(xx' + xvt' - x'vt - v^2tt')$

$$1 = \gamma^2\left(1 + \frac{vt'}{x'} - \frac{vt}{x} - \frac{v^2tt'}{xx'}\right)$$

$$1 = \gamma^2\left(1 + \cancel{\frac{v}{c}} - \cancel{\frac{v}{c}} - \frac{v^2}{c^2}\right)$$

$$1 = \gamma\sqrt{1 - \frac{v^2}{c^2}} \rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$\boxed{x' = \gamma(x - vt)}$ แทนค่าใน (2)

$$x = [\gamma(x - vt) + vt']\gamma$$

$$x = \gamma^2 x - \gamma^2 vt + \gamma vt'$$

$$t' = \frac{x}{\gamma v} - \frac{\gamma x}{v} + \gamma t$$

$$t' = \gamma\left[t + \frac{x}{\gamma^2 v} - \frac{x}{v}\right]$$

$$\frac{x}{\gamma^2 v} - \frac{x}{v} = \frac{x}{v}\left[\frac{1}{\gamma^2} - 1\right]$$

$$= \frac{x}{v}\cdot\left(-\frac{v^2}{c^2}\right)$$

$$= -\frac{xv}{c^2}$$

$$\boxed{t' = \gamma\left(t - \frac{xv}{c^2}\right)}$$

Inverse:

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + \frac{x'v}{c^2}\right)$$

Case example

$S(x, y, z, t)$, $S'(x', y', z', t')$

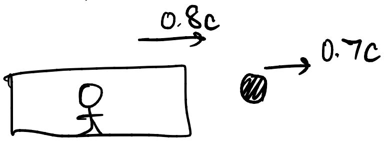
$\hookrightarrow$ $x' = \gamma(x - vt)$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$y' = y$$

$$z' = z$$

Example : object's motion



Gallileon : $\underline{1.5c}$ X

$\underline{\text{Lorentz}}$:

$$x_1' = \gamma(x_1 - vt_1) \qquad x_2' = \gamma(x_2 - vt_2)$$

$$t_1' = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) \qquad t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$\left.\begin{aligned} \Delta x' &= x_2' - x_1' = \gamma(\Delta x - v\Delta t) \\ \Delta t' &= t_2' - t_1' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) \end{aligned}\right\} \text{ ตาม Lorentz tf.}$$

$$u_x' = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x - v\Delta t}{\Delta t - \frac{v\Delta x}{c^2}} \cdot \frac{\frac{1}{\Delta t}}{\frac{1}{\Delta t}} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \rightarrow u_x = \frac{u_x' + v}{1 + \frac{u_x v}{c^2}}$$

For y, z components:

$$u_y' = \gamma^{-1}\left(\frac{u_y}{1 - \frac{u_x v}{c^2}}\right), \quad u_z' = \gamma^{-1}\left(\frac{u_z}{1 - \frac{u_x v}{c^2}}\right)$$

จาก Example case : (ของบอลเร็วเท่าไร เทียบ obs. ข้างนอก)

$$\boxed{u_x = \frac{u_x' + v}{1 + \frac{u_x v}{c^2}}} = \frac{0.7c + 0.8c}{1 + \frac{(0.7c)(0.8c)}{c^2}} = \frac{1.5c}{1.56} \approx 0.96c$$

$\underline{\text{Simultaneity - relativity of time}}$

- $\underline{\text{Time dilation}}$: optical clock (tick)

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

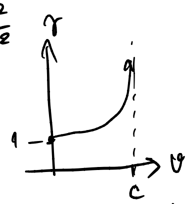
Clock at rest in S : $\Delta x = 0$

(1) $$\boxed{\Delta t' = \gamma \Delta t} = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

@ $v = 0$: $\gamma = 1$

@ $v > 0$: $\gamma > 1$

@ $v \to c$: $\gamma \to \infty$



∴ Observer in S' observes tick longer than S

→ $\Delta t'$ ช้ากว่า (>) Δt

• Another perspective

$$\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) ; \ \Delta x = 0 \text{ in S}$$

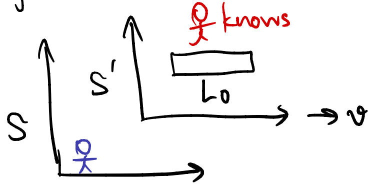
$$\rightarrow \boxed{\Delta t = \gamma \Delta t'} \ (2) \quad \text{vs} \quad \boxed{\Delta t' = \gamma \Delta t} \ (1)$$

proper time = time interval between 2 events as measured by an observer who sees the events occur at $\underline{\text{some point}}$ in space.

– Length contraction : Measurement of length : วัด Head, Tail @ t เดียวกัน

compare :

① length in S' known



$$\Delta x' = \gamma(\Delta x - v\overset{=0}{\Delta t})$$

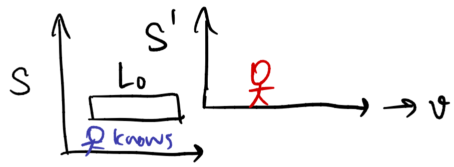
$$L_0 = \gamma L$$

S วัดได้

$$\rightarrow \boxed{L = \gamma^{-1} L_0}$$ proper length in its rest frame

$$(L < L_0)$$

② length in S known



$$\overset{=L}{\Delta x'} = \gamma(\Delta x - v\Delta t)$$ ($\Delta x = L_0$)

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

$= 0$ เพราะกรอบที่วัด $\Delta t = 0$

$$\Delta t = \frac{v\Delta x}{c^2}$$

$$\Delta x' = \gamma\left(L_0 - \frac{v^2}{c^2}L_0\right)$$

$$L = \gamma L_0(\gamma^{-2})$$

$$\boxed{L = \gamma^{-1}L_0} \rightarrow \text{same !!}$$

Doppler effect (of light)

