Rigid Elements

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# Multipoint Constraints

Multipoint constraints reduce the total number of degrees of freedom (DOF) by making a set of dependent DOF that is equal to linear combinations of the values of an independent set of DOF. In the most general form, multipoint constraints can be defined explicitly by directly providing the coefficients of the constraint equations. Rigid elements are a special case of multipoint constraints in which the user provides connection data and the program internally generates the required coefficients.

The global constraint equations can be expressed in the form

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where is the global[[1]](#footnote-1) displacement vector and is the constraint coefficient matrix. The vector is partitioned into independent and dependent sets

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

where is the dependent set and is the independent set. The matrix of constraint coefficients is similarly partitioned

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

resulting in the following relationship

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

is a square matrix. If nonsingular, the constraint matrix can be written as

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

and the dependent set values are determined directly from the independent set values through the constraint matrix

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

Details on how the constraint matrix is used to partition the finite element system matrices can be found in the *NASTRAN Theoretical Manual* [1]*.*

# RBE2 Element

The RBE2 element creates a perfectly rigid connection in which all dependent DOF are based on six independent DOF at a specified node. The constraint matrix for six dependent DOF at a single node can be calculated directly using rigid-body kinematics

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

where is the identity matrix, is a zero matrix, is a skew-symmetric matrix assembled based on the distance vector to the dependent node from the independent node (expressed in the dependent-node displacement reference frame), and is a matrix that transforms DOF expressed in the independent-node displacement reference frame to the dependent-node displacement reference frame. The relationship of Equation (7) assumes small angles in .

If more than one node with dependent DOF is specified on an RBE2 element, the relationship of Equation (7) is formed separately for each node with dependent DOF

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

where is the total number of nodes with dependent DOF, are dependent DOF corresponding to node , and are matrices that capture the rigid-body kinematic relationship at node

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

If the dependent set includes less than six DOF per node, than only the rows corresponding to dependent DOF are retained in Equation (8).

To further condense the notation, represents assembled matrices for a single RBE2 element

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

and Equation (8) can be written concisely as

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

where includes all dependent DOF corresponding to the RBE2 element. It is clear, based on (6), that for the element.

If the solver requires constraint coefficient matrices (i.e., the form of Equation (4)), they can be written as , and . The signs of the constraint coefficient matrices can be flipped while still remaining consistent with Equation (11).

With the RBE2 element, any number of dependent DOF are all exactly related to six independent DOF, as shown in Equation (8). The number of rows, or equations that make up the system, is always equal to the number of unknowns, which are the dependent DOF responses. The nature of rigid-body kinematics guarantees a unique relationship between and .

# RBE3 Element

Enforcement of rigid-body motion across all connected DOF may be undesirable in situations like:

* distributing applied loads from a single point to multiple points,
* distributing mass from a single point to multiple points, or
* monitoring the central tendency of the response at multiple points using a single point.

The RBE3 element is intended to connect a single point to multiple points without enforcing rigid-body motion across all connected DOF. This is accomplished by relating dependent DOF to independent DOF using a least-squares relationship.

An obsolete version of the RBE3 element was implemented in early versions of Nastran. The obsolete element does not uniformly dimensionalize displacement and rotation DOF, so it produces unit-system dependent results [2]. Unit-system dependent results are undesirable, so the element was ultimately modified and replaced by the modern RBE3 element. It is easier to comprehend the details of the modern RBE3 element after first reviewing the details of the obsolete RBE3 element, so both versions of the RBE3 element are discussed in the following subsections.

## *Obsolete RBE3 Element*

For the obsolete RBE3 element, a system analogous to Equation (8) is created

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

where are are analogous to

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

is a skew-symmetric matrix assembled based on the distance vector to independent node from the dependent node (expressed in the independent-node displacement reference frame), and is a matrix that transforms DOF expressed in the dependent-node displacement reference frame to the displacement reference frame of independent node .

The notation is adopted for conciseness. With concise notation, Equation (12) can be written . If the dependent set includes less than six DOF, than only the columns of corresponding to dependent DOF are retained. Like Equation (8), are known and are unknown; however, Equation (12) may represent an overdetermined system. Therefore, the least-squares approach is used to define in terms of

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

Thus, the element constraint matrix is . The least-squares solution is affected by the relative magnitude of displacements and rotations, which is why the response of the obsolete RBE3 element is unit-system dependent.

## *Modern RBE3 Element*

The modern RBE3 element scales each term of Equation (12) such that all dimensions are uniform. Additionally, relative weighting factors can be applied to each term. The weighting and uniform dimensionalization is accomplished using a diagonal matrix

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

where is the average of the distances from the dependent node to all independent nodes in the element, and are relative weight factors for each term, which are normally set to 1. This definition of differs slightly from that in Ref [2] (in which every term in the diagonal matrix, including , is the square of value presented here). The form used here was selected because the effect of uniform dimensionalization is considered more obvious in this form than in the Ref [2] form. The subsequent algebra is adjusted accordingly, but the final result is equivalent.

The uniform dimensionalization and scaling of Equation (12) is carried out by multiplying terms on both sides of the equation by

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

The equation is condensed to

|  |  |  |
| --- | --- | --- |
|  |  | (17) |

by defining

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | and |  | (18) |

The bar above is used to indicate that all six DOF at the dependent node are retained (and thus all six columns of ) while solving for the least-squares relationship; if the user specifies less than six nodal DOF are dependent, than the corresponding rows are removed from the constraint matrix after the least-squares solution. If the independent set includes less than six DOF per node, than only the relevant parts of and rows of are retained for the least-squares solution:

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

and, the interim element constraint matrix is . It is possible for the user to select options that result in a singular matrix. An error message is supplied if is singular.

Again, since has six rows, any rows corresponding to independent nodal DOF (i.e., DOF not listed in the RBE3 REFC field) are removed from the final element constraint matrix.

# References

1. Richard H. MacNeal and others. *The NASTRAN Theoretical Manual*. NASA SP-221(06). Scientific and Technical Information Office, National Aeronautics and Space Administration, 1981.
2. Timothy F. Walsh, Garth M. Reese, and Manoj K. Bhardwaj. *Salinas Theory Manual Version 4.22*. Sandia National Laboratories, 2011. DOI: 10.2172/1031884.

1. This document uses Nastran terminology and notation. In this context, the global DOF include all structural DOF; these DOF are expressed in their nodal-displacement reference frames. [↑](#footnote-ref-1)