Computational Fluid Dynamics

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The notation and process mostly follow Reference [1].

Mass conservation

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where the **.**

Momentum conservation

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Energy conservation

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where are velocity components, is density, is specific energy, is pressure, is absolute temperature, represent body forces and other source terms, is thermal conductivity, and are the deviatoric stress components.

# Relating pressure and density

The equation of state for a perfect gas is a popular choice. A perfect gas is defined as a gas in which the intermolecular forces are neglected [2]. The following is the equation of state for a perfect gas

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

where is the specific gas constant, which has a different value for different gasses.

# Compressibility

Reference [2] Figure 7.3 illustrates the definition of compressibility.

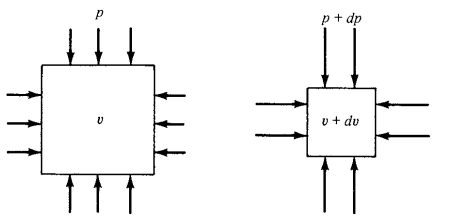


Figure 1. Definition of compressibility [2].

A pressure change results in a volume change (positive results in negative with this sign convention).

# Bernoulli

From momentum conservation

Assume steady, inviscid, and no body forces.

Chain rule

Assume 1D (this may force incompressibility)

|  |
| --- |
| 1D conservation of mass  Steady |

Apply conservation of mass to 1D momentum equation to simplify

Substitute

Move inside derivative

So

Integrate

# Characteristic-based split form A

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Separate contributions with and without pressure terms

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

With

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

From mass conservation

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

Mass conservation algebra

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

This is hairy AF. and are dependent on each other. Something like , where is the speed of sound. So

This seems to agree with the discretized form shown in Zienkiewicz Equation 3.67.

Expanded form of momentum conservation

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

Deviatoric stress components

# References

1. O. Zienkiewicz, R. L. Taylor, P. Nithiarasu. “The Finite Element Method for Fluid Dynamics.” Butterworth-Heinemann, Seventh Edition, 2013.
2. J. D. Anderson Jr. “Fundamentals of Aerodynamics.” McGraw-Hill, Fourth Edition, 2007.