↳ Relativistic doppler shift

ของ 2 wavefronts

$$\lambda' = (c - v)T'$$

$$v' = c = f'\lambda'$$

$$\rightarrow \lambda' = \frac{c}{f'}$$

$$\frac{c}{f'} = (c-v)T'$$

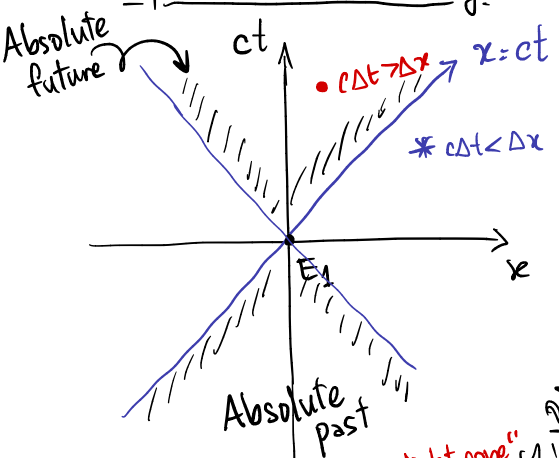
$$f' = \frac{c}{(c-v)\underset{=\gamma T}{T'}}$$

$$f' = \frac{c}{(c-v)\gamma} \cdot f$$

$$\boxed{f' = \frac{\sqrt{1 \pm \frac{v}{c}}}{\sqrt{1 \mp \frac{v}{c}}} f}$$

Term : $\beta = \frac{v}{c}$

Spacetime & Causality



" spacetime light cone "

event 1 : ทำข้อสอบ

event 2 : ได้เกรด

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

$$\Delta t = t_2 - t_1 > 0$$

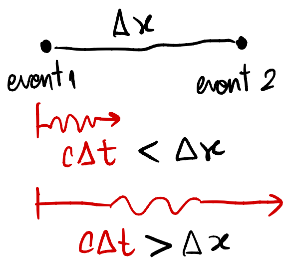
$\Delta t' > 0$? ไม่จำเป็น !

$\Delta t' < 0$ ก็ต่อเมื่อมี Observer ใน S' ที่เห็น event 2 ก่อน event 1 $(t_1' > t_2')$

$$\frac{v\Delta x}{c^2} > \Delta t$$

$$\frac{v}{c} > \frac{c\Delta t}{\Delta x}$$

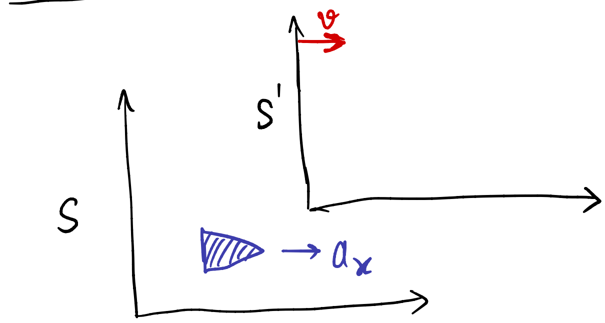
แต่ $v < c$ (post. 2)



จริง เมื่อ $c\Delta t < \Delta x$ ทำให้ $\Delta t' < 0$ ได้

ไม่จริง เมื่อ $c\Delta t ~~>~~ \Delta x \rightarrow \Delta t' ~~<~~ 0$

Acceleration in SR



$$a'_x = \frac{du'_x}{dt'} \quad ; \quad a_x = \frac{du_x}{dt}$$

From velocity transformation,

$$\boxed{u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}}$$

$$du'_x = \frac{du_x}{1-\frac{u_x v}{c^2}} \cdot \frac{(u_x - v)(-1)(-\frac{v}{c^2})\,du_x}{(1-\frac{u_x v}{c^2})^2}$$

$$du'_x = du_x\left[\frac{1-\cancel{\frac{u_x v}{c^2}}+\cancel{\frac{u_x v}{c^2}}-\frac{v^2}{c^2}}{(1-\frac{u_x v}{c^2})^2}\right]$$

$$du'_x = du_x\left[\frac{1-\frac{v^2}{c^2}}{(1-\frac{u_x v}{c^2})^2}\right] = \gamma^{-2} du_x\left(\frac{1}{1-\frac{u_x v}{c^2}}\right)^2$$

$$t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

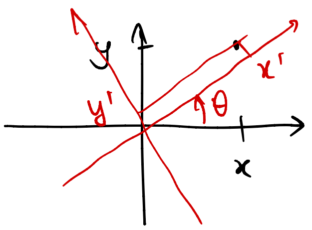
$$dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right) = \gamma\,dt\left(1 - \frac{u_x v}{c^2}\right)$$

$$\therefore \frac{du'_x}{dt'} = \frac{du_x}{dt}\left(\frac{1}{\gamma^2(1-\frac{u_x v}{c^2})^2}\right)\left(\frac{1}{\gamma(1-\frac{u_x v}{c^2})}\right)$$

$$\boxed{a'_x = a_x \cdot \gamma^{-3} \cdot \left(1 - \frac{u_x v}{c^2}\right)^{-3}}$$

Spacetime interval

$\xrightarrow{\text{relativity}}$



$$x' = x\cos\theta + y\sin\theta$$
$$y' = x\sin\theta - y\cos\theta$$
$$x^2 + y^2 = (x')^2 + (y')^2$$

$x_0 = ct_0$

$$x' = \gamma(x - vt) \rightarrow x' = \gamma(x - \tfrac{v}{c}\cdot ct)$$

with $\frac{v}{c} = \beta$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad \frac{vx}{c^2} \rightarrow \beta\frac{x}{c}$$

$$y' = y$$
$$z' = z$$

note $x'_1 = x'$, $x'_2 = y'$, $x'_3 = z'$

$$x'_1 = \gamma(x_1 - \beta x_0)$$
$$x'_0 = \gamma(x_0 - \beta x_1)$$
$$x'_2 = x_2$$
$$x'_3 = x_3$$

$$x_0'^2 - x_1'^2 = \gamma^2\left[x_1^2 - \cancel{2\beta x_0 x_1} + \beta^2 x_0^2 - x_0^2 + \cancel{2\beta x_0 x_1} - \beta^2 x_1^2\right]$$

$$= \underbrace{\cancel{\gamma^2}}_{\frac{1}{1-\beta^2}}(1-\beta^2)(x_0^2 - x_1^2)$$

$$x_0'^2 - x_1'^2 = x_0^2 - x_1^2 \quad \blacksquare$$

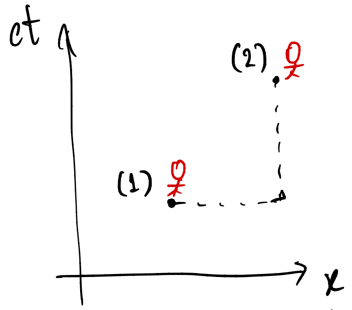
Generalized : $\boxed{\underbrace{x_0'^2 - x_1'^2 - x_2'^2 - x_3'^2}_{\text{space-time interval}} = x_0^2 - x_1^2 - x_2^2 - x_3^2}$

space-time interval : $X(x_0, \vec{r})$ or $X(x_0, x_1, x_2, x_3)$

$$\rightarrow X \cdot X = x_0^2 - \|\vec{r}\cdot\vec{r}\| = X' \cdot X'$$

Single Particle - Four momentum

Event : particle moves



Δx = ระยะอนุภาคเคลื่อนที่

Δt = ระยะเวลาอนุภาคเคลื่อนที่

$$(\Delta S)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$(\Delta S)^2 = (c\Delta t)^2\left(1-\frac{u_x^2}{c^2}\right)^2$$

$$\therefore \Delta S = c\Delta t\sqrt{1-\frac{u_x^2}{c^2}}$$

$$(\Delta S)^2 = (c\Delta\tau)^2$$

$$\therefore \Delta S = c\Delta\tau$$

τ Invariant เพราะ เป็น time ใน particle rest frame

$$\therefore \boxed{\Delta\tau = \Delta t\sqrt{1-\frac{u_x^2}{c^2}}}$$

$$\boxed{\frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{u_x^2}{c^2}}}}$$

* Proper time

$$\Delta\tau = \sqrt{\frac{\Delta S^2}{c^2}} \quad (\Delta S^2 > 0)$$

* Proper length

$$L = \sqrt{-\Delta S^2} \quad (\Delta S^2 < 0)$$

กำหนด $X(x_0, x_1, x_2, x_3)$

Derivative t ไม่ได้ เพราะ variant

แต่ τ ได้ :

$$\frac{\Delta X}{\Delta\tau} = \left(\frac{\Delta x_0}{\Delta\tau}, \frac{\Delta x_1}{\Delta\tau}, \frac{\Delta x_2}{\Delta\tau}, \frac{\Delta x_3}{\Delta\tau}\right)$$

$$V = \frac{dX}{d\tau} = \frac{dX}{dt}\cdot\frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{u_x^2}{c^2}}}(c, \vec{u})$$

$$mV = \frac{1}{\sqrt{1-\frac{u_x^2}{c^2}}}(mc, m\vec{u})$$

γmc $\gamma m u$

$$\vec{p} = \boxed{mV = \left(\frac{mc}{\sqrt{1-\frac{u_x^2}{c^2}}}, \frac{m\vec{u}}{\sqrt{1-\frac{u_x^2}{c^2}}}\right)}$$

* Relativistic linear momentum

$$\frac{m\vec{u}}{\sqrt{1-u_x^2/c^2}}$$ for $u_x \ll c \rightarrow m\vec{u}$ is momentum $(\vec{p})$

⊕ $a \propto (1-u^2/c^2)^{3/2}$ proof

Relativistic energy

$$\frac{mc}{\sqrt{1-\frac{u_x^2}{c^2}}}$$; p_0 for $u_x \ll c \rightarrow mc$ (unknown)

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots$$

$$p_0 = mc\left[1+\frac{1}{2}\frac{u_x^2}{c^2} - \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\frac{u_x^4}{c^4} + \dots\right]$$

$$p_0 = mc + \frac{1}{2}\frac{m}{c}u_x^2 + \dots$$

$$p_0 c = \underbrace{mc^2}_{\text{rest energy}} + \underbrace{\frac{1}{2}mu_x^2 + \dots}_{K}$$

"Energy"

Note เทอมหลัง... ใน classical ไม่มีให้เห็น เพราะ $u_x \ll c$

$$\therefore \boxed{K = \frac{mc^2}{\sqrt{1-\frac{u_x^2}{c^2}}} - mc^2}$$

$$\boxed{K = (\gamma - 1)mc^2}$$

ลอง $P = mV = \left(\frac{E}{c}, \vec{P}\right)$ "Four momentum"

ถ้ามองในมุม (คนที่อยู่บน particle) $P' = (\underset{\text{rest mass}}{mc}, 0)$

ใช้สมบัติ

$$P \cdot P = P' \cdot P'$$

$$\frac{E^2}{c^2} - p^2 = m^2c^2$$

$$E^2 = p^2c^2 + m^2c^4$$

or $$\boxed{E^2 = (pc)^2 + (mc^2)^2}$$

Electron volt

ประจุ e^-

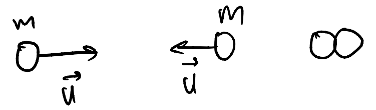
$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

Rest energy of e^- : $m_ec^2 = (9.11 \times 10^{-31})(3 \times 10^8)^2$

$$= \frac{[illegible]\ \text{J}}{1.602 \times 10^{-19}\ \text{J/eV}} = 0.511\ \text{MeV}$$

$$m_e = 0.511\ \frac{\text{MeV}}{c^2}$$ คุ้นเคย

Mass as energy



$E_{\text{ก่อน}} = E_{\text{หลัง}}$

$$\frac{m\cancel{c^2}}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{m\cancel{c^2}}{\sqrt{1-\frac{u^2}{c^2}}} = M\cancel{c^2}$$

$$\rightarrow M = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}(2m)$$

$$\rightarrow \boxed{M > 2m}$$

"มวลหลังชน เกินมวลรวมก่อนหน้า"

= มีพลังงานส่วนหนึ่ง แปลงไปเป็นมวล

$$\Delta M = M - 2m$$

$$\Delta M = 2\left(\frac{m}{\sqrt{1-\frac{u^2}{c^2}}} - m\right) \cdot \frac{c^2}{c^2}$$

$$= \frac{2}{c^2}\left[\ K\ \right]$$

$$\therefore \boxed{\Delta M = \frac{2}{c^2}K}$$

หมายถึง K ไม่ได้หายไปไหน

แต่เปลี่ยนไปเป็น composite mass

♡ ของ particle accelerator เพื่อสร้างอนุภาค ♡

App.

Fission :

$$Mc^2 = \gamma M_1c^2 + \gamma M_2c^2 + \gamma M_3c^2 + \ldots$$

$$M > M_1 + M_2 + M_3$$

loss of mass → energy

Quantum Mechanics

Blackbody Radiation : Stefan's law : $P = \sigma A e T^4$

โดย σ (Stefan-Boltzmann const.) $= 5.67 \times 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$

e (emissivity)

Wien's displacement law

$$\lambda_{max} T = 2.898 \times 10^{-3}\ m \cdot K$$

↳ UV catastrophe : Rayleigh-Jeans law $I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$ FAILED

$\lambda \to 0$ (high freq.): $I \to \infty$

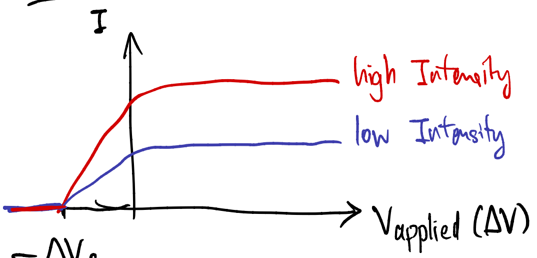
Plank : $E_n = nhf$ $\sim E = hf$

↳ Planck's const. (6.62×10^{-34} J.s)

↳ Quantum number

$$\to I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)}$$

Photoelectric effect



(stopping potential)

For $\Delta V \to \infty$: I ไม่เปลี่ยน

For $\Delta V < 0$: I decreases to 0

↳ $K_{max} = e \Delta V_s$

Note K_{max} เป็นของ e^- ว่าต้องการ K เท่าไร ถึงจะวิ่งไปอีก plate ได้ และ K_{max} คือไม่มี K สูงกว่านี้ได้อีกแล้ว

Consider 2 Intensity : $I \propto$ Intensity

$I \not\propto f$

$K \propto f$

$K \not\propto$ Intensity

Consider K-f graph (Einstein) : $K = (\text{slope}) f + K_0$; slope $= h$ related

↳ $-\frac{hc}{\lambda_c} = \phi$ (Work function)

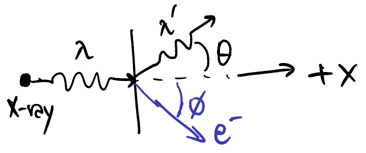
$hc = 1240$ eV·nm

เกิด concept เรื่อง "photon (γ)" และ photoelectric effect

↳ $E = hf + \phi$

The Compton effect

: scattering of X-ray from e^-



$E_{\text{ก่อน}} = hf + m_e c^2$

$E_{\text{หลัง}} = hf' + \dfrac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} = hf' + \sqrt{(pc)^2 + (mc^2)^2}$

$P_x = \dfrac{h}{\lambda}$ [momentum of photon]

$P_x : \dfrac{h}{\lambda} = \dfrac{h}{\lambda'}\cos\theta + p_e\cos\phi : \dfrac{h}{\lambda} - \dfrac{h}{\lambda'}\cos\theta = p_e\cos\phi$

$P_y : \dfrac{h}{\lambda'}\sin\theta = p_e\sin\phi$

$P_x^2 + P_y^2 : p_e^2 = \left(\dfrac{h}{\lambda} - \dfrac{h}{\lambda'}\cos\theta\right)^2 + \left(\dfrac{h}{\lambda'}\sin\theta\right)^2$

$p_e^2 = \left(\dfrac{h}{\lambda}\right)^2 + \left(\dfrac{h}{\lambda'}\right)^2 - \dfrac{2h^2}{\lambda\lambda'}\cos\theta$

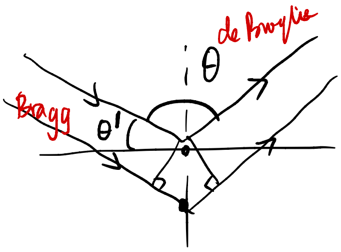
$p^2c^2 + m^2c^4 = \left(\dfrac{hc}{\lambda} - \dfrac{hc}{\lambda'} + mc^2\right)^2$

$p^2 = \left(\dfrac{h}{\lambda} - \dfrac{h}{\lambda'}\right)^2 + 2mc\left(\dfrac{h}{\lambda} - \dfrac{h}{\lambda'}\right)$

$\dfrac{h\cos\theta}{\cancel{\lambda\lambda'}} = -\dfrac{h}{\cancel{\lambda\lambda'}} + mc\left(\dfrac{\lambda' - \lambda}{\cancel{\lambda\lambda'}}\right)$

$h(1-\cos\theta) = mc(\lambda' - \lambda)$

$\therefore \boxed{\lambda' - \lambda = \dfrac{h}{m_e c}(1-\cos\theta)}$



สมมติฐาน de Broglie : $\lambda = \dfrac{h}{p}$ ← วทน

Generalization : $E = hf = \dfrac{hc}{\lambda} = pc$

de Broglie (vs.) Bragg

de Broglie

$\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2m_e eV}}$

$V = 54$ V $\rightsquigarrow$ $\underline{\lambda = 0.167 \text{ nm}}$

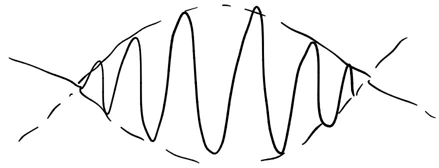
Bragg

$2d\sin\theta = n\lambda$

$\theta' = 90^\circ - \dfrac{\theta}{2} \rightsquigarrow \underline{\lambda = 0.165 \text{ nm}}$

เป็นการพิสูจน์ wave-particle duality ∎

Wave packet



$$y_1 = A\cos(k_1 x_1 - \omega_1 t)$$
$$y_2 = A\cos(k_2 x - \omega_2 t)$$
$$y_1 + y_2 = A[\cos(\ldots) + \cos(\ldots)]$$
$$= 2A\cos\left[\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right]\cos\left[\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right]$$

$$v_{phase} = \frac{\omega_1+\omega_2}{k_1+k_2}$$

$$v_{group} = \frac{\Delta\omega}{\Delta k} \quad \text{[of envelope function]}$$
$$= \frac{\hbar\, d\omega}{\hbar\, dk}$$
$$= \frac{d(\hbar\omega)}{d(\hbar k)} \qquad \frac{h}{2\pi}\cdot 2\pi f = E \qquad \frac{h}{2\pi}\cdot\frac{2\pi}{\lambda} = \frac{h}{\lambda} = p$$
$$v_g = \frac{dE}{dp}$$

Classical : $E = \frac{p^2}{2m}$

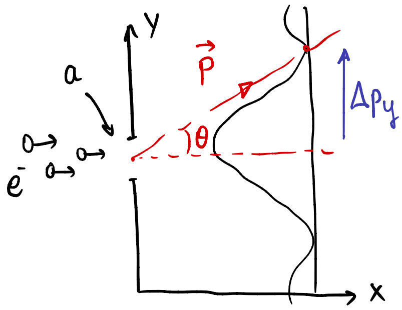
$$v_g = \frac{d}{dp}\frac{p^2}{2m} = \frac{p}{m} = u$$

SR : $\frac{d}{dp}\sqrt{p^2c^2 + m^2c^4} = \frac{1}{\cancel{2}\sqrt{p^2c^2+m^2c^4}}\cdot \cancel{2}pc^2$

$$= \frac{c}{\sqrt{1+\frac{m^2c^2}{p^2}}} \quad p^2 = \gamma m u \qquad = \frac{\gamma u c}{c\sqrt{\frac{1}{1-\frac{u^2}{c^2}}}}$$
$$= u$$

สามารถบรรยายคลื่นเป็นอนุภาคได้ (packet) ที่วิ่งด้วย v_g

Uncertainty Principle



$$a\sin\theta = n\lambda \quad (n = 1)$$
$$\Delta y \cdot \frac{\Delta p}{p} = \lambda$$
$$\Delta y\,\Delta p_y = p\lambda = h$$

" Position & Momentum cannot be simultaneously measured @ precision "

$$\rightarrow \boxed{\begin{aligned}\Delta x\Delta p &\geq \frac{\hbar}{2}\\ \Delta E\Delta t &\geq \frac{\hbar}{2}\end{aligned}}$$

Wave function (Ψ)

$|\Psi|^2$ or $\Psi^*\Psi \rightarrow$ probability

Classical : $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

EMW : $\dfrac{\partial^2 E}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 E}{\partial t^2}$

$$-\frac{\hbar^2}{2m}\overbrace{\frac{\partial^2}{\partial x^2}}^{\nabla^2}\Psi(x,t) + V(x)\Psi(x) = E\Psi(x,t) \quad \text{time independent}$$

$$= i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \text{time dependent}$$

$P(x)\,dx = |\Psi|^2 dx$

$P_{ab} = \int_a^b |\Psi|^2 dx$ Thus $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$

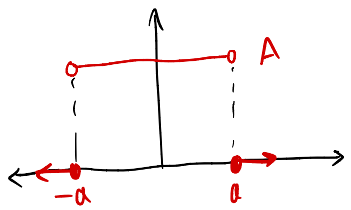
Expectation value

$$\langle x\rangle = \int_{-\infty}^{\infty} \Psi^* x \Psi \, dx$$

$$\langle f(x)\rangle = \int_{-\infty}^{\infty} \Psi^* f(x) \Psi \, dx$$

Example

$$\Psi(x) = \begin{cases} A & -a < x < a \\ 0 & \text{elsewhere} \end{cases}$$



On Normalization

$$1 = \int_{-\infty}^{\infty} |\Psi|^2 dx$$

$$\int_{-a}^{A} A^2 dx = 1$$

$$A^2(2a) = 1$$

$$A = \frac{1}{\sqrt{2a}}$$

$\Psi(x) = Ae^{-\lambda(x-a)^2}$; Determine $A, \langle x\rangle, \langle x^2\rangle$

$$1 = \int_{-\infty}^{\infty} (Ae^{-\lambda(x-a)^2})^2 dx$$

$$1 = A^2 \underbrace{\int_{-\infty}^{\infty} e^{-2\lambda(x-a)^2} d(x-a)}_{\text{Gaussian Integral } = \sqrt{\frac{\pi}{2\lambda}}}$$

$$\therefore A = \left(\frac{2\lambda}{\pi}\right)^{1/4}$$

$\langle x\rangle$: $\langle x\rangle = \int_{-\infty}^{\infty} [Ae^{-\lambda(x-a)^2}]^2 x \, dx$

$$\langle x\rangle = A^2 \int_{-\infty}^{\infty} x e^{-2\lambda(x-a)^2} dx$$

$$\langle x\rangle = a$$

Particle in a box

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi}$$

$k^2 = \frac{2mE}{\hbar^2}$

$E = \frac{1}{2m}\left(\frac{2\pi}{\lambda}\right)^2\left(\frac{h}{2\pi}\right)^2$

$E = \frac{p^2}{2m}$; $k\hbar = p$

$\psi(x) = Ae^{ikx} + Be^{-ikx}$

$= \underbrace{(A+B)}_{A_1}\cos(kx) + \underbrace{i(A-B)}_{A_2}\sin(kx)$

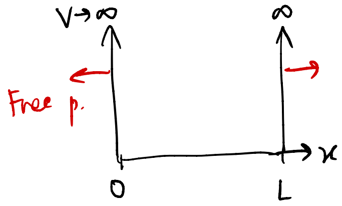
For "free particle": → Confined particle

$V = 0$,

$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E\psi = 0$

$\frac{\partial^2\psi}{\partial x^2} + \underbrace{\frac{2mE}{\hbar^2}}_{=k^2}\psi = 0$

Solution : SHM



$U = 0$; $\underbrace{0 < x < L}$ confined/boundary condition on top of free particle

↳ ยังคงได้

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\psi(x) = A_1\cos kx + A_2\sin kx \ ; \ 0 < x < L$$

ที่ $\psi(0) = 0 = A_1$

ที่ $\psi(L) = 0 = A_2\sin(kL)$: $A_2 \neq 0$

↳ $\sin(kL) = 0$, $kL = n\pi$ → $k = \frac{n\pi}{L}$

$\therefore \psi(x) = A_2\sin\left(\frac{n\pi}{L}x\right)$

$k^2 = \frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2}$ → $E_n = \frac{\hbar^2 n^2\pi^2}{2mL^2}$

$$\boxed{E_n = \left(\frac{h^2}{8mL^2}\right)n^2}$$

Energy of particle

Normalization

$$\int_0^L |\psi|^2 dx = 1$$

$$A_2^2\int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx = 1$$

$$A_2^2\frac{L}{n\pi}\left[\frac{1}{2}\cdot\frac{n\pi}{L}x - \frac{1}{4}\sin\left(\frac{2n\pi}{L}x\right)\right]_0^L = 1$$

$$\to A_2^2\left[\frac{\cancel{n\pi}}{2}\left(\frac{L}{\cancel{n\pi}}\right)\right] = 1$$

$$A_2 = \sqrt{\frac{2}{L}} \longrightarrow \psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) ※$$

Atomic Physics

1885: Balmer series : $\lambda = B\left(\frac{n^2}{n^2 - m^2}\right)$; $n, m > 2$

364.50682 nm

1888: Rydberg generalization :

$$\frac{1}{\lambda} = \underbrace{\frac{4}{B}}_{= R_H}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \; ; \; n > 2$$

Bohr's model : $\boxed{E_i - E_f = hf} \rightarrow \frac{hc}{\lambda}$

$\hookrightarrow \boxed{m_e v r = n\hbar}$; $n > 0$

$E = K + U$; $U = \frac{kq_1q_2}{r} = -\frac{ke^2}{r}$

$E = \frac{1}{2}mv^2 - \frac{ke^2}{r}$

และ: $F_c = \frac{mv^2}{r} = \frac{ke^2}{r^2} \rightarrow v^2 = \frac{ke^2}{mr}$

$\downarrow$ $U = 2K$

$\therefore \boxed{E = -\frac{ke^2}{2r}}$

$\hookrightarrow mvr = n\hbar$

$\hookrightarrow v = e\sqrt{\frac{k}{mr}}$

$mvr = n\hbar$

$v^2 = \frac{n^2\hbar^2}{m^2r^2} = \frac{ke^2}{mr}$

$\rightarrow r_n = \frac{n^2\hbar^2}{mke^2}$ (Bohr radius: a_0)

$\frac{1}{0.0529}$ nm^{-1}

$\therefore E = -\frac{ke^2}{2}\left[\frac{mke^2}{\hbar^2}\right]\left(\frac{1}{n^2}\right)$

$\rightarrow E = -\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right)$

$$\boxed{E = \frac{-13.606}{n^2}\ eV}$$

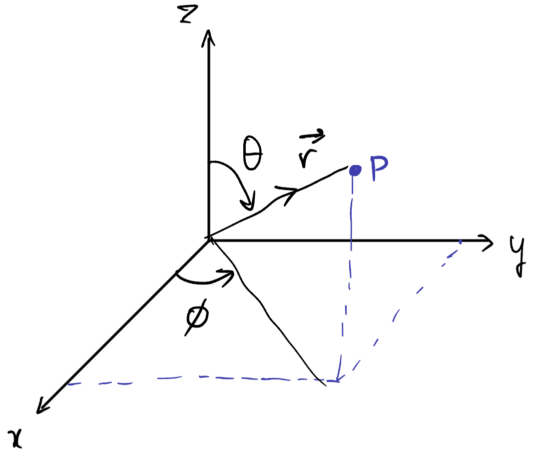
$$\therefore E_i - E_f = 13.606\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

และ: $f = \frac{c}{\lambda} = \frac{E_i - E_f}{h}$

$R_H = 1.097 \times 10^7\ m^{-1} = \frac{4}{B}$

$$\rightarrow \frac{1}{\lambda} = \frac{E_i - E_f}{hc} = \frac{13.606}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Quantum model of H-atom



$$-\frac{\hbar^2}{2m}\nabla^2\psi + U\psi = E\psi \quad ; \quad U = \frac{ke^2}{r}$$

$$(x,y,z) \rightarrow (r,\theta,\phi)$$

$$r^2 = x^2+y^2+z^2$$

$$\theta = \mathrm{proj}_z \vec{r} = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\phi = \mathrm{proj}_x \vec{r} = \cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$

Key idea : $\psi(x,y,z) = \psi(r,\theta,\phi) \rightarrow R(r)f(\theta)g(\phi)$

Results : n, l, m_l

	ค่า	จำนวนค่า
Principle Quantum No. (n)	$1, 2, 3, \ldots$	any
Orbital Quantum No. (l)	$0, 1, 2, \ldots, n-1$	n
O. Magnetic Quantum No. (m_l)	$-l, -l+1, \ldots, 0, \ldots, l$	$2l+1$

$\boxed{n:}$ ลำดับชั้นพลังงานใน Bohr

$\boxed{l:}$ ลำดับ s, p, d, f บอก angular momentum (L)

$\vec{L} = \vec{r} \times m\vec{v}$ [classical]

$mvr = n\hbar$ [Bohr's]

$\boxed{L = \sqrt{l(l+1)}\,\hbar}$ [Quantum]

$\boxed{m_l:}$ Magnetic moment (μ)

$\vec{\mu} = I\vec{A}$

ได้ $U_B = \vec{\mu}\cdot\vec{B}$

lowest : $-\mu B$ ($\vec{\mu}, \vec{B}$ ทิศเดียวกัน)

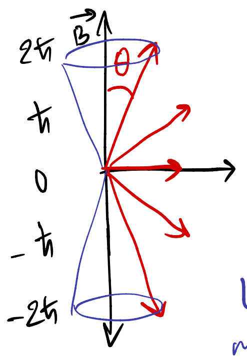
highest : μB ($\vec{\mu}, \vec{B}$ สวนทางกัน)

ใน [Quantum] $\vec{\mu}$ สัมพันธ์กับ $\vec{L}$

$\boxed{L_z = m_l\hbar}$

$\hookrightarrow \mathrm{proj}_z \vec{L}$ (ทิศของ $\vec{B}$-field)

$(l = 2)$



$$\cos\theta = \frac{L_z}{|L|} = \frac{m_l}{\sqrt{l(l+1)}}$$

L lies on cone's surface making θ with z-axis

Spin (Paul Dirac)

$\boxed{m_s}$ เกิดจาก $s = \frac{1}{2}$

$\hookrightarrow \vec{S} = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$

$\boxed{S_z = m_s \hbar} \rightarrow m_s = \pm \frac{1}{2}$

The exclusion principle (กฎการกีดกัน)

$n=1 - l=0 - m_l = 0 <$ $\frac{1}{2}$, $-\frac{1}{2}$

$n=2$:
- $l=0 - m_l=0 <$ $m_s=\frac{1}{2}$, $m_s=-\frac{1}{2}$
- $l=1$:
  - $m_l=1 <$ $\frac{1}{2}$, $-\frac{1}{2}$
  - $m_l=0 <$ $\frac{1}{2}$, $-\frac{1}{2}$
  - $m_l=-1 <$ $\frac{1}{2}$, $-\frac{1}{2}$

Total states $= 2n^2$

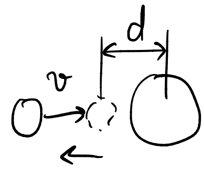
* จะไม่มี e^- ที่มี Quantum state เดียวกัน ในอะตอมเดียวกัน *

$n=3$:
- $0 - 0 <$
- 1: $1 <$, $0 <$, $-1 <$
- 2: $2 <$, $1 <$, $0 <$, $-1 <$, $-2 <$

each shell : it can accomodate up to $2n^2\ e^-$

Nuclear Physics

Mass unit :
$$\boxed{\begin{aligned} 1\,u &= 1.66\times10^{-27}\ \text{kg} \\ 1\,u &= 931.\overset{494}{5}\ \text{MeV}/c^2 \end{aligned}}$$



Nuclei separation distance: $K \to U$ (Stop momentarily)

$$\frac{1}{2}mv^2 = \frac{k(Ze)(2e)}{d}$$

$$\therefore d = \frac{4Zke^2}{mv^2} \quad ; \quad d \approx 3.2\times10^{-14}\ \text{m (Gold foil)}$$

ขนาด nucleus $< d$: $1\ \text{fm} = 10^{-15}\ \text{m}$

$$\boxed{r = aA^{1/3}} \ ; \ a = 1.2\ \text{fm}$$

Mass number [g/mol]

สื่อถึง Volume $\propto A$ และ $\rho \sim$ constant

Nuclear force : แรงมาก ๆ เหนือ Coulomb force (ภายใน nucleus เท่านั้น)

Nuclear binding energy

Rest energy (bound) < Rest energy (separated)

$$E_b = [Zm_p + (A-Z)m_n - m_{{}^A_Z X}]c^2 \quad [(\Delta m)c^2]$$

Liquid-drop model

Volume Energy : E_b roughly the same (overestimate) $= C_1 A$

Surface Energy : $4\pi r^2 \to A^{2/3}$ (Term ที่ผิวนอก) $= -C_2 A^{2/3}$

Coulomb Energy : Each proton repels every other protons. $\left[\frac{kq_i q_{other}}{r}\right]$ $= -C_3 \frac{Z(Z-1)}{A^{1/3}}$

Asymmetry Energy : $(N>Z)$ ทำให้ nucleus stable $= -C_4 \frac{(N-Z)^2}{A}$

$$\therefore \boxed{E_b = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(N-Z)^2}{A}}$$

Radioactivity

$$\boxed{N = N_0 e^{-\lambda t}} \qquad \left(\frac{dN}{dt} = -\lambda N \right)$$

Decay rate (activity) = No. of decay per second

$\hookrightarrow R = \left| \frac{dN}{dt} \right| = \lambda N = \lambda N_0 e^{-\lambda t}$

Half-life

$N = \frac{N_0}{2}$: $e^{-\lambda t} = \frac{1}{2}$; $\lambda t = \ln 2$; $t = T_{1/2} = \frac{\ln 2}{\lambda}$ or $\boxed{\lambda T_{1/2} = \overset{0.693}{\ln 2}}$

Unit of R

1 Bq. = 1 decay/s

1 Ci = 3.7×10^{10} decay/s

Alpha decay [α]

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \underbrace{{}^{4}_{2}He^{2+}}_{\alpha}$$

Disintegration energy

$$Q = (m_{N,X} - m_{N,Y} - m_{N\alpha})c^2$$

[amount of energy transformed appeared in the form of K in daughter nucleus & α-particle] } defect mass

Example

$${}^{226}_{88}\overset{(X)}{Ra} \rightarrow {}^{222}_{86}\overset{(Y)}{Rn} + {}^{4}_{2}He^{2+}$$

$$Q = (226.025410\,u - 222.017578\,u - 4.002603\,u) \times 931.494 \frac{MeV}{u}$$

$$Q = 4.87 \text{ MeV}$$

K of α : $0 = M_{Ra} v_{Ra} - M_\alpha v_\alpha$

$$Q = \frac{1}{2} M_{Ra} v_{Ra}^2 + \frac{1}{2} M_\alpha v_\alpha^2$$

$$Q = \frac{1}{2} M_{Ra} \left[\frac{M_\alpha}{M_{Ra}} v_\alpha \right]^2 + \frac{1}{2} M_\alpha v_\alpha^2$$

$$Q = \underbrace{\frac{1}{2} M_\alpha v_\alpha^2}_{K_\alpha} \left[\frac{M_\alpha + M_{Ra}}{M_{Ra}} \right] \rightarrow K_\alpha = Q \left[\frac{M_Y}{M_Y + M_\alpha} \right]$$

For $Q < 0$: ${}^{238}_{92}U \rightarrow {}^{237}_{91}Pa + {}^{1}_{1}H$ [Meaning : เกิดขึ้นเองไม่ได้]

Beta-decay $[\beta]$

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e$$

Charge conserved.
Nucleon no. conserved.

กรณี e^- : $n \rightarrow p + e^-$

กรณี e^+ : $p \rightarrow n + e^+$

Key : $n \leftrightarrow p$

$K_\alpha = Q[\dots]$ discrete แต่ K_β เป็น continuous spectrum พลังงานไม่ conserve (มีปัญหา)

จึงเกิด "Neutrino (ν)"

$$\text{I} \quad \boxed{^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e} \ \beta$$

ตอนหมายถึง $n \rightarrow p + \underbrace{e^- + \bar{\nu}_e}_{\text{ejected from nucleus}}$ (เกิดใน nucleus)

$$\therefore Q = [m_{N,X} - m_{N,Y} - m_e - \cancel{m_\nu}^{\,0}]c^2$$

$M_{neutral,X} = m_{N,X} + Zm_e$ [mass of neutral atom]

$$Q = [(m_X - Zm_e) - (m_Y - (Z+1)m_e) - m_e]$$

$$\therefore Q = [m_X - m_Y]c^2 \ ; \ m_X, m_Y = \text{neutral atom mass}$$

$$\text{II} \quad \boxed{^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e} \ \beta$$

หมายถึง $p \rightarrow n + e^- + \nu_e$

เกิดใน nucleus

not decay in isolated system ($m_p < m_n$)

$$\therefore Q = [m_{N,X} - m_{N,Y} - m_{e^+}]$$

$m_{e^+} = m_e$

$$Q = [(m_X - Zm_e) - (m_Y - (Z-1)m_e) - m_e]c^2$$

$$\therefore Q = [m_X - m_Y - 2m_e]c^2$$

or $^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu$ [electron capturing]

$$\therefore Q = [m_{N,X} + m_e - m_{N,Y}]c^2$$

$$Q = [(m_X - Zm_e) + m_e - (m_Y - (Z-1)m_e)]c^2$$

$$\therefore Q = [m_X - m_Y]c^2$$

App. "Carbon dating" When organism dies : Not absorb ^{14}C

- $r_0 \left[{}^{14}C / {}^{12}C \right] = 1.3 \times 10^{-12} \rightarrow r$ decreases
- $T_{1/2}$ ^{14}C = 5,730 years

$$\lambda = \ln 2 / T_{1/2}$$

$$N = N_0 e^{-\lambda t}$$

$$r_0 = \frac{N_0}{M_0} \rightarrow r = \frac{N}{M_0 + (N_0 - N)} \quad *$$

Gamma decay [γ]

$${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

$${}^{12}_{5}B \rightarrow {}^{12}_{6}C^{*} + e^{-} + \bar{\nu}$$

$$\hookrightarrow {}^{12}_{6}C + \gamma \quad \approx 4.4\ \text{MeV}$$

$$\hookrightarrow {}^{12}_{6}C + e^{-} + \bar{\nu} \quad \approx 13.4\ \text{MeV}$$

13.4 MeV

Nuclear Reaction

$$a + X \rightarrow b + Y \qquad \text{or} \qquad X(a,b)Y$$

$$\therefore \; Q = (m_a + m_X - m_b - m_Y)c^2$$

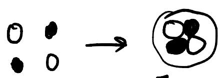
Example $\quad {}^{1}_{1}H + {}^{7}_{3}Li \rightarrow {}^{4}_{2}He + {}^{4}_{2}He^{2+}$

- Exothermic ($Q>0$)
- Endothermic ($Q<0$) does not occur unless bombarding particle has $K > |Q|$

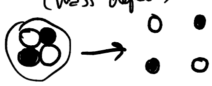
Threshold : $E_{th} = -Q\left(1 + \frac{m_a}{m_X}\right)$

Nuclear Fission & Fusion

$E_B \uparrow \quad |\Delta m| \uparrow$

Fusion  + E

$-\Delta m \rightarrow +E_B$ แทรก
(mass defect)

Fission  + E

มี mass defect
E_B สูง

Fission example : $\; {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{235}_{92}U^{*} \rightarrow X + Y + 3n$

$X \rightarrow {}^{141}_{56}Ba$, $\; Y \rightarrow {}^{92}_{36}Kr$

$E_B = 7.2$ MeV $\qquad E_B = 8.2$ MeV

ทุกๆ 1 nucleon $\rightarrow$ 1 MeV

U 1 nucleus $\rightarrow$ 235 MeV (prediction) / ideal

แต่ของจริง $(\text{mass defect})c^2 = 173.3$ MeV

Fusion

example

① ${}^{1}_{1}H + {}^{1}_{1}H \longrightarrow {}^{2}_{1}H + e^{+} + \nu_e$

② ${}^{1}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{2}He + \gamma$

③ ${}^{3}_{2}He + {}^{3}_{2}He \longrightarrow {}^{4}_{2}He + \underline{2\,{}^{1}_{1}H}$

"proton-proton cycle"

Total : $4\,{}^{1}_{1}H \longrightarrow {}^{4}_{2}He$

$4p = 4(1.007825\ u)$

${}^{4}_{2}He : 4.002603\ u$

$E = (4.031300 - 4.002603) \times 931.494$

$E = 26.7\ MeV/cycle$